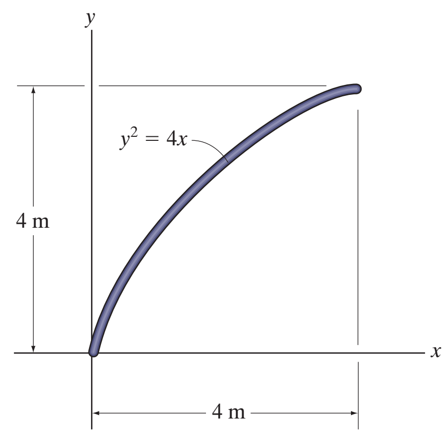
•**9–1.** Determine the mass and the location of the center of mass $(\bar{x}, \bar{y})$ of the uniform parabolic-shaped rod. The mass per unit length of the rod is 2 kg/m.



Differential Element. The length of the element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\right) dy$$

Here, $\frac{dx}{dy} = \frac{y}{2}$. Thus,

$$dL = \left(\sqrt{1 + \left(\frac{y}{2}\right)^2}\right) dy = \frac{1}{2}\sqrt{y^2 + 4}\, dy$$

The mass of the element is

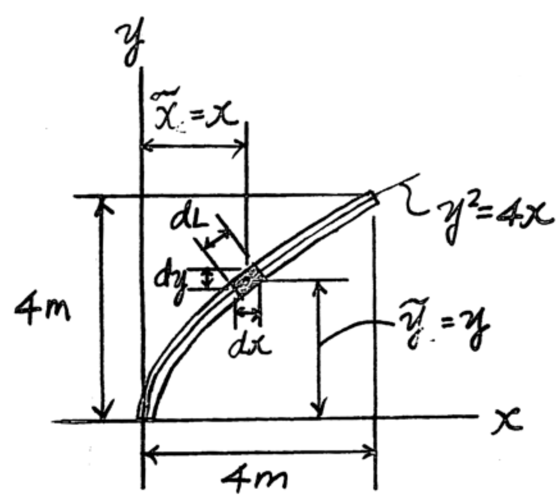
$$dm = \rho\, dL = 2\left(\frac{1}{2}\sqrt{y^2 + 4}\right) dy = \sqrt{y^2 + 4}\, dy$$

The centroid of the element is located at $\tilde{x} = x = \frac{y^2}{4}$ and $\tilde{y} = y$. Integrating,

$$m = \int_m dm = \int_0^{4\text{ m}} \sqrt{y^2 + 4}\, dy = 11.832 \text{ kg} = 11.8 \text{ kg} \qquad \textbf{Ans.}$$

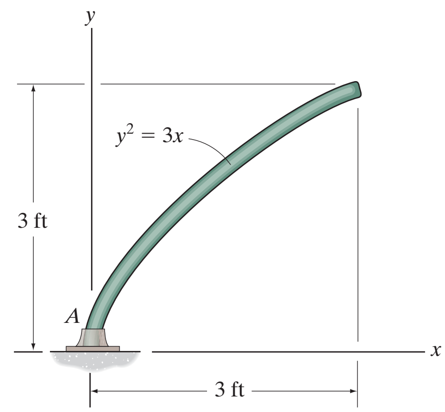
$$\bar{x} = \frac{\int_m \tilde{x}\, dm}{\int_m dm} = \frac{\int_0^{4\text{ m}} \frac{y^2}{4}\sqrt{y^2 + 4}\, dy}{\int_0^{4\text{ m}} \sqrt{y^2 + 4}\, dy} = \frac{19.403}{11.832} = 1.6399 \text{ m} = 1.64 \text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_m \tilde{y}\, dm}{\int_m dm} = \frac{\int_0^{4\text{ m}} y\sqrt{y^2 + 4}\, dy}{\int_0^{4\text{ m}} \sqrt{y^2 + 4}\, dy} = \frac{27.148}{11.832} = 2.2945 \text{ m} = 2.29 \text{ m} \qquad \textbf{Ans.}$$



(a)

9–2. The uniform rod is bent into the shape of a parabola and has a weight per unit length of 6 lb/ft. Determine the reactions at the fixed support A.



Differential Element. The length of the element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1+\left(\frac{dx}{dy}\right)^2}\right) dy$$

Here, $\frac{dx}{dy} = \frac{2}{3}y$. Thus,

$$dL = \left(\sqrt{1+\left(\frac{2}{3}y\right)^2}\right) dy = \frac{1}{3}\sqrt{9^2+4y^2}\, dy$$

The weight of the element is therefore

$$dW = \gamma dL = 6\left[\frac{1}{3}\sqrt{9+4y^2}\, dy\right] = 2\sqrt{9+4y^2}\, dy$$

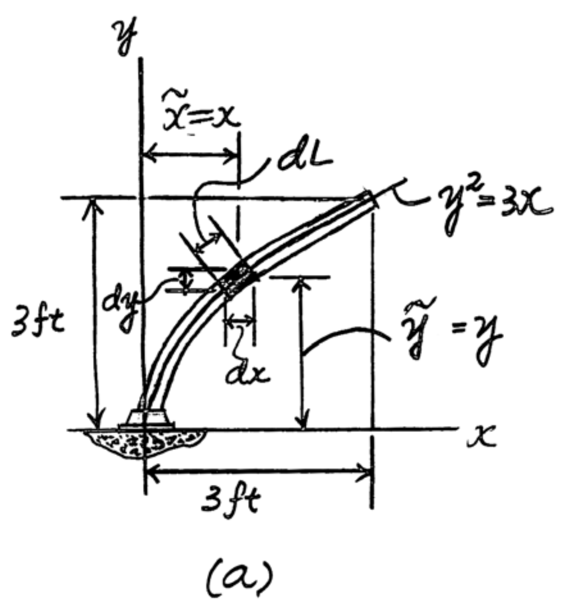
The centroid of the element is located at $\tilde{x} = x = \frac{y^2}{3}$. Integrating,

$$W = \int_W dW = \int_0^{3\text{ ft}} 2\sqrt{9+4y^2}\, dy = 26.621 \text{ lb}$$

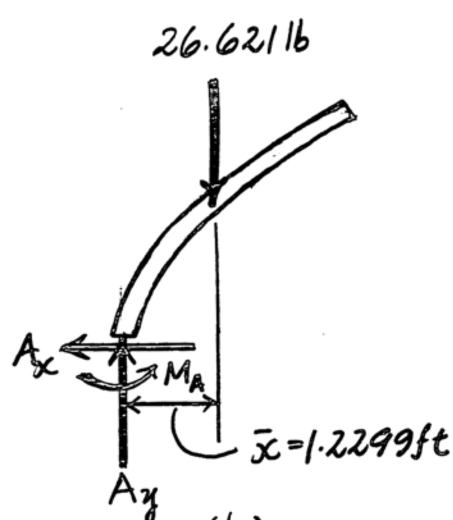
$$\bar{x} = \frac{\int_W \tilde{x}\, dW}{\int_W dW} = \frac{\int_0^{3\text{ ft}} \frac{y^2}{3}\left(2\sqrt{9+4y^2}\right) dy}{\int_0^{3\text{ ft}} 2\sqrt{9+4y^2}\, dy} = \frac{\frac{2}{3}\int_0^{3\text{ ft}} y^2\sqrt{9+4y^2}\, dy}{2\int_0^{3\text{ ft}} \sqrt{9+4y^2}\, dy} = \frac{32.742}{26.62} = 1.2299\text{ ft}$$

Equations of Equilibrium: By referring to the free - body diagram of the rod shown in Fig. *b*, yields

$\overset{+}{\rightarrow}\Sigma F_x = 0;$	$A_x = 0$		**Ans.**
$+\uparrow \Sigma F_y = 0;$	$A_y - 26.621 = 0$	$A_y = 26.621 \text{ lb} = 26.6 \text{ lb}$	**Ans.**
$\circlearrowleft + \Sigma M_A = 0;$	$M_A - 26.621(1.229) = 0$	$M_A = 32.74 \text{ lb}\cdot\text{ft} = 32.7 \text{ lb}\cdot\text{ft}$	**Ans.**

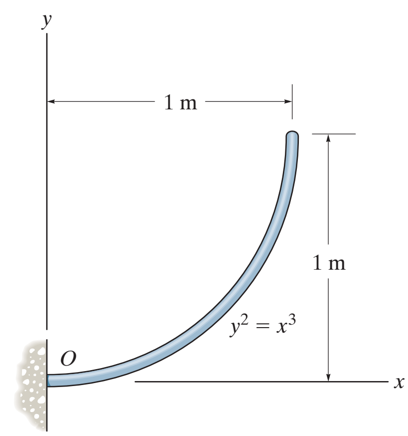


(a)

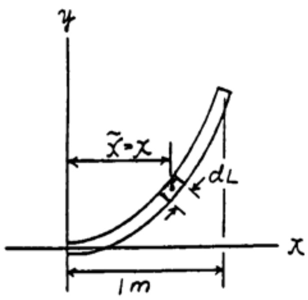


(b)

9–3. Determine the distance $\bar{x}$ to the center of mass of the homogeneous rod bent into the shape shown. If the rod has a mass per unit length of 0.5 kg/m, determine the reactions at the fixed support O.



Length and Moment Arm : The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$ and its centroid is $\tilde{x} = x$. Here, $\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$. Performing the integration, we have

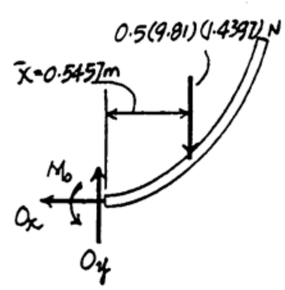


$$L = \int dL = \int_0^{1\,\text{m}} \left(\sqrt{1 + \frac{9}{4}x}\right) dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}} \Bigg|_0^{1\,\text{m}} = 1.4397 \text{ m}$$

$$\int_L \tilde{x}\,dL = \int_0^{1\,\text{m}} x\sqrt{1 + \frac{9}{4}x}\,dx$$
$$= \left[\frac{8}{27}x\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}} - \frac{64}{1215}\left(1 + \frac{9}{4}x\right)^{\frac{5}{2}}\right]\Bigg|_0^{1\,\text{m}}$$
$$= 0.7857$$

Centroid : Applying Eq. 9 – 7 , we have

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{0.7857}{1.4397} = 0.5457 \text{ m} = 0.546 \text{ m} \qquad \textbf{Ans}$$



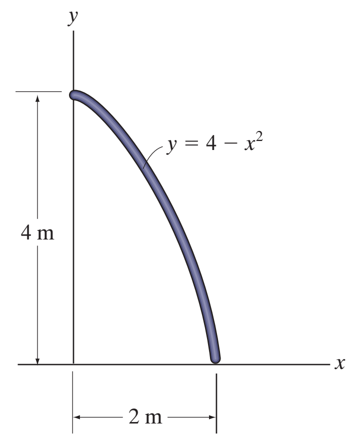
Equations of Equilibrium :

$\xrightarrow{+} \Sigma F_x = 0; \qquad O_x = 0 \qquad$ **Ans**

$+\uparrow \Sigma F_y = 0; \qquad O_y - 0.5(9.81)(1.4397) = 0$
$\qquad O_y = 7.06 \text{ N} \qquad$ **Ans**

$\curvearrowleft + \Sigma M_O = 0; \qquad M_O - 0.5(9.81)(1.4397)(0.5457) = 0$
$\qquad M_O = 3.85 \text{ N} \cdot \text{m} \qquad$ **Ans**

***9–4.** Determine the mass and locate the center of mass $(\bar{x}, \bar{y})$ of the uniform rod. The mass per unit length of the rod is 3 kg/m.



Differential Element. The length of the element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Here, $\frac{dy}{dx} = -2x$. Thus,

$$dL = \sqrt{1 + (-2x)^2}\,dx = \sqrt{1 + 4x^2}\,dx = 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx$$

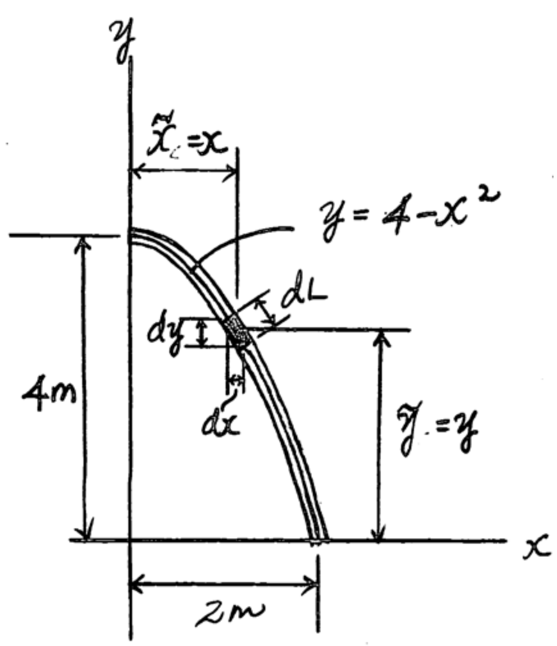
$$m = \int_0^{2\text{ m}} (3\text{ kg/m})\,2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx = 3(4.6468) = 13.9\text{ kg} \qquad \textbf{Ans.}$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y$.

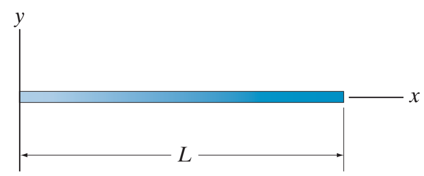
$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{2\text{ m}} x\left[2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx\right]}{\int_0^{2\text{ m}} 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx} = \frac{5.7577}{4.6468} = 1.24\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{2\text{ m}} (4 - x^2)\left[2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx\right]dx}{\int_0^{2\text{ m}} 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx} = \frac{\int_0^{2\text{ m}}\left[8\sqrt{\left(\frac{1}{2}\right)^2 + x^2} - 2x^2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\right]dx}{\int_0^{2\text{ m}} 2\sqrt{\left(\frac{1}{2}\right)^2 + x^2}\,dx}$$

$$= \frac{10.1160}{4.6468} = 2.18\text{ m} \qquad \textbf{Ans.}$$



•**9–5.** Determine the mass and the location of the center of mass $\bar{x}$ of the rod if its mass per unit length is $m = m_0(1 + x/L)$.

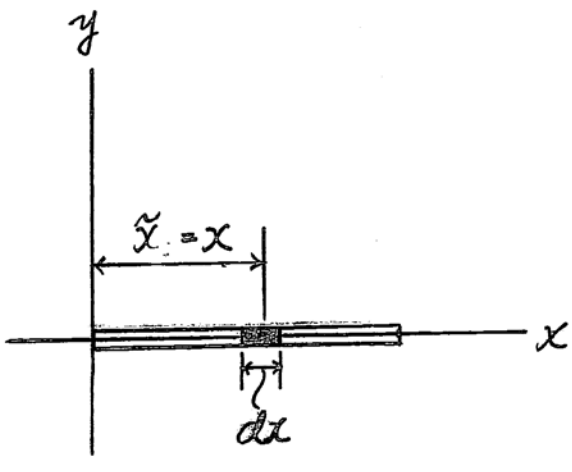


Differential Element. The element shown shaded in Fig. *a* has a mass of

$$\int_m dm = \int_0^L m_0\left(1+\frac{x}{L}\right)dx = \frac{3}{2}m_0 L \qquad \textbf{Ans.}$$

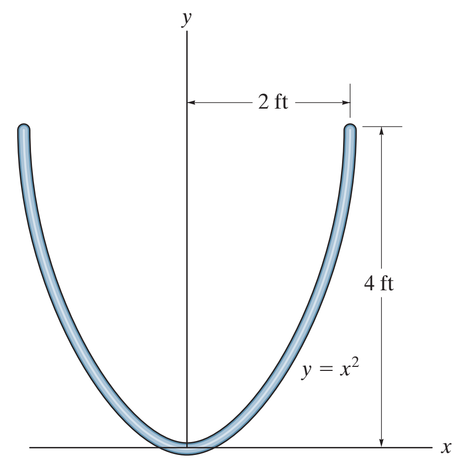
The centroid of the differential element is located at $x_c = x$.

$$\bar{x} = \frac{\int_m \tilde{x}\,dm}{\int_m dm} = \frac{\int_0^L x\left[m_0\left(1+\frac{x}{L}\right)dx\right]}{\int_0^L m_0\left(1+\frac{x}{L}\right)dx} = \frac{\int_0^L \left(x+\frac{x^2}{L}\right)dx}{\int_0^L \left(1+\frac{x}{L}\right)dx} = \frac{5}{9}L \qquad \textbf{Ans.}$$



(a)

9–6. Determine the location $(\bar{x}, \bar{y})$ of the centroid of the wire.



Length and Moment Arm : The length of the differential element is dL $= \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{y} = y = x^2$. Here, $\frac{dy}{dx} = 2x$.

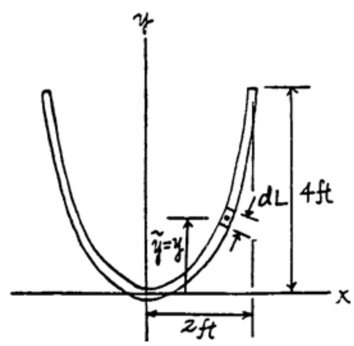
Centroid : Due to symmetry

$$\bar{x} = 0$$ **Ans**

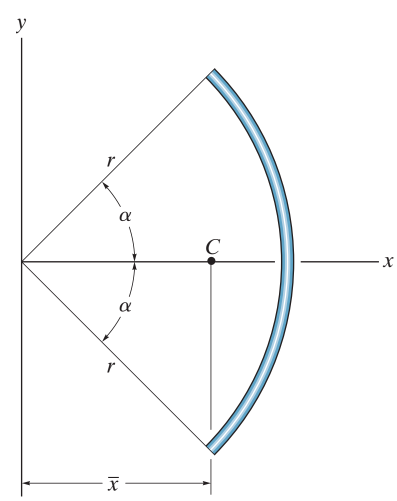
Applying Eq. 9–5 and performing the integration, we have

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_{-2\text{ ft}}^{2\text{ ft}} x^2\sqrt{1+4x^2}\,dx}{\int_{-2\text{ ft}}^{2\text{ ft}} \sqrt{1+4x^2}\,dx}$$

$$= \frac{16.9423}{9.2936} = 1.82 \text{ ft}$$ **Ans**



9–7. Locate the centroid $\bar{x}$ of the circular rod. Express the answer in terms of the radius r and semiarc angle α.



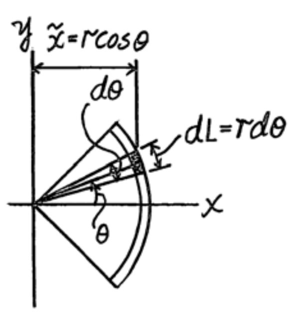
$$L = 2r\alpha$$

$$\tilde{x} = r\cos\theta$$

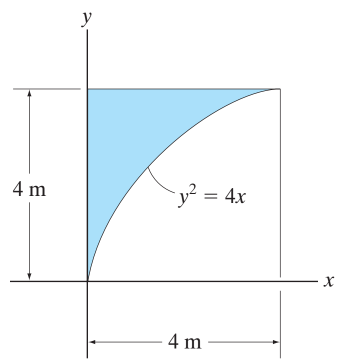
$$\int \tilde{x}\,dL = \int_{-\alpha}^{\alpha} r\cos\theta\; r\,d\theta$$

$$= 2r^2 \sin\alpha$$

$$\bar{x} = \frac{2r^2\sin\alpha}{2r\alpha} = \frac{r\sin\alpha}{\alpha}$$ **Ans**



***9–8.** Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the x axis shown shaded in Fig. *a* will be considered. The area of the element is

$$dA = x\,dy = \frac{y^2}{4}\,dy$$

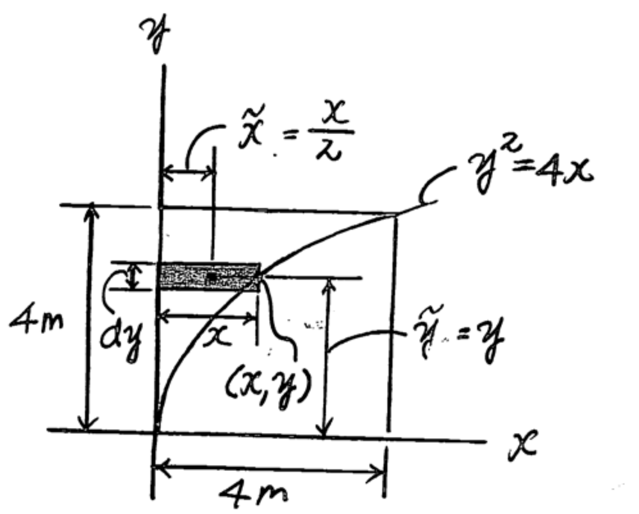
Centroid: The centroid of the element is located at $\tilde{x} = x/2 = \frac{(y^2/4)}{2} = \frac{y^2}{8}$ and $y_c = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^{4\text{ m}} \frac{y^2}{4}\,dy = \left.\frac{y^3}{12}\right|_0^{4\text{ m}} = 5.333\text{ m}^2 = 5.33\text{ m}^2 \qquad \textbf{Ans.}$$

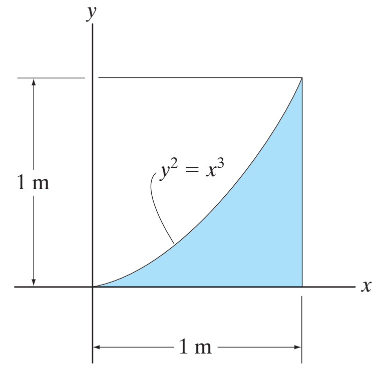
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} \frac{y^2}{8}\left(\frac{y^2}{4}\,dy\right)}{5.333} = \frac{\int_0^{4\text{ m}} \frac{y^4}{32}\,dy}{5.333} = \frac{\left.\left(\frac{y^5}{160}\right)\right|_0^{4\text{ m}}}{5.333} = 1.2\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} y\left(\frac{y^2}{4}\,dy\right)}{5.333} = \frac{\int_0^{4\text{ m}} \frac{y^3}{4}\,dy}{5.333} = \frac{\left.\left(\frac{y^4}{16}\right)\right|_0^{4\text{ m}}}{5.333} = 3\text{ m} \qquad \textbf{Ans.}$$



(a)

•9–9. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. *a* will be considered. The area of the element is

$$dA = y\,dx = x^{3/2}\,dx$$

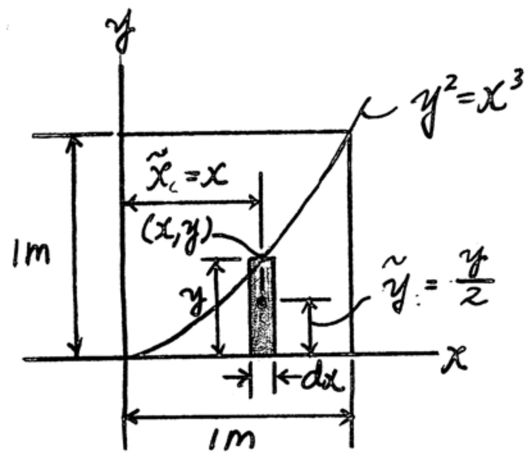
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \dfrac{x^{3/2}}{2}$.

Area: Integrating,

$$A = \int_A dA = \int_0^{1\text{ m}} x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^{1\text{ m}} = \frac{2}{5}\text{ m}^2 = 0.4\text{ m}^2$$ **Ans.**

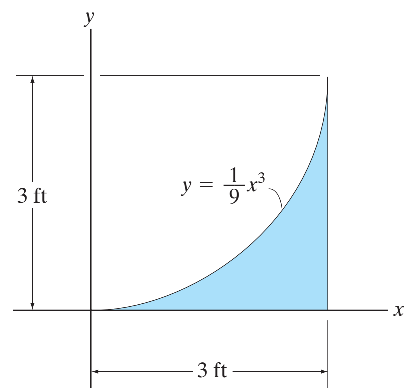
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} x\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} x^{5/2}\,dx}{2/5} = \frac{\left(\frac{2}{7}x^{7/2}\right)\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{7}\text{ m} = 0.714\text{ m}$$ **Ans.**

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} \left(\frac{x^{3/2}}{2}\right)\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} \frac{x^3}{2}\,dx}{2/5} = \frac{\frac{x^4}{8}\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{16}\text{ m} = 0.3125\text{ m}$$ **Ans.**



(a)

9–10. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{1}{9}x^3\,dx$$

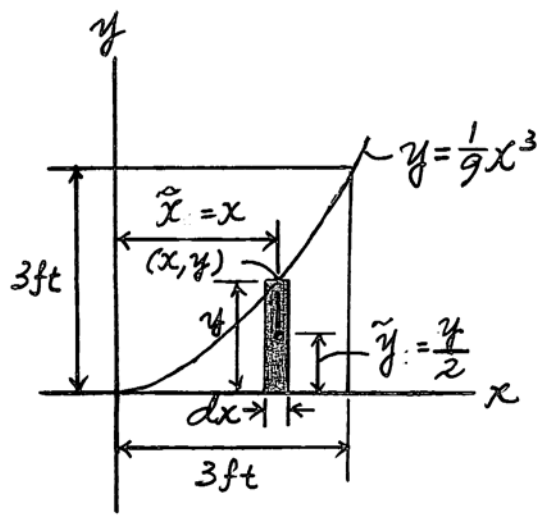
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \frac{1}{18}x^3$.

Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \frac{1}{9}x^3\,dx = \frac{1}{36}x^4\,dx\Big|_0^{3\text{ ft}} = 2.25\text{ft}^2 \qquad \textbf{Ans.}$$

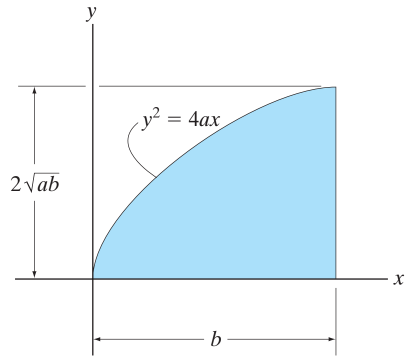
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} x\left(\frac{1}{9}x^3\,dx\right)}{2.25} = \frac{\int_0^{3\text{ ft}} \frac{1}{9}x^4\,dx}{2.25} = \frac{\frac{1}{45}x^5\Big|_0^{3\text{ ft}}}{2.25} = 2.4\text{ ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} \left(\frac{1}{18}x^3\right)\left(\frac{1}{9}x^3\,dx\right)}{2.25} = \frac{\int_0^{3\text{ ft}} \frac{1}{162}x^6\,dx}{2.25} = \frac{\frac{1}{1134}x^7\Big|_0^{3\text{ ft}}}{2.25} = 0.857\text{ ft} \qquad \textbf{Ans.}$$



(a)

9–11. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = 2a^{1/2}x^{1/2}\,dx$$

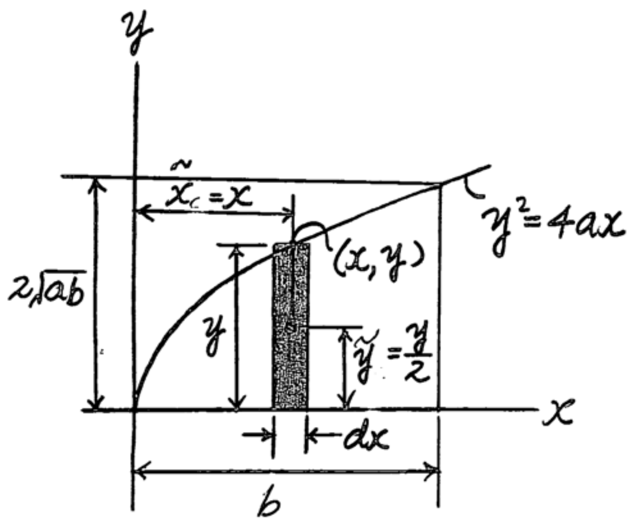
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = a^{1/2}x^{1/2}$.

Area: Integrating,

$$A = \int_A dA = \int_0^b 2a^{1/2}x^{1/2}\,dx = \frac{4}{3}a^{1/2}x^{3/2}\Big|_0^b = \frac{4}{3}a^{1/2}b^{3/2} \qquad \textbf{Ans.}$$

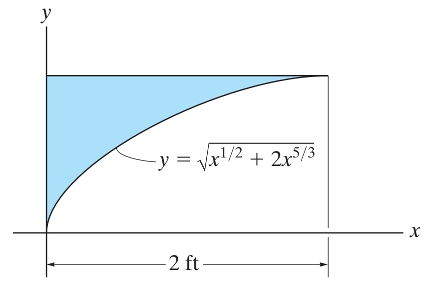
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b x\left(2a^{1/2}x^{1/2}\,dx\right)}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\int_0^b 2a^{1/2}x^{3/2}\,dx}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\frac{4}{5}a^{1/2}x^{5/2}\Big|_0^b}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{3}{5}b \qquad \textbf{Ans.}$$

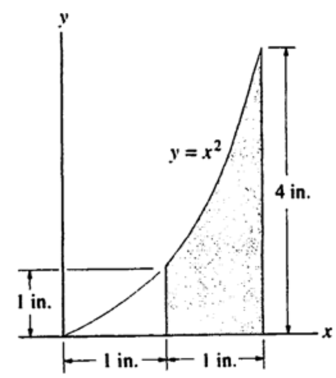
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \left(a^{1/2}x^{1/2}\right)\left(2a^{1/2}x^{1/2}\,dx\right)}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{\int_0^b 2ax\,dx}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{ax^2\Big|_0^b}{\frac{4}{3}a^{1/2}b^{3/2}} = \frac{3}{4}\sqrt{ab} \qquad \textbf{Ans.}$$



(a)

***■9–12.** Locate the centroid $\bar{x}$ of the area.

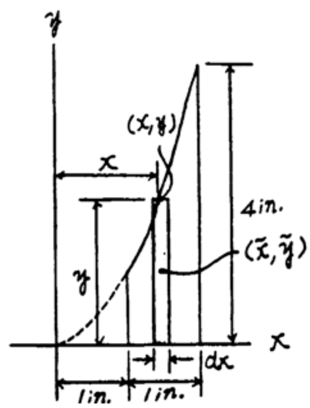




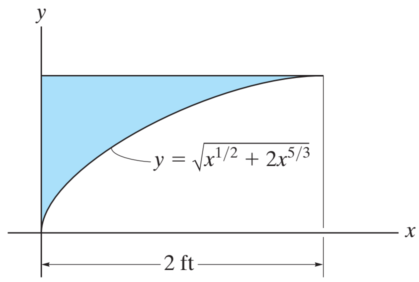
Area and Moment Arm : The area of the differential element is $dA = y\,dx = x^2 dx$ and its centroid is $\tilde{x} = x$.

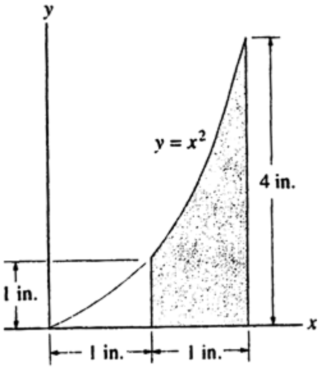
Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{1\text{ in}}^{2\text{ in}} x(x^2 dx)}{\int_{1\text{ in}}^{2\text{ in}} x^2 dx} = \frac{\left.\frac{x^4}{4}\right|_{1\text{ in}}^{2\text{ in}}}{\left.\frac{x^3}{3}\right|_{1\text{ in}}^{2\text{ in}}} = 1.61\text{ in} \qquad \textbf{Ans}$$



•■**9–13.** Locate the centroid $\bar{y}$ of the area.

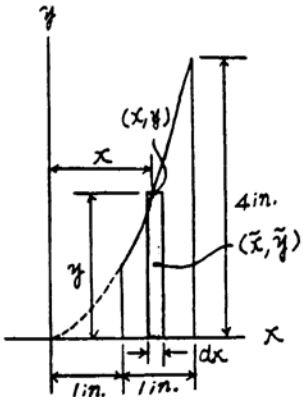




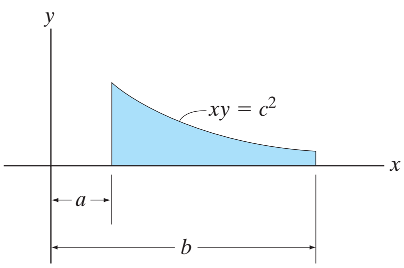
Area and Moment Arm : The area of the differential element is $dA = y\,dx = x^2\,dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}x^2$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{1\text{ in.}}^{2\text{ in.}} \frac{1}{2}x^2(x^2\,dx)}{\int_{1\text{ in.}}^{2\text{ in.}} x^2\,dx} = \frac{\left.\frac{x^5}{10}\right|_{1\text{ in.}}^{2\text{ in.}}}{\left.\frac{x^3}{3}\right|_{1\text{ in.}}^{2\text{ in.}}} = 1.33\text{ in.} \qquad \textbf{Ans}$$



9–14. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{c^2}{x}\,dx$$

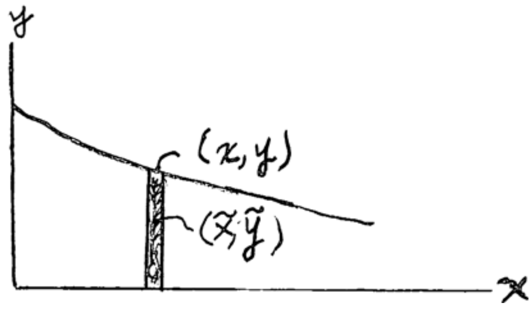
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2} = \frac{c^2}{2x}$.

Area: Integrating,

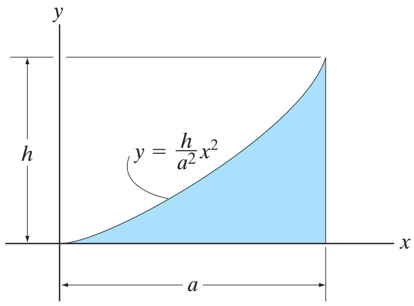
$$A = \int_A dA = \int_a^b \frac{c^2}{x}\,dx = c^2 \ln x\Big|_a^b = c^2 \ln\frac{b}{a} \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_a^b x\left(\frac{c^2}{x}\,dx\right)}{c^2 \ln\frac{b}{a}} = \frac{\int_a^b c^2\,dx}{c^2 \ln\frac{b}{a}} = \frac{c^2 x\Big|_a^b}{c^2 \ln\frac{b}{a}} = \frac{b-a}{\ln\frac{b}{a}} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_a^b \left(\frac{c^2}{2x}\right)\left(\frac{c^2}{x}\,dx\right)}{c^2 \ln\frac{b}{a}} = \frac{\int_a^b \frac{c^4}{2x^2}\,dx}{c^2 \ln\frac{b}{a}} = \frac{-\frac{c^4}{2x}\Big|_a^b}{c^2 \ln\frac{b}{a}} = \frac{c^2(b-a)}{2ab \ln\frac{b}{a}} \qquad \textbf{Ans.}$$



9–15. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{h}{a^2}x^2\,dx$$

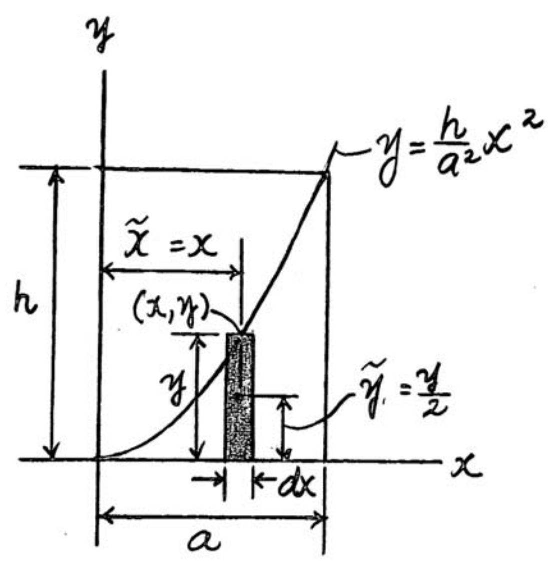
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \frac{1}{2}\left(\frac{h}{a^2}x^2\right) = \frac{h}{2a^2}x^2$.

Area: Integrating,

$$A = \int_A dA = \int_0^a \frac{h}{a^2}x^2\,dx = \frac{h}{a^2}\left(\frac{x^3}{3}\right)\Bigg|_0^a = \frac{1}{3}ah \qquad \textbf{Ans.}$$

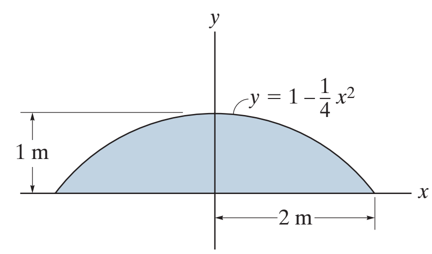
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left(\frac{h}{a^2}x^2\,dx\right)}{\frac{1}{3}ah} = \frac{\int_0^a \frac{h}{a^2}x^3\,dx}{\frac{1}{3}ah} = \frac{\frac{h}{a^2}\left(\frac{x^4}{4}\right)\Big|_0^a}{\frac{1}{3}ah} = \frac{3}{4}a \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{h}{2a^2}x^2\left(\frac{h}{a^2}x^2\,dx\right)}{\frac{1}{3}ah} = \frac{\int_0^a \frac{h^2}{2a^4}x^4\,dx}{\frac{1}{3}ah} = \frac{\frac{h^2}{2a^4}\left(\frac{x^5}{5}\right)\Big|_0^a}{\frac{1}{3}ah} = \frac{3}{10}h \qquad \textbf{Ans.}$$



(a)

***9–16.** Locate the centroid $(\bar{x}, \bar{y})$ of the area.



Area and Moment Arm : The area of the differential element is $dA = y\,dx = \left(1 - \frac{1}{4}x^2\right)dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)$.

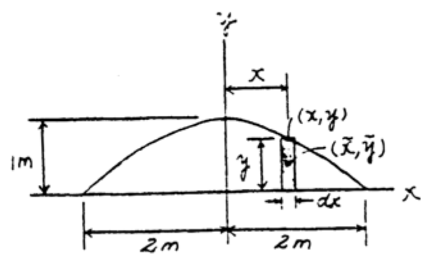
Centroid : Due to symmetry

$$\bar{x} = 0 \qquad \textbf{Ans}$$

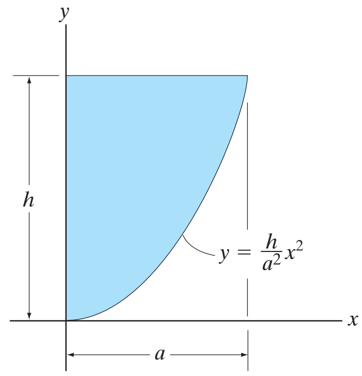
Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-2\,\mathrm{m}}^{2\,\mathrm{m}} \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)\left(1 - \frac{1}{4}x^2\right)dx}{\int_{-2\,\mathrm{m}}^{2\,\mathrm{m}} \left(1 - \frac{1}{4}x^2\right)dx}$$

$$= \frac{\left(\frac{x}{2} - \frac{x^3}{12} + \frac{x^5}{160}\right)\Big|_{-2\,\mathrm{m}}^{2\,\mathrm{m}}}{\left(x - \frac{x^3}{12}\right)\Big|_{-2\,\mathrm{m}}^{2\,\mathrm{m}}} = \frac{2}{5}\ \mathrm{m} \qquad \textbf{Ans}$$



•**9–17.** Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the x axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = x\,dy = \frac{a}{h^{1/2}}y^{1/2}\,dy$$

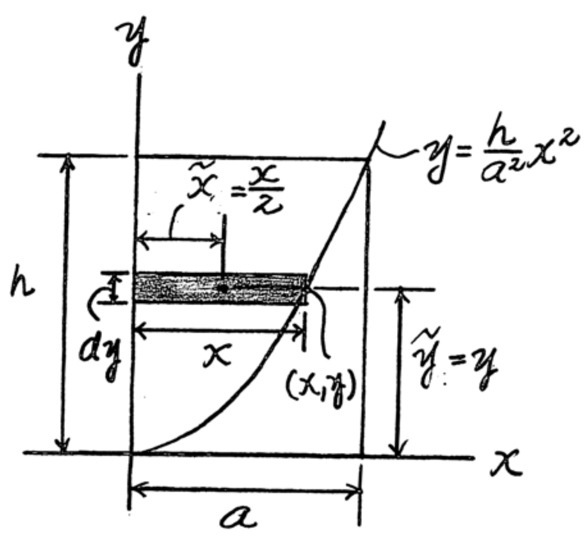
Centroid: The centroid of the element is located at $\tilde{x} = \frac{x}{2} = \frac{a}{2h^{1/2}}y^{1/2}$ and $\tilde{y} = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^h \frac{a}{h^{1/2}}y^{1/2}\,dy = \frac{2a}{3h^{1/2}}\left(y^{3/2}\right)\Big|_0^h = \frac{2}{3}ah \qquad \textbf{Ans.}$$

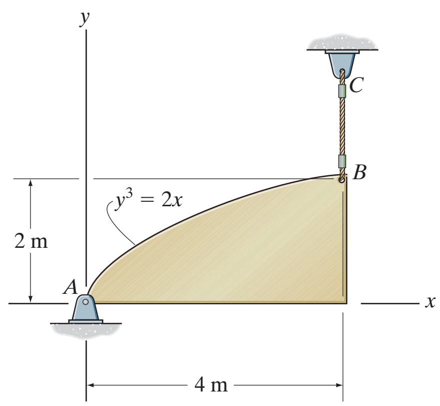
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{2h^{1/2}}y^{1/2}\right)\left(\frac{a}{h^{1/2}}y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a^2}{2h}y\,dy}{\frac{2}{3}ah} = \frac{\frac{a^2}{2h}\left(\frac{y^2}{2}\right)\Big|_0^h}{\frac{2}{3}ah} = \frac{3}{8}a \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(\frac{a}{h^{1/2}}y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a}{h^{1/2}}y^{3/2}\,dy}{\frac{2}{3}ah} = \frac{\frac{2a}{5h^{1/2}}y^{5/2}\Big|_0^h}{\frac{2}{3}ah} = \frac{3}{5}h \qquad \textbf{Ans.}$$



(a)

9–18. The plate is made of steel having a density of 7850 kg/m^3. If the thickness of the plate is 10 mm, determine the horizontal and vertical components of reaction at the pin A and the tension in cable BC.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of this element is given by

$$dA = y\,dx = 1.2599x^{1/3}\,dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2$.

