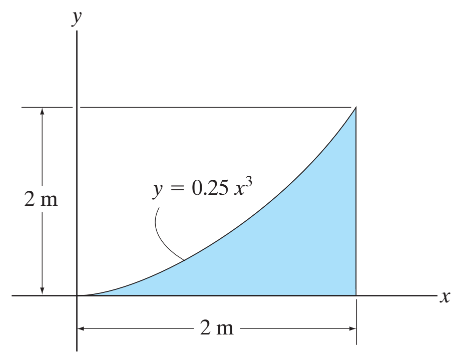
•10–1. Determine the moment of inertia of the area about the x axis.



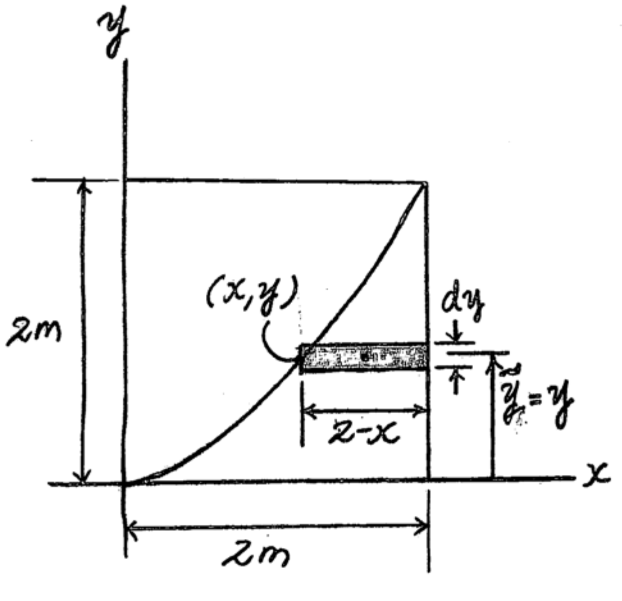
The area of the rectangular differential element in Fig. a is $dA = (2-x)\,dy$. Since $x = (4y)^{1/3}$ then $dA = \left[2-(4y)^{1/3}\right]dy$.

$$I_x = \int_A y^2 dA$$

$$= \int_0^{2\text{ m}} y^2\left[2-(4y)^{1/3}\right]dy$$

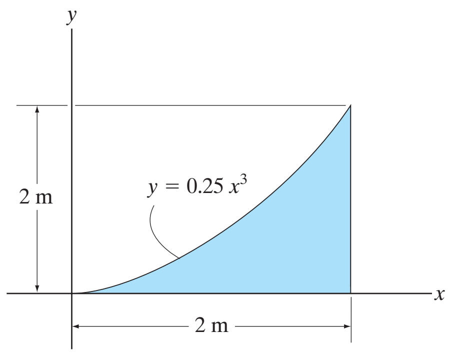
$$= \int_0^{2\text{ m}} \left(2y^2 - 4^{1/3}y^{7/3}\right)dy$$

$$= \left[\frac{2y^3}{3} - \frac{3}{10}\left(4^{1/3}\right)y^{10/3}\right]\Bigg|_0^{2\text{ m}} = 0.533\text{ m}^4 \qquad \textbf{Ans.}$$



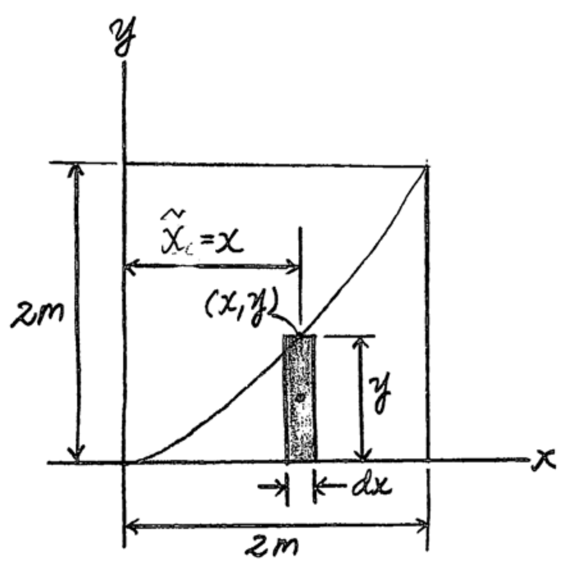
(a)

10–2. Determine the moment of inertia of the area about the *y* axis.



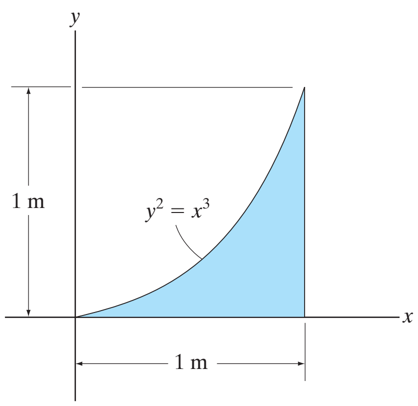
The area of the rectangular differential element in Fig. *a* is $dA = y\,dx = \dfrac{x^3}{4}\,dx$.

$$I_y = \int_A x^2\,dA$$
$$= \int_0^{2\text{ m}} x^2\left(\frac{x^3}{4}\right)dx$$
$$= \int_0^{2\text{ m}} \frac{x^5}{4}\,dx$$
$$= \left(\frac{x^6}{24}\right)\Bigg|_0^{2\text{ m}} = 2.67\text{ m}^4$$ **Ans.**



(a)

10–3. Determine the moment of inertia of the area about the x axis.



The area of the rectangular differential element in Fig. a is $dA = (1-x)\,dy$. Since $x = y^{2/3}$, then $dA = (1-y^{2/3})\,dy$.

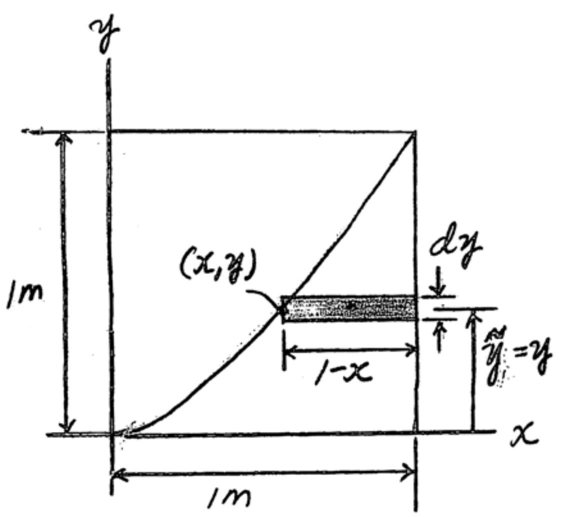
$$I_x = \int_A y^2 dA$$

$$= \int_0^{1\text{ m}} y^2\left[1-y^{2/3}\right]dy$$

$$= \int_0^{1\text{ m}} (y^2 - y^{8/3})\,dy$$

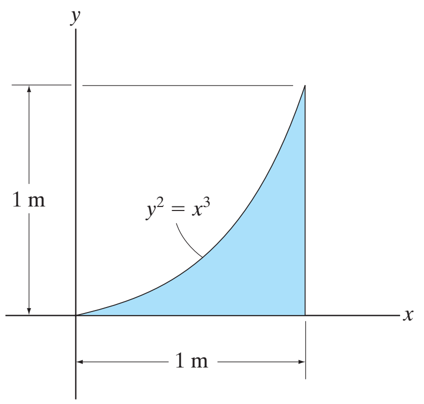
$$= \left(\frac{y^3}{3} - \frac{3}{11}y^{11/3}\right)\Bigg|_0^{1\text{ m}} = 0.0606\text{ m}^4$$

Ans.



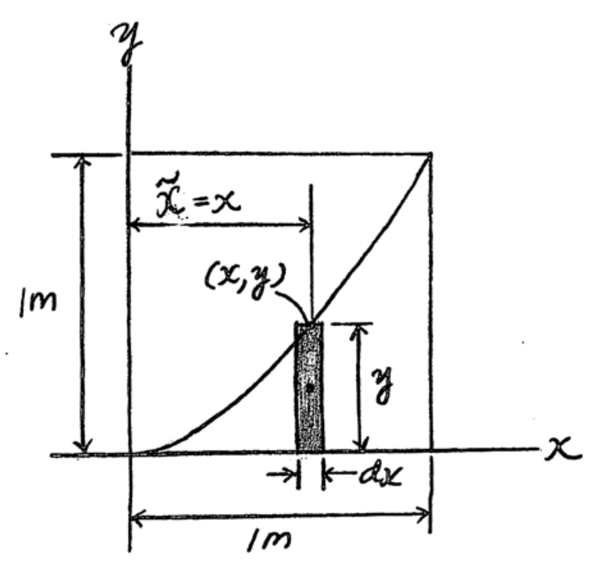
(a)

***10–4.** Determine the moment of inertia of the area about the y axis.



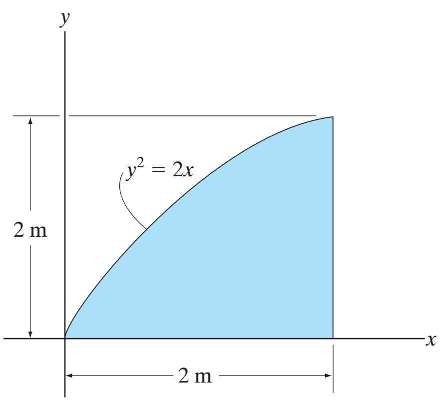
The area of the rectangular differential element in Fig. *a* is $dA = y\,dx = x^{3/2}\,dx$.

$$I_y = \int_A x^2 dA$$
$$= \int_0^{1\text{ m}} x^2\left(x^{3/2}\right)dx$$
$$= \int_0^{1\text{ m}} x^{7/2}\,dx$$
$$= \left(\frac{2}{9}x^{9/2}\right)\Bigg|_0^{1\text{ m}} = 0.222\text{ m}^4 \qquad \textbf{Ans.}$$



(a)

•**10–5.** Determine the moment of inertia of the area about the x axis.



