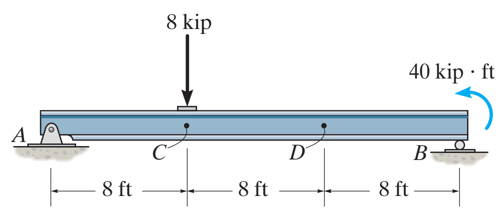
•7–1. Determine the internal normal force and shear force, and the bending moment in the beam at points *C* and *D*. Assume the support at *B* is a roller. Point *C* is located just to the right of the 8-kip load.

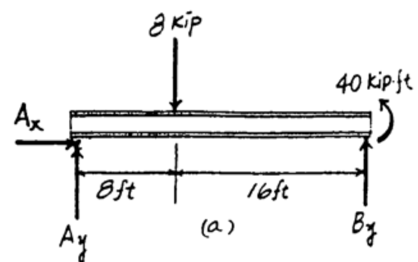


Support Reactions **: FBD (a).**

$\curvearrowleft + \Sigma M_A = 0;\quad B_y(24) + 40 - 8(8) = 0 \quad B_y = 1.00 \text{ kip}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 1.00 - 8 = 0 \quad A_y = 7.00 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0 \quad A_x = 0$

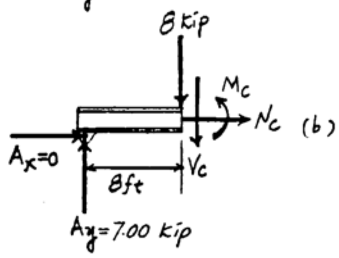


Internal Forces **: Applying the equations of equilibrium to segment *AC* [FBD (b)], we have**

$\xrightarrow{+} \Sigma F_x = 0;\quad N_C = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 7.00 - 8 - V_C = 0 \quad V_C = -1.00 \text{ kip}$ **Ans**

$\curvearrowleft + \Sigma M_C = 0;\quad M_C - 7.00(8) = 0 \quad M_C = 56.0 \text{ kip} \cdot \text{ft}$ **Ans**



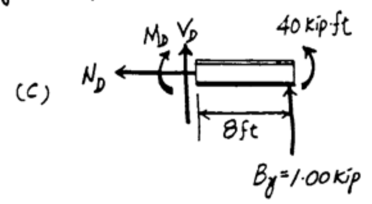
Applying the equations of equilibrium to segment *BD* [FBD (c)], we have

$\xrightarrow{+} \Sigma F_x = 0;\quad N_D = 0$ **Ans**

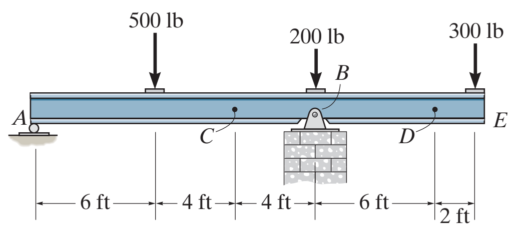
$+\uparrow \Sigma F_y = 0;\quad V_D + 1.00 = 0 \quad V_D = -1.00 \text{ kip}$ **Ans**

$\curvearrowleft + \Sigma M_D = 0;\quad 1.00(8) + 40 - M_D = 0$

$M_D = 48.0 \text{ kip} \cdot \text{ft}$ **Ans**



7–2. Determine the shear force and moment at points *C* and *D*.



Support Reactions : FBD (a).

$\circlearrowleft + \Sigma M_B = 0;\quad 500(8) - 300(8) - A_y(14) = 0$

$A_y = 114.29 \text{ lb}$

Internal Forces : Applying the equations of equilibrium to segment *AC* [FBD (b)], we have

$\xrightarrow{+} \Sigma F_x = 0;\quad N_C = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 114.29 - 500 - V_C = 0 \quad V_C = -386 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad M_C + 500(4) - 114.29(10) = 0$

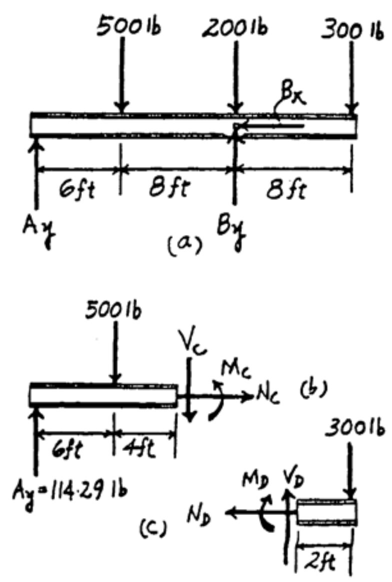
$M_C = -857 \text{ lb} \cdot \text{ft}$ **Ans**

Applying the equations of equilibrium to segment *ED* [FBD (c)], we have

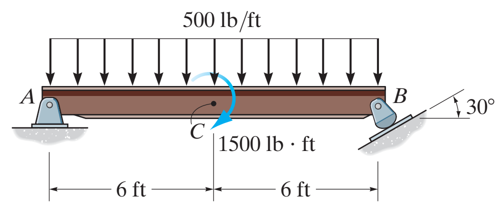
$\xrightarrow{+} \Sigma F_x = 0;\quad N_D = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad V_D - 300 = 0 \quad V_D = 300 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0;\quad -M_D - 300(2) = 0 \quad M_D = -600 \text{ lb} \cdot \text{ft}$ **Ans**



7–3. Determine the internal normal force, shear force, and moment at point C in the simply supported beam. Point C is located just to the right of the 1500-lb · ft couple moment.



Writing the moment equation of equilibrium about point A with reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;\quad F_B \cos 30°(12) - 500(12)(6) - 1500 = 0 \qquad F_B = 3608.44 \text{ lb}$

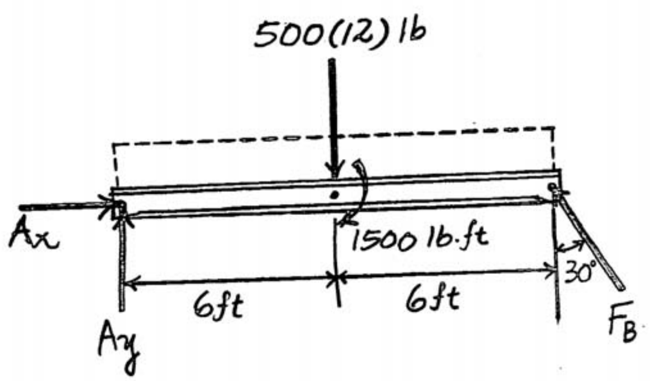
Using the result of $\mathbf{F}_B$ and referring to Fig. b,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -N_C - 3608.44 \sin 30° = 0 \qquad N_C = -1804 \text{ lb}$ **Ans.**

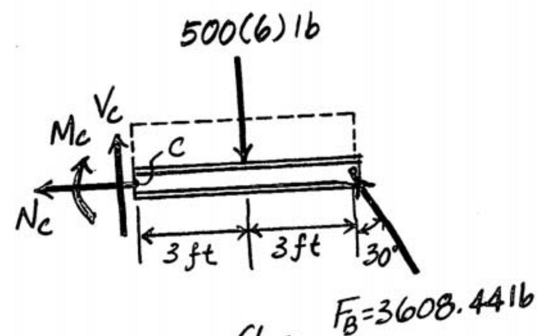
$+\uparrow \Sigma F_y = 0;\quad V_C + 3608.44 \cos 30° - 500(6) = 0 \qquad V_C = -125 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad 3608.44 \cos 30°(6) - 500(6)(3) - M_C = 0 \qquad M_C = 9750 \text{ lb} \cdot \text{ft}$ **Ans.**

The negative sign indicates that $\mathbf{N}_C$ and $\mathbf{V}_C$ act in the opposite sense to that shown on the free-body diagram.

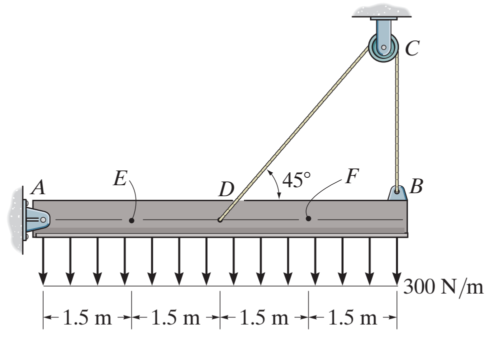


(a)



(b)

***7–4.** Determine the internal normal force, shear force, and moment at points E and F in the beam.



With reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad T(6) + T\sin 45°(3) - 300(6)(3) = 0 \quad T = 664.92\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 664.92\cos 45° - A_x = 0 \quad A_x = 470.17\text{ N}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 664.92\sin 45° + 664.92 - 300(6) = 0 \quad A_y = 664.92\text{ N}$

Using these results and referring to Fig. b,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_E - 470.17 = 0 \quad N_E = 470\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 664.92 - 300(1.5) - V_E = 0 \quad V_E = 215\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad M_E + 300(1.5)(0.75) - 664.92(1.5) = 0 \quad M_E = 660\text{ N}\cdot\text{m}$ **Ans.**

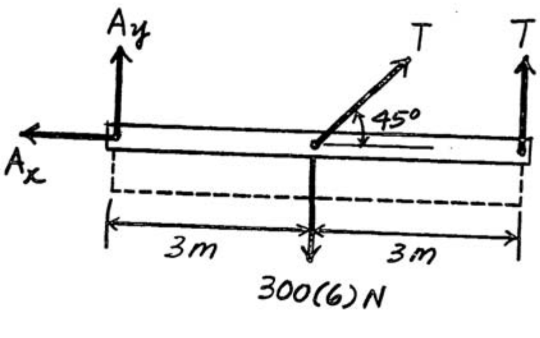
Also, by referring to Fig. c,

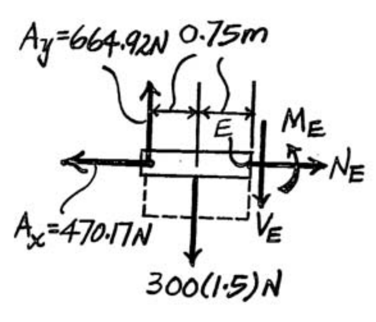
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_F = 0$ **Ans.**

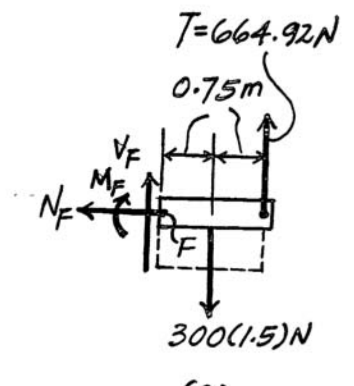
$+\uparrow \Sigma F_y = 0;$ $\quad V_F + 664.92 - 300(1.5) = 0 \quad V_F = -215\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0;$ $\quad 664.92(1.5) - 300(1.5)(0.75) - M_F = 0 \quad M_F = 660\text{ N}\cdot\text{m}$ **Ans.**

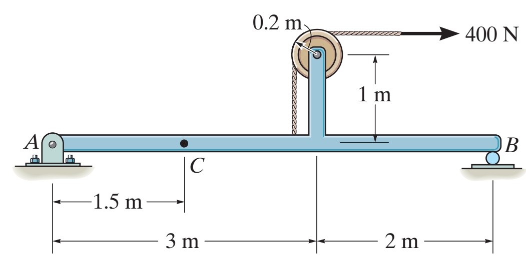
The negative sign indicates that $\mathbf{V}_F$ acts in the opposite sense to that shown on the free-body diagram.







•7–5. Determine the internal normal force, shear force, and moment at point C.



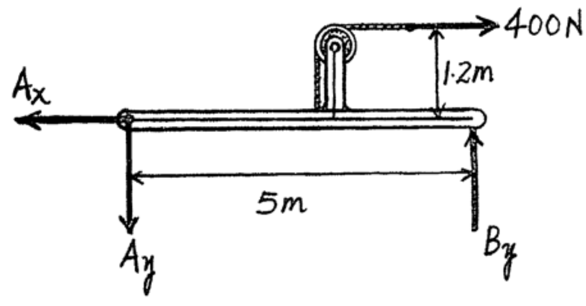
Beam :

$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 400 = 0$

$A_x = 400\text{ N}$

$\circlearrowleft + \Sigma M_B = 0; \quad A_y(5) - 400(1.2) = 0$

$A_y = 96\text{ N}$



Segment AC :

$\xrightarrow{+} \Sigma F_x = 0; \quad N_C - 400 = 0$

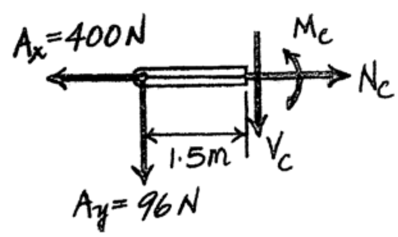
$N_C = 400\text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -96 - V_C = 0$

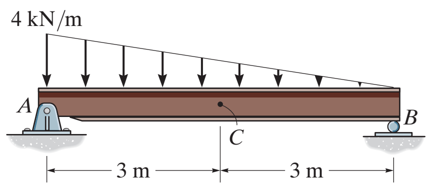
$V_C = -96\text{ N}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 96(1.5) = 0$

$M_C = -144\text{ N}\cdot\text{m}$ **Ans**



7–6. Determine the internal normal force, shear force, and moment at point C in the simply supported beam.



With reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(6) - \frac{1}{2}(4)(6)(2) = 0\qquad B_y = 4\text{ kN}$

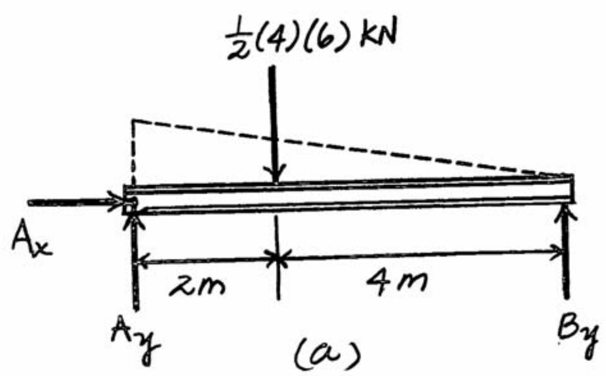
Using this result with reference to Fig. c,

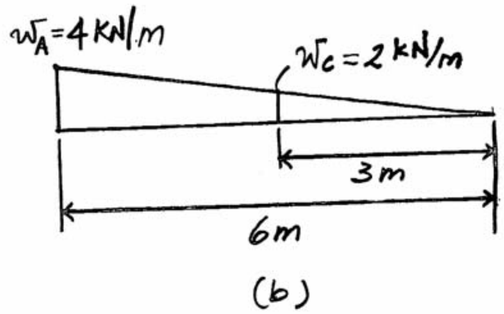
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_C = 0$ **Ans.**

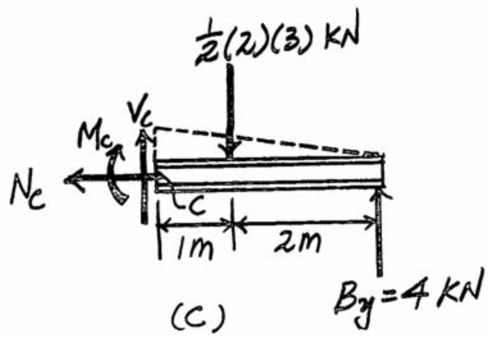
$+\uparrow \Sigma F_y = 0;\qquad 4 - \frac{1}{2}(2)(3) + V_C = 0\qquad V_C = -1\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\qquad 4(3) - \frac{1}{2}(2)(3)(1) - M_C = 0\qquad M_C = 9\text{ kN}\cdot\text{m}$ **Ans.**

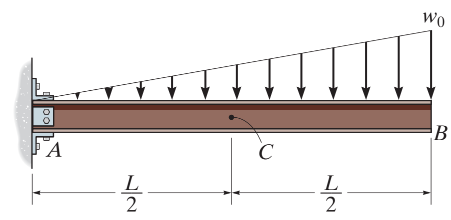
The negative sign indicates that $\mathbf{V}_C$ acts in the opposite sense to that shown on the free-body diagram.







7–7. Determine the internal normal force, shear force, and moment at point C in the cantilever beam.



The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. a,

$$\frac{w_C}{L/2} = \frac{w_0}{L} \text{ or } w_C = w_0/2$$

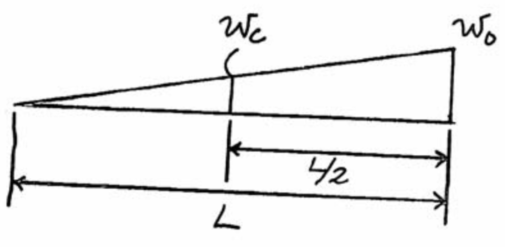
With reference to Fig. b,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_C = 0$ **Ans.**

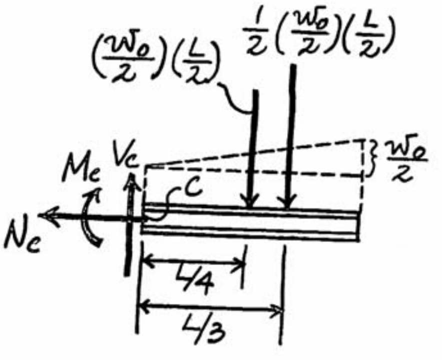
$+\uparrow \Sigma F_y = 0;$ $\quad V_C - \left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right) - \frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad V_C = \frac{3w_0 L}{8}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -M_C - \left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) - \frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = 0 \qquad M_C = -\frac{5}{48} w_0 L^2$ **Ans.**

The negative sign indicates that $\mathbf{M}_C$ acts in the opposite sense to that shown on the free-body diagram.

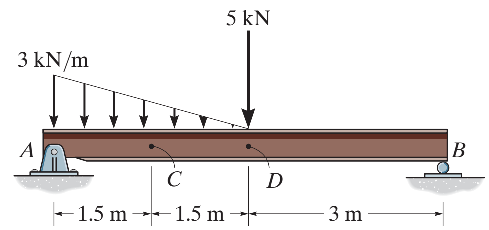


(a)



(b)

***7–8.** Determine the internal normal force, shear force, and moment at points C and D in the simply supported beam. Point D is located just to the left of the 5-kN force.



The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{1.5} = \frac{3}{3} \quad \text{or } w_C = 1.5 \text{ kN/m}$$

With reference to Fig. a,

$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(6) - 5(3) - \frac{1}{2}(3)(3)(1) = 0 \qquad B_y = 3.25 \text{ kN}$$

Using this result and referring to Fig. c,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_C + 3.25 - \frac{1}{2}(1.5)(1.5) - 5 = 0 \qquad V_C = 2.875 \text{ kN} \qquad \textbf{Ans.}$$

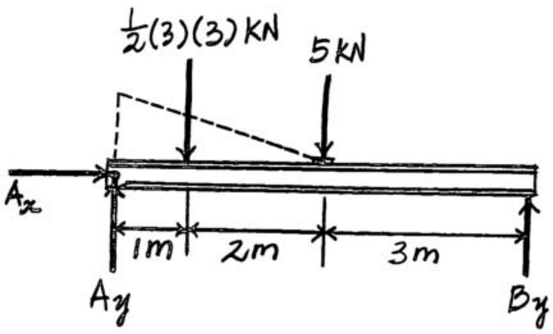
$$\curvearrowleft + \Sigma M_C = 0; \quad 3.25(4.5) - \frac{1}{2}(1.5)(1.5)(0.5) - 5(1.5) - M_C = 0 \qquad M_C = 6.56 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Also, referring to Fig. d,

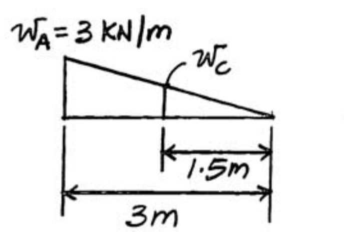
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_D + 3.25 - 5 = 0 \qquad V_D = 1.75 \text{ kN} \qquad \textbf{Ans.}$$

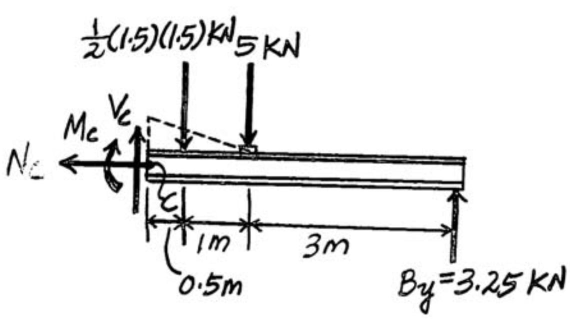
$$\curvearrowleft + \Sigma M_D = 0; \quad 3.25(3) - M_D \qquad M_D = 9.75 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



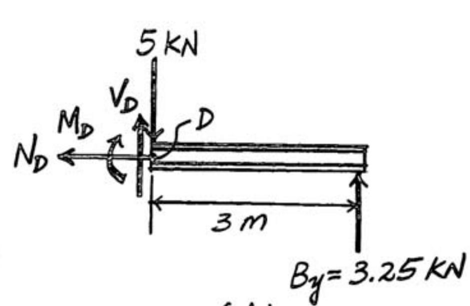
(a)



(b)

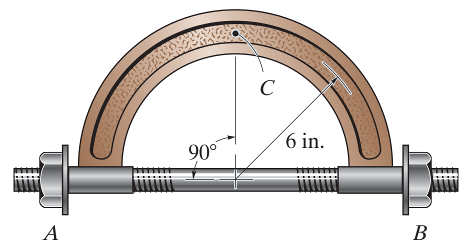


(c)



(d)

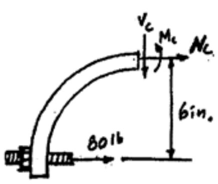
•7–9. The bolt shank is subjected to a tension of 80 lb. Determine the internal normal force, shear force, and moment at point *C*.



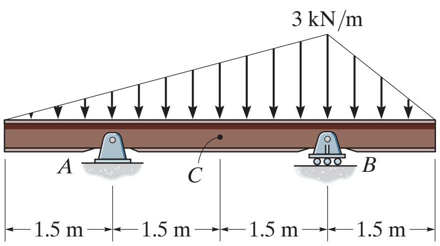
$\xrightarrow{+} \Sigma F_x = 0;\quad N_C + 80 = 0 \quad N_C = -80 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad V_C = 0$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad M_C + 80(6) = 0 \quad M_C = -480 \text{ lb} \cdot \text{in.}$ **Ans**



7–10. Determine the internal normal force, shear force, and moment at point C in the double-overhang beam.



The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{3} = \frac{3}{4.5} \quad \text{or} \quad w_C = 2\ \text{kN/m}$$

With reference to Fig. a,

$\curvearrowleft + \Sigma M_B = 0;\quad \frac{1}{2}(3)(4.5)(1.5) - \frac{1}{2}(3)(1.5)(0.5) - A_y(3) = 0 \qquad A_y = 3\ \text{kN}$

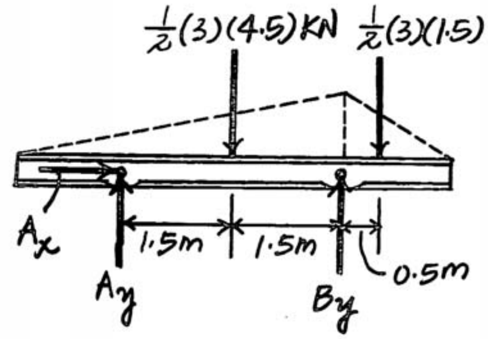
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$

Using the results of $\mathbf{A}_x$ and $\mathbf{A}_y$ and referring to Fig. c,

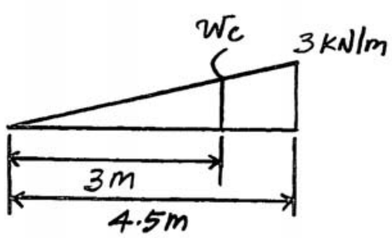
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 3 - \frac{1}{2}(2)(3) - V_C = 0 \qquad V_C = 0$ **Ans.**

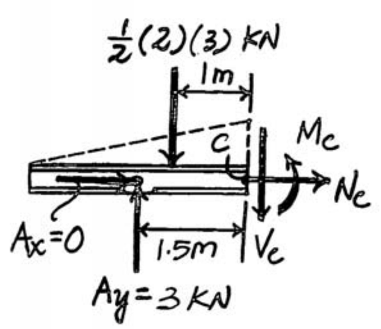
$\curvearrowleft + \Sigma M_C = 0;\quad M_C + \frac{1}{2}(2)(3)(1) - 3(1.5) = 0 \qquad M_C = 1.5\ \text{kN}\cdot\text{m}$ **Ans.**



(a)

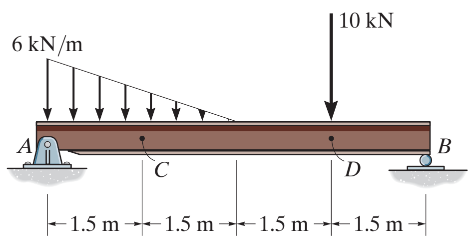


(b)



(c)

7–11. Determine the internal normal force, shear force, and moment at points C and D in the simply supported beam. Point D is located just to the left of the 10-kN concentrated load.



The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{1.5} = \frac{6}{3} \text{ or } w_C = 3 \text{kN / m}$$

With reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;\quad B_y(6) - 10(4.5) - \frac{1}{2}(6)(3)(1) = 0 \qquad B_y = 9 \text{ kN}$

$\circlearrowleft + \Sigma M_B = 0;\quad \frac{1}{2}(6)(3)(5) + 10(1.5) - A_y(6) = 0 \qquad A_y = 10 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$

Using these results and refering to Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 10 - \frac{1}{2}(3)(1.5) - 3(1.5) - V_C = 0 \qquad V_C = 3.25 \text{ kN}$ **Ans.**

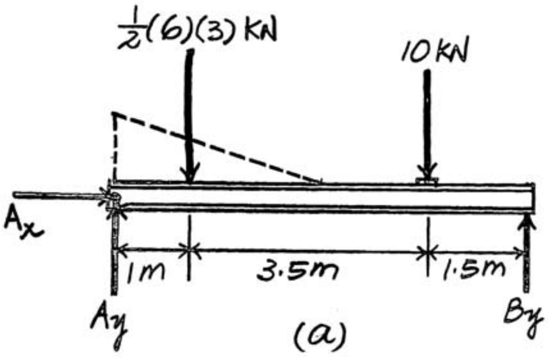
$\circlearrowleft + \Sigma M_C = 0;\quad M_C + 3(1.5)(0.75) + \frac{1}{2}(3)(1.5)(1) - 10(1.5) = 0 \qquad M_C = 9.375 \text{ kN} \cdot \text{m}$ **Ans.**

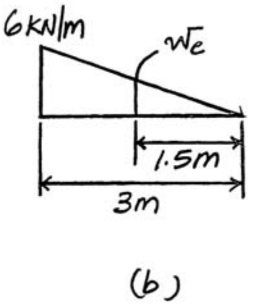
Also, by referring to Fig. d,

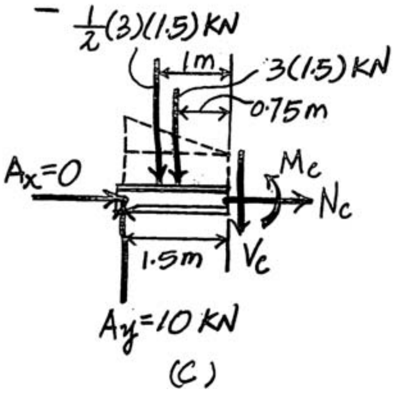
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans.**

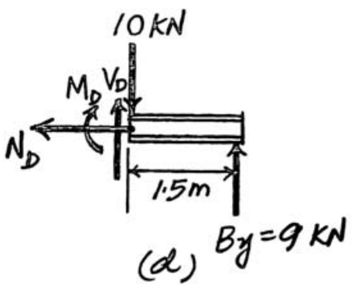
$+\uparrow \Sigma F_y = 0;\quad V_D + 9 - 10 = 0 \qquad V_D = 1 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;\quad 9(1.5) - M_D = 0 \qquad M_D = 13.5 \text{ kN} \cdot \text{m}$ **Ans.**

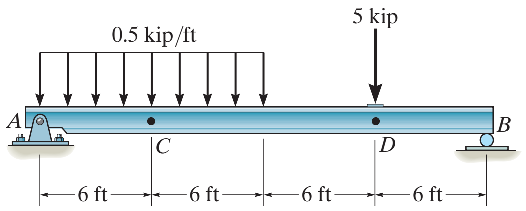








***7–12.** Determine the internal normal force, shear force, and moment in the beam at points C and D. Point D is just to the right of the 5-kip load.



Entire beam :

$\circlearrowleft + \Sigma M_B = 0; \quad 5(6) + 6(18) - A_y(24) = 0$

$A_y = 5.75 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

Segment AC :

$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0 \quad$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 5.75 - 3 - V_C = 0$

$V_C = 2.75 \text{ kip} \quad$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 3(3) - 5.75(6) = 0$

$M_C = 25.5 \text{ kip} \cdot \text{ft} \quad$ **Ans**

Segment AD :

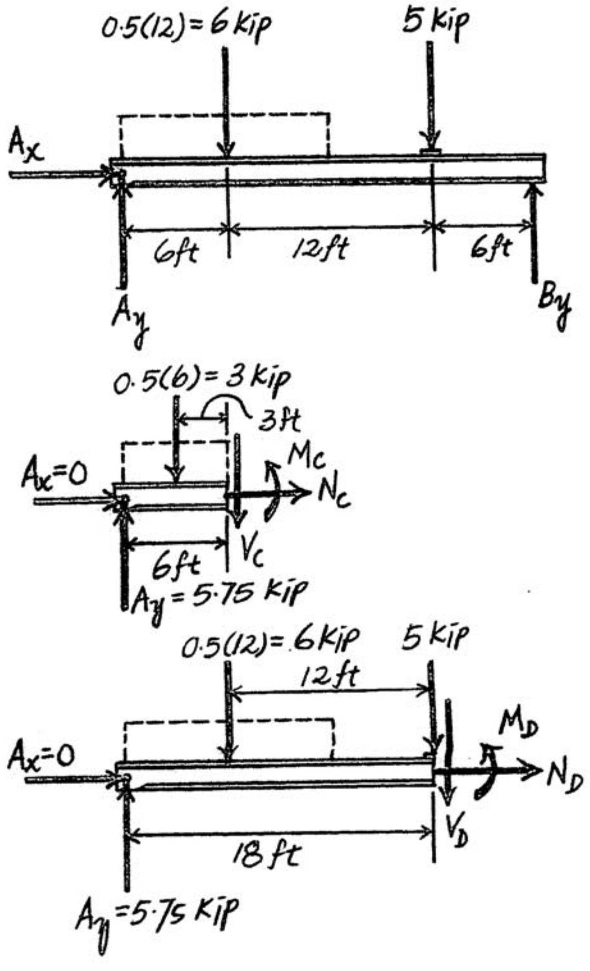
$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0 \quad$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 5.75 - 6 - 5 - V_D = 0$

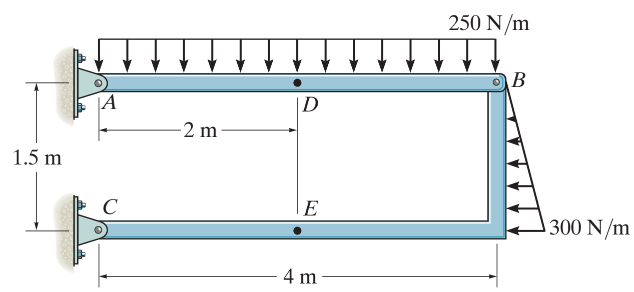
$V_D = -5.25 \text{ kip} \quad$ **Ans**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D + 6(12) - 5.75(18) = 0$

$M_D = 31.5 \text{ kip} \cdot \text{ft} \quad$ **Ans**



•**7–13.** Determine the internal normal force, shear force, and moment at point D of the two-member frame.



Member *AB*:

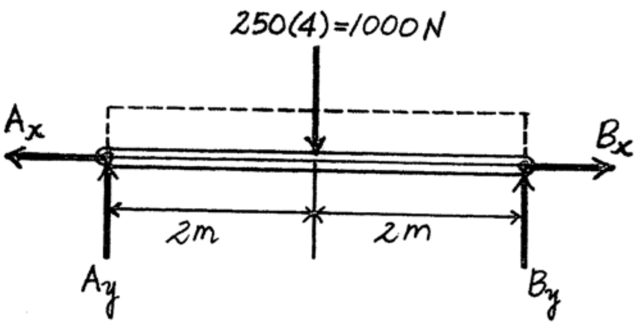
$\curvearrowleft + \Sigma M_A = 0; \quad B_y(4) - 1000(2) = 0$

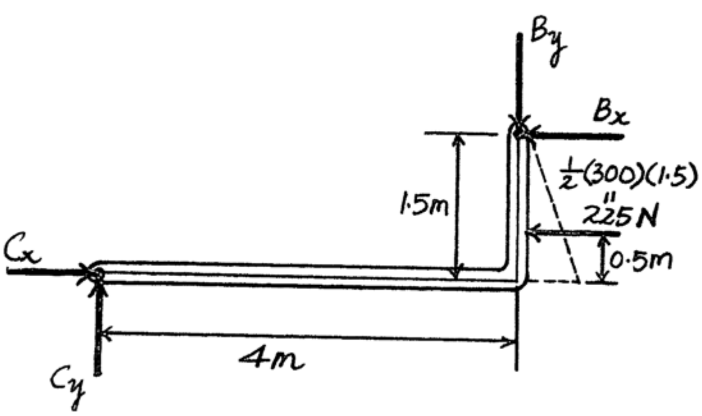
$B_y = 500 \text{ N}$

Member *BC*:

$\curvearrowleft + \Sigma M_C = 0; \quad -500(4) + 225(0.5) + B_x(1.5) = 0$

$B_x = 1258.33 \text{ N}$





Segment *DB*:

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_D + 1258.33 = 0$

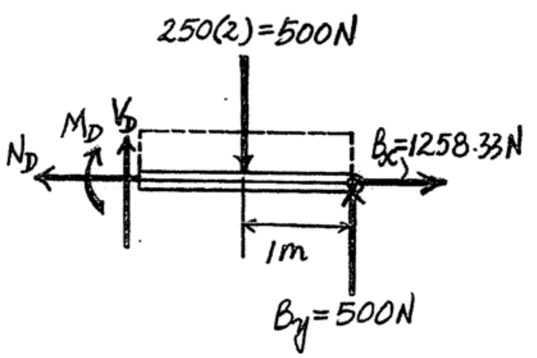
$N_D = 1.26 \text{ kN} \quad$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad V_D - 500 + 500 = 0$

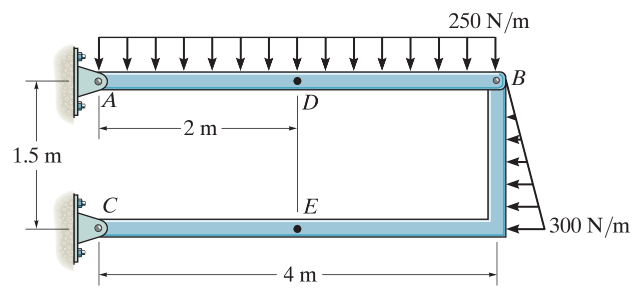
$V_D = 0 \quad$ **Ans**

$\curvearrowleft + \Sigma M_D = 0; \quad -M_D + 500(1) = 0$

$M_D = 500 \text{ N} \cdot \text{m} \quad$ **Ans**



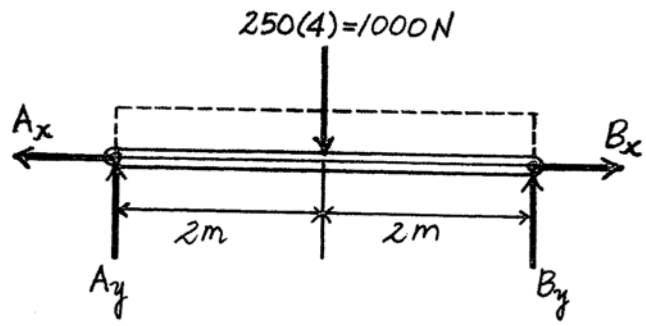
7–14. Determine the internal normal force, shear force, and moment at point E of the two-member frame.



Member AB:

$\circlearrowleft + \Sigma M_A = 0; \quad B_y (4) - 1000 (2) = 0$

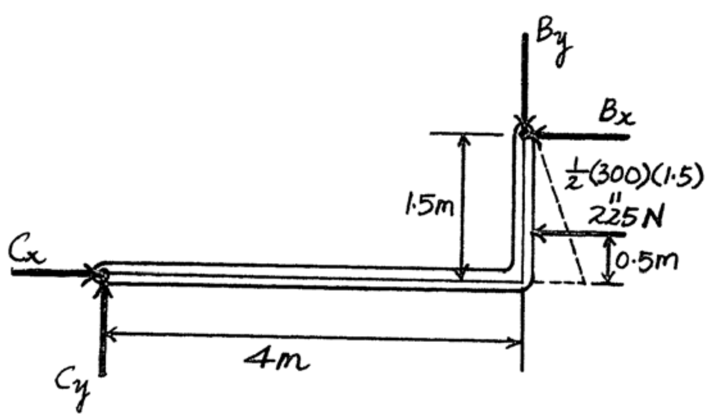
$B_y = 500 \text{ N}$



Member BC:

$\circlearrowleft + \Sigma M_C = 0; \quad -500 (4) + 225 (0.5) + B_x (1.5) = 0$

$B_x = 1258.33 \text{ N}$



Segment EB:

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_E - 1258.33 - 225 = 0$

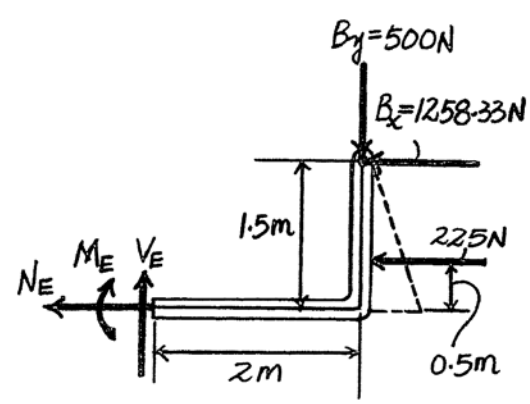
$N_E = -1.48 \text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad V_E - 500 = 0$

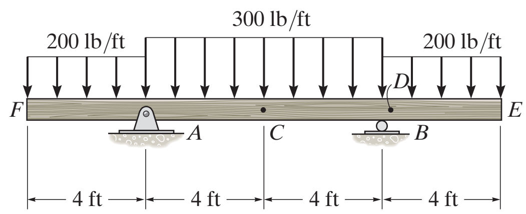
$V_E = 500 \text{ N}$ **Ans**

$\circlearrowleft + \Sigma M_E = 0; \quad -M_E + 225 (0.5) + 1258.33 (1.5) - 500 (2) = 0$

$M_E = 1000 \text{ N} \cdot \text{m}$ **Ans**



7–15. Determine the internal normal force, shear force, and moment acting at point C and at point D, which is located just to the right of the roller support at B.



Support Reactions : From FBD (a),

$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(8) + 800(2) - 2400(4) - 800(10) = 0$$
$$B_y = 2000 \text{ lb}$$

Internal Forces : Applying the equations of equilibrium to segment ED [FBD (b)], we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_D - 800 = 0 \quad V_D = 800 \text{ lb} \qquad \textbf{Ans}$$

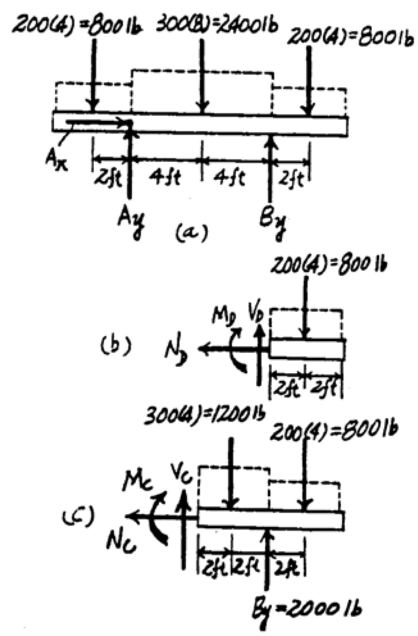
$$\curvearrowleft + \Sigma M_D = 0; \quad -M_D - 800(2) = 0$$
$$M_D = -1600 \text{ lb}\cdot\text{ft} = -1.60 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$

Applying the equations of equilibrium to segment EC [FBD (c)] , we have

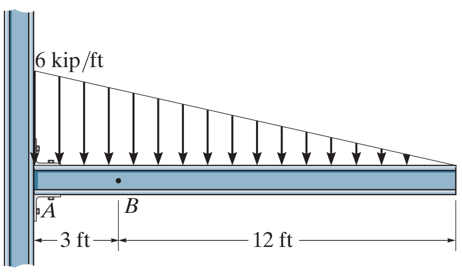
$$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_C + 2000 - 1200 - 800 = 0 \quad V_C = 0 \qquad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M_C = 0; \quad 2000(4) - 1200(2) - 800(6) - M_C = 0$$
$$M_C = 800 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$



***7–16.** Determine the internal normal force, shear force, and moment in the cantilever beam at point B.



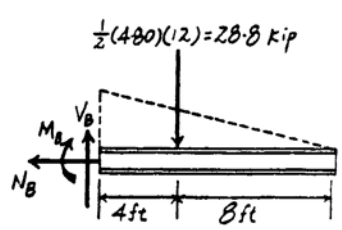
Free body Diagram : The support reactions at A need not be computed.

Internal Forces : Applying the equations of equilibrium to segment CB, we have

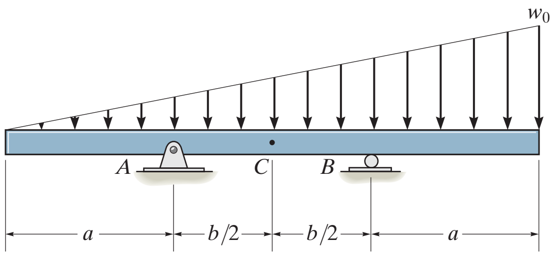
$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad V_B - 28.8 = 0 \quad V_B = 28.8 \text{ kip} \qquad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M_B = 0; \quad -28.8(4) - M_B = 0$$
$$M_B = -115 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$



•7–17. Determine the ratio of a/b for which the shear force will be zero at the midpoint C of the double-overhang beam.



Support Reactions : From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \quad \frac{1}{2}(2a+b)\,w\left[\frac{1}{3}(b-a)\right] - A_y\,(b) = 0$$

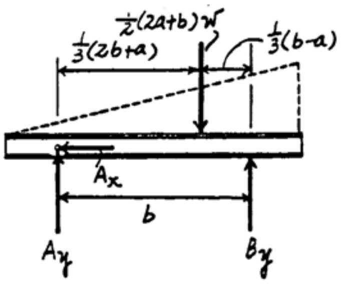
$$A_y = \frac{w}{6b}(2a+b)(b-a)$$

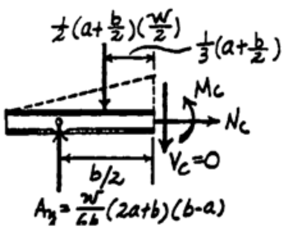
Internal Forces : This problem requires $V_C = 0$. Summing forces vertically [FBD (b)], we have

$$+\uparrow \Sigma F_y = 0; \quad \frac{w}{6b}(2a+b)(b-a) - \frac{1}{2}\left(a+\frac{b}{2}\right)\left(\frac{w}{2}\right) = 0$$

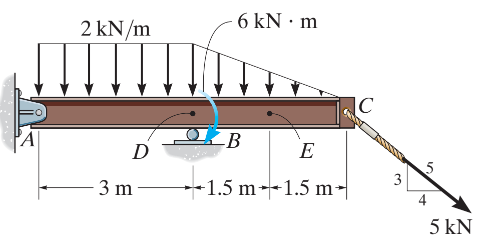
$$\frac{w}{6b}(2a+b)(b-a) = \frac{w}{8}(2a+b)$$

$$\frac{a}{b} = \frac{1}{4} \qquad \textbf{Ans}$$





7–18. Determine the internal normal force, shear force, and moment at points D and E in the overhang beam. Point D is located just to the left of the roller support at B, where the couple moment acts.



The intensity of the triangular distributed load at E can be found using the similar triangles in Fig. b.
With reference to Fig. a,

$\circlearrowleft +\Sigma M_A = 0;\quad B_y(3) - 2(3)(1.5) - 6 - \frac{1}{2}(2)(3)(4) - 5\left(\frac{3}{5}\right)(6) = 0$

$B_y = 15\text{ kN}$

Using this result and referring to Fig. c,

$\xrightarrow{+} \Sigma F_x = 0;\quad 5\left(\frac{4}{5}\right) - N_D = 0 \qquad N_D = 4\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_D + 15 - \frac{1}{2}(2)(3) - 5\left(\frac{3}{5}\right) = 0 \qquad V_D = -9\text{ kN}$ **Ans.**

$\circlearrowleft +\Sigma M_D = 0;\quad -M_D - 6 - \frac{1}{2}(2)(3)(1) - 5\left(\frac{3}{5}\right)(3) = 0 \qquad M_D = -18\text{ kN}\cdot\text{m}$ **Ans.**

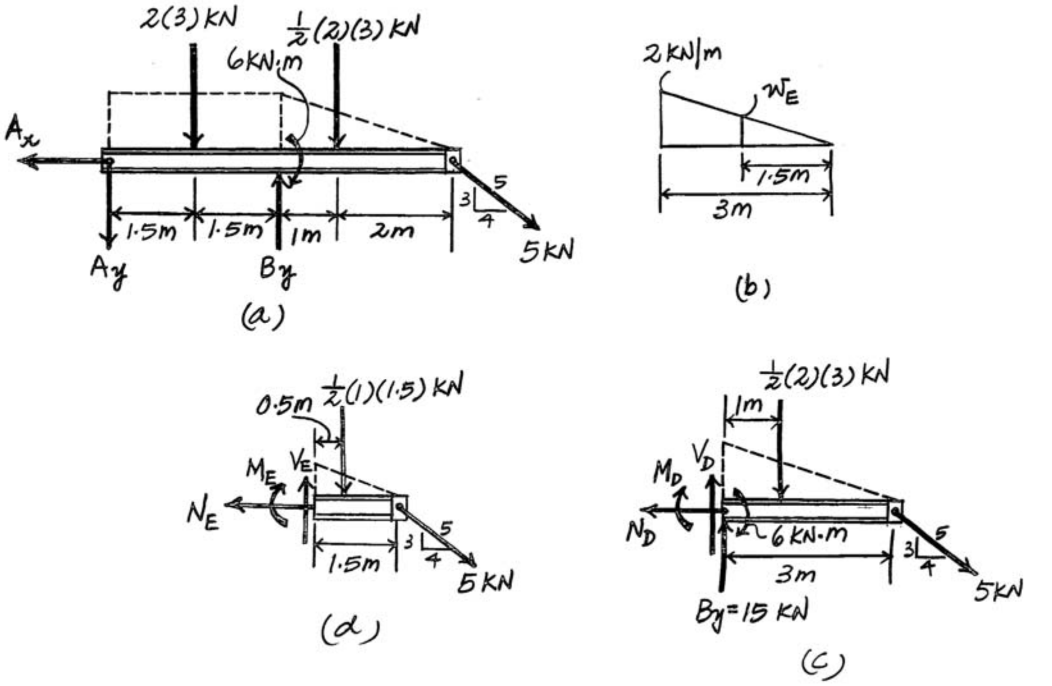
Also, by referring to Fig. d, we can write

$\xrightarrow{+} \Sigma F_x = 0;\quad 5\left(\frac{4}{5}\right) - N_E = 0 \qquad N_E = 4\text{ kN}$ **Ans.**

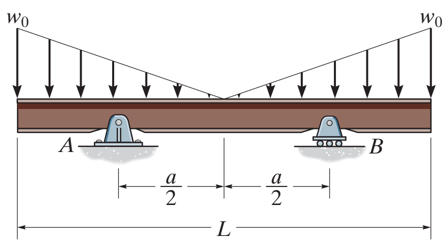
$+\uparrow \Sigma F_y = 0;\quad V_E - \frac{1}{2}(1)(1.5) - 5\left(\frac{3}{5}\right) = 0 \qquad V_E = 3.75\text{ kN}$ **Ans.**

$\circlearrowleft +\Sigma M_E = 0;\quad -M_E - \frac{1}{2}(1)(1.5)(0.5) - 5\left(\frac{3}{5}\right)(1.5) = 0 \qquad M_E = -4.875\text{ kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{V}_D$, $\mathbf{M}_D$, and $\mathbf{M}_E$ act in the opposite sense to that shown on the free - body diagram.