Area: Integrating,

$$A = \int_A dA = \int_0^{4\,\text{m}} 1.2599x^{1/3}\,dx = 0.9449x^{4/3}\Big|_0^{4\,\text{m}} = 6\text{ m}^2$$

Thus, the mass of the plate can be obtained from

$$m = \rho At = 7850(6)(0.01) = 471\text{ kg}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\,\text{m}} x\left(1.2599x^{1/3}\,dx\right)}{6} = \frac{\int_0^{4\,\text{m}} 1.2599x^{4/3}\,dx}{6} = \frac{0.5399x^{7/3}\Big|_0^{4\,\text{m}}}{6} = 2.2857\text{ m}$$

Since the plate has a uniform thickness, its center of gravity coincides with its centroid.

Equations of Equilibrium: By referring to the free body diagram shown in Fig. b,

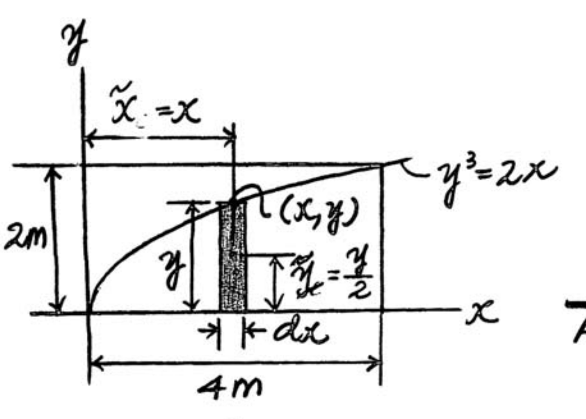
$\circlearrowleft +\Sigma M_A = 0;$ $\quad F_{BC}(4) - 471(9.81)(2.2857) = 0$

$F_{BC} = 2640.27\text{ N} = 2.64\text{ kN}$ **Ans.**

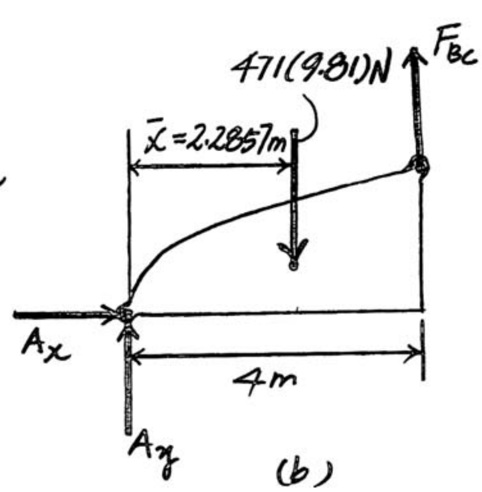
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 2640.27 - 471(9.81) = 0$

$A_y = 1980.24\text{ N} = 1.98\text{ kN}$ **Ans.**

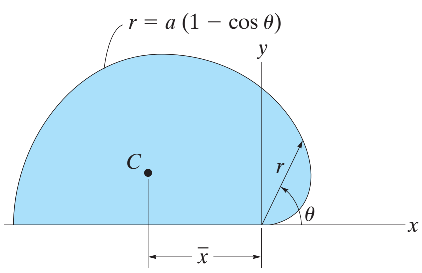


(a)



(b)

9–19. Determine the location $\bar{x}$ to the centroid C of the upper portion of the cardioid, $r = a(1 - \cos\theta)$.



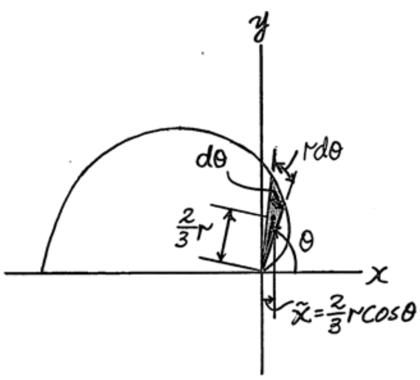
$$dA = \frac{1}{2} r^2\, d\theta$$

$$A = \int_0^{\pi} \frac{1}{2}(a^2)(1 - \cos\theta)^2 d\theta = \frac{3}{4}\pi a^2$$

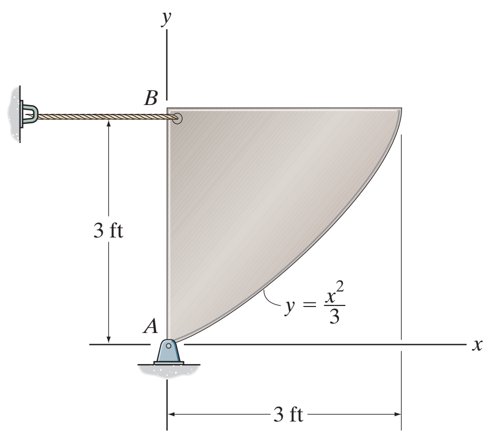
$$\int_A \tilde{x}\, dA = \int_0^{\pi}\left(\frac{2}{3} r\cos\theta\right)\left(\frac{1}{2}\right)(a^2)(1 - \cos\theta)^2\, d\theta$$

$$= \frac{2}{6} a^3 \int_0^{\pi} \cos\theta(1 - \cos\theta)^3\, d\theta = -1.9635\, a^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{-1.9635\, a^3}{\frac{3}{4}\pi a^2} = -0.833\, a \qquad \textbf{Ans}$$



***9–20.** The plate has a thickness of 0.5 in. and is made of steel having a specific weight of 490 lb/ft^3. Determine the horizontal and vertical components of reaction at the pin A and the force in the cord at B.



Differential Element: The element parallel to the x axis shown shaded in Fig. a will be considered. The area of this differential element is given by

$$dA = x\,dy = \sqrt{3}y^{1/2}\,dy$$

Centroid: The centroid of the element is located at $\tilde{x} = x/2 = \frac{\sqrt{3}}{2}y^{1/2}$ and $y_c = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \sqrt{3}y^{1/2}\,dy = \frac{2\sqrt{3}}{3}y^{3/2}\Big|_0^{3\text{ ft}} = 6\text{ ft}^2$$

Thus, the weight of the plate can be obtained from

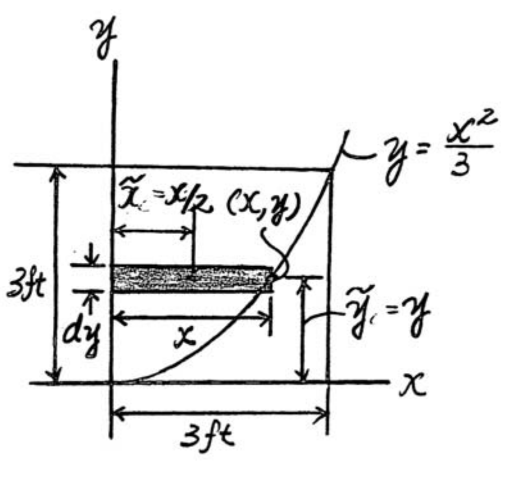
$$W = \gamma At = 490(6)\left(\frac{0.5}{12}\right) = 122.5\text{ lb}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}}\left(\frac{\sqrt{3}}{2}y^{1/2}\right)\left(\sqrt{3}y^{1/2}\,dy\right)}{6} = \frac{\int_0^{3\text{ ft}}\frac{3}{2}y\,dy}{6} = \frac{\frac{3}{4}y^2\Big|_0^{3\text{ ft}}}{6} = 1.125\text{ ft}$$

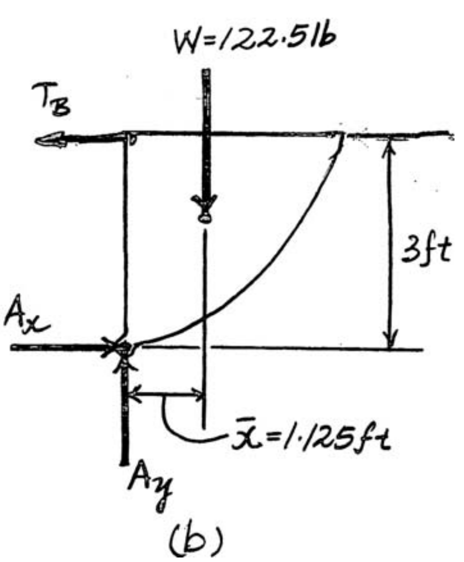
Since the plate has a uniform thickness, its center of gravity coincides with its centroid.

Equations of Equilibrium: By referring to the free body diagram shown in Fig. b,

$+\Sigma M_A = 0;$	$T_B(3) - 122.5(1.125) = 0$	$T_B = 45.94\text{ lb} = 45.9\text{ lb}$	**Ans.**
$\overset{+}{\rightarrow}\Sigma F_x = 0;$	$A_x - 45.94\text{ lb} = 0$	$A_x = 45.94\text{ lb} = 45.9\text{ lb}$	**Ans.**
$+\uparrow\Sigma F_y = 0;$	$A_y - 122.5 = 0$	$A_y = 122.5\text{ lb}$	**Ans.**

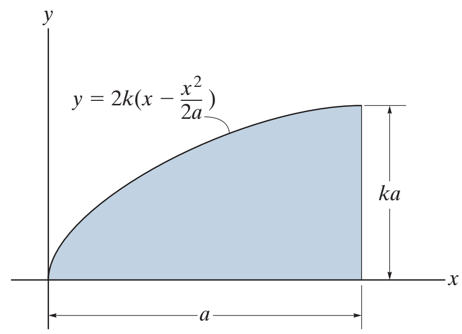


(a)



(b)

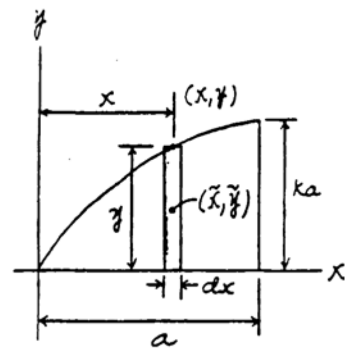
•9–21. Locate the centroid $\bar{x}$ of the shaded area.



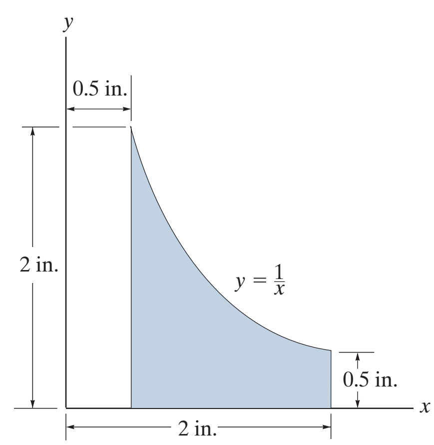
Area and Moment Arm : The area of the differential element is $dA = ydx = 2k\left(x - \frac{x^2}{2a}\right)dx$ and its centroid is $\tilde{x} = x$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left[2k\left(x - \frac{x^2}{2a}\right)dx\right]}{\int_0^a 2k\left(x - \frac{x^2}{2a}\right)dx} = \frac{2k\left(\frac{x^3}{3} - \frac{x^4}{8a}\right)\Big|_0^a}{2k\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\Big|_0^a} = \frac{5a}{8} \qquad \textbf{Ans}$$



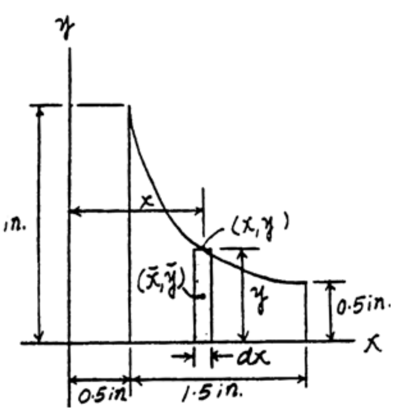
9–22. Locate the centroid $\bar{x}$ of the area.



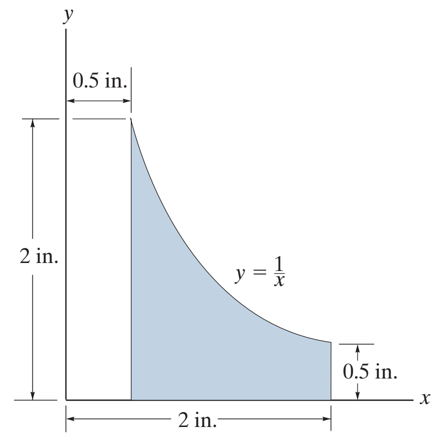
Area and Moment Arm : The area of the differential element is $dA = ydx = \frac{1}{x}dx$ and its centroid is $\tilde{x} = x$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{0.5\text{in}}^{2\text{in}} x\left(\frac{1}{x}dx\right)}{\int_{0.5\text{in}}^{2\text{in}} \frac{1}{x}dx} = \frac{x\big|_{0.5\text{in}}^{2\text{in}}}{\ln x\big|_{0.5\text{in}}^{2\text{in}}} = 1.08 \text{ in.} \qquad \textbf{Ans}$$



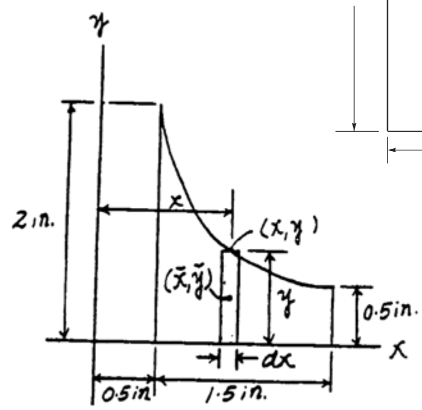
9–23. Locate the centroid $\overline{y}$ of the area.



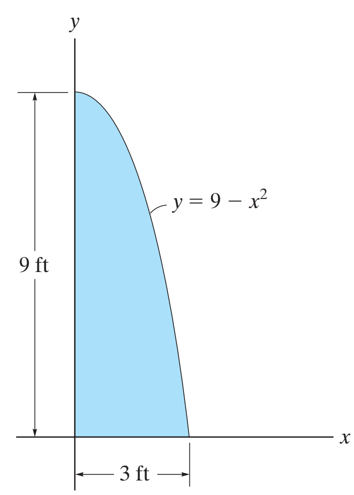
Area and Moment Arm : The area of the differential element is $dA = y\,dx = \frac{1}{x}dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2x}$.

Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{0.5\text{ in}}^{2\text{ in}} \frac{1}{2x}\left(\frac{1}{x}dx\right)}{\int_{0.5\text{ in}}^{2\text{ in}} \frac{1}{x}dx} = \frac{-\frac{1}{2x}\Big|_{0.5\text{ in}}^{2\text{ in}}}{\ln x\Big|_{0.5\text{ in}}^{2\text{ in}}} = 0.541\text{ in} \qquad \textbf{Ans}$$



***9–24.** Locate the centroid $(\overline{x}, \overline{y})$ of the area.



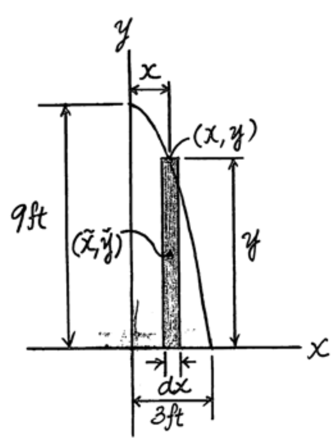
$$dA = y\,dx = (9 - x^2)dx$$

$$\tilde{x} = x$$

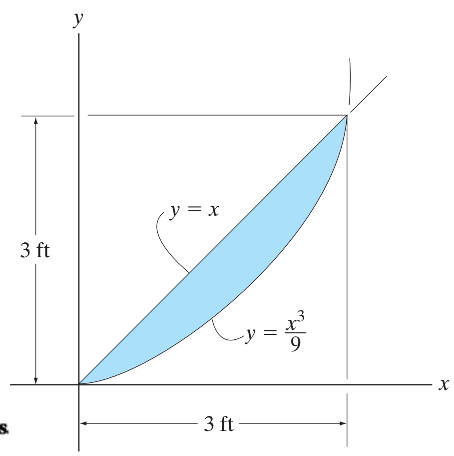
$$\tilde{y} = \frac{y}{2} = \frac{1}{2}(9 - x^2)dx$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^3 x(9 - x^2)\,dx}{\int_0^3 (9 - x^2)\,dx} = 1.125 \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^3 (9 - x^2)^2\,dx}{\int_0^3 (9 - x^2)\,dx} = 3.60\text{ ft} \qquad \textbf{Ans}$$



•9–25. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = (y_1 - y_2)\,dx = \left(x - \frac{x^3}{9}\right)dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}\left(x + \frac{x^3}{9}\right)$.

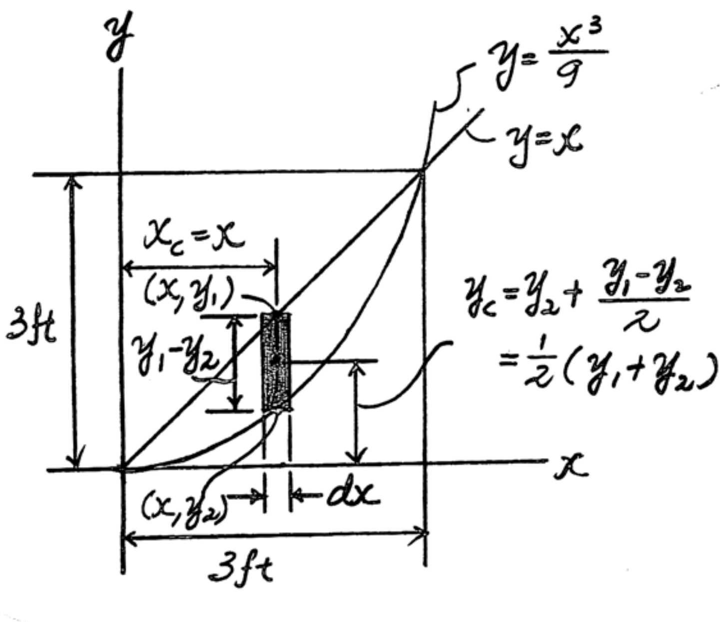
Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \left(x - \frac{x^3}{9}\right)dx = \left(\frac{x^2}{2} - \frac{x^4}{36}\right)\Bigg|_0^{3\text{ ft}} = 2.25\text{ ft}^2 \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} x\left(x - \frac{x^3}{9}\right)dx}{2.25} = \frac{\int_0^{3\text{ ft}} \left(x^2 - \frac{x^4}{9}\right)dx}{2.25} = \frac{\left(\frac{x^3}{3} - \frac{x^5}{45}\right)\Big|_0^{3\text{ ft}}}{2.25} = 1.6\text{ ft} \qquad \textbf{Ans.}$$

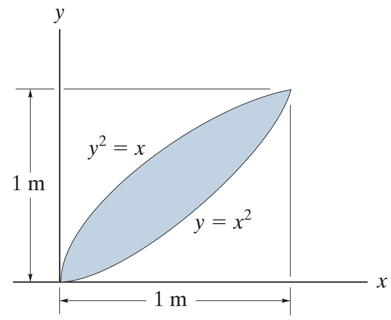
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}} \frac{1}{2}\left(x + \frac{x^3}{9}\right)\left(x - \frac{x^3}{9}\right)dx}{2.25} = \frac{\int_0^{3\text{ ft}} \frac{1}{2}\left(x^2 - \frac{x^6}{81}\right)dx}{2.25}$$

$$= \frac{\frac{1}{2}\left(\frac{x^3}{3} - \frac{x^7}{567}\right)\Big|_0^{3\text{ ft}}}{2.25} = 1.143\text{ ft} = 1.14\text{ ft} \qquad \textbf{Ans.}$$



(a)

9–26. Locate the centroid $\bar{x}$ of the area.

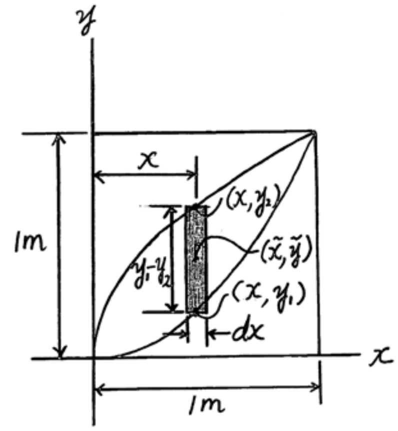


Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{x} = x$.

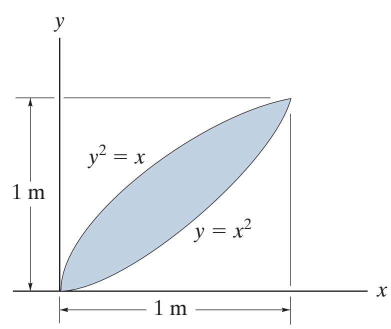
Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} x\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$

$$= \frac{\left(\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^4\right)\Big|_0^{1\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\text{m}}} = \frac{9}{20}\ \text{m} = 0.45\ \text{m} \qquad \textbf{Ans}$$



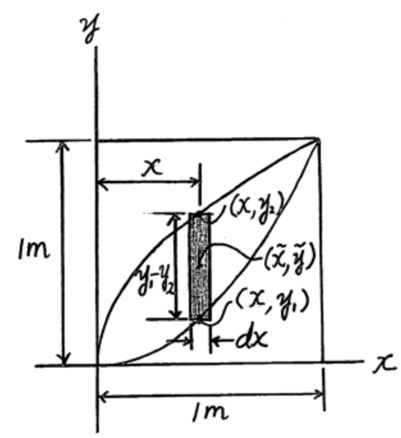
9–27. Locate the centroid $\bar{y}$ of the area.



Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{y} = y_2 + \frac{y_1 - y_2}{2} = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)$.

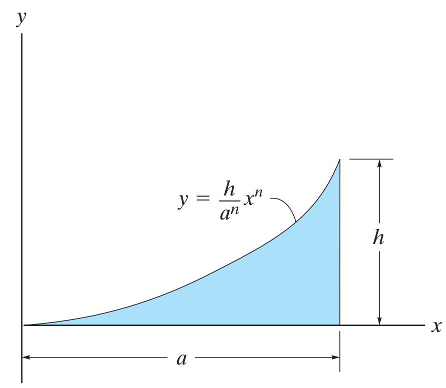
Centroid : Applying Eq. 9–4 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$



$$= \frac{\frac{1}{2}\left(\frac{1}{2}x^2 - \frac{1}{5}x^5\right)\Big|_0^{1\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\text{m}}} = \frac{9}{20}\ \text{m} = 0.45\ \text{m} \qquad \textbf{Ans}$$

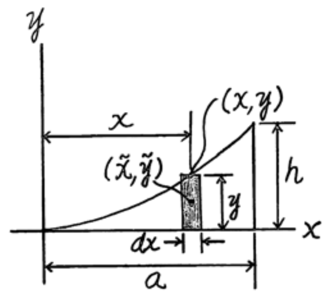
***9–28.** Locate the centroid $\bar{x}$ of the area.



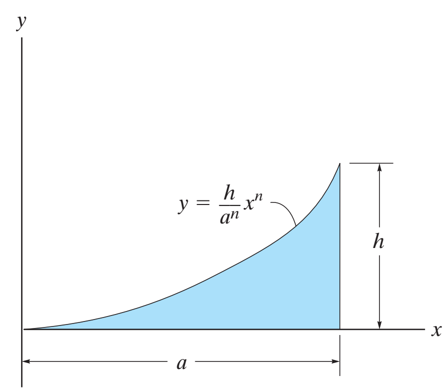
$dA = y\,dx$

$\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a \frac{h}{a^n}x^{n+1}\,dx}{\int_0^a \frac{h}{a^n}x^n\,dx} = \frac{\frac{h(a^{n+2})}{a^n(n+2)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{n+1}{n+2}a$$



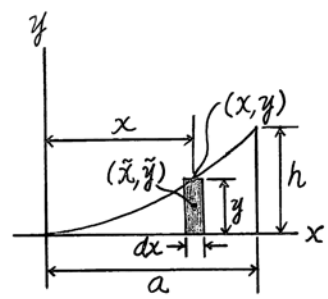
•9–29. Locate the centroid $\bar{y}$ of the area.



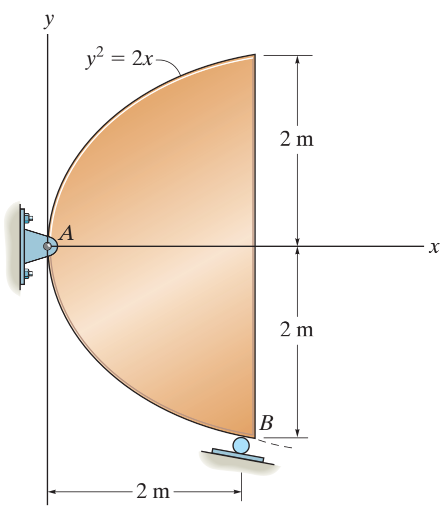
$dA = y\,dx$

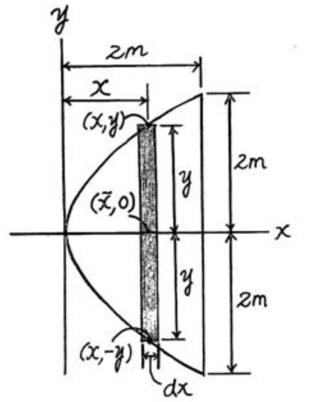
$\tilde{y} = \frac{y}{2}$

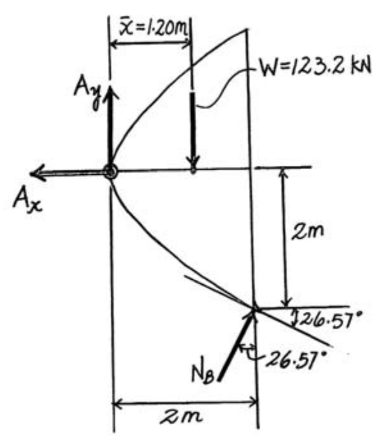
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \frac{h^2}{a^{2n}}x^{2n}\,dx}{\int_0^a \frac{h}{a^n}x^n\,dx} = \frac{\frac{h^2(a^{2n+1})}{2a^{2n}(2n+1)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{n+1}{2(2n+1)}h \qquad \text{Ans}$$



9–30. The steel plate is 0.3 m thick and has a density of 7850 kg/m^3. Determine the location of its center of mass. Also determine the horizontal and vertical reactions at the pin and the reaction at the roller support. *Hint:* The normal force at B is perpendicular to the tangent at B, which is found from $\tan\theta = dy/dx$.







$dA = 2y\,dx$

$\tilde{x} = x$

$\tilde{y} = 0$

$$\int_A dA = \int_0^2 2\sqrt{2x}\,dx = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}}\Big|_0^2 = 5.333\ \text{m}^2$$

$$\int_A \tilde{x}\,dA = \int_0^2 2\sqrt{2x}\,x\,dx = \frac{4\sqrt{2}}{5}x^{\frac{5}{2}}\Big|_0^2 = 6.40\ \text{m}^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{6.40}{5.333} = 1.20\ \text{m} \quad \text{Ans}$$

$\bar{y} = 0$ Ans (by symmetry)

$y^2 = 2x$

$2y\,dy = 2\,dx$

$\tan\theta = \frac{dy}{dx}\Big|_{(2,-2)} = -0.5$

$\theta = \tan^{-1}(-0.5) = -26.57°$

$W = 7850(0.3)(5.333)(9.81) = 123.2\ \text{kN}$

$\curvearrowleft + \Sigma M_A = 0;\quad -123.2(1.2) + N_B(\cos 26.57°)(2) + N_B(\sin 26.57°)(2) = 0$

$N_B = 55.10 = 55.1\ \text{kN}$ Ans

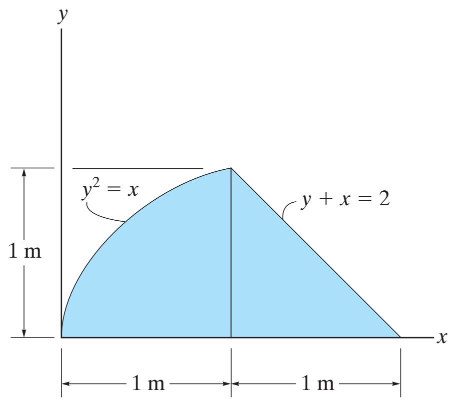
$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + 55.10 \sin 26.57° = 0$

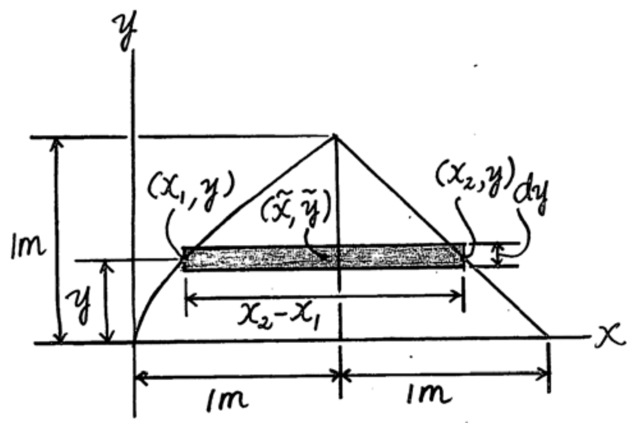
$A_x = 24.6\ \text{kN}$ Ans

$+\uparrow \Sigma F_y = 0;\quad A_y - 123.2 + 55.10 \cos 26.57° = 0$

$A_y = 73.9\ \text{kN}$ Ans

9–31. Locate the centroid of the area. *Hint:* Choose elements of thickness dy and length $[(2 - y) - y^2]$.





$$x_1 = y^2$$

$$x_2 = 2 - y$$

$$dA = (x_2 - x_1)\,dy = (2 - y - y^2)\,dy$$

$$\tilde{x} = \frac{x_2 + x_1}{2} = \frac{2 - y + y^2}{2}$$

$$\tilde{y} = y$$

$$\int_A dA = \int_0^1 (2 - y - y^2)\,dy = \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_0^1 = 1.167\text{ m}^2$$

$$\int_A \tilde{x}\,dA = \int_0^1 \frac{1}{2}(2 - y + y^2)(2 - y - y^2)\,dy$$

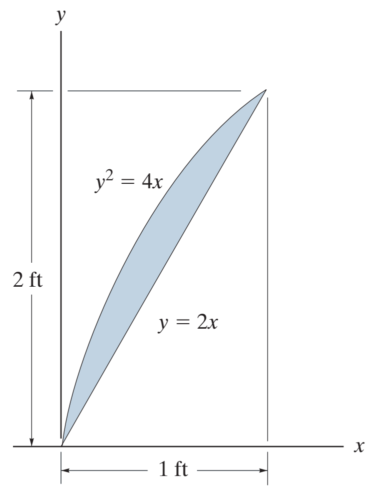
$$= \frac{1}{2}\left[4y - 2y^2 + \frac{y^3}{3} - \frac{y^5}{5}\right]_0^1 = 1.067\text{ m}^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{1.067}{1.167} = 0.914\text{ m} \qquad \textbf{Ans}$$

$$\int_A \tilde{y}\,dA = \int_0^1 y(2 - y - y^2)\,dy = \left[y^2 - \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = 0.4167\text{ m}^3$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{0.4167}{1.167} = 0.357\text{ m} \qquad \textbf{Ans}$$

***9–32.** Locate the centroid $\bar{x}$ of the area.

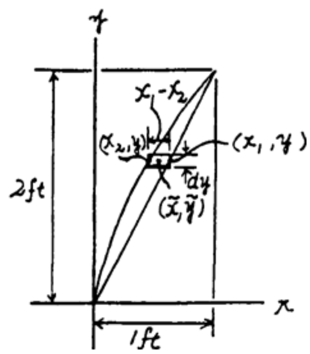


Area and Moment Arm : Here, $x_1 = \frac{y}{2}$ and $x_2 = \frac{y^2}{4}$. The area of the differential element is $dA = (x_1 - x_2)\,dy = \left(\frac{y}{2} - \frac{y^2}{4}\right)dy$ and its centroid is $\tilde{x} = x_2 + \frac{x_1 - x_2}{2} = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}\left(\frac{y}{2} + \frac{y^2}{4}\right)$.

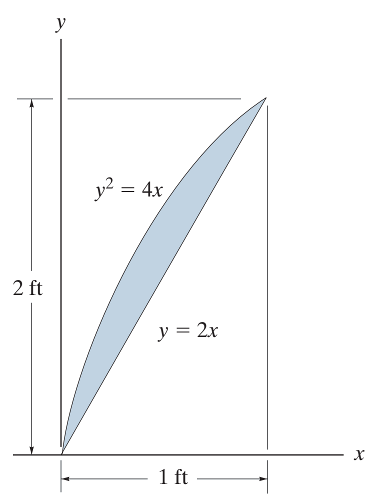
Centroid : Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{2\,\text{ft}} \frac{1}{2}\left(\frac{y}{2} + \frac{y^2}{4}\right)\left[\left(\frac{y}{2} - \frac{y^2}{4}\right)dy\right]}{\int_0^{2\,\text{ft}} \left(\frac{y}{2} - \frac{y^2}{4}\right)dy}$$

$$= \frac{\left[\frac{1}{2}\left(\frac{1}{12}y^3 - \frac{1}{80}y^5\right)\right]\Big|_0^{2\,\text{ft}}}{\left(\frac{1}{4}y^2 - \frac{1}{12}y^3\right)\Big|_0^{2\,\text{ft}}} = \frac{2}{5}\ \text{ft} = 0.4\ \text{ft} \qquad \textbf{Ans}$$



•9–33. Locate the centroid $\bar{y}$ of the area.

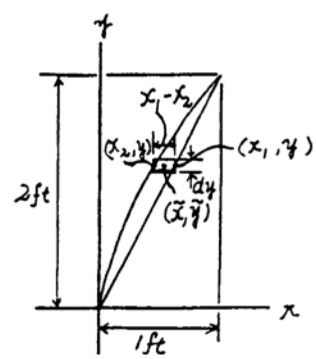


Area and Moment Arm : Here, $x_1 = \frac{y}{2}$ and $x_2 = \frac{y^2}{4}$. The area of the differential element is $dA = (x_1 - x_2)\,dy = \left(\frac{y}{2} - \frac{y^2}{4}\right)dy$ and its centroid is $\tilde{y} = y$.

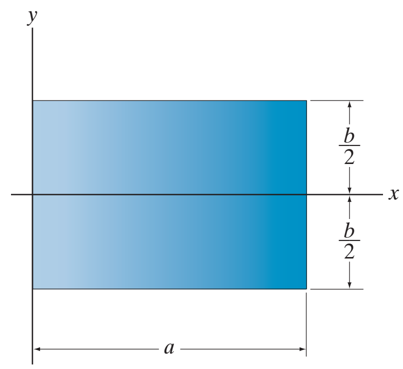
Centroid : Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{2\,\text{ft}} y\left[\left(\frac{y}{2} - \frac{y^2}{4}\right)dy\right]}{\int_0^{2\,\text{ft}} \left(\frac{y}{2} - \frac{y^2}{4}\right)dy}$$

$$= \frac{\left(\frac{1}{6}y^3 - \frac{1}{16}y^4\right)\Big|_0^{2\,\text{ft}}}{\left(\frac{1}{4}y^2 - \frac{1}{12}y^3\right)\Big|_0^{2\,\text{ft}}} = 1\ \text{ft} \qquad \textbf{Ans}$$



9–34. If the density at any point in the rectangular plate is defined by $\rho = \rho_0(1 + x/a)$, where ρ_0 is a constant, determine the mass and locate the center of mass $\bar{x}$ of the plate. The plate has a thickness t.



Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The mass of this element is

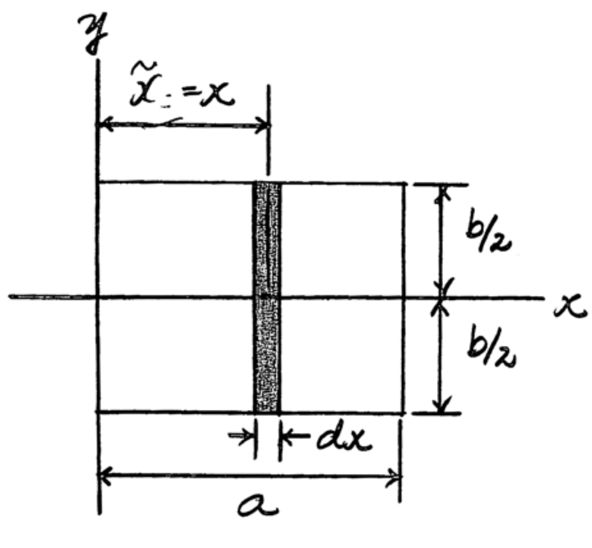
$$dm = \rho\, dV = \rho_0\left(1+\frac{x}{a}\right)t(b\,dx) = \rho_0 tb\left(1+\frac{x}{a}\right)dx$$

Mass: Integrating,

$$m = \int_m dm = \int_0^a \rho_0 tb\left(1+\frac{x}{a}\right)dx = \rho_0 tb\left(x+\frac{x^2}{2a}\right)dx\Bigg|_0^a = \frac{3}{2}\rho_0 abt \qquad \textbf{Ans.}$$

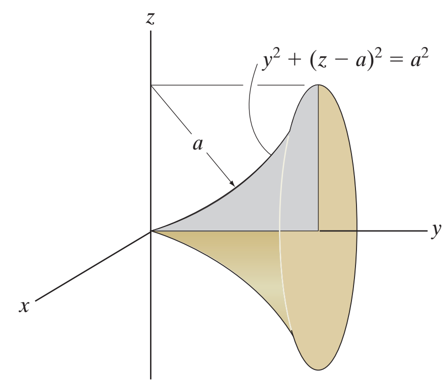
Center of Mass: The center of mass of the element is located at $\tilde{x} = x$.

$$\bar{x} = \frac{\int_m \tilde{x}\,dm}{\int_m dm} = \frac{\int_0^a x\left[\rho_0 tb\left(1+\frac{x}{a}\right)dx\right]}{\frac{3}{2}\rho_0 abt} = \frac{\int_0^a \rho_0 tb\left(x+\frac{x^2}{a}\right)dx}{\frac{3}{2}\rho_0 abt} = \frac{\rho_0 tb\left[\frac{x^2}{2}+\frac{x^3}{3a}\right]\Bigg|_0^a}{\frac{3}{2}\rho_0 abt} = \frac{5}{9}a \qquad \textbf{Ans.}$$



(a)

9–35. Locate the centroid $\bar{y}$ of the homogeneous solid formed by revolving the shaded area about the y axis.



Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is $dV = \pi z^2\, dy$.

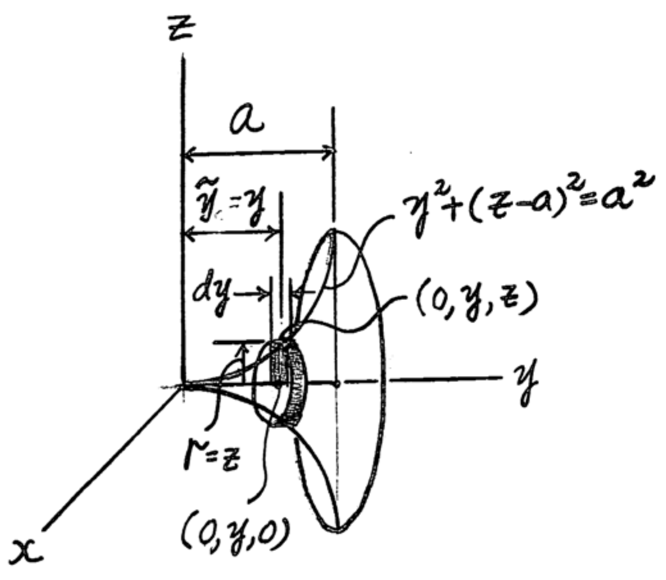
Here, $z = a - \sqrt{a^2 - y^2}$. Thus,

$$dV = \pi\left(a - \sqrt{a^2 - y^2}\right)^2 dy = \pi\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy$$

Centroid: The centroid of the element is located at $\tilde{y} = y$.

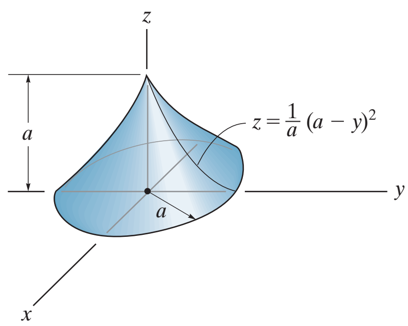
$$\bar{y} = \frac{\int_A \tilde{y}\, dV}{\int_A dV} = \frac{\int_0^a y\left[\pi\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy\right]}{\int_0^a \pi\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy} = \frac{\pi\int_0^a\left(2a^2 y - y^3 - 2ay\sqrt{a^2 - y^2}\right)dy}{\pi\int_0^a\left(2a^2 - y^2 - 2a\sqrt{a^2 - y^2}\right)dy}$$

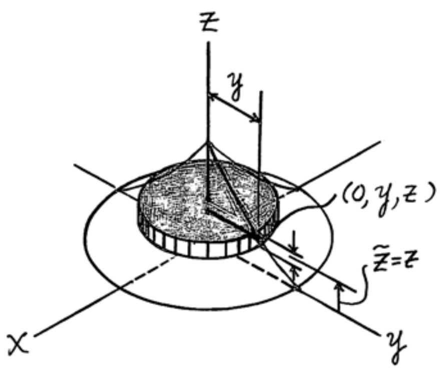
$$= \frac{\pi\left(a^2y^2 - \frac{y^4}{4} + \frac{2a}{3}\sqrt{\left(a^2 - y^2\right)^3}\right)\Bigg|_0^a}{\pi\left[2a^2y - \frac{y^3}{3} - a\left(y\sqrt{a^2 - y^2} + a^2\sin^{-1}\frac{y}{a}\right)\right]\Bigg|_0^a} = \frac{\frac{1}{12}a^4}{\left(\frac{10 - 3\pi}{6}\right)a^3} = \frac{a}{2(10 - 3\pi)} \qquad \textbf{Ans.}$$



(a)

***9–36.** Locate the centroid $\bar{z}$ of the solid.



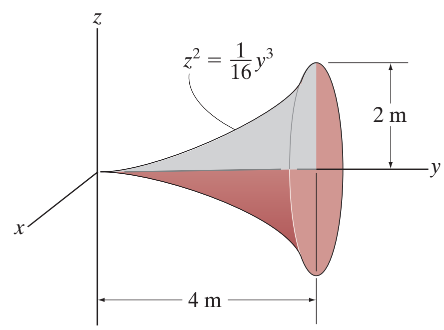


The volume of the differential thin- disk element $dV = \pi y^2 dz = \pi\left(a - \sqrt{az}\right)^2 dz$

$dV = \pi\left(a^2 + az - 2a^{\frac{3}{2}}z^{\frac{1}{2}}\right)dz$ and $\tilde{z} = z$.

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^a z\left[\pi\left(a^2 + az - 2a^{\frac{3}{2}}z^{\frac{1}{2}}\right)dz\right]}{\int_0^a \pi\left(a^2 + az - 2a^{\frac{3}{2}}z^{\frac{1}{2}}\right)dz} = \frac{1}{5}a \qquad \textbf{Ans}$$

•**9–37.** Locate the centroid $\bar{y}$ of the homogeneous solid formed by revolving the shaded area about the y axis.

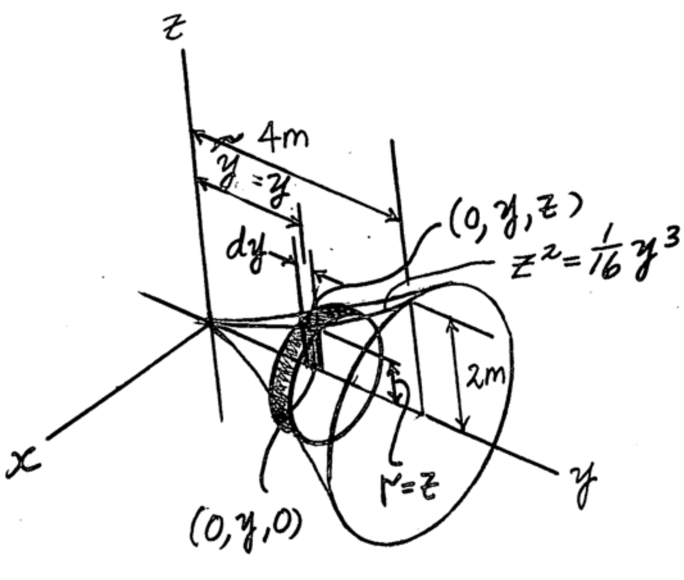


Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\, dy = \pi\left(\frac{1}{16}y^3\right) dy = \frac{\pi}{16}y^3\, dy$$

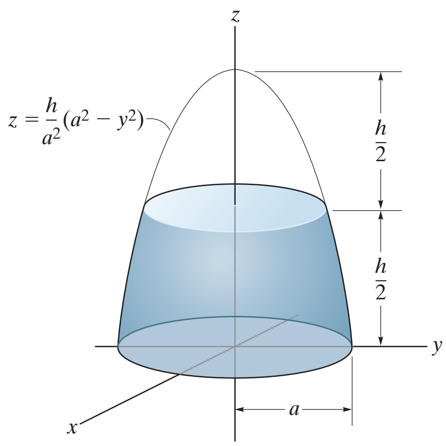
Centroid: The centroid of the element is located at $\tilde{y} = y$.

$$\bar{y} = \frac{\int_A \tilde{y}\, dV}{\int_A dV} = \frac{\int_0^{4\text{ m}} y\left(\frac{\pi}{16}y^3\, dy\right)}{\int_0^{4\text{ m}} \frac{\pi}{16}y^3\, dy} = \frac{\int_0^{4\text{ m}} \frac{\pi}{16}y^4\, dy}{\int_0^{4\text{ m}} \frac{\pi}{16}y^3\, dy} = \frac{\frac{\pi}{16}\left(\frac{y^5}{5}\right)\Big|_0^{4\text{ m}}}{\frac{\pi}{16}\left(\frac{y^4}{4}\right)\Big|_0^{4\text{ m}}} = 3.2\text{ m}$$ **Ans.**



(a)

9–38. Locate the centroid $\bar{z}$ of the homogeneous solid frustum of the paraboloid formed by revolving the shaded area about the z axis.

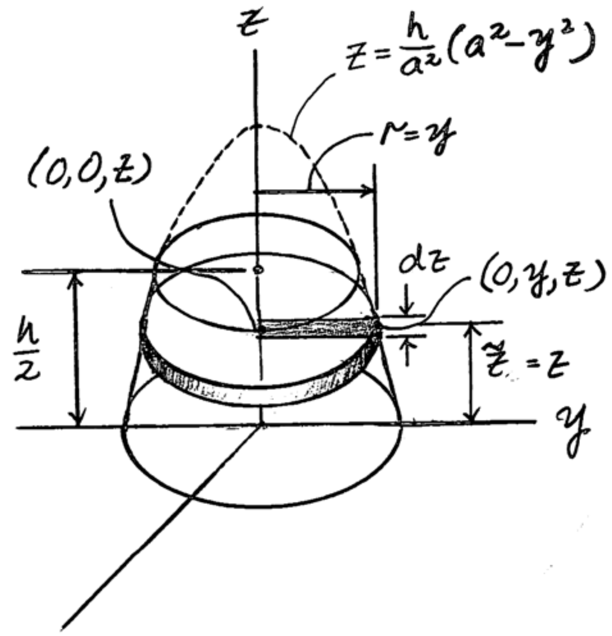


Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi y^2\,dz = \pi\left(a^2 - \frac{a^2}{h}z\right)dz$$

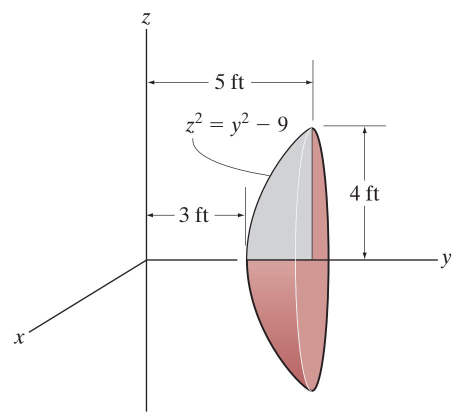
Centroid: The centroid of the element is located at $\tilde{z}_c = z$.

$$\bar{z} = \frac{\int_A \tilde{z}_c\,dV}{\int_A dV} = \frac{\int_0^{h/2} z\left[\pi\left(a^2 - \frac{a^2}{h}z\right)dz\right]}{\int_0^{h/2} \pi\left(a^2 - \frac{a^2}{h}z\right)dz} = \frac{\int_0^{h/2} \pi\left(a^2 z - \frac{a^2}{h}z^2\right)dz}{\int_0^{h/2} \pi\left(a^2 - \frac{a^2}{h}z\right)dz} = \frac{\pi\left(\frac{a^2}{2}z^2 - \frac{a^2}{3h}z^3\right)\Big|_0^{h/2}}{\pi\left(a^2 z - \frac{a^2}{2h}z^2\right)\Big|_0^{h/2}} = \frac{2}{9}h \qquad \textbf{Ans.}$$



(a)

9–39. Locate the centroid $\bar{y}$ of the homogeneous solid formed by revolving the shaded area about the y axis.

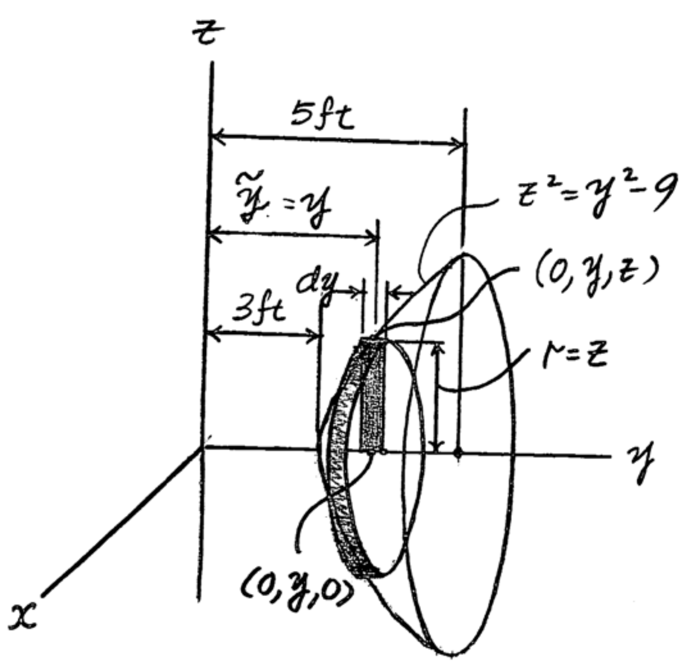


Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\,dy = \pi\left(y^2 - 9\right)dy$$

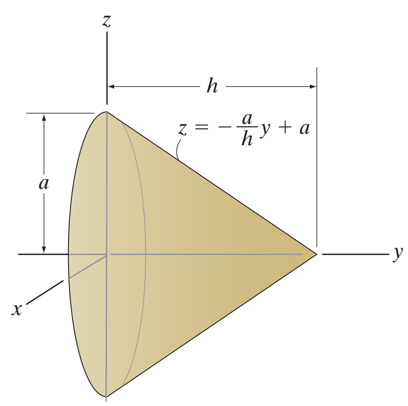
Centroid: The centroid of the element is located at $\tilde{y} = y$.

$$\bar{y} = \frac{\int_A \tilde{y}\,dV}{\int_A dV} = \frac{\int_{3\text{ ft}}^{5\text{ ft}} y\left[\pi\left(y^2 - 9\right)dy\right]}{\int_{3\text{ ft}}^{5\text{ ft}} \pi\left(y^2 - 9\right)dy} = \frac{\int_{3\text{ ft}}^{5\text{ ft}} \pi\left(y^3 - 9y\right)dy}{\int_{3\text{ ft}}^{5\text{ ft}} \pi\left(y^2 - 9\right)dy} = \frac{\pi\left(\frac{y^4}{4} - \frac{9y^2}{2}\right)\Big|_{3\text{ ft}}^{5\text{ ft}}}{\pi\left(\frac{y^3}{3} - 9y\right)\Big|_{3\text{ ft}}^{5\text{ ft}}} = 4.36\text{ ft} \qquad \textbf{Ans.}$$



(a)

***9–40.** Locate the center of mass $\bar{y}$ of the circular cone formed by revolving the shaded area about the y axis. The density at any point in the cone is defined by $\rho = (\rho_0/h)y$, where ρ_0 is a constant.



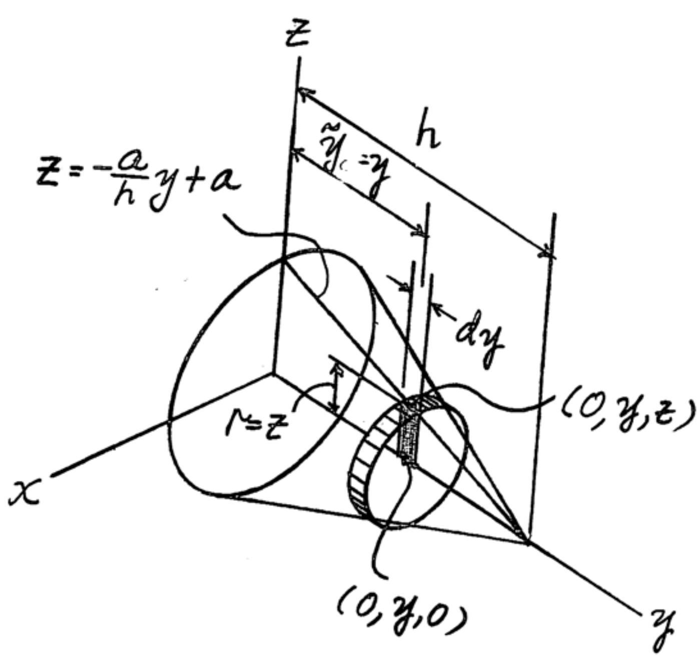
Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The mass of the element is

$$dm = \rho\, dV = \rho\pi z^2\, dy = \left(\frac{\rho_0}{h}\right)y\left[\pi\left(\frac{-a}{h}y + a\right)^2 dy\right] = \frac{\pi a^2 \rho_0}{h}\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy$$

Centroid: The centroid of the element is located at $\tilde{y} = y$.

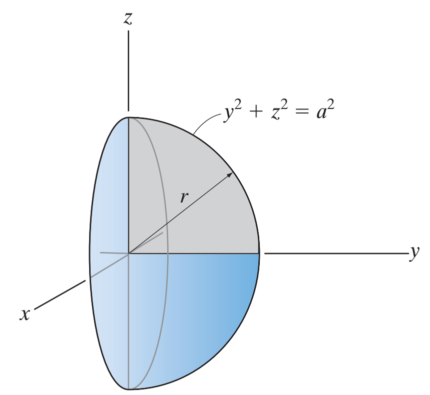
$$\bar{y} = \frac{\int_m \tilde{y}\, dm}{\int_m dm} = \frac{\int_0^h y\left[\frac{\pi a^2\rho_0}{h}\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy\right]}{\int_0^h \frac{\pi a^2\rho_0}{h}\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy} = \frac{\frac{\pi a^2\rho_0}{h}\int_0^h\left(\frac{y^4}{h^2} + y^2 - \frac{2y^3}{h}\right)dy}{\frac{\pi a^2\rho_0}{h}\int_0^h\left(\frac{y^3}{h^2} + y - \frac{2y^2}{h}\right)dy} = \frac{\left(\frac{y^5}{5h^2} + \frac{y^3}{3} - \frac{y^4}{2h}\right)\Big|_0^h}{\left(\frac{y^4}{4h^2} + \frac{y^2}{2} - \frac{2y^3}{3h}\right)\Big|_0^h}$$

$$= \frac{2}{5}h \qquad \textbf{Ans.}$$



(a)

•**9–41.** Determine the mass and locate the center of mass $\bar{y}$ of the hemisphere formed by revolving the shaded area about the y axis. The density at any point in the hemisphere can be defined by $\rho = \rho_0(1 + y/a)$, where ρ_0 is a constant.



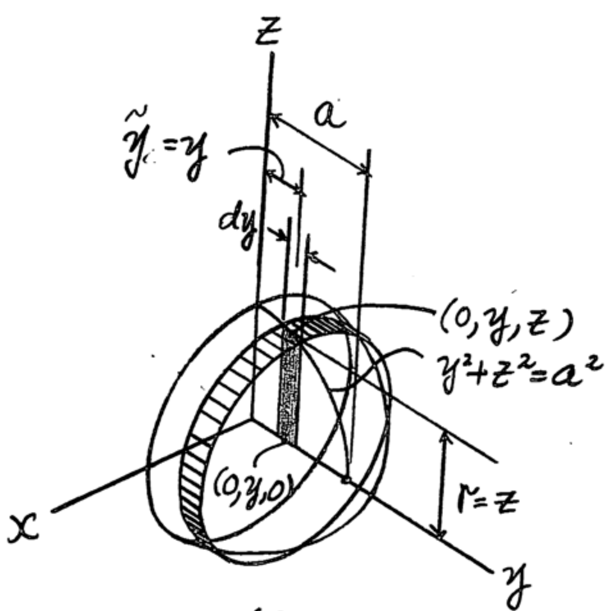
Differential Element: The thin disk element shown shaded in Fig. a will be considered. The mass of the element is

$$dm = \rho\, dV = \rho \pi z^2\, dy = \pi\rho_0\left(1+\frac{y}{a}\right)\left(a^2 - y^2\right)dy = \pi\rho_0\left(a^2 - y^2 + ay - \frac{y^3}{a}\right)dy$$

Centroid: The centroid of the element is located at $\tilde{y} = y$.

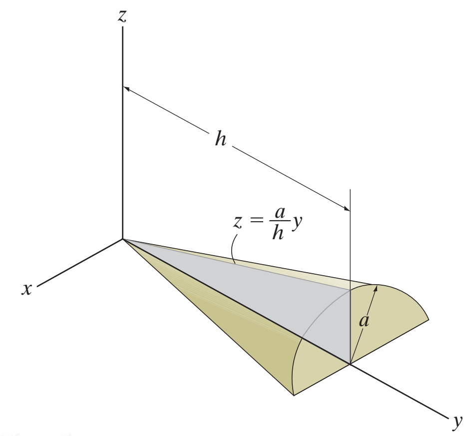
$$\bar{y} = \frac{\int_m \tilde{y}\,dm}{\int_m dm} = \frac{\int_0^a y\left[\pi\rho_0\left(a^2 - y^2 + ay - \frac{y^3}{a}\right)dy\right]}{\int_0^a \pi\rho_0\left(a^2 - y^2 + ay - \frac{y^3}{a}\right)dy} = \frac{\pi\rho_0\int_0^a\left(a^2y - y^3 + ay^2 - \frac{y^4}{a}\right)dy}{\pi\rho_0\int_0^a\left(a^2 - y^2 + ay - \frac{y^3}{a}\right)dy} = \frac{\left(\frac{a^2y^2}{2} - \frac{y^4}{4} + \frac{ay^3}{3} - \frac{y^5}{5a}\right)\Big|_0^a}{\left(a^2y - \frac{y^3}{3} + \frac{ay^2}{2} - \frac{y^4}{4a}\right)\Big|_0^a}$$

$$= \frac{23}{55}a \quad \textbf{Ans.}$$



(a)

9–42. Determine the volume and locate the centroid $(\bar{y}, \bar{z})$ of the homogeneous conical wedge.



Differential Element: The half - thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \frac{\pi}{2}z^2\,dy = \frac{\pi}{2}\left(\frac{a^2}{h^2}y^2\right)dy = \frac{a^2\pi}{2h^2}y^2\,dy$$

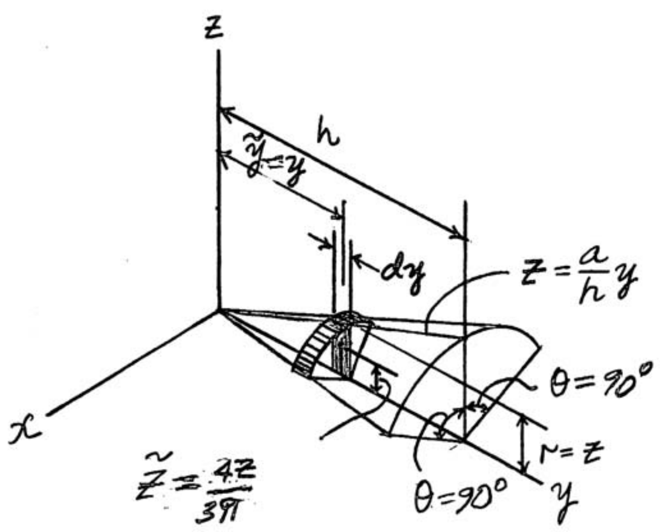
Volume: Integrating,

$$V = \int_V dV = \int_0^h \frac{a^2\pi}{2h^2}y^2\,dy = \frac{a^2\pi}{2h^2}\left(\frac{y^3}{3}\right)\Bigg|_0^h = \frac{\pi a^2 h}{6} \qquad \textbf{Ans.}$$

Centroid: The centroid of the element is located at $\tilde{y} = y$ and $\tilde{z} = \frac{4z}{3\pi} = \frac{4a}{3\pi h}y$.

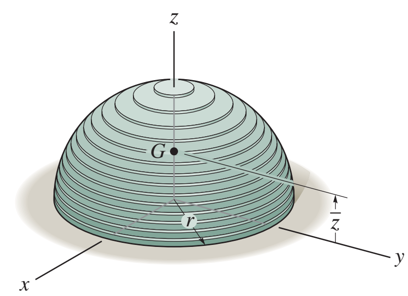
$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^h y\left(\frac{a^2\pi}{2h^2}y^2\,dy\right)}{\frac{\pi a^2 h}{6}} = \frac{\frac{a^2\pi}{2h^2}\int_0^h y^3\,dy}{\frac{\pi a^2 h}{6}} = \frac{\frac{a^2\pi}{2h^2}\left(\frac{y^4}{4}\right)\Big|_0^h}{\frac{\pi a^2 h}{6}} = \frac{3}{4}h \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^h \left(\frac{4a}{3\pi h}y\right)\left(\frac{\pi a^2}{2h^2}y^2\,dy\right)}{\frac{\pi a^2 h}{6}} = \frac{\frac{2a^3}{3h^3}\int_0^h y^3\,dy}{\frac{\pi a^2 h}{6}} = \frac{\frac{2a^3}{3h^3}\left(\frac{y^4}{4}\right)\Big|_0^h}{\frac{\pi a^2 h}{6}} = \frac{a}{\pi} \qquad \textbf{Ans.}$$

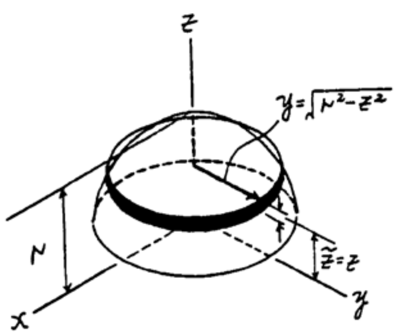


(a)

9–43. The hemisphere of radius r is made from a stack of very thin plates such that the density varies with height, $\rho = kz$, where k is a constant. Determine its mass and the distance $\bar{z}$ to the center of mass G.



Mass and Moment Arm : The density of the material is $\rho = kz$. The mass of the thin disk differential element is $dm = \rho dV = \rho \pi y^2 dz = kz\left[\pi\left(r^2 - z^2\right) dz\right]$ and its centroid $\tilde{z} = z$. Evaluating the integrals, we have



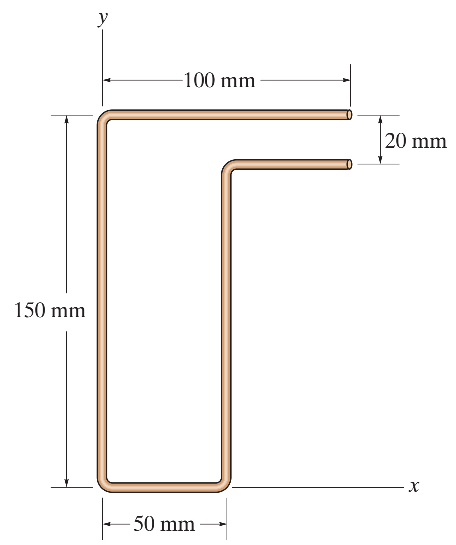
$$m = \int_m dm = \int_0^r kz\left[\pi\left(r^2 - z^2\right) dz\right]$$
$$= \pi k\left(\frac{r^2 z^2}{2} - \frac{z^4}{4}\right)\Bigg|_0^r = \frac{\pi k r^4}{4} \qquad \textbf{Ans}$$

$$\int_m \tilde{z}\, dm = \int_0^r z\left\{kz\left[\pi\left(r^2 - z^2\right) dz\right]\right\}$$
$$= \pi k\left(\frac{r^2 z^3}{3} - \frac{z^5}{5}\right)\Bigg|_0^r = \frac{2\pi k r^5}{15}$$

Centroid : Applying Eq. 9–3, we have

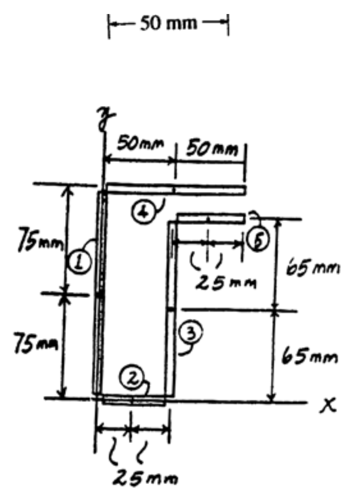
$$\bar{z} = \frac{\int_m \tilde{z}\, dm}{\int_m dm} = \frac{2\pi k r^5/15}{\pi k r^4/4} = \frac{8}{15} r \qquad \textbf{Ans}$$

***9–44.** Locate the centroid $(\bar{x}, \bar{y})$ of the uniform wire bent in the shape shown.



Centroid : The length of each segment and its respective centroid are tabulated below.

Segment	L(mm)	$\tilde{x}$(mm)	$\tilde{y}$(mm)	$\tilde{x}L$(mm²)	$\tilde{y}L$(mm²)
1	150	0	75	0	11250
2	50	25	0	1250	0
3	130	50	65	6500	8450
4	100	50	150	5000	15000
5	50	75	130	3750	6500
Σ	480			16500	41200

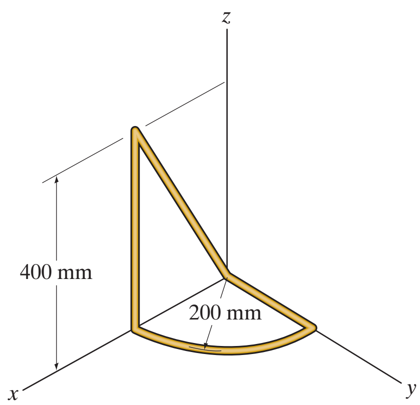


Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{16500}{480} = 34.375 \text{ mm} = 34.4 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{41200}{480} = 85.83 \text{ mm} = 85.8 \text{ mm} \qquad \textbf{Ans}$$

•9–45. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.

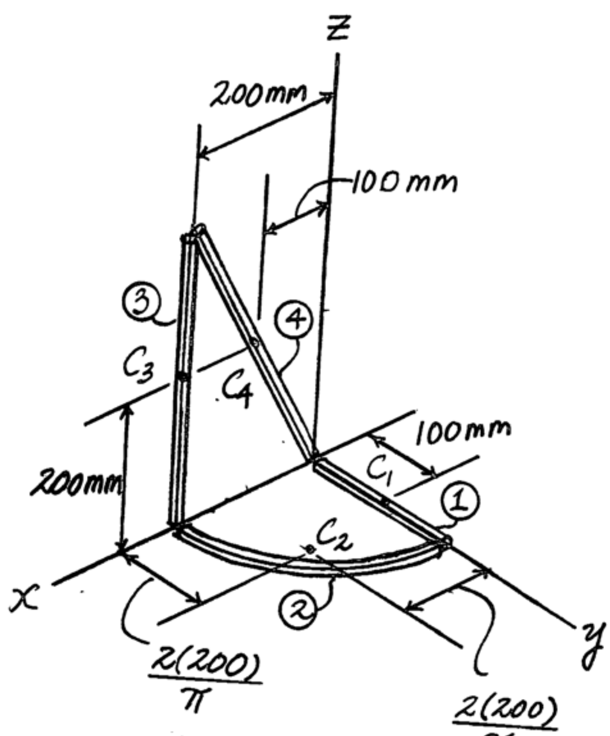


Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{0(200) + \frac{2(200)}{\pi}\left(\frac{\pi(200)}{2}\right) + 200(400) + 100\left(\sqrt{200^2 + 400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2 + 400^2}} = \frac{164.72(10^3)}{1361.37} = 121 \text{ mm} \qquad \textbf{Ans.}$$

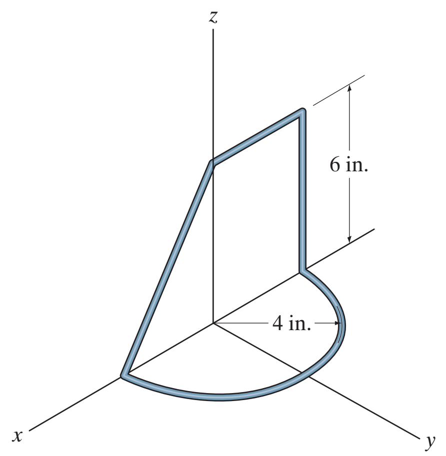
$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{100(200) + \frac{2(200)}{\pi}\left(\frac{\pi(200)}{2}\right) + 0(400) + 0\left(\sqrt{200^2 + 400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2 + 400^2}} = \frac{60(10^3)}{1361.37} = 44.1 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{0(200) + 0\left(\frac{\pi(200)}{2}\right) + 200(400) + 200\left(\sqrt{200^2 + 400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2 + 400^2}} = \frac{169.44(10^3)}{1361.37} = 124 \text{ mm} \qquad \textbf{Ans.}$$



(a)

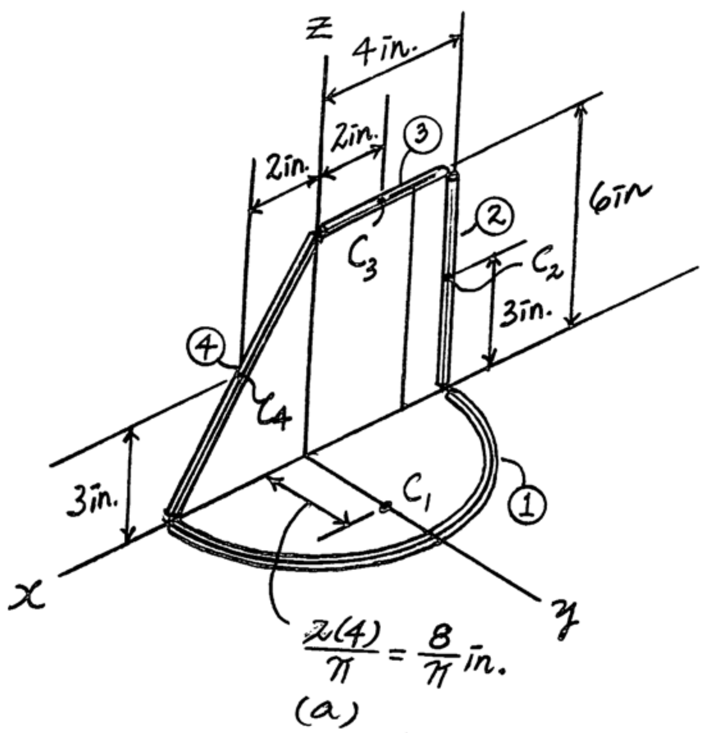
9–46. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.



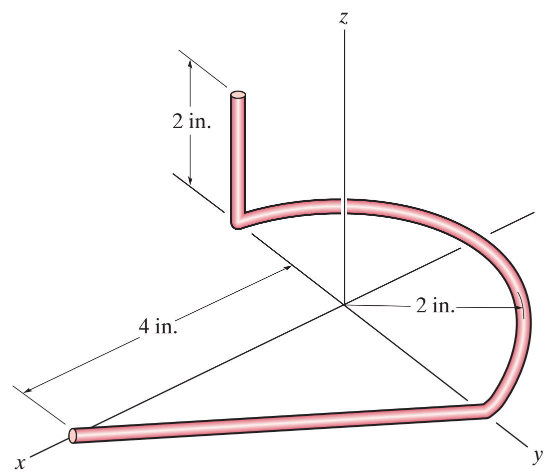
$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{(0)\pi(4) + (-4)(6) + (-2)(4) + (2)\left(\sqrt{4^2+6^2}\right)}{\pi(4) + 6 + 4 + \sqrt{4^2+6^2}} = \frac{-17.58}{29.78} = -0.590 \text{ in.} \qquad \textbf{Ans.}$$

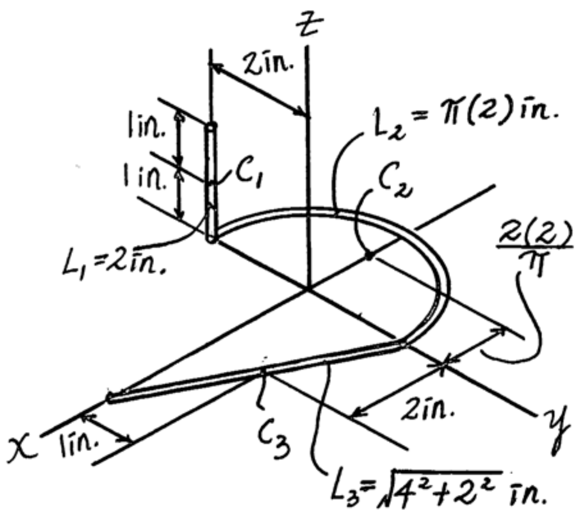
$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{(8/\pi)\pi(4) + 0(6) + 0(4) + 0\left(\sqrt{4^2+6^2}\right)}{\pi(4) + 6 + 4 + \sqrt{4^2+6^2}} = \frac{32}{29.78} = 1.07 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{(0)\pi(4) + 3(6) + 6(4) + 3\left(\sqrt{4^2+6^2}\right)}{\pi(4) + 6 + 4 + \sqrt{4^2+6^2}} = \frac{63.63}{29.78} = 2.14 \text{ in.} \qquad \textbf{Ans.}$$



9–47. Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire which is bent in the shape shown.





$$\Sigma L = 2 + \pi(2) + \sqrt{4^2 + 2^2} = 12.7553 \text{ in.}$$

$$\Sigma \tilde{x} L = 0(2) - \frac{2(2)}{\pi}(\pi 2) + 2\left(\sqrt{4^2 + 2^2}\right) = 0.94427 \text{ in}^2$$

$$\Sigma \tilde{y} L = (-2)(2) - 0(\pi 2) + 1\left(\sqrt{4^2 + 2^2}\right) = 0.47214 \text{ in}^2$$

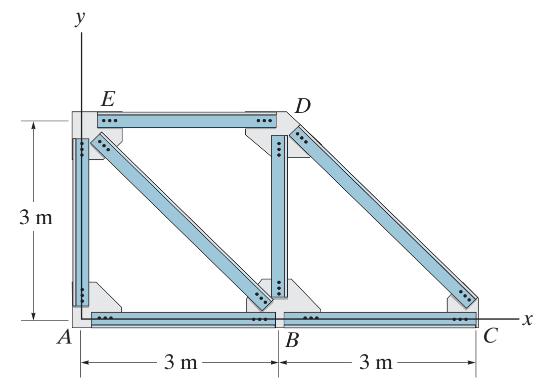
$$\Sigma \tilde{z} L = 1(2) - 0(\pi 2) + 0\left(\sqrt{4^2 + 2^2}\right) = 2 \text{ in}^2$$

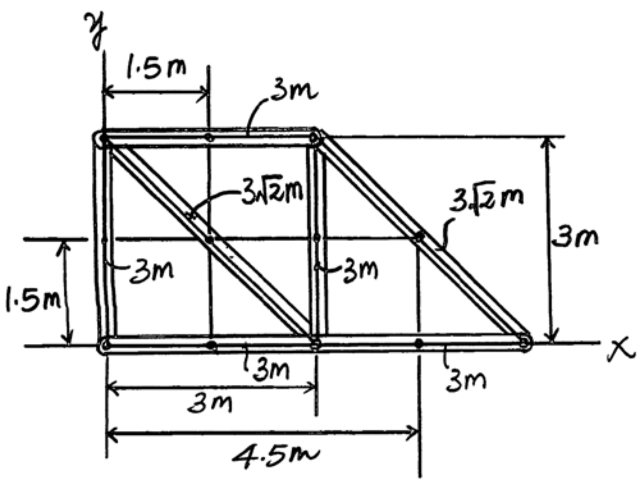
$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{0.94427}{12.7553} = 0.0740 \text{ in.} \qquad \text{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{0.47214}{12.7553} = 0.0370 \text{ in.} \qquad \text{Ans}$$

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{2}{12.7553} = 0.157 \text{ in.} \qquad \text{Ans}$$

***9–48.** The truss is made from seven members, each having a mass per unit length of 6 kg/m. Locate the position $(\bar{x}, \bar{y})$ of the center of mass. Neglect the mass of the gusset plates at the joints.





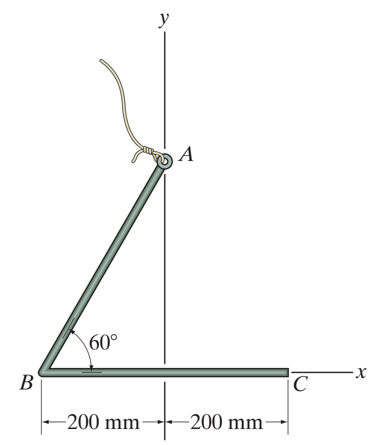
$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{(1.5)(3) + 4.5(3) + 4.5(3\sqrt{2}) + 1.5(3) + 3(3) + 1.5(3\sqrt{2})}{5(3) + 2(3\sqrt{2})}$$

$= 2.43$ m **Ans**

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{(1.5)(3) + 3(3) + 1.5(3) + 1.5(3\sqrt{2}) + 1.5(3\sqrt{2})}{5(3) + 2(3\sqrt{2})}$$

$= 1.31$ m **Ans**

•**9–49.** Locate the centroid $(\bar{x}, \bar{y})$ of the wire. If the wire is suspended from A, determine the angle segment AB makes with the vertical when the wire is in equilibrium.



Centroid: The length of segment AB is $l_{AB} = \frac{200}{\cos 60°} = 400$ mm. The centroid C_{AB} of this segment is located at $\tilde{x} = -\frac{400 \cos 60°}{2} = -100$ mm and $\tilde{y} = \frac{400 \sin 60°}{2} = 173.21$ mm as indicated in Fig. a. The centroid of segment BC is located at the origin of the coordinate axes.