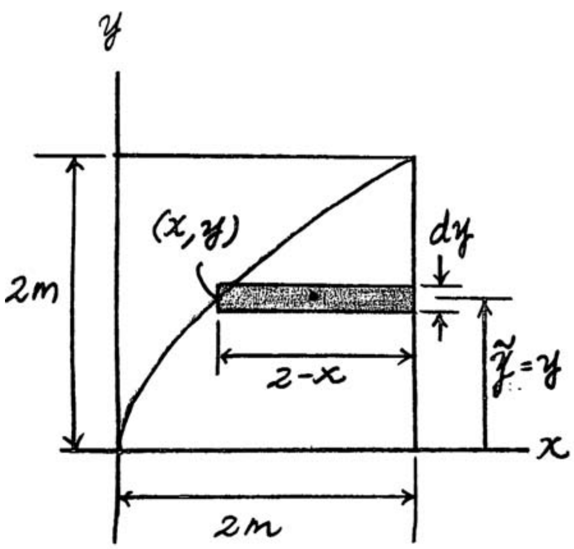
The area of the rectangular differential element in Fig. *a* is $dA = (2 - x)\,dy$. Since $x = \frac{y^2}{2}$, then $dA = \left(2 - \frac{y^2}{2}\right)dy$.

$$I_x = \int_A y^2 dA$$

$$= \int_0^{2\,\text{m}} y^2\left(2 - \frac{y^2}{2}\right)dy$$

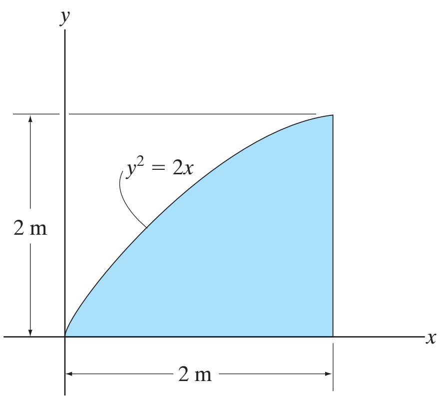
$$= \int_0^{2\,\text{m}} \left(2y^2 - \frac{y^4}{2}\right)dy$$

$$= \left(\frac{2}{3}y^3 - \frac{y^5}{10}\right)\Bigg|_0^{2\,\text{m}} = 2.13\ \text{m}^4$$ **Ans.**



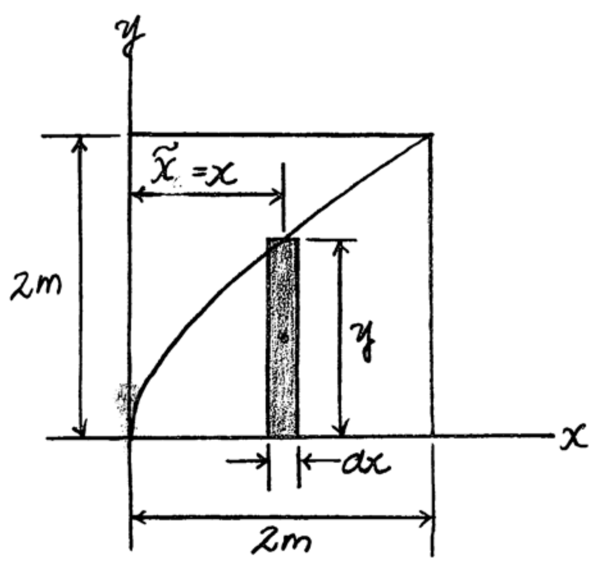
(a)

10–6. Determine the moment of inertia of the area about the y axis.



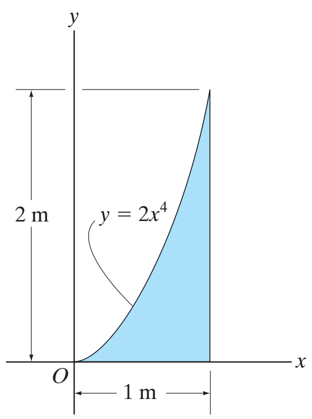
The area of the rectangular differential element in Fig. a is $dA = y\,dx = (2x)^{1/2}\,dx$.

$$I_y = \int_A x^2 dA$$
$$= \int_0^{2\,\text{m}} x^2 (2x)^{1/2}\, dA$$
$$= \int_0^{2\,\text{m}} \sqrt{2}x^{5/2}\, dx$$
$$= \left[\sqrt{2}\left(\frac{2}{7}x^{7/2}\right)\right]_0^{2\,\text{m}} = 4.57\ \text{m}^4 \qquad \textbf{Ans.}$$



(a)

10–7. Determine the moment of inertia of the area about the x axis.

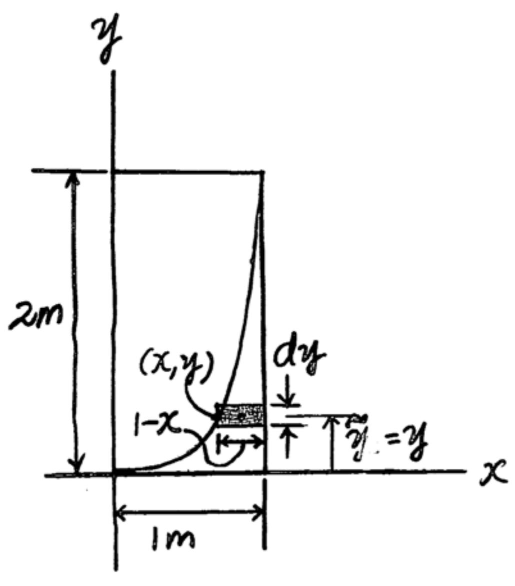


The moment of inertia of the area about the x axis will be determined using the rectangular differential element in Fig. a. This area is

$$dA = (1 - x)\,dy = \left[1 - \left(\frac{y}{2}\right)^{1/4}\right]dy$$

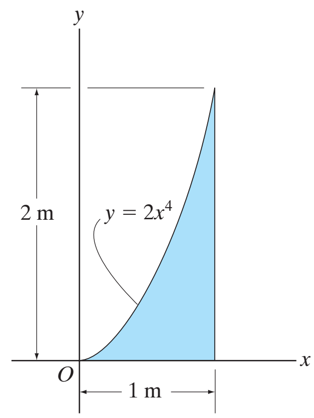
$$I_x = \int_A y^2 dA = \int_0^{2\text{ m}} y^2\left[1 - \left(\frac{y}{2}\right)^{1/4}\right]dy = \int_0^{2\text{ m}} \left[y^2 - \left(\frac{1}{2}\right)^{1/4} y^{9/4}\right]dy$$

$$= \left[\frac{y^3}{3} - \left(\frac{1}{2}\right)^{1/4}\left(\frac{4}{13}\right)y^{13/4}\right]\Bigg|_0^{2\text{ m}} = 0.205\text{ m}^4 \qquad \textbf{Ans.}$$



(a)

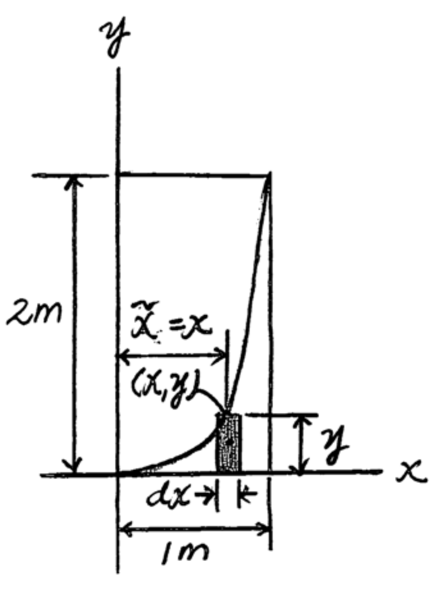
***10–8.** Determine the moment of inertia of the area about the y axis.



The moment of inertia of the area about the y axis will be determined using the rectangular differential element in Fig. *a*. This area is

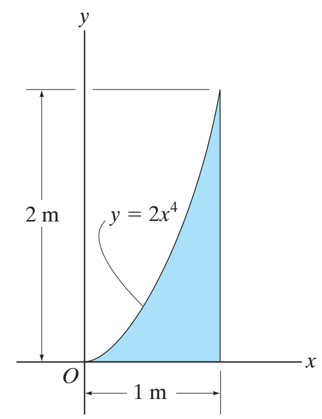
$$dA = y\,dx = 2x^4\,dx$$

$$I_y = \int_A x^2 dA = \int_0^{1\text{ m}} x^2\left(2x^4 dx\right) = \int_0^{1\text{ m}} 2x^6\,dx = \left(\frac{2}{7}x^7\right)\Bigg|_0^{1\text{ m}} = 0.286\text{ m}^4 \quad \textbf{Ans.}$$



(b)

•**10–9.** Determine the polar moment of inertia of the area about the z axis passing through point O.