7–19. Determine the distance a in terms of the beam's length L between the symmetrically placed supports A and B so that the internal moment at the center of the beam is zero.



In this problem, it is required that the internal moment at point C be equal to zero. With reference to Fig. a,

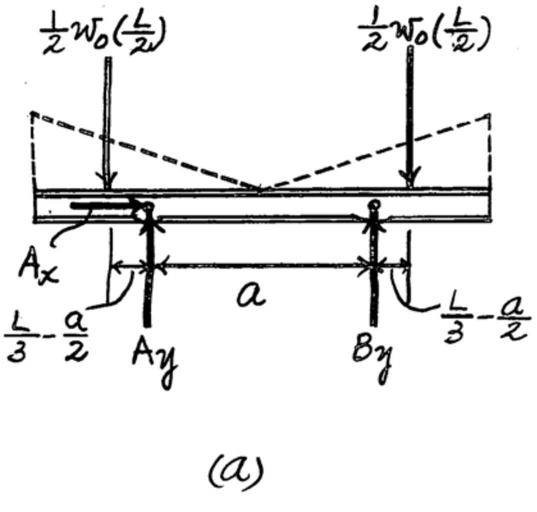
$$\curvearrowleft + \Sigma M_A = 0; \qquad B_y(a) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left[a + \left(\frac{L}{3} - \frac{a}{2}\right)\right] + \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{3} - \frac{a}{2}\right) = 0$$

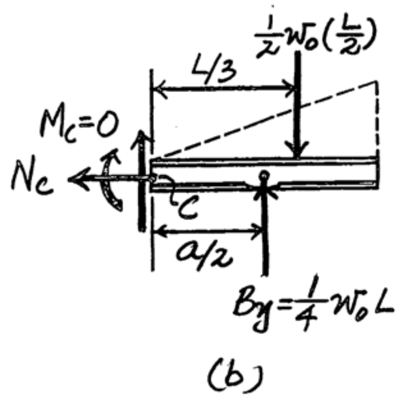
$$B_y = \frac{1}{4}w_0 L$$

Using this result and referring to Fig. b,

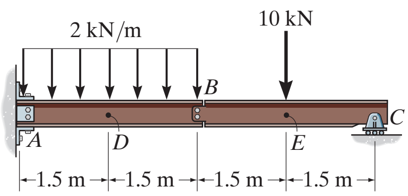
$$\curvearrowleft + \Sigma M_C = 0; \qquad \frac{1}{4}w_0 L\left(\frac{a}{2}\right) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = 0$$

$$a = \frac{2}{3}L$$ **Ans.**





***7–20.** Determine the internal normal force, shear force, and moment at points D and E in the compound beam. Point E is located just to the left of the 10-kN concentrated load. Assume the support at A is fixed and the connection at B is a pin.



With reference to Fig. b,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $B_x = 0$

$\curvearrowleft + \Sigma M_B = 0;$ $C_y(3) - 10(1.5) = 0$ $C_y = 5\text{ kN}$

$\curvearrowleft + \Sigma M_C = 0;$ $10(1.5) - B_y(3) = 0$ $B_y = 5\text{ kN}$

Using these results and referring to Fig. c,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $V_D - 2(1.5) - 5 = 0$ $V_D = 8\text{ kN}$ **Ans.**

$\curvearrowleft + \Sigma M_D = 0;$ $-M_D - 2(1.5)(0.75) - 5(1.5) = 0$ $M_D = -9.75\text{ kN}\cdot\text{m}$ **Ans.**

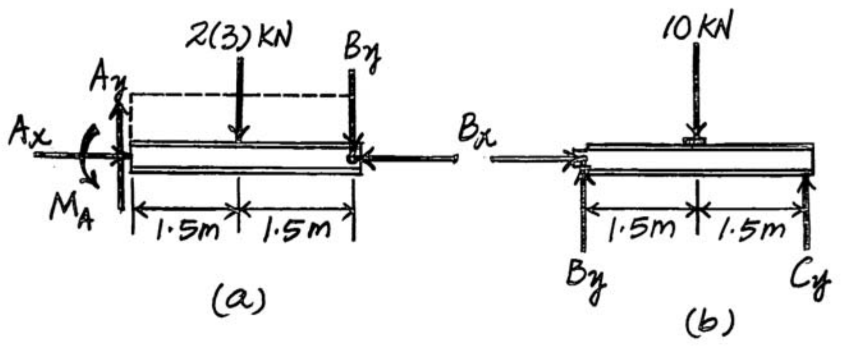
Also, by referring to Fig. d,

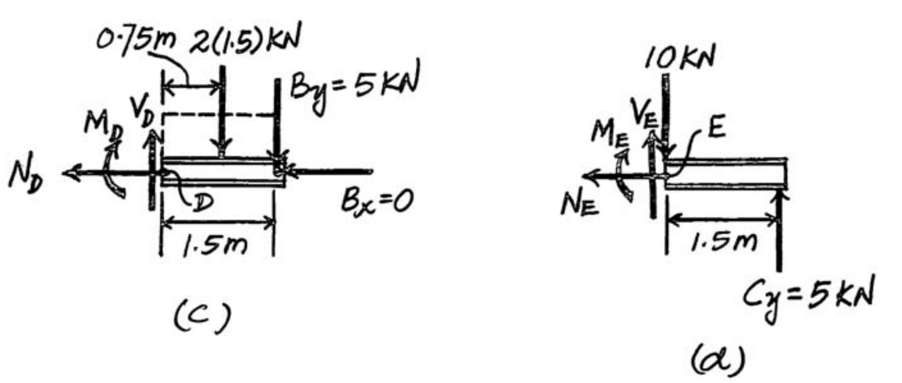
$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $N_E = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $V_E - 10 + 5 = 0$ $V_E = 5\text{ kN}$ **Ans.**

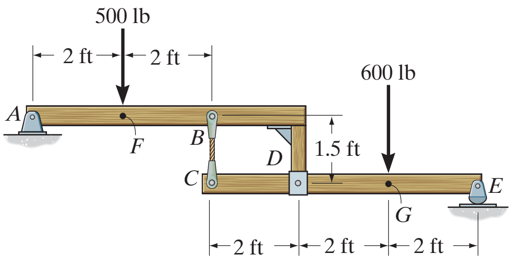
$\curvearrowleft + \Sigma M_E = 0;$ $5(1.5) - M_E = 0$ $M_E = 7.5\text{ kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{M}_D$ acts in the opposite sense to that shown in the free - body diagram.





•7–21. Determine the internal normal force, shear force, and moment at points F and G in the compound beam. Point F is located just to the right of the 500-lb force, while point G is located just to the right of the 600-lb force.



With reference to Fig. b,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad D_x = 0$

Using this result and writing the moment equation of equilibrium about point A, Fig. a, and about point E, Fig. b, we have

$\circlearrowleft + \Sigma M_A = 0;\qquad D_y(6) - F_{BC}(4) - 500(2) = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_E = 0;\qquad 600(2) + D_y(4) - F_{BC}(6) = 0 \qquad (2)$

Solving Eqs. (1) and (2)

$F_{BC} = 560 \text{ lb} \qquad D_y = 540 \text{ lb}$

Using these results and referring to Fig. b,

$+\uparrow \Sigma F_y = 0;\qquad E_y - 600 - 540 + 560 = 0 \qquad E_y = 580 \text{ lb}$

Again, using the results of $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{F}_{BC}$, the force equation of equilibrium written along the x and y axes, Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad A_x = 0$

$+\uparrow \Sigma F_y = 0;\qquad A_y - 500 - 560 + 540 = 0 \qquad A_y = 520 \text{ lb}$

Using these results and referring to Fig. c,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad N_F = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 520 - 500 - V_F = 0 \quad V_F = 20 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0;\qquad M_F - 520(2) = 0 \qquad M_F = 1040 \text{ lb}\cdot\text{ft}$ **Ans.**

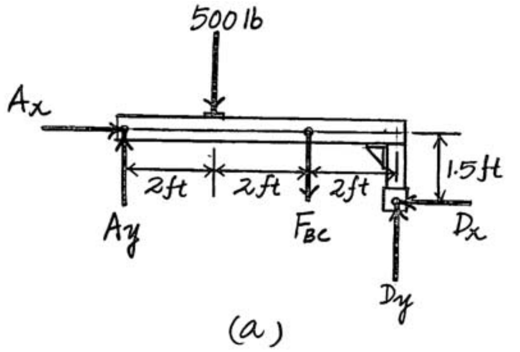
Using the result for $\mathbf{E}_y$ and referring to Fig. d

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad N_G = 0$ **Ans.**

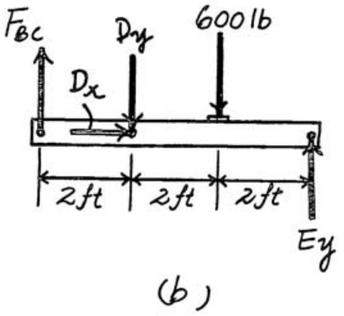
$+\uparrow \Sigma F_y = 0;\qquad V_G + 580 = 0 \qquad V_G = -580 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_G = 0;\qquad 580(2) - M_G = 0 \qquad M_G = 1160 \text{ lb}\cdot\text{ft}$ **Ans.**

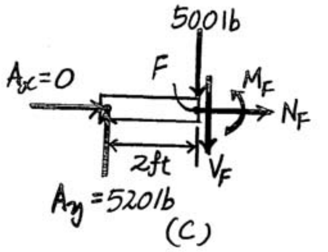
The negative sign indicates that $\mathbf{V}_G$ acts in the opposite sense to that shown in the free-body diagram.



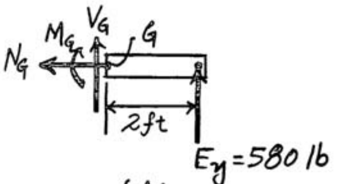
(a)



(b)

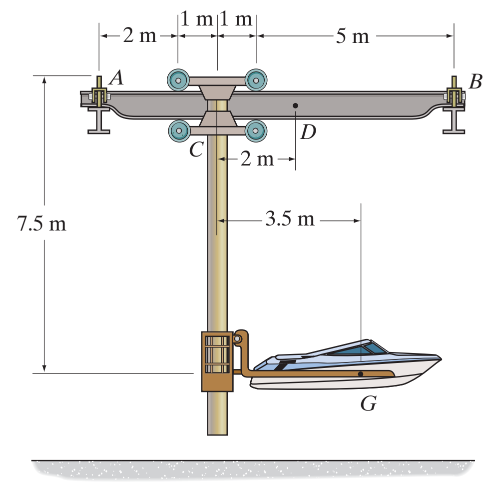


(c)



(d)

7–22. The stacker crane supports a 1.5-Mg boat with the center of mass at G. Determine the internal normal force, shear force, and moment at point D in the girder. The trolley is free to roll along the girder rail and is located at the position shown. Only vertical reactions occur at A and B.



With reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(9) - 1500(9.81)(3.5 + 3) = 0 \quad B_y = 10627.5\text{ N}$

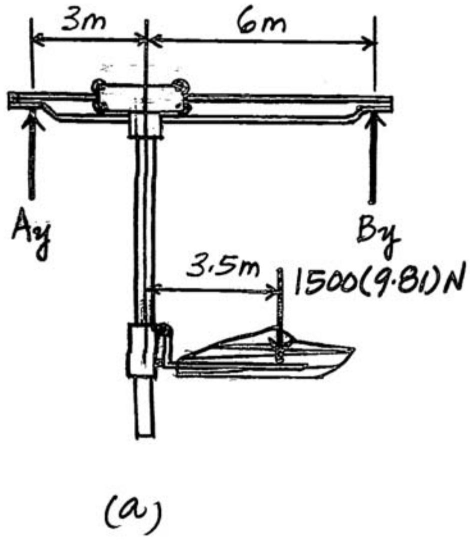
Using this result and referring to Fig. b,

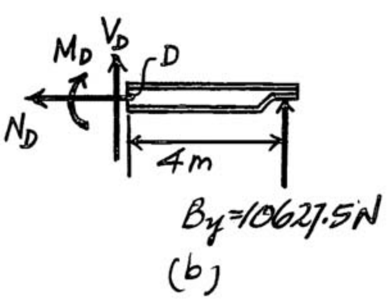
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_D + 10627.5 = 0$ $\quad V_D = -10627.5\text{ N} = -10.6\text{ kN}$ **Ans.**

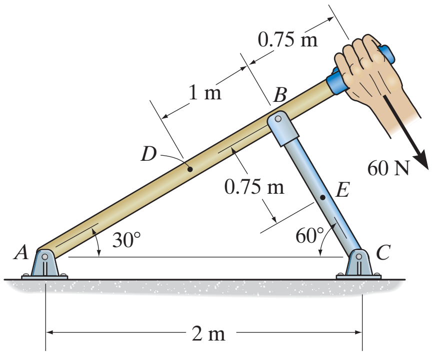
$\circlearrowleft + \Sigma M_D = 0;$ $\quad 10627.5(4) - M_D = 0$ $\quad M_D = 42510\text{ N}\cdot\text{m} = 42.5\text{ kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{V}_D$ acts in the opposite sense to that shown on the free-body diagram.





7–23. Determine the internal normal force, shear force, and moment at points D and E in the two members.



With reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC}\,(2\cos 30°) - 60(2\cos 30° + 0.75) = 0$

$F_{BC} = 85.98\text{ N}$

Using this result and referring to Fig. b,

$+\swarrow \Sigma F_{x'} = 0;\quad N_D = 0$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0;\quad 85.98 - 60 - V_D = 0 \qquad V_D = 26.0\text{ N}$ **Ans.**

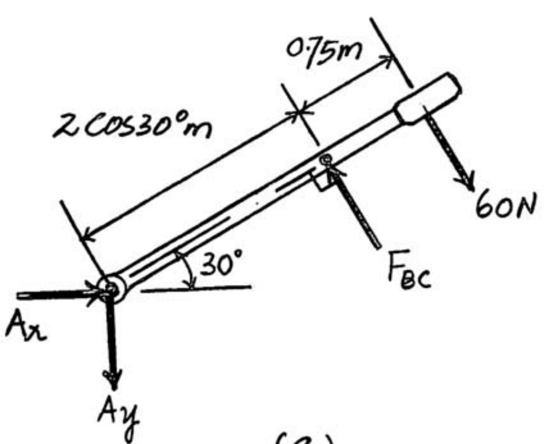
$\circlearrowleft + \Sigma M_D = 0;\quad 85.98(1) - 60(1.75) + M_D = 0 \qquad M_D = 19.0\text{ N}\cdot\text{m}$ **Ans.**

Also, be referring to Fig. c,

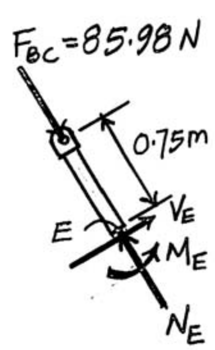
$+\nearrow \Sigma F_{x'} = 0;\quad V_E = 0$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0;\quad N_E - 85.98 = 0 \qquad N_E = 86.0\text{ N}$ **Ans.**

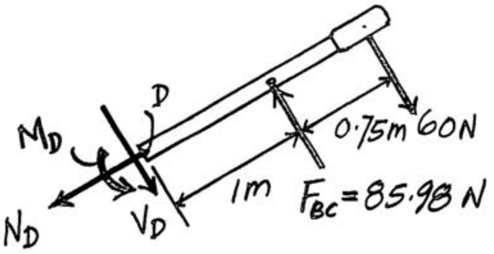
$\circlearrowleft + \Sigma M_E = 0;\quad M_E = 0$ **Ans.**



(a)

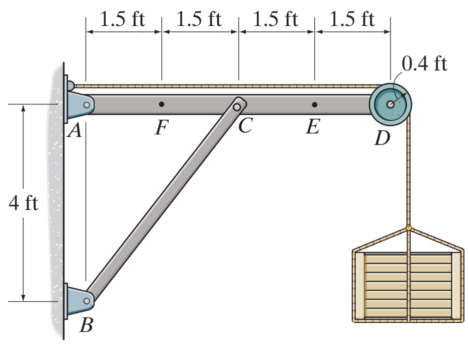


(c)



(b)

***7–24.** Determine the internal normal force, shear force, and moment at points *F* and *E* in the frame. The crate weighs 300 lb.



With reference to Fig. *a*,

$\curvearrowleft + \Sigma M_A = 0; \quad F_{BC}\left(\frac{4}{5}\right)(3) + 300(0.4) - 300(6.4) = 0 \qquad F_{BC} = 750 \text{ lb}$

Referring to Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -N_E - 300 = 0 \qquad N_E = -300 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_E - 300 = 0 \qquad V_E = 300 \text{ lb}$ **Ans.**

$\curvearrowleft + \Sigma M_E = 0; \quad -M_E + 300(0.4) - 300(1.9) = 0 \quad M_E = -450 \text{ lb} \cdot \text{ft}$ **Ans.**

Using the result of $\mathbf{F}_{BC}$ and referring to Fig. *c*,

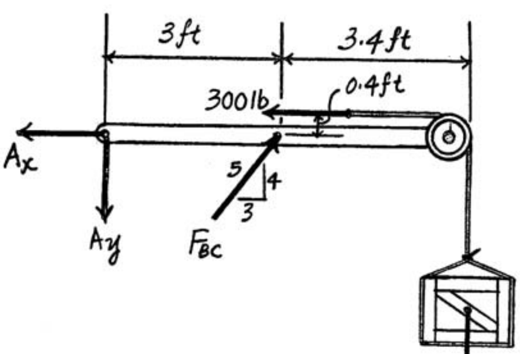
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 750\left(\frac{3}{5}\right) - 300 - N_F = 0 \qquad N_F = 150 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_F + 750\left(\frac{4}{5}\right) - 300 = 0 \qquad V_F = -300 \text{ lb}$ **Ans.**

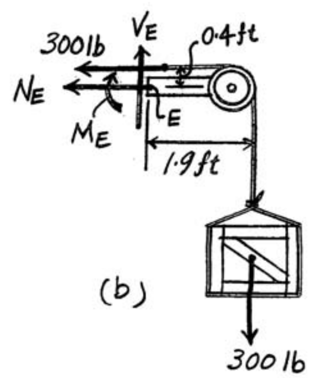
$\curvearrowleft + \Sigma M_F = 0; \quad 750\left(\frac{4}{5}\right)(1.5) + 300(0.4) - 300(4.9) - M_F = 0$

$M_F = -450 \text{ lb} \cdot \text{ft}$ **Ans.**

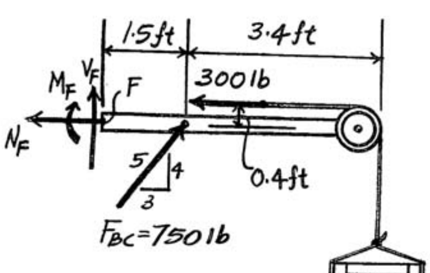
The negative sign indicates that $\mathbf{N}_E$, $\mathbf{V}_F$, and $\mathbf{M}_F$ act in the opposite sense to that shown in the free-body diagram.



(a)

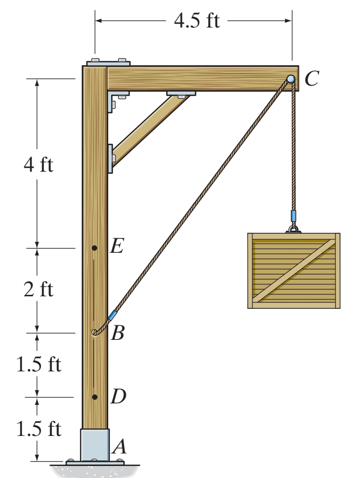


(b)



(C)

•7–25. Determine the internal normal force, shear force, and moment at points *D* and *E* of the frame which supports the 200-lb crate. Neglect the size of the smooth peg at *C*.



Referring to Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad V_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad N_D - 200 = 0 \qquad N_D = 200 \text{ lb}$ **Ans.**

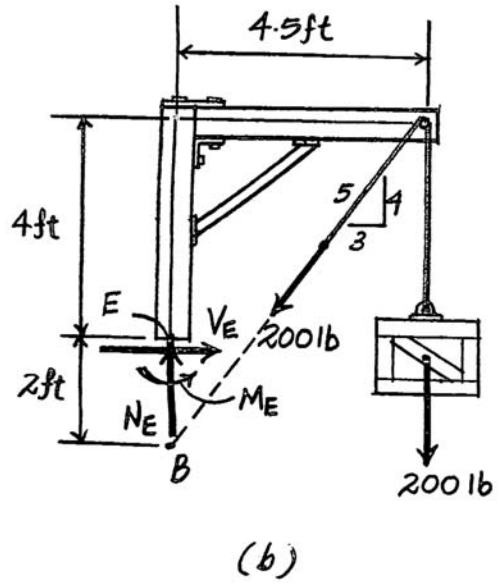
$\circlearrowleft + \Sigma M_F = 0;$ $\quad M_D - 200(4.5) = 0 \qquad M_D = 900 \text{ lb} \cdot \text{ft}$ **Ans.**

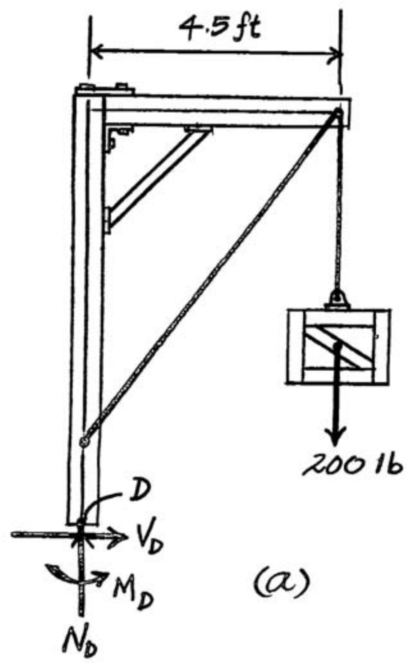
Also, by referring to Fig. *b*,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad V_E - 200\left(\frac{3}{5}\right) = 0 \qquad V_E = 120 \text{ lb}$ **Ans.**

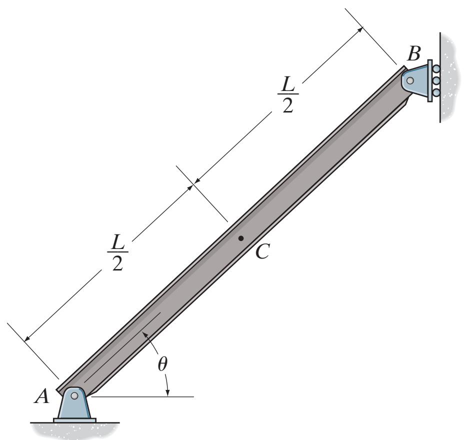
$+\uparrow \Sigma F_y = 0;$ $\quad N_E - 200\left(\frac{4}{5}\right) - 200 = 0 \qquad N_E = 360 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad M_E - 200(4.5) - 200\left(\frac{3}{5}\right)(2) = 0 \qquad M_E = 1140 \text{ lb} \cdot \text{ft}$ **Ans.**





7–26. The beam has a weight w per unit length. Determine the internal normal force, shear force, and moment at point C due to its weight.



With reference to Fig. a,

$$\circlearrowleft +\Sigma M_A = 0; \qquad B_x(L\sin\theta) - wL\cos\theta\left(\frac{L}{2}\right) = 0 \quad B_x = \frac{wL\cos\theta}{2\sin\theta}$$

Using this result and referring to Fig. b,

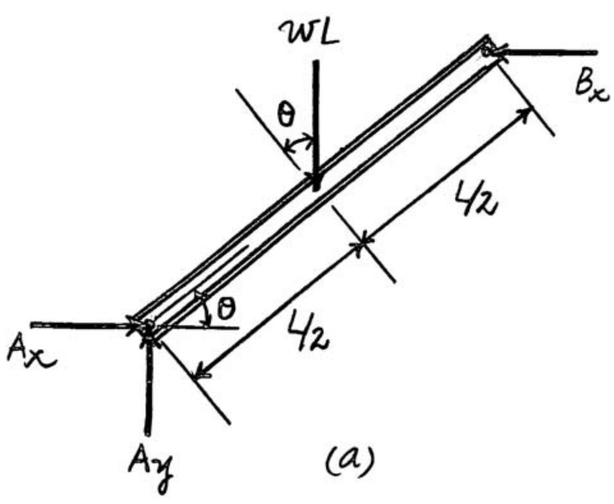
$$+\nearrow \Sigma F_{x'} = 0; \quad -N_C - \frac{wL\cos\theta}{2\sin\theta}(\cos\theta) - w\left(\frac{L}{2}\right)\sin\theta = 0 \quad N_C = -\frac{wL}{2}\csc\theta \qquad \textbf{Ans.}$$

$$+\nwarrow \Sigma F_{y'} = 0; \quad V_C - w\left(\frac{L}{2}\right)\cos\theta + \frac{wL\cos\theta}{2\sin\theta}\sin\theta = 0 \qquad V_C = 0 \qquad \textbf{Ans.}$$

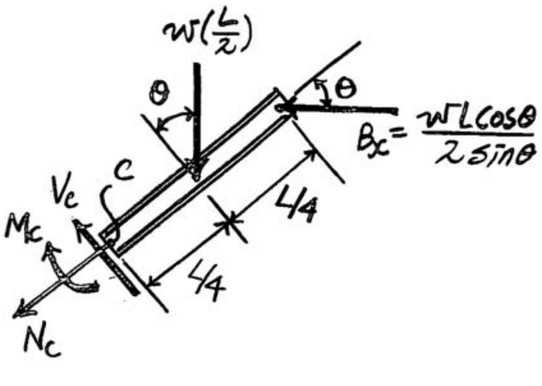
$$\circlearrowleft +\Sigma M_C = 0; \qquad \frac{wL\cos\theta}{2\sin\theta}\left(\frac{L}{2}\sin\theta\right) - w\left(\frac{L}{2}\right)\cos\theta\left(\frac{L}{4}\right) - M_C = 0$$

$$M_C = \frac{wL^2}{8}\cos\theta \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{N}_C$ acts in the opposite sense to that shown on the free-body diagram.

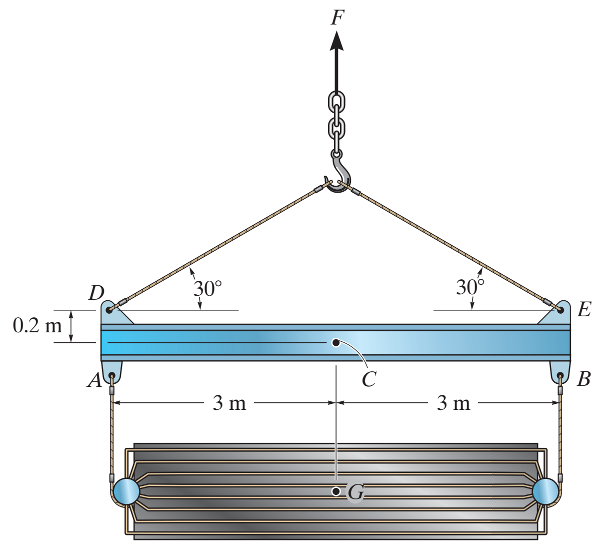


(a)



(b)

7–27. Determine the internal normal force, shear force, and moment acting at point C. The cooling unit has a total mass of 225 kg with a center of mass at G.



From FBD (a)

$\circlearrowleft + \Sigma M_A = 0; \quad T_B(6) - 225(9.81)(3) = 0 \quad T_B = 1103.625 \text{ N}$

From FBD (b)

$\circlearrowleft + \Sigma M_D = 0; \quad T_E \sin 30°(6) - 1103.625(6) = 0 \quad T_E = 2207.25 \text{ N}$

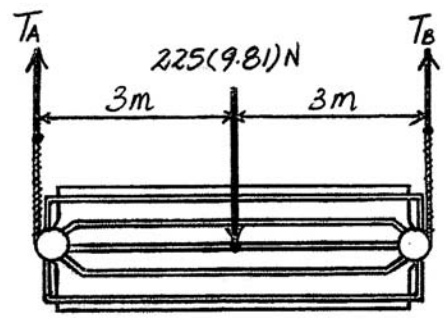
From FBD (c)

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_C - 2207.25 \cos 30° = 0 \quad N_C = -1.91 \text{ kN}$ **Ans**

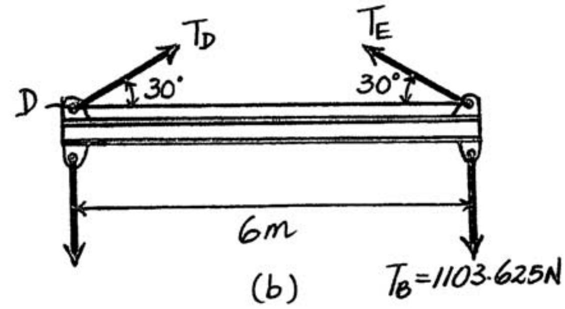
$+\uparrow \Sigma F_y = 0; \quad V_C + 2207.25 \sin 30° - 1103.625 = 0 \quad V_C = 0$ **Ans**

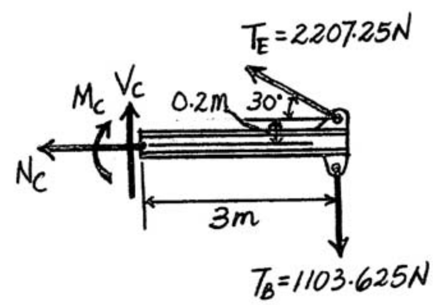
$\circlearrowleft + \Sigma M_C = 0; \quad 2207.25 \cos 30°(0.2) + 2207.25 \sin 30°(3) - 1103.625(3) - M_C = 0$

$M_C = 382 \text{ N} \cdot \text{m}$ **Ans**

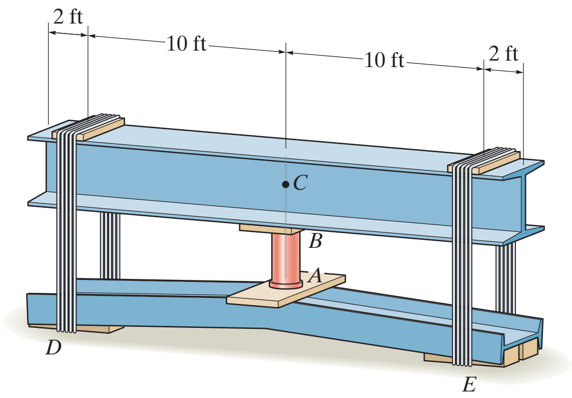


(a)





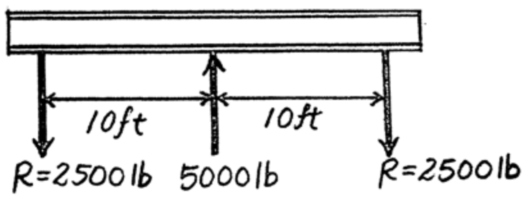
***7–28.** The jack *AB* is used to straighten the bent beam *DE* using the arrangement shown. If the axial compressive force in the jack is 5000 lb, determine the internal moment developed at point *C* of the top beam. Neglect the weight of the beams.

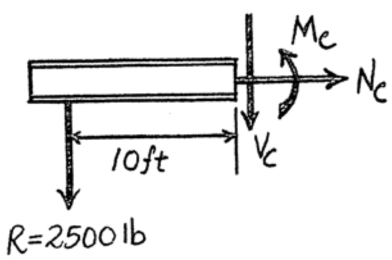


Segment :

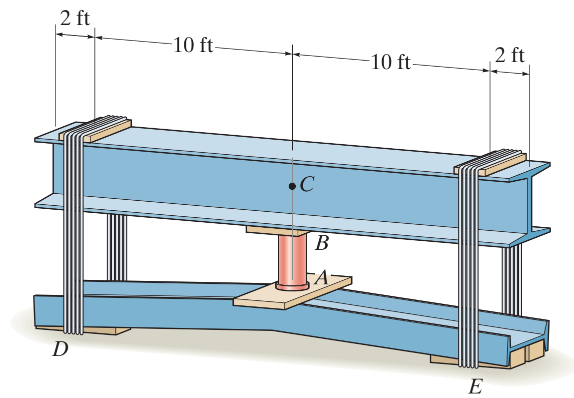
$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 2500(10) = 0$

$M_C = -25.0 \text{ kip} \cdot \text{ft}$ **Ans**





•7–29. Solve Prob. 7–28 assuming that each beam has a uniform weight of 150 lb/ft.



Beam:

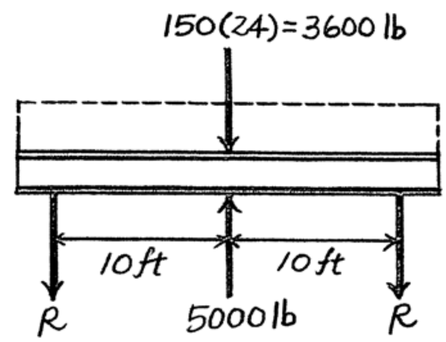
$+\uparrow \Sigma F_y = 0; \quad 5000 - 3600 - 2R = 0$

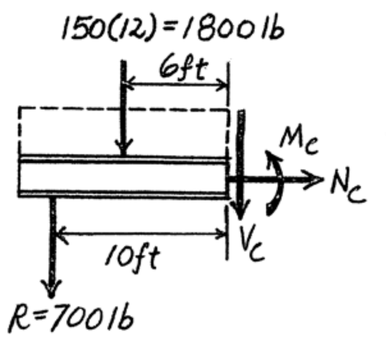
$R = 700 \text{ lb}$

Segment:

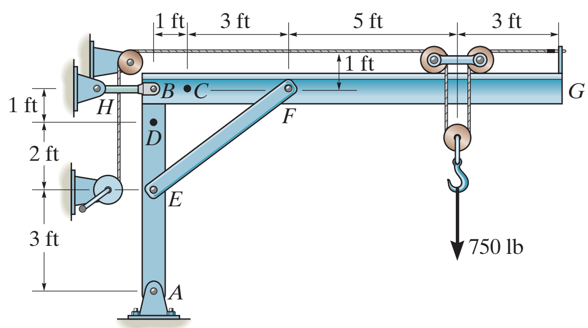
$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 700(10) + 1800(6) = 0$

$M_C = -17.8 \text{ kip} \cdot \text{ft}$ **Ans**





7–30. The jib crane supports a load of 750 lb from the trolley which rides on the top of the jib. Determine the internal normal force, shear force, and moment in the jib at point C when the trolley is at the position shown. The crane members are pinned together at B, E and F and supported by a short link BH.



Member BFG:

$\circlearrowleft + \Sigma M_B = 0; \quad F_{EF}\left(\frac{3}{5}\right)(4) - 750(9) + 375(1) = 0$

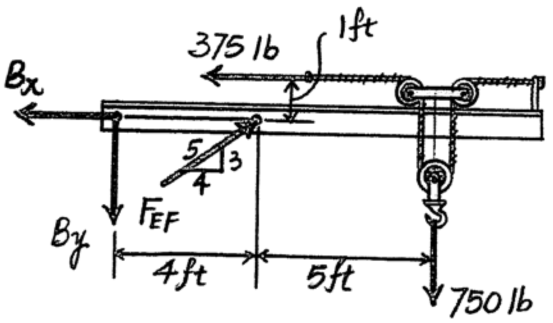
$F_{EF} = 2656.25 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad -B_x + 2656.25\left(\frac{4}{5}\right) - 375 = 0$

$B_x = 1750 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -B_y + 2656.25\left(\frac{3}{5}\right) - 750 = 0$

$B_y = 843.75 \text{ lb}$



Segment BC:

$\xrightarrow{+} \Sigma F_x = 0; \quad N_C - 1750 = 0$

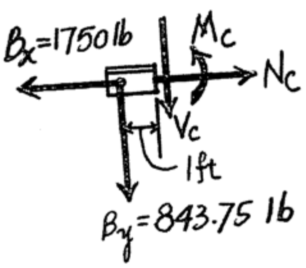
$N_C = 1.75 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -843.75 - V_C = 0$

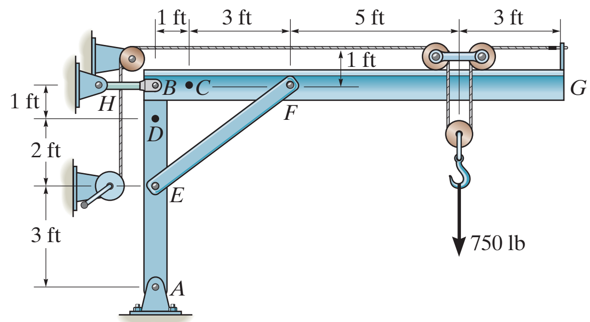
$V_C = -844 \text{ lb}$ **Ans**

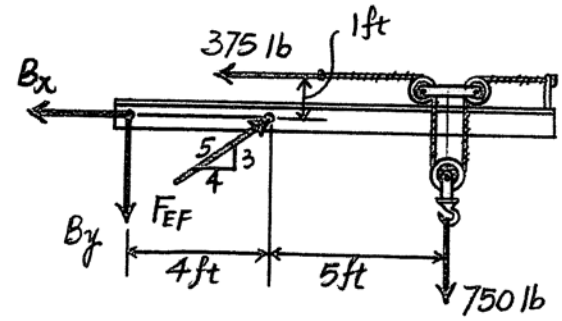
$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 843.75(1) = 0$

$M_C = -844 \text{ lb} \cdot \text{ft}$ **Ans**



7–31. The jib crane supports a load of 750 lb from the trolley which rides on the top of the jib. Determine the internal normal force, shear force, and moment in the column at point *D* when the trolley is at the position shown. The crane members are pinned together at *B*, *E* and *F* and supported by a short link *BH*.

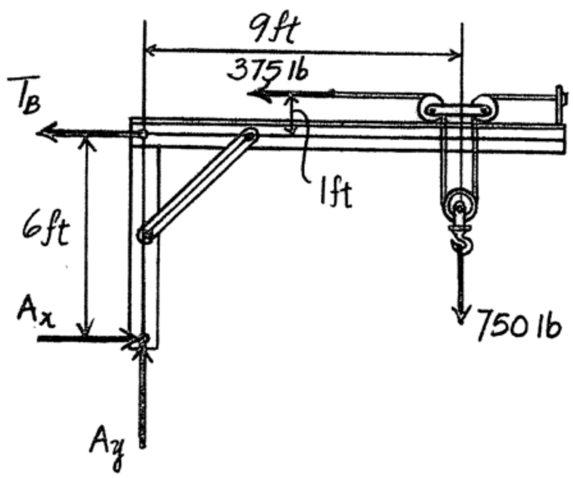




Member *BFG*:

$\circlearrowleft +\Sigma M_B = 0;\quad F_{EF}\left(\frac{3}{5}\right)(4) - 750(9) + 375(1) = 0$

$F_{EF} = 2656.25\text{ lb}$



Entire Crane:

$\circlearrowleft +\Sigma M_A = 0;\quad T_B(6) - 750(9) + 375(7) = 0$

$T_B = 687.5\text{ lb}$

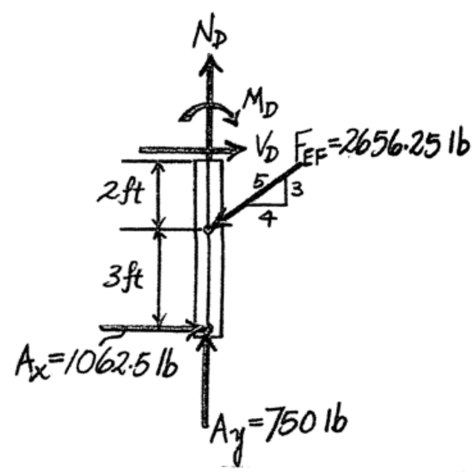
$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 687.5 - 375 = 0$

$A_x = 1062.5\text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 750 = 0$

$A_y = 750\text{ lb}$

Segment *AED*:



$+\uparrow \Sigma F_y = 0;\quad N_D + 750 - 2656.25\left(\frac{3}{5}\right) = 0$

$N_D = 844\text{ lb}$ **Ans**

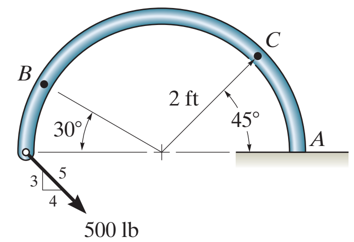
$\xrightarrow{+} \Sigma F_x = 0;\quad 1062.5 - 2656.25\left(\frac{4}{5}\right) + V_D = 0$

$V_D = 1.06\text{ kip}$ **Ans**

$\circlearrowleft +\Sigma M_D = 0;\quad -M_D - 2656.25\left(\frac{4}{5}\right)(2) + 1062.5(5) = 0$

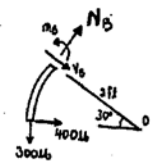
$M_D = 1.06\text{ kip}\cdot\text{ft}$ **Ans**

***7–32.** Determine the internal normal force, shear force, and moment acting at points *B* and *C* on the curved rod.



$\nearrow + \Sigma F_x = 0;$ $\quad 400 \sin 30° - 300 \cos 30° + N_B = 0$

$N_B = 59.81 \text{ lb} = 59.8 \text{ lb}$ **Ans**



$\searrow + \Sigma F_y = 0;$ $\quad V_B + 400 \cos 30° + 300 \sin 30° = 0$

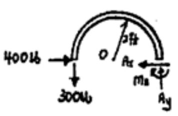
$V_B = -496 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_B = 0;$ $\quad M_B + 400(2 \sin 30°) + 300(2 - 2 \cos 30°) = 0$

$M_B = -480 \text{ lb} \cdot \text{ft}$ **Ans**

Also,

$\circlearrowleft + \Sigma M_O = 0;$ $\quad -59.81(2) + 300(2) + M_B = 0$



$M_B = -480 \text{ lb} \cdot \text{ft}$ **Ans**

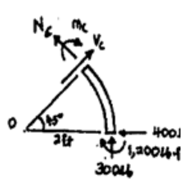
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 400 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y = 300 \text{ lb}$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - 300(4) = 0$

$M_A = 1200 \text{ lb} \cdot \text{ft}$

$+\nwarrow \Sigma F_x = 0;$ $\quad N_C + 400 \sin 45° + 300 \cos 45° = 0$



$N_C = -495 \text{ lb}$ **Ans**

$+\nearrow \Sigma F_y = 0;$ $\quad V_C - 400 \cos 45° + 300 \sin 45° = 0$

$V_C = 70.7 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -M_C - 1200 - 400(2 \sin 45°) + 300(2 - 2 \cos 45°) = 0$

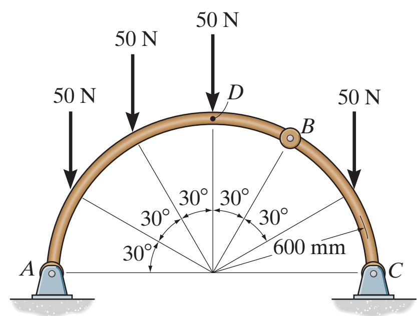
$M_C = -1590 \text{ lb} \cdot \text{ft} = -1.59 \text{ kip} \cdot \text{ft}$ **Ans**

Also,

$\circlearrowleft + \Sigma M_O = 0;$ $\quad -1200 - 495(2) + 300(2) - M_C = 0$

$M_C = -1590 \text{ lb} \cdot \text{ft} = -1.59 \text{ kip} \cdot \text{ft}$ **Ans**

•**7–33.** Determine the internal normal force, shear force, and moment at point D which is located just to the right of the 50-N force.



Referring to Figs. *a* and *b*, respectively,

$\circlearrowleft + \Sigma M_A = 0;$ $B_y(0.6 + 0.6\sin 30°) + B_x(0.6\cos 30°) - 50(0.6 - 0.6\cos 30°) = 0$
$-50(0.6 - 0.6\cos 60°) - 50(0.6) = 0$ (1)

$\circlearrowleft + \Sigma M_C = 0;$ $B_y(0.6 - 0.6\cos 60°) - B_x(0.6\sin 60°) + 50(0.6 - 0.6\cos 30°) = 0$ (2)

Solving Eqs. (1) and (2) yields

$B_x = 29.39\text{ N}$ $B_y = 37.5\text{ N}$

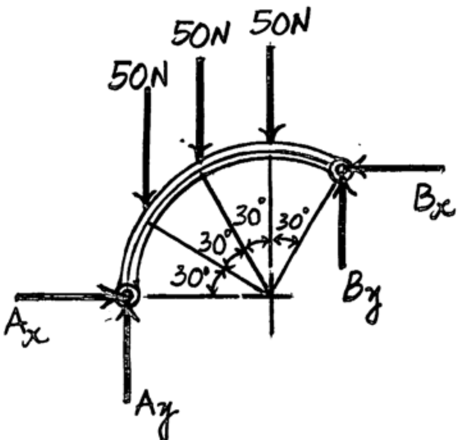
Using these results and referring to Fig. *c*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $-N_D - 29.39 = 0$ $N_D = -29.4\text{ N}$ **Ans.**

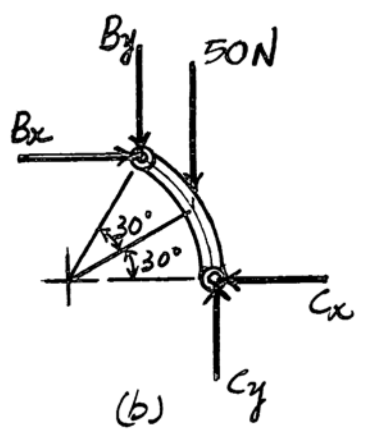
$+\uparrow \Sigma F_y = 0;$ $37.5 - V_D = 0$ $V_D = 37.5\text{N}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $37.5(0.6\sin 30°) - 29.39(0.6 - 0.6\cos 30°) - M_D = 0$

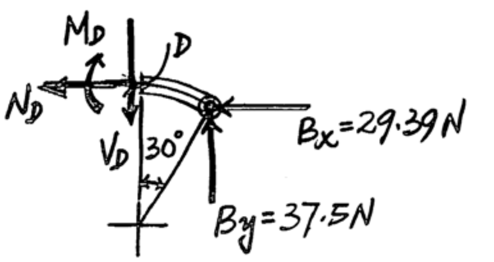
$M_D = 8.89\text{ N}\cdot\text{m}$ **Ans.**



(a)

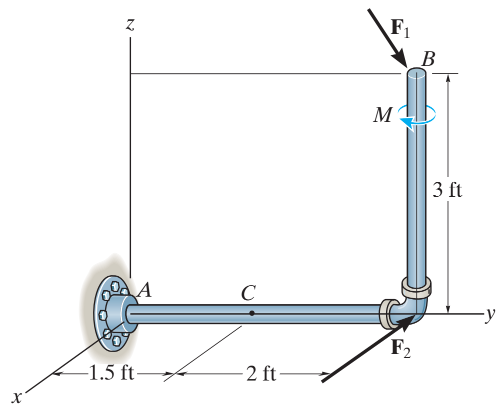


(b)



(c)

7–34. Determine the x, y, z components of internal loading at point C in the pipe assembly. Neglect the weight of the pipe. The load is $\mathbf{F}_1 = \{-24\mathbf{i} - 10\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{-80\mathbf{i}\}$ lb, and $\mathbf{M} = \{-30\mathbf{k}\}$ lb · ft.



Free body Diagram : The support reactions need not be computed.