$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{400(-100) + 400(0)}{400 + 400} = -50 \text{ mm} \qquad \textbf{Ans.}$$

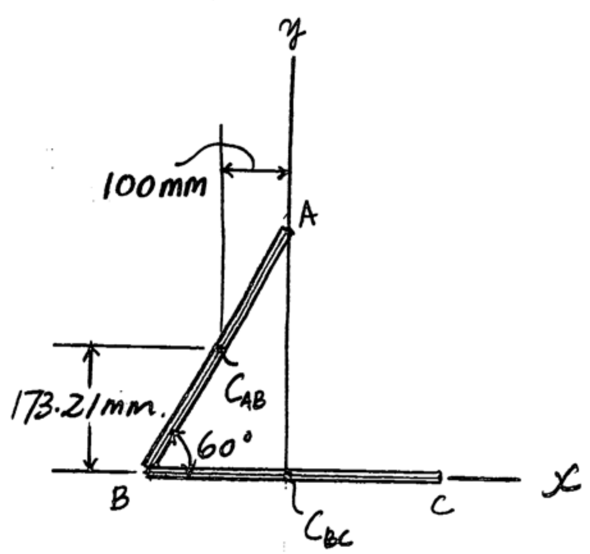
$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{400(173.21) + 400(0)}{400 + 400} = 86.60 \text{ mm} = 86.6 \text{ mm} \qquad \textbf{Ans.}$$

Geometry: When the bent wire hangs freely from A line AG will be vertical as shown in Fig. b. From the geometry of this figure, we have

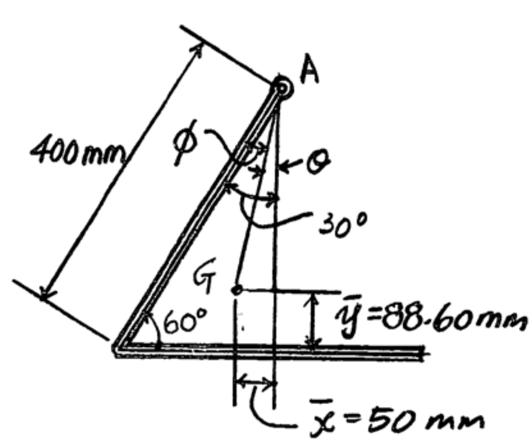
$$\tan\theta = \frac{50}{400 \sin 60° - 86.60} \qquad \theta = 10.89°$$

Thus, the angle ϕ that segment AB makes with the vertical is

$$\phi = 30° - 10.89° = 19.1° \qquad \textbf{Ans.}$$

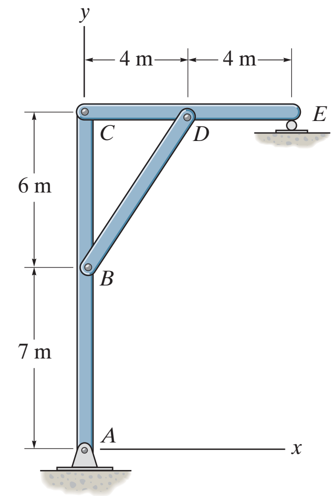


(a)



(b)

9–50. Each of the three members of the frame has a mass per unit length of 6 kg/m. Locate the position $(\bar{x}, \bar{y})$ of the center of mass. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the pin A and roller E.



Centroid : The length of each segment and its respective centroid are tabulated below.

Segment	L(m)	$\bar{x}$(m)	$\bar{y}$(m)	$\bar{x}L$(m²)	$\bar{y}L$(m²)
1	8	4	13	32.0	104.0
2	7.211	2	10	14.42	72.11
3	13	0	6.5	0	84.5
Σ	28.211			46.42	260.61

Thus,

$$\bar{x} = \frac{\Sigma \bar{x}L}{\Sigma L} = \frac{46.42}{28.211} = 1.646 \text{ m} = 1.65 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{260.61}{28.211} = 9.238 \text{ m} = 9.24 \text{ m} \qquad \textbf{Ans}$$

Equations of Equilibrium : The total weight of the frame is $W = 28.211(6)(9.81) = 1660.51$ N.

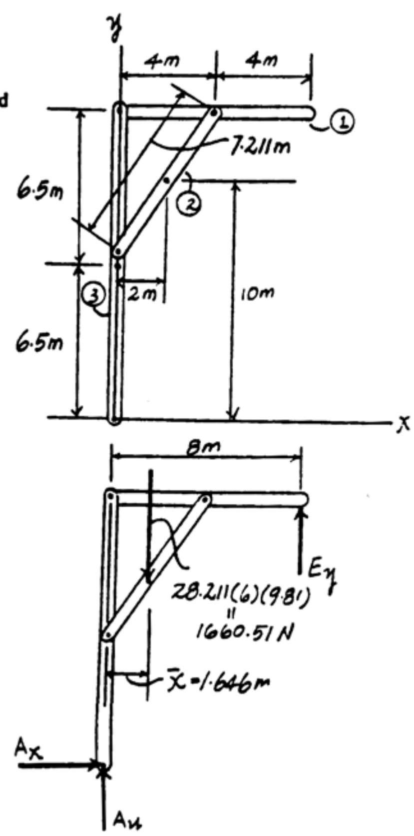
$$+\Sigma M_A = 0; \quad E_y(8) - 1660.51(1.646) = 0$$

$$E_y = 341.55 \text{ N} = 342 \text{ N} \qquad \textbf{Ans}$$

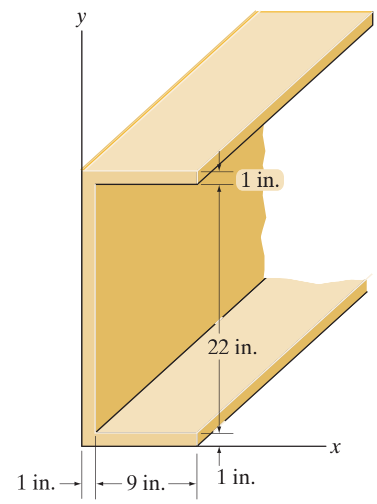
$$+\uparrow \Sigma F_y = 0; \quad A_y + 341.55 - 1660.51 = 0$$

$$A_y = 1318.95 \text{ N} = 1.32 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0 \qquad \textbf{Ans}$$



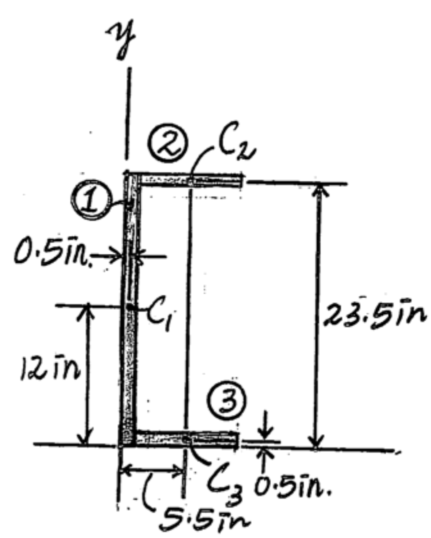
9–51. Locate the centroid $(\bar{x}, \bar{y})$ of the cross-sectional area of the channel.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

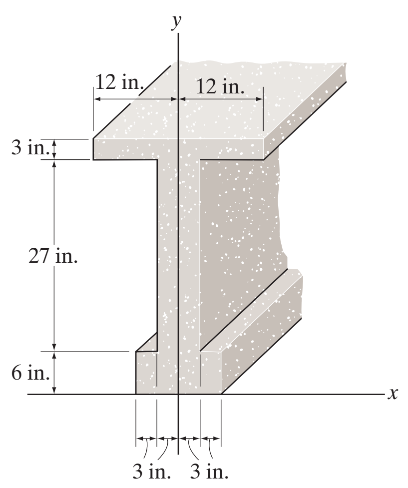
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.5(24(1)) + 5.5(9(1)) + 5.5(9(1))}{24(1) + 9(1) + 9(1)} = \frac{111}{42} = 2.64 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{12(24(1)) + 23.5(9(1)) + 0.5(9(1))}{24(1) + 9(1) + 9(1)} = \frac{504}{42} = 12 \text{ in.} \qquad \textbf{Ans.}$$



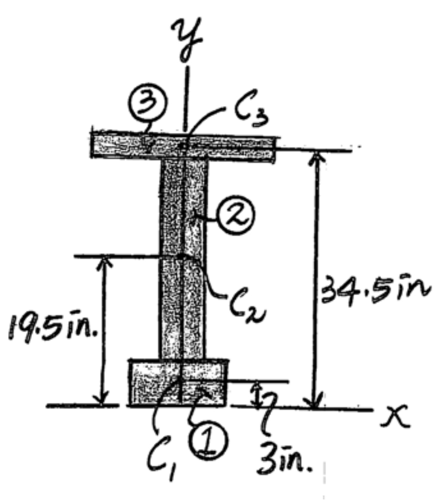
(a)

***9–52.** Locate the centroid $\bar{y}$ of the cross-sectional area of the concrete beam.



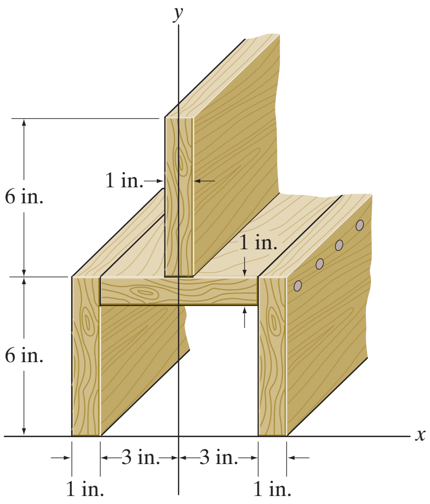
Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{3(12)(6) + 19.5(27)(6) + 34.5(24)(3)}{12(6) + 27(6) + 24(3)} = 19.1 \text{ in.} \qquad \textbf{Ans.}$$



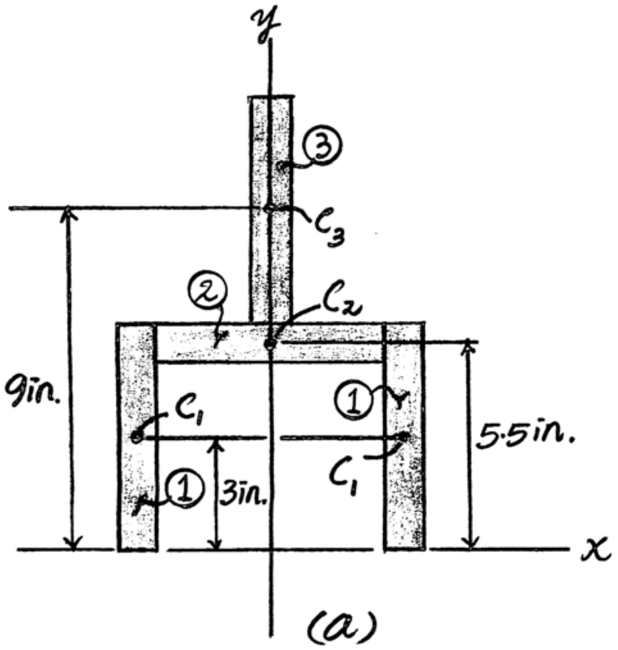
(a)

•**9–53.** Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.

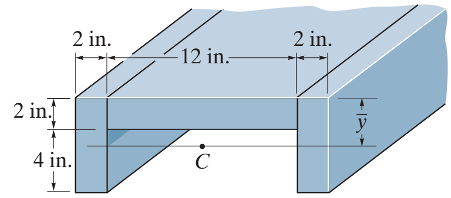


Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{3[2(6)(1)] + 5.5(6)(1) + 9(6)(1)}{2(6)(1) + 6(1) + 6(1)} = 5.125 \text{ in.} \qquad \textbf{Ans.}$$

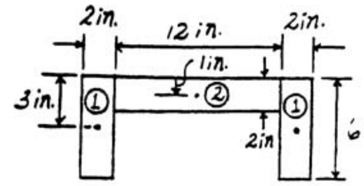


9–54. Locate the centroid $\bar{y}$ of the channel's cross-sectional area.



Centroid **: The area of each segment and its respective centroid are tabulated below.**

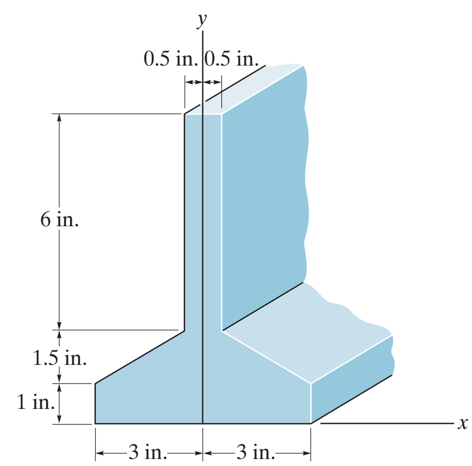
Segment	A (in^2)	$\tilde{y}$ (in.)	$\tilde{y}A$ (in^3)
1	6(4)	3	72.0
2	12(2)	1	24.0
Σ	48.0		96.0

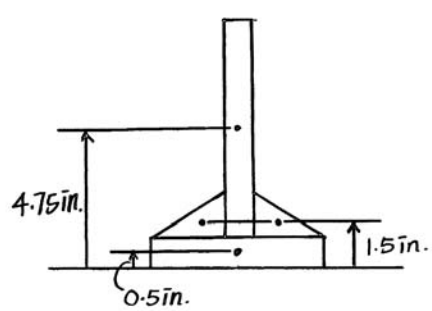


Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{96.0}{48.0} = 2.00 \text{ in.} \qquad \textbf{Ans}$$

9–55. Locate the distance $\bar{y}$ to the centroid of the member's cross-sectional area.





$$\Sigma\tilde{y}A = 0.5(6)(1) + 2(1.5)\left(\frac{1}{2}\right)(2.5)(1.5) + 4.75(7.5)(1)$$

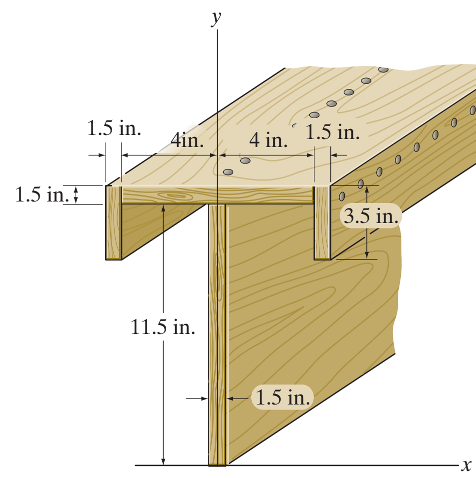
$$= 44.25 \text{ in}^3$$

$$\Sigma A = 6(1) + (2)\left(\frac{1}{2}\right)(2.5)(1.5) + 7.5(1)$$

$$= 17.25 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{44.25}{17.25} = 2.57 \text{ in.} \qquad \textbf{Ans}$$

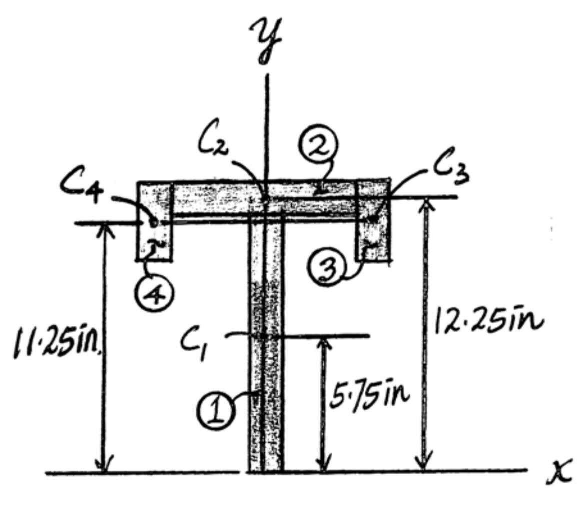
***9–56.** Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

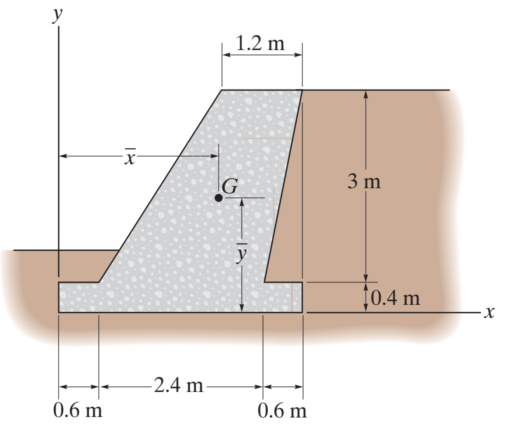
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{5.75(11.5)(1.5) + 12.25(8)(1.5) + 11.25(3.5)(1.5) + 11.25(3.5)(1.5)}{11.5(1.5) + 8(1.5) + 3.5(1.5) + 3.5(1.5)}$$

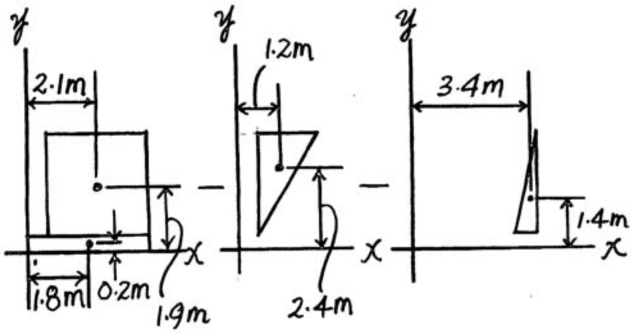
$= 9.17$ in. **Ans.**



(a)

•9–57. The gravity wall is made of concrete. Determine the location $(\bar{x}, \bar{y})$ of the center of mass G for the wall.





$$\Sigma \tilde{x} A = 1.8(3.6)(0.4) + 2.1(3)(3) - 3.4\left(\frac{1}{2}\right)(3)(0.6) - 1.2\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 15.192 \text{ m}^3$$

$$\Sigma \tilde{y} A = 0.2(3.6)(0.4) + 1.9(3)(3) - 1.4\left(\frac{1}{2}\right)(3)(0.6) - 2.4\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 9.648 \text{ m}^3$$

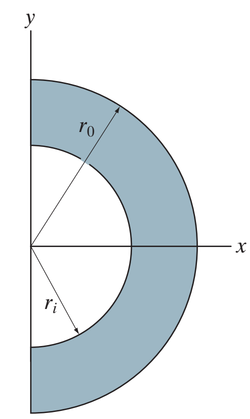
$$\Sigma A = 3.6(0.4) + 3(3) - \frac{1}{2}(3)(0.6) - \frac{1}{2}(1.8)(3)$$

$$= 6.84 \text{ m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{15.192}{6.84} = 2.22 \text{ m} \qquad \textbf{Ans}$$

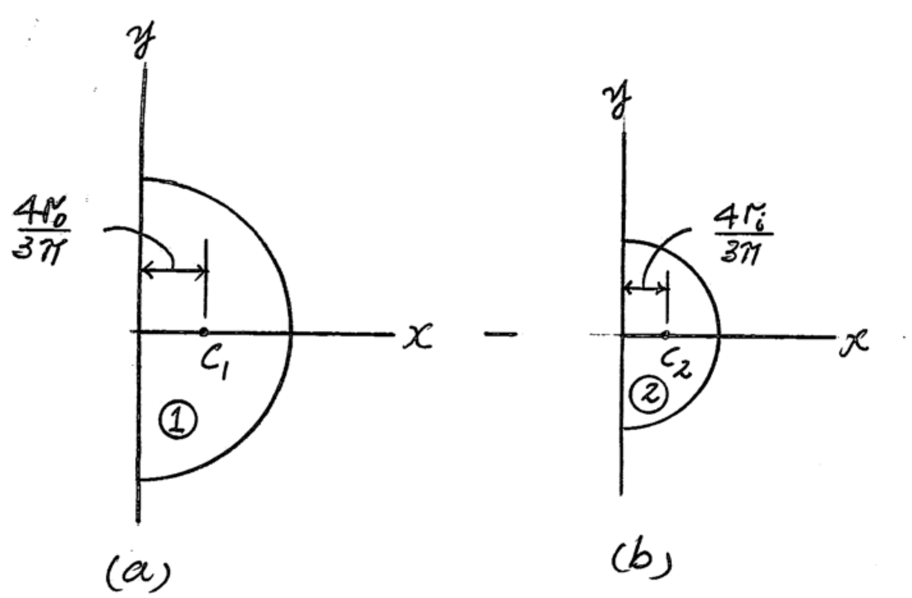
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{9.648}{6.84} = 1.41 \text{ m} \qquad \textbf{Ans}$$

9–58. Locate the centroid $\bar{x}$ of the composite area.

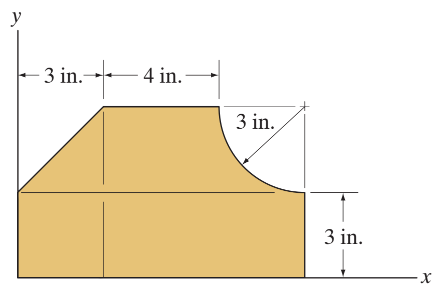


Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. Since segment (2) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\left(\frac{4r_o}{3\pi}\right)\left(\frac{\pi r_o^2}{2}\right) + \left(\frac{4r_i}{3\pi}\right)\left(-\frac{\pi r_i^2}{2}\right)}{\frac{\pi r_o^2}{2} + \left(-\frac{\pi r_i^2}{2}\right)} = \frac{4\left(r_o^3 - r_i^3\right)}{3\pi\left(r_o^2 - r_i^2\right)} \qquad \textbf{Ans.}$$



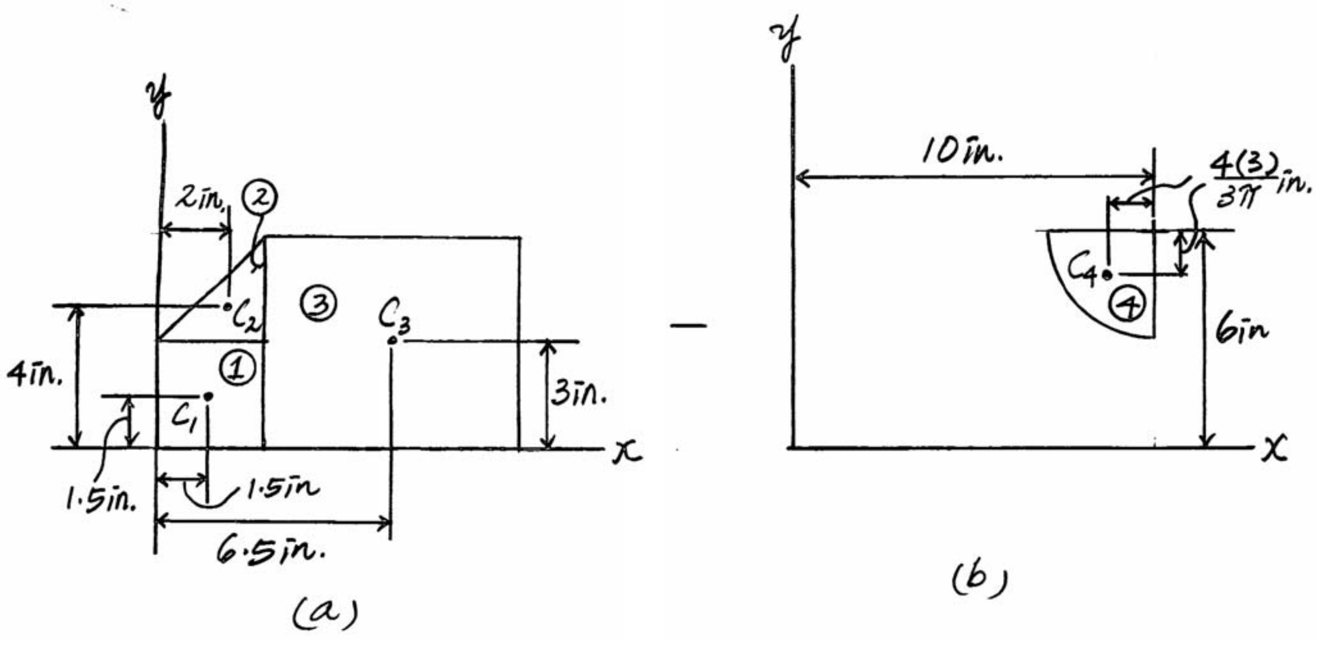
9–59. Locate the centroid $(\bar{x}, \bar{y})$ of the composite area.



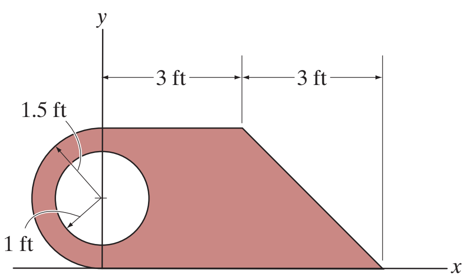
Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. since segment (4) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{1.5(3(3)) + 2\left(\frac{1}{2}(3)(3)\right) + 6.5(7(6)) + \left(10 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{233.81}{48.43} = 4.83 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 3(7(6)) + \left(6 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{124.09}{48.43} = 2.56 \text{ in.} \qquad \textbf{Ans.}$$



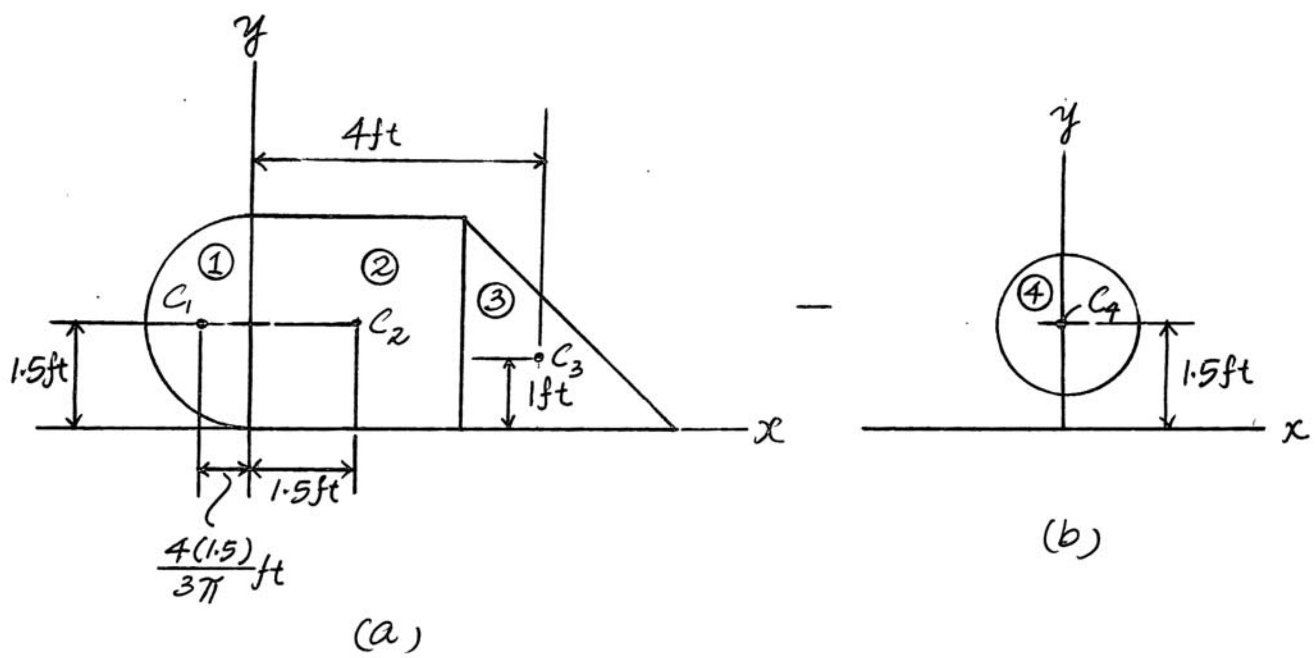
***9–60.** Locate the centroid $(\bar{x}, \bar{y})$ of the composite area.



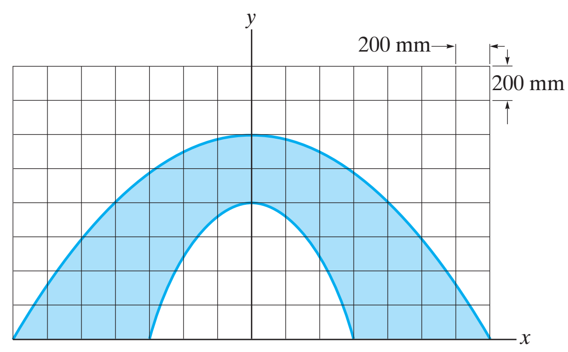
Centroid: The centroid of each composite segment is shown in Figs. *a* and *b*. Since segment (4) is a hole, its area should be considered negative.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\left(-\frac{4(1.5)}{3\pi}\right)\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 0\left(-\frac{\pi(1^2)}{4}\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\frac{\pi(1^2)}{4}\right)} = \frac{29.25}{13.89} = 2.11 \text{ ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 1\left(\frac{1}{2}(3)(3)\right) + 1.5\left(-\frac{\pi(1^2)}{4}\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\frac{\pi(1^2)}{4}\right)} = \frac{18.59}{13.89} = 1.34 \text{ ft} \qquad \textbf{Ans.}$$



•9–61. Divide the plate into parts, and using the grid for measurement, determine approximately the location $(\bar{x}, \bar{y})$ of the centroid of the plate.

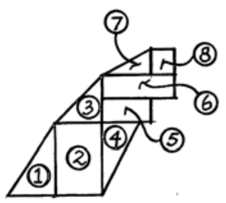


Due to symmetry.

$\bar{x} = 0$ **Ans**

Divide half the area into 8 segments as shown.

	A (Approx. 10^4)	$\tilde{y}$ (Approx. 10^2)	$\tilde{y}A$ (10^6)
1)	$\frac{1}{2}(6)(4)$	2	24
2)	4(6)	3	72
3)	$\frac{1}{2}(4)(4)$	7.32	58.56
4)	$\frac{1}{2}(3)(6)$	4	36
5)	8	7	56
6)	6(2)	9	108
7)	$\frac{1}{2}(4)(2)$	10.66	42.64
8)	(2)(2)	11	44



$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{441.2(10^6)}{81(10^4)} = 544 \text{ mm} \quad \textbf{Ans}$$

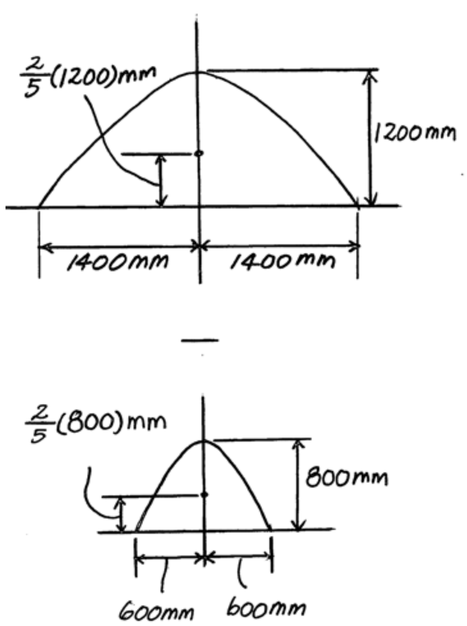
A simpler solution consists of dividing the area into two parabolas.

For parabola :

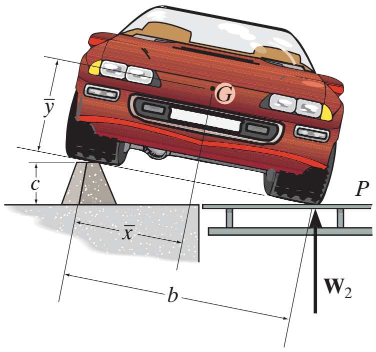
$$\Sigma \tilde{y}A = \frac{2}{5}(1200)\left(\frac{4}{3}\right)(2800)(1200) - \frac{2}{5}(800)\left(\frac{4}{3}\right)(1200)(800)$$

$$\Sigma A = \frac{4}{3}(2800)(1200) - \frac{4}{3}(1200)(800)$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = 544 \text{ mm} \quad \textbf{Ans}$$



9–62. To determine the location of the center of gravity of the automobile it is first placed in a *level position*, with the two wheels on one side resting on the scale platform P. In this position the scale records a reading of W_1. Then, one side is elevated to a convenient height c as shown. The new reading on the scale is W_2. If the automobile has a total weight of W, determine the location of its center of gravity $G(\bar{x}, \bar{y})$.



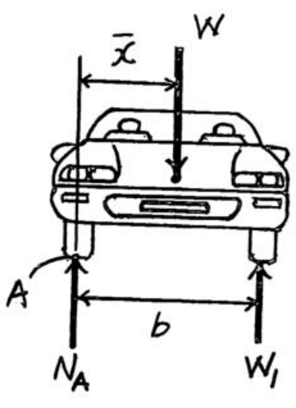
Equation of Equilibrium: First, we will consider the case in which the automobile is in a level position. Referring to the free-body diagram in Fig. a and writing the moment equation of equilibrium about point A,

$$\curvearrowleft +\Sigma M_A = 0; \qquad W_1(b) - W(\bar{x}) = 0 \qquad \bar{x} = \frac{W_1}{W}b \qquad \textbf{Ans.}$$

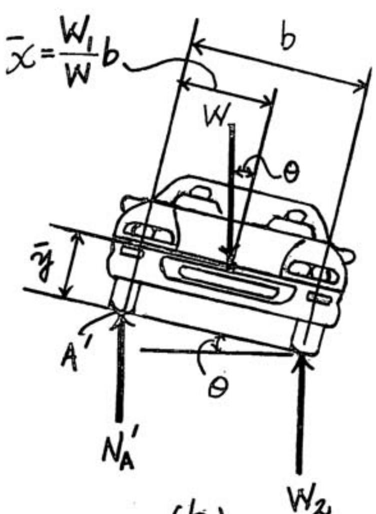
From the geometry in Fig. c, $\sin\theta = \dfrac{c}{b}$ and $\cos\theta = \dfrac{\sqrt{b^2 - c^2}}{b}$. Using the result of $\bar{x}$ and referring to the free-body diagram in Fig. b, we can write the moment equation of equilibrium about point A'.

$$\curvearrowleft +\Sigma M_{A'} = 0; \qquad W_2\left[b\left(\frac{\sqrt{b^2-c^2}}{b}\right)\right] - W\left(\frac{\sqrt{b^2-c^2}}{b}\right)\left(\frac{W_1}{W}b\right) - W\left(\frac{c}{b}\right)\bar{y} = 0$$

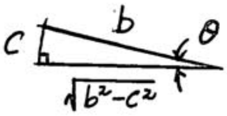
$$\bar{y} = \frac{b(W_2 - W_1)\sqrt{b^2-c^2}}{cW} \qquad \textbf{Ans.}$$



(a)

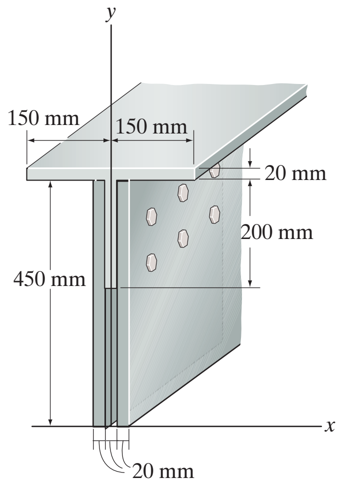


(b)



(c)

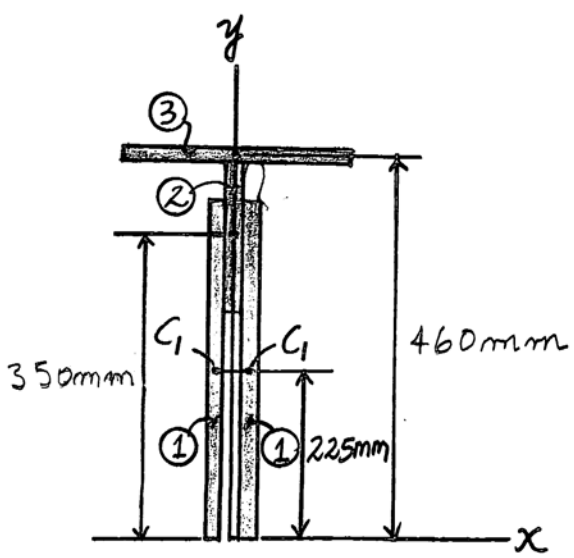
9–63. Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

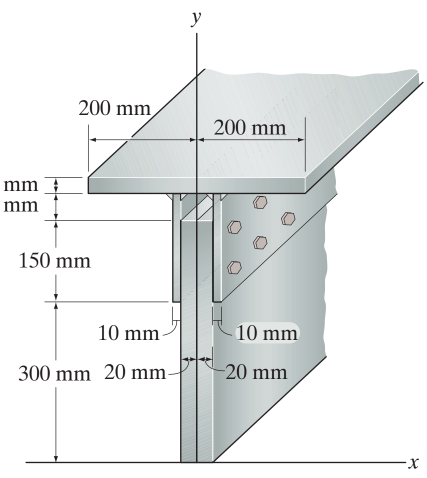
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[225(450)(20)] + 350(200)(20) + 460(300)(20)}{2(450)(20) + 200(20) + 300(20)}$$

$$= 293 \text{ mm}$$ **Ans.**



(a)

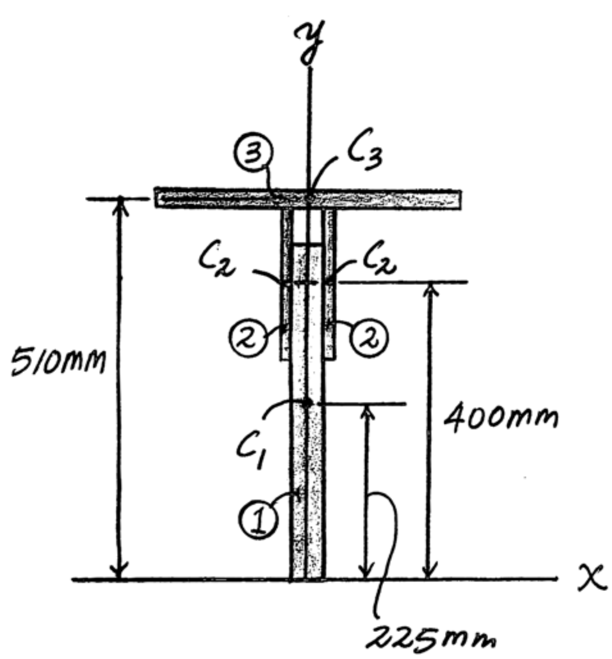
***9–64.** Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.



Centroid: The centroid of each composite segment is shown in Fig. *a*.