The moment of inertia of the area about the x and y axes will be determined using the rectangular differential element in Figs. a and b. The area of these two elements are

$$dA = (1 - x)\,dy = \left[1 - \left(\frac{y}{2}\right)^{1/4}\right] dy \text{ and } dA = y\,dx = 2x^4\,dx.$$

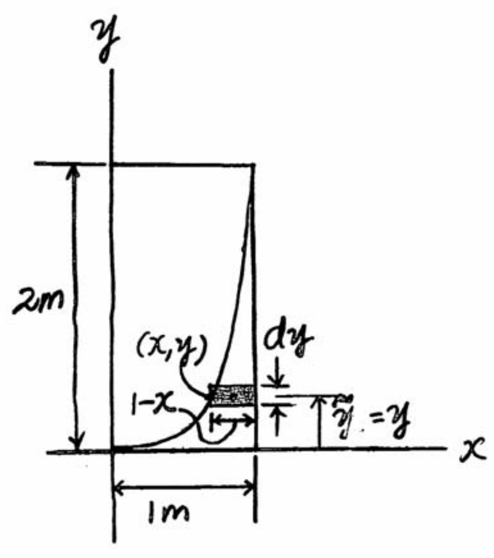
$$I_x = \int_A y^2 dA = \int_0^{2\text{ m}} y^2\left[1 - \left(\frac{y}{2}\right)^{1/4}\right] dy = \int_0^{2\text{ m}} \left[y^2 - \left(\frac{1}{2}\right)^{1/4} y^{9/4}\right] dy$$

$$= \left[\frac{y^3}{3} - \left(\frac{1}{2}\right)^{1/4}\left(\frac{4}{13}\right) y^{13/4}\right]\Bigg|_0^{2\text{ m}} = 0.2051\text{ m}^4$$

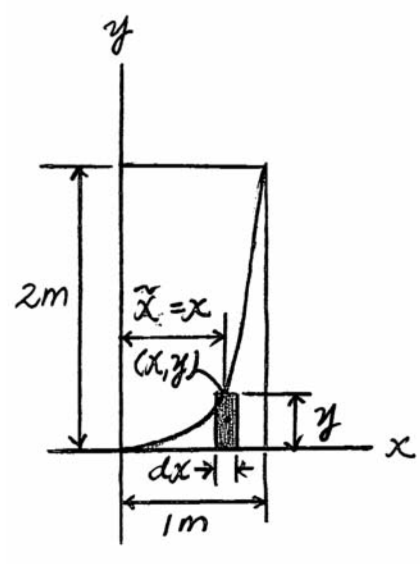
$$I_y = \int_A x^2 dA = \int_0^{1\text{ m}} x^2\left(2x^4 dx\right) = \int_0^{1\text{ m}} 2x^6\,dx = \left(\frac{2}{7}x^7\right)\Bigg|_0^{1\text{ m}} = 0.2857\text{ m}^4$$

Thus, the polar moment of inertia of the area about the z axis is

$$J_O = I_x + I_y = 0.2051 + 0.2857 = 0.491\text{ m}^4$$ **Ans.**

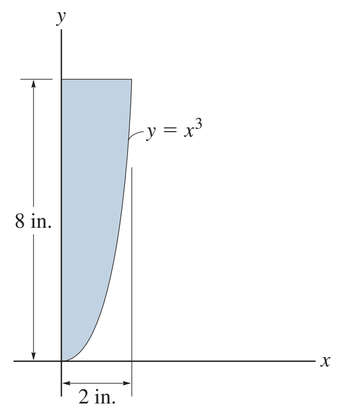


(a)



(b)

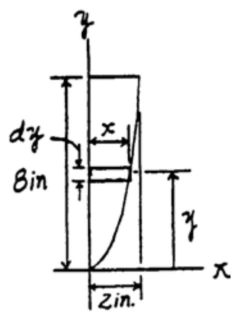
10–10. Determine the moment of inertia of the area about the x axis.



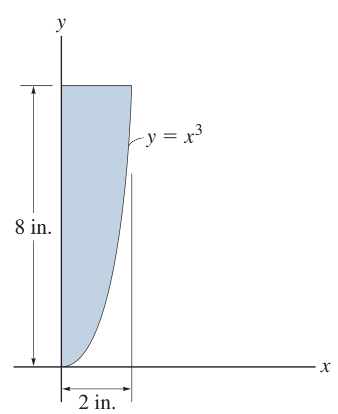
Differential Element : Here, $x = y^{\frac{1}{3}}$. The area of the differential element parallel to x axis is $dA = x\,dy = y^{\frac{1}{3}}dy$.

Moment of Inertia : Applying Eq. 10–1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^{8\text{ in.}} y^2\left(y^{\frac{1}{3}}\right)dy$$
$$= \left[\frac{3}{10}y^{\frac{10}{3}}\right]\Big|_0^{8\text{ in.}}$$
$$= 307\text{ in}^4 \qquad \textbf{Ans}$$



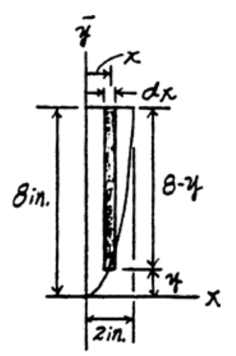
10–11. Determine the moment of inertia of the area about the y axis.



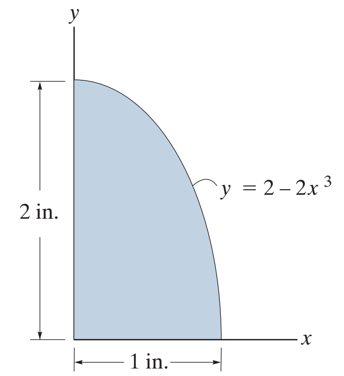
Differential Element : The area of the differential element parallel to y axis is $dA = (8 - y)\,dx = \left(8 - x^3\right)dx$.

Moment of Inertia : Applying Eq. 10–1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^{2\text{ in.}} x^2\left(8 - x^3\right)dx$$
$$= \left(\frac{8}{3}x^3 - \frac{1}{6}x^6\right)\Big|_0^{2\text{ in.}}$$
$$= 10.7\text{ in}^4 \qquad \textbf{Ans}$$

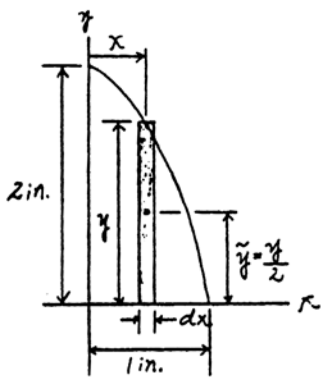


***10–12.** Determine the moment of inertia of the area about the x axis.



Differential Element : The area of the differential element parallel to y axis is $dA = y\,dx$. The moment of inertia of this element about x axis is

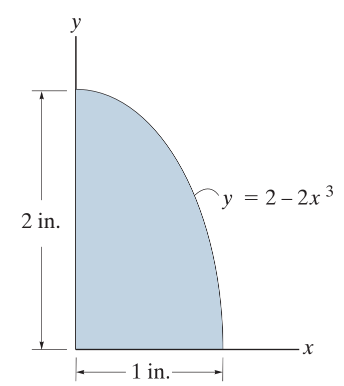
$$dI_x = d\bar{I}_{x'} + dA\,\tilde{y}^2$$
$$= \frac{1}{12}(dx)y^3 + y\,dx\left(\frac{y}{2}\right)^2$$
$$= \frac{1}{3}\left(2-2x^3\right)^3 dx$$
$$= \frac{1}{3}\left(-8x^9+24x^6-24x^3+8\right)dx$$



Moment of Inertia : Performing the integration, we have

$$I_x = \int dI_x = \frac{1}{3}\int_0^{1\text{in.}}\left(-8x^9+24x^6-24x^3+8\right)dx$$
$$= \frac{1}{3}\left(-\frac{4}{5}x^{10}+\frac{24}{7}x^7-6x^4+8x\right)\Big|_0^{1\text{in.}}$$
$$= 1.54\ \text{in}^4 \qquad \textbf{Ans}$$

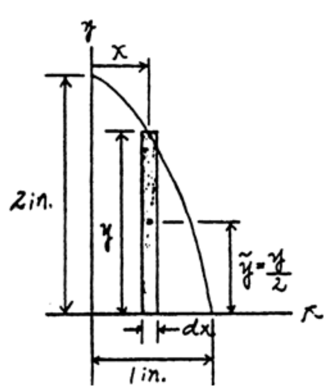
•10–13. Determine the moment of inertia of the area about the y axis.



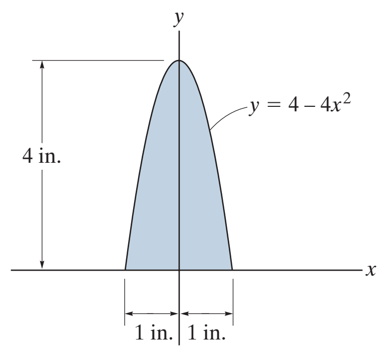
Differential Element : The area of the differential element parallel to yaxis is $dA = y\,dx = \left(2-2x^3\right)dx$.

Moment of Inertia : Applying Eq. 10–1and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^{1\text{in.}} x^2\left(2-2x^3\right)dx$$
$$= \left[\frac{2}{3}x^3 - \frac{1}{3}x^6\right]\Big|_0^{1\text{in.}}$$
$$= 0.333\ \text{in}^4 \qquad \textbf{Ans}$$

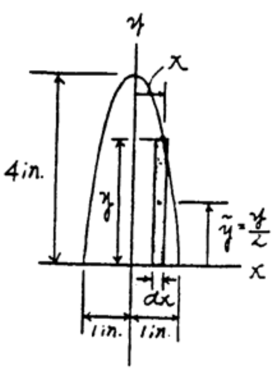


10–14. Determine the moment of inertia of the area about the x axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of dx, and (b) having a thickness of dy.



a) *Differential Element :* The area of the differential element parallel to y axis is $dA = y\,dx$. The moment of inertia of this element about x axis is

$$
\begin{aligned}
dI_x &= d\bar{I}_{x'} + dA\,\tilde{y}^2 \\
&= \frac{1}{12}(dx)\,y^3 + y\,dx\left(\frac{y}{2}\right)^2 \\
&= \frac{1}{3}\left(4-4x^2\right)^3 dx \\
&= \frac{1}{3}\left(-64x^6 + 192x^4 - 192x^2 + 64\right) dx
\end{aligned}
$$

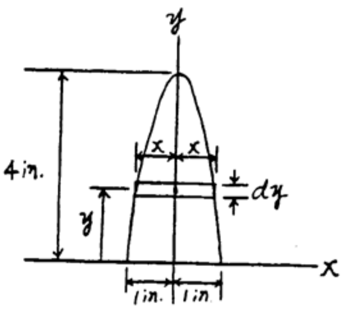


(a)

Moment of Inertia : Performing the integration, we have

$$
\begin{aligned}
I_x = \int dI_x &= \frac{1}{3}\int_{-1\text{in.}}^{1\text{in.}} \frac{1}{3}\left(-64x^6 + 192x^4 - 192x^2 + 64\right) dx \\
&= \frac{1}{3}\left(-\frac{64}{7}x^7 + \frac{192}{5}x^5 - \frac{192}{3}x^3 + 64x\right)\Bigg|_{-1\text{in.}}^{1\text{in.}} \\
&= 19.5\ \text{in}^4
\end{aligned}
$$