Internal Forces : Applying the equations of equilibrium to segment BC, we have

$\Sigma F_x = 0;\quad (V_C)_x - 24 - 80 = 0 \quad (V_C)_x = 104 \text{ lb}$ **Ans**

$\Sigma F_y = 0;\quad N_C = 0$ **Ans**

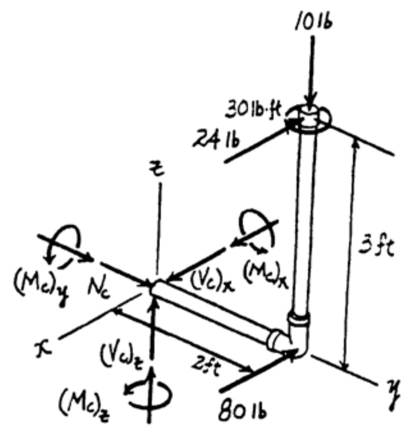
$\Sigma F_z = 0;\quad (V_C)_z - 10 = 0 \quad (V_C)_z = 10.0 \text{ lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_C)_x - 10(2) = 0 \quad (M_C)_x = 20.0 \text{ lb}\cdot\text{ft}$ **Ans**

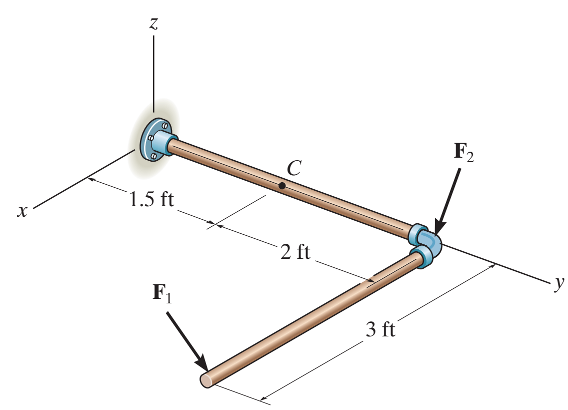
$\Sigma M_y = 0;\quad (M_C)_y - 24(3) = 0 \quad (M_C)_y = 72.0 \text{ lb}\cdot\text{ft}$ **Ans**

$\Sigma M_z = 0;\quad (M_C)_z + 24(2) + 80(2) - 30 = 0$

$(M_C)_z = -178 \text{ lb}\cdot\text{ft}$ **Ans**



7–35. Determine the x, y, z components of internal loading at a section passing through point C in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{350\mathbf{j} - 400\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{150\mathbf{i} - 300\mathbf{k}\}$ lb.



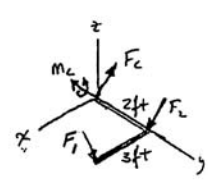
$\Sigma \mathbf{F}_R = 0;\quad \mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$

$\mathbf{F}_C = \{-150\mathbf{i} - 350\mathbf{j} + 700\mathbf{k}\}$ lb

$C_x = -150$ lb **Ans**

$C_y = -350$ lb **Ans**

$C_z = 700$ lb **Ans**



$\Sigma \mathbf{M}_R = 0;\quad \mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = \mathbf{0}$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 0 & 350 & -400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 150 & 0 & -300 \end{vmatrix} = \mathbf{0}$$

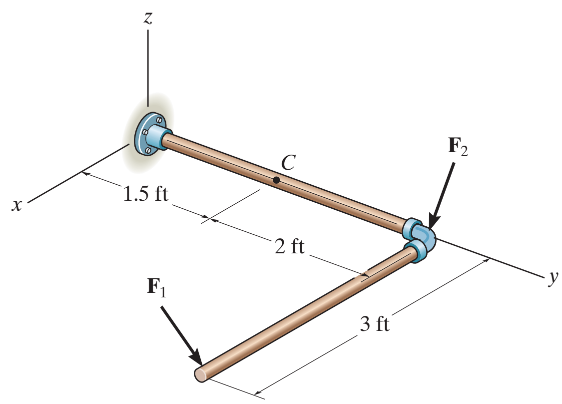
$\mathbf{M}_C = \{1400\mathbf{i} - 1200\mathbf{j} - 750\mathbf{k}\}$ lb·ft

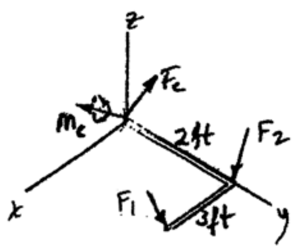
$M_{Cx} = 1.40$ kip·ft **Ans**

$M_{Cy} = -1.20$ kip·ft **Ans**

$M_{Cz} = -750$ lb·ft **Ans**

***7–36.** Determine the *x*, *y*, *z* components of internal loading at a section passing through point *C* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{-80\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{250\mathbf{i} - 150\mathbf{j} - 200\mathbf{k}\}$ lb.





$\Sigma \mathbf{F}_R = 0;\qquad \mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{0}$

$\mathbf{F}_C = \{-170\mathbf{i} - 50\mathbf{j} + 500\mathbf{k}\}$ lb

$C_x = -170$ lb **Ans**

$C_y = -50$ lb **Ans**

$C_z = 500$ lb **Ans**

$\Sigma \mathbf{M}_R = 0;\qquad \mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = \mathbf{0}$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ -80 & 200 & -300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 250 & -150 & -200 \end{vmatrix} = \mathbf{0}$$

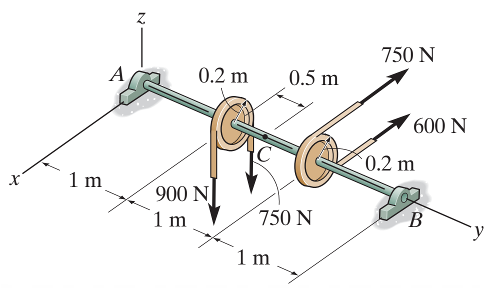
$\mathbf{M}_C = \{1000\mathbf{i} - 900\mathbf{j} - 260\mathbf{k}\}\ \text{lb}\cdot\text{ft}$

$M_{Cx} = 1\ \text{kip}\cdot\text{ft}$ **Ans**

$M_{Cy} = 900\ \text{lb}\cdot\text{ft}$ **Ans.**

$M_{Cz} = -260\ \text{lb}\cdot\text{ft}$ **Ans.**

•**7–37.** The shaft is supported by a thrust bearing at *A* and a journal bearing at *B*. Determine the *x*, *y*, *z* components of internal loading at point *C*.



With reference to Fig. *a*,

$\Sigma M_x = 0;\quad B_z(3) - 900(1) - 750(1) = 0 \quad B_z = 550\text{ N}$

$\Sigma M_z = 0;\quad 750(2) + 600(2) - B_x(3) = 0 \quad B_x = 900\text{ N}$

Using these results and referring to Fig. *b*,

$\Sigma F_x = 0;\quad (V_C)_x + 900 - 750 - 600 = 0 \quad (V_C)_x = 450\text{ N}$ **Ans.**

$\Sigma F_y = 0;\quad N_C = 0$ **Ans.**

$\Sigma F_z = 0;\quad (V_C)_z + 550 = 0 \quad (V_C)_z = -550\text{ N}$ **Ans.**

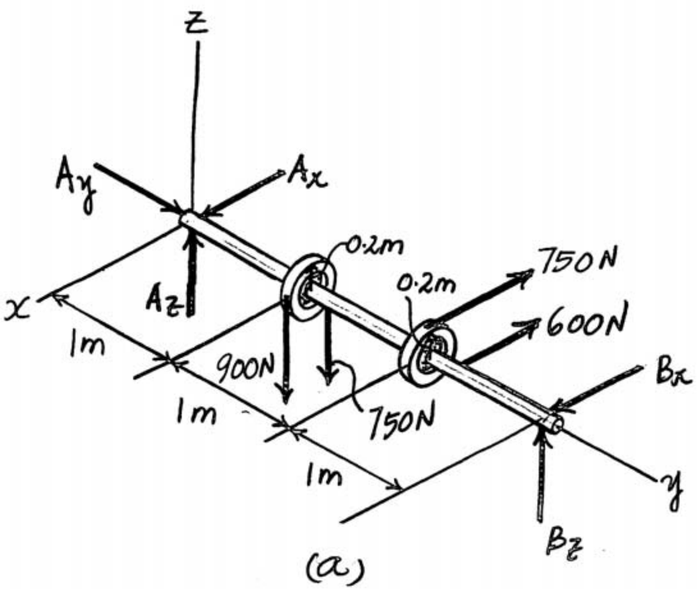
$\Sigma M_x = 0;\quad (M_C)_x + 550(1.5) = 0 \quad (M_C)_x = -825\text{ N}\cdot\text{m}$ **Ans.**

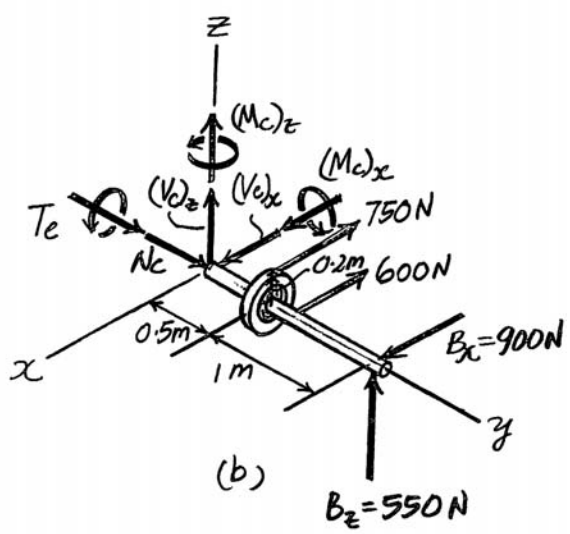
$\Sigma M_y = 0;\quad T_C + 600(0.2) - 750(0.2) = 0 \quad T_C = 30\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0;\quad (M_C)_z + 750(0.5) + 600(0.5) - 900(1.5) = 0$

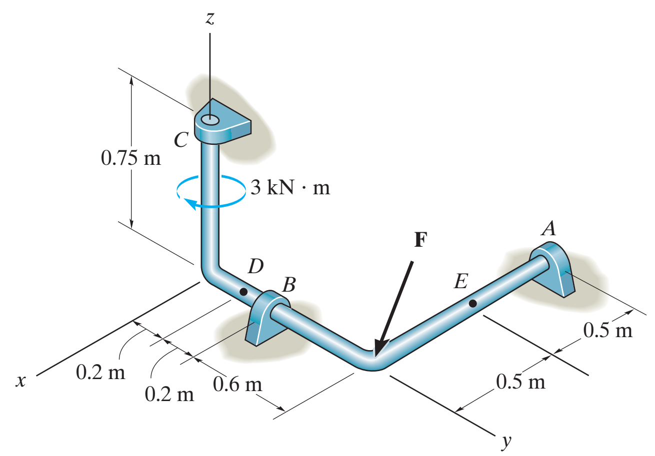
$(M_C)_z = 675\text{ N}\cdot\text{m}$ **Ans.**

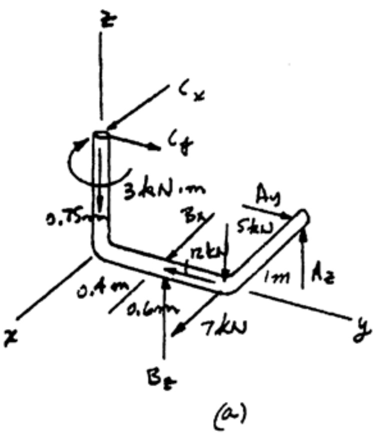
The negative signs indicate that $(\mathbf{V}_C)_z$ and $(\mathbf{M}_C)_z$ act in the opposite sense to those shown in the free-body diagram.

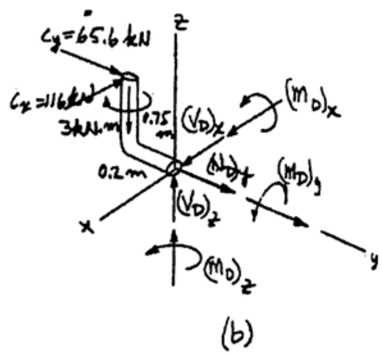




7–38. Determine the *x, y, z* components of internal loading in the rod at point *D*. There are journal bearings at *A*, *B*, and *C*. Take $\mathbf{F} = \{7\mathbf{i} - 12\mathbf{j} - 5\mathbf{k}\}$ kN.







Support reactions : FBD (a)

$\Sigma M_x = 0;\quad B_z(0.4) + A_z(1) - C_y(0.75) - 5(1) = 0 \quad (1)$

$\Sigma M_y = 0;\quad A_x(1) + C_x(0.75) = 0 \quad (2)$

$\Sigma M_z = 0;\quad -B_x(0.4) - A_y(1) - 7(1) - 3 = 0 \quad (3)$

$\Sigma F_x = 0;\quad C_x + B_x + 7 = 0 \quad (4)$

$\Sigma F_y = 0;\quad C_y + A_y - 12 = 0 \quad (5)$

$\Sigma F_z = 0;\quad B_z + A_z - 5 = 0 \quad (6)$

Solving Eqs. (1) to (6) yields :

$C_x = -116$ kN $\quad B_x = 109$ kN $\quad A_z = 87.0$ kN

$A_y = -53.6$ kN $\quad C_y = 65.6$ kN $\quad B_z = -82.0$ kN

Negative signs indicate that the support reactions act in the opposite sense to those shown on FBD (a).
From FBD (b)

$\Sigma F_x = 0;\quad (V_D)_x - 116 = 0;\quad (V_D)_x = 116$ kN **Ans**

$\Sigma F_y = 0;\quad (N_D)_y + 65.6 = 0;\quad (N_D)_y = -65.6$ kN **Ans**

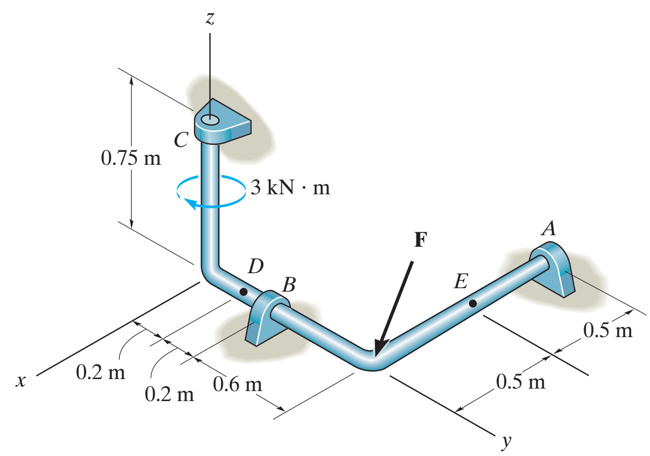
$\Sigma F_z = 0;\quad (V_D)_z = 0$ **Ans**

$\Sigma M_x = 0;\quad (M_D)_x - 65.6(0.75) = 0;\quad (M_D)_x = 49.2$ kN · m **Ans**

$\Sigma M_y = 0;\quad (M_D)_y - 116(0.75) = 0;\quad (M_D)_y = 87.0$ kN · m **Ans**

$\Sigma M_z = 0;\quad (M_D)_z - 116(0.2) - 3 = 0;\quad (M_D)_z = 26.2$ kN · m **Ans**

7–39. Determine the x, y, z components of internal loading in the rod at point E. Take $\mathbf{F} = \{7\mathbf{i} - 12\mathbf{j} - 5\mathbf{k}\}$ kN.



Support reactions : FBD (a)

$\Sigma M_x = 0;\quad B_z(0.4) + A_z(1) - C_y(0.75) - 5(1) = 0 \quad (1)$

$\Sigma M_y = 0;\quad A_z(1) + C_x(0.75) = 0 \quad (2)$

$\Sigma M_z = 0;\quad -B_x(0.4) - A_y(1) - 7(1) - 3 = 0 \quad (3)$

$\Sigma F_x = 0;\quad C_x + B_x + 7 = 0 \quad (4)$

$\Sigma F_y = 0;\quad C_y + A_y - 12 = 0 \quad (5)$

$\Sigma F_z = 0;\quad B_z + A_z - 5 = 0 \quad (6)$

Solving Eqs. (1) to (6) yields :

$C_x = -116$ kN $\quad B_x = 109$ kN $\quad A_z = 87.0$ kN

$A_y = -53.6$ kN $\quad C_y = 65.6$ kN $\quad B_z = -82.0$ kN

Negative signs indicate that the support reactions act in the opposite sense to those shown on FBD (a).

From FBD (b)

$\Sigma F_x = 0;\quad (N_E)_x = 0$ **Ans**

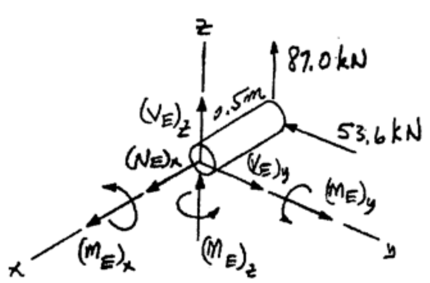
$\Sigma F_y = 0;\quad (V_E)_y - 53.6 = 0;\quad (V_E)_y = 53.6$ kN **Ans**

$\Sigma F_z = 0;\quad (V_E)_z + 87.0 = 0;\quad (V_E)_z = -87.0$ kN **Ans**

$\Sigma M_x = 0;\quad (M_E)_x = 0$ **Ans**

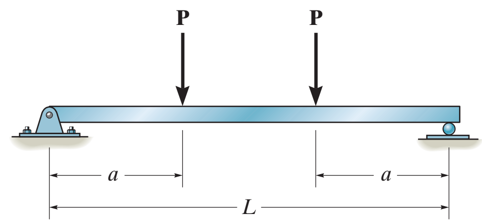
$\Sigma M_y = 0;\quad (M_E)_y + 87.0(0.5) = 0;\quad (M_E)_y = -43.5$ kN · m **Ans**

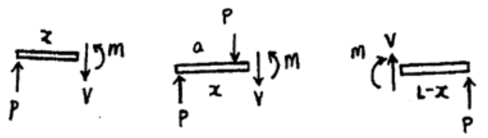
$\Sigma M_z = 0;\quad (M_E)_z + 53.6(0.5) = 0;\quad (M_E)_z = -26.8$ kN · m **Ans**



(b)

***7–40.** Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft.





(a) For $0 \le x < a$

$+\uparrow \Sigma F_y = 0;$ $V = P$ **Ans**

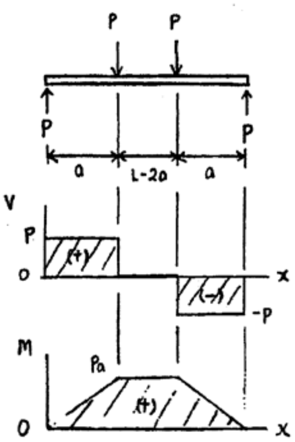
$\circlearrowleft + \Sigma M = 0;$ $M = Px$ **Ans**

For $a < x < L - a$

$+\uparrow \Sigma F_y = 0;$ $V = 0$ **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-Px + P(x - a) + M = 0$

$M = Pa$ **Ans**

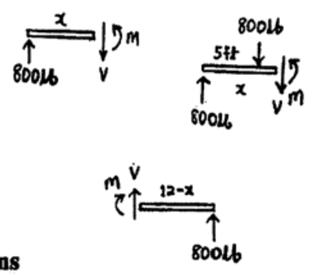


For $L - a < x \le L$

$+\uparrow \Sigma F_y = 0;$ $V = -P$ **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-M + P(L - x) = 0$

$M = P(L - x)$ **Ans**



(b) Set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft

For $0 \le x < 5$ ft

$+\uparrow \Sigma F_y = 0;$ $V = 800$ lb **Ans**

$\circlearrowleft + \Sigma M = 0;$ $M = 800x$ lb·ft **Ans**

For 5 ft $< x <$ 7 ft

$+\uparrow \Sigma F_y = 0;$ $V = 0$ **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-800x + 800(x - 5) + M = 0$

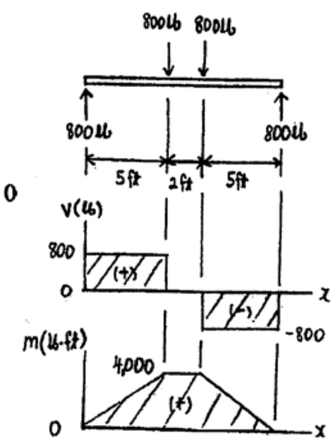
$M = 4000$ lb·ft **Ans**

For 7 ft $< x \le$ 12 ft

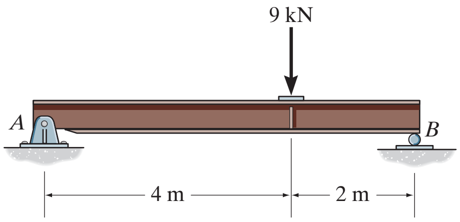
$+\uparrow \Sigma F_y = 0;$ $V = -800$ lb **Ans**

$\circlearrowleft + \Sigma M = 0;$ $-M + 800(12 - x) = 0$

$M = (9600 - 800x)$ lb·ft **Ans**



•7–41. Draw the shear and moment diagrams for the simply supported beam.



Since the loading discontinues at the 9-kN concentrated force, the shear and moment equations must be written for the regions $0 \leq x < 4$ m and 4 m $< x \leq 6$ m of the beam. The free-body diagrams of the beam's segment sectioned through the arbitrary points in these two regions are shown in Figs. *b* and *c*.

Region $0 \leq x < 4$ m, Fig. *b*

$+\uparrow \Sigma F_y = 0;\quad 3 - V = 0 \qquad V = 3\text{kN} \qquad (1)$

$\circlearrowleft + \Sigma M = 0;\quad M - 3x = 0 \qquad M = \{3x\}\text{kN}\cdot\text{m} \qquad (2)$

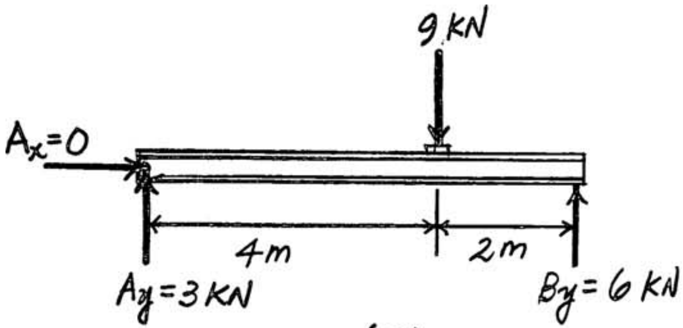
Region 4 m $< x \leq 6$ m, Fig. *c*

$+\uparrow \Sigma F_y = 0;\quad V + 6 = 0 \qquad V = -6\text{ kN} \qquad (3)$

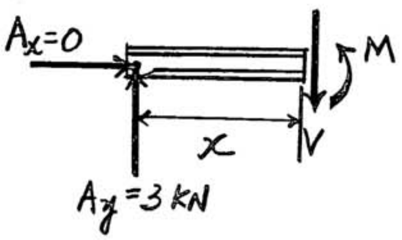
$\circlearrowleft + \Sigma M = 0;\quad 6(6 - x) - M = 0 \qquad M = \{36 - 6x\}\text{ kN}\cdot\text{m} \qquad (4)$

The shear and moment diagrams in Figs. *d* and *e* are plotted using Eqs. (1) and (3), and Eqs. (3) and (4), respectively. The values of the moment at $x = 4$ m are evaluated using either Eqs. (2) or (4),

$M\big|_{x=4\text{ m}} = 3(4) = 12\text{ kN}\cdot\text{m}$ or $M\big|_{x=4\text{m}} = 36 - 6(4) = 12\text{ kN}\cdot\text{m}$

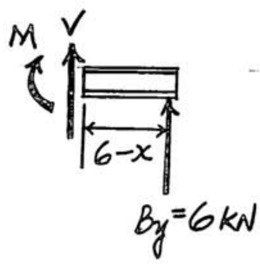


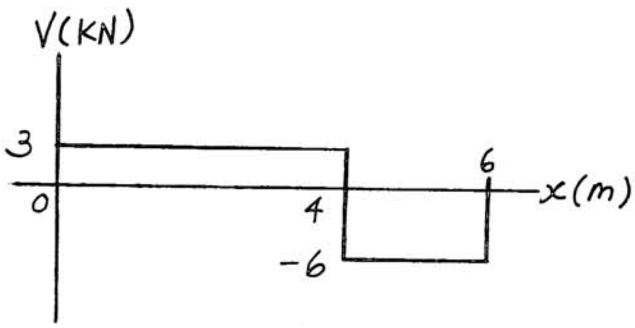
(a)



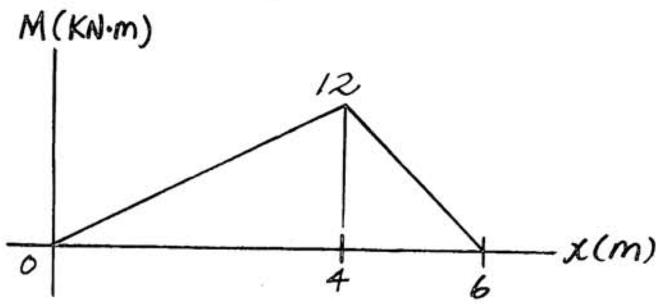
$0 \le x < 4m$ (b)

(c) $4m < x \le 6m$



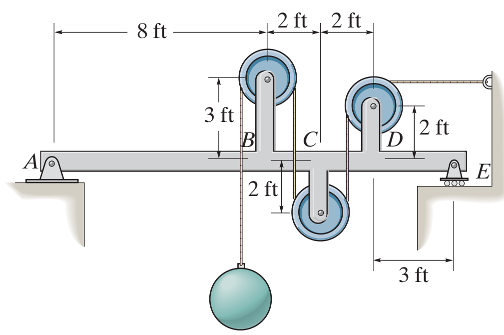


(d)



(e)

7–42. Draw the shear and moment diagrams for the beam *ABCDE*. All pulleys have a radius of 1 ft. Neglect the weight of the beam and pulley arrangement. The load weighs 500 lb.

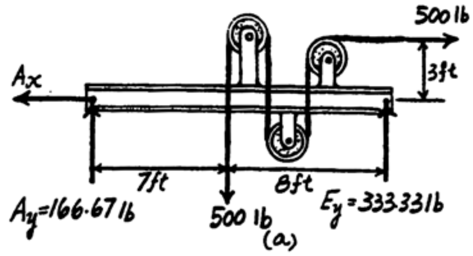


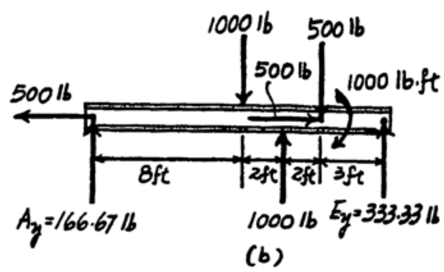
Support Reactions : From FBD (a),

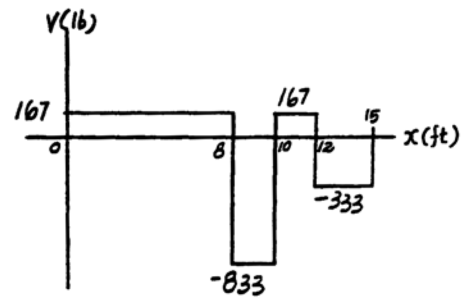
$$\curvearrowleft + \Sigma M_A = 0; \quad E_y(15) - 500(7) - 500(3) = 0 \qquad E_y = 333.33 \text{ lb}$$

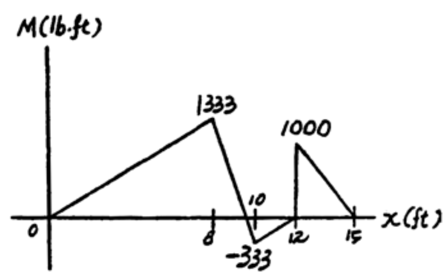
$$+\uparrow \Sigma F_y = 0; \quad A_y + 333.33 - 500 = 0 \qquad A_y = 166.67 \text{ lb}$$

Shear and Moment Diagrams : The load on the pulley at *D* can be replaced by equivalent force and couple moment at *D* as shown on FBD (b).

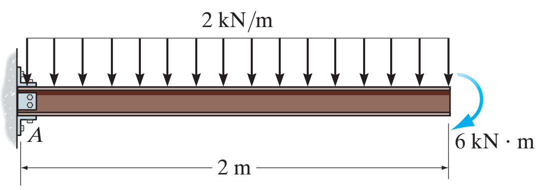








7–43. Draw the shear and moment diagrams for the cantilever beam.



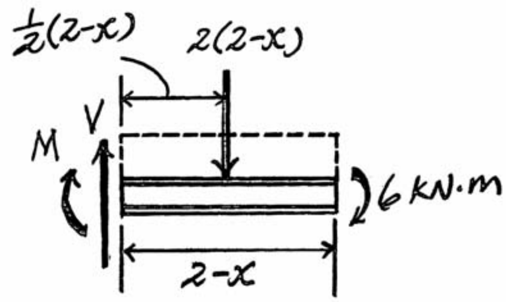
The free - body diagram of the beam's right segment sectioned through an arbitrary point shown in Fig. *a* will be used to write the shear and moment equations of the beam.

$+\uparrow \Sigma F_y = 0; \quad V - 2(2-x) = 0 \qquad V = \{4 - 2x\}\ \text{kN} \qquad (1)$

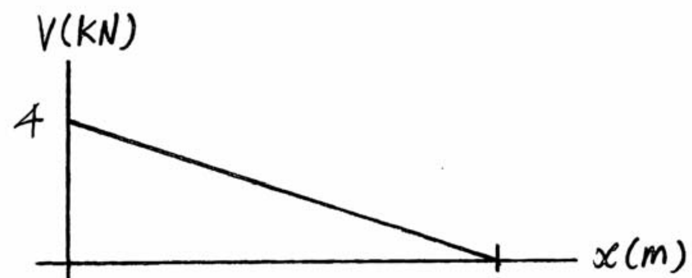
$+\Sigma M = 0; \quad -M - 2(2-x)\left[\frac{1}{2}(2-x)\right] - 6 = 0 \qquad M = \{-x^2 + 4x - 10\}\ \text{kN}\cdot\text{m} \qquad (2)$

The shear and moment diagrams shown in Figs. *b* and *c* are plotted using Eqs. (1) and (2), respectively. The value of the shear and moment at $x = 0$ is evaluated using Eqs. (1) and (2).

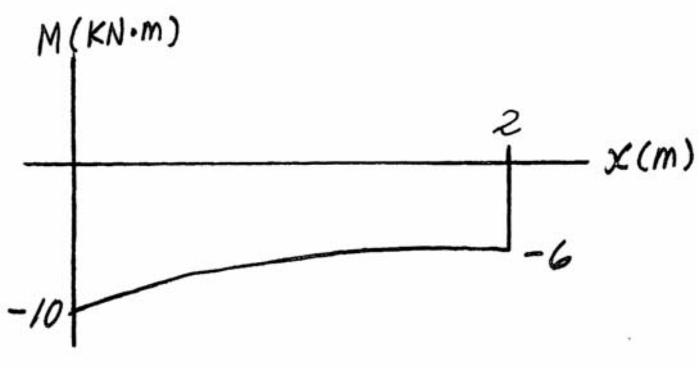
$$V\big|_{x=0} = 4 - 2(0) = 4\ \text{kN}$$
$$M\big|_{x=0} = [-0 + 4(0) - 10] = -10\ \text{kN}\cdot\text{m}$$



(a)

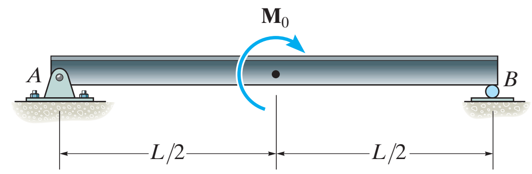


(b)



(c)

***7–44.** Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $M_0 = 500\ \text{N}\cdot\text{m}$, $L = 8$ m.



(a) $\quad 0 \le x < \frac{L}{2}$

$+\uparrow \Sigma F_y = 0; \quad -\frac{M_0}{L} - V = 0$

$V = -\frac{M_0}{L} \quad$ **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + \frac{M_0}{L}x = 0$

$M = -\frac{M_0}{L}x \quad$ **Ans**

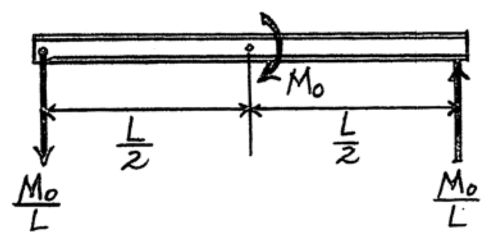
$\frac{L}{2} < x \le L$

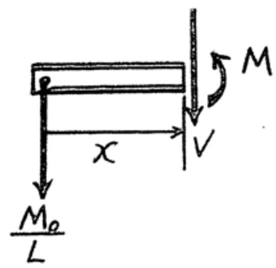
$+\uparrow \Sigma F_y = 0; \quad -\frac{M_0}{L} - V = 0$

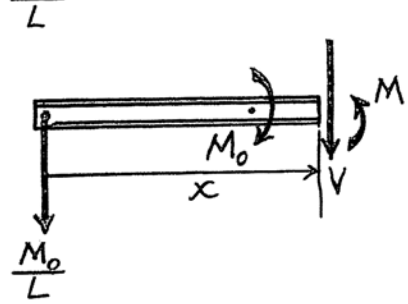
$V = -\frac{M_0}{L} \quad$ **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + \frac{M_0}{L}x - M_0 = 0$

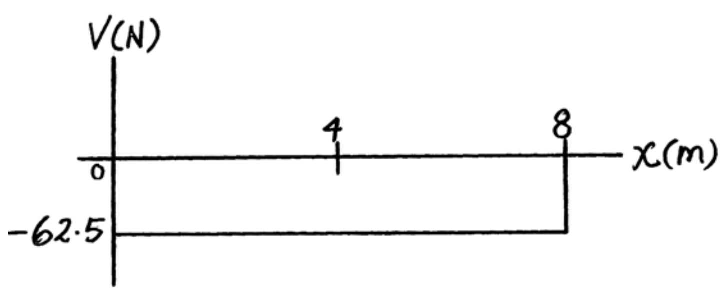
$M = M_0\left(1 - \frac{x}{L}\right) \quad$ **Ans**

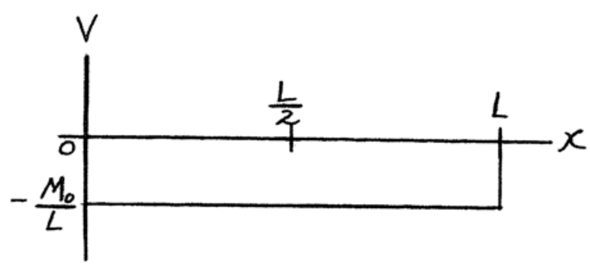


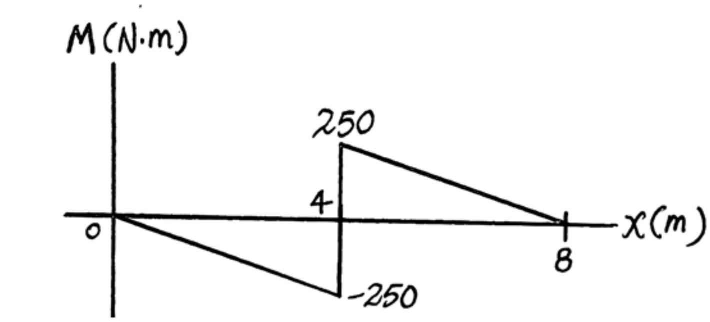


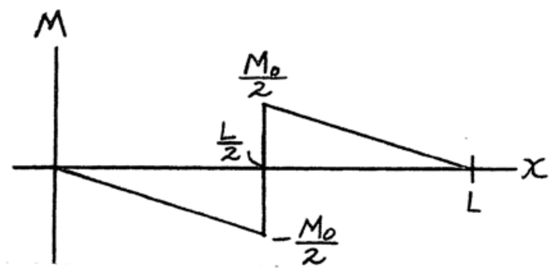


(b) When $M_0 = 500\ \text{N}\cdot\text{m}$, and $L = 8$ m

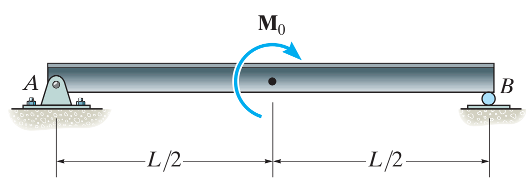








•7–45. If $L = 9$ m, the beam will fail when the maximum shear force is $V_{max} = 5$ kN or the maximum bending moment is $M_{max} = 22$ kN·m. Determine the largest couple moment M_0 the beam will support.



(a) $0 \le x < \frac{L}{2}$

$+\uparrow \Sigma F_y = 0; \quad -\frac{M_0}{L} - V = 0$

$V = -\frac{M_0}{L}$ **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + \frac{M_0}{L}x = 0$

$M = -\frac{M_0}{L}x$ **Ans**

$\frac{L}{2} < x \le L$

$+\uparrow \Sigma F_y = 0; \quad -\frac{M_0}{L} - V = 0$

$V = -\frac{M_0}{L}$ **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + \frac{M_0}{L}x - M_0 = 0$

$M = M_0\left(1 - \frac{x}{L}\right)$ **Ans**

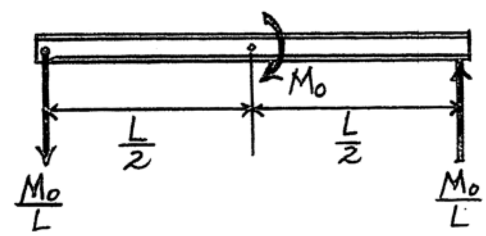
(b) When $M_0 = 500$ N·m, and $L = 8$ m

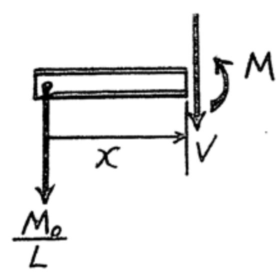
$V_{max} = \frac{M_0}{L}; \quad 5 = \frac{M_0}{9}; \quad M_0 = 45$ kN·m

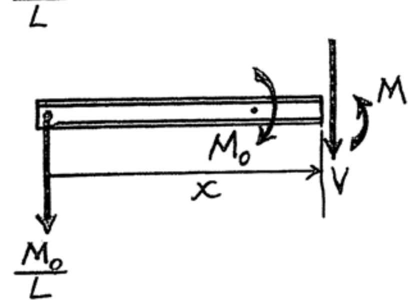
$M_{max} = \frac{M_0}{2}; \quad 22 = \frac{M_0}{2}; \quad M_0 = 44$ kN·m

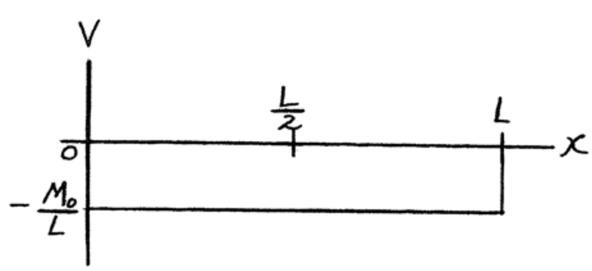
Thus,

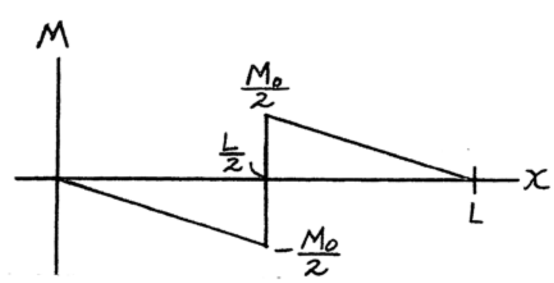
$M_0 = 44$ kN·m **Ans**



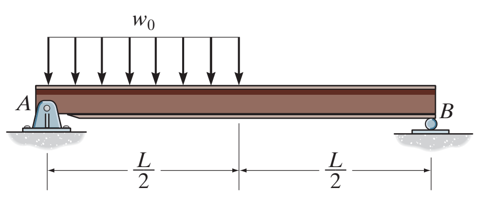








7–46. Draw the shear and moment diagrams for the simply supported beam.



Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < L/2$ and $L/2 < x \le L$ of the beam. The free-body diagram of the beam's segments sectioned through arbitrary points in these two regions are shown in Figs. *b* and *c*.

Region $0 \le x < \frac{L}{2}$, Fig. *b*

$+\uparrow \Sigma F_y = 0;\quad \frac{3}{8}w_0L - w_0x - V = 0\qquad V = w_0\left(\frac{3}{8}L - x\right)\qquad (1)$

$\circlearrowleft + \Sigma M = 0;\quad M + w_0x\left(\frac{x}{2}\right) - \frac{3}{8}w_0L(x) = 0\qquad M = \frac{w_0}{8}(3Lx - 4x^2)\qquad (2)$

Region $L/2 < x \le L$, Fig. *c*

$+\uparrow \Sigma F_y = 0;\quad V + \frac{w_0L}{8} = 0\qquad V = -\frac{w_0L}{8}\qquad (3)$

$\circlearrowleft + \Sigma M = 0;\quad \frac{w_0L}{8}(L - x) - M = 0\qquad M = \frac{w_0L}{8}(L - x)\qquad (4)$

The shear diagram is plotted using Eqs. (1) and (3). The location at where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

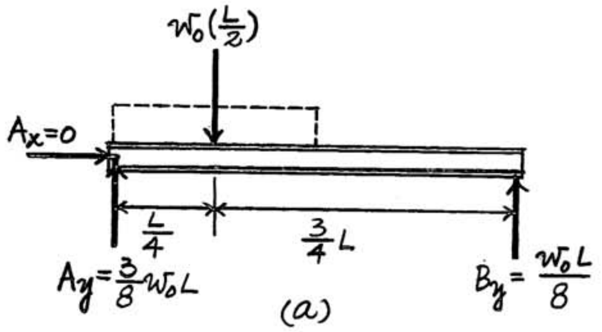
$$0 = w_0\left(\frac{3}{8}L - x\right)\qquad x = \frac{3}{8}L$$

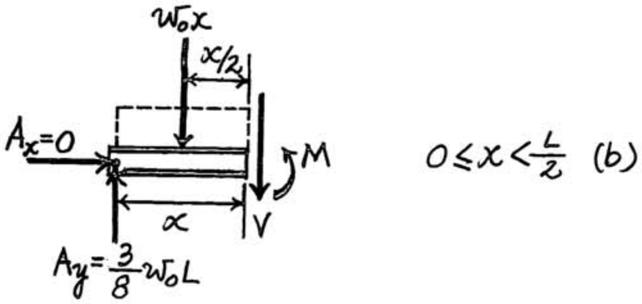
The moment diagram is plotted using Eqs. (2) and (4). The value of the moment at $x = \frac{3}{8}L$ ($V = 0$) can be evaluated using Eq. (2).

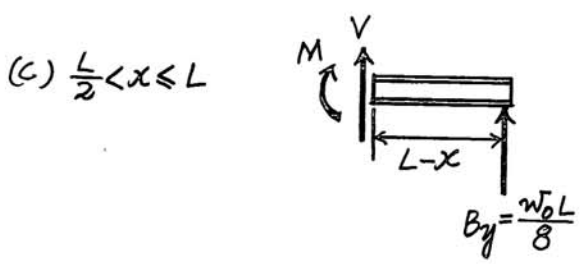
$$M\Big|_{x=\frac{3}{8}L} = \frac{w_0}{8}\left(3L\left(\frac{3}{8}L\right) - 4\left(\frac{3}{8}L\right)^2\right) = \frac{9}{128}w_0L^2$$

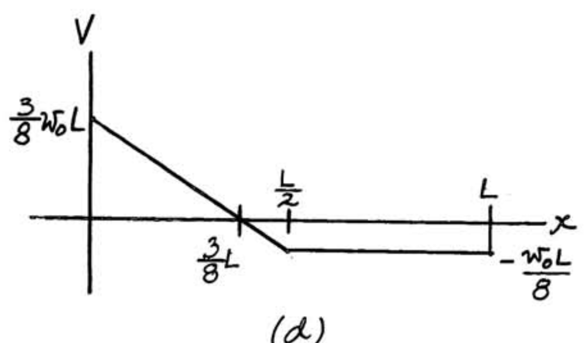
The value of the moment at $x = L/2$ is evaluated using either Eqs. (3) or (4).

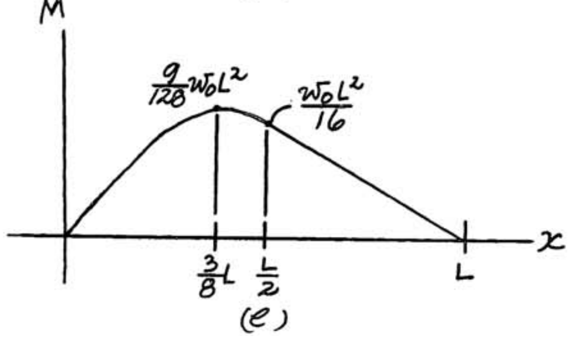
$$M\Big|_{x=\frac{L}{2}} = \frac{w_0L}{8}\left(L - \frac{L}{2}\right) = \frac{w_0L^2}{16}$$


$w_0(\frac{L}{2})$
$A_x = 0$
$\frac{L}{4}$
$\frac{3}{4}L$
$A_y = \frac{3}{8}w_0 L$
$B_y = \frac{w_0 L}{8}$
(a)

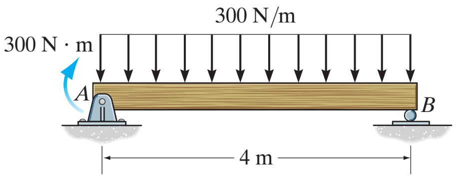

$w_0 x$
$x/2$
$A_x = 0$
M
x
V
$A_y = \frac{3}{8}w_0 L$
$0 \le x < \frac{L}{2}$ (b)


(c) $\frac{L}{2} < x \le L$
V
M
$L - x$
$B_y = \frac{w_0 L}{8}$


V
$\frac{3}{8}w_0 L$
$\frac{L}{2}$
L
x
$\frac{3}{8}L$
$-\frac{w_0 L}{8}$
(d)


M
$\frac{9}{128}w_0 L^2$
$\frac{w_0 L^2}{16}$
x
$\frac{3}{8}L$
$\frac{L}{2}$
L
(e)

7–47. Draw the shear and moment diagrams for the simply supported beam.



The free - body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations.

$+\uparrow \Sigma F_y = 0; \quad 525 - 300x - V = 0 \quad V = \{525 - 300x\}\text{kN} \quad (1)$

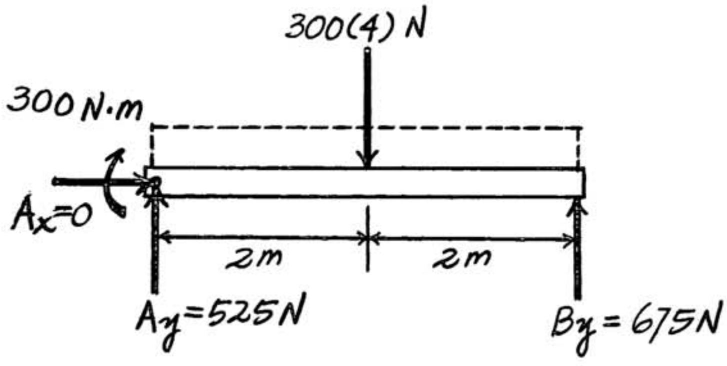
$+\Sigma M = 0; \quad M + 300x\left(\frac{x}{2}\right) - 525x - 300 = 0 \quad M = \{-150x^2 + 525x + 300\}\text{N}\cdot\text{m} \quad (2)$

The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively. The location where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

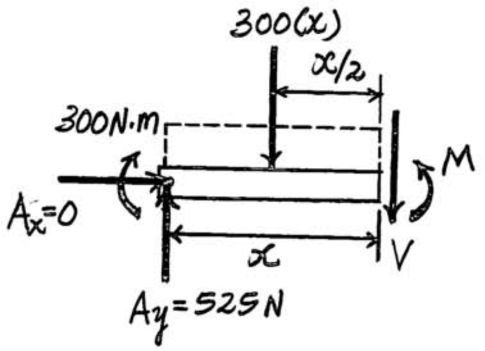
$0 = 525 - 300x \quad x = 1.75\text{ m}$

The value of the moment at $x = 1.75\text{ m}$ ($V = 0$) can be evaluated using Eq. (2).