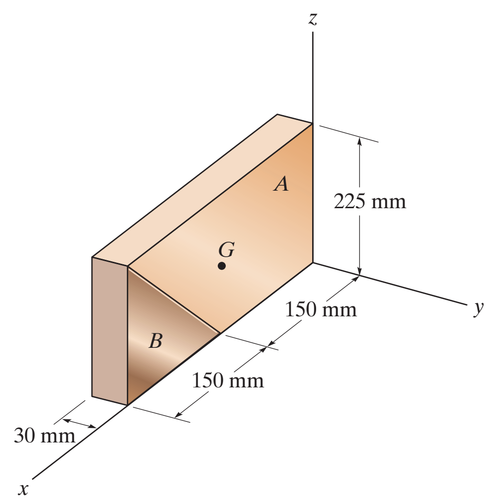
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{225(450)(40) + 2[400(200)(10)] + 510(400)(20)}{450(40) + 2(200)(10) + 400(20)}$$

$= 324$ mm **Ans.**



(a)

•**9–65.** The composite plate is made from both steel (A) and brass (B) segments. Determine the mass and location $(\bar{x}, \bar{y}, \bar{z})$ of its mass center G. Take $\rho_{st} = 7.85\ \text{Mg/m}^3$ and $\rho_{br} = 8.74\ \text{Mg/m}^3$.



$$\Sigma m = \Sigma\rho V = \left[8.74\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right] + \left[7.85\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right]$$

$$+ [7.85(0.15)(0.225)(0.03)]$$

$$= \left[4.4246(10^{-3})\right] + \left[3.9741(10^{-3})\right] + \left[7.9481(10^{-3})\right]$$

$$= 16.347(10^{-3}) = 16.4\ \text{kg} \qquad \textbf{Ans}$$

$$\Sigma\tilde{x}m = \left(0.150 + \frac{2}{3}(0.150)\right)(4.4246)(10^{-3}) + \left(0.150 + \frac{1}{3}(0.150)\right)(3.9741)(10^{-3})$$

$$+ \frac{1}{2}(0.150)(7.9481)(10^{-3}) = 2.4971(10^{-3})\text{kg}\cdot\text{m}$$

$$\Sigma\tilde{z}m = \left(\frac{1}{3}(0.225)\right)(4.4246)(10^{-3}) + \left(\frac{2}{3}(0.225)\right)(3.9471)(10^{-3}) + \left(\frac{0.225}{2}\right)(7.9481)(10^{-3})$$

$$= 1.8221(10^{-3})\text{kg}\cdot\text{m}$$

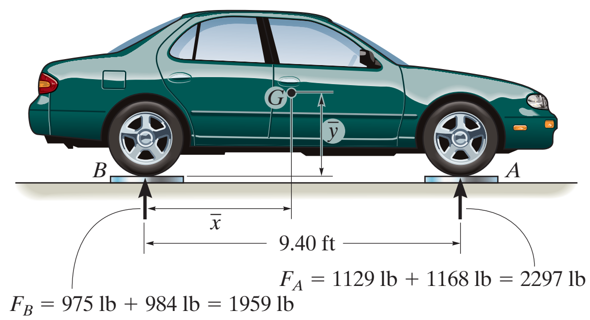
$$\bar{x} = \frac{\Sigma\tilde{x}m}{\Sigma m} = \frac{2.4971(10^{-3})}{16.347(10^{-3})} = 0.153\ \text{m} = 153\ \text{mm} \qquad \textbf{Ans}$$

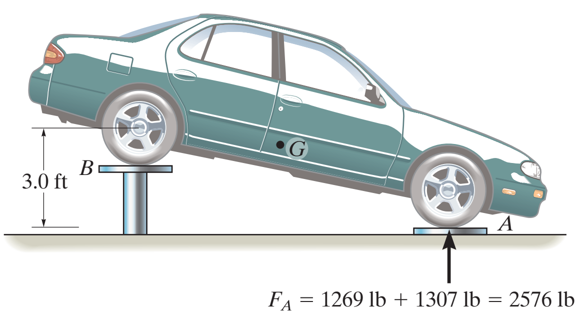
Due to symmetry :

$$\bar{y} = -15\ \text{mm} \qquad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma\tilde{z}m}{\Sigma m} = \frac{1.8221(10^{-3})}{16.347(10^{-3})} = 0.1115\ \text{m} = 111\ \text{mm} \qquad \textbf{Ans}$$

9–66. The car rests on four scales and in this position the scale readings of both the front and rear tires are shown by F_A and F_B. When the rear wheels are elevated to a height of 3 ft above the front scales, the new readings of the front wheels are also recorded. Use this data to compute the location $\bar{x}$ and $\bar{y}$ to the center of gravity G of the car. The tires each have a diameter of 1.98 ft.





In horizontal position

$W = 1959 + 2297 = 4256$ lb

$\curvearrowleft + \Sigma M_B = 0; \quad 2297(9.40) - 4256\,\bar{x} = 0$

$\bar{x} = 5.0733 = 5.07$ ft **Ans**

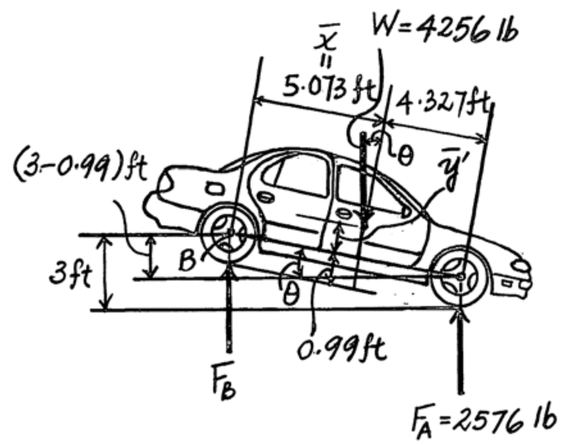
$\theta = \sin^{-1}\left(\dfrac{3 - 0.990}{9.40}\right) = 12.347°$

$\curvearrowleft + \Sigma M_B = 0; \quad 2576(9.40 \cos 12.347°) - 4256 \cos 12.347°(5.0733)$

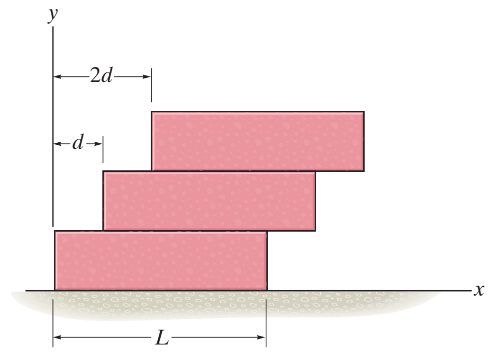
$- 4256 \sin 12.347°\,\bar{y}' = 0$

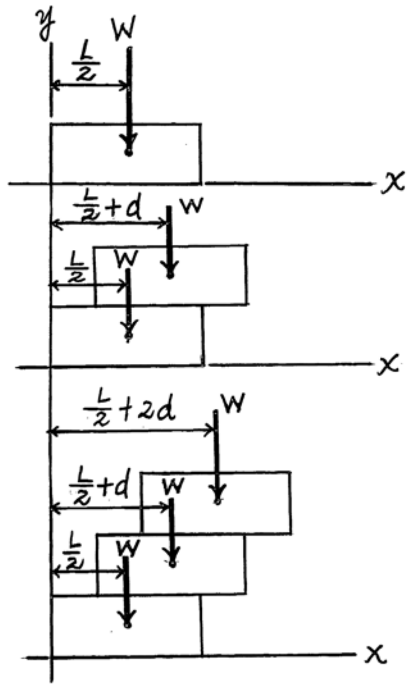
$\bar{y}' = 2.815$ ft

$\bar{y} = 2.815 + 0.990 = 3.81$ ft **Ans**



9–67. Uniform blocks having a length L and mass m are stacked one on top of the other, with each block overhanging the other by a distance d, as shown. If the blocks are glued together, so that they will not topple over, determine the location $\bar{x}$ of the center of mass of a pile of n blocks.





$n = 1: \quad \bar{x} = \frac{L}{2} = \frac{L}{2} + 0\left(\frac{d}{2}\right)$

$n = 2: \quad \bar{x} = \dfrac{\frac{L}{2}(W) + \left(\frac{L}{2} + d\right)W}{2W}$

$= \frac{L}{4} + \frac{L}{4} + \frac{d}{2} = \frac{L}{2} + (1)\frac{d}{2}$

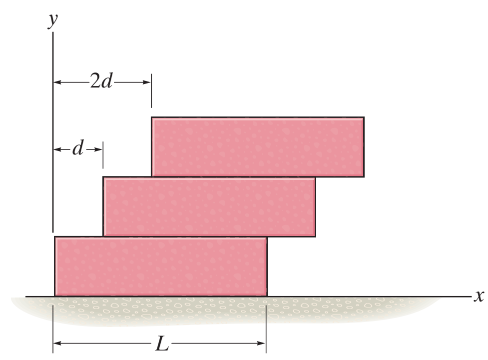
$n = 3: \quad \bar{x} = \dfrac{\frac{L}{2}(W) + \left(\frac{L}{2} + d\right)W + \left(\frac{L}{2} + 2d\right)W}{3W}$

$= \frac{L}{6} + \frac{L}{6} + \frac{d}{3} + \frac{L}{6} + \frac{2}{3}d = \frac{L}{2} + 2\left(\frac{d}{2}\right)$

In general :

$\bar{x} = \frac{L}{2} + (n-1)\left(\frac{d}{2}\right) = \frac{L + (n-1)d}{2}$ **Ans**

***9–68.** Uniform blocks having a length L and mass m are stacked one on top of the other, with each block overhanging the other by a distance d, as shown. Show that the maximum number of blocks which can be stacked in this manner is $n < L/d$.



$n = 2$: $\quad \bar{x} = \frac{L}{2} + d = \frac{L}{2} + 2\left(\frac{d}{2}\right)$

$n = 3$: $\quad \bar{x} = \dfrac{\left(d + \frac{L}{2}\right)W + \left(2d + \frac{L}{2}\right)W}{2W} = \frac{L}{2} + 3\left(\frac{d}{2}\right)$

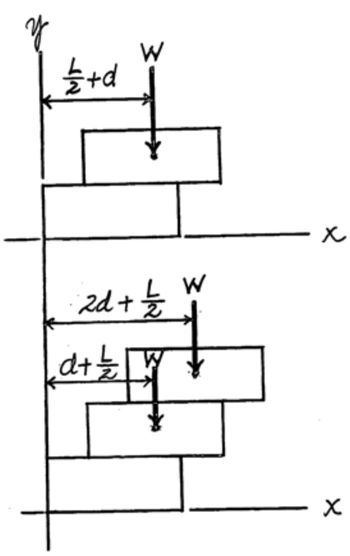
In general :

$$\bar{x} = \frac{L}{2} + n\left(\frac{d}{2}\right)$$

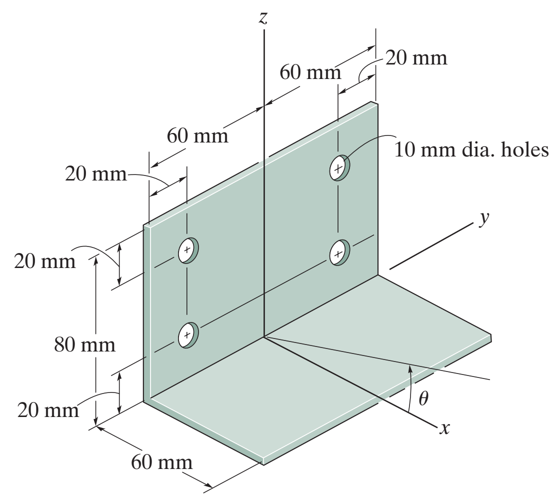
For stable stack :

$$\bar{x} = \frac{L}{2} + n\left(\frac{d}{2}\right) \le L$$

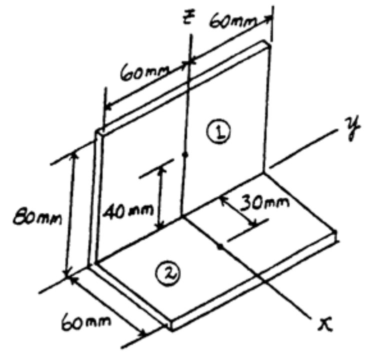
$$n \le \frac{L}{d} \qquad \textbf{Ans}$$



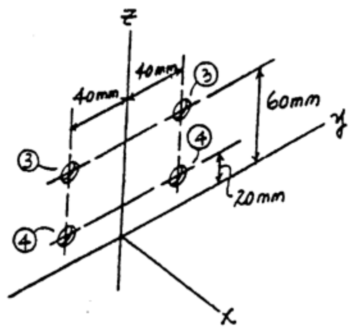
•9–69. Locate the center of gravity ($\bar{x}$, $\bar{z}$) of the sheet-metal bracket if the material is homogeneous and has a constant thickness. If the bracket is resting on the horizontal x–y plane shown, determine the maximum angle of tilt θ which it can have before it falls over, i.e., begins to rotate about the y axis.



Centroid : The area of each segment and its respective centroid are tabulated below.



Segment	A (mm²)	$\tilde{x}$ (mm)	$\tilde{z}$ (mm)	$\tilde{x}A$ (mm³)	$\tilde{z}A$ (mm³)
1	120(80)	0	40	0	384 000
2	120(60)	30	0	216 000	0
3	$-2\left[\frac{\pi}{4}(10^2)\right]$	0	60	0	−9424.78
4	$-2\left[\frac{\pi}{4}(10^2)\right]$	0	20	0	−3141.59
Σ	16 485.84			216 000	371 433.63



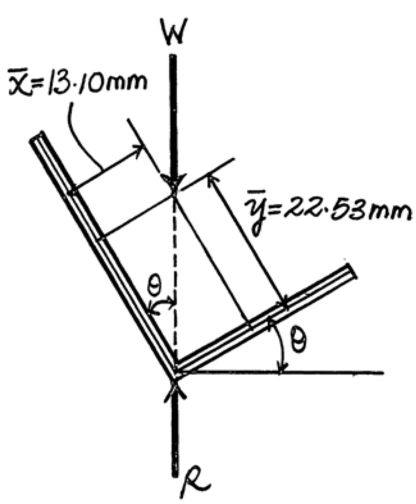
Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{216\,000}{16\,485.84} = 13.10 \text{ mm} = 13.1 \text{ mm} \qquad \textbf{Ans}$$

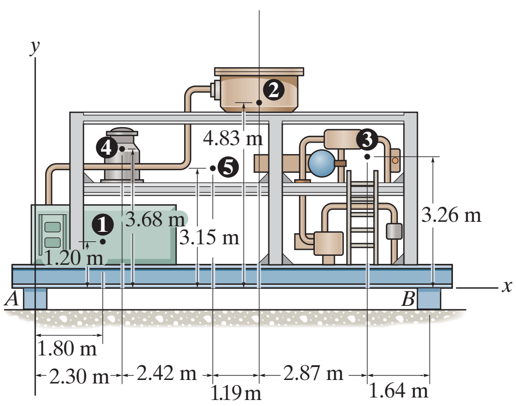
$$\bar{z} = \frac{\Sigma\tilde{z}A}{\Sigma A} = \frac{371\,433.63}{16\,485.84} = 22.53 \text{ mm} = 22.5 \text{ mm} \qquad \textbf{Ans}$$

Equilibrium : In order for the bracket not to rotate about y axis, the weight of the bracket must coincide with the reaction. From the FBD,

$$\theta = \tan^{-1}\frac{13.10}{22.53} = 30.2° \qquad \textbf{Ans}$$



9–70. Locate the center of mass for the compressor assembly. The locations of the centers of mass of the various components and their masses are indicated and tabulated in the figure. What are the vertical reactions at blocks A and B needed to support the platform?



1 Instrument panel 230 kg
2 Filter system 183 kg
3 Piping assembly 120 kg
4 Liquid storage 85 kg
5 Structural framework 468 kg

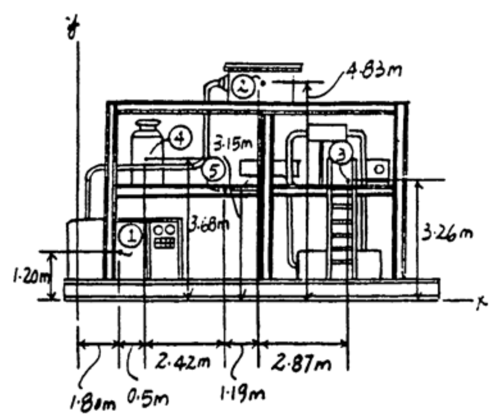
Centroid : The mass of each component of the compressor and its respective centroid are tabulated below.

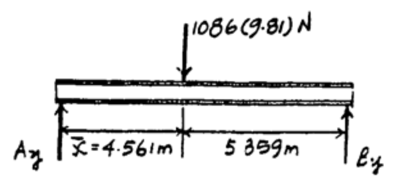
Component	m (kg)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}m$ (kg · m)	$\bar{y}m$ (kg · m)
1	230	1.80	1.20	414.00	276.00
2	183	5.91	4.83	1081.53	883.89
3	120	8.78	3.26	1053.60	391.20
4	85	2.30	3.68	195.50	312.80
5	468	4.72	3.15	2208.96	1474.20
Σ	1086			4953.59	3338.09

Thus,

$$\bar{x} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{4953.59}{1086} = 4.561 \text{ m} = 4.56 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y} m}{\Sigma m} = \frac{3338.09}{1086} = 3.074 \text{ m} = 3.07 \text{ m} \qquad \textbf{Ans}$$



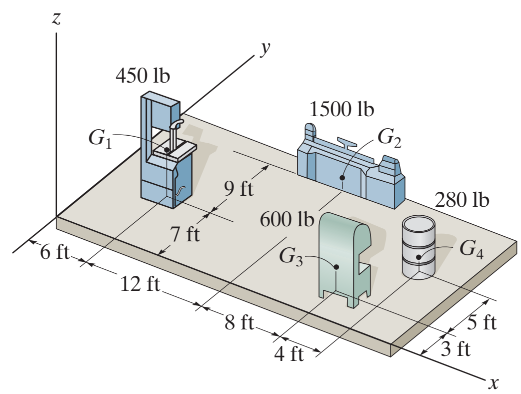


Equations of Equilibrium :

$$+\Sigma M_A = 0; \quad B_y(10.42) - 1086(9.81)(4.561) = 0$$
$$B_y = 4663.60 \text{ N} = 4.66 \text{ kN} \qquad \textbf{Ans}$$

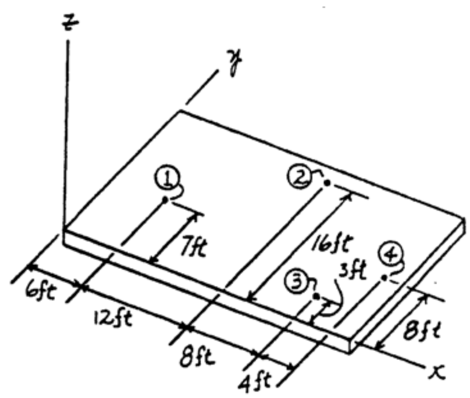
$$+\uparrow \Sigma F_y = 0; \quad A_y + 4663.60 - 1086(9.81) = 0$$
$$A_y = 5990.06 \text{ N} = 5.99 \text{ kN} \qquad \textbf{Ans}$$

9–71. Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity G. Locate the center of gravity $(\bar{x}, \bar{y})$ of all these components.



Centroid : The floor loadings on the floor and its respective centroid are tabulated below.

Loading	W (lb)	$\bar{x}$ (ft)	$\bar{y}$ (ft)	$\bar{x}W$ (lb·ft)	$\bar{y}W$ (lb·ft)
1	450	6	7	2700	3150
2	1500	18	16	27000	24000
3	600	26	3	15600	1800
4	280	30	8	8400	2240
Σ	2830			53700	31190

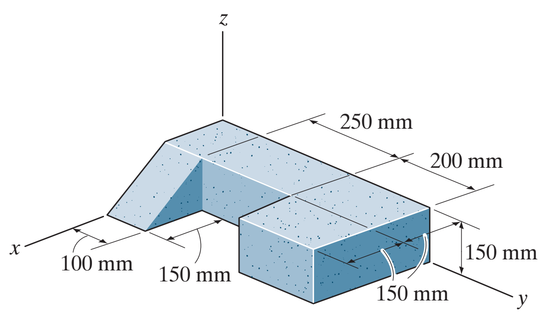


Thus,

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma W} = \frac{53700}{2830} = 18.98 \text{ ft} = 19.0 \text{ ft} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y} W}{\Sigma W} = \frac{31190}{2830} = 11.02 \text{ ft} = 11.0 \text{ ft} \qquad \textbf{Ans}$$

***9–72.** Locate the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly.

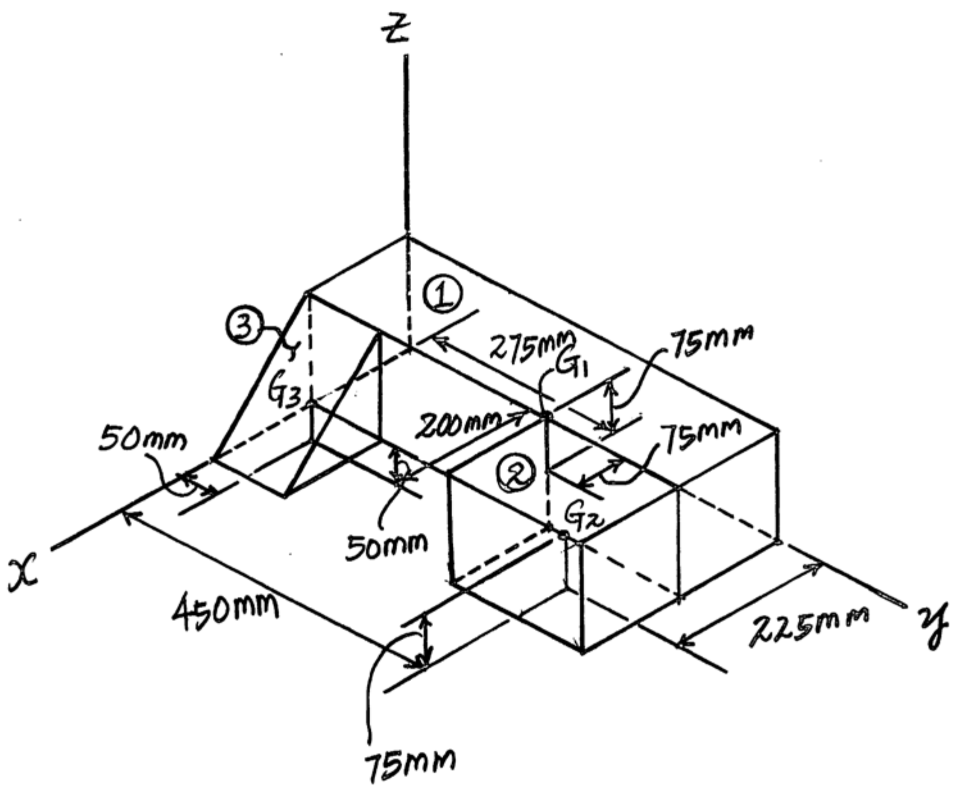


Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{(75)(150)(150)(550) + (225)(150)(150)(200) + (200)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{2.165625(10^9)}{18(10^6)} = 120 \text{ mm}$$ **Ans.**

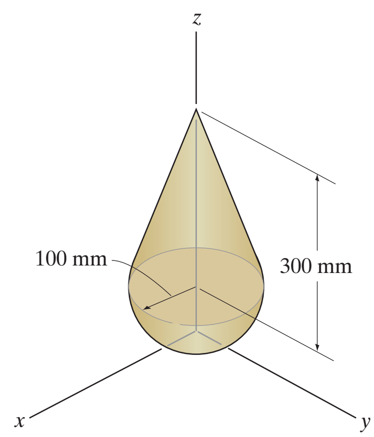
$$\bar{y} = \frac{\Sigma \tilde{y} V}{\Sigma V} = \frac{(275)(150)(150)(550) + (450)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{5.484375(10^9)}{18(10^6)} = 305 \text{ mm}$$ **Ans.**

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{(75)(150)(150)(550) + (75)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{1.321875(10^9)}{18(10^6)} = 73.4 \text{ mm}$$ **Ans.**



(a)

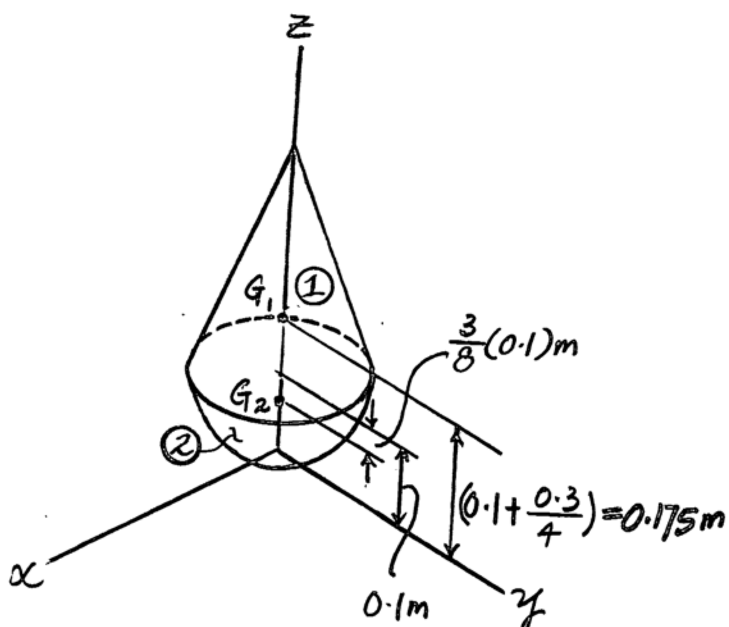
•9–73. Locate the center of mass $\bar{z}$ of the assembly. The hemisphere and the cone are made from materials having densities of 8 $\mathrm{Mg/m^3}$ and 4 $\mathrm{Mg/m^3}$, respectively.



Centroid: The center of mass of each composite segment is shown in Fig. *a*.

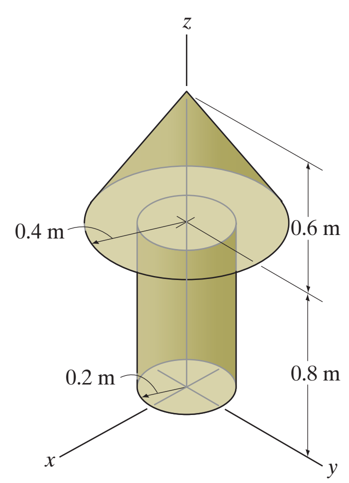
$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{4000(0.175)\left[\frac{1}{3}\pi(0.1^2)(0.3)\right] + 8000\left(0.1 - \frac{3}{8}(0.1)\right)\left[\frac{2}{3}\pi(0.1^3)\right]}{4000\left[\frac{1}{3}\pi(0.1^2)(0.3)\right] + 8000\left[\frac{2}{3}\pi(0.1^3)\right]}$$

$$= \frac{1.0333\pi}{9.3333\pi} = 0.1107\text{ m} = 111\text{ mm} \qquad \textbf{Ans.}$$



(a)

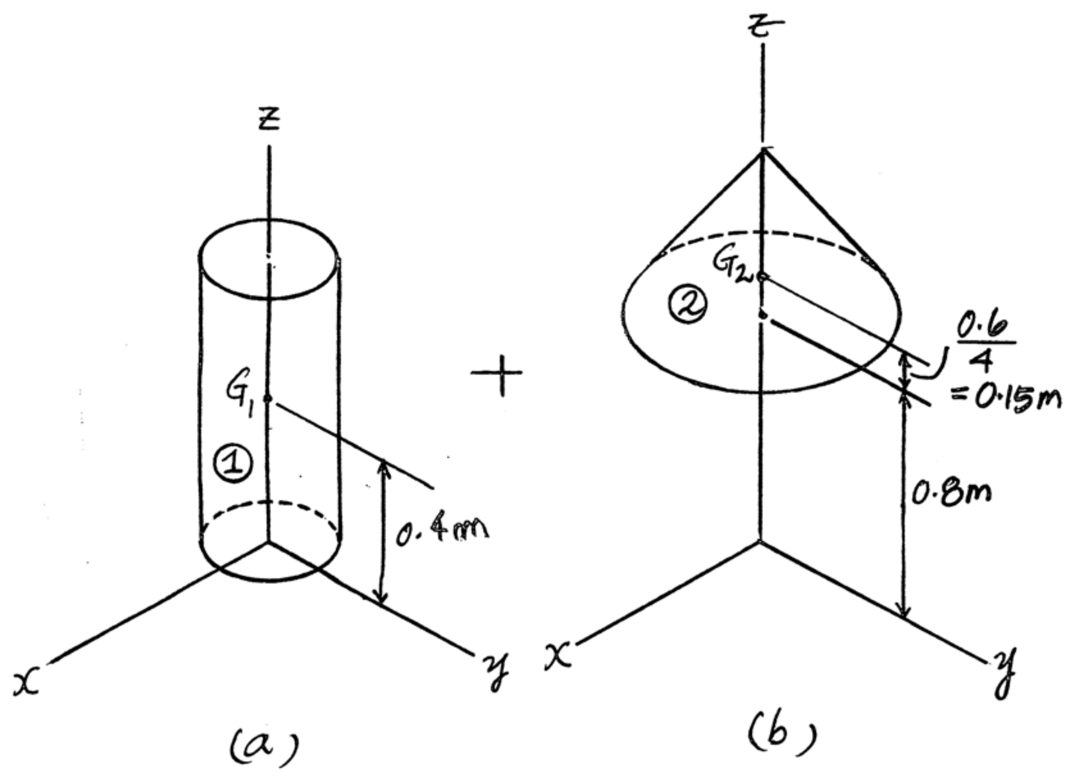
9–74. Locate the center of mass $\bar{z}$ of the assembly. The cylinder and the cone are made from materials having densities of 5 Mg/m^3 and 9 Mg/m^3, respectively.



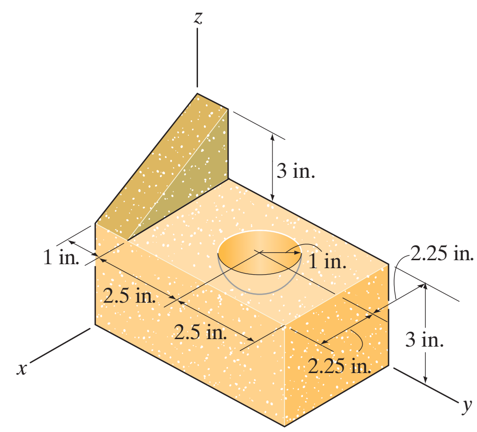
Center of mass: The assembly is broken into two composite segments, as shown in Figs. *a* and *b*.

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{5000(0.4)\left[\pi(0.2^2)(0.8)\right] + 9000(0.8 + 0.15)\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}{5000\left[\pi(0.2^2)(0.8)\right] + 9000\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}$$

$$= \frac{1060.60}{1407.4} = 0.754 \text{ m} = 754 \text{ mm} \qquad \textbf{Ans.}$$



9–75. Locate the center of gravity $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly having a hemispherical hole.

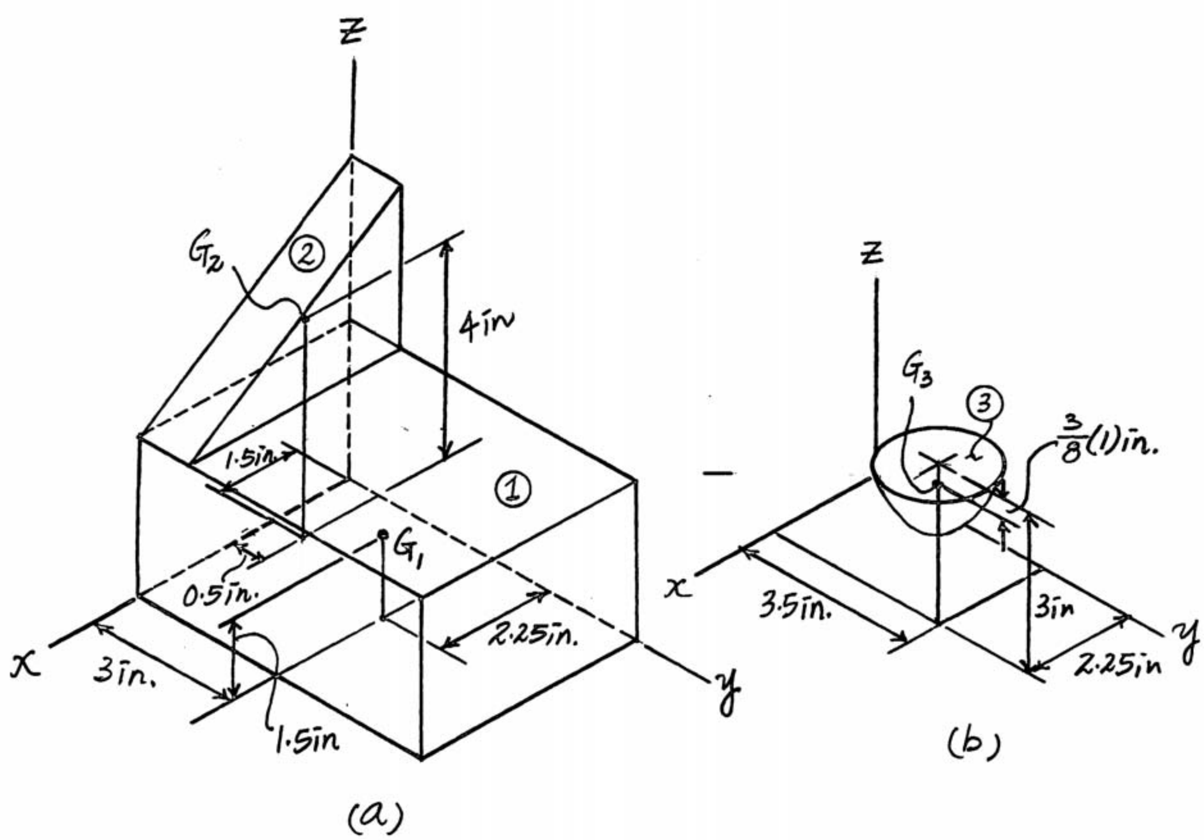


Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Figs. *a* and *b*. Since segment (3) is a hole, its volume should be considered negative.

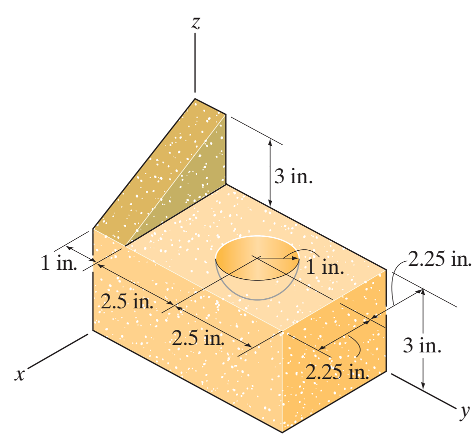
$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{2.25(3)(4.5)(6) + (1.5)\left(\frac{1}{2}\right)(3)(4.5)(1) + 2.25\left(-\frac{2}{3}\pi(1^3)\right)}{(3)(4.5)(6) + \frac{1}{2}(3)(4.5)(1) + \left(-\frac{2}{3}\pi(1^3)\right)} = \frac{187.663}{85.656} = 2.19 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} V}{\Sigma V} = \frac{3(3)(4.5)(6) + (0.5)\left(\frac{1}{2}\right)(3)(4.5)(1) + 3.5\left(-\frac{2}{3}\pi(1^3)\right)}{(3)(4.5)(6) + \frac{1}{2}(3)(4.5)(1) + \left(-\frac{2}{3}\pi(1^3)\right)} = \frac{239.045}{85.656} = 2.79 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{1.5(3)(4.5)(6) + (4)\left(\frac{1}{2}\right)(3)(4.5)(1) + \left(3 - \frac{3}{8}(1)\right)\left(-\frac{2}{3}\pi(1^3)\right)}{(3)(4.5)(6) + \frac{1}{2}(3)(4.5)(1) + \left(-\frac{2}{3}\pi(1^3)\right)} = \frac{143.002}{85.656} = 1.67 \text{ in.} \qquad \textbf{Ans.}$$



***9–76.** Locate the center of gravity $(\bar{x}, \bar{y}, \bar{z})$ of the assembly. The triangular and the rectangular blocks are made from materials having specific weights of 0.25 lb/in^3 and 0.1 lb/in^3, respectively.

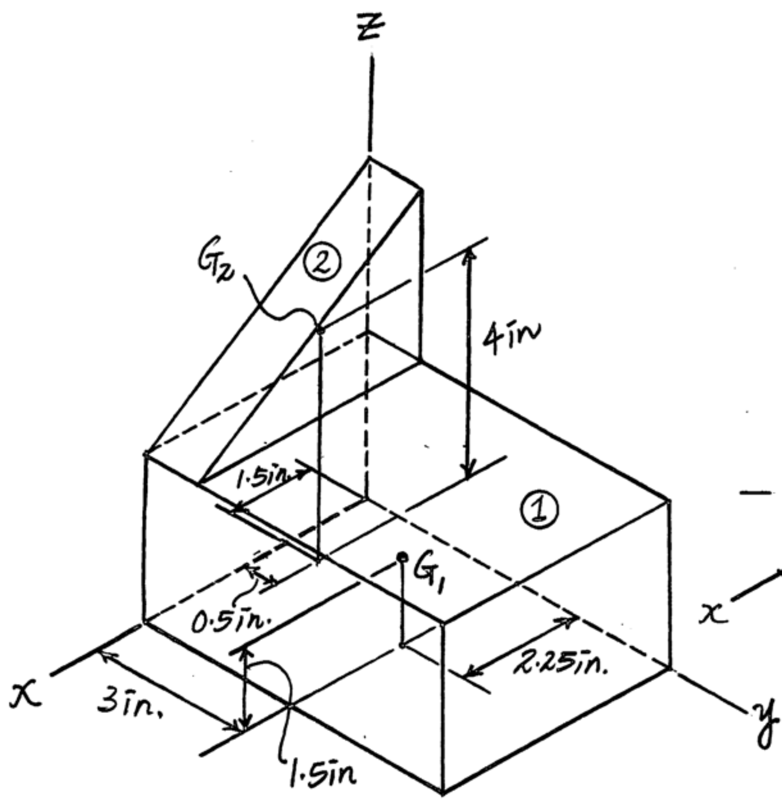


Center of Gravity: The center of gravity for each composite segment is shown in Figs. *a* and *b*. Since segment (3) is a hole, its weight should be considered negative.

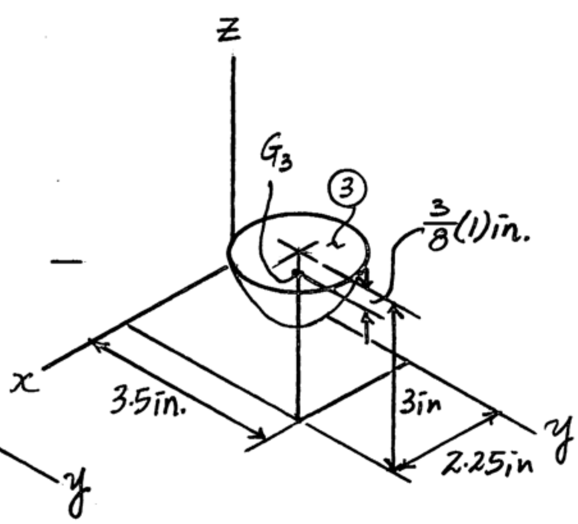
$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{(2.25)(0.1)(3)(4.5)(6) + 0.25(1.5)\left[\frac{1}{2}(3)(4.5)(1)\right] + 2.25\left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]}{(0.1)(3)(4.5)(6) + 0.25\left[\frac{1}{2}(3)(4.5)(1)\right] + \left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]} = \frac{20.2850}{9.5781} = 2.12 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{3(0.1)(3)(4.5)(6) + 0.25(0.5)\left[\frac{1}{2}(3)(4.5)(1)\right] + 3.5\left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]}{(0.1)(3)(4.5)(6) + 0.25\left[\frac{1}{2}(3)(4.5)(1)\right] + \left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]} = \frac{24.4107}{9.5781} = 2.55 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W} = \frac{1.5(0.1)(3)(4.5)(6) + 0.25(4)\left[\frac{1}{2}(3)(4.5)(1)\right] + \left(3 - \frac{3}{8}(1)\right)\left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]}{(0.1)(3)(4.5)(6) + 0.25\left[\frac{1}{2}(3)(4.5)(1)\right] + \left[-0.1\left(\frac{2}{3}\pi(1^3)\right)\right]} = \frac{18.3502}{9.5781} = 1.92 \text{ in.} \qquad \textbf{Ans.}$$

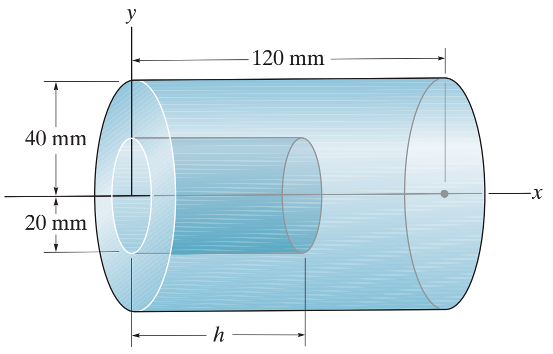


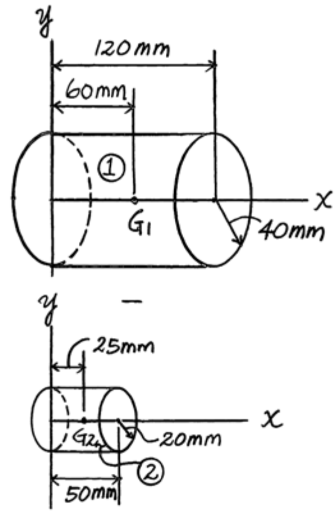
(a)



(b)

•9–77. Determine the distance $\bar{x}$ to the centroid of the solid which consists of a cylinder with a hole of length $h = 50$ mm bored into its base.