Ans



(b)

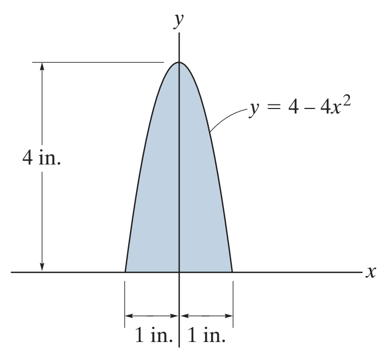
b) *Differential Element :* Here, $x = \frac{1}{2}\sqrt{4-y}$. The area of the differential element parallel to x axis is $dA = 2x\,dy = \sqrt{4-y}\,dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

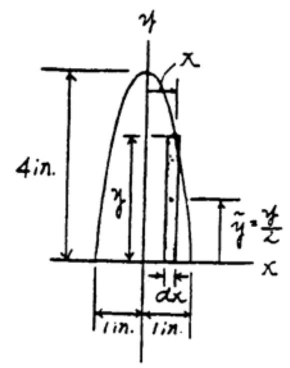
$$
\begin{aligned}
I_x &= \int_A y^2\,dA \\
&= \int_0^{4\text{in.}} y^2\sqrt{4-y}\,dy \\
&= \left[-\frac{2y^2}{3}(4-y)^{\frac{3}{2}} - \frac{8y}{15}(4-y)^{\frac{5}{2}} - \frac{16}{105}(4-y)^{\frac{7}{2}}\right]\Bigg|_0^{4\text{in.}} \\
&= 19.5\ \text{in}^4
\end{aligned}
$$

Ans

10–15. Determine the moment of inertia of the area about the y axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of dx, and (b) having a thickness of dy.



a) ***Differential Element :*** The area of the differential element parallel to yaxis is $dA = ydx = \left(4-4x^2\right) dx$.



Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

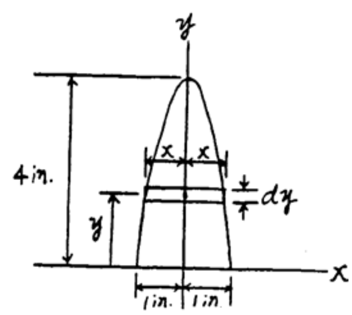
$$I_y = \int_A x^2 dA = \int_{-1\text{ in.}}^{1\text{ in.}} x^2\left(4-4x^2\right) dx$$
$$= \left[\frac{4}{3}x^3 - \frac{4}{5}x^5\right]\Bigg|_{-1\text{ in.}}^{1\text{ in.}}$$
$$= 1.07 \text{ in}^4 \qquad \textbf{Ans}$$

b) ***Differential Element :*** Here, $x = \frac{1}{2}\sqrt{4-y}$. The moment of inertia of the differential element about y axis is

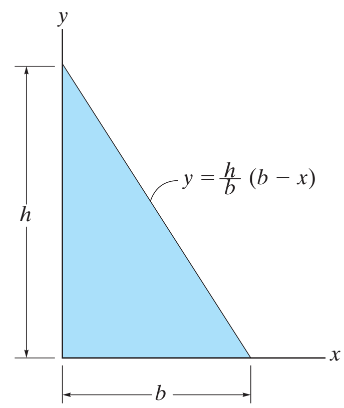
$$dI_y = \frac{1}{12}(dy)\left(2x\right)^3 = \frac{2}{3}x^3 dy = \frac{1}{12}(4-y)^{\frac{3}{2}} dy$$

Moment of Inertia : Performing the integration, we have

$$I_y = \int dI_y = \frac{1}{12}\int_0^{4\text{ in.}} (4-y)^{\frac{3}{2}} dy$$
$$= \frac{1}{12}\left[-\frac{2}{5}(4-y)^{\frac{5}{2}}\right]\Bigg|_0^{4\text{ in.}}$$
$$= 1.07 \text{ in}^4 \qquad \textbf{Ans}$$

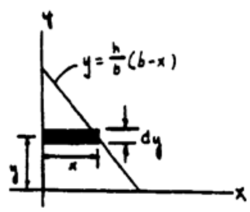


***10–16.** Determine the moment of inertia of the triangular area about the x axis.



Area of the differential element (shaded) $dA = x\,dy$ where $x = b - \frac{b}{h}y$, hence, $dA = x\,dy = \left(b - \frac{b}{h}y\right)dy$.

$$I_x = \int_A y^2\,dA = \int_0^h y^2\left(b - \frac{b}{h}y\right)dy$$

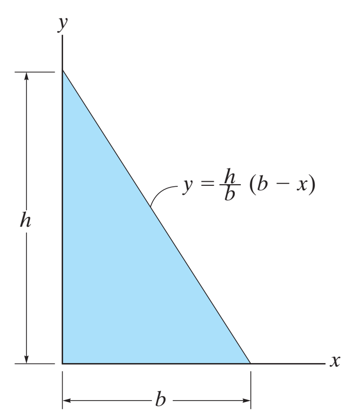


$$= \int_0^h \left(by^2 - \frac{b}{h}y^3\right)dy$$

$$= \frac{b}{3}y^3 - \frac{b}{4h}y^4\Big|_0^h$$

$$= \frac{1}{12}bh^3 \qquad \textbf{Ans}$$

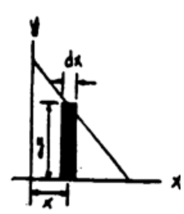
•10–17. Determine the moment of inertia of the triangular area about the y axis.



Area of the differential element (shaded) $dA = y\,dx$ where $y = h - \frac{h}{b}x$, hence, $dA = y\,dx = \left(h - \frac{h}{b}x\right)dx$.

$$I_y = \int_A x^2\,dA = \int_0^b x^2\left(h - \frac{h}{b}x\right)dx$$

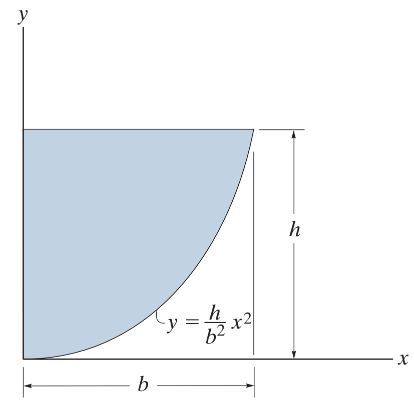
$$= \int_0^b \left(hx^2 - \frac{h}{b}x^3\right)dx$$



$$= \frac{h}{3}x^3 - \frac{h}{4b}x^4\Big|_0^b$$

$$= \frac{1}{12}hb^3 \qquad \textbf{Ans}$$

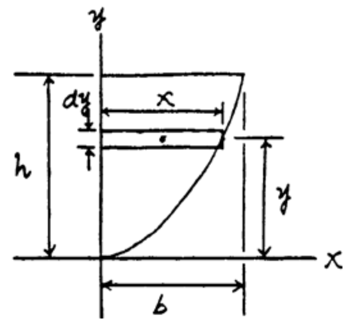
10–18. Determine the moment of inertia of the area about the x axis.



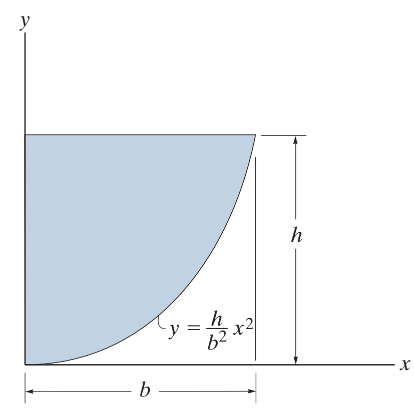
Differential Element : Here, $x = \frac{b}{\sqrt{h}}y^{\frac{1}{2}}$. The area of the differential element parallel to x axis is $dA = x\,dy = \left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^h y^2\left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$$
$$= \frac{b}{\sqrt{h}}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Big|_0^h$$
$$= \frac{2}{7}bh^3 \qquad \textbf{Ans}$$



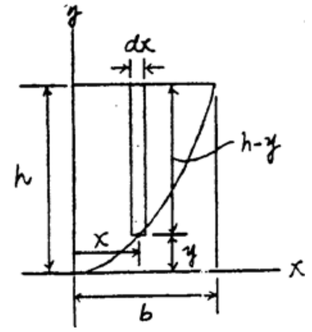
10–19. Determine the moment of inertia of the area about the y axis.



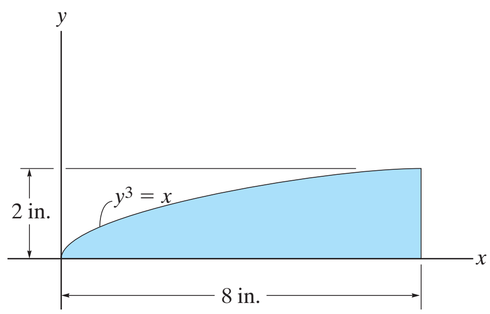
Differential Element : The area of the differential element parallel to y axis is $dA = (h - y)\,dx = \left(h - \frac{h}{b^2}x^2\right)dx$.