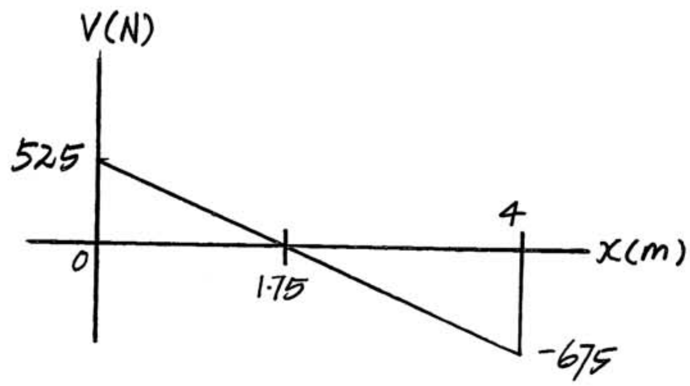
$$M\big|_{x=1.75\text{ m}} = -150(1.75^2) + 525(1.75) + 300 = 759\text{ N}\cdot\text{m}$$


300(4) N
300 N·m
Ax=0
2m
2m
Ay=525N
By=675N

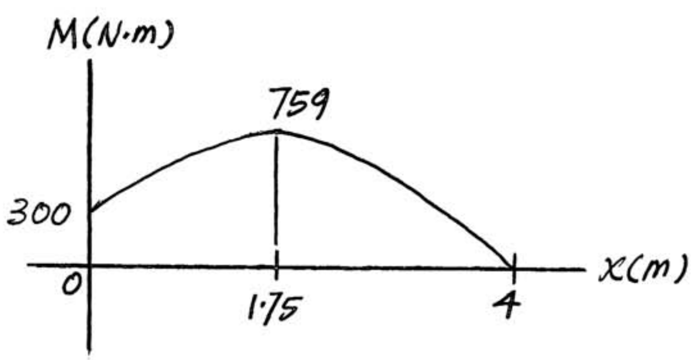
(a)


300(x)
x/2
300N·m
M
Ax=0
x
V
Ay=525N

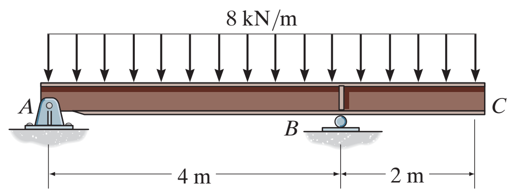
(b)

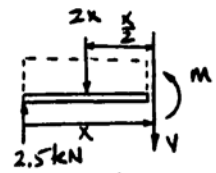

V(N)
525
4
0
x(m)
1.75
-675

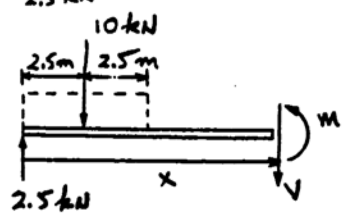
(c)


M(N·m)
759
300
0
x(m)
1.75
4

***7–48.** Draw the shear and moment diagrams for the overhang beam.







$0 \le x < 5\ \text{m}$:

$+\uparrow \Sigma F_y = 0; \quad 2.5 - 2x - V = 0$

$V = 2.5 - 2x$

$\circlearrowleft + \Sigma M = 0; \quad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$M = 2.5x - x^2$

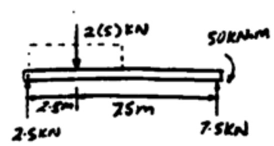
$5\ \text{m} < x \le 10\ \text{m}$:

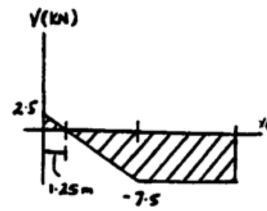
$+\uparrow \Sigma F_y = 0; \quad 2.5 - 10 - V = 0$

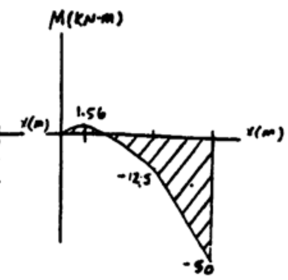
$V = -7.5$

$\circlearrowleft + \Sigma M = 0; \quad M + 10(x - 2.5) - 2.5x = 0$

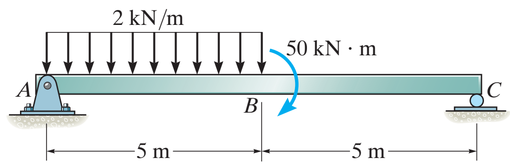
$M = -7.5x + 25$







•7–49. Draw the shear and moment diagrams for the beam.



$0 \leq x < 5\text{ m}$:

$+\uparrow \Sigma F_y = 0; \quad 2.5 - 2x - V = 0$

$$V = 2.5 - 2x$$

$\circlearrowleft + \Sigma M = 0; \quad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$$M = 2.5x - x^2$$

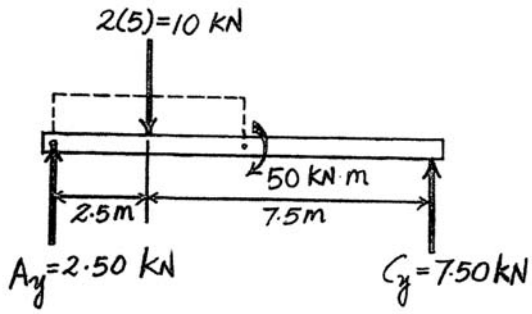
$5\text{ m} < x < 10\text{ m}$:

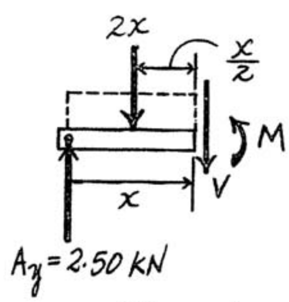
$+\uparrow \Sigma F_y = 0; \quad 2.5 - 10 - V = 0$

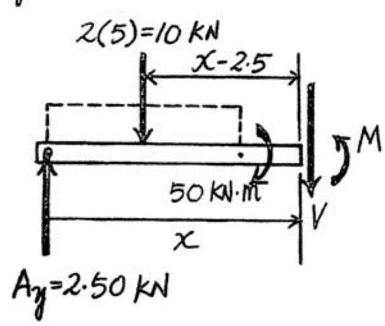
$$V = -7.5$$

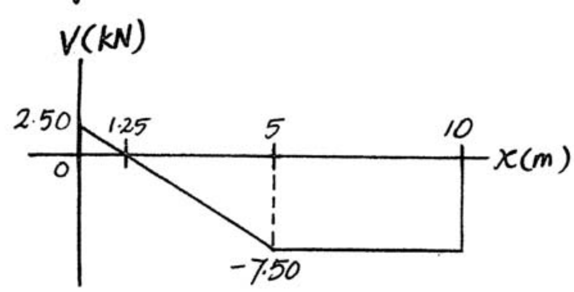
$\circlearrowleft + \Sigma M = 0; \quad M + 10(x - 2.5) - 2.5x - 50 = 0$

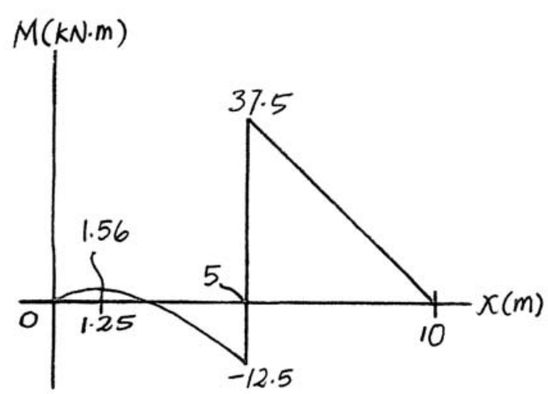
$$M = -7.5x + 75$$



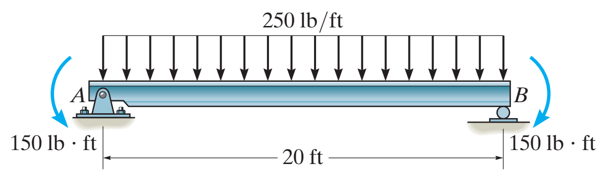








7–50. Draw the shear and moment diagrams for the beam.



$\circlearrowleft + \Sigma M_A = 0;\quad -5000(10) + B_y(20) = 0$

$B_y = 2500\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$

$+\uparrow \Sigma F_y = 0;\quad A_y - 5000 + 2500 = 0$

$A_y = 2500\text{ lb}$

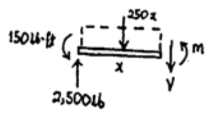
For $0 \le x \le 20\text{ ft}$

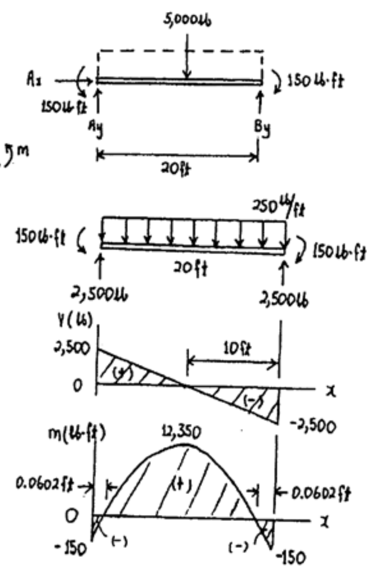
$+\uparrow \Sigma F_y = 0;\quad 2500 - 250x - V = 0$

$V = 250(10 - x)$ **Ans**

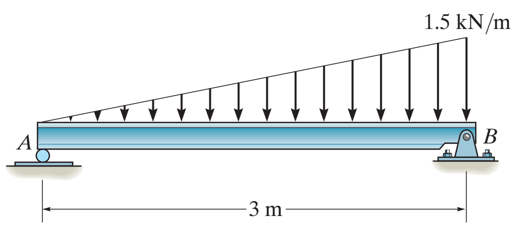
$\circlearrowleft + \Sigma M = 0;\quad -2500(x) + 150 + 250x(\frac{x}{2}) + M = 0$

$M = 25(100x - 5x^2 - 6)$ **Ans.**





7–51. Draw the shear and moment diagrams for the beam.

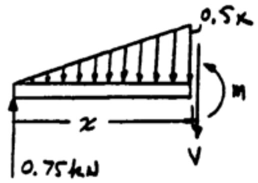


$+\uparrow \Sigma F_y = 0; \quad 0.75 - \frac{1}{2}x(0.5x) - V = 0$

$V = 0.75 - 0.25x^2$

$V = 0 = 0.75 - 0.25x^2$

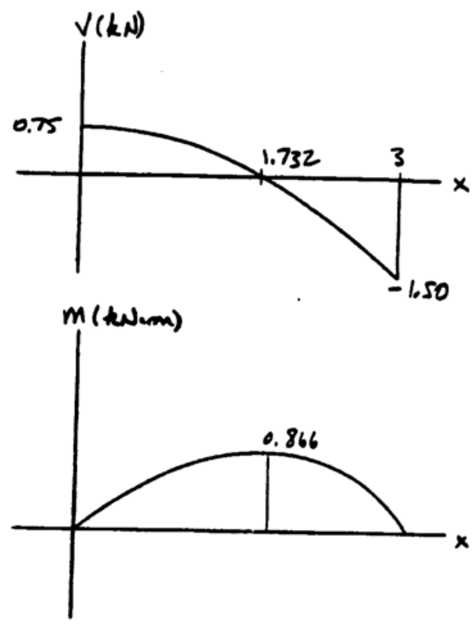
$x = 1.732$ m



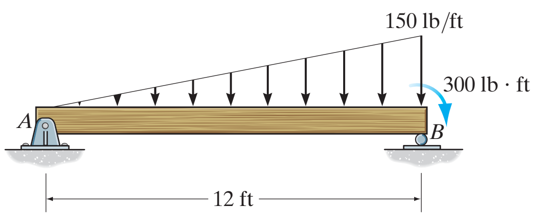
$\curvearrowleft + \Sigma M = 0; \quad M + \left(\frac{1}{2}\right)(0.5x)(x)\left(\frac{1}{3}x\right) - 0.75x = 0$

$M = 0.75x - 0.08333x^3$

$M_{max} = 0.75(1.732) - 0.08333(1.732)^3 = 0.866$



***7–52.** Draw the shear and moment diagrams for the simply supported beam.



The free - body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

$$w = 150\left(\frac{x}{12}\right) = 12.5x$$

Referring to Fig. *b*,

$+\uparrow \Sigma F_y = 0;\quad 275 - \frac{1}{2}(12.5x)(x) - V = 0 \qquad V = \{275 - 6.25x^2\}\ \text{lb} \qquad (1)$

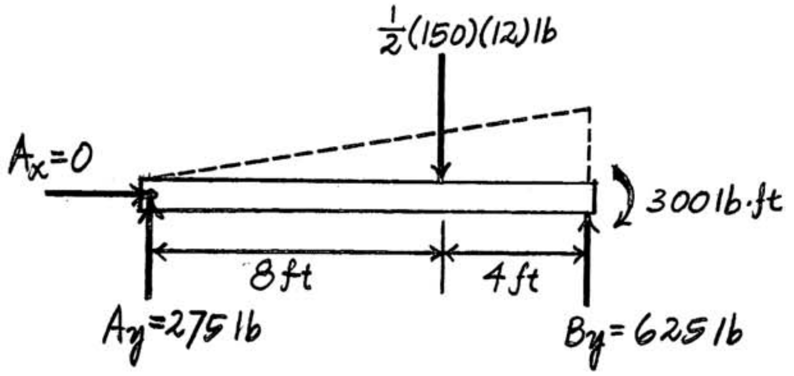
$\circlearrowleft + \Sigma M = 0;\quad M + \frac{1}{2}(12.5x)(x)\left(\frac{x}{3}\right) - 275x = 0 \qquad M = \{275x - 2.083x^2\}\ \text{lb}\cdot\text{ft} \qquad (2)$

The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively. The location where the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

$0 = 275 - 6.25x^2 \qquad x = 6.633\ \text{ft}$

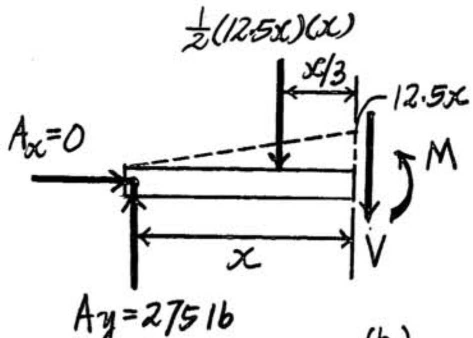
The value of the moment at $x = 6.633$ ft ($V = 0$) is evaluated using Eq. (2).

$$M\big|_{x=6.633\ \text{ft}} = 275(6.633) - 2.083(6.633)^3 = 1216\ \text{lb}\cdot\text{ft}$$



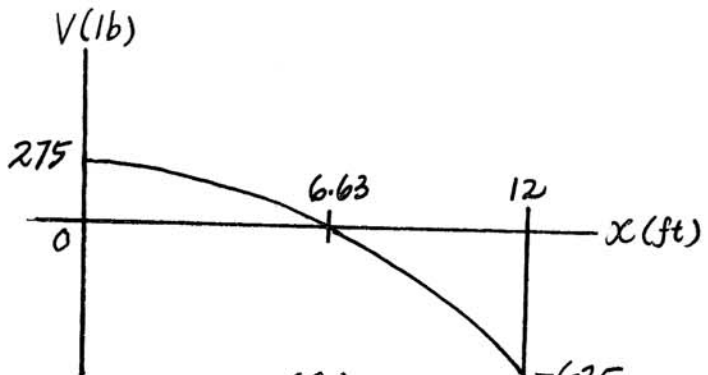
½(150)(12) lb
$A_x=0$
300 lb·ft
8 ft
4 ft
$A_y=275$ lb
$B_y=625$ lb

(a)



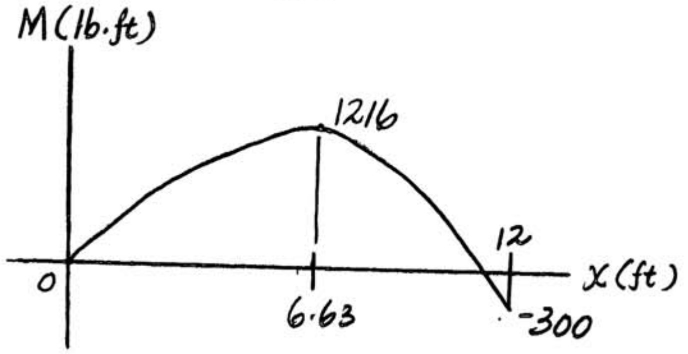
½(12.5x)(x)
x/3
12.5x
$A_x=0$
M
x
V
$A_y=275$ lb

(b)



V (lb)
275
6.63
12
0
x (ft)
-625

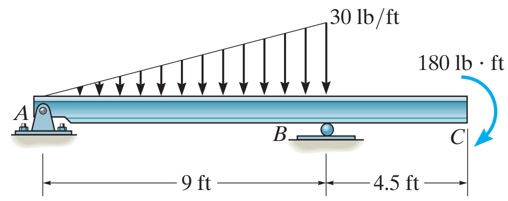
(c)

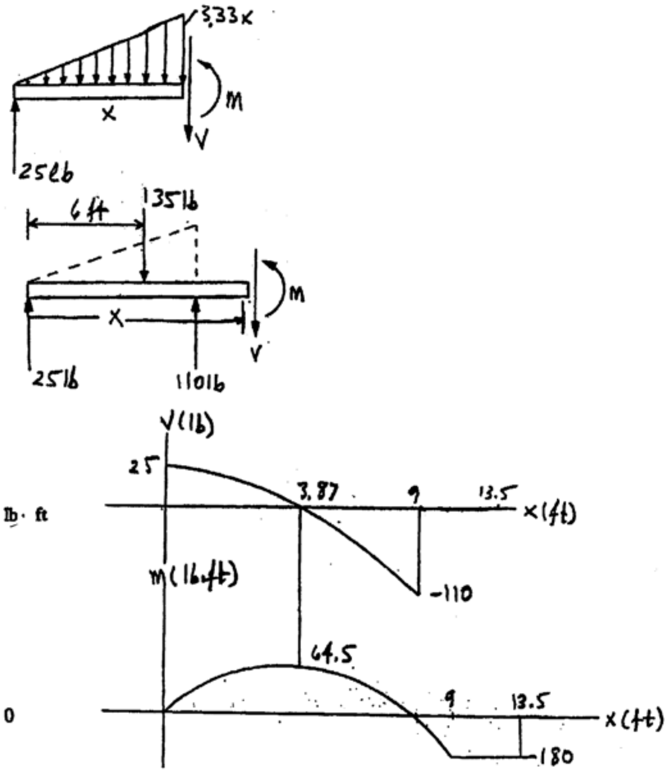


M (lb·ft)
1216
12
0
x (ft)
6.63
-300

(d)

•7–53. Draw the shear and moment diagrams for the beam.





$0 \le x < 9$ ft:

$+\uparrow \Sigma F_y = 0; \quad 25 - \frac{1}{2}(3.33x)(x) - V = 0$

$V = 25 - 1.667x^2$ **Ans**

$V = 0 = 25 - 1.667x^2$

$x = 3.87$ ft

$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(3.33x)(x)\left(\frac{x}{3}\right) - 25x = 0$

$M = 25x - 0.5556x^3$ **Ans**

$M_{max} = 25(3.87) - 0.5556(3.87)^3 = 64.5 \text{ lb} \cdot \text{ft}$

$9 \text{ ft} < x < 13.5$ ft:

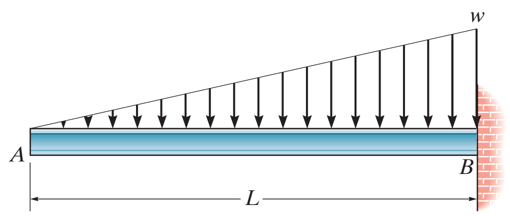
$+\uparrow \Sigma F_y = 0; \quad 25 - 135 + 110 - V = 0$

$V = 0$ **Ans**

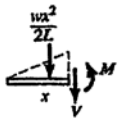
$\circlearrowleft + \Sigma M = 0; \quad -25x + 135(x - 6) - 110(x - 9) + M = 0$

$M = -180$ **Ans**

7–54. If $L = 18$ ft, the beam will fail when the maximum shear force is $V_{max} = 800$ lb, or the maximum moment is $M_{max} = 1200$ lb·ft. Determine the largest intensity w of the distributed loading it will support.



For $0 \le x \le L$



$+\uparrow \Sigma F_y = 0; \qquad V = -\frac{wx^2}{2L}$

$\circlearrowleft + \Sigma M = 0; \qquad M = -\frac{wx^3}{6L}$

$V_{max} = \frac{-wL}{2}$

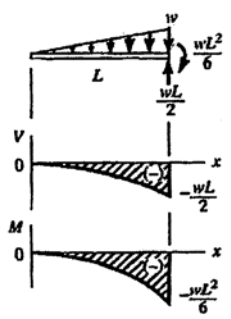
$-800 = \frac{-w(18)}{2}$

$w = 88.9$ lb/ft

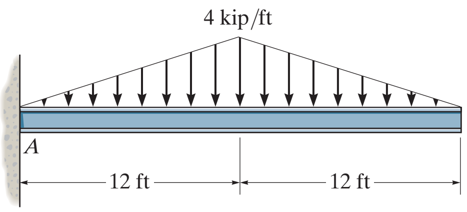
$M_{max} = -\frac{wL^2}{6};$

$-1200 = \frac{-w(18)^2}{6}$

$w = 22.2$ lb/ft **Ans**



7–55. Draw the shear and moment diagrams for the beam.



Support Reactions : From FBD (a),

$$\curvearrowleft + \Sigma M_A = 0; \quad M_A - 48.0(12) = 0 \quad M_A = 576 \text{ kip} \cdot \text{ft}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 48.0 = 0 \quad A_y = 48.0 \text{ kip}$$

Shear and Moment Functions : For $0 \le x < 12$ ft [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \quad 48.0 - \frac{x^2}{6} - V = 0$$

$$V = \left\{48.0 - \frac{x^2}{6}\right\} \text{ kip} \qquad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M = 0; \quad M + \frac{x^2}{6}\left(\frac{x}{3}\right) + 576 - 48.0x = 0$$

$$M = \left\{48.0x - \frac{x^3}{18} - 576\right\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

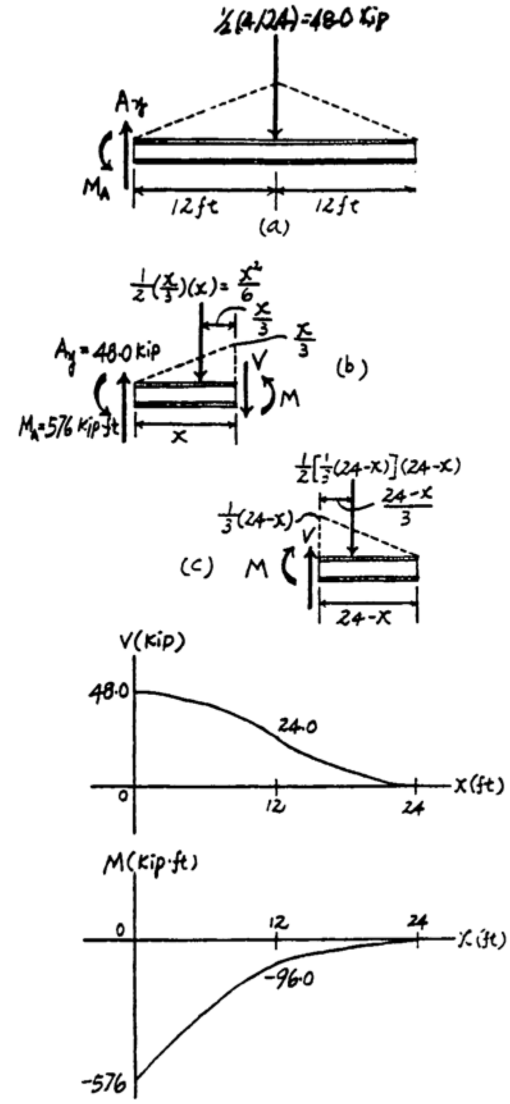
For $12 \text{ ft} < x \le 24$ ft [FBD (c)],

$$+\uparrow \Sigma F_y = 0; \quad V - \frac{1}{2}\left[\frac{1}{3}(24-x)\right](24-x) = 0$$

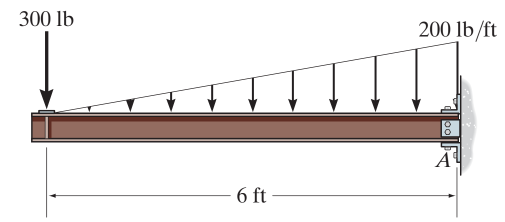
$$V = \left\{\frac{1}{6}(24-x)^2\right\} \text{ kip} \qquad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M = 0; \quad -\frac{1}{2}\left[\frac{1}{3}(24-x)\right](24-x)\left(\frac{24-x}{3}\right) - M = 0$$

$$M = \left\{-\frac{1}{18}(24-x)^3\right\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$



***7–56.** Draw the shear and moment diagrams for the cantilevered beam.



The free-body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

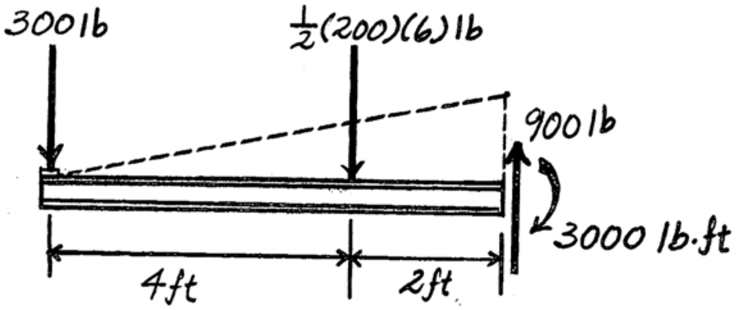
$$w = 200\left(\frac{x}{6}\right) = 33.33x$$

Referring to Fig. *b*,

$+\uparrow \Sigma F_y = 0; \quad -300 - \frac{1}{2}(33.33x)(x) - V = 0 \qquad V = \{-300 - 16.67x^2\}\text{ lb} \qquad (1)$

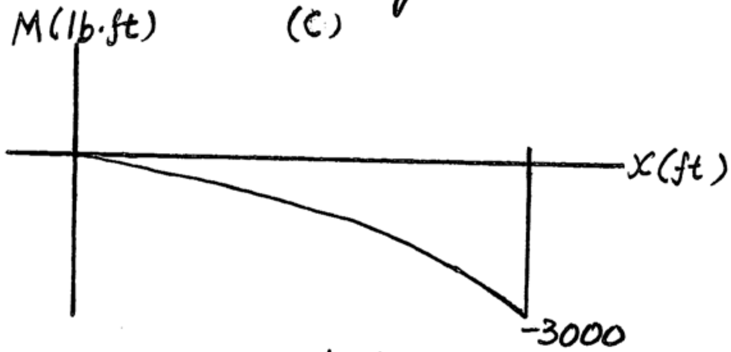
$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(33.33x)(x)\left(\frac{x}{3}\right) + 300x = 0 \qquad M = \{-300x - 5.556x^3\}\text{ lb}\cdot\text{ft} \qquad (2)$

The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively.

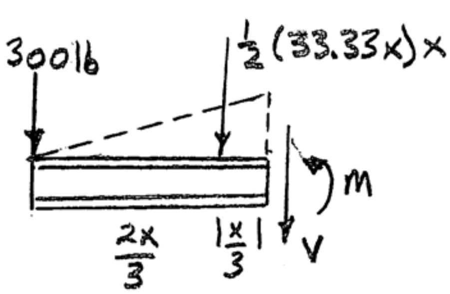


(a)

Shear diagram
(c)

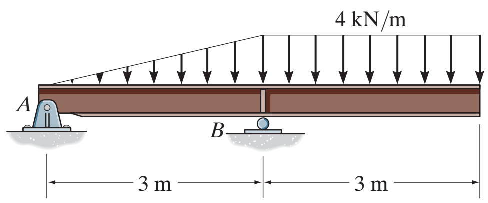


Moment diagram
(d)



(b)

•7–57. Draw the shear and moment diagrams for the overhang beam.



Since the loading is discontinuous at support B, the shear and moment equations must be written for regions $0 \leq x < 3$ m and $3 \text{ m} < x \leq 6$ m of the beam. The free - body diagram of the beam' s segment sectioned through an arbitrary point within these two regions is shown in Figs. b and c.

Region $0 \leq x < 3$ m, Fig. b

$$+\uparrow \Sigma F_y = 0; \qquad -4-\frac{1}{2}\left(\frac{4}{3}x\right)(x)-V=0 \qquad V=\left\{-\frac{2}{3}x^2-4\right\}\text{ kN} \qquad (1)$$

$$\circlearrowleft+\Sigma M = 0; \quad M+\frac{1}{2}\left(\frac{4}{3}x\right)(x)\left(\frac{x}{3}\right)+4x=0 \qquad M=\left\{-\frac{2}{9}x^3-4x\right\}\text{ kN}\cdot\text{m} \qquad (2)$$

Region $3\text{ m} < x \leq 6$ m, Fig. c

$$+\uparrow \Sigma F_y = 0; \qquad V-4(6-x)=0 \qquad V=\{24-4x\}\text{kN} \qquad (3)$$

$$\circlearrowleft+\Sigma M = 0; \quad -M-4(6-x)\left[\frac{1}{2}(6-x)\right]=0 \qquad M=\{-2(6-x)^2\}\text{ kN}\cdot\text{m} \qquad (4)$$

The shear diagram shown in Fig. d is plotted using Eqs. (1) and (3). The value of shear just to the left and just to the right of the support is evaluated using Eqs. (1) and (3), respectively.

$$V\big|_{x=3\text{ m}-}=-\frac{2}{3}(3^2)-4=-10\text{ kN}$$

$$V\big|_{x=3\text{ m}+}=24-4(3)=12\text{ kN}$$

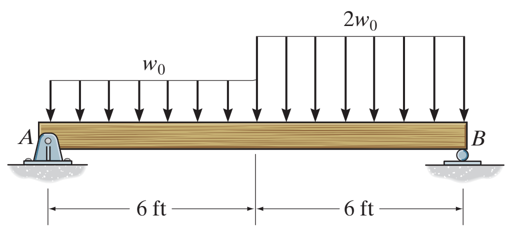
The moment diagram shown in Fig. e is plotted using Eqs. (2) and (4). The value of the moment at support B is evaluated using either Eq. (2) or Eq. (4).

$$M\big|_{x=3\text{ m}}=-\frac{2}{9}(3^3)-4(3)=-18\text{ kN}\cdot\text{m}$$

or

$$M\big|_{x=3\text{ m}}=-2(6-3)^2=-18\text{ kN}\cdot\text{m}$$

7–58. Determine the largest intensity w_0 of the distributed load that the beam can support if the beam can withstand a maximum shear force of $V_{max} = 1200$ lb and a maximum bending moment of $M_{max} = 600$ lb · ft.



Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < 6$ ft and $6\text{ ft} < x \le 12$ ft of the beam. The free - body diagram of the beam's segment sectioned through the arbitrary point within these two regions are shown in Figs. *b* and *c*.

Region $0 \le x \le 6$ ft, Fig. *b*

$+\uparrow \Sigma F_y = 0; \quad 7.5w_0 - w_0 x - V = 0 \qquad V = w_0(7.5 - x) \qquad (1)$

$\circlearrowleft + \Sigma M = 0; \; M + w_0 x\left(\frac{x}{2}\right) - 7.5w_0 x = 0 \qquad M = \frac{w_0}{2}(15x - x^2) \qquad (2)$

Region $6\text{ ft} < x \le 12$ ft, Fig. *c*

$+\uparrow \Sigma F_y = 0; \quad 10.5w_0 - 2w_0(12 - x) + V = 0 \qquad V = w_0(13.5 - 2x) \qquad (3)$

$\circlearrowleft + \Sigma M = 0; \; 10.5w_0(12 - x) - 2w_0(12 - x)\left[\frac{1}{2}(12 - x)\right] - M = 0$

$$M = w_0(-x^2 + 13.5x - 18) \qquad (4)$$

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The value of the shear at $x = 6$ ft is evaluated using either Eq. (1) or Eq. (3).

$$V\big|_{x=6\text{ ft}} = w_0(7.5 - 6) = 1.5w_0$$

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (3).

$$0 = w_0(13.5 - 2x) \qquad x = 6.75\text{ ft}$$

The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at $x = 6$ ft is evaluated using either Eqs. (2) or (4).

$$M\big|_{x=6\text{ ft}} = \frac{w_0}{2}(15.6 - 6^2) = 27w_0$$

The value of the moment at $x = 6.75$ ft (where $V = 0$) is evaluated using Eq. (4).

$$M\big|_{x=6.75\text{ ft}} = w_0\left[-6.75^2 + 13.5(6.75) - 18\right] = 27.5625w_0$$

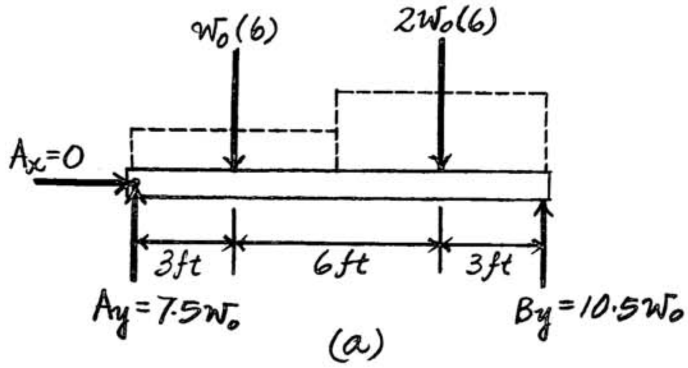
By observing the shear and moment diagrams, we notice that $V_{max} = 10.5w_0$ and $M_{max} = 27.56w_0$. Thus,

$V_{max} = 1200 = 10.5w_0$

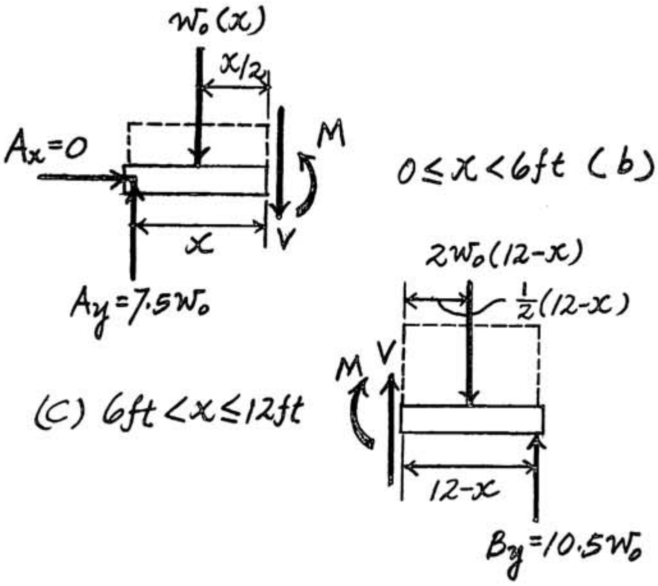
$w_0 = 114.29$ lb·ft

$M_{max} = 600 = 27.56w_0$

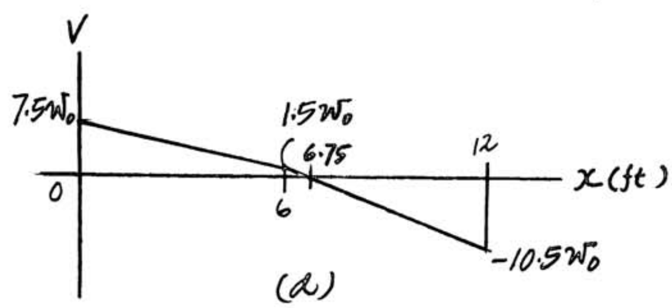
$w_0 = 21.8$ lb/ft (control!) **Ans.**



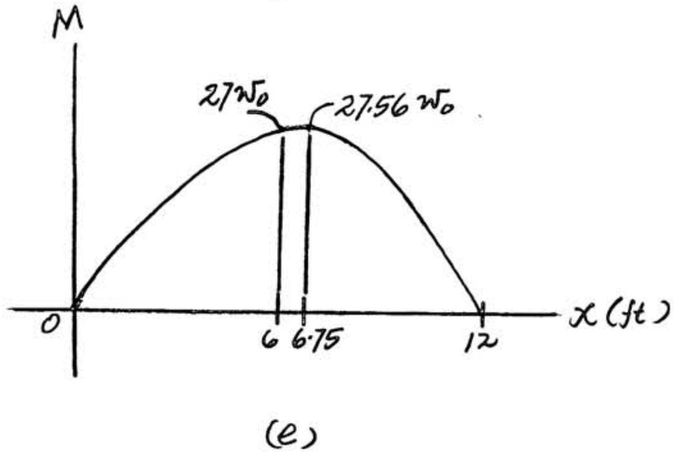
w₀(6)
2w₀(6)
Ax=0
3ft
6ft
3ft
Ay=7.5w₀
By=10.5w₀
(a)



w₀(x)
x/2
Ax=0
M
V
x
Ay=7.5w₀
0≤x<6ft (b)
2w₀(12-x)
½(12-x)
M V
(c) 6ft<x≤12ft
12-x
By=10.5w₀

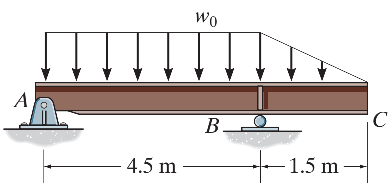


V
7.5w₀
1.5w₀
6.75
12
x (ft)
0
6
-10.5w₀
(d)



M
27w₀
27.56 w₀
0
6
6.75
12
x (ft)
(e)

7–59. Determine the largest intensity w_0 of the distributed load that the beam can support if the beam can withstand a maximum bending moment of $M_{max} = 20$ kN·m and a maximum shear force of $V_{max} = 80$ kN.



Since the loading is discontinuous at support B, the shear and moment equations must be written for regions $0 \le x < 4.5$ m and $4.5 \text{ m} < x \le 6$ m of the beam. The free-body diagram of the beam's segment sectioned through the arbitrary points within these two regions are shown in Figs. b and c.

Region $0 \le x < 4.5$ m, Fig. b

$+\uparrow \Sigma F_y = 0;\quad 2.167w_0 - w_0 x - V = 0 \qquad V = w_0(2.167 - x) \qquad (1)$

$\curvearrowleft + \Sigma M = 0;\quad M + w_0 x\left(\frac{x}{2}\right) - 2.167 w_0 x = 0 \qquad M = w_0(2.167x - 0.5x^2) \qquad (2)$

Region $4.5 \text{ m} < x \le 6$ m, Fig. c

$+\uparrow \Sigma F_y = 0;\quad V - \frac{1}{2}\left[\left(\frac{6-x}{1.5}\right)w_0\right](6-x) = 0 \qquad V = \frac{w_0}{3}(6-x)^2 \qquad (3)$

$\curvearrowleft + \Sigma M = 0;\quad -M - \frac{1}{2}\left[\left(\frac{6-x}{1.5}\right)w_0\right](6-x)\left[\frac{1}{3}(6-x)\right] = 0$

$$M = -\frac{w_0}{9}(6-x)^3 \qquad (4)$$

The shear diagram shown in Fig. d is plotted using Eqs. (1) and (3). The value of the shear just to the left and right of support B is evaluated using either Eq. (1) or Eq. (3), respectively.

$$V\big|_{x=4.5\text{ m}-} = w_0(2.167 - 4.5) = -2.333w_0$$

$$V\big|_{x=4.5\text{ m}+} = \frac{w_0}{3}(6 - 4.5)^2 = 0.75w_0$$

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

$$0 = w_0(2.167 - x) \qquad x = 2.167 \text{ m}$$

The moment diagram shown in Fig. e is plotted using Eqs. (2) and (4). The value of the moment at $x = 2.167$ m ($V = 0$) is evaluated using Eq. (2).

$$M\big|_{x=2.167\text{ m}} = w_0\left[2.167(2.167) - 0.5(2.167^2)\right] = 2.347w_0$$

The value of the moment at support B is evaluated using Eqs. (2) or (4).

$$M\big|_{x=4.5\text{ m}} = -\frac{w_0}{9}(6 - 4.5)^3 = -0.375w_0$$

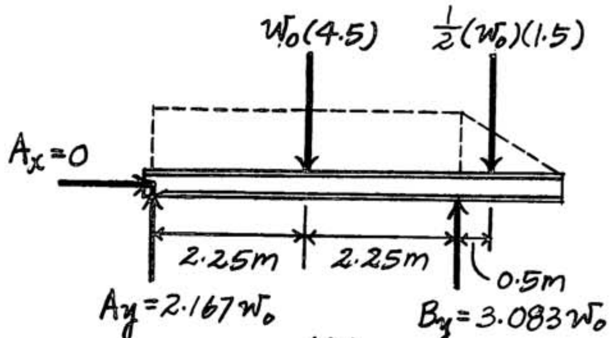
By observing the shear and moment diagrams, we notice that $V_{max} = 2.333w_0$ and $M_{max} = 2.347w_0$. Thus,

$V_{max} = 80 = 2.333w_0$

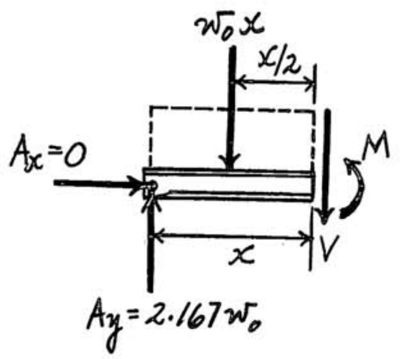
$w_0 = 34.29$ kN/m

$M_{max} = 20 = 2.347w_0$

$w_0 = 8.52$ kN/m (control!) **Ans.**

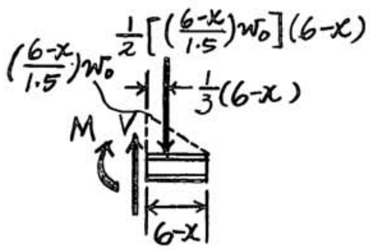


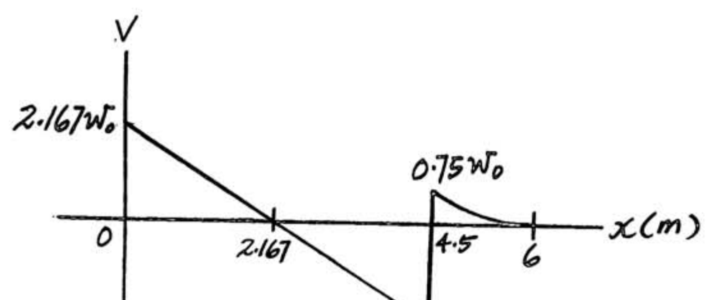
(a)



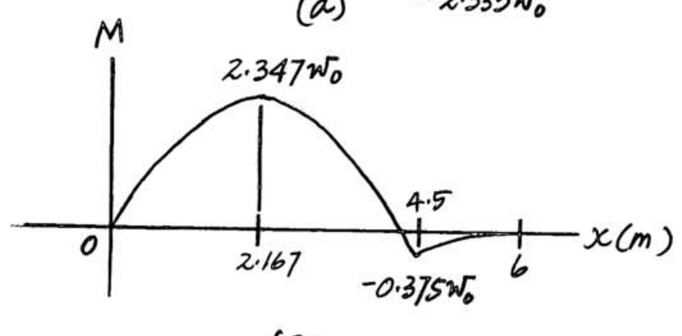
$0 \le x < 4.5\,\text{m}$ (b)

(C) $4.5\,\text{m} < x \le 6\,\text{m}$



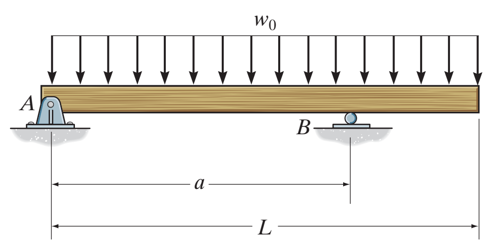


(d)



(e)

***7–60.** Determine the placement a of the roller support B so that the maximum moment within the span AB is equivalent to the moment at the support B.



Since the loading is discontinuous at support B, the shear and moment equations must be written for regions $0 \le x < a$ and $a < x \le L$. The free - body diagram of the beam's segment sectioned through the arbitrary points within these two regions are shown in Figs. b and c.

Region $0 \le x < a$, Fig. b

$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0}{2a}(2aL - L^2) - w_0 x - V = 0 \qquad V = \frac{w_0}{2a}(2aL - L^2 - 2ax) \quad (1)$$

$$\circlearrowleft + \Sigma M = 0; \quad M + w_0 x\left(\frac{x}{2}\right) - \frac{w_0}{2a}(2aL - L^2)x = 0 \qquad M = \frac{w_0}{2a}\left[(2aL - L^2)x - ax^2\right] \quad (2)$$

Region $a < x \le L$, Fig. c

$$+\uparrow \Sigma F_y = 0; \quad V - w_0(L - x) = 0 \qquad V = w_0(L - x) \quad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad -M - w_0(L - x)\left[\frac{1}{2}(L - x)\right] = 0 \quad M = -\frac{w_0}{2}(L - x)^2 \quad (4)$$

The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

$$0 = \frac{w_0}{2a}(2aL - L^2 - 2ax) \qquad x = \frac{2aL - L^2}{2a}$$

The maximum span moment occurs at the position at which $V = 0$. Thus, using Eq. (2), we obtain

$$\left(M_{\text{span}}\right)_{\text{max}} = \frac{w_0}{2a}\left[(2aL - L^2)\left(\frac{2aL - L^2}{2a}\right) - a\left(\frac{2aL - L^2}{2a}\right)^2\right] = \frac{w_0}{8a^2}\left[\left(2aL - L^2\right)^2\right]$$

The support moment at B is evaluated using Eq. (2).

$$M_{\text{suppport}} = \frac{w_0}{2a}\left[(2aL - L^2)a - a^3\right] = \frac{w_0}{2}(2aL - L^2 - a^2) = -\frac{w_0}{2}(L - a)^2$$

The support moment at B can also be computed from Eq. (4).

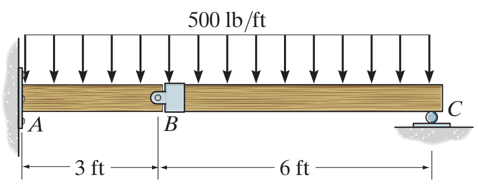
$$M_{\text{suppport}} = -\frac{w_0}{2}(L - a)^2$$

Here, we require $\left|(M_{\text{max}})_{\text{span}}\right| = \left|M_{\text{support}}\right|$ Thus,

$$\frac{w_0}{8a^2}(2aL - L^2)^2 = \frac{w_0}{2}(L - a)^2$$

$$a = \frac{L}{\sqrt{2}} \qquad \textbf{Ans.}$$

•7–61. The compound beam is fix supported at A, pin connected at B and supported by a roller at C. Draw the shear and moment diagrams for the beam.



The support reactions at A and C and the interaction force at pin connection B are indicated on the free-body diagram of members AB and BC of the compound beam shown in Figs. a and b. Since the loading is continuous through the entire beam and the interaction force at the pin connection at B is internal to the beam, the shear and moment equations can be described by a single function. The free-body diagram of the beam's left segment sectioned through an arbitrary point is shown in Fig. c.