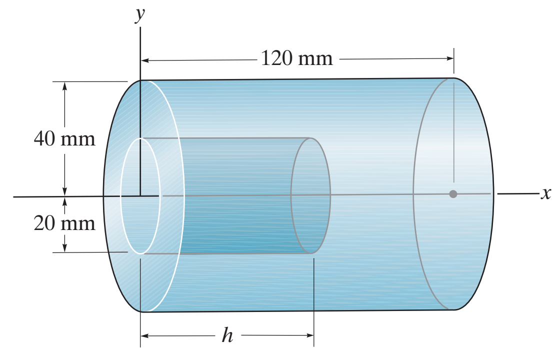


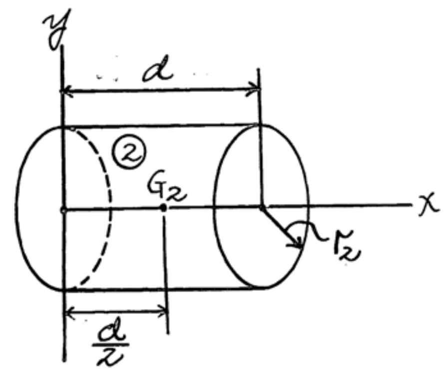
$$\Sigma V = \pi(40)^2(120) - \pi(20)^2(50) = 172\left(10^3\right)\pi \text{ mm}^3$$

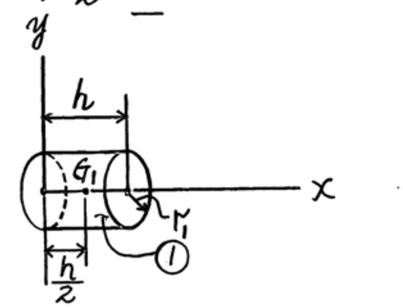
$$\Sigma \tilde{x} V = 60(\pi)(40)^2(120) - 25(\pi)(20)^2(50) = 11.02\left(10^6\right)\pi \text{ mm}^4$$

$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{11.02(10^6)\pi}{172(10^3)\pi} = 64.1 \text{ mm} \qquad \textbf{Ans}$$

9–78. Determine the distance h to which a hole must be bored into the cylinder so that the center of mass of the assembly is located at $\bar{x} = 64$ mm. The material has a density of 8 Mg/m^3.







$$\Sigma V = \pi r_2^2 d - \pi r_1^2 h$$

$$\Sigma \tilde{x} V = \frac{d}{2}(\pi)\left(r_2^2\right) d - \frac{h}{2}(\pi)\left(r_1^2\right) h$$

$$\bar{x} = \frac{\Sigma \tilde{x} V}{\Sigma V} = \frac{\frac{d^2}{2}(\pi)\left(r_2^2\right) - \frac{h^2}{2}(\pi)\left(r_1^2\right)}{\pi r_2^2 d - \pi r_1^2 h}$$

$$2\bar{x}\left(r_2^2 d\right) - 2\bar{x}\left(r_1^2 h\right) = d^2 r_2^2 - h^2 r_1^2$$

$$h^2 - 2\bar{x}h + d\,(2\bar{x} - d)\left(\frac{r_2}{r_1}\right)^2 = 0$$

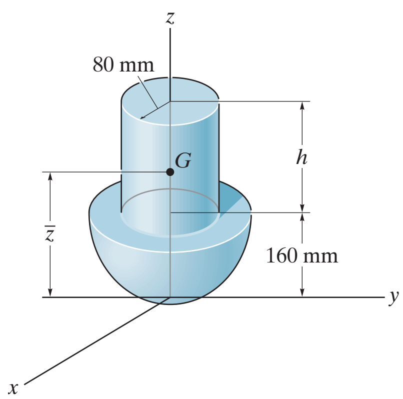
Set $\bar{x} = 64$ mm, $r_2 = 40$ mm, $r_1 = 20$ mm, $d = 120$ mm

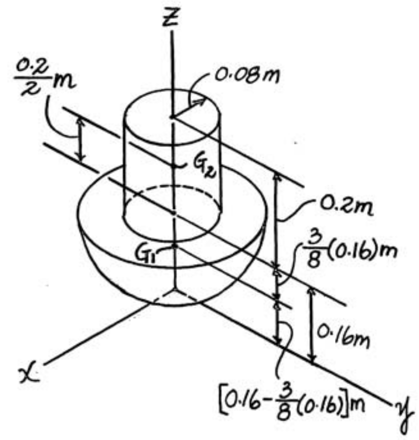
$$h^2 - 128h + 3840 = 0$$

Solving,

$h = 80$ mm **Ans** or $h = 48$ mm **Ans**

9–79. The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the mass center of the assembly if the height of the cylinder is $h = 200$ mm.



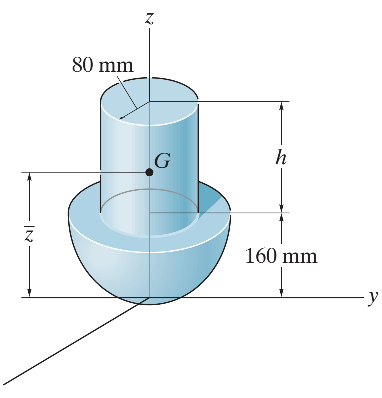


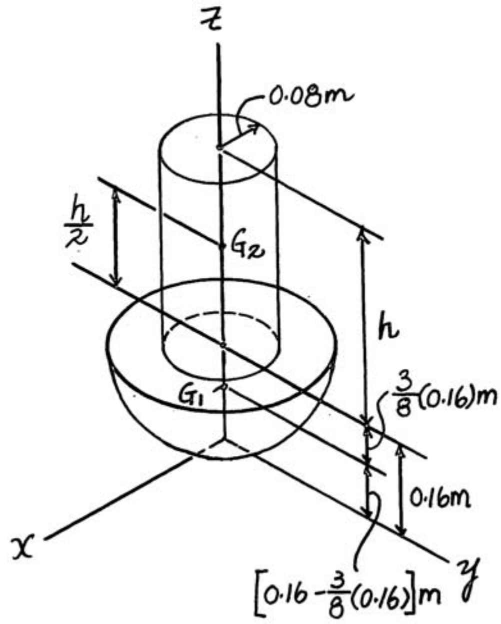
$$\Sigma \tilde{z} m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{0.2}{2}\right)\pi(0.2)(0.08)^2(2.70)$$
$$= 9.51425(10^{-3})\ \text{Mg}\cdot\text{m}$$

$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(0.2)(0.08)^2(2.70)$$
$$= 77.7706(10^{-3})\ \text{Mg}$$

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{9.51425(10^{-3})}{77.7706(10^{-3})} = 0.122\ \text{m} = 122\ \text{mm} \qquad \textbf{Ans}$$

***9–80.** The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the height h of the cylinder so that the mass center of the assembly is located at $\bar{z} = 160$ mm.





$$\Sigma \tilde{z} m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{h}{2}\right)\pi(h)(0.08)^2(2.70)$$
$$= 6.691(10^{-3}) + 8.686(10^{-3})h + 27.143(10^{-3})h^2$$

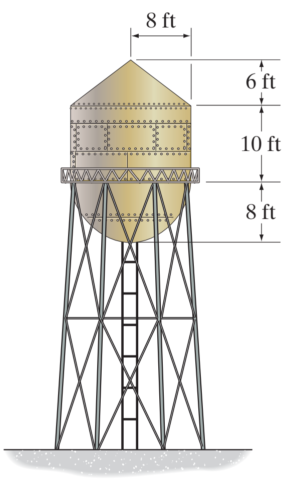
$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(h)(0.08)^2(2.70)$$
$$= 66.91(10^{-3}) + 54.29(10^{-3})h$$

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{6.691(10^{-3}) + 8.686(10^{-3})h + 27.143(10^{-3})h^2}{66.91(10^{-3}) + 54.29(10^{-3})h} = 0.160$$

Solving

$h = 0.385\ \text{m} = 385\ \text{mm}$ **Ans**

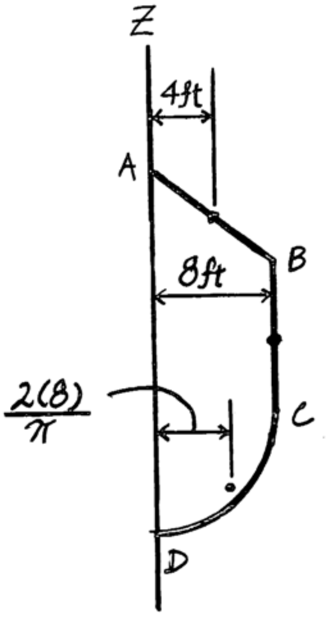
•9–81. The elevated water storage tank has a conical top and hemispherical bottom and is fabricated using thin steel plate. Determine how many square feet of plate is needed to fabricate the tank.



Surface Area: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. *a*.

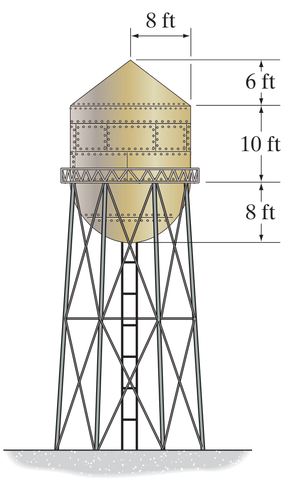
$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[4\left(\sqrt{8^2+6^2}\right)+8(10)+\left(\frac{2(8)}{\pi}\right)\left(\frac{\pi(8)}{2}\right)\right]$$

$$= 2\pi(184) = 1156\,\text{ft}^2$$ **Ans.**



(a)

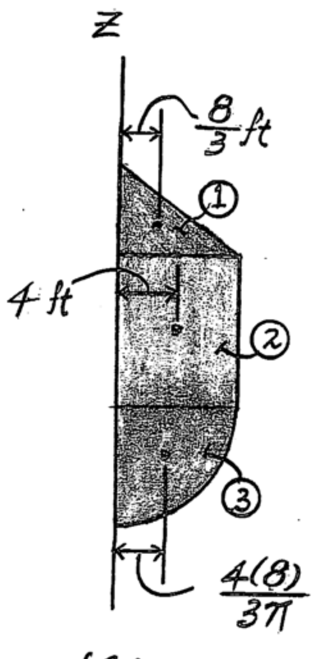
9–82. The elevated water storage tank has a conical top and hemispherical bottom and is fabricated using thin steel plate. Determine the volume within the tank.



Volume: The perpendicular distance measured from the z axis to the centroid of each area segment is indicated in Fig. *a*.

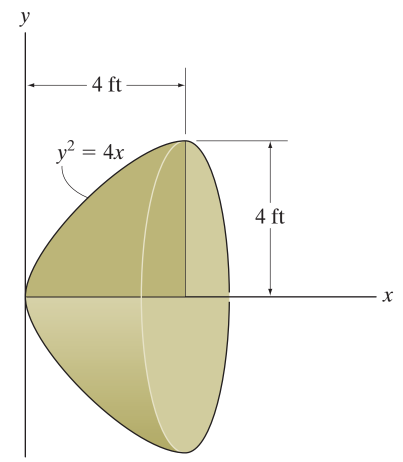
$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(\frac{8}{3}\right)\left(\frac{1}{2}\right)(6)(8) + 4(10)(8) + \left(\frac{4(8)}{3\pi}\right)\left(\frac{\pi(8^2)}{4}\right)\right]$$

$$= 2\pi(554.67) = 3485\text{ ft}^3$$ **Ans.**



(a)

9–83. Determine the volume of the solid formed by revolving the shaded area about the x axis using the second theorem of Pappus–Guldinus. The area and centroid $\overline{y}$ of the shaded area should first be obtained by using integration.



Area and Centroid: The differential element parallel to the x axis is shown shaded in Fig. a. The area of this element is given by

$$dA = (4 - x)\,dy = \left(4 - \frac{y^2}{4}\right)dy$$

Integrating,

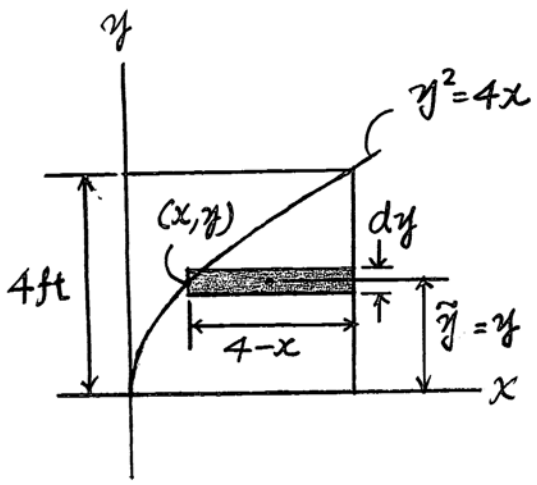
$$A = \int_A dA = \int_0^{4\,\text{ft}} \left(4 - \frac{y^2}{4}\right)dy = 4y - \frac{y^3}{12}\Bigg|_0^{4\,\text{ft}} = 10.67\,\text{ft}^2$$

With $\tilde{y} = y$,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\,\text{ft}} y\left[\left(4 - \frac{y^2}{4}\right)dy\right]}{10.67} = \frac{\int_0^{4\,\text{ft}}\left(4y - \frac{y^3}{4}\right)dy}{10.67} = \frac{\left(2y^2 - \frac{y^4}{16}\right)\Big|_0^{4\,\text{ft}}}{10.67} = 1.5\,\text{ft}$$

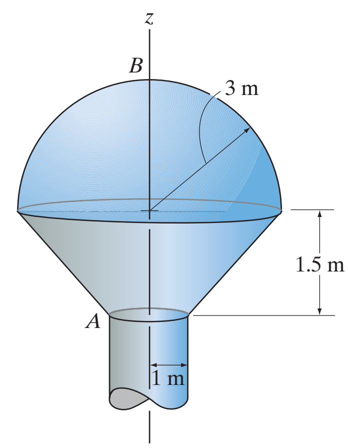
Volume: Applying the second theorem of Pappus - Guldinus and using the results obtained above,

$$V = 2\pi\bar{r}A = 2\pi(1.5)(10.67) = 101\,\text{ft}^3$$ **Ans.**



(a)

***9–84.** Determine the surface area from A to B of the tank.

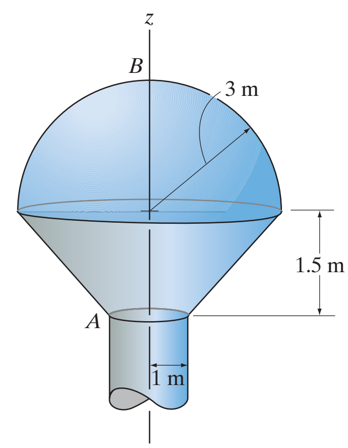


Surface Area: The perpendicular distance to the centroid of each of three line segments.

$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[\left(\frac{2(3)}{\pi}\right)\frac{2\pi(3)}{4} + 2\sqrt{1.5^2 + 2^2}\right]$$

$$= 88.0\ \text{m}^2 \qquad \textbf{Ans}$$

•9–85. Determine the volume within the thin-walled tank from A to B.

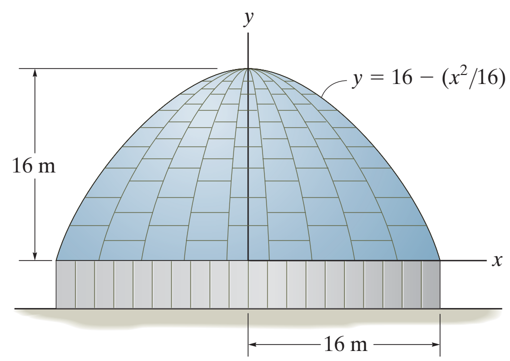


Volume: The perpendicular distance measured to the centroid of each of three area segments.

$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(\frac{4(3)}{3\pi}\right)\left(\frac{\pi(3^2)}{4}\right) + 0.5(1.5)(1) + 1.667\left(\frac{2(1.5)}{2}\right)\right]$$

$$= 77.0\ \text{m}^3 \qquad \textbf{Ans.}$$

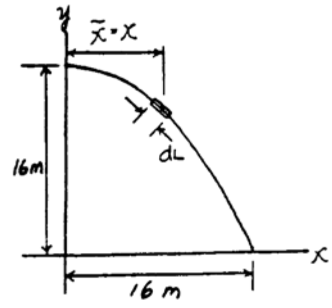
9–86. Determine the surface area of the roof of the structure if it is formed by rotating the parabola about the y axis.



Centroid : The length of the differential element is $dL = \sqrt{dx^2 + dy^2}$ $= \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{x} = x$. Here, $\frac{dy}{dx} = -\frac{x}{8}$. Evaluating the integrals, we have

$$L = \int dL = \int_0^{16\text{m}} \left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 23.663 \text{ m}$$

$$\int_L \tilde{x}\,dL = \int_0^{16\text{m}} x\left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 217.181 \text{ m}^2$$



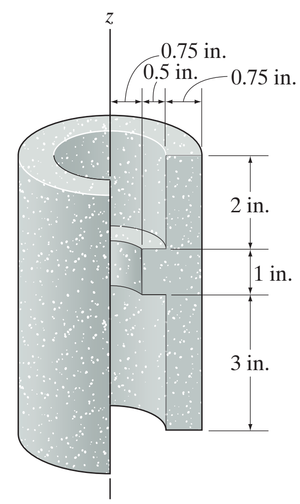
Applying Eq. 9–5, we have

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{217.181}{23.663} = 9.178 \text{ m}$$

Surface Area : Applying the theorem of Pappus and Guldinus, Eq. 9–7, with $\theta = 2\pi$, $L = 23.663$ m, $\bar{r} = \bar{x} = 9.178$, we have

$$A = \theta \bar{r} L = 2\pi(9.178)(23.663) = 1365 \text{ m}^2 \qquad \textbf{Ans}$$

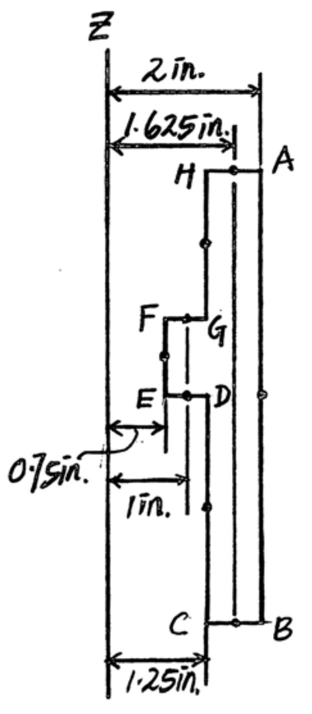
9–87. Determine the surface area of the solid formed by revolving the shaded area 360° about the z axis.



Surface Area: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. a.

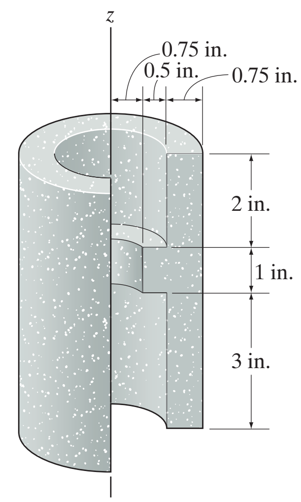
$$A = 2\pi\Sigma\bar{r}L = 2\pi[2(6) + 1.625(0.75) + 1.625(0.75) + 1.25(3) + 1.25(2) + 1(0.5) + 1(0.5) + 0.75(1)]$$

$$= 2\pi(22.4375) = 44.875\pi\,\text{in}^2 = 141\,\text{in}^2 \qquad \textbf{Ans.}$$



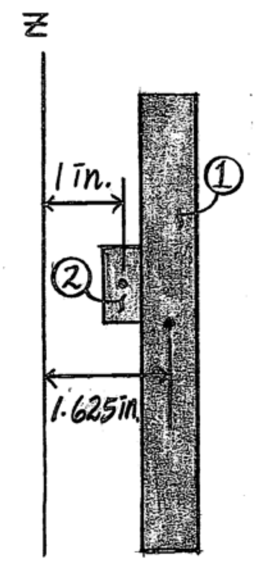
(a)

***9–88.** Determine the volume of the solid formed by revolving the shaded area 360° about the z axis.



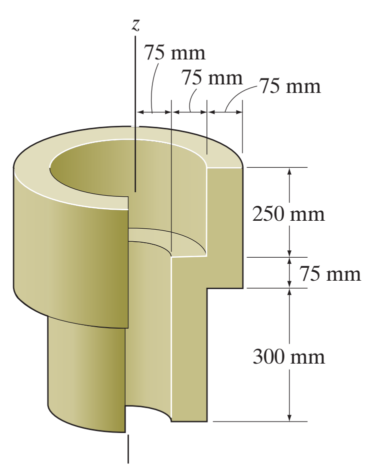
Volume: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. *a*.

$$V = 2\pi\Sigma\bar{r}A = 2\pi[1.625(6)(0.75) + 1(1)(0.5)] = 2\pi(7.8125) = 49.1\,\text{in}^3$$ **Ans.**



(a)

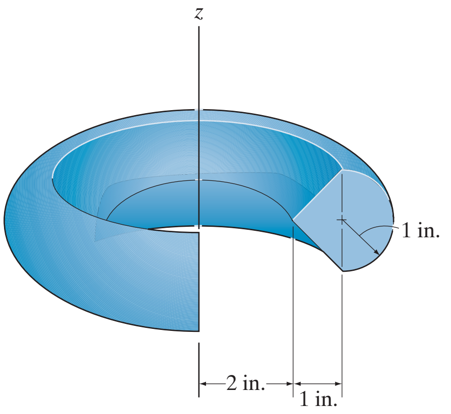
•9–89. Determine the volume of the solid formed by revolving the shaded area 360° about the z axis.



Volume: The perpendicular distance measured to the centroid of each of two area segments.

$$V = 2\pi\Sigma\bar{r}A = 2\pi[(112.5)(75)(375) + (187.5)(325)(75)]$$
$$= 0.0486\,\text{m}^3$$ **Ans.**

9–90. Determine the surface area and volume of the solid formed by revolving the shaded area 360° about the z axis.



Surface Area: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. a.

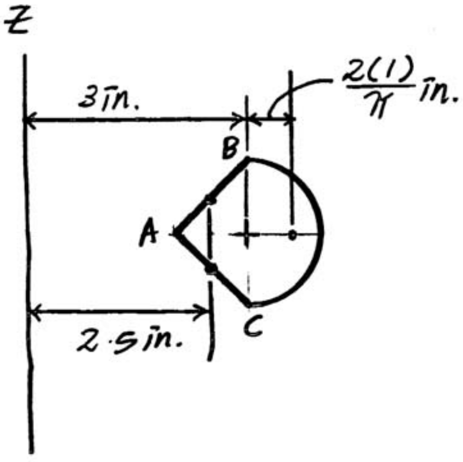
$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[(2.5)\left(\sqrt{1^2+1^2}\right)+(2.5)\left(\sqrt{1^2+1^2}\right)+\left(3+\frac{2(1)}{\pi}\right)\pi(1)\right]$$

$$= 2\pi(18.4958) = 116\text{ in}^2$$ **Ans.**

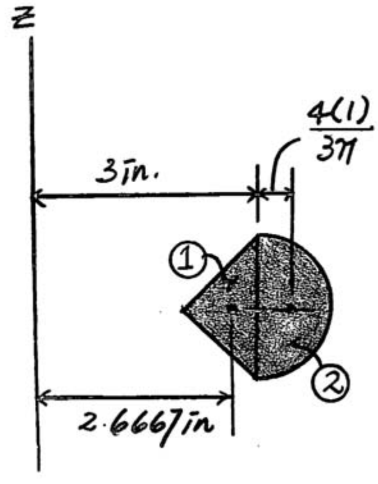
Volume: The perpendicular distance measured from the z axis to the centroid of each area segment is indicated in Fig. a.

$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[(2.667)\left(\frac{1}{2}(2)(1)\right)+\left(3+\frac{4(1)}{3\pi}\right)\left(\frac{\pi(1)}{2}\right)\right]$$

$$= 2\pi(8.0457) = 50.6\text{ in}^3$$ **Ans.**

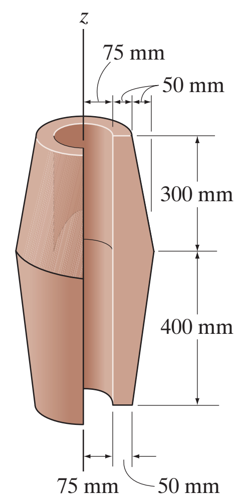


(a)



(b)

9–91. Determine the surface area and volume of the solid formed by revolving the shaded area 360° about the z axis.



Surface Area: The perpendicular distance measured from the x axis to the centroid of each of four line segments..

$$A = 2\pi\left[150\sqrt{(400)^2+(50)^2} + 150\sqrt{(300)^2+(50)^2} + 75(700) + 2(100)(50)\right]$$

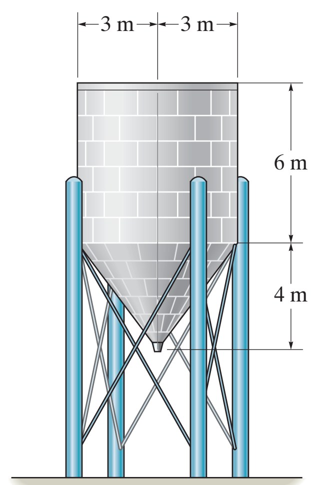
$= 1.06\ \text{m}^2$ **Ans.**

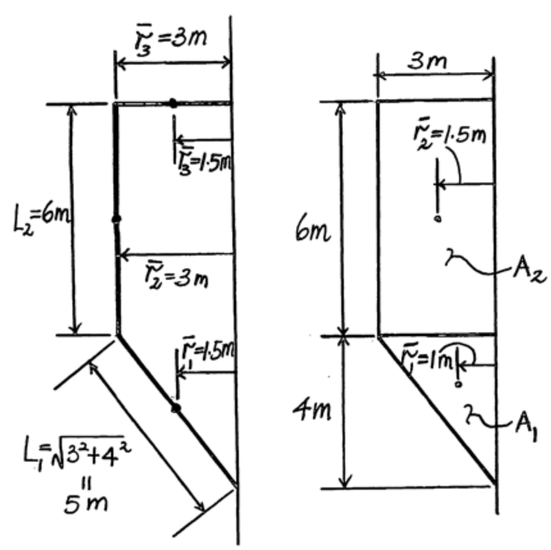
Volume: The perpendicular distance measured from the x axis to the centroid of each of two area segments.

$$V = 2\pi\left[(100)(700)(50) + 141.667\left(\frac{1}{2}(700)(50)\right)\right]$$

$= 0.0376\ \text{m}^3$ **Ans.**

***9–92.** The process tank is used to store liquids during manufacturing. Estimate both the volume of the tank and its surface area. The tank has a flat top and a thin wall.

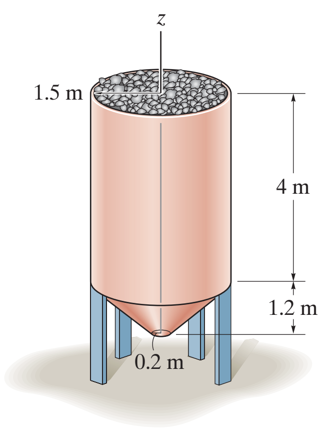




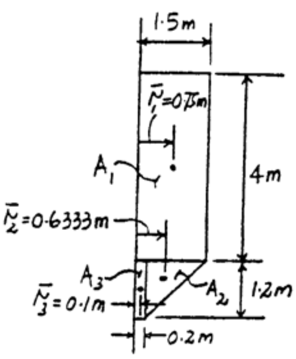
$V = \Sigma\theta\bar{r}A = 2\pi\left[1\left(\frac{1}{2}\right)(3)(4) + 1.5(3)(6)\right] = 207\ \text{m}^3$ **Ans**

$A = \Sigma\theta\bar{r}L = 2\pi[1.5(5) + 3(6) + 1.5(3)] = 188\ \text{m}^2$ **Ans**

•9–93. The hopper is filled to its top with coal. Estimate the volume of coal if the voids (air space) are 35 percent of the volume of the hopper.



Volume : The volume of the hopper can be obtained by applying the theorem of Pappus and Guldinus, Eq. 9–18 with $\theta = 2\pi$, $\bar{r}_1 = 0.75$ m, $\bar{r}_2 = 0.6333$ m, $\bar{r}_3 = 0.1$ m, $A_1 = 1.5(4) = 6.00\ \text{m}^2$, $A_2 = \dfrac{1}{2}(1.3)(1.2) = 0.780\ \text{m}^2$ and $A_3 = (0.2)(1.2) = 0.240\ \text{m}^2$.

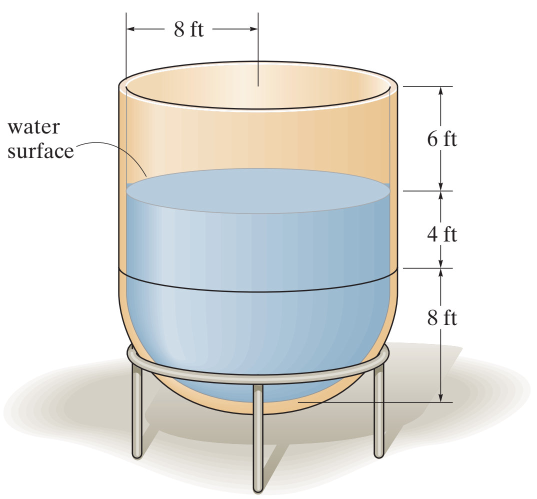


$$V_h = \theta\Sigma\bar{r}A = 2\pi[0.75(6.00) + 0.6333(0.780) + 0.1(0.240)]$$
$$= 10.036\pi\ \text{m}^3$$

The volume of the coal is

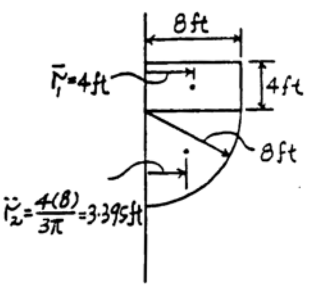
$$V_c = 0.65V_h = 0.65(10.036\pi) = 20.5\ \text{m}^3 \qquad \textbf{Ans}$$

9–94. The thin-wall tank is fabricated from a hemisphere and cylindrical shell. Determine the vertical reactions that each of the four symmetrically placed legs exerts on the floor if the tank contains water which is 12 ft deep in the tank. The specific gravity of water is 62.4 lb/ft^3. Neglect the weight of the tank.



Volume : The volume of the water can be obtained by applying the theorem of Pappus and Guldinus, Eq. 9–10, with $\theta = 2\pi$, $\bar{r}_1 = 4$ ft, $\bar{r}_2 = 3.395$ ft, $A_1 = 8(4) = 32.0\ \text{ft}^2$ and $A_2 = \dfrac{1}{4}\pi(8^2) = 50.27\ \text{ft}^2$.

$$V = \theta\Sigma\bar{r}A = 2\pi[4(32.0) + 3.395(50.27)] = 1876.58\ \text{ft}^3$$



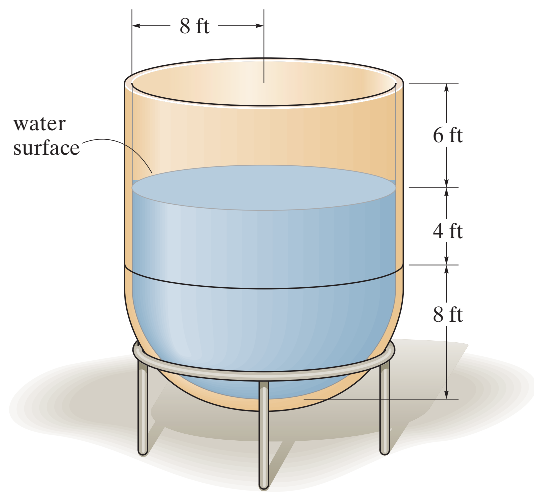
The weight of the water is

$$W = \gamma_w V = 62.4(1876.58) = 117098.47\ \text{lb}$$

Thus, the reaction of each leg on the floor is

$$R = \frac{W}{4} = \frac{117098.47}{4} = 29274.62\ \text{lb} = 29.3\ \text{kip} \qquad \textbf{Ans}$$

9–95. Determine the approximate amount of paint needed to cover the outside surface of the open tank. Assume that a gallon of paint covers 400 ft^2.

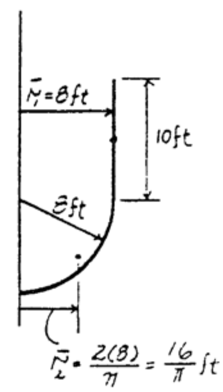


Surface Area: Applying the theorem of Pappus and Guldinus, Eq. 9–9, with $\theta = 2\pi$, $L_1 = 10$ ft, $L_2 = \dfrac{\pi(8)}{2} = 4\pi$ ft, $\bar{r}_1 = 8$ ft and $\bar{r}_2 = \dfrac{16}{\pi}$ ft, we have

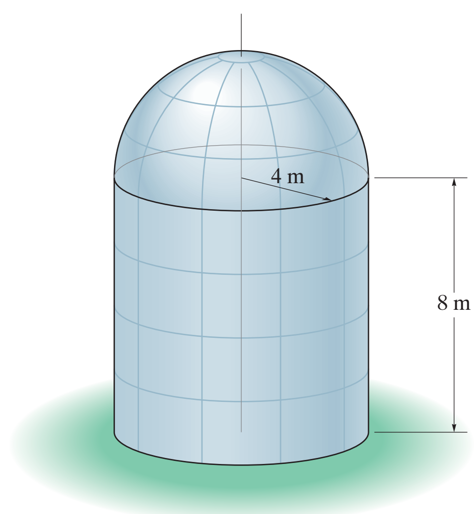
$$A = \theta \Sigma \bar{r} L = 2\pi\left[8(10) + \frac{16}{\pi}(4\pi)\right] = 288\pi \text{ ft}^2$$

Thus,

$$\text{The required amount paint} = \frac{288\pi}{400} = 2.26 \text{ gallon} \qquad \textbf{Ans}$$

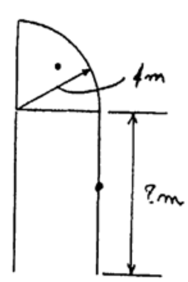


***9–96.** Determine the surface area of the tank, which consists of a cylinder and hemispherical cap.

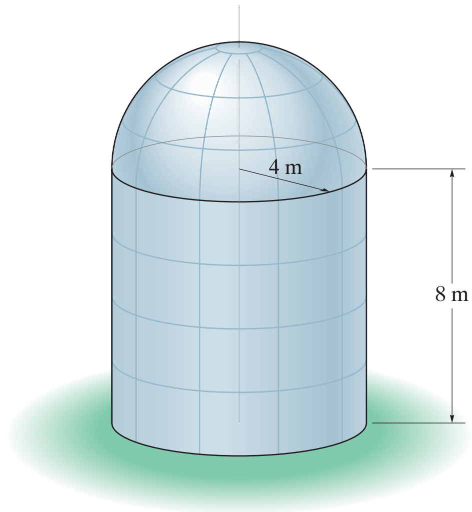


$$A = \Sigma\theta\,\bar{r}\,L = 2\pi\left[(4)(8) + \left(\frac{2(4)}{\pi}\right)\left(\frac{1}{4}(2\pi(4))\right)\right]$$

$$= 302 \text{ m}^2 \qquad \textbf{Ans}$$

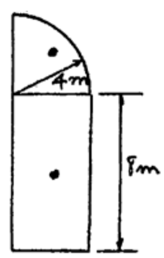


•9–97. Determine the volume of the thin-wall tank, which consists of a cylinder and hemispherical cap.

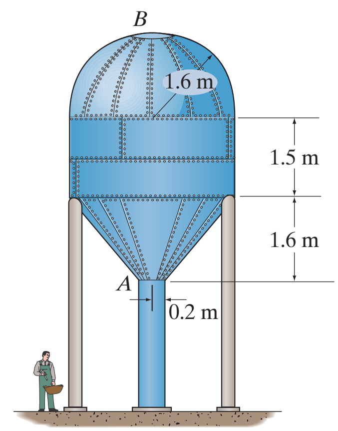


$$V = \Sigma\theta\,\bar{r}\,A = 2\pi\left[\left(\frac{4(4)}{3\pi}\right)\left(\frac{1}{4}\pi\,(4)^2\right) + (2)(8)(4)\right]$$

$$= 536\ \text{m}^3 \qquad \textbf{Ans}$$



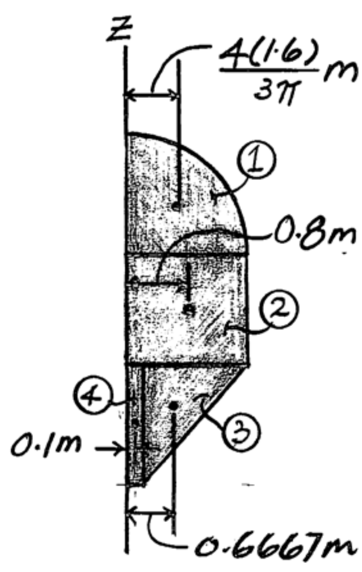
9–98. The water tank AB has a hemispherical top and is fabricated from thin steel plate. Determine the volume within the tank.



Volume: The perpendicular distance measured from the z axis to the centroid of each area segment is indicated in Fig. *a*.

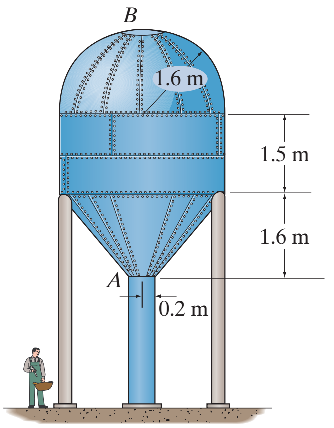
$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(\frac{4(1.6)}{3\pi}\right)\left(\frac{\pi(1.6^2)}{4}\right) + 0.8(1.6)(1.5) + 0.6667\left(\frac{1}{2}\right)(1.4)(1.6) + 0.1(0.2)(1.6)\right]$$

$$= 2\pi(4.064) = 25.5\ \text{m}^3 \qquad \textbf{Ans.}$$



(a)

9–99. The water tank AB has a hemispherical roof and is fabricated from thin steel plate. If a liter of paint can cover 3 m^2 of the tank's surface, determine how many liters are required to coat the surface of the tank from A to B.



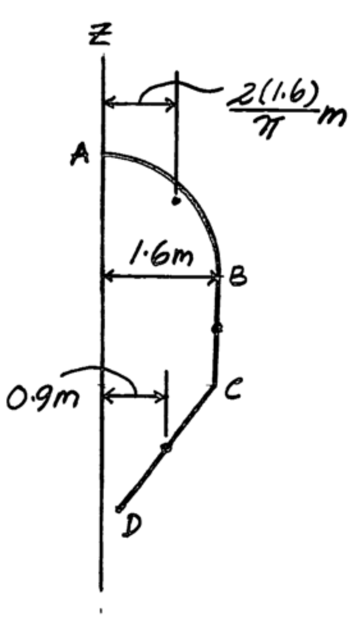
Surface Area: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. a.

$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[\left(\frac{2(1.6)}{\pi}\right)\left(\frac{\pi(1.6)}{2}\right) + 1.6(1.5) + 0.9\left(\sqrt{1.4^2 + 1.6^2}\right)\right]$$

$$= 2\pi(6.8734) = 43.18 \text{ m}^2$$

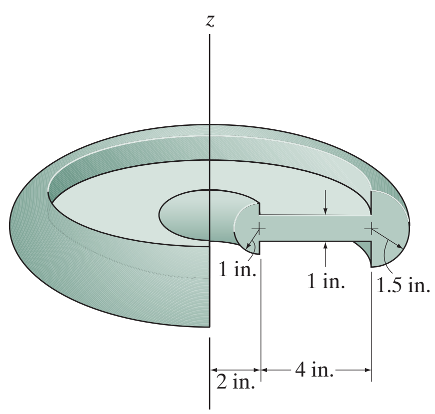
Thus, the amount of paint required is

$$\text{Number of liters} = \frac{43.18}{3} = 14.4 \text{ liters}$$ **Ans.**



(a)

***9–100.** Determine the surface area and volume of the wheel formed by revolving the cross-sectional area 360° about the z axis.



Surface Area: The perpendicular distance measured from the z axis to the centroid of each line segment is indicated in Fig. *a*.