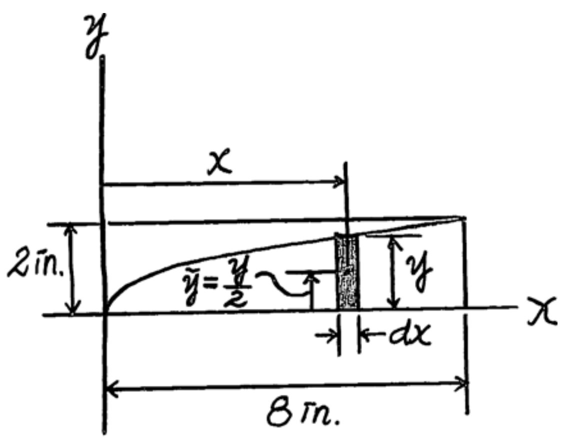
Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^b x^2\left(h - \frac{h}{b^2}x^2\right)dx$$
$$= \left(\frac{h}{3}x^3 - \frac{h}{5b^2}x^5\right)\Big|_0^b$$
$$= \frac{2}{15}hb^3 \qquad \textbf{Ans}$$



***10–20.** Determine the moment of inertia of the area about the x axis.





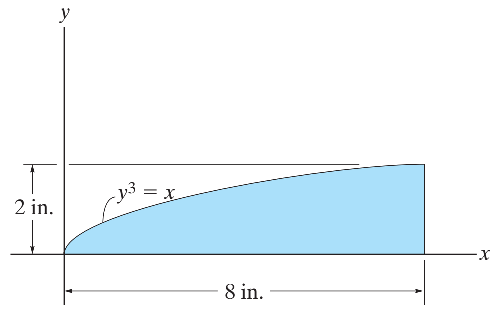
$$dI_x = dI_{\bar{x}'} + dA\,\tilde{y}^2$$

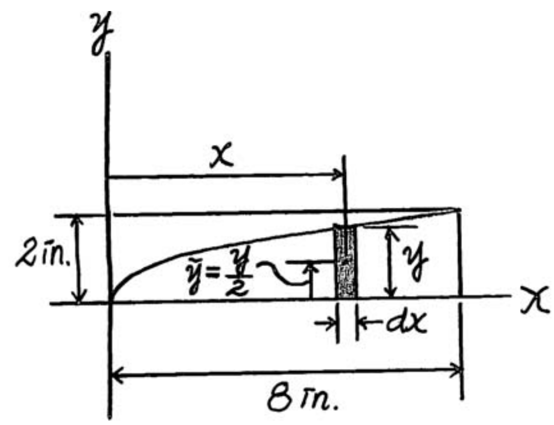
$$= \frac{1}{12}dx\,y^3 + y\,dx\left(\frac{y}{2}\right)^2$$

$$= \frac{1}{3}y^3\,dx$$

$$I_x = \int_A dI_x = \int_0^8 \frac{1}{3}y^3\,dx = \int_0^8 \frac{1}{3}x\,dx = \left.\frac{x^2}{6}\right]_0^8 = 10.7\text{ in}^4 \qquad \textbf{Ans}$$

•10–21. Determine the moment of inertia of the area about the y axis.





$$dI_y = dI_{\bar{y}} + dA\,\tilde{x}^2$$

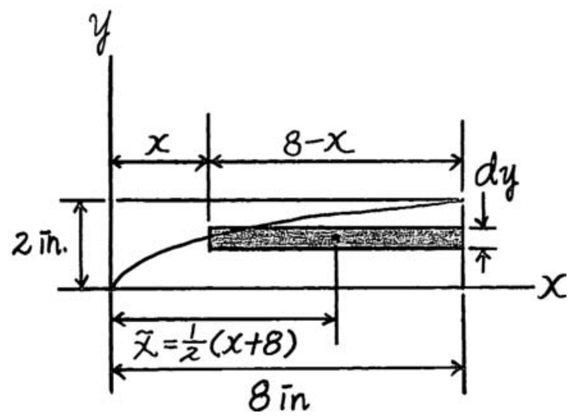
$$= \frac{1}{12}dy\,(8 - y^3)^3 + (8 - y^3)\,dy\left(y^3 + \frac{1}{2}(8 - y^3)\right)^2$$

$$= \left[\frac{1}{12}(8 - y^3)^3 + (8 - y^3)\left(\frac{1}{4}\right)(y^3 + 8)^2\right]dy$$

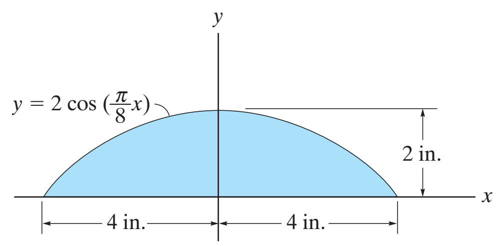
$$I_y = \int_A dI_y = \int_0^2 \left[\frac{1}{12}(8 - y^3)^3 + (8 - y^3)\left(\frac{1}{4}\right)(y^3 + 8)^2\right]dy = 307\ \text{in}^4 \qquad \text{Ans}$$

Also,

$$I_y = \int_A x^2\,dA = \int_A x^2 y\,dx = \int_0^8 x^{\frac{7}{3}}\,dx = \frac{3}{10}x^{\frac{10}{3}}\Big]_0^8 = 307\ \text{in}^4 \qquad \text{Ans}$$



10–22. Determine the moment of inertia of the area about the x axis.

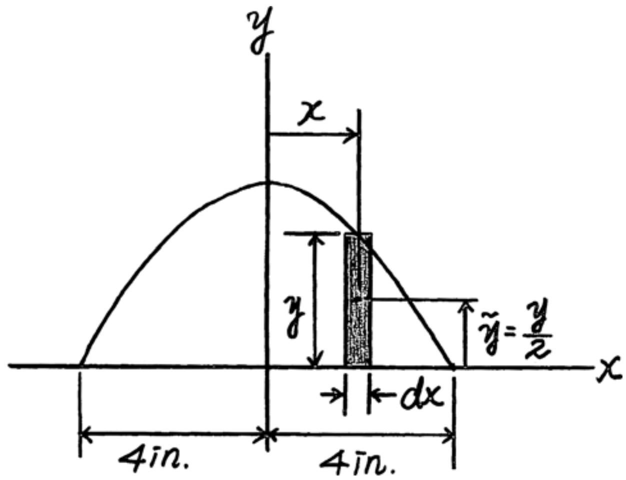


$$dI_x = dI_{\bar{x}'} + dA\,\tilde{y}^2$$

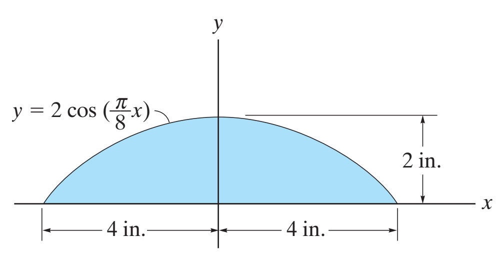
$$= \frac{1}{12}dx\,y^3 + y\,dx\left(\frac{y}{2}\right)^2 = \frac{1}{3}y^3\,dx$$

$$I_x = \int_A dI_x = \int_{-4}^{4}\frac{8}{3}\cos^3\left(\frac{\pi}{8}x\right)dx$$

$$= \frac{8}{3}\left[\frac{\sin\left(\frac{\pi}{8}x\right)}{\frac{\pi}{8}} - \frac{\sin^3\left(\frac{\pi}{8}x\right)}{\frac{3\pi}{8}}\right]_{-4}^{4} = \frac{256}{9\pi} = 9.05\text{ in}^4 \quad \textbf{Ans}$$



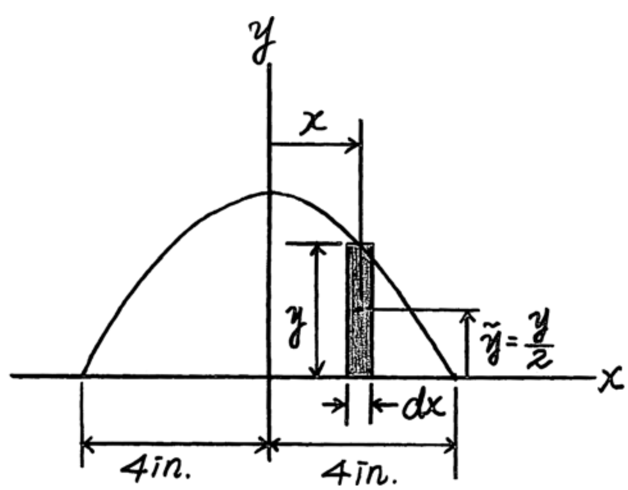
10–23. Determine the moment of inertia of the area about the y axis.



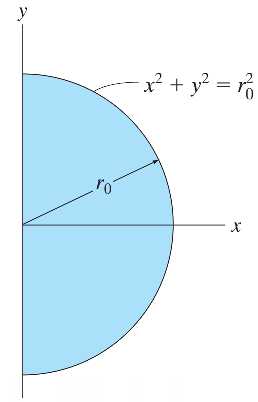
$$I_y = \int_A x^2\,dA = \int_{-4}^{4} x^2\,2\cos\left(\frac{\pi}{8}x\right)dx$$

$$= 2\left[\frac{x^2\sin\left(\frac{\pi}{8}x\right)}{\frac{\pi}{8}} + \frac{2x\cos\left(\frac{\pi}{8}x\right)}{\left(\frac{\pi}{8}\right)^2} - \frac{2\sin\left(\frac{\pi}{8}x\right)}{\left(\frac{\pi}{8}\right)^3}\right]_{-4}^{4}$$

$$= 4\left(\frac{128}{\pi} - \frac{1024}{\pi^3}\right) = 30.9\text{ in}^4 \quad \textbf{Ans}$$



***10–24.** Determine the moment of inertia of the area about the x axis.



Differential Element: The area of the differential element shown shaded in Fig. *a* is $dA = (rd\theta)\,dr$.

Moment of Inertia:

$$I_x = \int_A y^2 dA = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^2 \sin^2\theta\,(rd\theta)dr$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3 \sin^2\theta\,dr\,d\theta$$

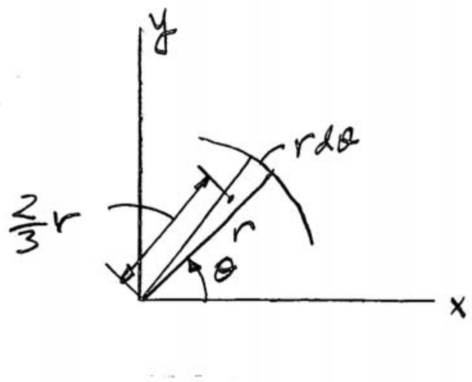
$$= \int_{-\pi/2}^{\pi/2}\left(\frac{r^4}{4}\right)\Bigg|_0^{r_0} \sin^2\theta\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\frac{r_0{}^4}{4}\sin^2\theta\,d\theta$$

However, $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$. Thus,

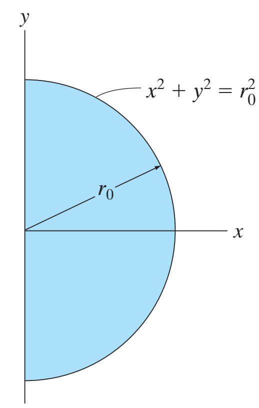
$$I_x = \int_{-\pi/2}^{\pi/2}\frac{r_0{}^4}{4}(1-\cos 2\theta)\,d\theta$$

$$= \frac{r_0{}^4}{8}\left[\theta - \frac{1}{2}\sin 2\theta\right]\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi r_0{}^4}{8}$$ **Ans.**



(a)

•10–25. Determine the moment of inertia of the area about the y axis.