By referring to Fig. c, we have

$+\uparrow \Sigma F_y = 0; \quad 3000 - 500x - V = 0 \qquad V = \{3000 - 500x\}\ \text{lb} \qquad (1)$

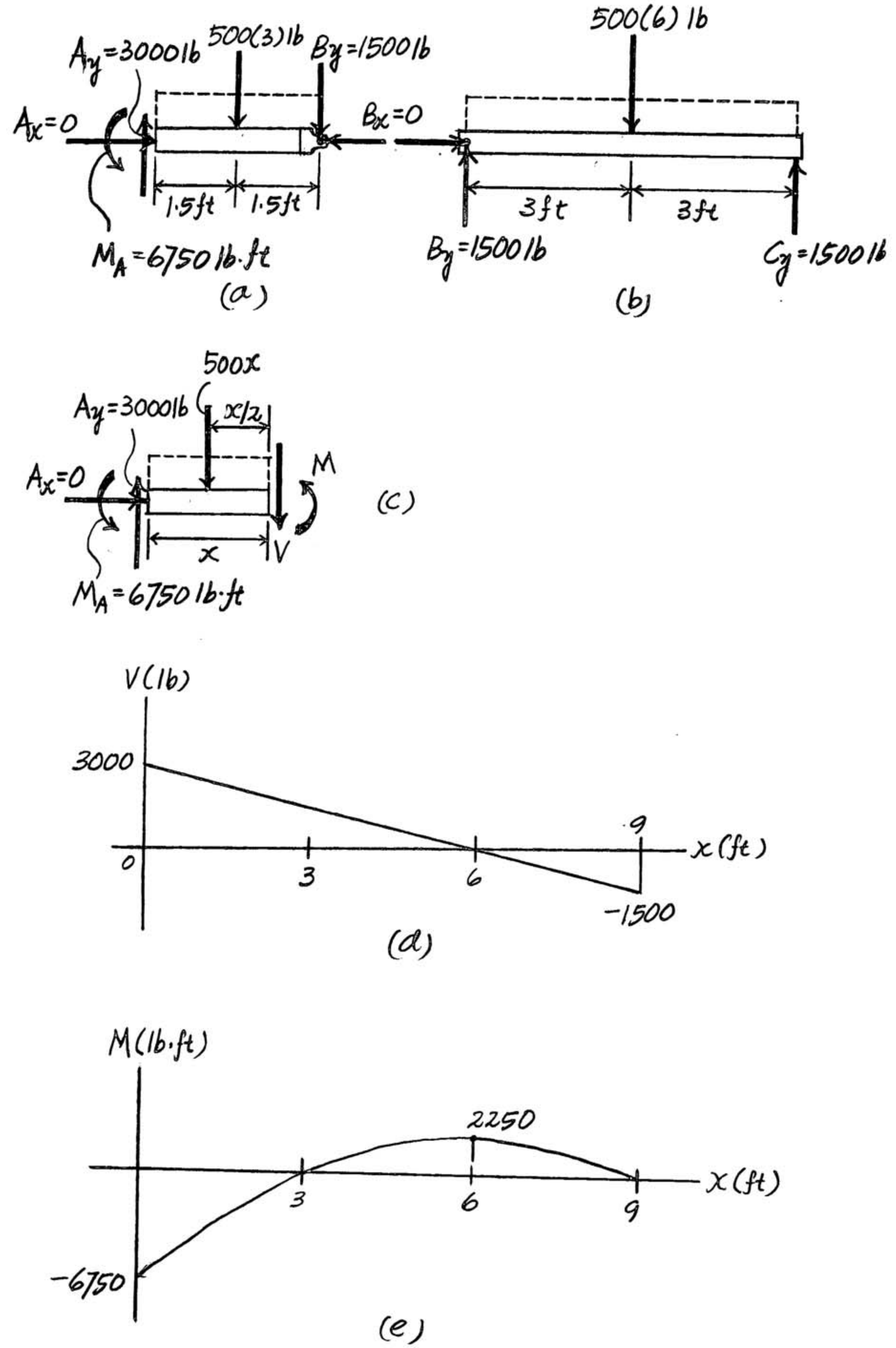
$\circlearrowleft + \Sigma M = 0; \quad M + 500x\left(\frac{x}{2}\right) + 6750 - 3000x = 0 \qquad M = \{3000x - 250x^2 - 6750\}\ \text{lb}\cdot\text{ft} \qquad (2)$

The shear and moment diagram shown in Figs. d and e are plotted using Eqs. (1) and (2), respectively. The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

$0 = 3000 - 500x \qquad x = 6\ \text{ft}$

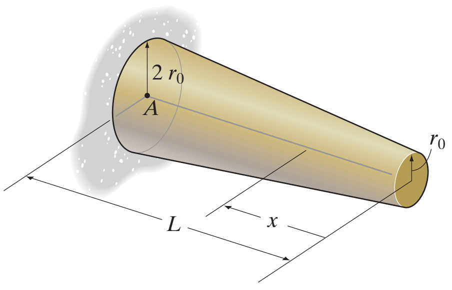
The value of the moment at $x = 6$ ft ($V = 0$) is computed using Eq. (2).

$$M\big|_{x=6\ \text{ft}} = 3000(6) - 250(6^2) - 6750 = 2250\ \text{lb}\cdot\text{ft}$$



Ay=3000 lb
500(3) lb
By=1500 lb
Ax=0
MA=6750 lb·ft
1.5ft
1.5ft
(a)
500(6) lb
Bx=0
3ft
3ft
By=1500 lb
Cy=1500 lb
(b)
500x
Ay=3000 lb
x/2
Ax=0
M
V
x
MA=6750 lb·ft
(c)
V(lb)
3000
0
3
6
9
x(ft)
-1500
(d)
M(lb·ft)
2250
3
6
9
x(ft)
-6750
(e)

7–62. The frustum of the cone is cantilevered from point A. If the cone is made from a material having a specific weight of γ, determine the internal shear force and moment in the cone as a function of x.



Using the similar triangles shown in Fig. *a*,

$$r = r_0 + \frac{r_0}{L}x = \frac{r_0}{L}(L+x)$$

$$\frac{L'}{r_0} = \frac{L+L'}{2r_0} \qquad L' = L$$

Thus, the volume of the frustrum of the cone shown shaded in Fig. *a* is

$$V = \frac{1}{3}\pi\left[\frac{r_0}{L}(L+x)\right]^2(L+x) - \frac{1}{3}\pi r_0^2 L$$

$$= \frac{\pi r_0^2}{3L^2}\left[(L+x)^3 - L^3\right]$$

The weight of the frustrum is

$$W = \gamma V = \frac{\pi\gamma r_0^2}{3L^2}\left[(L+x)^3 - L^3\right]$$

The location $\bar{x}$ of the center of gravity of the frustrum is

$$\bar{x} = \frac{\frac{1}{3}\pi\left[\frac{r_0}{L}(L+x)\right]^2(L+x)\left[\frac{1}{4}(L+x)\right] - \frac{1}{3}\pi r_0^2 L\left(x+\frac{L}{4}\right)}{\frac{\pi r_0^2}{3L^2}\left[(L+x)^3 - L^3\right]} = \frac{(L+x)^4 - L^3(4x+L)}{4\left[(L+x)^3 - L^3\right]}$$

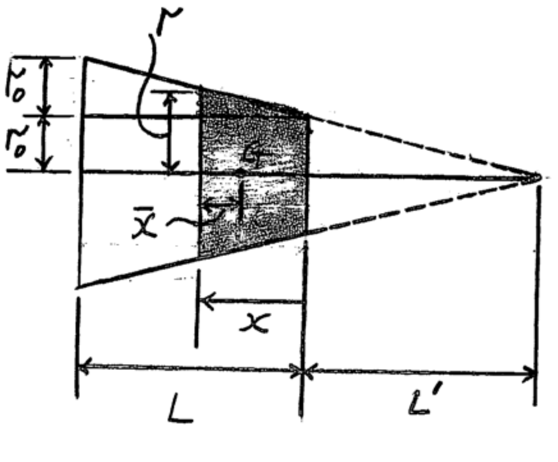
Using these results and referring to the free - body diagram of the frustrum shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad V - \frac{\pi\gamma r_0^2}{3L^2}\left[(L+x)^3 - L^3\right] = 0$

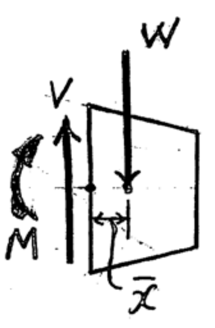
$$V = \frac{\pi\gamma r_0^2}{3L^2}\left[(L+x)^3 - L^3\right] \qquad \textbf{Ans.}$$

$\circlearrowleft +\Sigma M = 0; \quad -M - \left\{\frac{\pi\gamma r_0^2}{3L^2}\left[(L+x)^3 - L^3\right]\right\}\left\{\frac{(L+x)^4 - L^3(4x+L)}{4[(L+x)^3 - L^3]}\right\} = 0$

$$M = -\frac{\pi\gamma r_0^2}{12L^2}\left[(L+x)^4 - L^3(4x+L)\right] \qquad \textbf{Ans.}$$

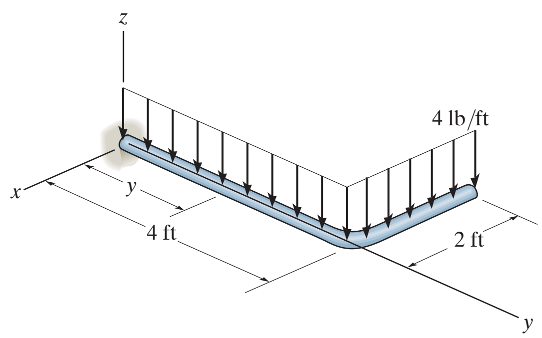


(a)



(b)

7–63. Express the internal shear and moment components acting in the rod as a function of y, where $0 \leq y \leq 4$ ft.



Shear and Moment Functions :

$\Sigma F_x = 0;\quad V_x = 0$ **Ans**

$\Sigma F_z = 0;\quad V_z - 4(4-y) - 8.00 = 0$

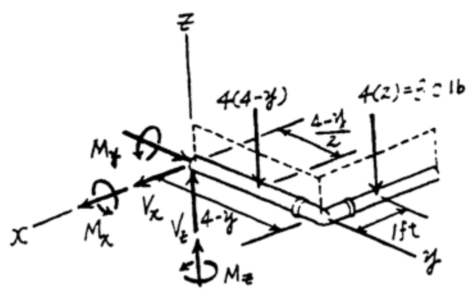
$V_z = \{24.0 - 4y\}$ lb **Ans**

$\Sigma M_x = 0;\quad M_x - 4(4-y)\left(\frac{4-y}{2}\right) - 8.00(4-y) = 0$

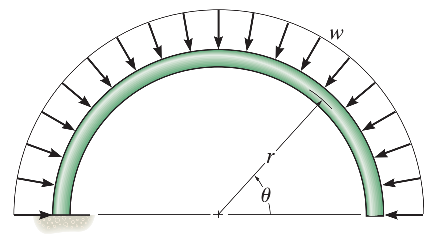
$M_x = \{2y^2 - 24y + 64.0\}$ lb · ft **Ans**

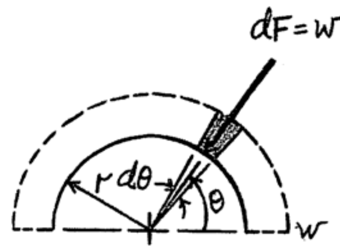
$\Sigma M_y = 0;\quad M_y - 8.00(1) = 0 \quad M_y = 8.00$ lb · ft . **Ans**

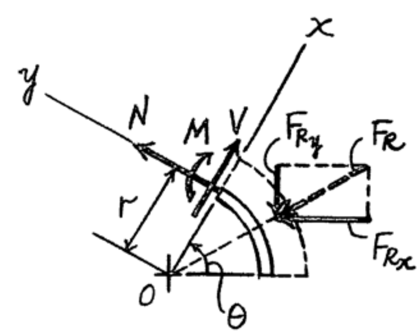
$\Sigma M_z = 0;\quad M_z = 0$ **Ans**

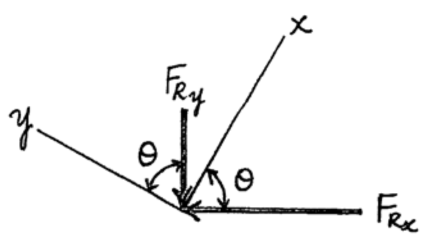


***7–64.** Determine the normal force, shear force, and moment in the curved rod as a function of θ.









$$F_{Rx} = \int_0^{\theta} w(r\,d\theta)\cos\theta = w\,r\sin\theta$$

$$F_{Ry} = \int_0^{\theta} w(r\,d\theta)\sin\theta = w\,r(1-\cos\theta)$$

$$\nearrow \Sigma F_x = 0; \quad V - (w\,r\sin\theta)\cos\theta - r\,w(1-\cos\theta)\sin\theta = 0$$

$$V = w\,r\sin\theta \qquad \textbf{Ans}$$

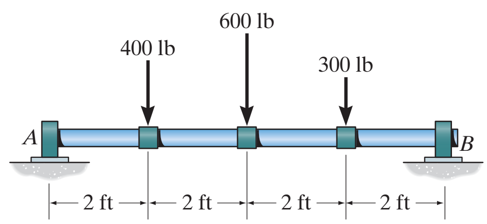
$$+\nwarrow \Sigma F_y = 0; \quad N + (w\,r\sin\theta)\sin\theta - r\,w(1-\cos\theta)\cos\theta = 0$$

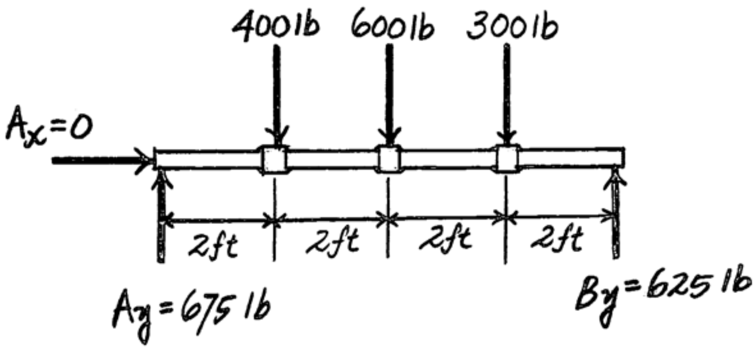
$$N = w\,r(\cos\theta - 1) \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_O = 0; \quad w\,r(\cos\theta - 1)\,r - M = 0$$

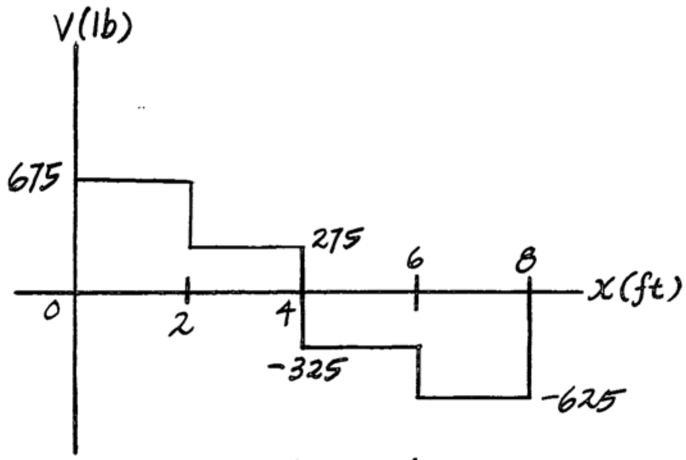
$$M = w\,r^2(\cos\theta - 1) \qquad \textbf{Ans}$$

•7–65. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



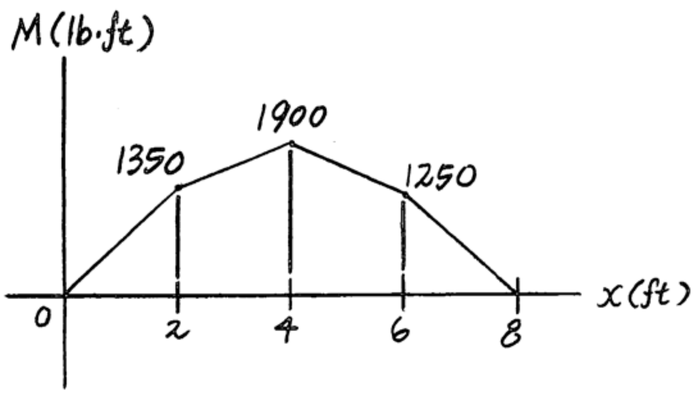


(a)



Shear diagram

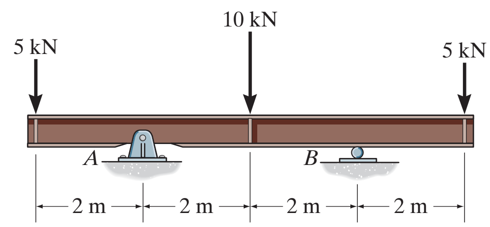
(b)

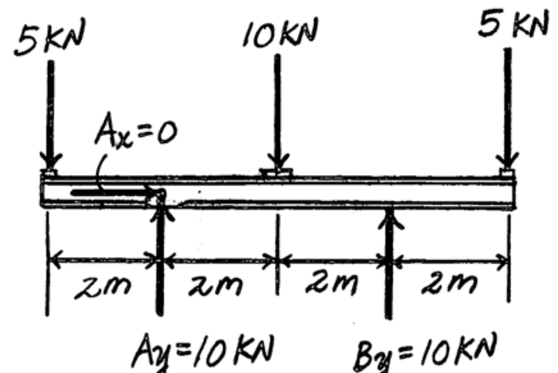


Moment diagram

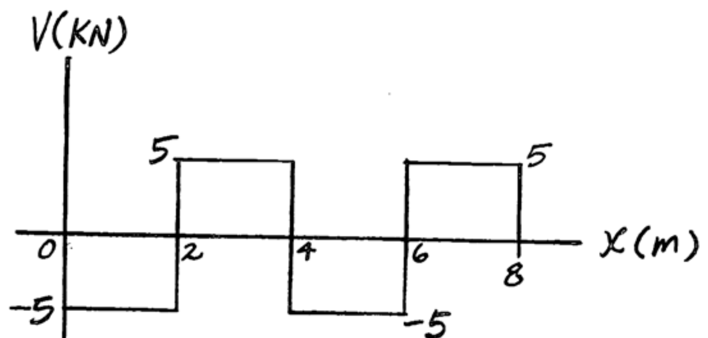
(c)

7–66. Draw the shear and moment diagrams for the double overhang beam.

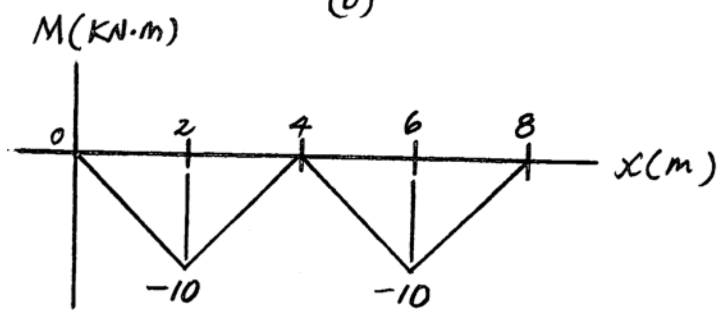




(a)

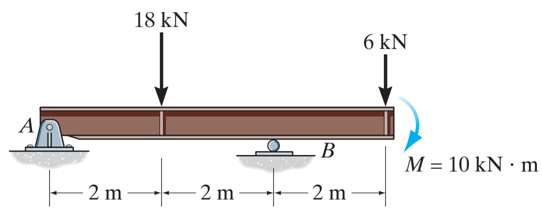


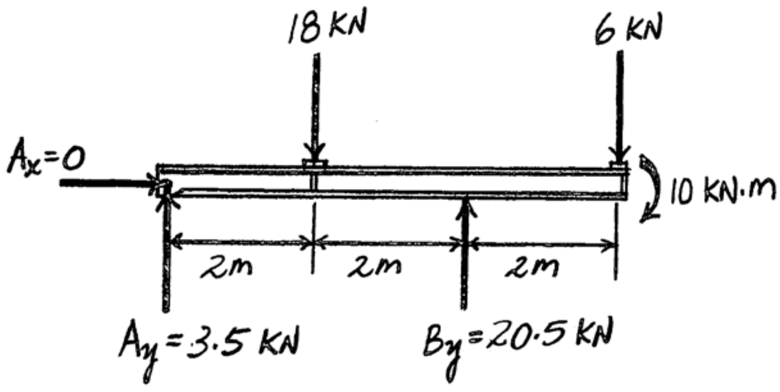
shear diagram
(b)



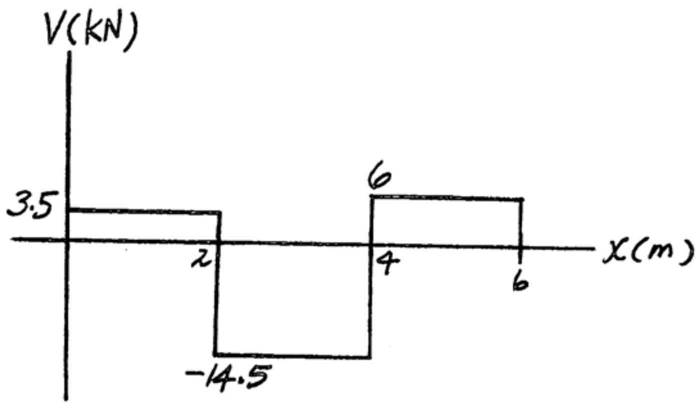
moment diagram
(c)

7–67. Draw the shear and moment diagrams for the overhang beam.



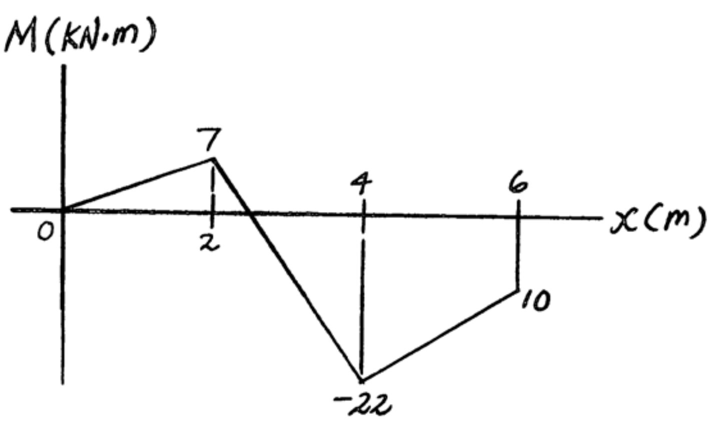


(a)



shear diagram

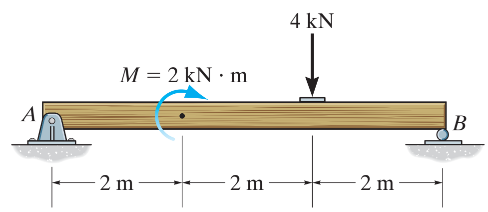
(b)

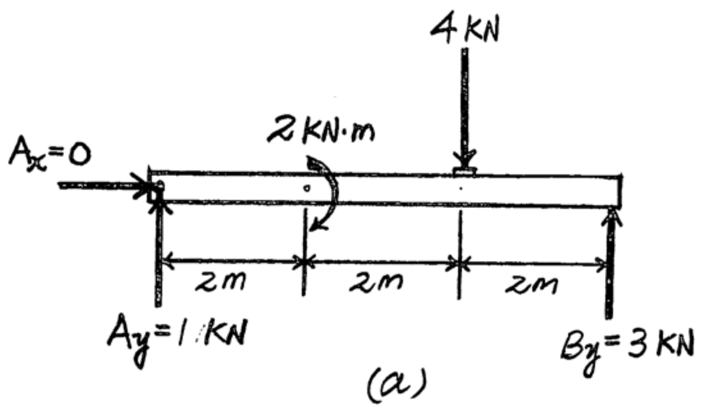


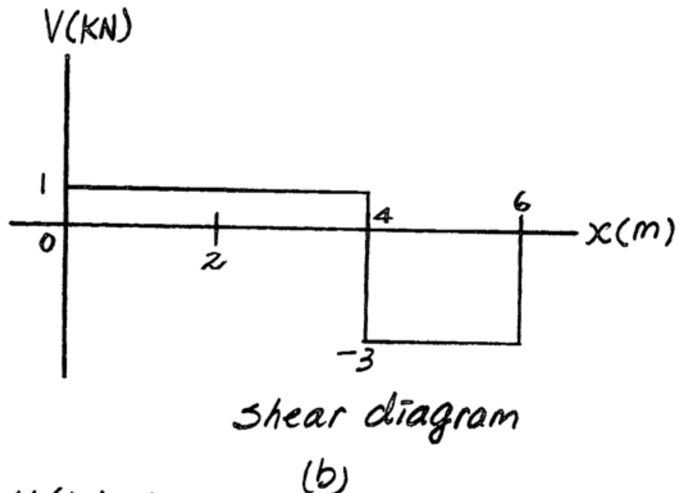
moment diagram

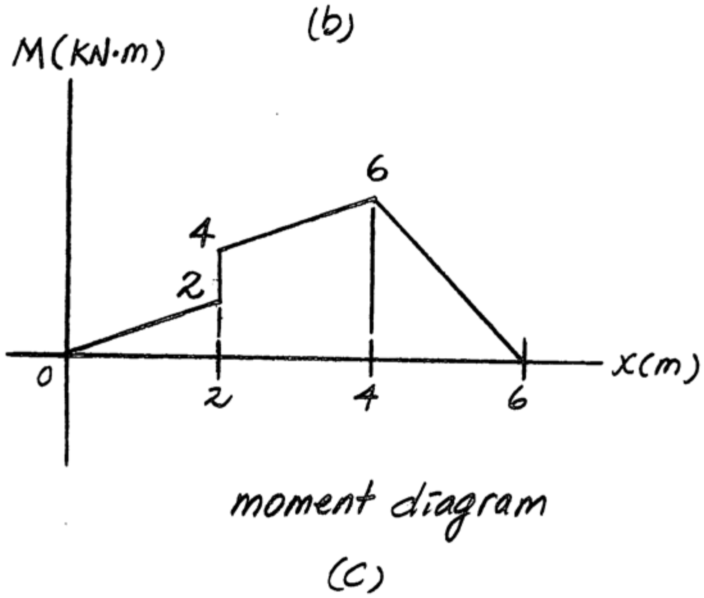
(c)

***7–68.** Draw the shear and moment diagrams for the simply supported beam.

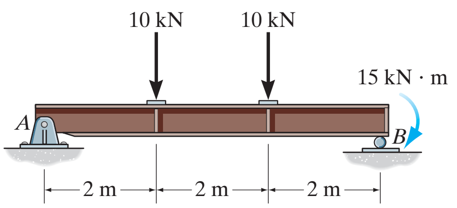


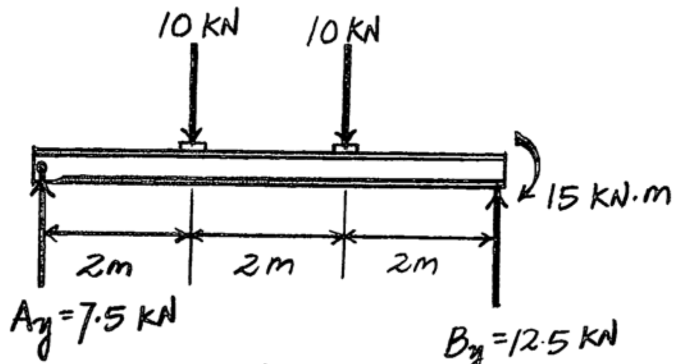




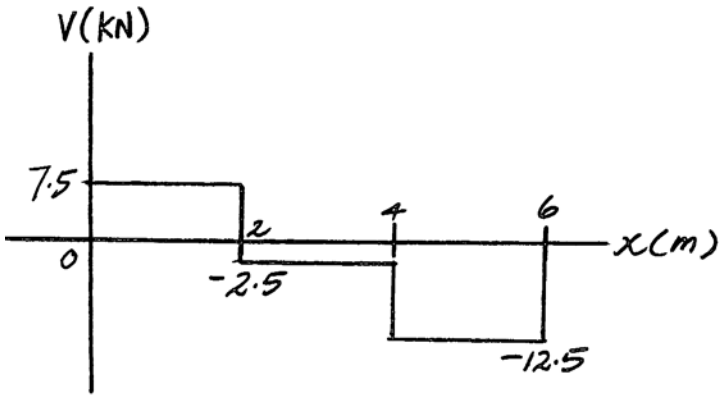


•7–69. Draw the shear and moment diagrams for the simply supported beam.

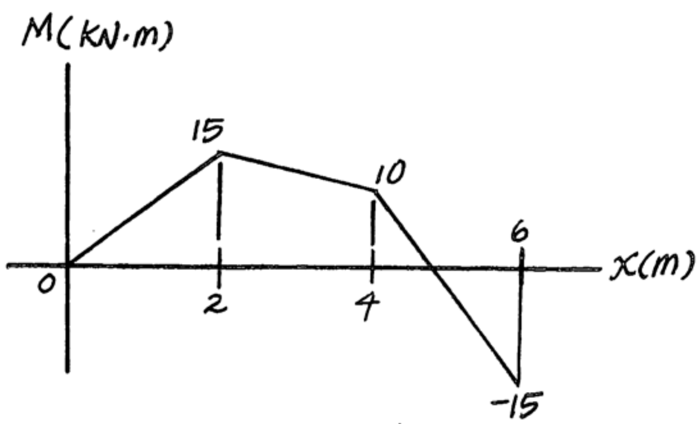




(a)

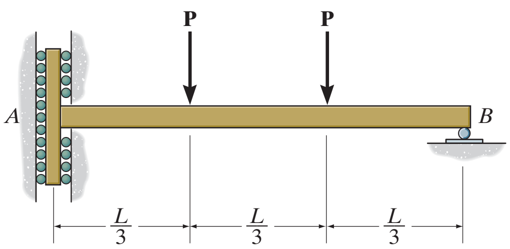


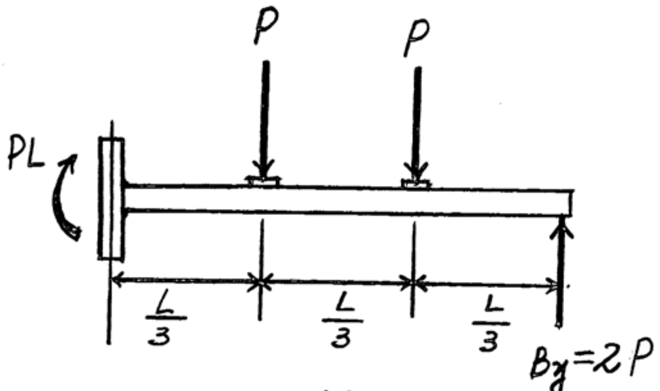
Shear diagram
(b)



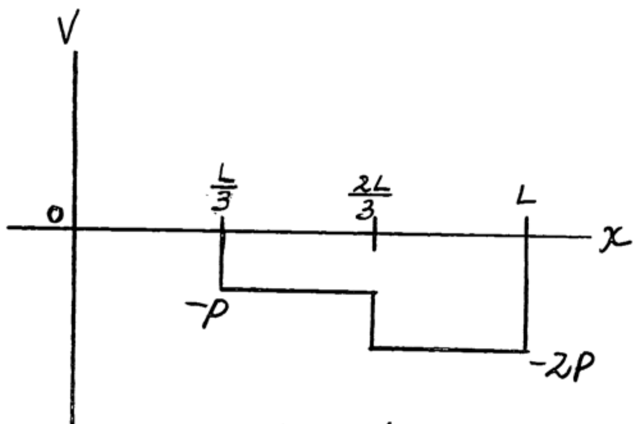
moment diagram
(c)

7–70. Draw the shear and moment diagrams for the beam. The support at A offers no resistance to vertical load.



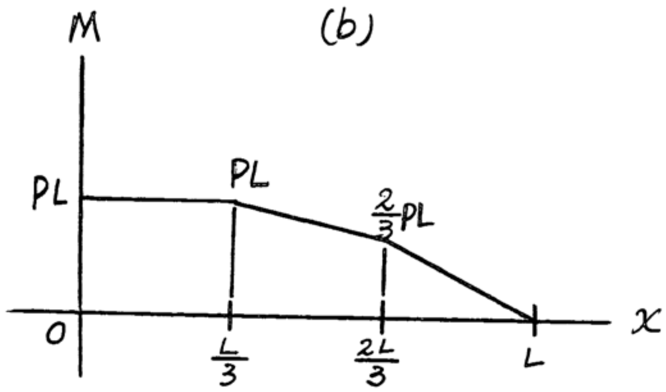


(a)



shear diagram

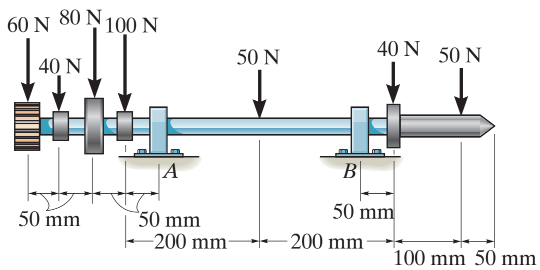
(b)

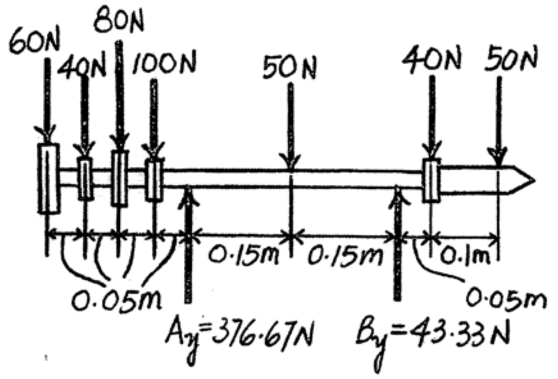


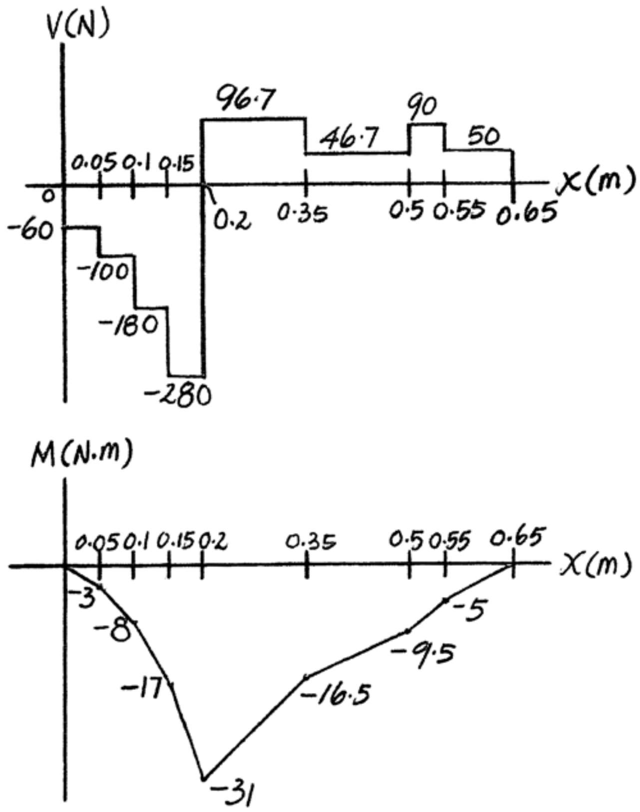
Moment diagram

(c)

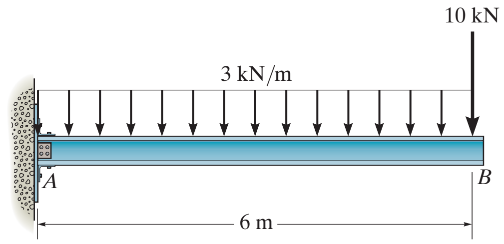
7–71. Draw the shear and moment diagrams for the lathe shaft if it is subjected to the loads shown. The bearing at A is a journal bearing, and B is a thrust bearing.

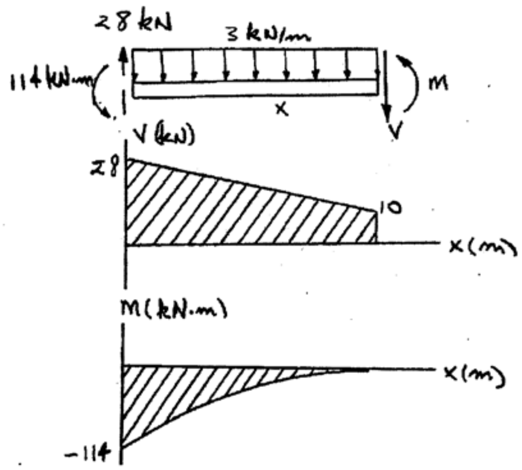




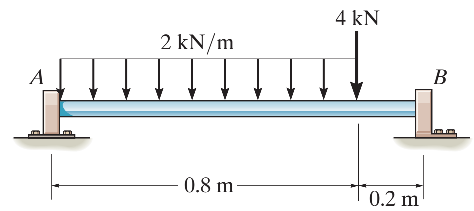


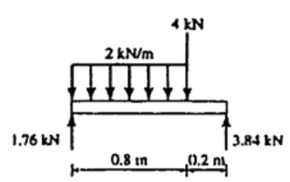
***7–72.** Draw the shear and moment diagrams for the beam.

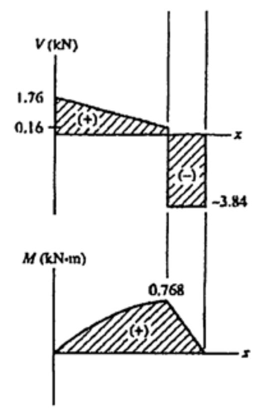




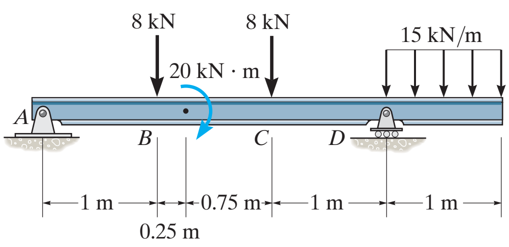
•7–73. Draw the shear and moment diagrams for the shaft. The support at A is a thrust bearing and at B it is a journal bearing.







7–74. Draw the shear and moment diagrams for the beam.



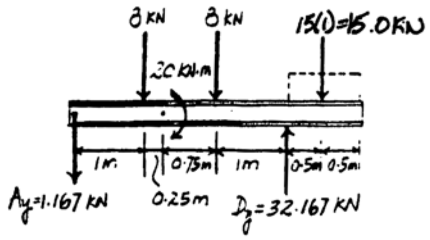
Support Reactions :

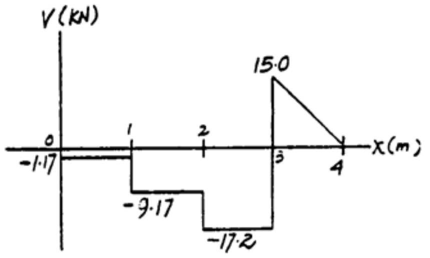
$\curvearrowleft + \Sigma M_A = 0;\quad D_y(3) - 8(1) - 8(2) - 15.0(3.5) - 20 = 0$

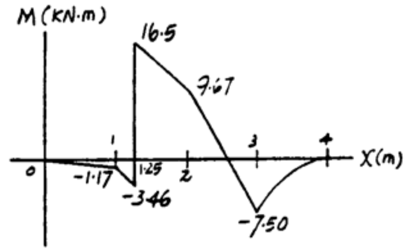
$D_y = 32.167 \text{ kN}$

$+\uparrow \Sigma F_y = 0;\quad 32.167 - 8 - 8 - 15.0 - A_y = 0$

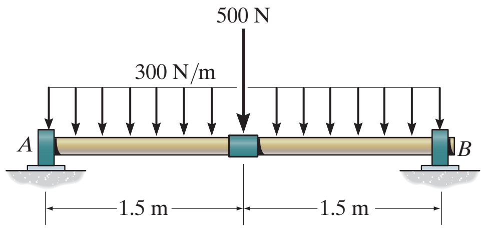
$A_y = 1.167 \text{ kN}$

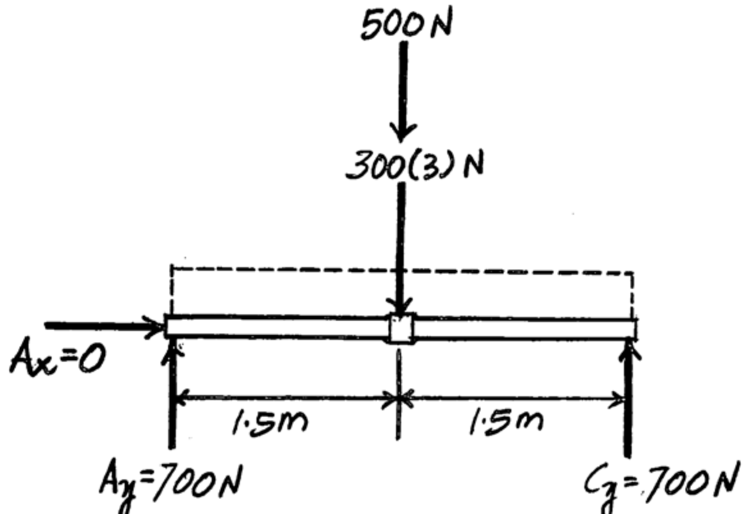




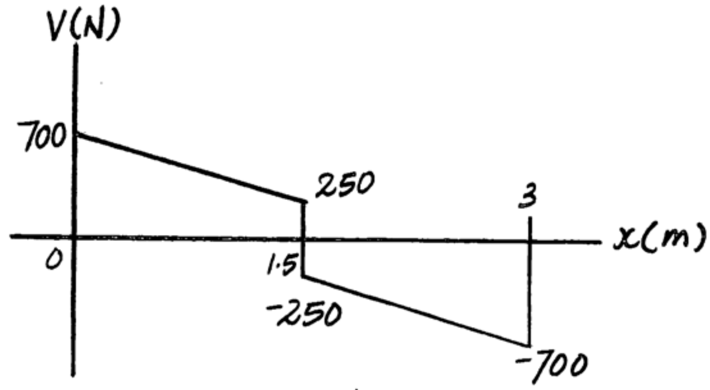


7–75. The shaft is supported by a smooth thrust bearing at *A* and a smooth journal bearing at *B*. Draw the shear and moment diagrams for the shaft.

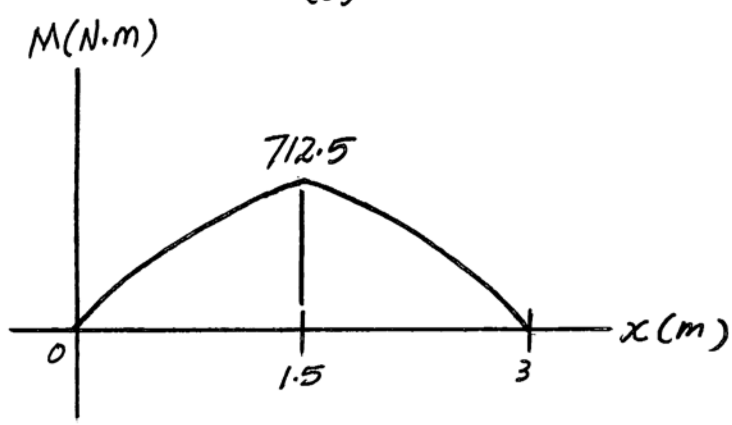




(a)

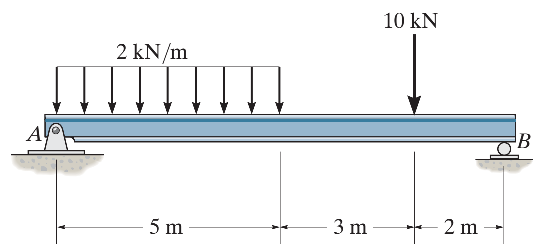


Shear diagram
(b)



moment diagram
(c)

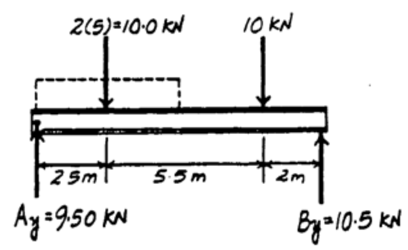
***7–76.** Draw the shear and moment diagrams for the beam.

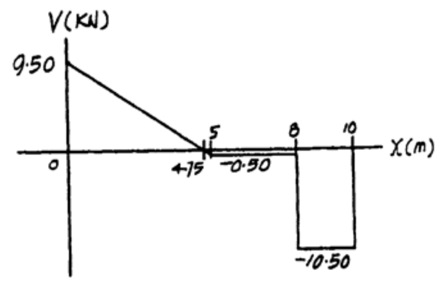


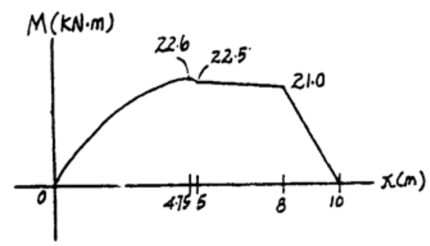
Support Reactions :

$\curvearrowleft + \Sigma M_A = 0;\quad B_y(10) - 10.0(2.5) - 10(8) = 0 \quad B_y = 10.5\text{ kN}$

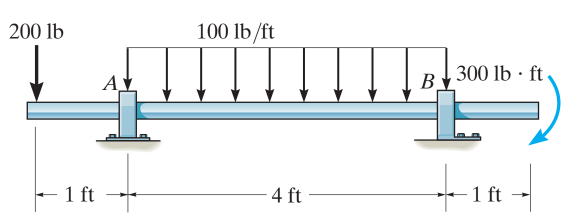
$+\uparrow \Sigma F_y = 0;\quad A_y + 10.5 - 10.0 - 10 = 0 \quad A_y = 9.50\text{ kN}$

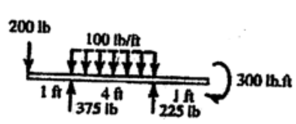


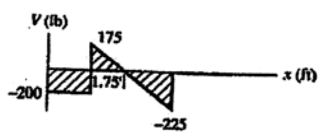


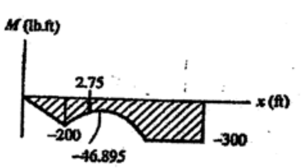


•7–77. Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.

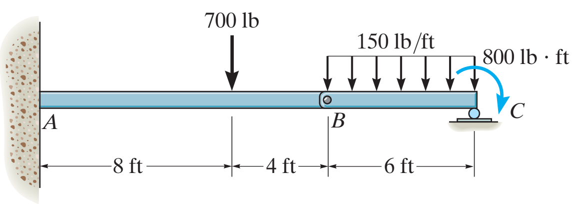


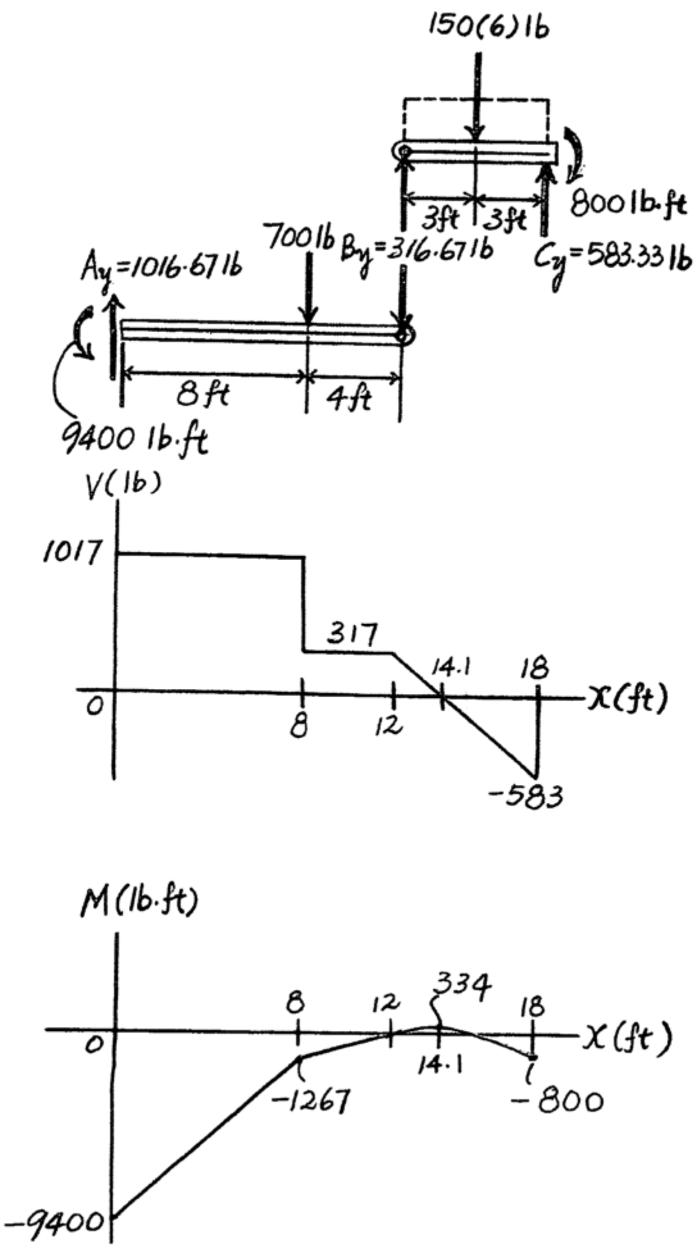




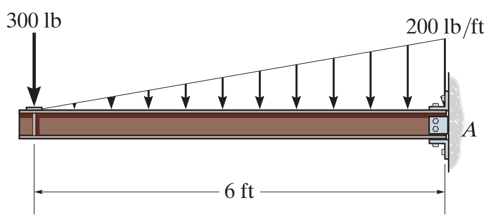


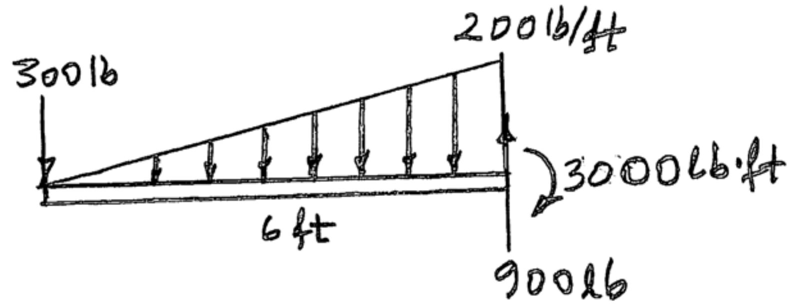
7–78. The beam consists of two segments pin connected at *B*. Draw the shear and moment diagrams for the beam.