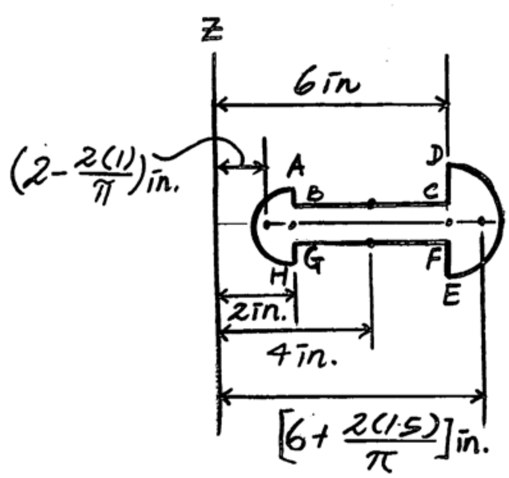
$$A = 2\pi\Sigma\bar{r}L = 2\pi\left[\left(2 - \frac{2(1)}{\pi}\right)\pi(1) + 2(1) + 4(2)(4) + 6(2) + \left(6 + \frac{2(1.5)}{\pi}\right)\pi(1.5)\right]$$

$$= 2\pi(83.0575) = 522\ \text{in}^2$$ **Ans.**

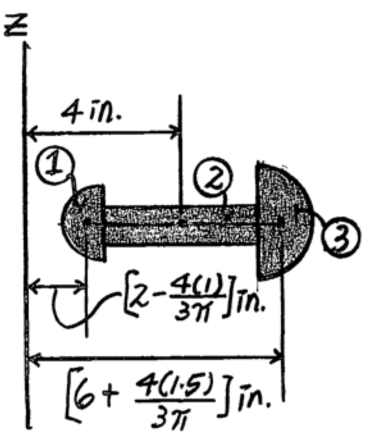
Volume: The perpendicular distance measured from the z axis to the centroid of each area segment is indicated in Fig. *a*.

$$V = 2\pi\Sigma\bar{r}A = 2\pi\left[\left(2 - \frac{4(1)}{3\pi}\right)\left(\frac{\pi(1^2)}{2}\right) + 4(4)(1) + \left(6 + \frac{4(1.5)}{3\pi}\right)\left(\frac{\pi(1.5^2)}{2}\right)\right]$$

$$= 2\pi(41.9307) = 263\ \text{in}^3$$ **Ans.**

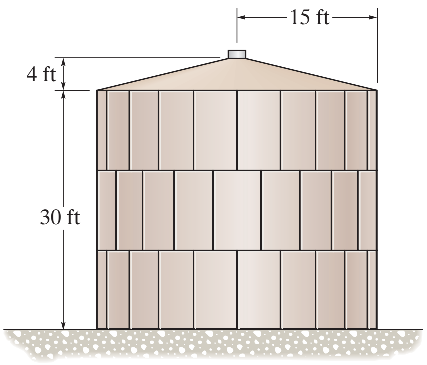


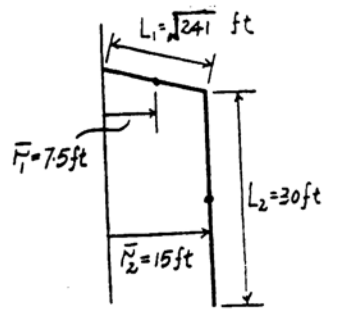
(a)



(b)

•9–101. Determine the outside surface area of the storage tank.

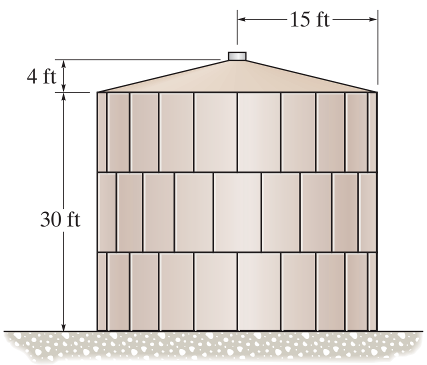


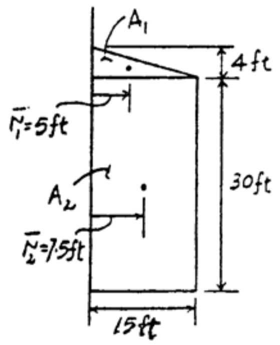


Surface Area: Applying the theorem of Pappus and Guldinus, Eq. 9–9, with $\theta = 2\pi$, $L_1 = \sqrt{15^2 + 4^2} = \sqrt{241}$ ft, $L_2 = 30$ ft, $\bar{r}_1 = 7.5$ ft and $\bar{r}_2 = 15$ ft, we have

$$A = \theta \Sigma \bar{r} L = 2\pi\left[7.5\left(\sqrt{241}\right) + 15(30)\right] = 3.56\left(10^3\right)\ \text{ft}^2 \qquad \textbf{Ans}$$

9–102. Determine the volume of the thin-wall storage tank.

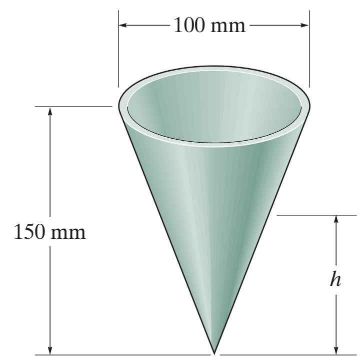




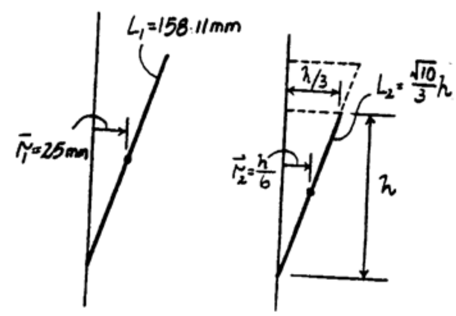
Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–10, with $\theta = 2\pi$, $\bar{r}_1 = 5$ ft, $\bar{r}_2 = 7.5$ ft, $A_1 = \frac{1}{2}(15)(4) = 30.0\ \text{ft}^2$ and $A_2 = 30(15) = 450\ \text{ft}^2$, we have

$$V = \theta \Sigma \bar{r} A = 2\pi[5(30.0) + 7.5(450)] = 22.1\left(10^3\right)\ \text{ft}^3 \qquad \textbf{Ans}$$

9–103. Determine the height h to which liquid should be poured into the conical paper cup so that it contacts half the surface area on the inside of the cup.



Surface Area : This problem requires that $\frac{1}{2}A_1 = A_2$. Applying the theorem of Pappus and Guldinus, Eq. 9–7, with $\theta = 2\pi$, $L_1 = \sqrt{50^2 + 150^2} = 158.11$ mm, $L_2 = \sqrt{h^2 + \left(\frac{h}{3}\right)^2} = \frac{\sqrt{10}}{3}h$, $\bar{r}_1 = 25$ mm and $\bar{r}_2 = \frac{h}{6}$, we have

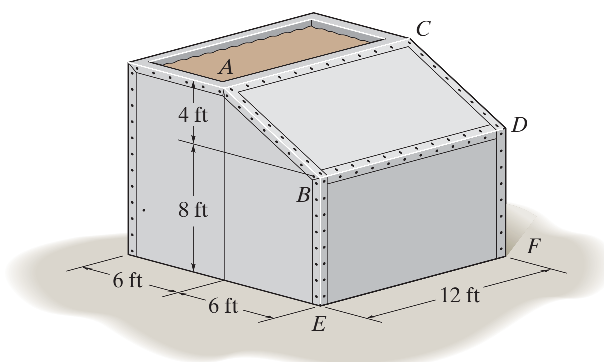


$$\frac{1}{2}(\theta \bar{r}_1 L_1) = \theta \bar{r}_2 L_2$$

$$\frac{1}{2}[2\pi(25)(158.11)] = 2\pi\left(\frac{h}{6}\right)\left(\frac{\sqrt{10}}{3}h\right)$$

$$h = 106 \text{ mm} \qquad \textbf{Ans}$$

***9–104.** The tank is used to store a liquid having a specific weight of 80 lb/ft^3. If it is filled to the top, determine the magnitude of the force the liquid exerts on each of its two sides $ABDC$ and $BDFE$.



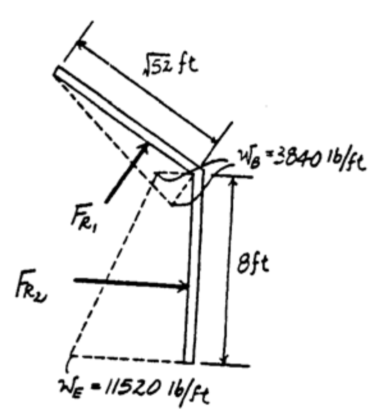
Fluid Pressure : The fluid pressure at points B and E can be determined using Eq. 9–13, $p = \gamma z$.

$$p_B = 80(4) = 320 \text{ lb/ft}^2 \qquad p_E = 80(12) = 960 \text{ lb/ft}^2$$

Thus,

$$w_B = 320(12) = 3840 \text{ lb/ft} \qquad w_E = 960(12) = 11520 \text{ lb/ft}$$

Resultant Forces : The resultant force acts on suface $ABCD$ is

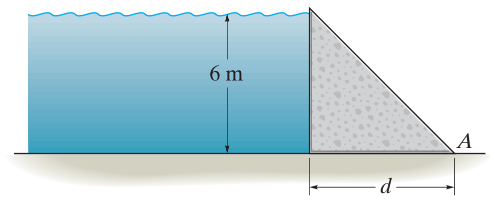


$$F_{R_1} = \frac{1}{2}(3840)\left(\sqrt{52}\right) = 13\,845.31 \text{ lb} = 13.8 \text{ kip} \qquad \textbf{Ans}$$

and acts on surface $BDFE$ is

$$F_{R_2} = \frac{1}{2}(3840 + 11520)(8) = 61\,440 \text{ lb} = 61.4 \text{ kip} \qquad \textbf{Ans}$$

•9–105. The concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \text{Mg/m}^3$, and water has a density of $\rho_w = 1.0\ \text{Mg/m}^3$, determine the smallest dimension d that will prevent the dam from overturning about its end A.



Consider a 1 – m width of dam.

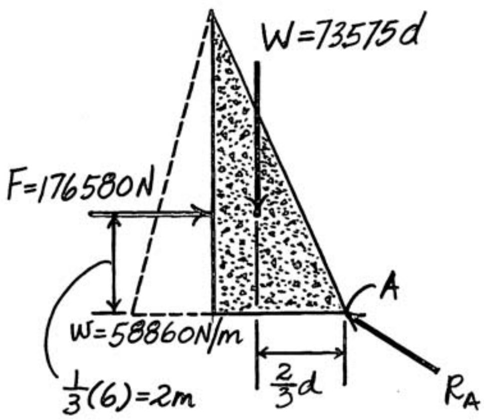
$$w = 1000(9.81)(6)(1) = 58\,860\ \text{N/m}$$

$$F = \frac{1}{2}(58\,860)(6) = 176\,580\ \text{N}$$

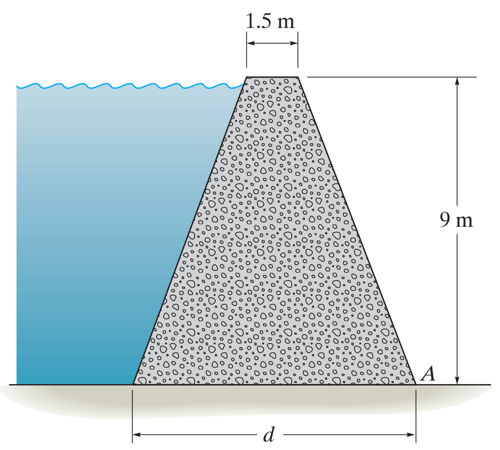
$$W = \frac{1}{2}(d)(6)(1)(2500)(9.81) = 73\,575d\ \text{N}$$

$$\curvearrowleft +\Sigma M_A = 0; \qquad -176\,580(2) + 73\,575d\left(\frac{2}{3}d\right) = 0$$

$$d = 2.68\ \text{m} \qquad \text{Ans}$$



9–106. The symmetric concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \text{Mg/m}^3$, and water has a density of $\rho_w = 1.0\ \text{Mg/m}^3$, determine the smallest distance d at its base that will prevent the dam from overturning about its end A. The dam has a width of 8 m.



$$w = b\rho_w gh = 8(1000)(9.81)(9) = 706.32(10^3)\ \text{N/m}$$

$$F_h = \tfrac{1}{2}\left[706.32(10^3)\right](9) = 3178.44(10)^3\ \text{N}$$

$$F_v = 1000(9.81)(\tfrac{1}{2})(\tfrac{d-1.5}{2})(9)(8) = (176.58d - 264.87)(10^3)$$

$$W = 2.5(10^3)(9.81)\left[\tfrac{1}{2}(d+1.5)(9)(8)\right] = (882.9d + 1324.35)(10^3)$$

$$x_1 = d - \tfrac{1}{3}(\tfrac{d-1.5}{2}) = \tfrac{5}{6}d + 0.25$$

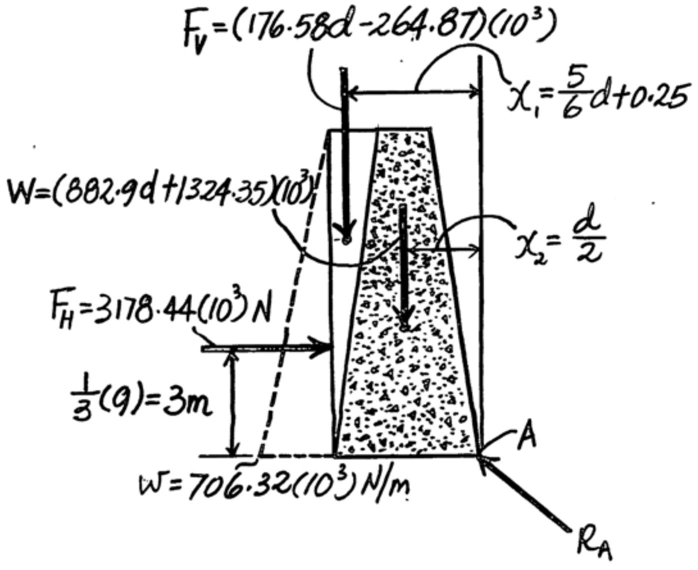
$$x_2 = \tfrac{d}{2}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad (176.58d - 264.87)(10^3)(\tfrac{5}{6}d + 0.25)$$

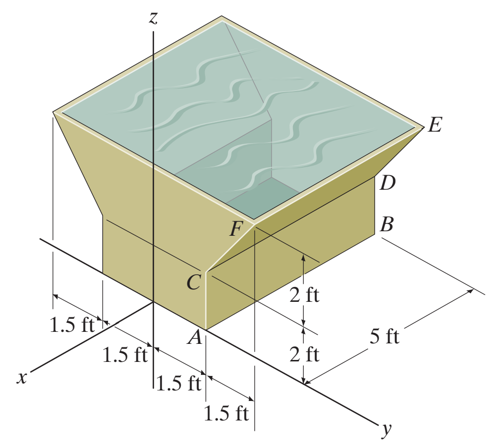
$$+ (882.9d + 1324.35)(10^3)(\tfrac{d}{2}) - 3178.44(10^3)(3) = 0$$

$$588.6d^2 + 485.595d - 9601.54 = 0$$

$$d = 3.65\ \text{m}$$



9–107. The tank is used to store a liquid having a specific weight of 60 lb/ft^3. If the tank is full, determine the magnitude of the hydrostatic force on plates *CDEF* and *ABDC*.



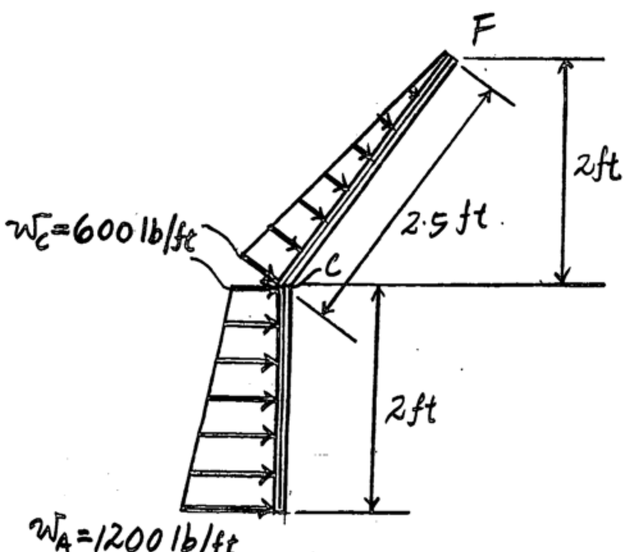
Loading: Since walls *CDEF* and *ABDC* have a constant width, the loading due to the fluid pressure on the walls can be represented by a two dimensional distributed loading. The intensity of the distributed load at points F, C, and A are given by

$$w_F = \gamma h_F b = 60(0)(5) = 0$$
$$w_C = \gamma h_C b = 60(2)(5) = 600 \text{ lb/ft}$$
$$w_A = \gamma h_A b = 60(4)(5) = 1200 \text{ lb/ft}$$

Resultant Force: The distributed loading acting on walls *CDEF* and *ABDC* is shown in Fig. *a*. Thus, the magnitude of the hydrostatic force on these two walls are

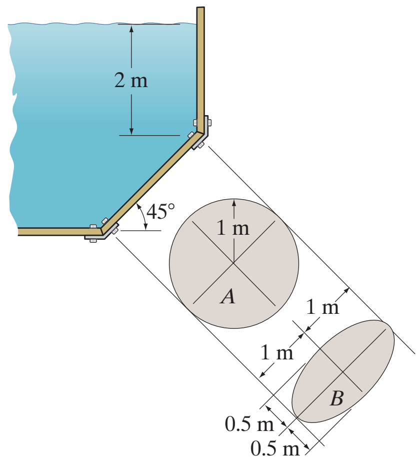
$$F_{CDEF} = \frac{1}{2}(600)(2.5) = 750 \text{ lb} \qquad \textbf{Ans.}$$

$$F_{ABDC} = \frac{1}{2}(600 + 1200)(2) = 1800 \text{ lb} \qquad \textbf{Ans.}$$



(a)

***9–108.** The circular steel plate A is used to seal the opening on the water storage tank. Determine the magnitude of the resultant hydrostatic force that acts on it. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth *h* expressed in terms of *y* is

$$h = \left(\frac{2}{\cos 45°} + 1 - y\right)\sin 45° = 2.7071 - 0.7071y$$

Thus, the water pressure at the depth *h* is

$$p = \rho_w g h = \frac{1000(9.81)(2.7071 - 0.7071y)}{1000} = (26.5567 - 6.9367y)\ \text{kN/m}^2$$

The differential force dF_R acting on the differential area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = (26.5567 - 6.9367y)\left(2\sqrt{1-y^2}\right)dy$$

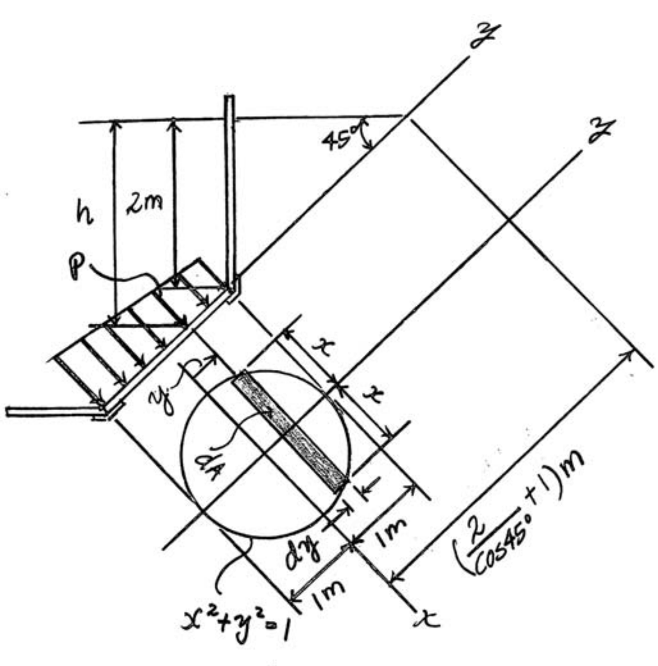
$$= \left(53.1134\sqrt{1-y^2} - 13.8734y\sqrt{1-y^2}\right)dy$$

Resultant Force: Integrating dF_R from $y = -1$ m to $y = 1$ m,

$$F_R = \int dF_R = \int_{-1\,\text{m}}^{1\,\text{m}}\left(53.1134\sqrt{1-y^2} - 13.8734y\sqrt{1-y^2}\right)dy$$

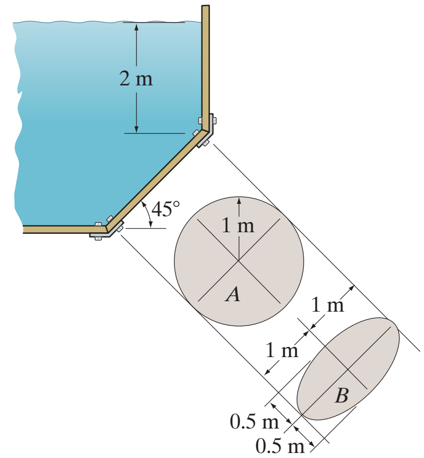
$$= \left[26.5567\left(y\sqrt{1-y^2} + \sin^{-1} y\right) + 4.6245\sqrt{(1-y^2)^3}\right]_{-1\,\text{m}}^{1\,\text{m}}$$

$$= 83.4\ \text{kN}$$



(a)

•9–109. The elliptical steel plate B is used to seal the opening on the water storage tank. Determine the magnitude of the resultant hydrostatic force that acts on it. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth h expressed in terms of y is

$$h = \left(\frac{2}{\cos 45°} + 1 - y\right)\sin 45° = 2.7071 - 0.7071y$$

Thus, the water pressure at the depth h is

$$p = \rho_w g h = \frac{1000(9.81)(2.7071 - 0.7071y)}{1000} = (26.5567 - 6.9367y)\ \text{kN/m}^2$$

The differential force $d\mathbf{F}_R$ acting on the differential area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = (26.5567 - 6.9367y)\left[2\left(0.5\sqrt{1-y^2}\right)\right]dy$$

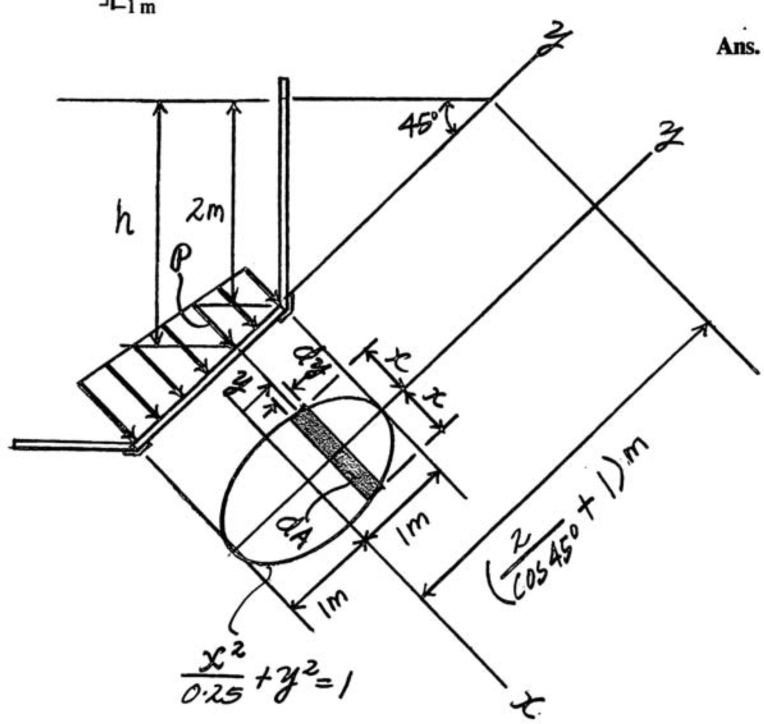
$$= \left(26.5567\sqrt{1-y^2} - 6.9367y\sqrt{1-y^2}\right)dy$$

Resultant Force: Integrating $d\mathbf{F}_R$ from $y = -1$ m to $y = 1$ m,

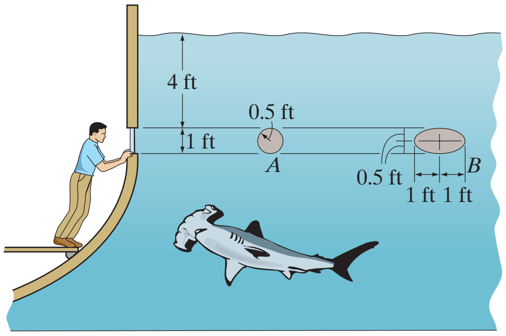
$$F_R = \int dF_R = \int_{-1\text{ m}}^{1\text{ m}} \left(26.5567\sqrt{1-y^2} - 6.9367y\sqrt{1-y^2}\right)dy$$

$$= \left[13.2784\left(y\sqrt{1-y^2} + \sin^{-1} y\right) + 2.3122\sqrt{(1-y^2)^3}\right]_{-1\text{ m}}^{1\text{ m}}$$

$$= 41.7\ \text{kN} \qquad \textbf{Ans.}$$



9–110. Determine the magnitude of the hydrostatic force acting on the glass window if it is circular, A. The specific weight of seawater is $\gamma_w = 63.6\ \mathrm{lb/ft^3}$.



Loading: By referring to the geometry of Fig. *a*, the depth h expressed in terms of y is

$$h = 4 + 0.5 - y = (4.5 - y)\ \mathrm{ft}$$

Thus, the water pressure at the depth h is

$$p = \gamma_w h = 63.6(4.5 - y)\ \mathrm{lb/ft^2}$$

Resultant Force: The differential force $d\mathbf{F}_R$ acting on the differential area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = 63.6(4.5 - y)\left(2\sqrt{0.25 - y^2}\right)dy$$

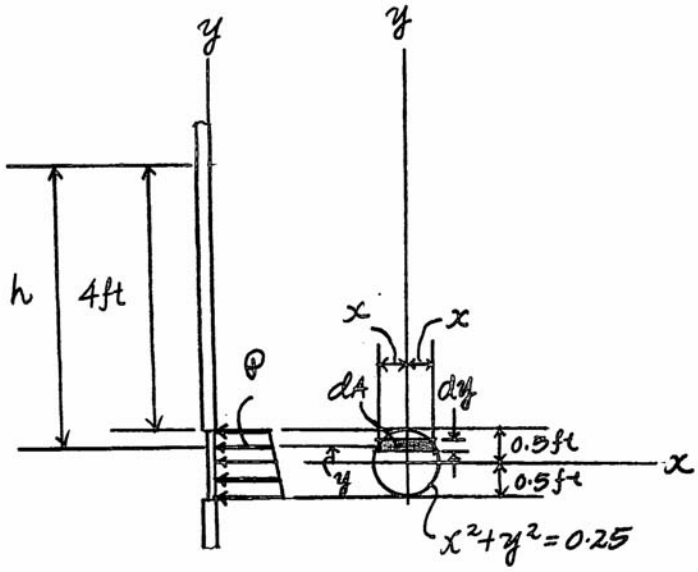
$$= \left(572.4\sqrt{0.25 - y^2} - 127.2y\sqrt{0.25 - y^2}\right)dy$$

Integrating $d\mathbf{F}_R$ from $y = -0.5$ ft to $y = 0.5$ ft,

$$F_R = \int dF_R = \int_{-0.5\ \mathrm{ft}}^{0.5\ \mathrm{ft}} \left(572.4\sqrt{0.25 - y^2} - 127.2y\sqrt{0.25 - y^2}\right)dy$$

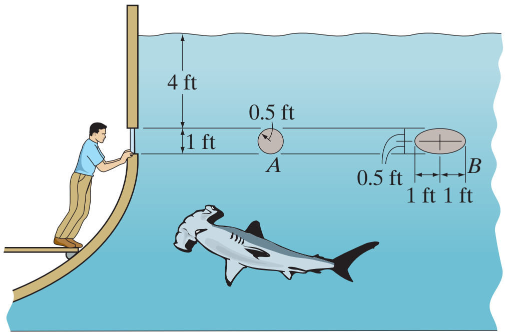
$$= \left[286.2\left(y\sqrt{0.25 - y^2} + 0.25\sin^{-1}\frac{y}{0.5}\right) + 42.4\sqrt{(0.25 - y^2)^3}\right]_{-0.5\ \mathrm{ft}}^{0.5\ \mathrm{ft}}$$

$$= 224.78\ \mathrm{lb} = 225\ \mathrm{lb}$$ **Ans.**



(a)

9–111. Determine the magnitude and location of the resultant hydrostatic force acting on the glass window if it is elliptical, B. The specific weight of seawater is $\gamma_w = 63.6\ \text{lb/ft}^3$.



Loading: By referring to the geometry of Fig. *a*, the depth h expressed in terms of y is

$$h = 4 + 0.5 - y = (4.5 - y)\ \text{ft}$$

Thus, the water pressure at the depth h is

$$p = \gamma_w h = 63.6(4.5 - y)\ \text{lb/ft}^2$$

Resultant Force: The differential force $d\mathbf{F}_R$ acting on the area dA shown shaded in Fig. *a* is

$$dF_R = p\,dA = p(2x)\,dy = 63.6(4.5 - y)\left[2\left(2\sqrt{0.25 - y^2}\right)\right]dy$$

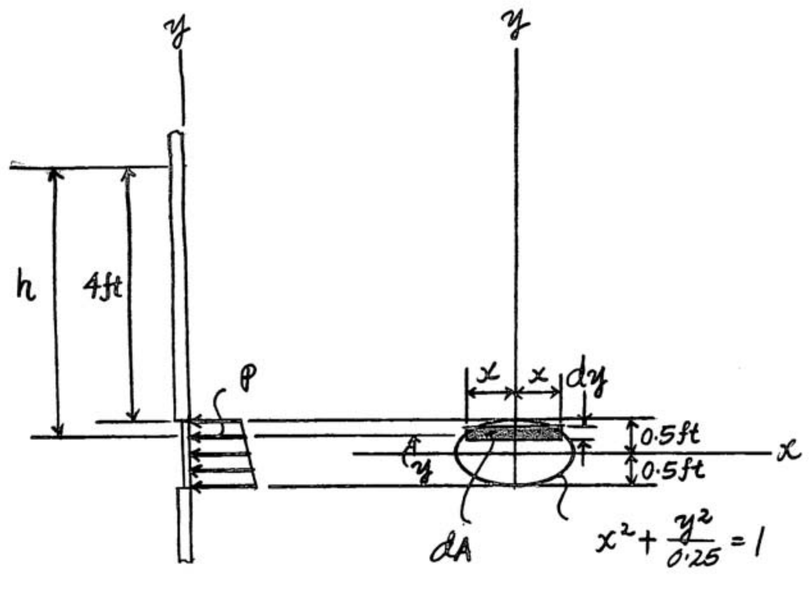
$$= \left(1144.8\sqrt{0.25 - y^2} - 254.4y\sqrt{0.25 - y^2}\right)dy$$

Integrating $d\mathbf{F}_R$ from $y = -0.5$ ft to $y = 0.5$ ft,

$$F_R = \int dF_R = \int_{-0.5\text{ ft}}^{0.5\text{ ft}} \left(1144.8\sqrt{0.25 - y^2} - 254.4y\sqrt{0.25 - y^2}\right)dy$$

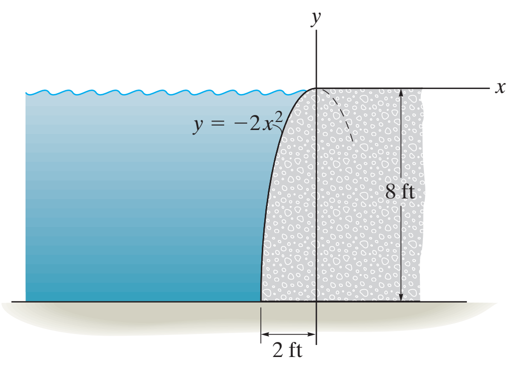
$$= \left[572.4\left(y\sqrt{0.25 - y^2} + 0.25\sin^{-1}\frac{y}{0.5}\right) + 84.8\sqrt{(0.25 - y^2)^3}\right]_{-0.5\text{ ft}}^{0.5\text{ ft}}$$

$$= 449.56\ \text{lb} = 450\ \text{lb}$$ **Ans.**



(a)

***9–112.** Determine the magnitude of the hydrostatic force acting per foot of length on the seawall. $\gamma_w = 62.4\ \text{lb/ft}^3$.



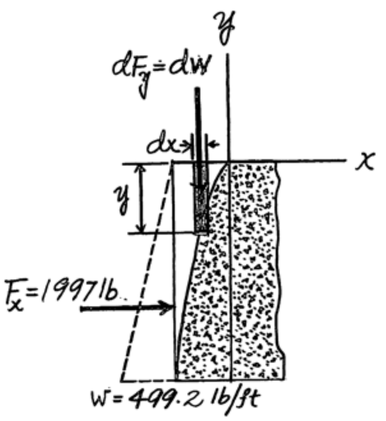
$$A = \int_A dA = \int_{-2}^{0} -y\,dx = \int_{-2}^{0} 2x^2\,dx = \frac{2}{3}x^3\Big|_{-2}^{0} = 5.333\ \text{ft}^2$$

$$w = b\gamma h = 1(62.4)(8) = 499.2\ \text{lb/ft}$$

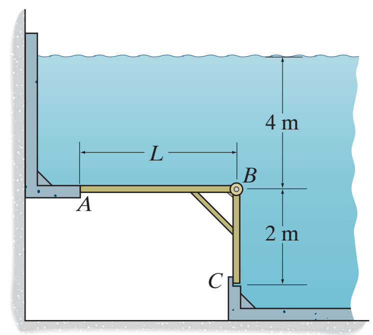
$$F_y = 5.333(1)(62.4) = 332.8\ \text{lb}$$

$$F_x = \frac{1}{2}(499.2)(8) = 1997\ \text{lb}$$

$$F_R = \sqrt{(332.8)^2 + (1997)^2} = 2024\ \text{lb} = 2.02\ \text{kip} \qquad \text{Ans}$$



•**9–113.** If segment AB of gate ABC is long enough, the gate will be on the verge of opening. Determine the length L of this segment in order for this to occur. The gate is hinged at B and has a width of 1 m. The density of water is $\rho_w = 1\ \mathrm{Mg/m^3}$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points B and C are

$$w_B = \rho_w g h_B b = 1000(9.81)(4)(1) = 39\ 240\ \mathrm{N} = 39.24\ \mathrm{kN}$$

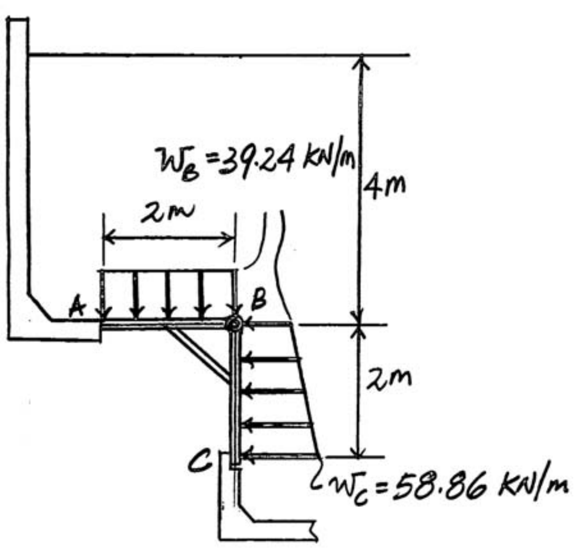
$$w_C = \rho_w g h_C b = 1000(9.81)(6)(1) = 58\ 860\ \mathrm{N} = 58.86\ \mathrm{kN}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

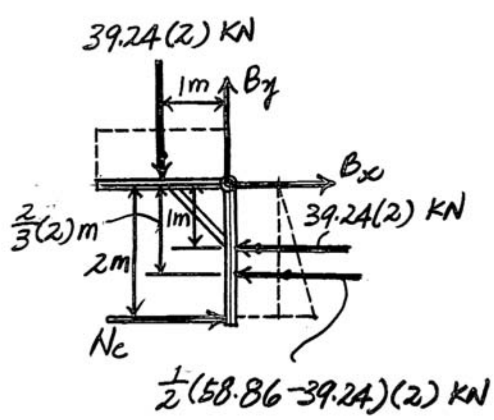
Equations of Equilibrium: Writing the moment equation of equilibrium about point B,

$$\circlearrowleft + \Sigma M_B = 0; \qquad N_C(2) + 39.24(2)(1) - 39.24(2)(1) - \frac{1}{2}(58.86 - 39.24)(2)\left(\frac{2}{3}\right)(2) = 0$$

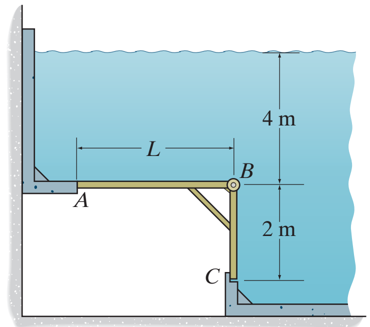
$$N_C = 13.08\ \mathrm{kN} = 13.1\ \mathrm{kN}$$ **Ans.**



(a)



9–114. If $L = 2$ m, determine the force the gate ABC exerts on the smooth stopper at C. The gate is hinged at B, free at A, and is 1 m wide. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points B and C are

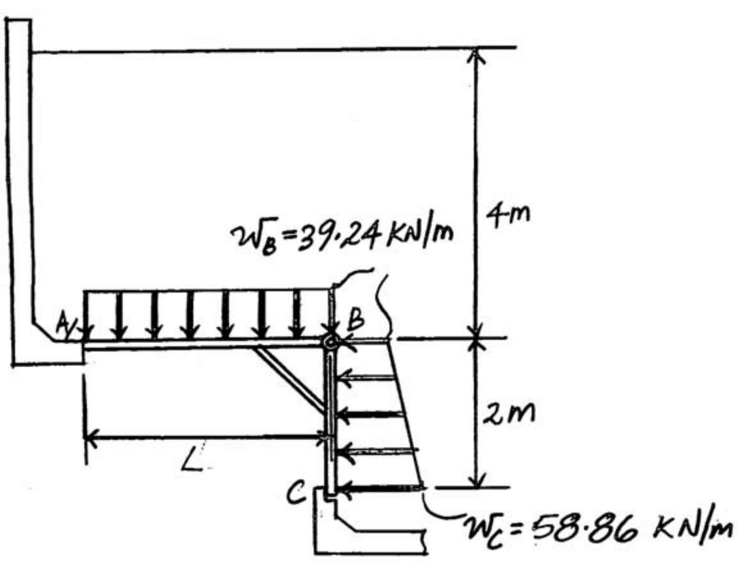
$$w_B = \rho_w g h_B b = 1000(9.81)(4)(1) = 39\ 240\ \text{N} = 39.24\ \text{kN}$$
$$w_C = \rho_w g h_C b = 1000(9.81)(6)(4+2) = 58\ 860\ \text{N} = 58.86\ \text{kN}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

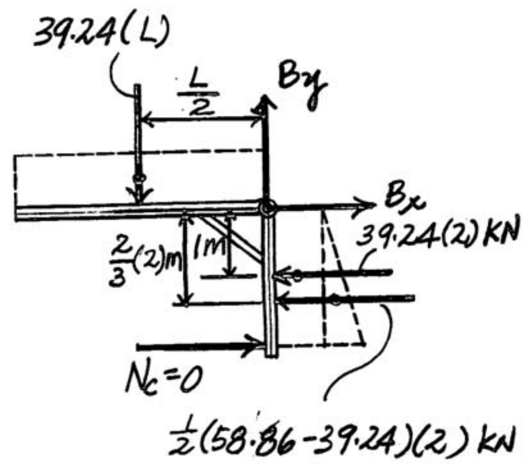
Equations of Equilibrium: Since the gate is on the verge of opening, $N_C = 0$. Writing the moment equation of equilibrium about point B,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 39.24(L)\left(\frac{L}{2}\right) - 39.24(2)(1) - \frac{1}{2}(58.86 - 39.24)(2)\left(\frac{2}{3}\right)(2) = 0$$

$$L = 2.31\ \text{m}$$ **Ans.**

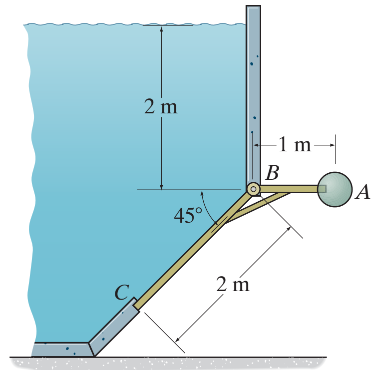


(a)



(b)

9–115. Determine the mass of the counterweight A if the 1-m-wide gate is on the verge of opening when the water is at the level shown. The gate is hinged at B and held by the smooth stop at C. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points B and C are

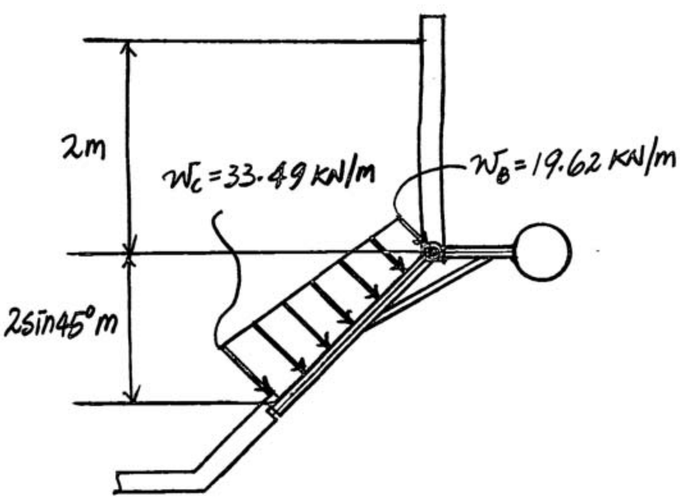
$$w_B = \rho_w g h_B b = 1000(9.81)(2)(1) = 19620\ \text{N/m} = 19.62\ \text{kN/m}$$
$$w_C = \rho_w g h_C b = 1000(9.81)(2 + 2\sin 45°)(1) = 33493.44\ \text{N/m} = 33.49\ \text{kN/m}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. a. This distributed loading is replaced by its resultant force on the free - body diagram of the gate, Fig. b.