Differential Element: The area of the differential element shown shaded in Fig. *a* is $dA = (rd\theta)\,dr$.

Moment of Inertia:

$$I_y = \int_A x^2 dA = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^2\cos^2\theta(rd\theta)dr$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3\cos^2\theta drd\theta$$

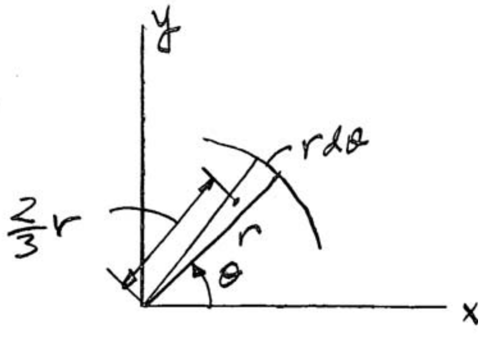
$$= \int_{-\pi/2}^{\pi/2}\left(\frac{r^4}{4}\right)\Bigg|_0^{r_0}\cos^2\theta d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\frac{r_0^4}{4}\cos^2\theta d\theta$$

However, $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$. Thus,

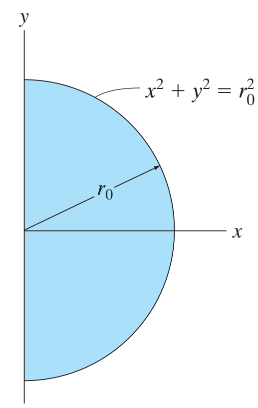
$$I_y = \int_{-\pi/2}^{\pi/2}\frac{r_0^4}{8}(\cos 2\theta + 1)d\theta$$

$$= \frac{r_0^4}{8}\left[\frac{1}{2}\sin 2\theta + \theta\right]\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi r_0^4}{8}$$ **Ans.**



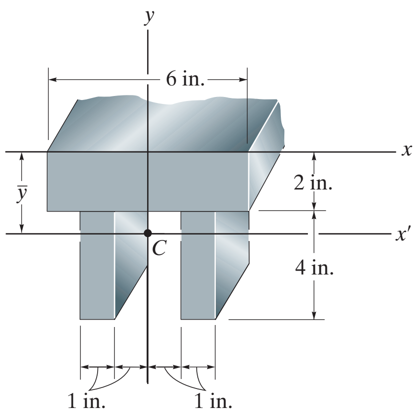
(a)

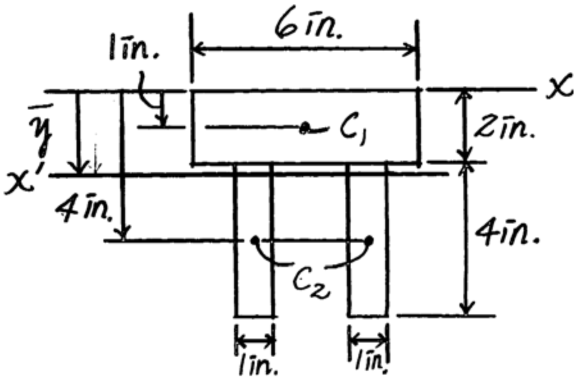
10–26. Determine the polar moment of inertia of the area about the z axis passing through point O.



$$J_O = I_x + I_y = \frac{\pi r_0^4}{8} + \frac{\pi r_0^4}{8} = \frac{\pi r_0^4}{4}$$

10–27. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x' axis.





Centroid :

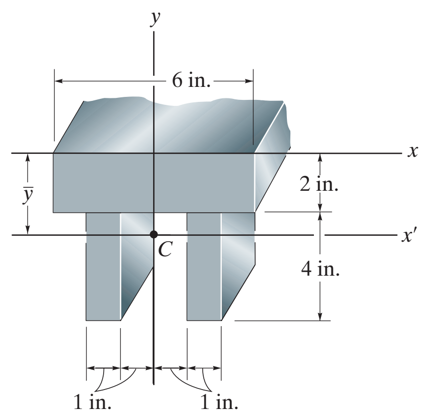
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{1(6)(2) + 2[4(4)(1)]}{6(2) + 2[4(1)]} = 2.20 \text{ in.} \qquad \textbf{Ans}$$

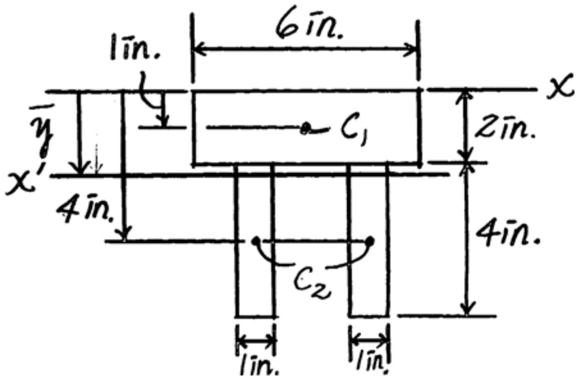
Moment inertia :

$$I_{x'} = \frac{1}{12}(6)(2)^3 + 6(2)(2.20 - 1)^2 + 2\left[\frac{1}{12}(1)(4)^3 + 1(4)(4 - 2.20)^2\right]$$

$$= 57.9 \text{ in}^4 \qquad \textbf{Ans}$$

***10–28.** Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.

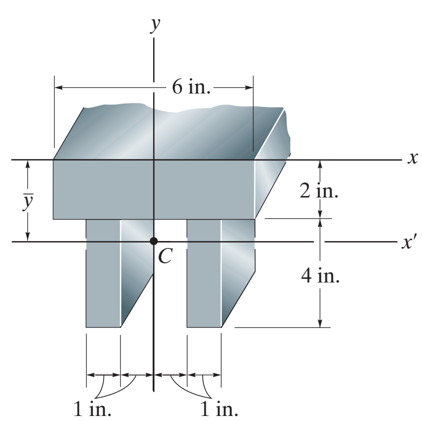


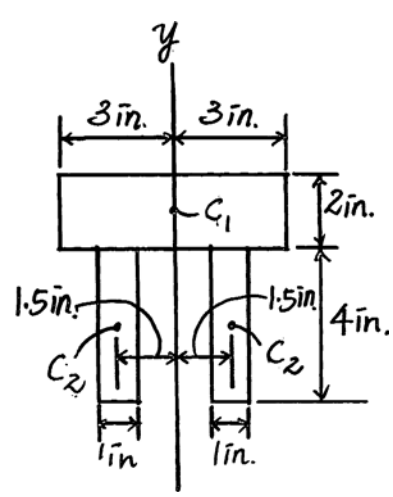


$$I_x = \left[\frac{1}{12}(6)(2)^3 + (6)(2)(1)^2\right] + 2\left[\frac{1}{12}(1)(4)^3 + (4)(1)(4)^2\right]$$

$$= 155 \text{ in.}^4 \qquad \text{Ans}$$

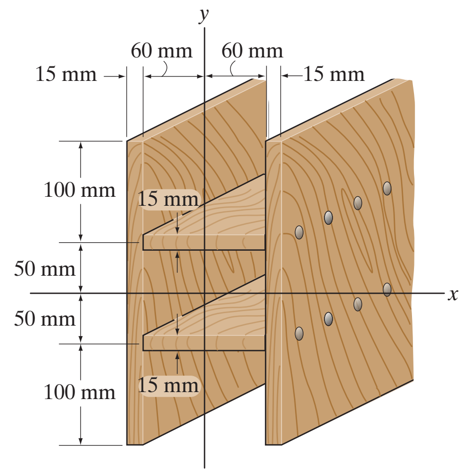
•10–29. Determine the moment of inertia of the beam's cross-sectional area about the *y* axis.





$$I_y = \frac{1}{12}(2)(6)^3 + 2\left[\frac{1}{12}(4)(1)^3 + 1(4)(1.5)^2\right] = 54.7 \text{ in}^4 \qquad \text{Ans}$$

10–30. Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.