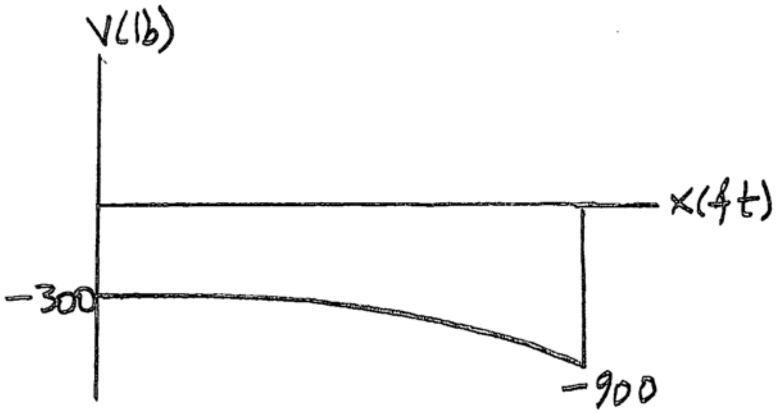


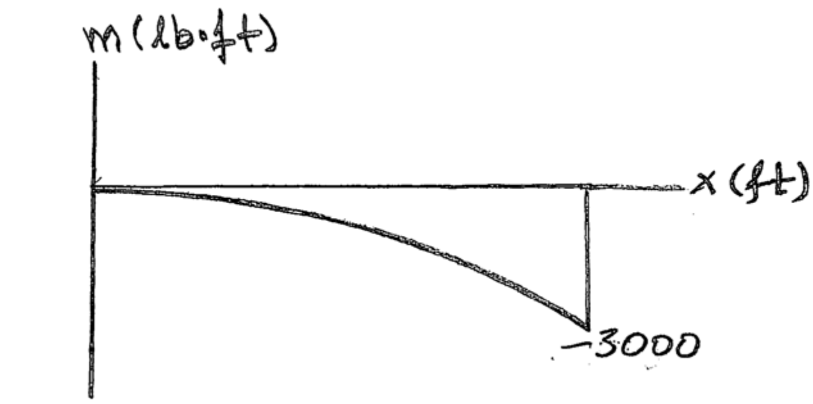


7–79. Draw the shear and moment diagrams for the cantilever beam.

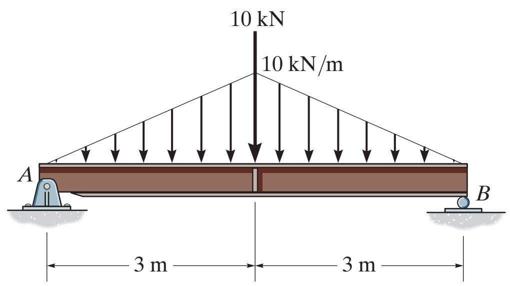






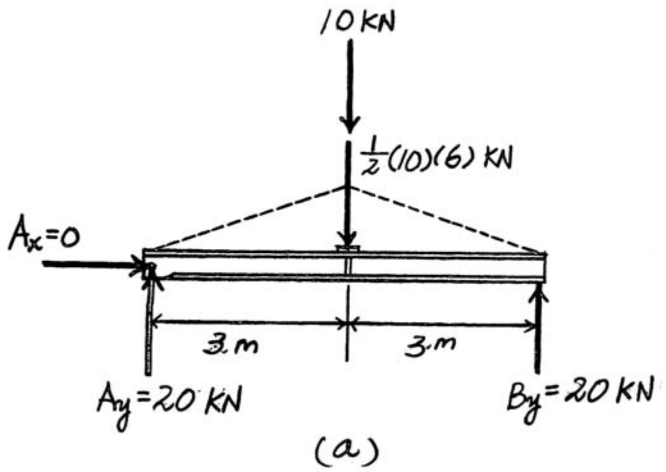


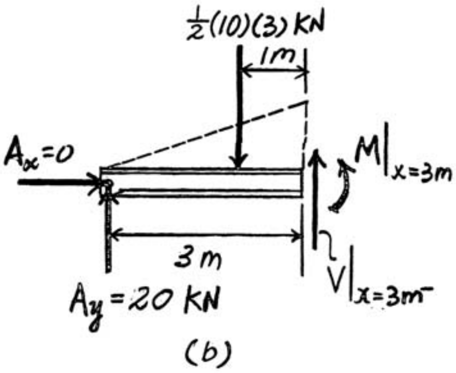
***7–80.** Draw the shear and moment diagrams for the simply supported beam.

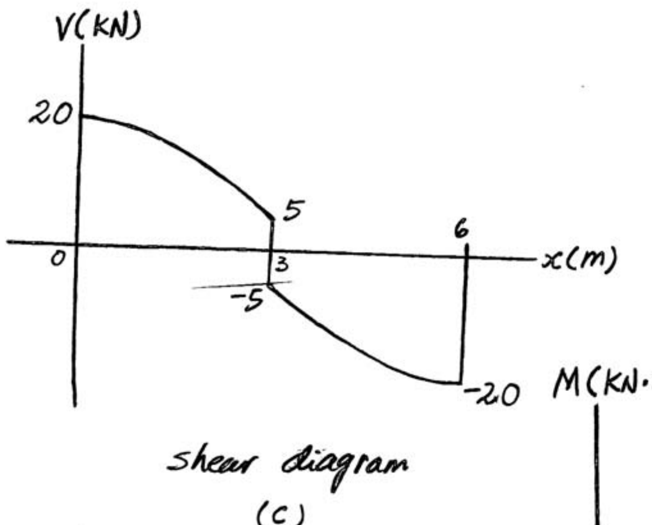


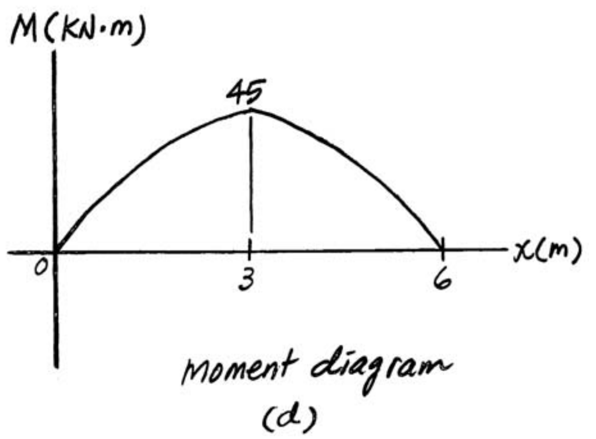
Since the area under the curved shear diagram can not be computed directly, the value of the moment at $x = 3$ m will be computed using the method of sections. By referring to the free-body diagram shown in Fig. *b*,

$\circlearrowleft + \Sigma M = 0;\ M\big|_{x=3\text{ m}} + \frac{1}{2}(10)(3)(1) - 20(3) = 0 \qquad M\big|_{x=3\text{ m}} = 45\text{ kN}\cdot\text{m}$ **Ans.**

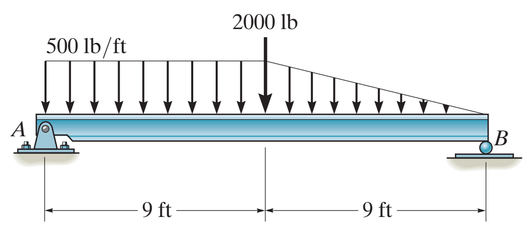


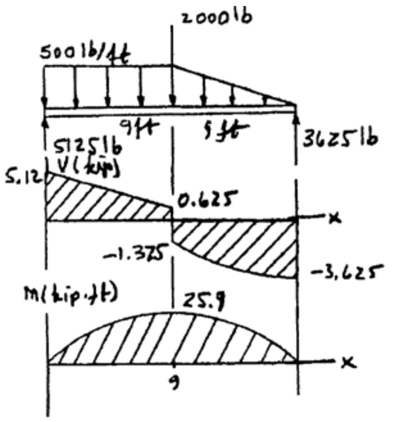




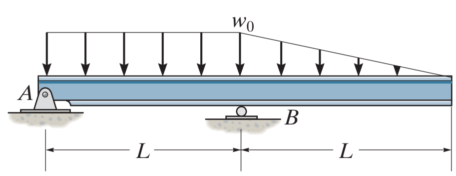


•7–81. Draw the shear and moment diagrams for the beam.





7–82. Draw the shear and moment diagrams for the beam.



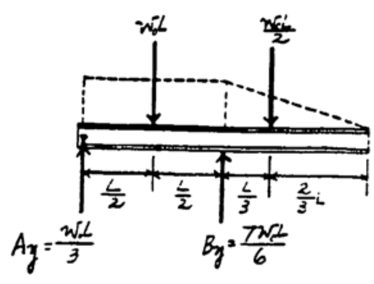
Support Reactions:

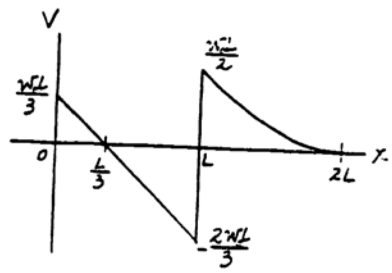
$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(L) - w_0 L\left(\frac{L}{2}\right) - \frac{w_0 L}{2}\left(\frac{4L}{3}\right) = 0$$

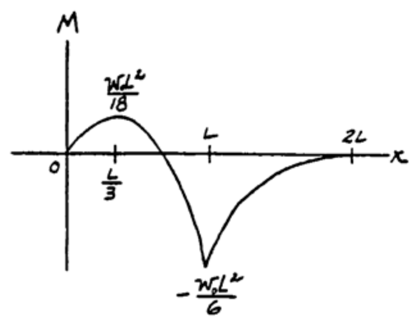
$$B_y = \frac{7w_0 L}{6}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + \frac{7w_0 L}{6} - w_0 L - \frac{w_0 L}{2} = 0$$

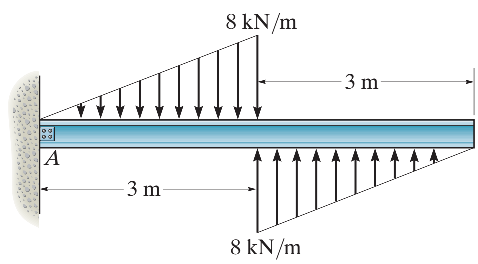
$$A_y = \frac{w_0 L}{3}$$

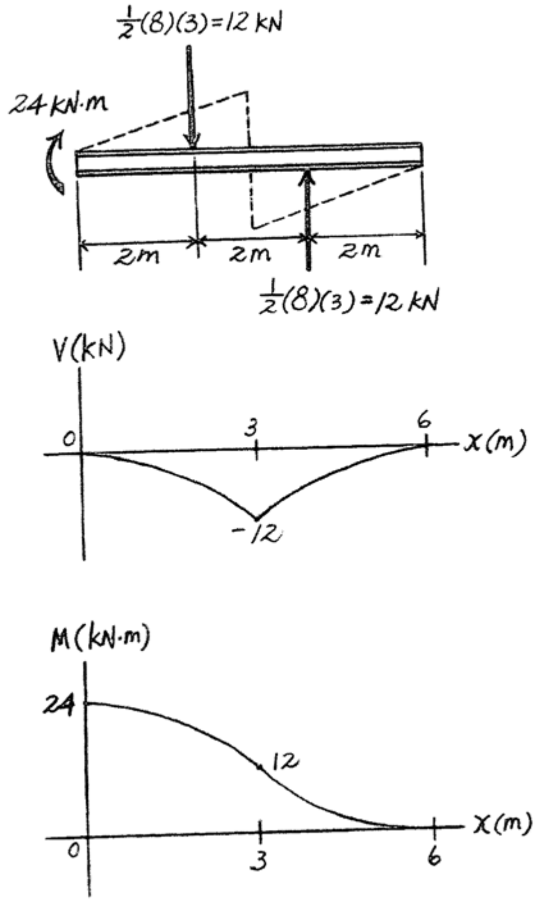




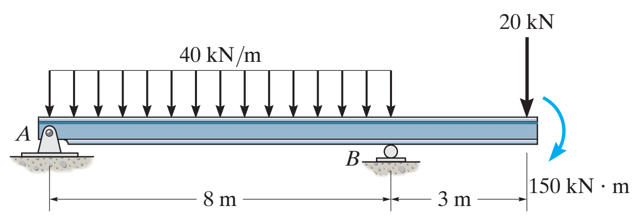


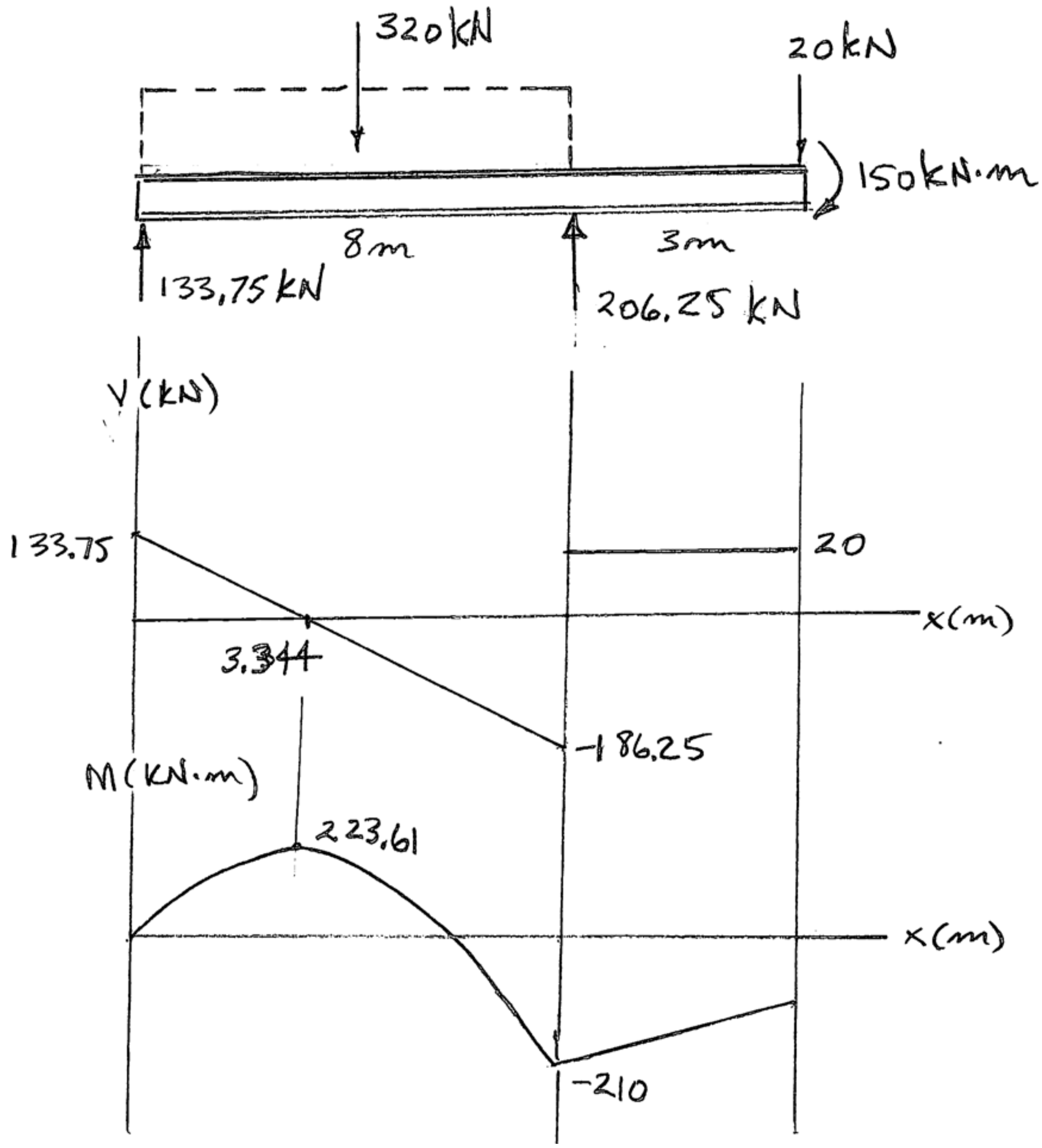
7–83. Draw the shear and moment diagrams for the beam.



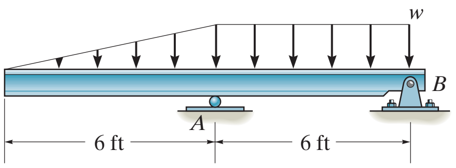


***7–84.** Draw the shear and moment diagrams for the beam.





•7–85. The beam will fail when the maximum moment is $M_{max} = 30$ kip · ft or the maximum shear is $V_{max} = 8$ kip. Determine the largest intensity w of the distributed load the beam will support.



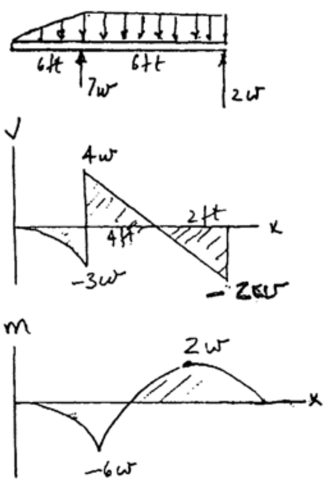
$V_{max} = 4w; \quad 8 = 4w$

$w = 2$ kip/ft

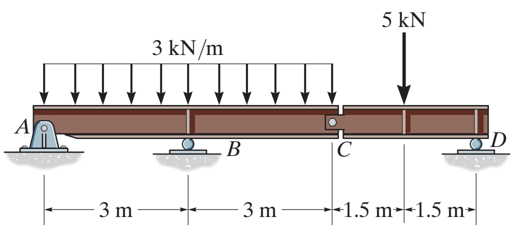
$M_{max} = -6w; \quad -30 = -6w$

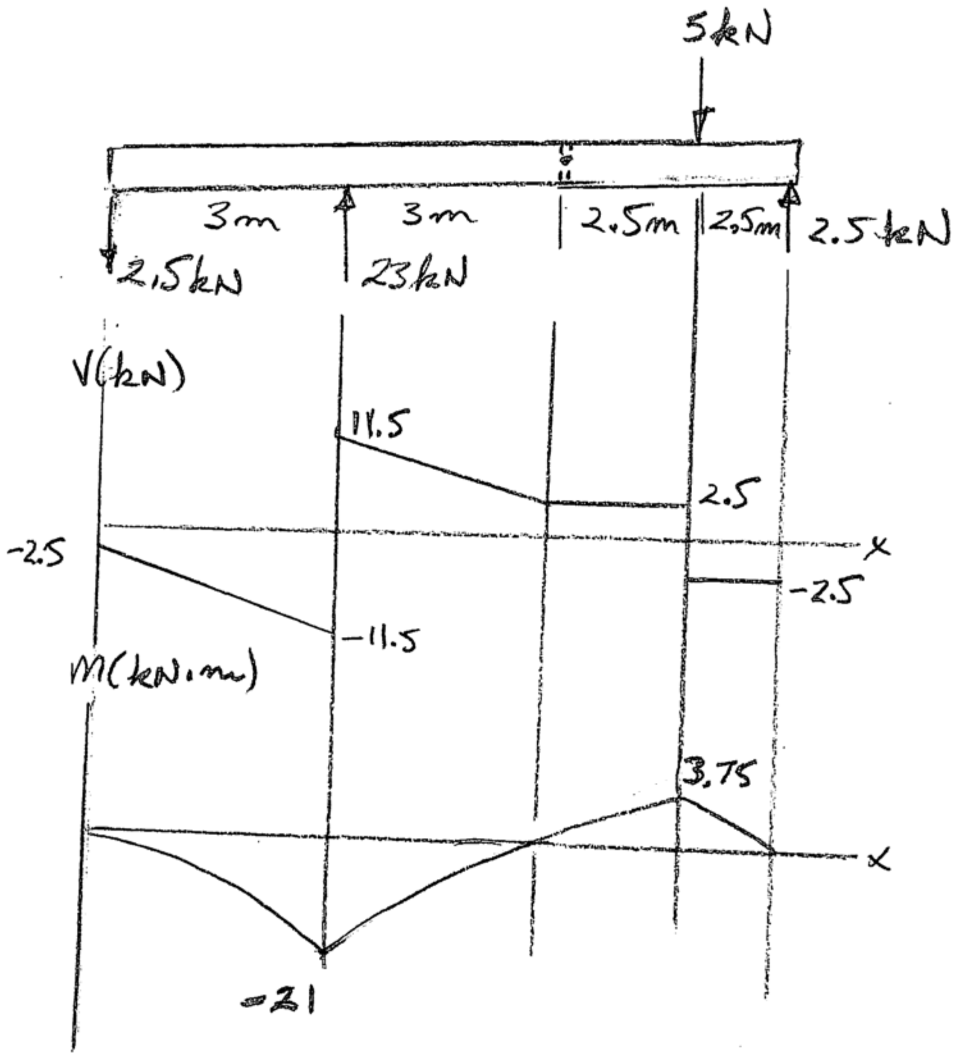
$w = 5$ kip/ft

Thus, $w = 2$ kip/ft **Ans**

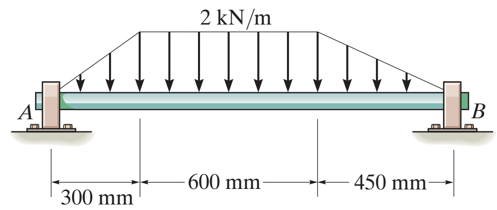


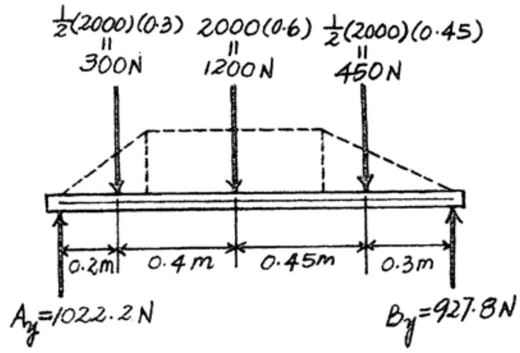
7–86. Draw the shear and moment diagrams for the compound beam.





7–87. Draw the shear and moment diagrams for the shaft. The supports at A and B are journal bearings.



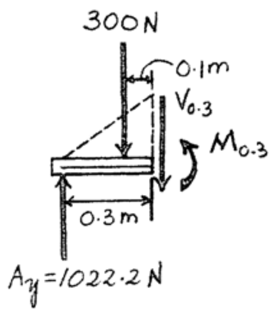


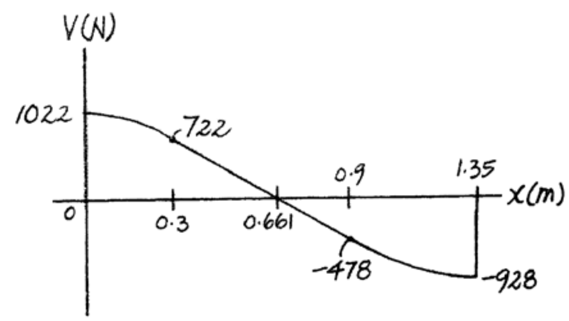
$+\uparrow \Sigma F_y = 0;\quad 1022.2 - \frac{1}{2}(2000)(0.3) - V_{0.3} = 0$

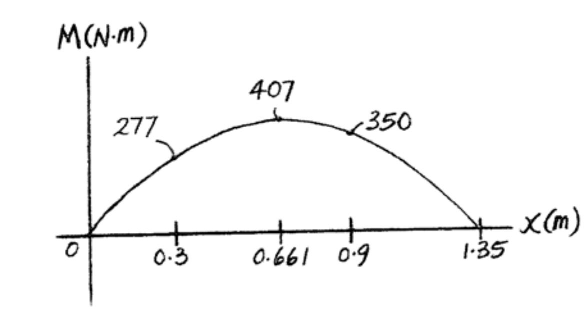
$V_{0.3} = 722\text{ N}$

$\circlearrowleft + \Sigma M = 0;\quad M_{0.3} + \frac{1}{2}(2000)(0.3)(0.1) - 1022.2(0.3) = 0$

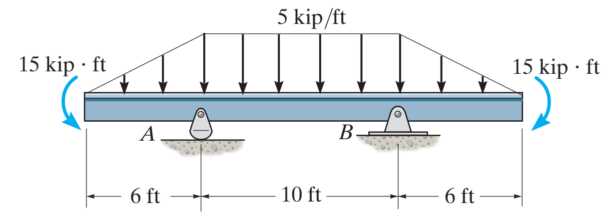
$M_{0.3} = 277\text{ N}\cdot\text{m}$







***7–88.** Draw the shear and moment diagrams for the beam.



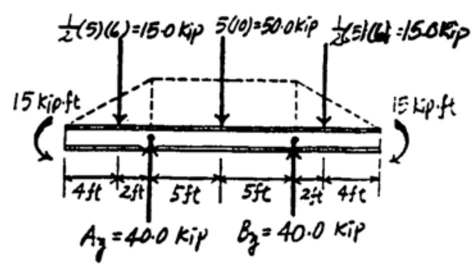
Support Reactions **: From FBD (a),**

$$\curvearrowleft + \Sigma M_A = 0; \quad B_y(10) + 15.0(2) + 15 - 50.0(5) - 15.0(12) - 15 = 0$$

$$B_y = 40.0 \text{ kip}$$

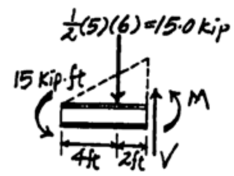
$$+\uparrow \Sigma F_y = 0; \quad A_y + 40.0 - 15.0 - 50.0 - 15.0 = 0$$

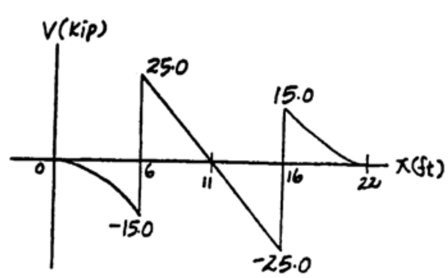
$$A_y = 40.0 \text{ kip}$$

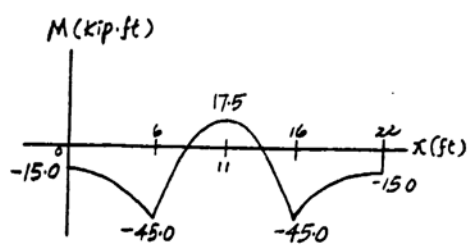


Shear and Moment Diagrams **: The value of the moment at supports *A* and *B* can be evaluated using the method of sections [FBD (c)].**

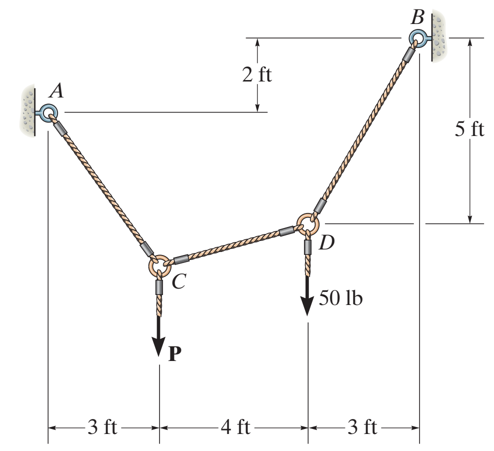
$$\curvearrowleft + \Sigma M = 0; \quad M + 15.0(2) + 15 = 0 \qquad M = -45.0 \text{ kip} \cdot \text{ft}$$

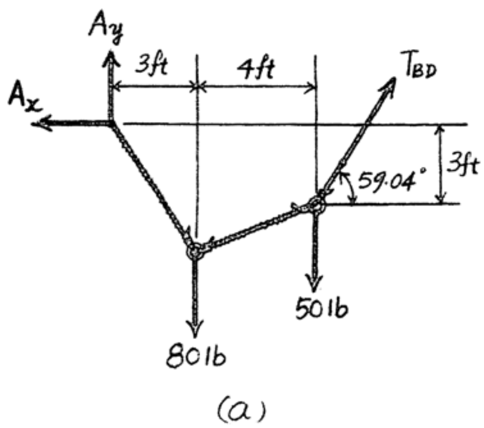






•**7–89.** Determine the tension in each segment of the cable and the cable's total length. Set $P = 80$ lb.





From FBD (a)

$\curvearrowleft + \Sigma M_A = 0;\quad T_{BD}\cos 59.04°(3) + T_{BD}\sin 59.04°(7) - 50(7) - 80(3) = 0$

$T_{BD} = 78.188 \text{ lb} = 78.2 \text{ lb}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad 78.188\cos 59.04° - A_x = 0 \quad A_x = 40.227 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 78.188\sin 59.04° - 80 - 50 = 0 \quad A_y = 62.955 \text{ lb}$

Joint A :

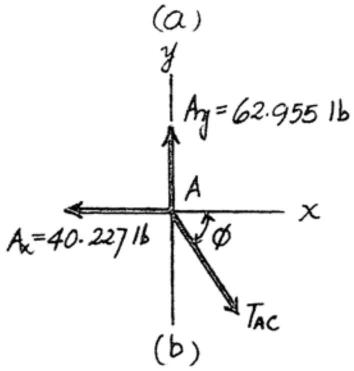
$\xrightarrow{+} \Sigma F_x = 0;\quad T_{AC}\cos\phi - 40.227 = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0;\quad -T_{AC}\sin\phi + 62.955 = 0 \qquad (2)$

Solving Eqs.(1) and (2) yields :

$\phi = 57.42°$

$T_{AC} = 74.7 \text{ lb}$ **Ans**



Joint D :

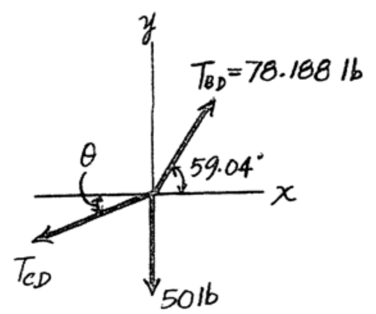
$\xrightarrow{+} \Sigma F_x = 0;\quad 78.188\cos 59.04° - T_{CD}\cos\theta = 0 \qquad (3)$

$+\uparrow \Sigma F_y = 0;\quad 78.188\sin 59.04° - T_{CD}\sin\theta - 50 = 0 \qquad (4)$

Solving Eqs. (3) and (4) yields :

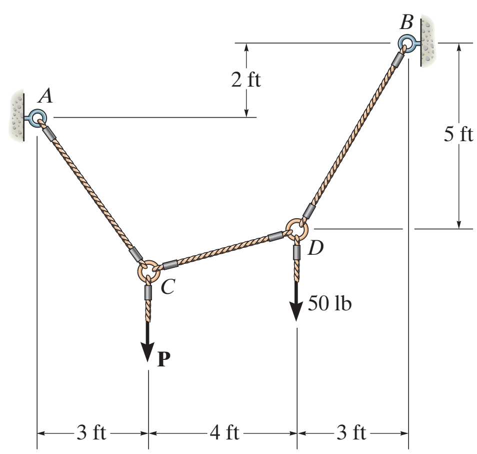
$\theta = 22.96°$

$T_{CD} = 43.7 \text{ lb}$ **Ans**



Total length of the cable : $l_T = \dfrac{5}{\sin 59.04°} + \dfrac{4}{\cos 22.96°} + \dfrac{3}{\cos 57.42°} = 15.7 \text{ ft}$ **Ans**

7–90. If each cable segment can support a maximum tension of 75 lb, determine the largest load *P* that can be applied.



$\circlearrowleft + \Sigma M_A = 0;\quad -T_{BD}(\cos 59.04°)\,2 + T_{BD}(\sin 59.04°)(10) - 50(7) - P(3) = 0$

$T_{BD} = 0.39756\,P + 46.383$

$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + T_{BD}\cos 59.04° = 0$

$+\uparrow \Sigma F_y = 0;\quad A_y - P - 50 + T_{BD}\sin 59.04° = 0$

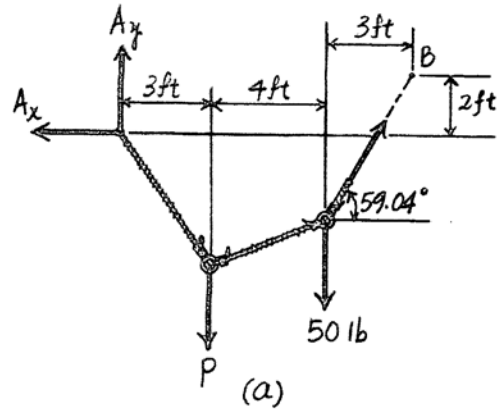
Assume maximum tension is in cable *BD*.

$T_{BD} = 75\text{ lb}$

$P = 71.98\text{ lb}$

$A_x = 38.59\text{ lb}$

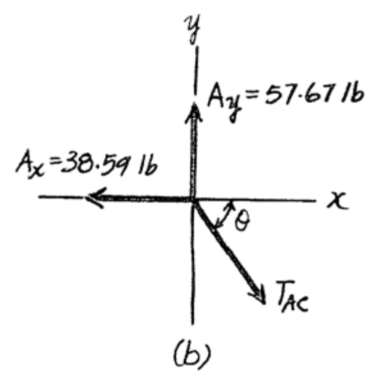
$A_y = 57.670\text{ lb}$



Pin *A*:

$T_{AC} = \sqrt{(38.59)^2 + (57.670)^2} = 69.39\text{ lb} < 75\text{ lb}\quad \text{OK}$

$\theta = \tan^{-1}\left(\frac{57.670}{38.59}\right) = 56.21°$



Joint *C*:

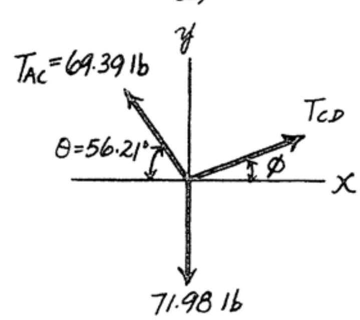
$\xrightarrow{+} \Sigma F_x = 0;\quad T_{CD}\cos\phi - 69.39\cos 56.21° = 0$

$+\uparrow \Sigma F_y = 0;\quad T_{CD}\sin\phi + 69.39\sin 56.21° - 71.98 = 0$

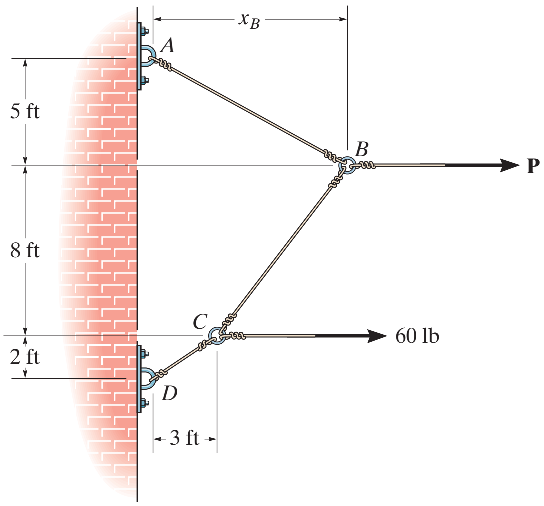
$T_{CD} = 41.2\text{ lb} < 75\text{ lb}\quad \text{OK}$

$\phi = 20.3°$

Thus, $\quad P = 72.0\text{ lb}\quad$ **Ans**



7–91. The cable segments support the loading shown. Determine the horizontal distance x_B from the force at B to point A. Set $P = 40$ lb.



$\circlearrowleft + \Sigma M_A = 0; \quad -T_{CD}\cos 33.69°(13) - T_{CD}\sin 33.69°(3) + 60(13) + 40(5) = 0$

$T_{CD} = 78.521$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad 40 + 60 - 78.521\cos 33.69° - A_x = 0$

$A_x = 34.667$ lb

$+\uparrow \Sigma F_y = 0; \quad A_y - 78.521\sin 33.69° = 0$

$A_y = 43.555$ lb

Joint A:

$\xrightarrow{+} \Sigma F_x = 0; \quad T_{AB}\cos\theta - 34.667 = 0 \qquad (1)$

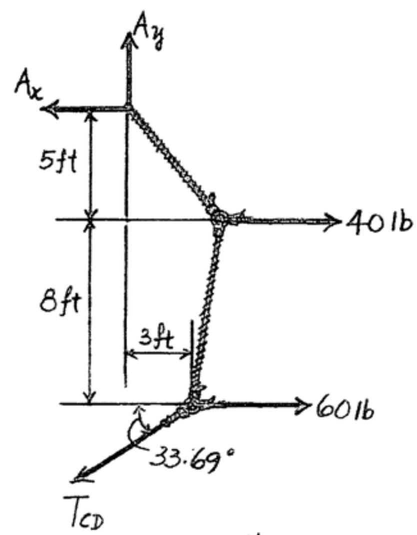
$+\uparrow \Sigma F_y = 0; \quad 43.555 - T_{AB}\sin\theta = 0 \qquad (2)$

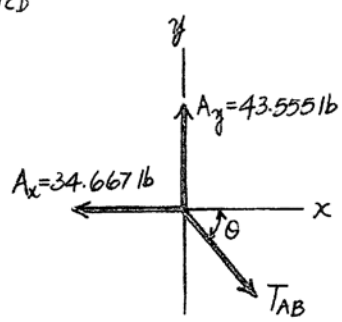
Solving Eqs. (1) and (2) yields:

$\theta = 51.48°$

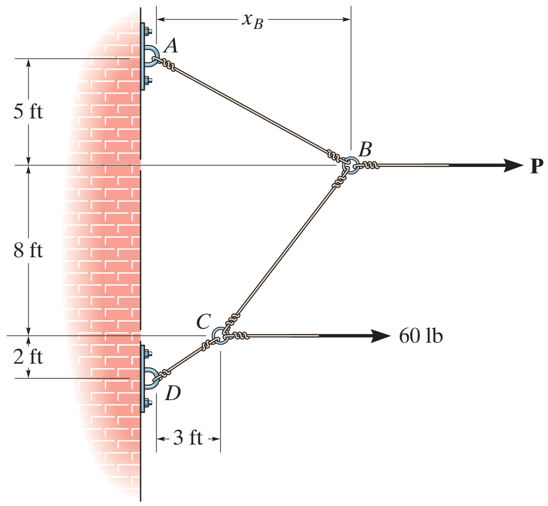
$T_{AB} = 55.67$ lb

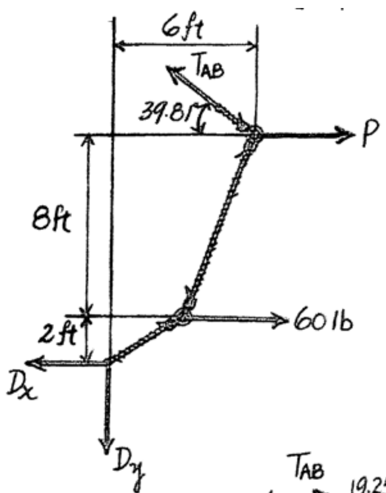
$x_B = \dfrac{5}{\tan 51.48°} = 3.98$ ft **Ans**





***7–92.** The cable segments support the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.





$\curvearrowleft + \Sigma M_D = 0;\quad T_{AB}\cos 39.81°(10) + T_{AB}\sin 39.81°(6) - 60(2) - P(10) = 0$

$$11.523T_{AB} - 10P = 120 \qquad (1)$$

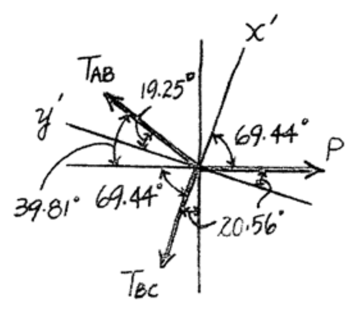
Joint B :

$+\nwarrow \Sigma F_{y'} = 0;\quad T_{AB}\cos 19.25° - P\sin 69.44° = 0 \qquad (2)$

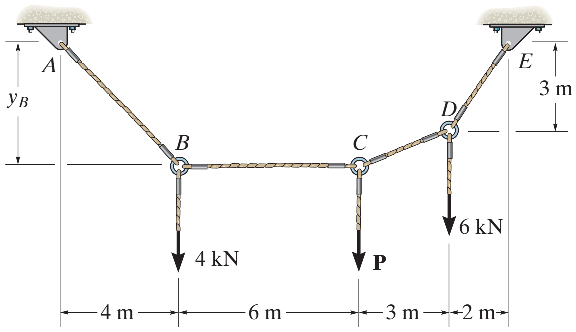
Solving Eqs. (1) and (2) yields :

$P = 84.0$ lb **Ans**

$T_{AB} = 83.32$ lb



•7–93. Determine the force P needed to hold the cable in the position shown, i.e., so segment BC remains horizontal. Also, compute the sag y_B and the maximum tension in the cable.

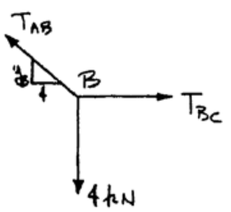


Joint B:

$$\xrightarrow{+} \Sigma F_x = 0; \quad T_{BC} - \frac{4}{\sqrt{y_B^2 + 16}} T_{AB} = 0$$

$$+\uparrow \Sigma F_y = 0, \quad \frac{y_B}{\sqrt{y_B^2 + 16}} T_{AB} - 4 = 0$$

$$y_B\, T_{BC} = 16 \qquad (1)$$

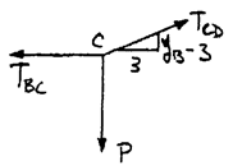


Joint C:

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - T_{BC} = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - P = 0$$

$$(y_B - 3) T_{BC} = 3P \qquad (3)$$



Combining Eqs. (1) and (2):

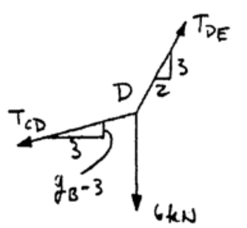
$$\frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = \frac{16}{y_B} \qquad (4)$$

Joint D:

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{2}{\sqrt{13}} T_{DE} - \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{\sqrt{13}} T_{DE} - \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - 6 = 0$$

$$\frac{15 - 2y_B}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 12 \qquad (5)$$



From Eqs. (1) and (3): $\quad 3y_B P - 16y_B + 48 = 0$

From Eqs. (4) and (5): $\quad y_B = 3.53\text{ m}$ **Ans**

$P = 0.8\text{ kN}$ **Ans**

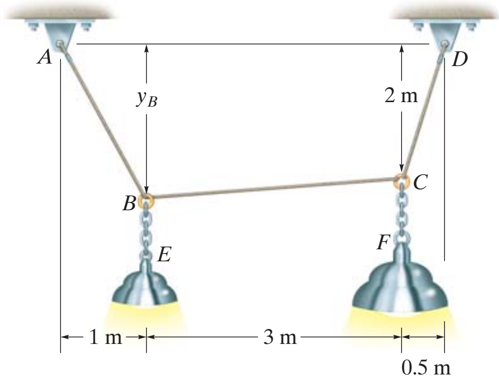
$T_{AB} = 6.05\text{ kN}$

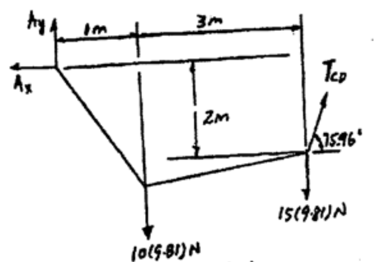
$T_{BC} = 4.53\text{ kN}$

$T_{CD} = 4.60\text{ kN}$

$T_{max} = T_{DE} = 8.17\text{ kN}$ **Ans**

7–94. Cable *ABCD* supports the 10-kg lamp *E* and the 15-kg lamp *F*. Determine the maximum tension in the cable and the sag y_B of point *B*.





(a)

From FBD (a)

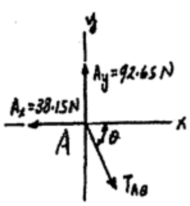
$$\circlearrowleft + \Sigma M_A = 0; \quad T_{CD}\cos 75.96°(2) + T_{CD}\sin 75.96°(4) - 15(9.81)(4) - 10(9.81)(1) = 0$$

$$T_{CD} = 157.30 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 157.30\cos 75.96° - A_x = 0 \qquad A_x = 38.15 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 157.30\sin 75.96° - 15(9.81) - 10(9.81) = 0$$

$$A_y = 92.65 \text{ N}$$



Joint A :

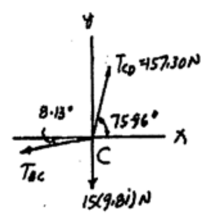
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T_{AB}\cos\theta - 38.15 = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad 92.65 - T_{AB}\sin\theta = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$\theta = 67.62°$ $\qquad T_{AB} = 100.2 \text{ N}$

$y_B = (1)\tan 67.62° = 2.43 \text{ m}$ **Ans**



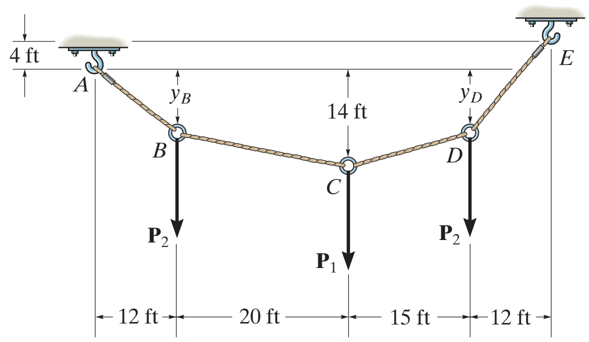
Joint C :

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 157.30\cos 75.96° - T_{BC}\cos 8.13° = 0 \qquad T_{BC} = 38.54 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 157.3\sin 75.96° - 38.54\sin 8.13° - 15(9.81) = 0 \quad \text{(Check)}$$

$$T_{max} = T_{CD} = 157 \text{ N} \qquad \textbf{Ans}$$

7–95. The cable supports the three loads shown. Determine the sags y_B and y_D of points B and D. Take $P_1 = 400$ lb, $P_2 = 250$ lb.