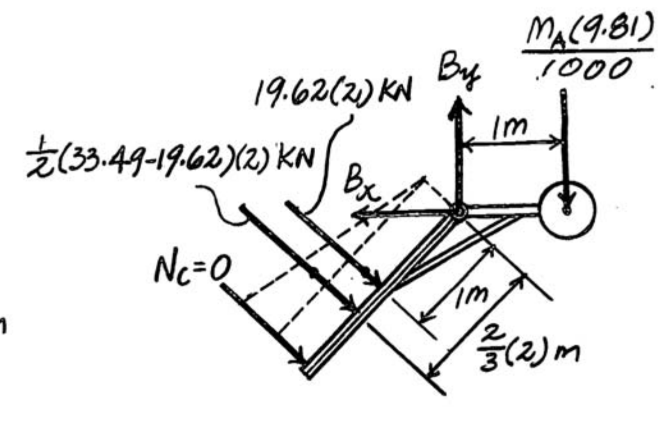
Equations of Equilibrium: Since the gate is on the verge of opening, $N_C = 0$. Writing the moment equation of equilibrium about point B,

$$\curvearrowleft + \Sigma M_B = 0; \qquad 19.62(2)(1) + \frac{1}{2}(33.49 - 19.62)(2)\left(\frac{2}{3}\right)(2) - \frac{m_A(9.81)(1)}{1000} = 0$$

$$m_A = 5885.62\ \text{kg} = 5.89\ \text{Mg}$$ **Ans.**

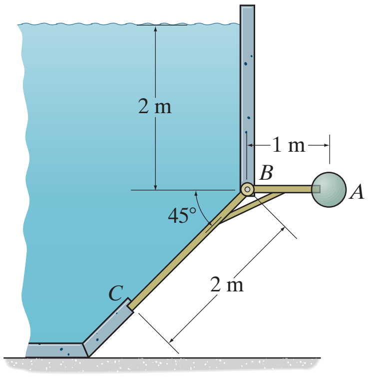


(a)



(b)

***9–116.** If the mass of the counterweight at A is 6500 kg, determine the force the gate exerts on the smooth stop at C. The gate is hinged at B and is 1-m wide. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



Loading: Since the gate has a constant width, the hydrostatic loading on the gate can be represented by a two dimensional distributed loading. The intensity of the distributed loading at points B and C are

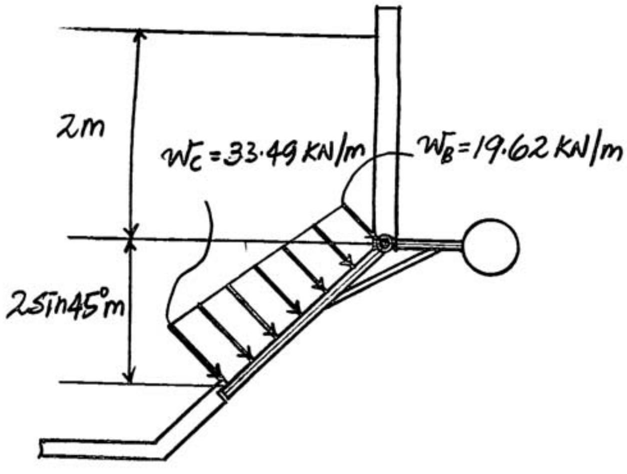
$$w_B = \rho_w g h_B b = 1000(9.81)(2)(1) = 19620\ \text{N/m} = 19.62\ \text{kN/m}$$
$$w_C = \rho_w g h_C b = 1000(9.81)(2 + 2\sin 45°)(1) = 33493.44\ \text{N/m} = 33.49\ \text{kN/m}$$

Free - Body Diagram: The distributed loading acting on the gate is shown in Fig. *a*. This distributed loading is replaced by its resultant force on the free - body diagram of the gate, Fig. *b*.

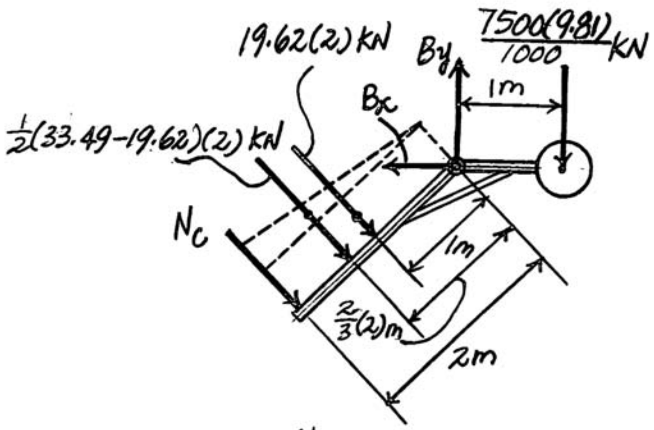
Equations of Equilibrium: Writing the moment equation of equilibrium about point B,

$$\circlearrowleft + \Sigma M_B = 0; \qquad N_C(2) + \frac{1}{2}(33.49 - (19.62)(2)\left(\frac{2}{3}\right)(2) + 19.62(2)(1) - \frac{6500(9.81)}{1000}(1) = 0$$

$$N_C = 3.02\ \text{kN}$$ **Ans.**

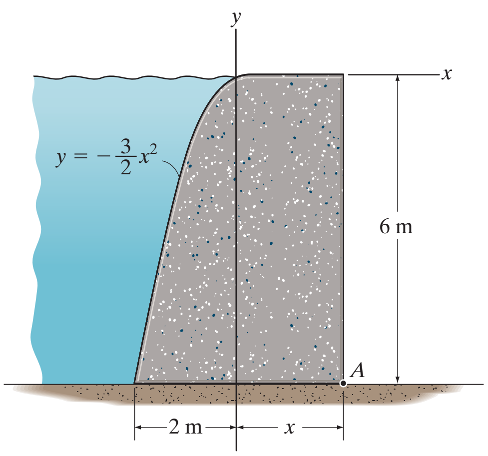


(a)



(b)

•9–117. The concrete gravity dam is designed so that it is held in position by its own weight. Determine the factor of safety against overturning about point A if $x = 2$ m. The factor of safety is defined as the ratio of the stabilizing moment divided by the overturning moment. The densities of concrete and water are $\rho_{conc} = 2.40\ \text{Mg/m}^3$ and $\rho_w = 1\ \text{Mg/m}^3$, respectively. Assume that the dam does not slide.



Resultant Force Component: The analysis of this problem will be based on a per meter width of the dam. The hydrostatic force acting on the parabolic surface of the dam consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*. The vertical component $\mathbf{F}_v$ consists of the weight of water contained in the shaded area shown in Fig. *a*.

$$F_v = \rho_w g A_{BCD}(1) = (1000)(9.81)\left[\frac{1}{3}(2)(6)(1)\right] = 39240\text{ N} = 39.24\text{ kN}$$

The horizontal component $\mathbf{F}_h$ consists of the horizontal hydrostatic pressure, which can be represented by a triangular distributed loading shown in Fig. *a*. The intensity of the distributed loading at point B is $w_B = \rho_w g h_B b = 1000(9.81)(6)(1) = 58860\text{ N/m} = 58.86\text{ kN/m}$. Thus,

$$F_h = \frac{1}{2}(58.86)(6) = 176.58\text{ kN}$$

The weight of the parabolic shaped concrete dam is

$$(W_{con})_p = \rho_{con} gV = 2400(9.81)\left[\frac{2}{3}(2)(6)(1)\right] = 188\,352\text{ N} = 188.352\text{ kN}$$

The weight of the rectangular shaped concrete dam is

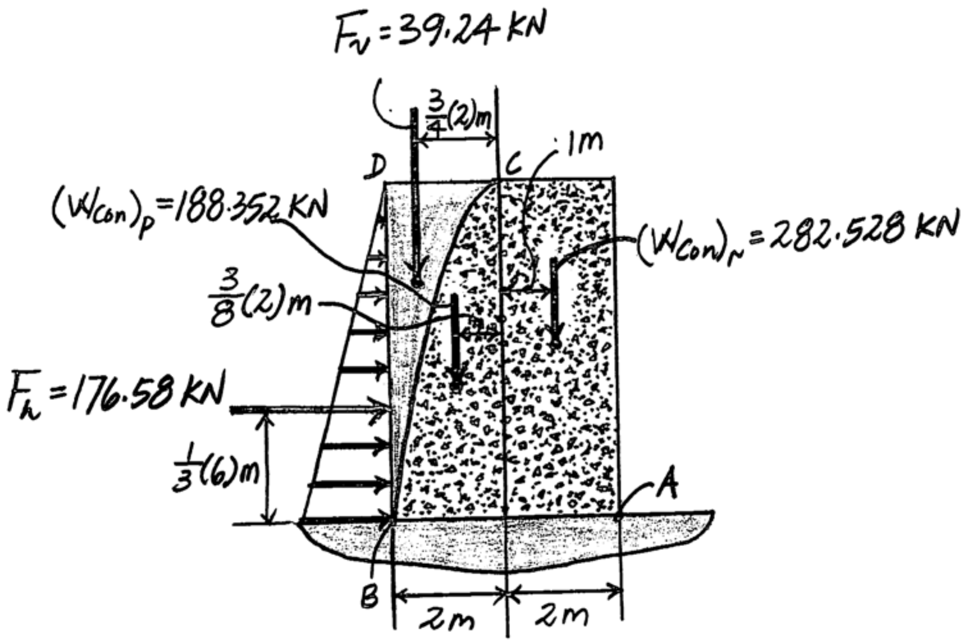
$$(W_{con})_r = \rho_{con} gV = 2400(9.81)[2(6)(1)] = 282\,528\text{ N} = 282.528\text{ kN}$$

Location: The location of each of the above forces are indicated in Fig. *a*. Here, $\mathbf{F}_h$ creates the overturning moment M_t about point A, while $\mathbf{F}_v$, $(\mathbf{W}_{con})_p$, and $(\mathbf{W}_{con})_r$ contribute to the stabilizing moment M_s about point A. Thus

$$\text{F.S.} = \frac{M_s}{M_t}$$

$$= \frac{282.528(1) + 188.352\left[2 + \frac{3}{8}(2)\right] + 39.24\left[2 + \frac{3}{4}(2)\right]}{176.58\left[\frac{1}{3}(6)\right]}$$

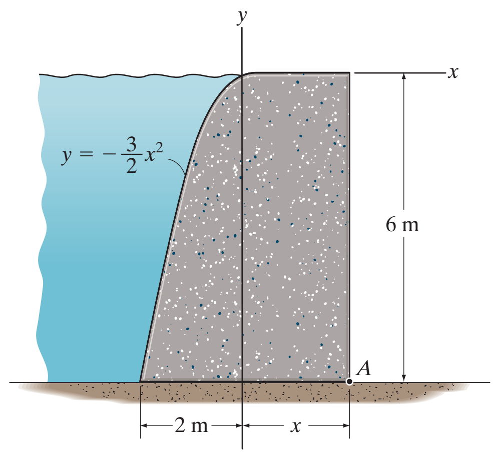
$$= 2.66 \qquad \textbf{Ans.}$$



$F_v = 39.24$ kN
$\frac{3}{4}(2)$m
1m
D
C
$(W_{con})_p = 188.352$ kN
$(W_{con})_r = 282.528$ kN
$\frac{3}{8}(2)$m
$F_h = 176.58$ kN
$\frac{1}{3}(6)$m
A
B
2m
2m

(a)

9–118. The concrete gravity dam is designed so that it is held in position by its own weight. Determine the minimum dimension x so that the factor of safety against overturning about point A of the dam is 2. The factor of safety is defined as the ratio of the stabilizing moment divided by the overturning moment. The densities of concrete and water are $\rho_{conc} = 2.40\ \text{Mg/m}^3$ and $\rho_w = 1\ \text{Mg/m}^3$, respectively. Assume that the dam does not slide.



Resultant Force Component: The analysis of this problem will be based on a per meter width of the dam. The hydrostatic force acting on the parabolic surface of the dam consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*. The vertical component $\mathbf{F}_v$ consists of the weight of water contained in the shaded area shown in Fig. *a*.

$$F_v = \rho_w g A_{BCD} b = (1000)(9.81)\left[\frac{1}{3}(2)(6)\right](1) = 39240\ \text{N} = 39.24\ \text{kN}$$

The horizontal component $\mathbf{F}_h$ consists of the horizontal hydrostatic pressure, which can be represented by a triangular distributed loading shown in Fig. *a*. The intensity of the distributed loading at point B is $w_B = \rho_w g h_B b = 1000(9.81)(6)(1) = 58860\ \text{N/m} = 58.86\ \text{kN/m}$. Thus,

$$F_h = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

The weight of the parabolic shaped concrete dam is

$$(W_{con})_p = \rho_{con} g V = 2400(9.81)\left[\frac{2}{3}(2)(6)(1)\right] = 188\,352\ \text{N} = 188.352\ \text{kN}$$

The weight of the rectangular shaped concrete dam is

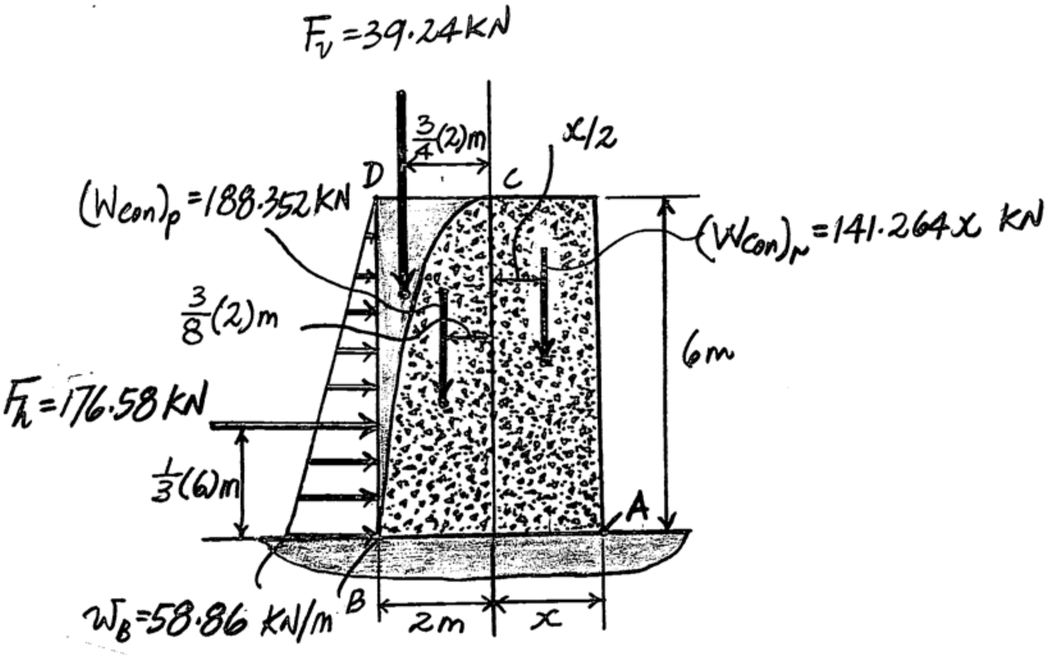
$$(W_{con})_r = \rho_{con} g V = 2400(9.81)(6)(x)(1) = 141\,264x\ \text{N} = 141.264x\ \text{kN}$$

Location: The location of each of the above forces are indicated in Fig. *a*. Here, $\mathbf{F}_h$ creates the overturning moment M_t about point A, while $\mathbf{F}_v$, $(\mathbf{W}_{con})_p$, and $(\mathbf{W}_{con})_r$ contribute to the stabilizing moment M_s about point A. Thus

$$\text{F.S.} = \frac{M_s}{M_t}$$

$$2 = \frac{141.264(x)\left(\frac{x}{2}\right) + 188.352\left[\frac{3}{8}(2) + x\right] + 39.24\left[\frac{3}{4}(2) + x\right]}{176.58\left[\frac{1}{3}(6)\right]}$$

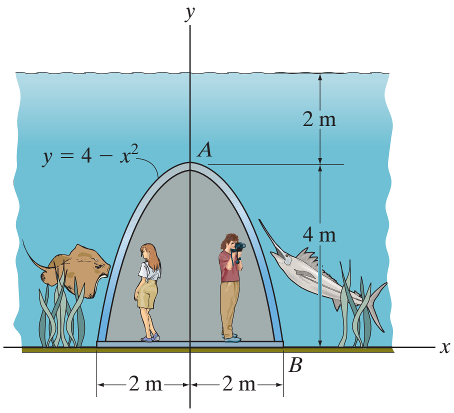
$$x = 1.51\ \text{m}$$ **Ans.**



$F_v = 39.24$ kN
$\frac{3}{4}(2)$ m
$x/2$
D
C
$(W_{con})_p = 188.352$ kN
$(W_{con})_r = 141.264x$ kN
$\frac{3}{8}(2)$ m
6m
$F_h = 176.58$ kN
$\frac{1}{3}(6)$ m
A
$w_B = 58.86$ kN/m
B
2m
x

(a)

9–119. The underwater tunnel in the aquatic center is fabricated from a transparent polycarbonate material formed in the shape of a parabola. Determine the magnitude of the hydrostatic force that acts per meter length along the surface AB of the tunnel. The density of the water is $\rho_w = 1000\ \text{kg/m}^3$.



Resultant Force Component: The hydrostatic force acting on the parabolic surface AB of the tunnel consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*.
The vertical component $\mathbf{F}_v$ represents the weight of water contained in the shaded area shown in Fig. *a*

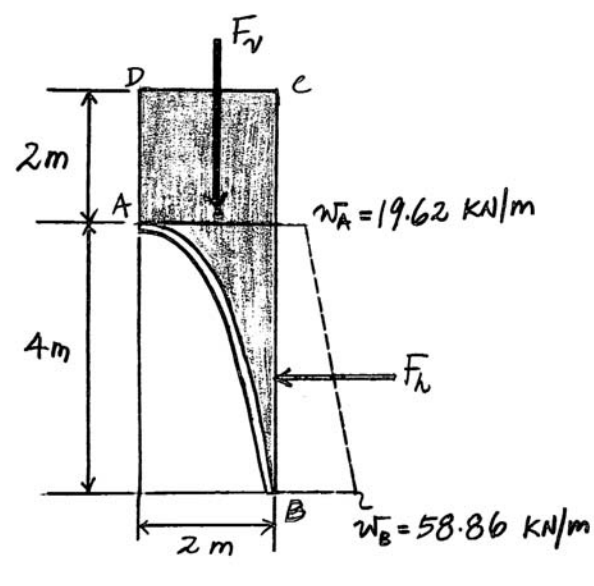
$$F_v = \rho_w g A_{ABCD} b = (1000)(9.81)\left[2(2) + \frac{1}{3}(2)(4)\right](1) = 65\,400\ \text{N} = 65.4\ \text{kN}$$

The horizontal component $\mathbf{F}_h$ represents the horizontal hydrostatic pressure. Since the width of the tunnel is constant (1 m), this horizontal loading can be represented by a trapezoidal distributed loading shown in Fig. *a*. The intensity of this distributed loading at points A and B are $w_A = \rho_w g h_A b = 1000(9.81)(2)(1) = 19\,620\ \text{N/m}$ and $w_B = \rho_w g h_B b = 1000(9.81)(6)(1) = 58860\ \text{N/m} = 58.86\ \text{kN/m}$. Thus,

$$F_h = \frac{1}{2}(19.62 + 58.86)(4) = 156.96\ \text{kN}$$

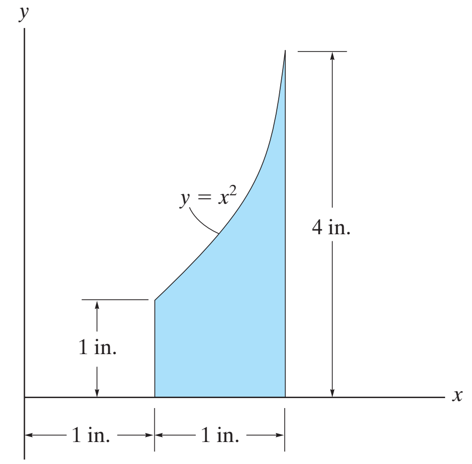
Resultant: The resultant hydrostatic force acting on the surface AB of the tunnel is therefore

$$F_R = \sqrt{F_h{}^2 + F_v{}^2} = \sqrt{156.96^2 + 65.4^2} = 170\ \text{kN}$$ **Ans.**



(a)

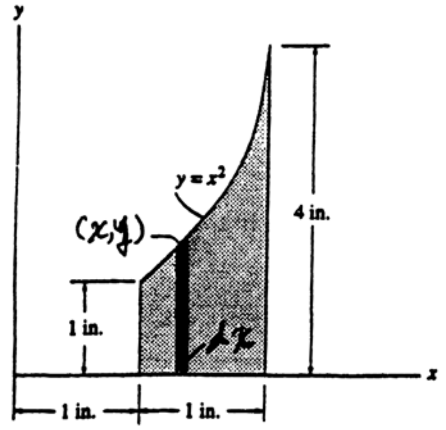
***9–120.** Locate the centroid $\bar{x}$ of the shaded area.



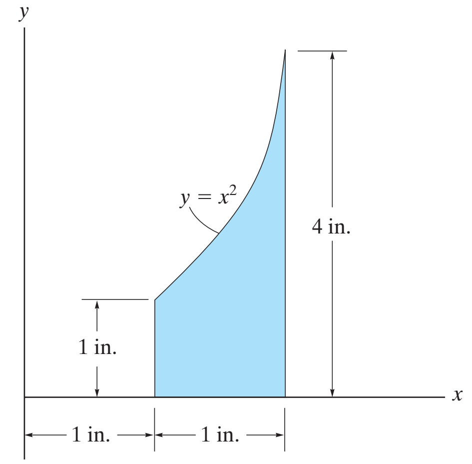
$dA = y\,dx = x^2\,dx$

$\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_1^2 x^3\,dx}{\int_1^2 x^2\,dx} = 1.61 \text{ in.} \quad \text{Ans}$$



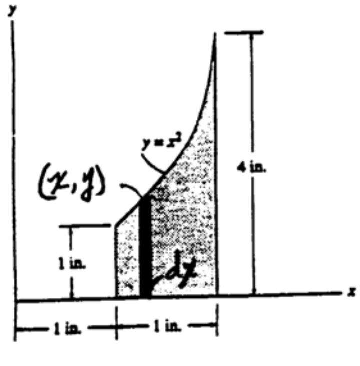
•9–121. Locate the centroid $\bar{y}$ of the shaded area.



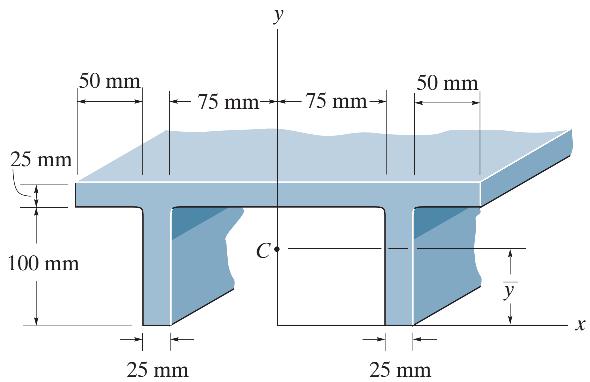
$dA = y\,dx = x^2\,dx$

$\tilde{y} = \frac{y}{2} = \frac{x^2}{2}$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_1^2 x^4\,dx}{\int_1^2 x^2\,dx} = \frac{\frac{1}{10}\left[(2)^5 - (1)^5\right]}{\frac{1}{3}\left[(2)^3 - (1)^3\right]} = 1.33 \text{ in.} \quad \text{Ans}$$

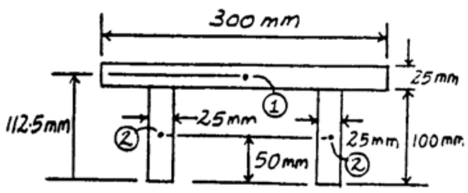


9–122. Locate the centroid $\bar{y}$ of the beam's cross-sectional area.



Centroid : The area of each segment and its respective centroid are tabulated below.

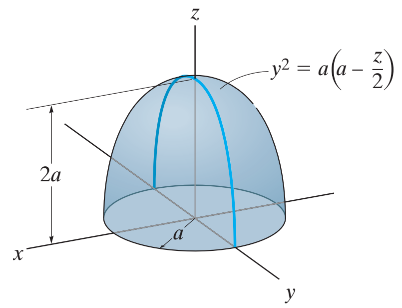
Segment	$A\,(\text{mm}^2)$	$\tilde{y}\,(\text{mm})$	$\tilde{y}A\,(\text{mm}^3)$
1	300(25)	112.5	843 750
2	100(50)	50	250 000
Σ	12 500		1 093 750



Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{1\,093\,750}{12\,500} = 87.5 \text{ mm} \qquad \textbf{Ans}$$

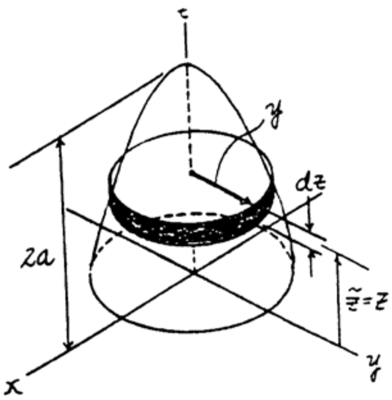
9–123. Locate the centroid $\bar{z}$ of the solid.



Volume and Moment Arm : The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi\left[a\left(a - \frac{z}{2}\right)\right]dz = \pi a\left(a - \frac{z}{2}\right)dz$ and its centroid is at $\tilde{z} = z$.

Centroid : Due to symmetry about the z axis

$$\bar{x} = \bar{y} = 0 \qquad \textbf{Ans}$$

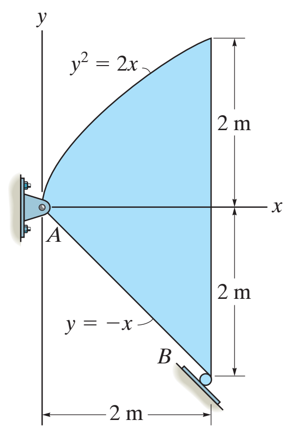


Applying Eq. 9–3 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{2a} z\left[\pi a\left(a - \frac{z}{2}\right)dz\right]}{\int_0^{2a} \pi a\left(a - \frac{z}{2}\right)dz}$$

$$= \frac{\pi a\left(\frac{az^2}{2} - \frac{z^3}{6}\right)\Big|_0^{2a}}{\pi a\left(az - \frac{z^2}{4}\right)\Big|_0^{2a}} = \frac{2}{3}a \qquad \textbf{Ans}$$

***9–124.** The steel plate is 0.3 m thick and has a density of 7850 kg/m^3. Determine the location of its center of mass. Also compute the reactions at the pin and roller support.



Fluid Pressure : The fluid pressure at the toe of the dam can be determined using Eq. 9 – 15, $p = \gamma z$.

$$p = 62.4(8) = 499.2 \text{ lb/ft}^2$$

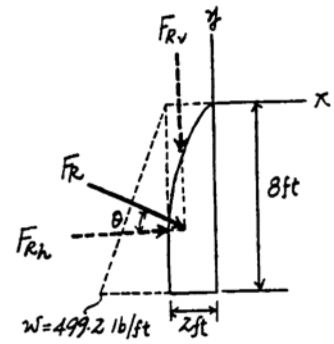
Thus,

$$w = 499.2(1) = 499.2 \text{ lb/ft}$$

Resultant Forces : From the inside back cover of the text, the exparabolic area is $A = \frac{1}{3}ab = \frac{1}{3}(8)(2) = 5.333 \text{ ft}^2$. Then, the vertical and horizontal components of the resultant force are

$$F_{R_v} = \gamma V = 62.4[5.333(1)] = 332.8 \text{ lb}$$

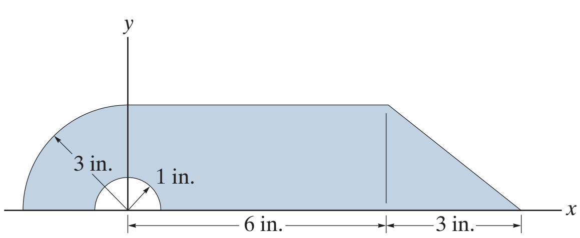
$$F_{R_h} = \frac{1}{2}(499.2)(8) = 1996.8 \text{ lb}$$



The resultant force and is

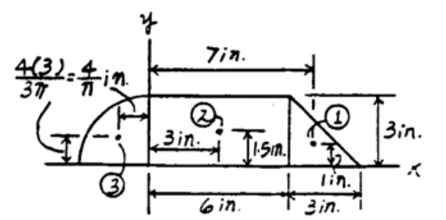
$$F_R = \sqrt{F_{R_v}^2 + F_{R_h}^2} = \sqrt{332.8^2 + 1996.8^2}$$
$$= 2024.34 \text{ lb} = 2.02 \text{ kip} \qquad \textbf{Ans}$$

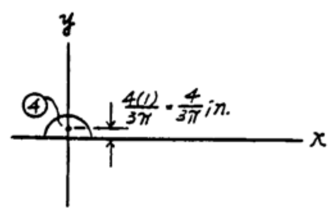
•9–125. Locate the centroid $(\bar{x}, \bar{y})$ of the area.



Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (in^2)	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$ (in^3)	$\tilde{y}A$ (in^3)
1	$\frac{1}{2}(3)(3)$	7	1	31.5	4.50
2	6(3)	3	1.5	54.0	27.0
3	$\frac{\pi}{4}(3^2)$	$-\frac{4}{\pi}$	$\frac{4}{\pi}$	−9.00	9.00
4	$-\frac{\pi}{2}(1^2)$	0	$\frac{4}{3\pi}$	0	−0.667
Σ	27.998			76.50	39.833



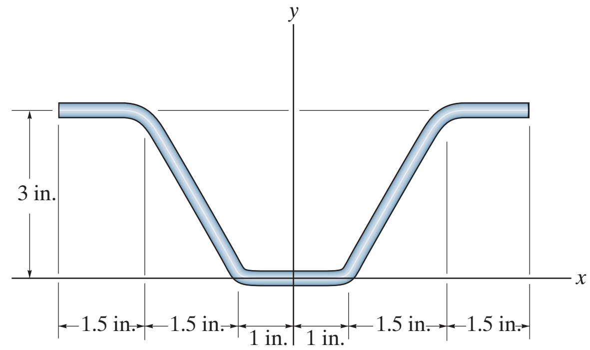


Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{76.50}{27.998} = 2.73 \text{ in.} \qquad \textbf{Ans}$$

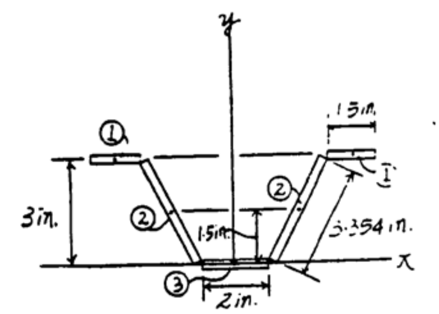
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{39.833}{27.998} = 1.42 \text{ in.} \qquad \textbf{Ans}$$

9–126. Determine the location $(\bar{x}, \bar{y})$ of the centroid for the structural shape. Neglect the thickness of the member.



Centroid : The length of each segment and its respective centroid are tabulated below.

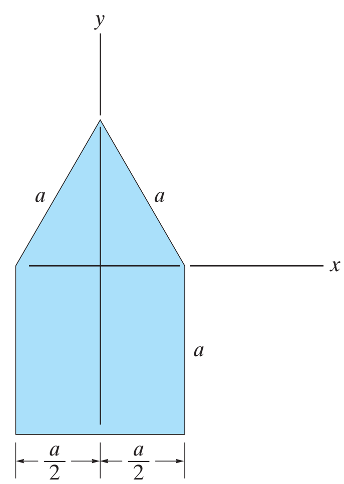
Segment	L(in.)	$\tilde{y}$(in.)	$\tilde{y}L$(in²)
1	2(1.5)	3	9.00
2	2(3.354)	1.5	10.06
3	2	0	0
Σ	11.71		19.06

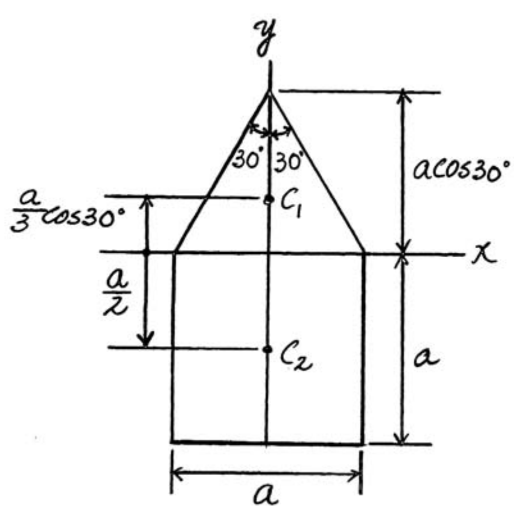


Due to symmetry about y axis, $\bar{x} = 0$ **Ans**

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{19.06}{11.71} = 1.628 \text{ in.} = 1.63 \text{ in.}$$ **Ans**

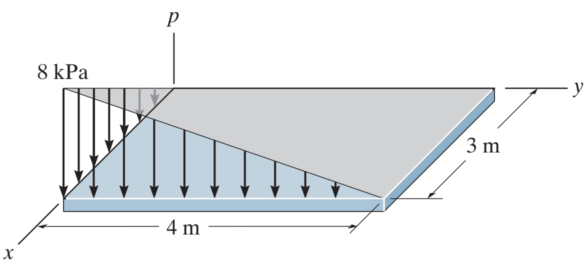
9–127. Locate the centroid $\bar{y}$ of the shaded area.



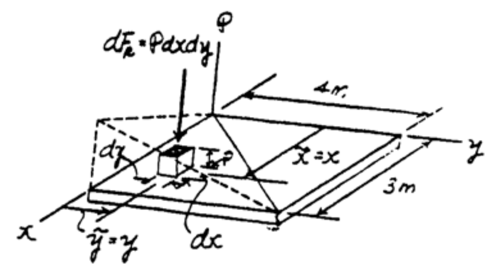


$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{a}{3}\cos 30°\left[\frac{1}{2}(a)(a\cos 30°)\right] - \frac{a}{2}[a(a)]}{\frac{1}{2}(a)(a\cos 30°) + [a(a)]} = -0.262a$$

***9–128.** The load over the plate varies linearly along the sides of the plate such that $p = \frac{2}{3}[x(4 - y)]$ kPa. Determine the resultant force and its position $(\bar{x}, \bar{y})$ on the plate.



Resultant Force and its Location : The volume of the differential element is $dV = dF_R = pdxdy = \frac{2}{3}(xdx)[(4-y)\,dy]$ and its centroid are $\tilde{x} = x$ and $\tilde{y} = y$.



$$F_R = \int_{F_R} dF_R = \int_0^{3\text{m}} \frac{2}{3}(xdx)\int_0^{4\text{m}}(4-y)\,dy$$
$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{m}}\right] = 24.0\text{ kN} \qquad \textbf{Ans}$$

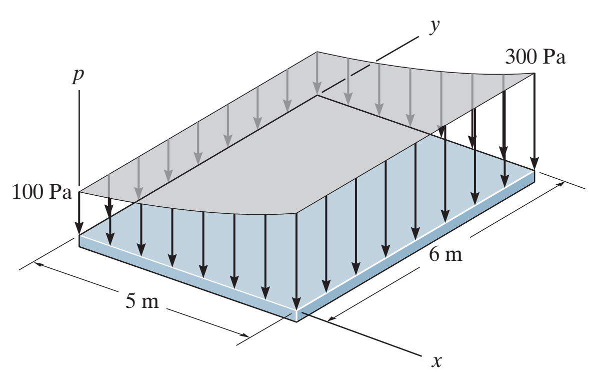
$$\int_{F_R} \tilde{x}dF_R = \int_0^{3\text{m}} \frac{2}{3}(x^2dx)\int_0^{4\text{m}}(4-y)\,dy$$
$$= \frac{2}{3}\left[\left(\frac{x^3}{3}\right)\Big|_0^{3\text{m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{m}}\right] = 48.0\text{ kN}\cdot\text{m}$$

$$\int_{F_R} \tilde{y}dF_R = \int_0^{3\text{m}} \frac{2}{3}(xdx)\int_0^{4\text{m}}y(4-y)\,dy$$
$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{m}}\left(2y^2 - \frac{y^3}{3}\right)\Big|_0^{4\text{m}}\right] = 32.0\text{ kN}\cdot\text{m}$$

$$\bar{x} = \frac{\int_{F_R}\tilde{x}dF_R}{\int_{F_R}dF_R} = \frac{48.0}{24.0} = 2.00\text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\int_{F_R}\tilde{y}dF_R}{\int_{F_R}dF_R} = \frac{32.0}{24.0} = 1.33\text{ m} \qquad \textbf{Ans}$$

•9–129. The pressure loading on the plate is described by the function $p = \{-240/(x + 1) + 340\}$ Pa. Determine the magnitude of the resultant force and coordinates of the point where the line of action of the force intersects the plate.

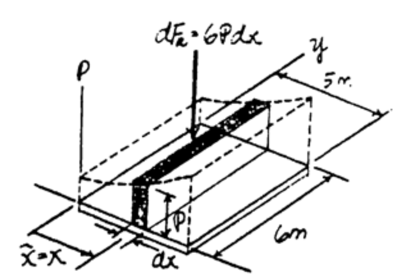


Resultant Force and its Location : The volume of the differential element is $dV = dF_R = 6pdx = 6\left(-\frac{240}{x+1} + 340\right)dx$ and its centroid is $\tilde{x} = x$.

$$F_R = \int_{F_R} dF_R = \int_0^{5\text{m}} 6\left(-\frac{240}{x+1} + 340\right)dx$$
$$= 6[-240\ln(x+1) + 340x]\big|_0^{5\text{m}}$$
$$= 7619.87\text{ N} = 7.62\text{ kN} \qquad \textbf{Ans}$$

$$\int_{F_R} \tilde{x}dF_R = \int_0^{5\text{m}} 6x\left(-\frac{240}{x+1} + 340\right)$$
$$= \left[-1440[x - \ln(x+1)] + 1020x^2\right]\big|_0^{5\text{m}}$$
$$= 20880.13\text{ N}\cdot\text{m}$$

$$\bar{x} = \frac{\int_{F_R}\tilde{x}dF_R}{\int_{F_R}dF_R} = \frac{20880.13}{7619.87} = 2.74\text{ m} \qquad \textbf{Ans}$$



Due to symmetry,

$$\bar{y} = 3.00\text{ m} \qquad \textbf{Ans}$$