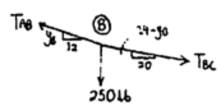


At B

$\xrightarrow{+}\Sigma F_x = 0;\qquad \dfrac{20}{\sqrt{(14-y_B)^2+400}}T_{BC} - \dfrac{12}{\sqrt{y_B^2+144}}T_{AB} = 0$

$+\uparrow \Sigma F_y = 0;\qquad -\dfrac{14-y_B}{\sqrt{(14-y_B)^2+400}}T_{BC} + \dfrac{y_B}{\sqrt{y_B^2+144}}T_{AB} - 250 = 0$

$$\frac{32y_B - 168}{\sqrt{(14-y_B)^2+400}}T_{BC} = 3000 \qquad (1)$$

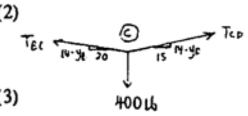


At C

$\xrightarrow{+}\Sigma F_x = 0;\qquad \dfrac{15}{\sqrt{(14-y_D)^2+225}}T_{CD} - \dfrac{20}{\sqrt{(14-y_B)^2+400}}T_{BC} = 0$

$+\uparrow \Sigma F_y = 0;\qquad \dfrac{14-y_D}{\sqrt{(14-y_D)^2+225}}T_{CD} + \dfrac{14-y_B}{\sqrt{(14-y_B)^2+400}}T_{BC} - 400 = 0$

$$\frac{-20y_D + 490 - 15y_B}{\sqrt{(14-y_B)^2+400}}T_{BC} = 6000 \qquad (2)$$



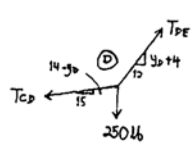
$$\frac{-20y_D + 490 - 15y_B}{\sqrt{(14-y_D)^2+225}}T_{CD} = 8000 \qquad (3)$$

At D

$\xrightarrow{+}\Sigma F_x = 0;\qquad \dfrac{12}{\sqrt{(4+y_D)^2+144}}T_{DE} - \dfrac{15}{\sqrt{(14-y_D)^2+225}}T_{CD} = 0$

$+\uparrow \Sigma F_y = 0;\qquad \dfrac{4+y_D}{\sqrt{(4+y_D)^2+144}}T_{DE} - \dfrac{14-y_D}{\sqrt{(14-y_D)^2+225}}T_{CD} - 250 = 0$

$$\frac{-108 + 27y_D}{\sqrt{(14-y_D)^2+225}}T_{CD} = 3000 \qquad (4)$$



Combining Eqs. (1) & (2)

$$79y_B + 20y_D = 826$$

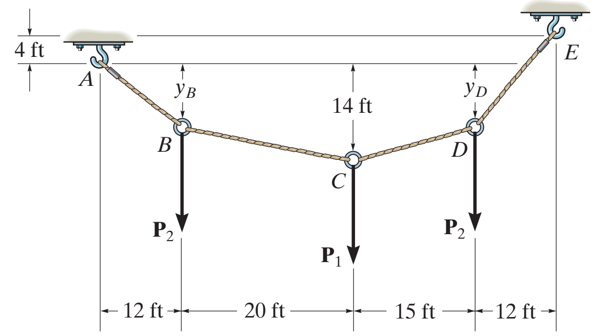
Combining Eqs. (3) & (4)

$$45y_B + 276y_D = 2334$$

$y_B = 8.67$ ft **Ans**

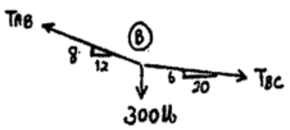
$y_D = 7.04$ ft **Ans**

***7–96.** The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 300$ lb and $y_B = 8$ ft. Also find the sag y_D.



At *B*

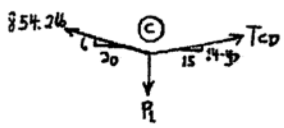
$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{20}{\sqrt{436}}T_{BC} - \frac{12}{\sqrt{208}}T_{AB} = 0$$



$$+\uparrow\Sigma F_y = 0; \qquad \frac{-6}{\sqrt{436}}T_{BC} + \frac{8}{\sqrt{208}}T_{AB} - 300 = 0$$

$$T_{AB} = 983.3 \text{ lb}$$

$$T_{BC} = 854.2 \text{ lb}$$



At *C*

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{-20}{\sqrt{436}}(854.2) + \frac{15}{\sqrt{(14-y_D)^2 + 225}}T_{CD} = 0 \qquad (1)$$

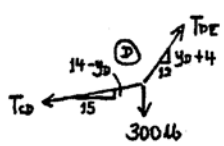
$$+\uparrow\Sigma F_y = 0; \qquad \frac{6}{\sqrt{436}}(854.2) + \frac{14 - y_D}{\sqrt{(14-y_D)^2 + 225}}T_{CD} - P_1 = 0 \qquad (2)$$

At *D*

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{12}{\sqrt{(4+y_D)^2 + 144}}T_{DE} - \frac{15}{\sqrt{(14-y_D)^2 + 225}}T_{CD} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{4 + y_D}{\sqrt{(4+y_D)^2 + 144}}T_{DE} - \frac{14 - y_D}{\sqrt{(14-y_D)^2 + 225}}T_{CD} - 300 = 0$$

$$T_{CD} = \frac{3600\sqrt{225 + (14 - y_D)^2}}{27y_D - 108}$$



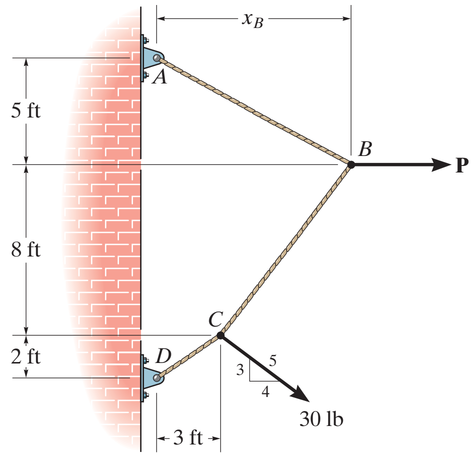
Substitute into Eq. (1) :

$$y_D = 6.44 \text{ ft} \qquad \textbf{Ans}$$

$$T_{CD} = 916.1 \text{ lb}$$

$$P_1 = 658 \text{ lb} \qquad \textbf{Ans}$$

•7–97. The cable supports the loading shown. Determine the horizontal distance x_B the force at point B acts from A. Set $P = 40$ lb.

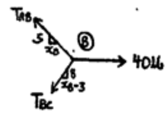


At B

$$\xrightarrow{+}\Sigma F_x = 0; \qquad 40 - \frac{x_B}{\sqrt{x_B^2 + 25}}T_{AB} - \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{5}{\sqrt{x_B^2 + 25}}T_{AB} - \frac{8}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 0$$

$$\frac{13x_B - 15}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 200 \qquad (1)$$

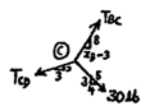


At C

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{4}{5}(30) + \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} - \frac{3}{\sqrt{13}}T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{8}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} - \frac{2}{\sqrt{13}}T_{CD} - \frac{3}{5}(30) = 0$$

$$\frac{30 - 2x_B}{\sqrt{(x_B - 3)^2 + 64}}T_{BC} = 102 \qquad (2)$$

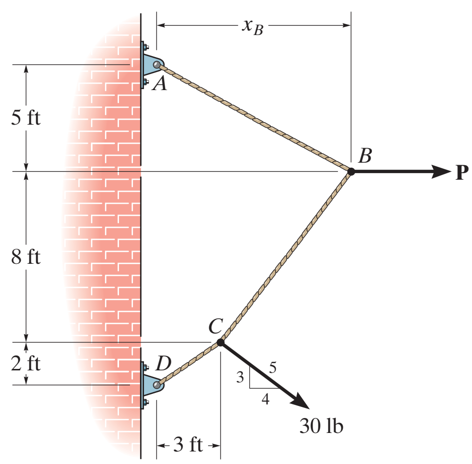


Solving Eqs. (1) & (2)

$$\frac{13x_B - 15}{30 - 2x_B} = \frac{200}{102}$$

$$x_B = 4.36 \text{ ft} \qquad \textbf{Ans}$$

7–98. The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.



At B

$$\xrightarrow{+}\Sigma F_x = 0; \qquad P - \frac{6}{\sqrt{61}}T_{AB} - \frac{3}{\sqrt{73}}T_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{5}{\sqrt{61}}T_{AB} - \frac{8}{\sqrt{73}}T_{BC} = 0$$

$$5P - \frac{63}{\sqrt{73}}T_{BC} = 0 \qquad (1)$$

At C

$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{4}{5}(30) + \frac{3}{\sqrt{73}}T_{BC} - \frac{3}{\sqrt{13}}T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{8}{\sqrt{73}}T_{BC} - \frac{2}{\sqrt{13}}T_{CD} - \frac{3}{5}(30) = 0$$

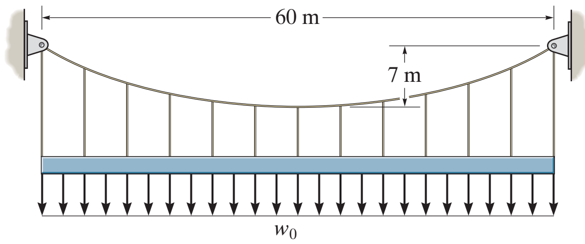
$$\frac{18}{\sqrt{73}}T_{BC} = 102 \qquad (2)$$

Solving Eqs. (1) & (2)

$$\frac{63}{18} = \frac{5P}{102}$$

$$P = 71.4 \text{ lb} \qquad \textbf{Ans}$$

7–99. Determine the maximum uniform distributed loading w_0 N/m that the cable can support if it is capable of sustaining a maximum tension of 60 kN.



The Equation of The Cable :

$$y = \frac{1}{F_H}\int\left(\int w(x)\,dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1x + C_2\right) \qquad [1]$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0x + C_1) \qquad [2]$$

Boundary Conditions :

$y = 0$ at $x = 0$, then from Eq.[1] $\quad 0 = \frac{1}{F_H}(C_2) \quad C_2 = 0$

$\frac{dy}{dx} = 0$ at $x = 0$, then from Eq.[2] $\quad 0 = \frac{1}{F_H}(C_1) \quad C_1 = 0$

Thus,

$$y = \frac{w_0}{2F_H}x^2 \qquad [3]$$

$$\frac{dy}{dx} = \frac{w_0}{F_H}x \qquad [4]$$

$y = 7$ m at $x = 30$ m, then from Eq.[3] $\quad 7 = \frac{w_0}{2F_H}(30^2) \quad F_H = \frac{450}{7}w_0$

$\theta = \theta_{max}$ at $x = 30$ m and the maximum tension occurs when $\theta = \theta_{max}$. From Eq.[4]

$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x=30\text{ m}} = \frac{w_0}{\frac{450}{7}w_0}x = 0.01556(30) = 0.4667$$

$$\theta_{max} = 25.02°$$

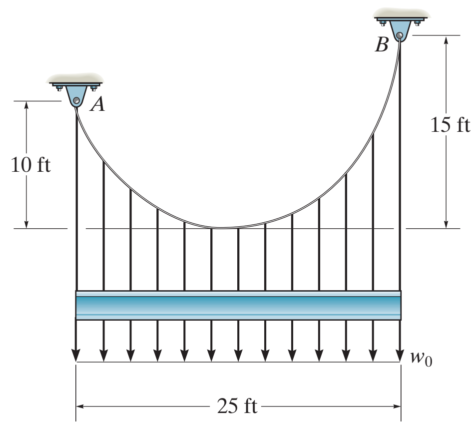
The maximum tension in the cable is

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

$$60 = \frac{\frac{450}{7}w_0}{\cos 25.02°}$$

$$w_0 = 0.846 \text{ kN/m} \qquad \textbf{Ans}$$

***7–100.** The cable supports the uniform distributed load of $w_0 = 600$ lb/ft. Determine the tension in the cable at each support A and B.



Use the equations of Example 7-12.

$$y = \frac{w_0}{2F_H}x^2$$

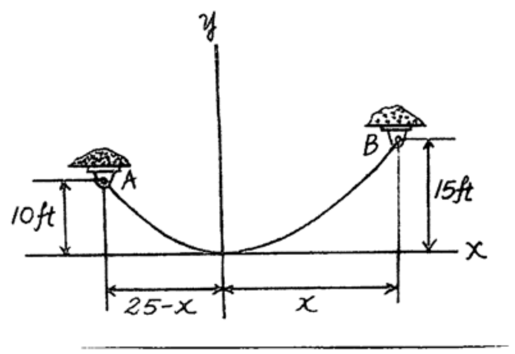
$$15 = \frac{600}{2F_H}x^2$$

$$10 = \frac{600}{2F_H}(25 - x)^2$$

$$\frac{600}{2(15)}x^2 = \frac{600}{2(10)}(25 - x)^2$$

$$x^2 = 1.5(625 - 50x + x^2)$$

$$0.5x^2 - 75x + 937.50 = 0$$



Choose root < 25 ft.

$$x = 13.76 \text{ ft}$$

$$F_H = \frac{w_0}{2y}x^2 = \frac{600}{2(15)}(13.76)^2 = 3788 \text{ lb}$$

At B:

$$y = \frac{w_0}{2F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_B = 0.15838x\Big|_{x=13.76} = 2.180$$

$$\theta_B = 65.36^\circ$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{3788}{\cos 65.36^\circ} = 9085 \text{ lb} = 9.09 \text{ kip} \quad \textbf{Ans}$$

At A

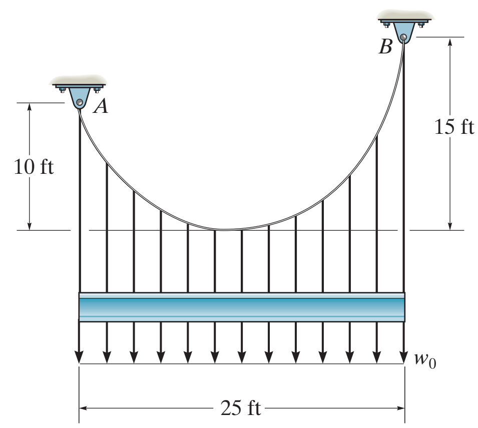
$$y = \frac{w_0}{2F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_A = 0.15838x\Big|_{x=(25-13.76)} = 1.780$$

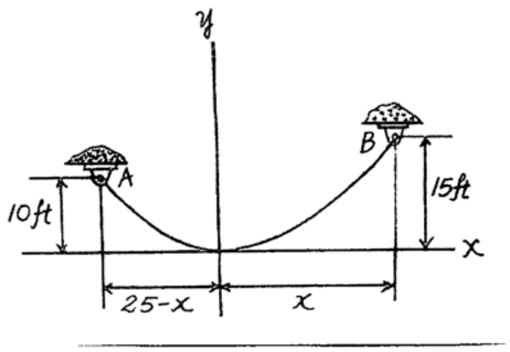
$$\theta_A = 60.67^\circ$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{3788}{\cos 60.67^\circ} = 7733 \text{ lb} = 7.73 \text{ kip} \quad \textbf{Ans}$$

•7–101. Determine the maximum uniform distributed load w_0 the cable can support if the maximum tension the cable can sustain is 4000 lb.



Use the equations of Example 7 - 12.



$$y = \frac{w_0}{2F_H}x^2$$

$$15 = \frac{w_0}{2F_H}x^2$$

$$10 = \frac{w_0}{2F_H}(25 - x)^2$$

$$\frac{x^2}{15} = \frac{1}{10}(25 - x)^2$$

$$x^2 = 1.5(625 - 50x + x^2)$$

$$0.5x^2 - 75x + 937.50 = 0$$

Choose root < 25 ft.

$$x = 13.76 \text{ ft}$$

$$F_H = \frac{w_0}{2y}x^2 = \frac{w_0}{2(15)}(13.76)^2 = 6.31378\,w_0$$

Maximum tension occurs at B since the slope y of the cable is greatest there.

$$y = \frac{w_0}{2F_H}x^2$$

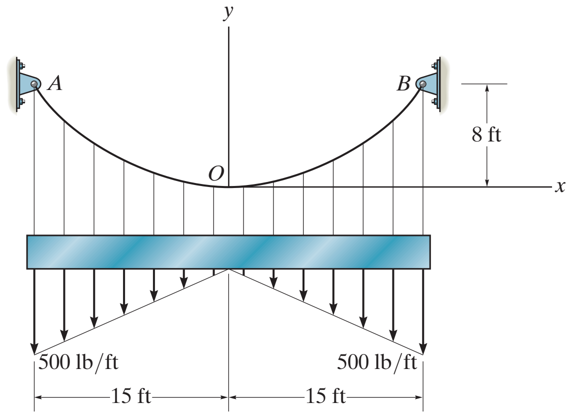
$$\left.\frac{dy}{dx}\right|_{x = 13.76 \text{ ft}} = \tan\theta_{max} = \frac{w_0 x}{F_H} = \frac{w_0(13.76)}{6.31378\,w_0}$$

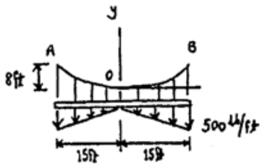
$$\theta_{max} = 65.36°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

$$4000 = \frac{6.31378\,w_0}{\cos 65.36°} \qquad w_0 = 264 \text{ lb/ft} \qquad \textbf{Ans}$$

7–102. The cable is subjected to the triangular loading. If the slope of the cable at point O is zero, determine the equation of the curve $y = f(x)$ which defines the cable shape OB, and the maximum tension developed in the cable.





$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\int\left(\int \frac{500}{15}x\,dx\right)dx$$

$$= \frac{1}{F_H}\int\left(\frac{50}{3}x^2 + C_1\right)dx$$

$$= \frac{1}{F_H}\left(\frac{50}{9}x^3 + C_1x + C_2\right)$$

$$\frac{dy}{dx} = \frac{50}{3F_H}x^2 + \frac{C_1}{F_H}$$

At $x = 0$, $\frac{dy}{dx} = 0$ $\quad C_1 = 0$

At $x = 0$, $y = 0$ $\quad C_2 = 0$

$$y = \frac{50}{9F_H}x^3$$

At $x = 15$ ft, $y = 8$ ft $\quad F_H = 2344$ lb

$y = 2.37(10^{-3})x^3$ **Ans**

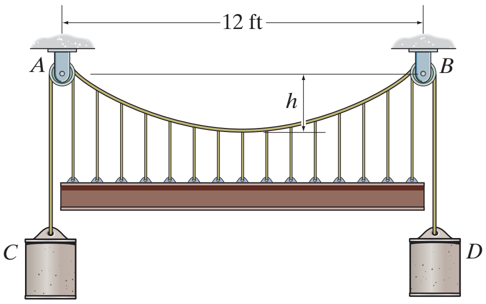
$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{50}{3(2344)}x^2\right|_{x=15\text{ ft}}$$

$$\theta_{max} = \tan^{-1}(1.6) = 57.99°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2344}{\cos 57.99°} = 4422 \text{ lb}$$

$T_{max} = 4.42$ kip **Ans**

7–103. If cylinders C and D each weigh 900 lb, determine the maximum sag h, and the length of the cable between the smooth pulleys at A and B. The beam has a weight per unit length of 100 lb/ft.



Since the loading and system are symmetric as indicated in the free-body diagram shown in Fig. a,

$+\uparrow \Sigma F_y = 0;$ $\quad 2(900\sin\theta_{max}) - 100(12) = 0$

$$\theta_{max} = 41.81°$$

Thus,

$$F_H = T_{max}\cos\theta_{max} = 900\cos 41.81° = 670.82 \text{ lb}$$

As shown in Fig. a, the origin of the $x-y$ coordinate system will be set at the lowest point of the cable.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{100}{670.82} = 0.1491$$

Integrating the above equation,

$$\frac{dy}{dx} = 0.1491x + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus, Eq. (1) becomes

$$\frac{dy}{dx} = 0.1491x$$

Integrating,

$$y = 0.07454x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus, Eq. (1) becomes

$$y = 0.07454x^2$$

Applying another boundary condition, $y = h$, at $x = 6$ ft,

$$h = 0.07454(6^2) = 2.68 \text{ ft}$$ **Ans.**

The differential length of the cable is

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + 0.02222x^2}\,dx$$

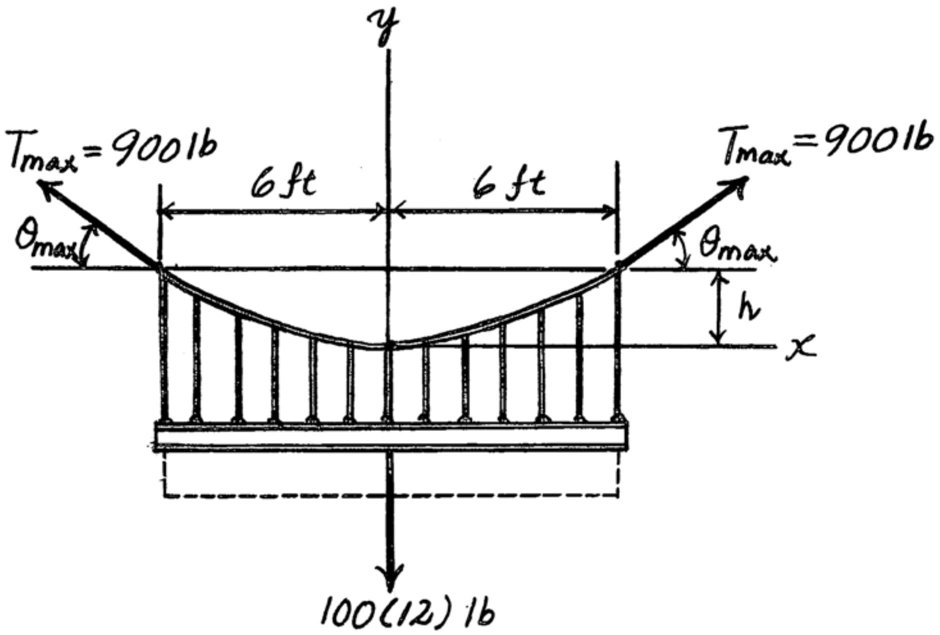
Thus, the total length of the cable is

$$L = \int ds = 2\int_0^{6\text{ ft}} \sqrt{1 + 0.02222x^2}\,dx$$

$$= 0.2981\int_0^{6\text{ ft}} \sqrt{45 + x^2}\,dx$$

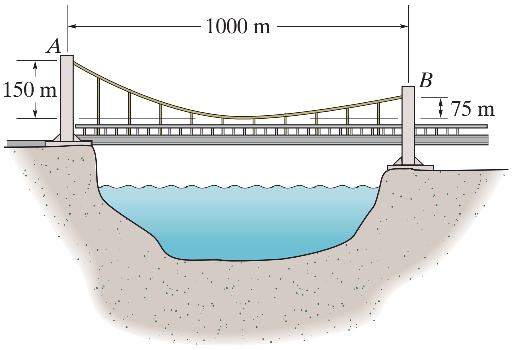
$$= 0.2981\left\{\frac{1}{2}\left[x\sqrt{45 + x^2} + 45\ln\left(x + \sqrt{45 + x^2}\right)\right]\right\}\Bigg|_0^{6\text{ ft}}$$

$$= 13.4\text{ ft}$$ **Ans.**



(a)

***7–104.** The bridge deck has a weight per unit length of 80 kN/m. It is supported on each side by a cable. Determine the tension in each cable at the piers A and B.



As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Since the bridge deck is supported by two cables, $w(x) = \frac{80}{2} = 40\text{ kN/m}$.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{40(10^3)}{F_H}$$

Integrating,

$$\frac{dy}{dx} = \frac{40(10^3)}{F_H} + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{40(10^3)}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{20(10^3)}{F_H}x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{20(10^3)}{F_H}x^2$$

Applying two other boundary conditions $y = 75$ m at $x = x_0$ and $y = 150$ m at $x = -(1000 - x_0)$,

$$75 = \frac{20(10^3)}{F_H}x_0^2$$

$$150 = \frac{20(10^3)}{F_H}[-(1000 - x_0)]^2$$

Solving these equations

$$x_0 = 414.21\text{ m} \qquad F_H = 45.75(10^6)\text{ N}$$

Substituting the result for $\mathbf{F}_H$ into Eq. (1),

$$\frac{dy}{dx} = \frac{40(10^3)}{45.75(10^6)}x = 0.8743(10^{-3})x$$

Thus, the angles the cables make with the horizontal at A and B are

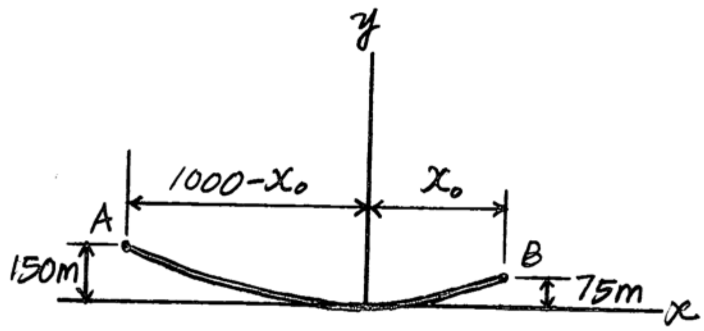
$$\theta_B = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_B}\right)\right| = \left|\tan^{-1}\left[0.8743(10^{-3})(414.21)\right]\right| = 19.91°$$

$$\theta_A = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_A}\right)\right| = \left|\tan^{-1}\left\{0.8743(10^{-3})\left[-(1000 - 414.21)\right]\right\}\right| = 27.12°$$

Thus,

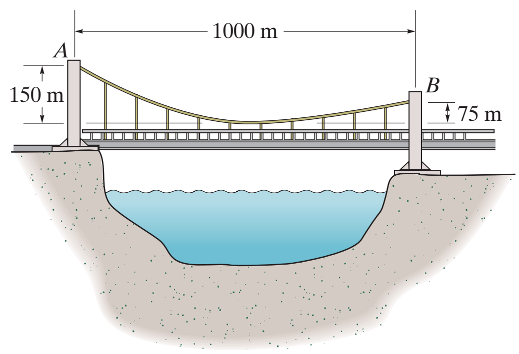
$$T_B = \frac{F_H}{\cos\theta_B} = \frac{45.75(10^6)}{\cos 19.91°} = 48.66(10^6)\text{ N} = 48.7\text{ MN}$$ **Ans.**

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{45.75(10^6)}{\cos 27.12°} = 51.40(10^6)\text{ N} = 51.4\text{ MN}$$ **Ans.**



(a)

•7–105. If each of the two side cables that support the bridge deck can sustain a maximum tension of 50 MN, determine the allowable uniform distributed load w_0 caused by the weight of the bridge deck.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Since the bridge deck is supported by two cables, $w(x) = \frac{w_0}{2}$.

$$\frac{d^2y}{dx^2} = \frac{w_0/2}{F_H} = \frac{w_0}{2F_H}$$

Integrating,

$$\frac{dy}{dx} = \frac{w_0}{2F_H} + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_0}{2F_H}x \qquad (1)$$

Integrating,

$$y = \frac{w_0}{4F_H}x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{w_0}{4F_H}x^2$$

Applying two other boundary conditions $y = 75$ m at $x = x_0$ and $y = 150$ m at $x = -(1000 - x_0)$,

$$75 = \frac{w_0}{4F_H}x^2$$

$$150 = \frac{w_0}{4F_H}[-(1000 - x_0)]^2$$

Solving these equations

$$x_0 = 414.21\text{ m} \qquad F_H = 571.91w_0$$

Substituting the result for $\mathbf{F}_H$ into Eq. (1),

$$\frac{dy}{dx} = \frac{w_0}{2(571.91w_0)}x = 0.8743(10^{-3})x$$

By observation, the angle the cable makes with the horizontal at $A\,(\theta_A)$ is greater than that at $B\,(\theta_B)$. Thus, the cable tension at A is the greatest.

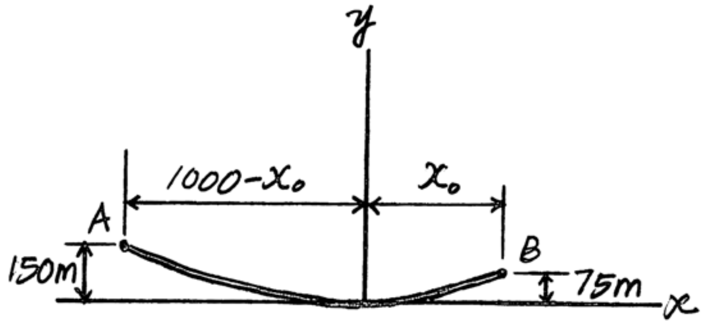
$$\theta_A = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_A}\right)\right| = \left|\tan^{-1}\left\{0.8743(10^{-3})[-(1000-414.21)]\right\}\right| = 27.12^\circ$$

By setting $T_A = 50(10^6)$ N,

$$T_A = \frac{F_H}{\cos\theta_A}$$

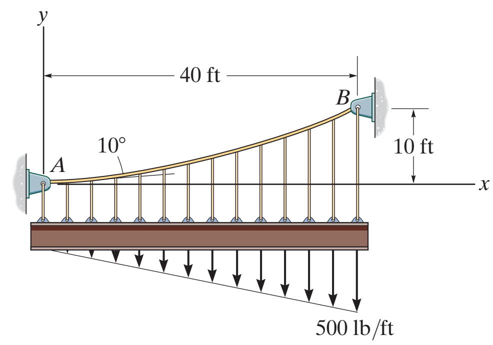
$$50(10^6) = \frac{571.91w_0}{\cos 27.12^\circ}$$

$$w_0 = 77.82(10^3)\text{ N/m} = 77.8\text{ kN/m} \qquad \textbf{Ans.}$$



(a)

7–106. If the slope of the cable at support A is 10°, determine the deflection curve $y = f(x)$ of the cable and the maximum tension developed in the cable.



The triangular distributed load is described by $w(x) = \frac{500}{40}x = 12.5x$.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{12.5}{F_H}x$$

Integrating,

$$\frac{dy}{dx} = \frac{6.25}{F_H}x^2 + C_1$$

Applying the boundary condition $\frac{dy}{dx} = \tan 10°$ at $x = 0$ results in $C_1 = \tan 10°$. Thus,

$$\frac{dy}{dx} = \frac{6.25}{F_H}x^2 + \tan 10° \qquad (1)$$

Integrating,

$$y = \frac{2.0833}{F_H}x^3 + \tan 10° x + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{2.0833}{F_H}x^3 + \tan 10° x \qquad (2)$$

Applying the boundary condition $y = 10$ ft at $x = 40$ ft,

$$10 = \frac{2.0833}{F_H}(40)^3 + \tan 10°(40)$$

$$F_H = 45.245(10^3) \text{ lb}$$

Substituting the result into Eqs. (1) and (2),

$$\frac{dy}{dx} = \frac{6.25}{45.245(10^3)}x^2 + \tan 10°$$

$$= 0.1381(10^{-3})x^2 + \tan 10°$$

and

$$y = \frac{2.0833}{45.245(10^3)}x^3 + \tan 10° x$$

$$= 46.0(10^{-6})x^3 + 0.176x \qquad \textbf{Ans.}$$

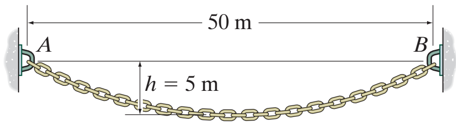
The maximum tension occurs at point B, where the cable makes the greatest angle with the horizontal. Here,

$$\theta_{max} = \tan^{-1}\left(\frac{dy}{dx}\bigg|_{40 \text{ ft}}\right) = \tan^{-1}\left[\frac{6.25}{45.245(10^3)}(40^2) + \tan 10°\right] = 21.67°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{45.245(10^3)}{\cos 21.67°} = 48.69(10^3) \text{ lb} = 48.7 \text{ kip} \qquad \textbf{Ans.}$$

7–107. If $h = 5$ m, determine the maximum tension developed in the chain and its length. The chain has a mass per unit length of 8 kg/m.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 8(9.81)\text{N / m} = 78.48 \text{ N / m}$.

$$\frac{d^2y}{dx^2} = \frac{78.48}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{78.48}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{78.48}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{78.48}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{78.48}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{78.48}{F_H}x} - e^{-\frac{78.48}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{78.48}{F_H}x \qquad (1)$$

Integrating Eq. (1),

$$y = \frac{F_H}{78.48}\cosh\left(\frac{78.48}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 5$ m at $x = 25$ m,

$$5 = \frac{F_H}{78.48}\left\{\cosh\left(\frac{78.48}{F_H}(25)\right) - 1\right\}.$$

Solving by trial and error,

$$F_H = 4969.06 \text{ N}$$

The maximum tension occurs at either points A or B where the chain makes the greatest angle with the horizontal. Here,

$$\theta_{max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=25\text{ m}}\right) = \tan^{-1}\left\{\sinh\left(\frac{78.48}{F_H}(25)\right)\right\} = 22.06°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{4969.06}{\cos 22.06°} = 5361.46\text{ N} = 5.36\text{ kN} \qquad \textbf{Ans.}$$

Referring to the free-body diagram shown in Fig. b,

$+\uparrow \Sigma F_y = 0;\qquad T\sin\theta - 8(9.81)s = 0$

$\xrightarrow{+} \Sigma F_y = 0;\qquad T\cos\theta - 4969.06 = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = 0.015794s$$

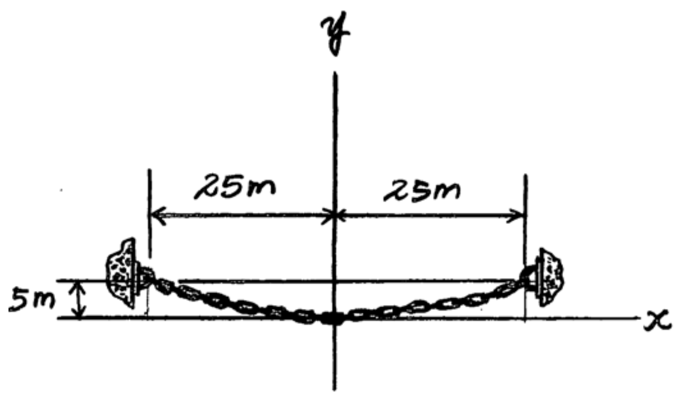
Equating Eqs. (1) and (2),

$$\sinh\left[\frac{78.48}{4969.06}x\right] = 0.015794s$$

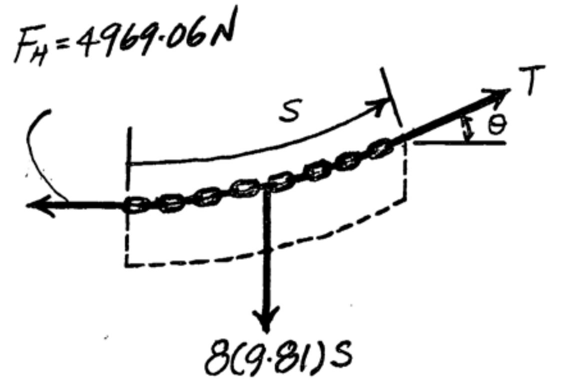
$$s = 63.32\sinh[0.01579x]$$

Thus, the length of the chain is

$$L = 2\{63.32\sinh[0.01579(25)]\} = 51.3\text{ m} \qquad \textbf{Ans.}$$

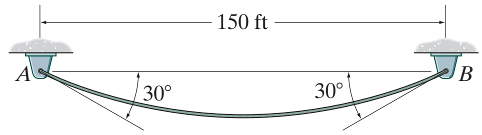


(a)



(b)

***7–108.** A cable having a weight per unit length of 5 lb/ft is suspended between supports A and B. Determine the equation of the catenary curve of the cable and the cable's length.



As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 5\text{ lb/ft}$.

$$\frac{d^2y}{dx^2} = \frac{5}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Substituting these two values into the equation,

$$\frac{du}{\sqrt{1+u^2}} = \frac{5}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{5}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{5}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{5}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{5}{F_H}x} - e^{-\frac{5}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{5}{F_H}x \qquad (1)$$

Applying the boundary equation $\frac{dy}{dx} = \tan 30°$ at $x = 75\text{ ft}$,

$$\tan 30° = \sinh\left[\frac{5}{F_H}(75)\right]$$

$$F_H = 682.68\text{ lb}$$

Substituting this result into Eq. (1),

$$\frac{dy}{dx} = \sinh\left[7.324(10^{-3})x\right] \qquad (2)$$

Integrating,

$$y = 136.54\cosh\left[7.324(10^{-3})x\right] + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -136.54$. Thus,

$$y = 137\left\{\cosh\left[7.324(10^{-3})x\right] - 1\right\}\text{ ft} \qquad \textbf{Ans.}$$

If we write the force equation of equilibrium along the x and y axes by referring to the free - body diagram shown in Fig. b, we have

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T\cos\theta - 682.68 = 0$

$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta - 5s = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = 7.324(10^{-3})s \qquad (3)$$

Equating Eqs. (2) and (3),

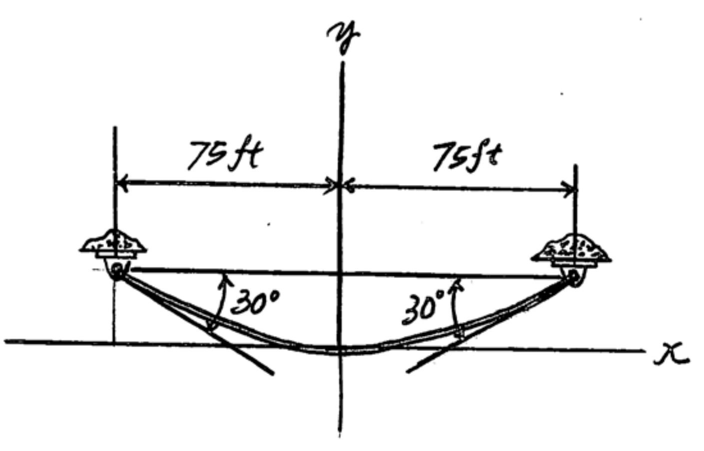
$$7.324(10^{-3})s = \sinh\left[7.324(10^{-3})x\right]$$

$$s = 136.54\sinh\left[7.324(10^{-3})x\right]\text{ft}$$

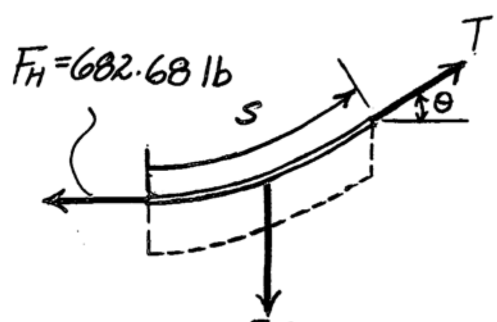
Thus, the length of the cable is

$$L = 2\left\{136.54\sinh\left[7.324(10^{-3})(75)\right]\right\}$$

$$= 157.66\text{ ft} = 158\text{ ft} \qquad \textbf{Ans.}$$

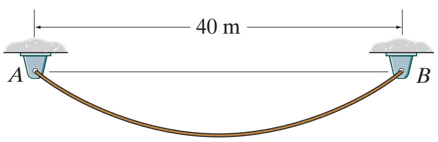


(a)



(b)

•7–109. If the 45-m-long cable has a mass per unit length of 5 kg/m, determine the equation of the catenary curve of the cable and the maximum tension developed in the cable.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 5(9.81)\text{ N/m} = 49.05\text{ N/m}$.

$$\frac{d^2y}{dx^2} = \frac{49.05}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{49.05}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{49.05}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{49.05}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{49.05}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{49.05}{F_H}x} - e^{-\frac{49.05}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{49.05}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{F_H}{49.05}\cosh\left(\frac{49.05}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -\frac{F_H}{49.05}$. Thus,

$$y = \frac{F_H}{49.05}\left[\cosh\left(\frac{49.05}{F_H}x\right) - 1\right]\text{ m}$$

If we write the force equation of equilibrium along the x and y axes by referring to the free - body diagram shown in Fig. b,

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad T\cos\theta - F_H = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad T\sin\theta - 5(9.81)s = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = \frac{49.05s}{F_H} \qquad (3)$$

Equating Eqs. (1) and (3) yields

$$\frac{49.05s}{F_H} = \sinh\left(\frac{49.05}{F_H}x\right)$$

$$s = \frac{F_H}{49.05}\sinh\left(\frac{49.05}{F_H}\right)$$

Thus, the length of the cable is

$$L = 45 = 2\left\{\frac{F_H}{49.05}\sinh\left(\frac{49.05}{F_H}(20)\right)\right\}$$

Solving by trial and error,

$$F_H = 1153.41\text{ N}$$

Substituting this result into Eq. (2),

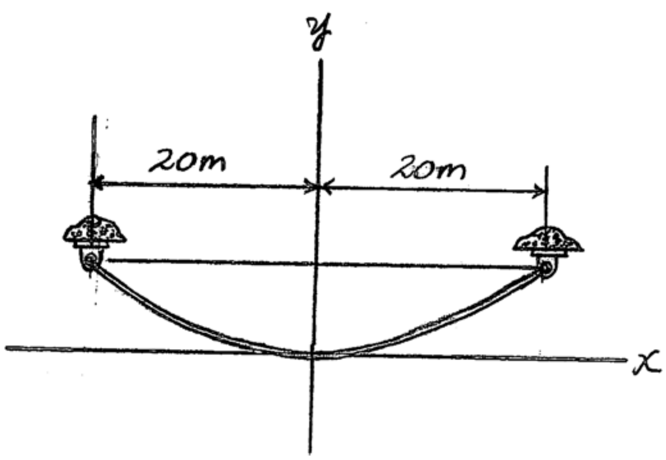
$$y = 23.5[\cos h\,0.0425x - 1]\text{ m} \qquad \textbf{Ans.}$$

The maximum tension occurs at either points A or B where the cable makes the greatest angle with the horizontal. Here

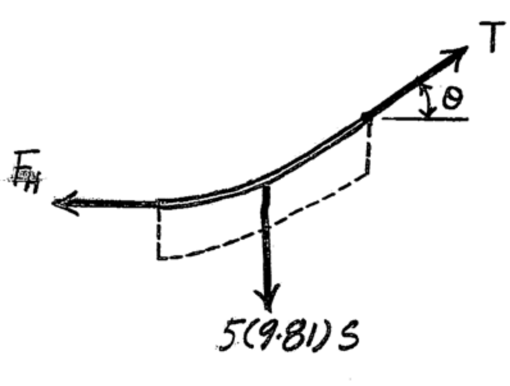
$$\theta_{max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=20\text{ m}}\right) = \tan^{-1}\left\{\sinh\left(\frac{49.05}{F_H}(20)\right)\right\} = 43.74°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{1153.41}{\cos 43.74°} = 1596.36\text{ N} = 1.60\text{ kN} \qquad \textbf{Ans.}$$



(a)



(b)

7–110. Show that the deflection curve of the cable discussed in Example 7–13 reduces to Eq. 4 in Example 7–12 when the *hyperbolic cosine function* is expanded in terms of a series and only the first two terms are retained. (The answer indicates that the *catenary* may be replaced by a parabola in the analysis of problems in which the sag is small. In this case, the cable weight is assumed to be uniformly distributed along the horizontal.)

$$\cosh x = 1 + \frac{x^2}{2!} + \ldots$$

Substituting into

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$= \frac{F_H}{w_0}\left[1 + \frac{w_0^2 x^2}{2F_H^2} + \ldots - 1\right]$$

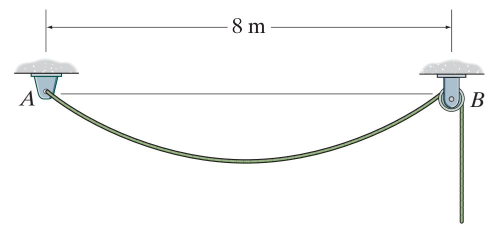
$$\approx \frac{w_0 x^2}{2F_H}$$

Using Eq. (3) in Example 7 – 12,

$$F_H = \frac{w_0 L^2}{8h}$$

We get $y = \dfrac{4h}{L^2}x^2$ **QED**

7–111. The cable has a mass per unit length of 10 kg/m. Determine the shortest total length L of the cable that can be suspended in equilibrium.



As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. $w(s) = 10(9.81)\text{N}/\text{m} = 98.1\text{ N}/\text{m}$.

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$, then

$$\frac{du}{\sqrt{1+u^2}} = \frac{98.1}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{98.1}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{98.1}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{98.1}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{98.1}{F_H}x} - e^{-\frac{98.1}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{98.1}{F_H}x \qquad (1)$$

Referring to the free-body diagram shown in Fig. *b*,

$\xrightarrow{+}\Sigma F_x = 0;\qquad T\cos\theta - F_H = 0$

$+\uparrow\Sigma F_y = 0;\qquad T\sin\theta - 10(9.81)s = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = \frac{98.1s}{F_H} \qquad (2)$$

Equating Eqs. (1) and (2),

$$\frac{98.1s}{F_H} = \sinh\left(\frac{98.1}{F_H}x\right)$$

$$s = \frac{F_H}{98.1}\sinh\left(\frac{98.1}{F_H}x\right)$$

The length of the cable between A and B is therefore

$$L' = 2\left\{\frac{F_H}{98.1}\sinh\left(\frac{98.1}{F_H}(4)\right)\right\} = 0.02039F_H \sinh\left(\frac{392.4}{F_H}\right)$$

Thus, the length of the overhanging cable is

$$L - L' = L - 0.2039F_H \sinh\left(\frac{392.4}{F_H}\right)$$

The tension developed in the cable at B is equal to the weight of the overhanging cable.

$$T_B = 10(9.81)\left[L - 0.2039F_H \sinh\left(\frac{392.4}{F_H}\right)\right]$$

$$= 98.1L - 2F_H \sinh\left(\frac{392.4}{F_H}\right) \qquad (3)$$

Using Eq. (1), the angle that the cable makes with the horizontal at B is

$$\tan\theta_B = \sinh\left(\frac{98.1}{F_H}(4)\right) = \sinh\left(\frac{392.4}{F_H}\right)$$

From the geometry of Fig. c,

$$\cos\theta_B = \frac{1}{\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)}}$$

$$T_B = \frac{F_H}{\cos\theta_B} = F_H\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)} \qquad (4)$$

Equating Eqs. (3) and (4),

$$F_H\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)} = 98.1L - 2F_H \sin\left(\frac{392.4}{F_H}\right)$$

$$L = \frac{1}{98.1}\left[F_H\sqrt{1+\sinh^2\left(\frac{392.4}{F_H}\right)} + 2F_H \sin\left(\frac{392.4}{F_H}\right)\right]$$

However, $\cosh^2\left(\frac{392.4}{F_H}\right) = 1+\sinh^2\left(\frac{392.4}{F_H}\right)$. Thus,

$$L = \frac{1}{98.1}\left[F_H \cosh\left(\frac{392.4}{F_H}\right) + 2F_H \sinh\left(\frac{392.4}{F_H}\right)\right] \qquad (5)$$

In order for L to be minimum, $\frac{dL}{dF_H}$ must be equal to zero.

$$\frac{dL}{dF_H} = \frac{1}{98.1}\left[F_H \sinh\left(\frac{392.4}{F_H}\right)\left(-\frac{392.4}{F_H{}^2}\right) + \cosh\left(\frac{392.4}{F_H}\right) + 2F_H \cosh\left(\frac{392.4}{F_H}\right)\left(-\frac{392.4}{F_H{}^2}\right) + 2\sinh\left(\frac{392.4}{F_H}\right)\right]$$

$$= \frac{1}{98.1}\left[\cosh\left(\frac{392.4}{F_H}\right) + 2\sinh\left(\frac{392.4}{F_H}\right) - \frac{392.4}{F_H}\sinh\left(\frac{392.4}{F_H}\right) - \frac{784.8}{F_H}\cosh\left(\frac{392.4}{F_H}\right)\right]$$

Setting $\frac{dL}{dF_H} = 0.$

$$\sinh\left(\frac{392.4}{F_H}\right)\left[2-\frac{392.4}{F_H}\right] + \cosh\left(\frac{392.4}{F_H}\right)\left[1-\frac{784.8}{F_H}\right] = 0$$

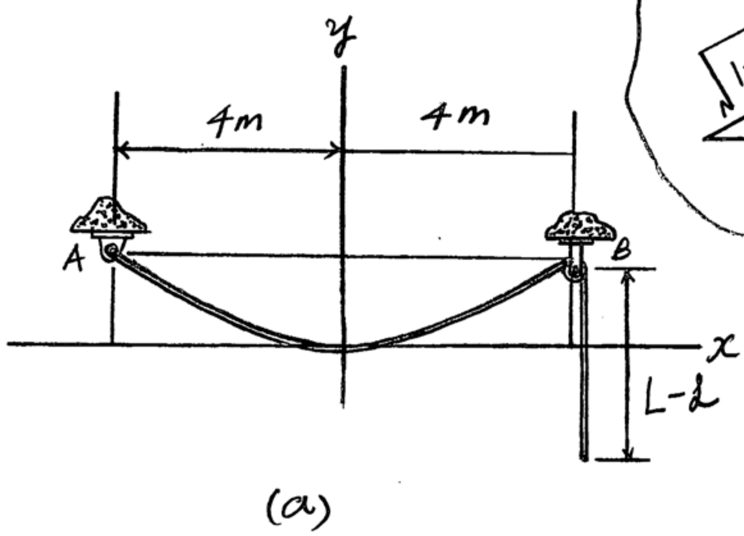
$$\tanh\left(\frac{392.4}{F_H}\right)(2F_H - 392.4) + (F_H - 784.8) = 0$$

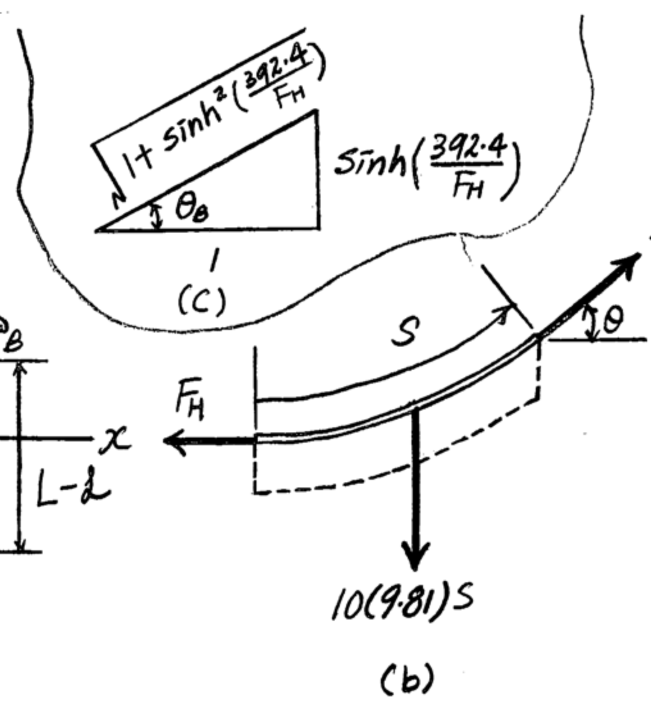
Solving by trial and error,

$$F_H = 438.70\text{ N}$$

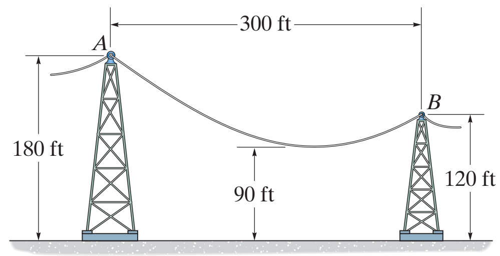
Substituting this result into Eq. (5) yields

$L = 15.5\text{ m}$ **Ans.**





***7–112.** The power transmission cable has a weight per unit length of 15 lb/ft. If the lowest point of the cable must be at least 90 ft above the ground, determine the maximum tension developed in the cable and the cable's length between A and B.



As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 15\text{ lb/ft}$.

$$\frac{d^2y}{dx^2} = \frac{15}{F_H}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

If we set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Thus,

$$\frac{du}{\sqrt{1+u^2}} = \frac{15}{F_H}dx$$

Integrating,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{15}{F_H}x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = \frac{15}{F_H}x$$

$$u+\sqrt{1+u^2} = e^{\frac{15}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{15}{F_H}x} - e^{-\frac{15}{F_H}x}}{2}$$

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{15}{F_H}x \qquad (1)$$

Integrating,

$$y = \frac{F_H}{15}\cosh\left(\frac{15}{F_H}x\right) + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -\frac{F_H}{15}$. Thus,

$$y = \frac{F_H}{15}\left[\cosh\left(\frac{15}{F_H}x\right) - 1\right]$$

Applying the boundary equation $y = 30$ ft at $x = x_0$ and $y = 90$ ft at $x = -(300 - x_0)$,

$$30 = \frac{F_H}{15}\left[\cosh\left(\frac{15x_0}{F_H}\right) - 1\right] \qquad (2)$$

$$90 = \frac{F_H}{15}\left\{\cosh\left[\frac{-15(300 - x_0)}{F_H}\right] - 1\right\}$$

Since $\cosh(a - b) = \cosh a \cosh b - \sinh a \sinh b$, then

$$90 = \frac{F_H}{15}\left(\cosh\frac{15x_0}{F_H}\cosh\frac{4500}{F_H} - \sinh\frac{15x_0}{F_H}\sinh\frac{4500}{F_H} - 1\right) \qquad (3)$$

Eq. (2) can be rewritten as

$$\cosh\frac{15x_0}{F_H} = \frac{450 + F_H}{F_H} \qquad (4)$$

Since $\sinh a = \sqrt{\cosh^2 a - 1}$, then

$$\sinh\frac{15x_0}{F_H} = \sqrt{\left(\frac{450 + F_H}{F_H}\right)^2 - 1} = \frac{1}{F_H}\sqrt{202500 + 900F_H} \qquad (5)$$

Substituting Eqs. (4) and (5) into Eq. (3),

$$1350 = (450 + F_H)\cosh\frac{4500}{F_H} - \sqrt{202500 + 900F_H}\sinh\frac{4500}{F_H} - F_H$$

Solving by trial and error,

$$F_H = 3169.58 \text{ lb}$$

Substituting this result into Eq. (4),

$$x_0 = 111.31 \text{ ft}$$

The maximum tension occurs at point A where the cable makes the greatest angle with the horizontal. Here,

$$\theta_{max} = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x=-188.69 \text{ ft}}\right)\right| = \tan^{-1}\left\{\sinh\left(\frac{15}{3169.58}(-188.69)\right)\right\} = 45.47°$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{3169.58}{\cos 45.47°} = 4519.58 \text{ lb} = 4.52 \text{ kip} \qquad \textbf{Ans.}$$

Referring to the free-body diagram shown in Fig. *b*,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad T\cos\theta - 3169.58 = 0$

$+\uparrow\Sigma F_y = 0;\qquad T\sin\theta - 15s = 0$

Eliminating T,

$$\frac{dy}{dx} = 4.732(10^{-3})s \qquad (6)$$

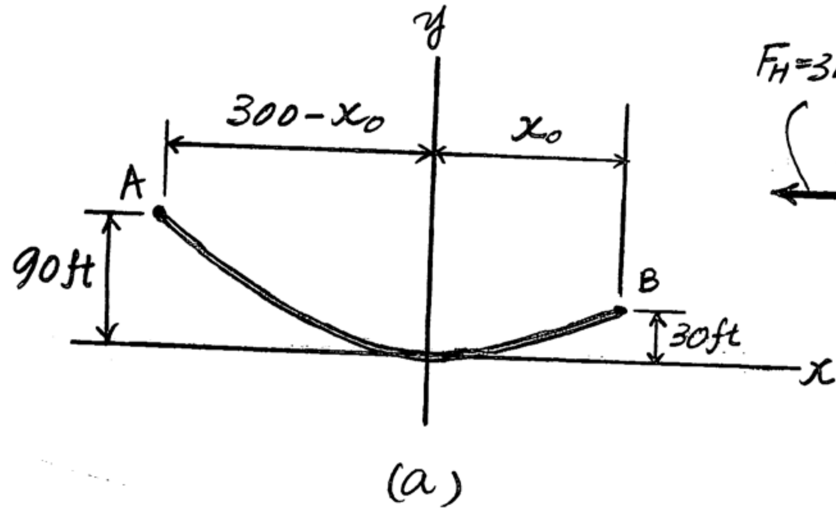
Equating Eqs. (1) and (6) yields

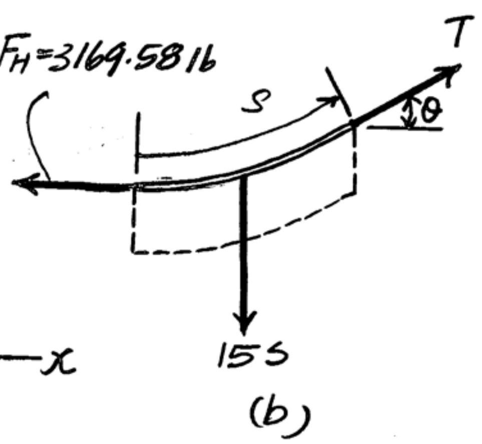
$$4.732(10^{-3})s = \sinh\left[4.732(10^{-3})x\right]$$

$$s = 211.31\sinh\left[4.732(10^{-3})x\right]$$

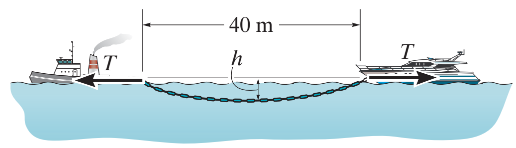
Thus, the length of the cable is

$$L = 211.31\sinh\left[4.732(10^{-3})(111.31)\right] + 211.31\sinh\left[4.732(10^{-3})(188.69)\right] = 331\text{ ft} \qquad \textbf{Ans.}$$





•7–113. If the horizontal towing force is $T = 20$ kN and the chain has a mass per unit length of 15 kg/m, determine the maximum sag h. Neglect the buoyancy effect of the water on the chain. The boats are stationary.



As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the chain. Here, $F_H = T = 20(10^3)$ N and $w(s) = 15(9.81)$ N/m $= 147.15$ N/m.

$$\frac{d^2y}{dx^2} = \frac{147.15}{20(10^3)}\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 7.3575(10^{-3})\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Thus,

$$\frac{du}{\sqrt{1+u^2}} = 7.3575(10^{-3})dx$$

Integrating,

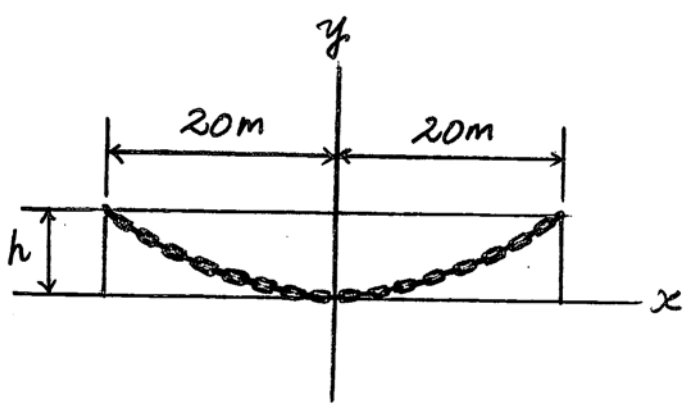
$$\ln\left(u+\sqrt{1+u^2}\right) = 7.3575(10^{-3})x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u+\sqrt{1+u^2}\right) = 7.3575(10^{-3})x$$

$$u+\sqrt{1+u^2} = e^{7.3575(10^{-3})x}$$

$$\frac{dy}{dx} = u = \frac{e^{7.3575(10^{-3})x} - e^{-7.3575(10^{-3})x}}{2}$$



(a)

Since $\sinh x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh 7.3575(10^{-3})x$$

Integrating,

$$y = 135.92\cosh 7.3575(10^{-3})x + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -135.92$. Thus,

$$y = 135.92\left[\cosh 7.3575(10^{-3})x - 1\right]$$

Applying the boundary equation $y = h$ at $x = 20$ m,

$$h = 135.92\left[\cosh 7.3575(10^{-3})(20) - 1\right] = 1.47 \text{ m}$$ **Ans.**

7–114. A 100-lb cable is attached between two points at a distance 50 ft apart having equal elevations. If the maximum tension developed in the cable is 75 lb, determine the length of the cable and the sag.

From Example 7 - 15,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = 75 \text{ lb}$$

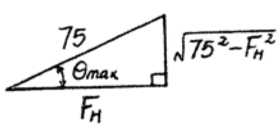
$$\cos\theta_{max} = \frac{F_H}{75}$$

For $\frac{1}{2}$ of cable,

$$w_0 = \frac{\frac{100}{2}}{s} = \frac{50}{s}$$

$$\tan\theta_{max} = \frac{w_0\, s}{F_H} = \frac{\sqrt{(75)^2 - F_H^2}}{F_H} = \frac{50}{F_H}$$

Thus,



$$\sqrt{(75)^2 - F_H^2} = 50; \qquad F_H = 55.9 \text{ lb}$$

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right) = \frac{55.9}{\left(\frac{50}{s}\right)}\sinh\left\{\left(\frac{50}{s\,(55.9)}\right)\left(\frac{50}{2}\right)\right\}$$

$s = 27.8$ ft

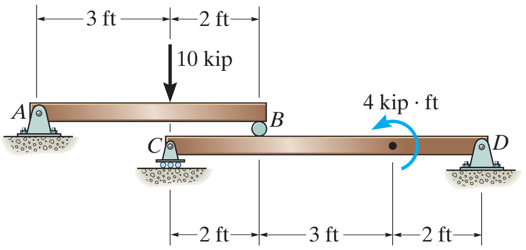
$$w_0 = \frac{50}{27.8} = 1.80 \text{ lb/ft}$$

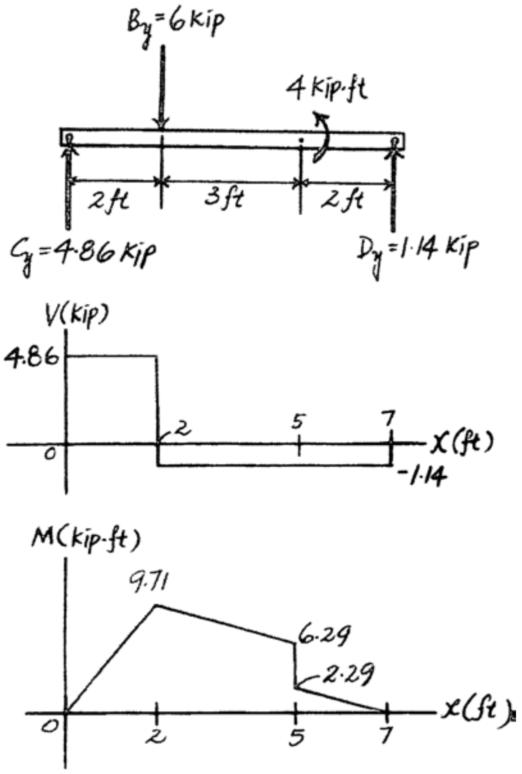
Total length $= 2\,s = 55.6$ ft **Ans**

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2 F_H}\right) - 1\right] = \frac{55.9}{1.80}\left[\cosh\left(\frac{1.80\,(50)}{2\,(55.9)}\right) - 1\right]$$

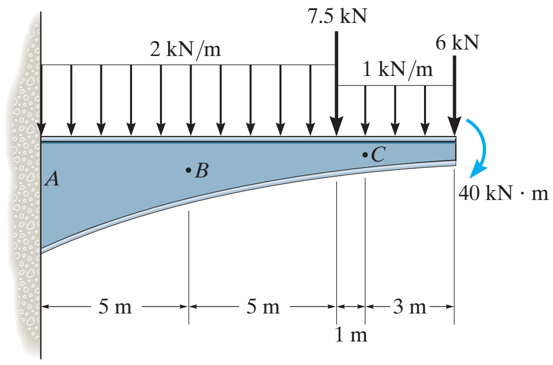
$= 10.6$ ft **Ans**

7–115. Draw the shear and moment diagrams for beam *CD*.





***7–116.** Determine the internal normal force, shear force, and moment at points B and C of the beam.



Free body Diagram : The Support reactions need not be computed for this case.

Internal Forces : Applying the equations of equilibrium to [FBD (a)], we have

$\xrightarrow{+} \Sigma F_x = 0;$ $N_C = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $V_C - 3.00 - 6 = 0$ $V_C = 9.00$ kN **Ans**

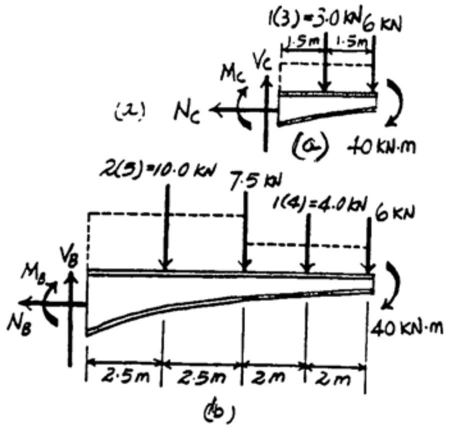
$\circlearrowleft + \Sigma M_C = 0;$ $-M_C - 3.00(1.5) - 6(3) - 40 = 0$
$M_C = -62.5$ kN · m **Ans**

Applying the equations of equilibrium to segment *DB* [FBD (b)], we have

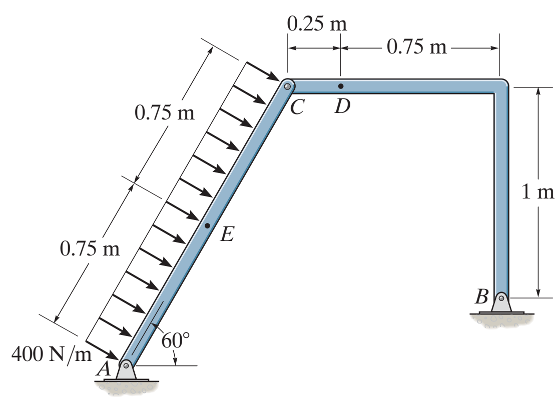
$\xrightarrow{+} \Sigma F_x = 0;$ $N_B = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $V_B - 10.0 - 7.5 - 4.00 - 6 = 0$
$V_D = 27.5$ kN **Ans**

$\circlearrowleft + \Sigma M_B = 0;$ $-M_B - 10.0(2.5) - 7.5(5)$
$-4.00(7) - 6(9) - 40 = 0$
$M_B = -184.5$ kN · m **Ans**



•7–117. Determine the internal normal force, shear force and moment at points D and E of the frame.



Support Reactions : Member BC is a two force member. From FBD (a),

$$\curvearrowleft + \Sigma M_B = 0; \quad F_{BC} \cos 15°(1.5) - 600(0.75) = 0$$
$$F_{BC} = 310.58 \text{ N}$$

Internal Forces : Applying the equations of equilibrium to segment CE [FBD (b)], we have

$$+ \Sigma F_{x'} = 0; \quad 310.58 \sin 15° - N_E = 0 \quad N_E = 80.4 \text{ N} \quad \textbf{Ans}$$

$$+ \Sigma F_{y'} = 0; \quad V_E + 310.58 \cos 15° - 300 = 0 \quad V_E = 0 \quad \textbf{Ans}$$

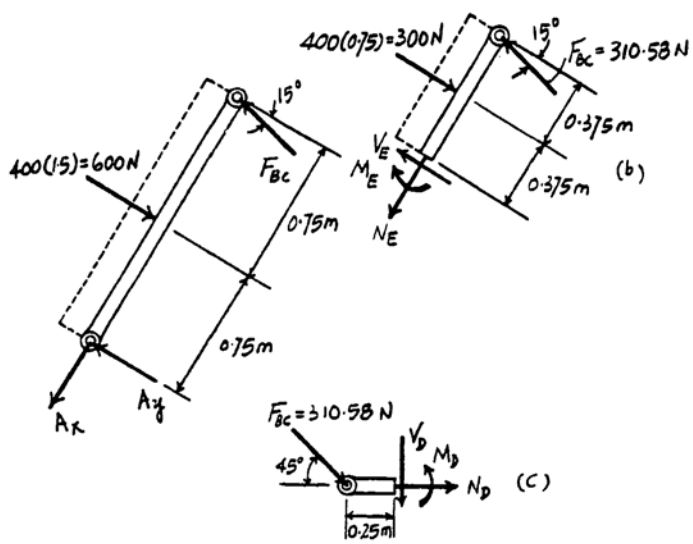
$$\curvearrowleft + \Sigma M_E = 0; \quad 310.58 \cos 15°(0.75) - 300(0.375) - M_E = 0$$
$$M_E = 112.5 \text{ N} \cdot \text{m} \quad \textbf{Ans}$$

Applying the equations of equilibrium to segment CD[FBD (c)], we have

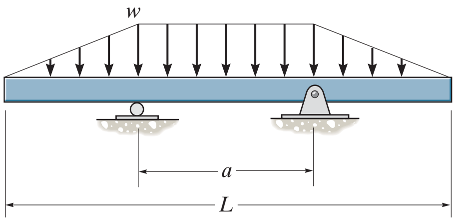
$$\xrightarrow{+} \Sigma F_x = 0; \quad N_D + 310.58 \cos 45° = 0 \quad N_D = -220 \text{ N} \quad \textbf{Ans}$$

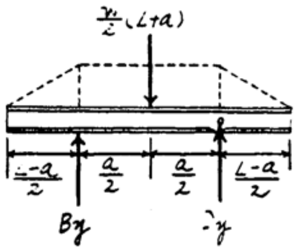
$$+\uparrow \Sigma F_y = 0; \quad -310.58 \sin 45° - V_D = 0 \quad V_D = -220 \text{ N} \quad \textbf{Ans}$$

$$\curvearrowleft + \Sigma M_D = 0; \quad M_D + 310.58 \sin 45°(0.25) = 0$$
$$M_D = -54.9 \text{ N} \cdot \text{m} \quad \textbf{Ans}$$



7–118. Determine the distance a between the supports in terms of the beam's length L so that the moment in the *symmetric* beam is zero at the beam's center.



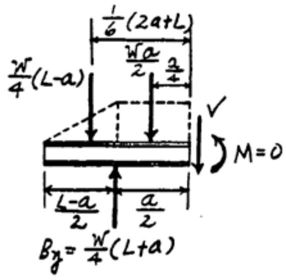


Support Reactions : From FBD (a),

$$\circlearrowleft + \Sigma M_C = 0; \quad \frac{w}{2}(L+a)\left(\frac{a}{2}\right) - B_y(a) = 0 \qquad B_y = \frac{w}{4}(L+a)$$

Free body Diagram : The FBD for segment AC sectioned through point C is drawn.

Internal Forces : This problem requires $M_C = 0$. Summing moments about point C[FBD (b)], we have



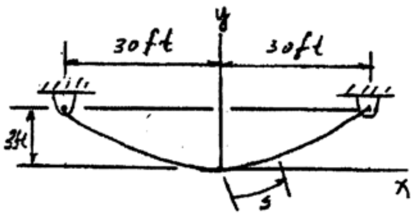
$$\circlearrowleft + \Sigma M_C = 0; \quad \frac{wa}{2}\left(\frac{a}{4}\right) + \frac{w}{4}(L-a)\left[\frac{1}{6}(2a+L)\right] - \frac{w}{4}(L+a)\left(\frac{a}{2}\right) = 0$$

$$2a^2 + 2aL - L^2 = 0$$

$$a = 0.366L \qquad \textbf{Ans}$$

7–119. A chain is suspended between points at the same elevation and spaced a distance of 60 ft apart. If it has a weight per unit length of 0.5 lb/ft and the sag is 3 ft, determine the maximum tension in the chain.

$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}\left(\int w_0\, ds\right)^2\right\}^{\frac{1}{2}}}$$



Performing the integration yields :

$$x = \frac{F_H}{0.5}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0.5s + C_1)\right] + C_2\right\} \qquad [1]$$

From Eq. 7-14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(0.5s + C_1)$$

At $s = 0$; $\frac{dy}{dx} = 0$ hence $C_1 = 0$

$$\frac{dy}{dx} = \tan\theta = \frac{0.5s}{F_H} \qquad [2]$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.[1] and using the result $C_1 = 0$ yields $C_2 = 0$. Hence

$$s = \frac{F_H}{0.5}\sinh\left(\frac{0.5}{F_H}x\right) \qquad [3]$$

Substituting Eq.[3] into [2] yields :

$$\frac{dy}{dx} = \sinh\left(\frac{0.5x}{F_H}\right) \qquad [4]$$

Performing the integration

$$y = \frac{F_H}{0.5}\cosh\left(\frac{0.5}{F_H}x\right) + C_3$$

Applying boundary conditions at $x = 0$; $y = 0$ yields $C_3 = -\frac{F_H}{0.5}$. Therefore

$$y = \frac{F_H}{0.5}\left[\cosh\left(\frac{0.5}{F_H}x\right) - 1\right]$$

At $x = 30$ ft; $y = 3$ ft $\qquad 3 = \frac{F_H}{0.5}\left[\cosh\left(\frac{0.5}{F_H}(30)\right) - 1\right]$

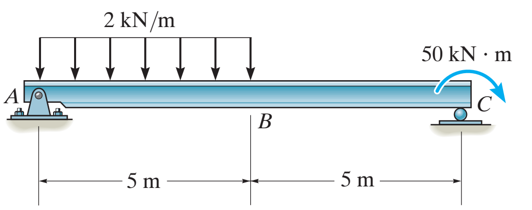
By trial and error $\qquad F_H = 75.25$ lb

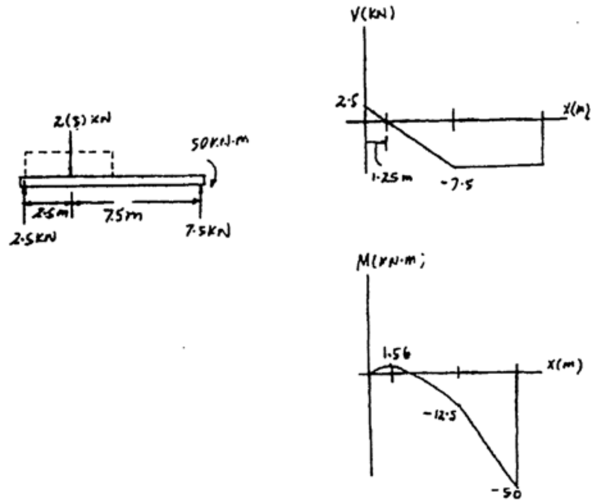
At $x = 30$ ft; $\theta = \theta_{max}$. From Eq.[4]

$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x=30\text{ ft}} = \sinh\left(\frac{0.5(30)}{75.25}\right) \qquad \theta_{max} = 11.346°$$

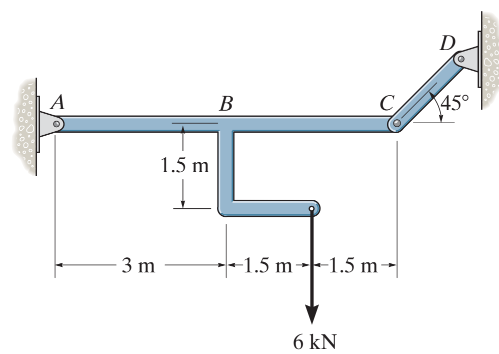
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{75.25}{\cos 11.346°} = 76.7\text{ lb.} \qquad \textbf{Ans}$$

***7–120.** Draw the shear and moment diagrams for the beam.





•7–121. Determine the internal shear and moment in member *ABC* as a function of *x*, where the origin for *x* is at *A*.



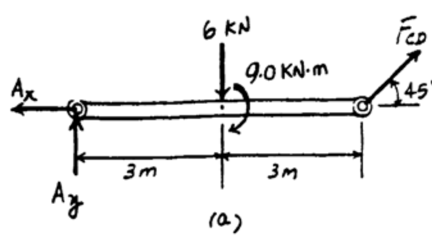
Support Reactions : The 6 kN load can be replaced by an equivalent force and couple moment at *B* as shown on FBD (a).

$\circlearrowleft + \Sigma M_A = 0;\quad F_{CD}\sin 45°(6) - 6(3) - 9.00 = 0 \quad F_{CD} = 6.364\text{ kN}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 6.364\sin 45° - 6 = 0 \quad A_y = 1.50\text{ kN}$

Shear and Moment Functions : For $0 \le x < 3$ m [FBD (b)],

$+\uparrow \Sigma F_y = 0;\quad 1.50 - V = 0 \quad V = 1.50\text{ kN}$ **Ans**



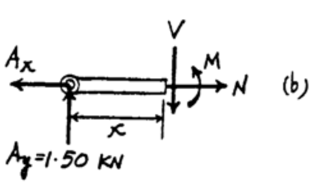
$\circlearrowleft + \Sigma M = 0;\quad M - 1.50x = 0 \quad M = \{1.50x\}\text{ kN}\cdot\text{m}$ **Ans**

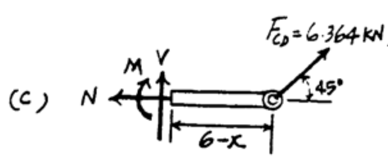
For $3\text{ m} < x \le 6$ m [FBD (c)],

$+\uparrow \Sigma F_y = 0;\quad V + 6.364\sin{}°45 = 0 \quad V = -4.50\text{ kN}$ **Ans**

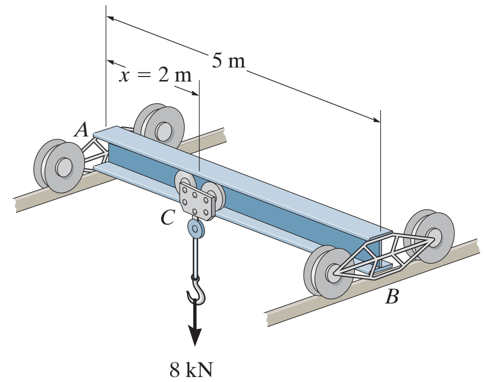
$\circlearrowleft + \Sigma M = 0;\quad 6.364\sin 45°(6 - x) - M = 0$

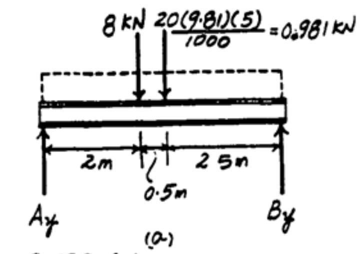
$M = \{27.0 - 4.50x\}\text{ kN}\cdot\text{m}$ **Ans**





7–122. The traveling crane consists of a 5-m-long beam having a uniform mass per unit length of 20 kg/m. The chain hoist and its supported load exert a force of 8 kN on the beam when $x = 2$ m. Draw the shear and moment diagrams for the beam. The guide wheels at the ends A and B exert only vertical reactions on the beam. Neglect the size of the trolley at C.



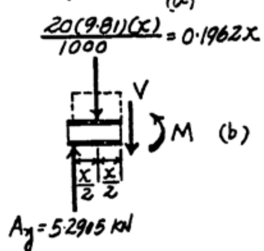


Support Reactions : From FBD (a),

$\curvearrowleft + \Sigma M_A = 0; \quad B_y(5) - 8(2) - 0.981(2.5) = 0 \qquad B_y = 3.6905 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 3.6905 - 8 - 0.981 = 0 \qquad A_y = 5.2905 \text{ kN}$

Shear and Moment Functions : For $0 \le x < 2$ m [FBD (b)],



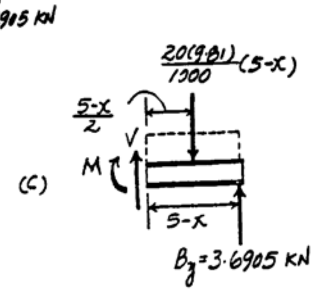
$+\uparrow \Sigma F_y = 0; \quad 5.2905 - 0.1962x - V = 0$

$V = \{5.29 - 0.196x\} \text{ kN}$ **Ans**

$\curvearrowleft + \Sigma M = 0; \quad M + 0.1962x\left(\frac{x}{2}\right) - 5.2905x = 0$

$M = \{5.29x - 0.0981x^2\} \text{ kN}\cdot\text{m}$ **Ans**

For $2 \text{ m} < x \le 5$ m [FBD (c)],

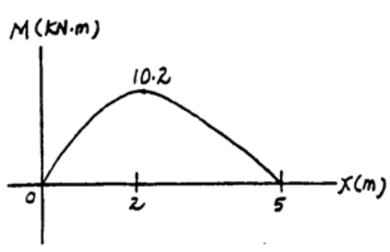


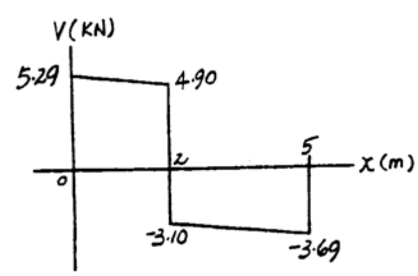
$+\uparrow \Sigma F_y = 0; \quad V + 3.6905 - \frac{20(9.81)}{1000}(5 - x) = 0$

$V = \{-0.196x - 2.71\} \text{ kN}$ **Ans**

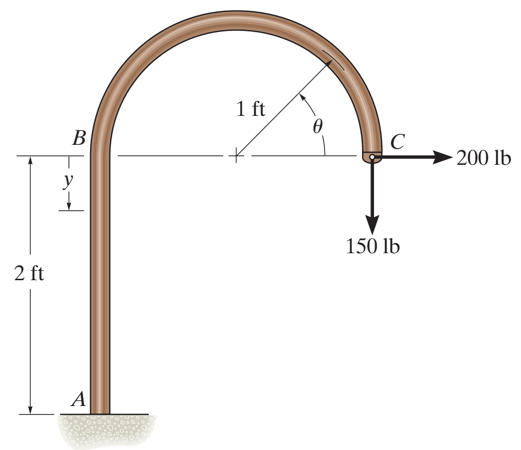
$\curvearrowleft + \Sigma M = 0; \quad 3.6905(5 - x) - \frac{20(9.81)}{1000}(5 - x)\left(\frac{5 - x}{2}\right) - M = 0$

$M = \{16.0 - 2.71x - 0.0981x^2\} \text{ kN}\cdot\text{m}$ **Ans**





***7–123.** Determine the internal normal force, shear force, and the moment as a function of $0° \le \theta \le 180°$ and $0 \le y \le 2$ ft for the member loaded as shown.



For $0° \le \theta \le 180°$:

$\nearrow\!+ \Sigma F_x = 0;\quad V + 200\cos\theta - 150\sin\theta = 0$

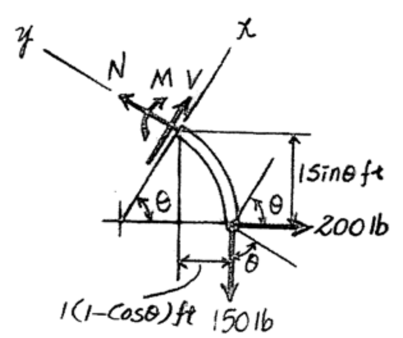
$V = 150\sin\theta - 200\cos\theta \quad$ **Ans**

$+\nwarrow \Sigma F_y = 0;\quad N - 200\sin\theta - 150\cos\theta = 0$

$N = 150\cos\theta + 200\sin\theta \quad$ **Ans**

$\circlearrowleft\!+\Sigma M = 0;\quad -M - 150(1)(1 - \cos\theta) + 200(1)\sin\theta = 0$

$M = 150\cos\theta + 200\sin\theta - 150 \quad$ **Ans**

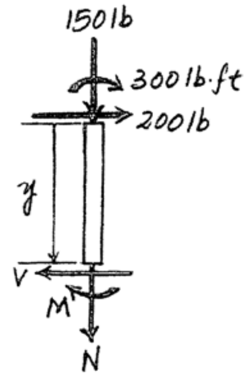


At section B, $\theta = 180°$, thus

$V_B = 200$ lb

$N_B = -150$ lb

$M_B = -300$ lb · ft



For $0 \le y \le 2$ ft:

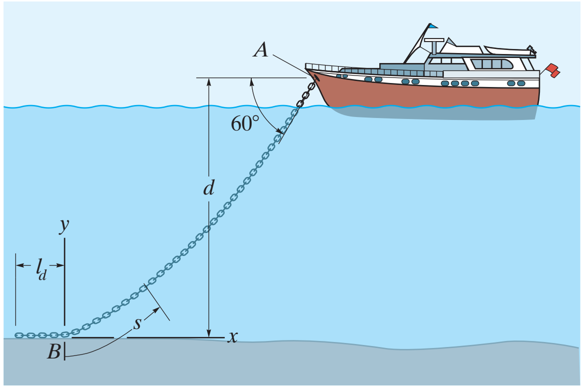
$\xrightarrow{+} \Sigma F_x = 0;\quad V = 200 \text{ lb} \quad$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad N = -150 \text{ lb} \quad$ **Ans**

$\circlearrowleft\!+\Sigma M = 0;\quad -M - 300 - 200y = 0$

$M = -300 - 200y \quad$ **Ans**

***7–124.** The yacht is anchored with a chain that has a total length of 40 m and a mass per unit length of 18 kg/m, and the tension in the chain at A is 7 kN. Determine the length of chain l_d which is lying at the bottom of the sea. What is the distance d? Assume that buoyancy effects of the water on the chain are negligible. *Hint:* Establish the origin of the coordinate system at B as shown in order to find the chain length BA.



Component of force at A is

$$F_H = T\cos\theta = 7000\cos 60^\circ = 3500\text{ N}$$

From Eq. (1) of Example 7-13

$$x = \frac{3500}{18(9.81)}\left(\sinh^{-1}\left[\frac{1}{3500}(18)(9.81)s + C_1\right] + C_2\right)$$

Since $\frac{dy}{dx} = 0$, $s = 0$, then

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0 s + C_1); \qquad C_1 = 0$$

Also $x = 0$, $s = 0$, so that $C_2 = 0$ and the above equation becomes

$$x = 19.82\left(\sinh^{-1}\left(\frac{s}{19.82}\right)\right) \qquad (1)$$

or,

$$s = 19.82\left(\sinh\left(\frac{x}{19.82}\right)\right) \qquad (2)$$

From Example 7-13

$$\frac{dy}{dx} = \frac{w_0 s}{F_H} = \frac{18(9.81)}{3500}s = \frac{s}{19.82} \qquad (3)$$

Substituting Eq. (2) into Eq. (3),

$$\frac{dy}{dx} = \sinh\left(\frac{x}{19.82}\right)$$

Integrating,

$$y = 19.82\cosh\left(\frac{x}{19.82}\right) + C_3$$

Since $x = 0$, $y = 0$, then $C_3 = -19.82$

Thus,

$$y = 19.82\left(\cosh\left(\frac{x}{19.82}\right) - 1\right) \qquad (4)$$

Slope of the cable at point A is

$$\frac{dy}{dx} = \tan 60^\circ = 1.732$$

Using Eq. (3),

$$s_{AB} = 19.82(1.732) = 34.33\text{ m}$$

Length of chain on the ground is thus

$$l_d = 40 - 34.33 = 5.67\text{ m} \qquad \textbf{Ans}$$

From Eq. (1), with $s = 34.33$ m

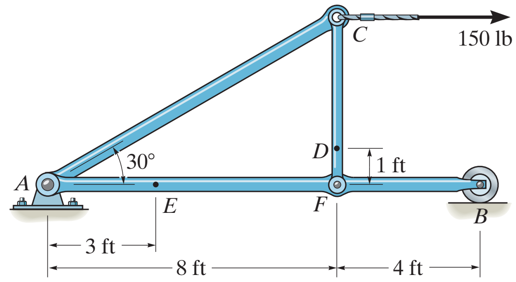
$$x = 19.82\left(\sinh^{-1}\left(\frac{34.33}{19.82}\right)\right) = 26.10\text{ m}$$

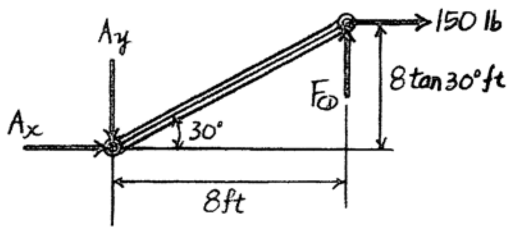
Using Eq. (4),

$$y = 19.82\left(\cosh\left(\frac{26.10}{19.82}\right) - 1\right)$$

$$d = y = 19.8\text{ m} \qquad \textbf{Ans}$$

•7–125. Determine the internal normal force, shear force, and moment at points D and E of the frame.

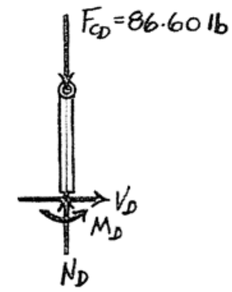




$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(8) - 150(8\tan 30°) = 0$

$F_{CD} = 86.60 \text{ lb}$

Since member CF is a two-force member



$V_D = M_D = 0$ **Ans**

$N_D = F_{CD} = 86.6 \text{ lb}$ **Ans**

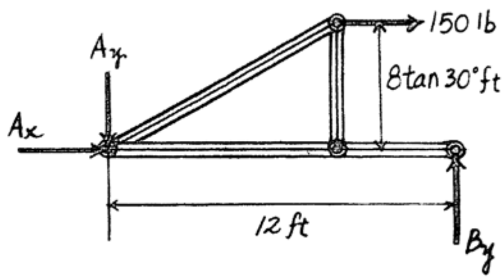
$\circlearrowleft + \Sigma M_A = 0; \quad B_y(12) - 150(8\tan 30°) = 0$

$B_y = 57.735 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad N_E = 0$ **Ans**

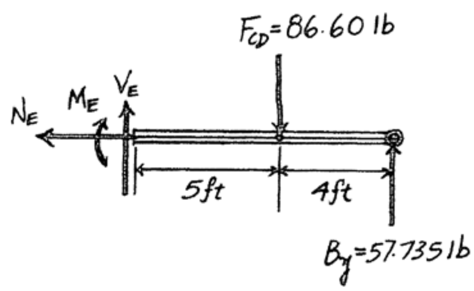
$+\uparrow \Sigma F_y = 0; \quad V_E + 57.735 - 86.60 = 0$

$V_E = 28.9 \text{ lb}$ **Ans**

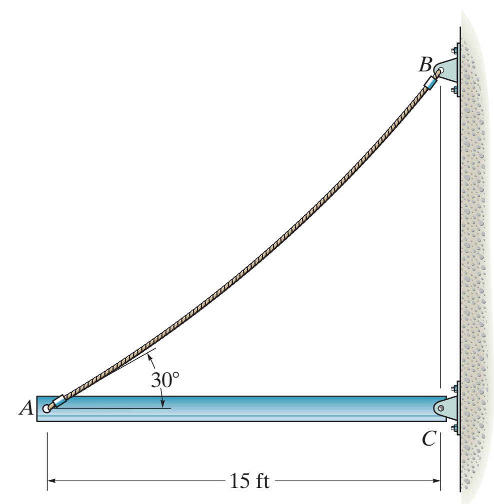


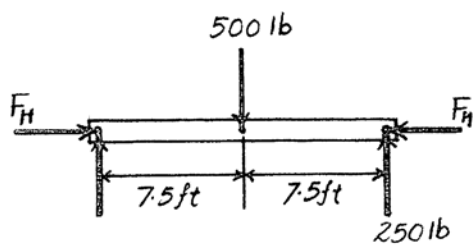
$\circlearrowleft + \Sigma M_E = 0; \quad 57.735(9) - 86.60(5) - M_E = 0$

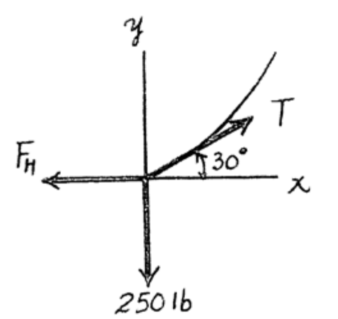
$M_E = 86.6 \text{ lb} \cdot \text{ft}$ **Ans**



7–126. The uniform beam weighs 500 lb and is held in the horizontal position by means of cable *AB*, which has a weight of 5 lb/ft. If the slope of the cable at *A* is 30°, determine the length of the cable.







$$T = \frac{250}{\sin 30^\circ} = 500 \text{ lb}$$

$$F_H = 500 \cos 30^\circ = 433.0 \text{ lb}$$

From Example 7-13

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0 s + C_1)$$

At $s = 0$, $\frac{dy}{dx} = \tan 30^\circ = 0.577$

$$\therefore C_1 = 433.0\,(0.577) = 250$$

$$x = \frac{F_H}{w_0}\left\{\sinh^{-1}\left[\frac{1}{F_H}(w_0 s + C_1)\right] + C_2\right\}$$

$$= \frac{433.0}{5}\left\{\sinh^{-1}\left[\frac{1}{433.0}(5s + 250)\right] + C_2\right\}$$

$s = 0$ at $x = 0$, $\quad C_2 = -0.5493$

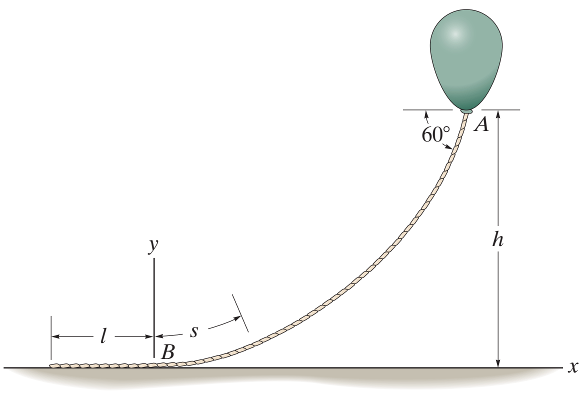
Thus,

$$x = 86.6\left\{\sinh^{-1}\left[\frac{1}{433.0}(5s + 250)\right] - 0.5493\right\}$$

When $x = 15$ ft,

$$s = 18.2 \text{ ft} \qquad \textbf{Ans}$$

7–127. The balloon is held in place using a 400-ft cord that weighs 0.8 lb/ft and makes a 60° angle with the horizontal. If the tension in the cord at point *A* is 150 lb, determine the length of the cord, *l*, that is lying on the ground and the height *h*. *Hint*: Establish the coordinate system at *B* as shown.



Deflection Curve of The Cable:

$$x = \int \frac{ds}{\left[1 + \left(1/F_H^2\right)\left(\int w_0\, ds\right)^2\right]^{\frac{1}{2}}} \qquad \text{where } w_0 = 0.8 \text{ lb/ft}$$

Performing the integration yields

$$x = \frac{F_H}{0.8}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0.8s + C_1)\right] + C_2\right\} \qquad [1]$$

From Eq. 7 – 14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds = \frac{1}{F_H}(0.8s + C_1) \qquad [2]$$

Boundary Conditions:

$\frac{dy}{dx} = 0$ at $s = 0$. From Eq. [2] $\quad 0 = \frac{1}{F_H}(0 + C_1) \qquad C_1 = 0$

Then, Eq. [2] becomes

$$\frac{dy}{dx} = \tan\theta = \frac{0.8s}{F_H} \qquad [3]$$

$s = 0$ at $x = 0$ and use the result $C_1 = 0$. From Eq. [1]

$$x = \frac{F_H}{3}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0 + 0)\right] + C_2\right\} \qquad C_2 = 0$$

Rearranging Eq. [1], we have

$$s = \frac{F_H}{0.8}\sinh\left(\frac{0.8}{F_H}x\right) \qquad [4]$$

Substituting Eq. [4] into [3] yields

$$\frac{dy}{dx} = \sinh\left(\frac{0.8}{F_H}x\right)$$

Performing the integration

$$y = \frac{F_H}{0.8}\cosh\left(\frac{0.8}{F_H}x\right) + C_3 \qquad [5]$$

$y = 0$ at $x = 0$. From Eq. [5] $\quad 0 = \frac{F_H}{0.8}\cosh 0 + C_3$, thus, $C_3 = -\frac{F_H}{0.8}$

Then, Eq. [5] becomes

$$y = \frac{F_H}{0.8}\left[\cosh\left(\frac{0.8}{F_H}x\right) - 1\right] \qquad [6]$$

The tension developed at the end of the cord is $T = 150$ lb and $\theta = 60°$. Thus

$$T = \frac{F_H}{\cos\theta} \qquad 150 = \frac{F_H}{\cos 60°} \qquad F_H = 75.0 \text{ lb}$$

From Eq. [3]

$$\frac{dy}{dx} = \tan 60° = \frac{0.8s}{75} \qquad s = 162.38 \text{ ft}$$

Thus, $\quad l = 400 - 162.38 = 238$ ft **Ans**

Substituting $s = 162.38$ ft into Eq. [4],

$$162.38 = \frac{75}{0.8}\sinh\left(\frac{0.8}{75}x\right)$$
$$x = 123.46 \text{ ft}$$

$y = h$ at $x = 123.46$ ft. From Eq. [6]

$$h = \frac{75.0}{0.8}\left[\cosh\left[\frac{0.8}{75.0}(123.46)\right] - 1\right] = 93.75 \text{ ft} \qquad \textbf{Ans}$$