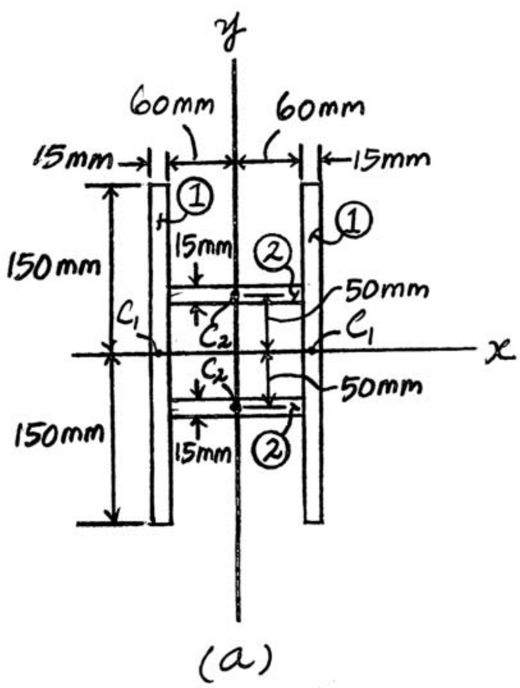
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the *x* axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the *x* axis can be determined using the parallel - axis theorem. Thus,

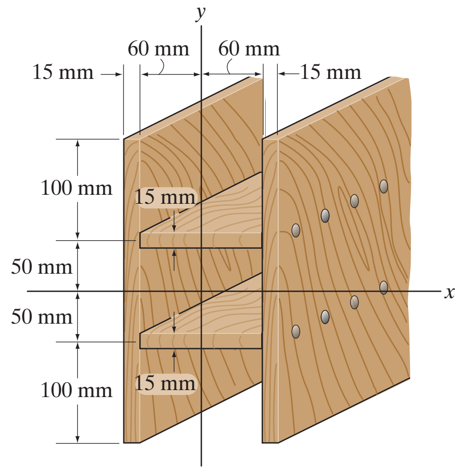
$$I_x = \bar{I}_{x'} + A(d_y)^2$$

$$= \left[2\left(\frac{1}{12}(15)(300^3)\right) + 2(15)(300)(0)^2\right] + \left[2\left(\frac{1}{12}(120)(15^3)\right) + 2(120)(15)(50)^2\right]$$

$$= 67.5(10^6) + 9.0675(10^6) = 76.6(10^6)\ \text{mm}^4$$ **Ans.**



10–31. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



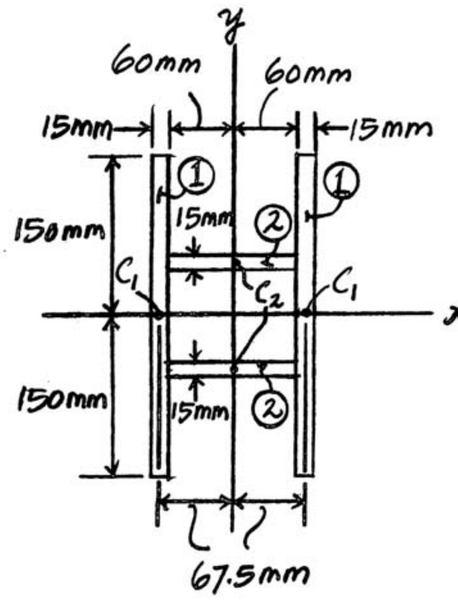
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

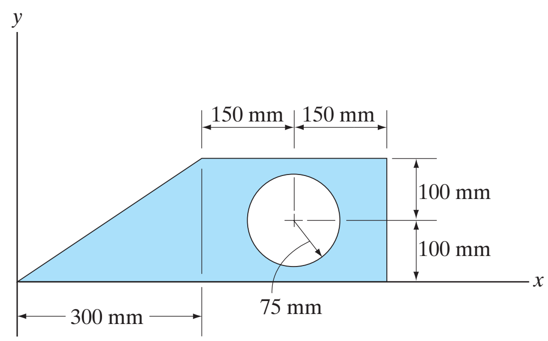
$$= \left[2\left(\frac{1}{12}(300)(15^3)\right) + 2(300)(15)(67.5)^2\right] + \left[2\left(\frac{1}{12}(15)(120^3)\right) + 2(120)(15)(0)^2\right]$$

$$= 41.175(10^6) + 4.32(10^6) = 45.5(10^6)\,\text{mm}^4$$ **Ans.**



(a)

***10–32.** Determine the moment of inertia of the composite area about the x axis.



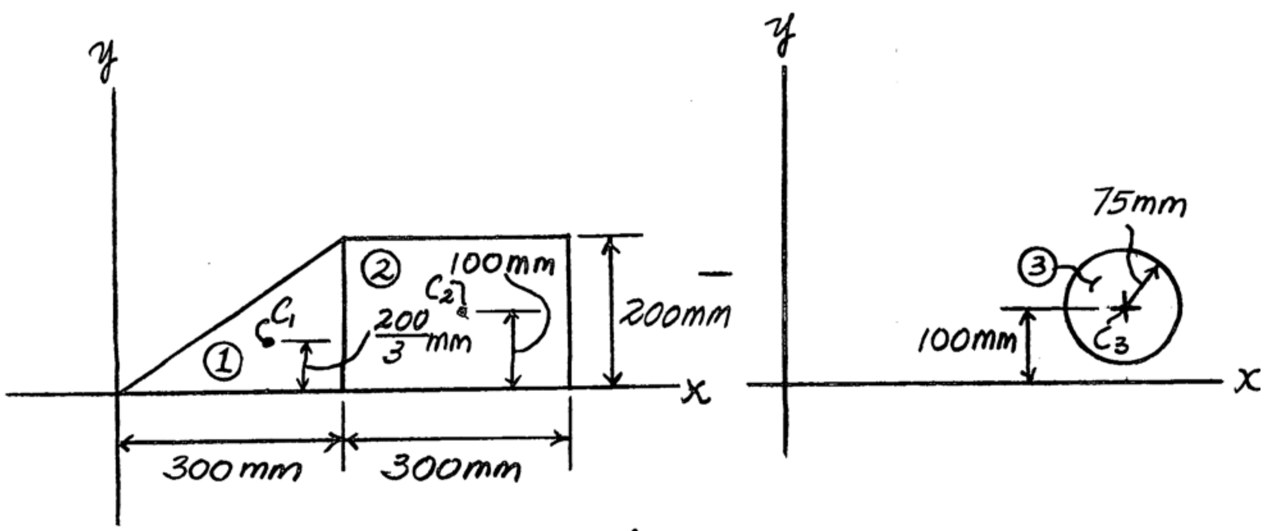
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

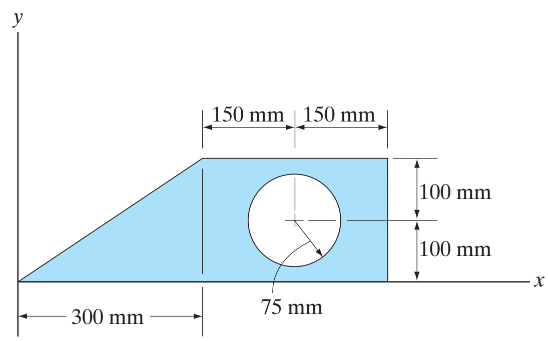
$$= \left[\frac{1}{36}(300)(200^3) + \frac{1}{2}(300)(200)\left(\frac{200}{3}\right)^2\right] + \left[\frac{1}{12}(300)(200^3) + 300(200)(100)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(100)^2\right]$$

$$= 798(10^6)\ \text{mm}^4$$ **Ans.**



(a)

•10–33. Determine the moment of inertia of the composite area about the y axis.



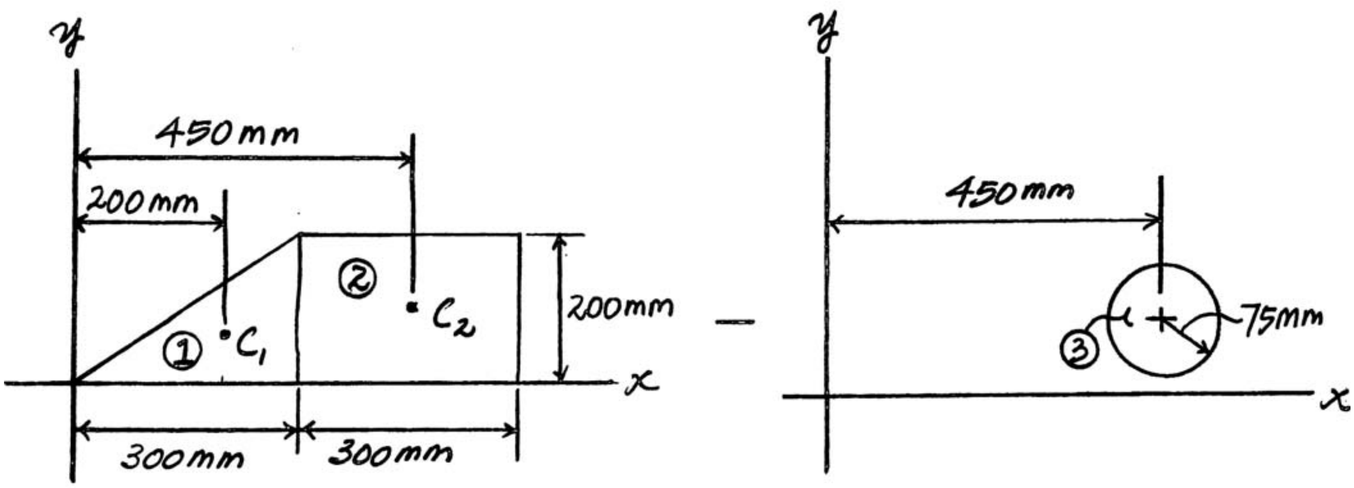
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

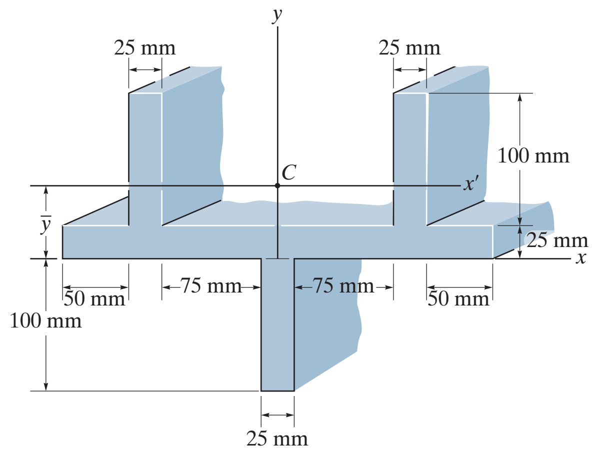
$$= \left[\frac{1}{36}(200)(300^3) + \frac{1}{2}(200)(300)(200)^2\right] + \left[\frac{1}{12}(200)(300^3) + 200(300)(450)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(450)^2\right]$$

$$= 10.3(10^9)\ \text{mm}^4 \qquad \textbf{Ans.}$$



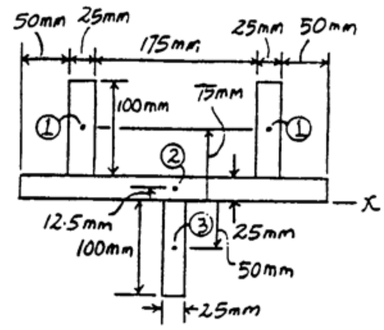
(a)

10–34. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then determine the moment of inertia about the x' axis.



Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	$A\ (\text{mm}^2)$	$\tilde{y}\ (\text{mm})$	$\tilde{y}A\ (\text{mm}^3)$
1	50(100)	75	$375(10^3)$
2	325(25)	12.5	$101.5625(10^3)$
3	25(100)	−50	$-125(10^3)$
Σ	$15.625(10^3)$		$351.5625(10^3)$

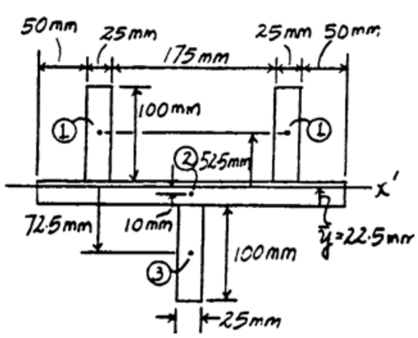


Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{351.5625(10^3)}{15.625(10^3)} = 22.5 \text{ mm} \qquad \textbf{Ans}$$

Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

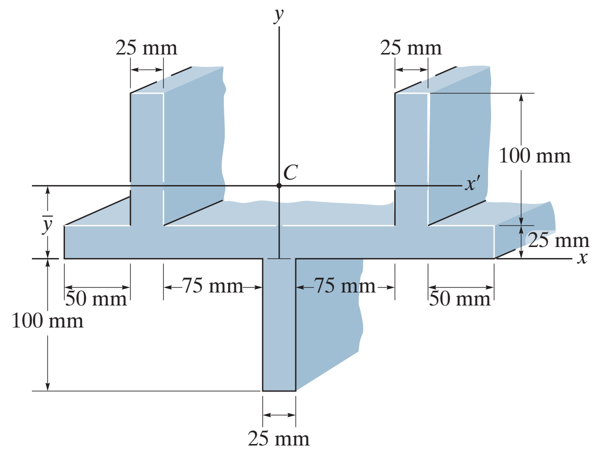
Segment	$A_i\ (\text{mm}^2)$	$(d_y)_i\ (\text{mm})$	$(\bar{I}_{x'})_i\ (\text{mm}^4)$	$(Ad_y^2)_i\ (\text{mm}^4)$	$(I_{x'})_i\ (\text{mm}^4)$
1	50(100)	52.5	$\frac{1}{12}(50)(100^3)$	$13.781(10^6)$	$17.948(10^6)$
2	325(25)	10	$\frac{1}{12}(325)(25^3)$	$0.8125(10^6)$	$1.236(10^6)$
3	25(100)	72.5	$\frac{1}{12}(25)(100^3)$	$13.141(10^6)$	$15.224(10^6)$



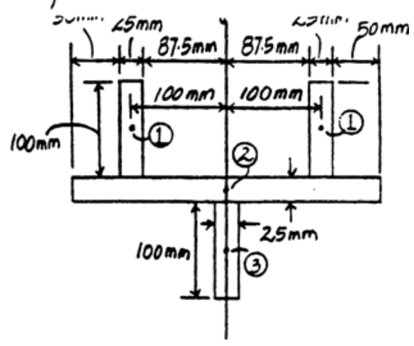
Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 34.41(10^6) \text{ mm}^4 = 34.4(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

10–35. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



Moment of Inertia : The moment of inertia about the y' axis for each segment can be determined using the parallel – axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

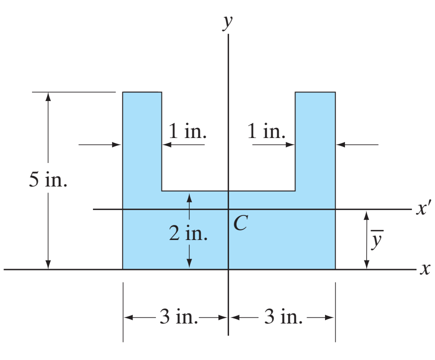


Segment	A_i (mm²)	$(d_x)_i$ (mm)	$(\bar{I}_{y'})_i$ (mm⁴)	$(Ad_x^2)_i$ (mm⁴)	$(I_{y'})_i$ (mm⁴)
1	2[100(25)]	100	$2\left[\frac{1}{12}(100)(25^3)\right]$	$50.0(10^6)$	$50.130(10^6)$
2	25(325)	0	$\frac{1}{12}(25)(325^3)$	0	$71.519(10^6)$
3	100(25)	0	$\frac{1}{12}(100)(25^3)$	0	$0.130(10^6)$

Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 121.78(10^6)\ \text{mm}^4 = 122(10^6)\ \text{mm}^4 \quad \textbf{Ans}$$

*__10–36.__ Locate the centroid $\bar{y}$ of the composite area, then determine the moment of inertia of this area about the centroidal x' axis.



Composite Parts: The composite area can be subdivided into three segments. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Centroid: The perpendicular distances measured from the centroid of each segment to the x axis are indicated in Fig. a.

$$\bar{y} = \frac{\Sigma \tilde{y}_C A}{\Sigma A} = \frac{(1)(6)(2) + 2[3.5(3)(1)]}{(6)(2) + 2[(3)(1)]} = 1.833\ \text{in.} = 1.83\ \text{in.} \qquad \textbf{Ans.}$$

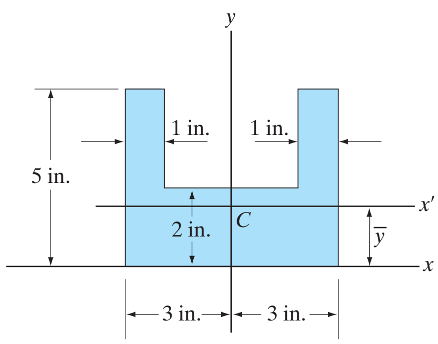
Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. Thus,

$$I_{x'} = \bar{I}_x + A(d_y)^2$$

$$= \left[\frac{1}{12}(6)(2^3) + 6(2)(1.833 - 1)^2\right] + 2\left[\frac{1}{12}(1)(3^3) + 1(3)(3.5 - 1.833)^2\right]$$

$$= 33.5\ \text{in}^4 \qquad \textbf{Ans.}$$

•10–37. Determine the moment of inertia of the composite area about the centroidal y axis.



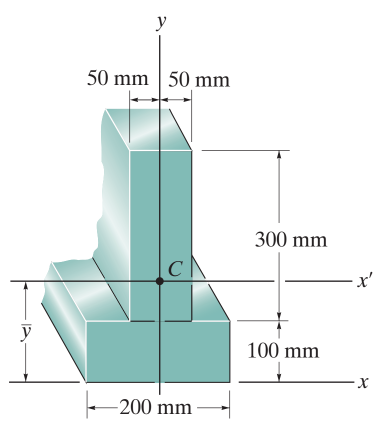
Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

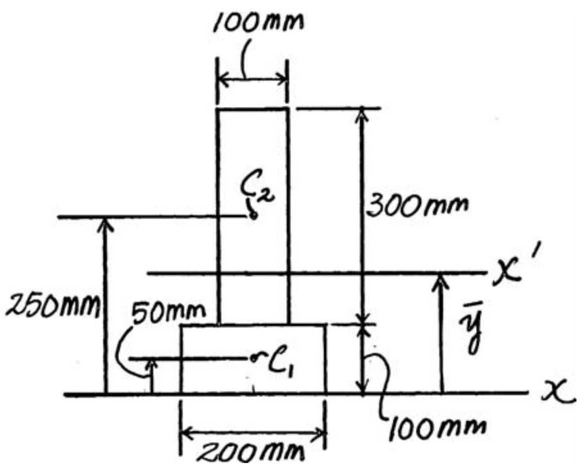
$$I_y = \bar{I}_y + A(d_x)^2$$

$$= \left[\frac{1}{12}(2)(6^3)\right] + 2\left[\frac{1}{12}(3)(1^3) + 3(1)(2.5)^2\right]$$

$$= 74 \text{ in}^4 \qquad \textbf{Ans.}$$

10–38. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x' axis.





Centroid :

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{50(100)(200) + 250(100)(300)}{100(200) + 100(300)} = 170 \text{ mm} \qquad \textbf{Ans}$$

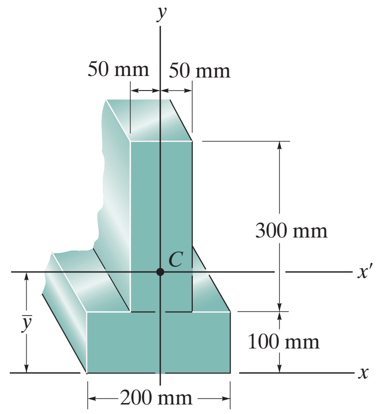
Moment of inertia :

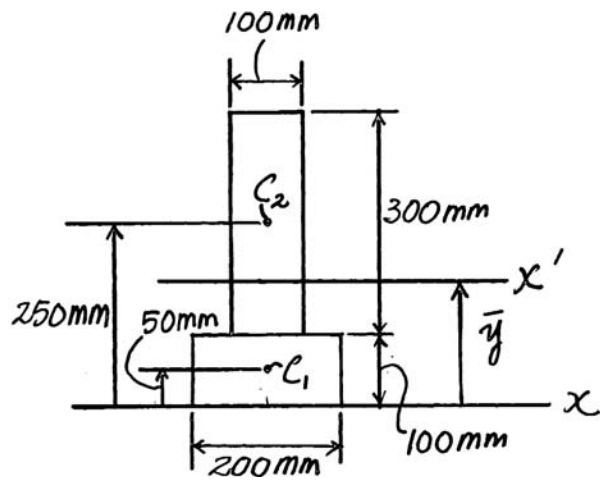
$$I_{x'} = \frac{1}{12}(200)(100)^3 + 200(100)(170-50)^2$$

$$+ \frac{1}{12}(100)(300)^3 + 100(300)(250-170)^2$$

$$= 722(10)^6 \text{ mm}^4 \qquad \textbf{Ans}$$

10–39. Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.

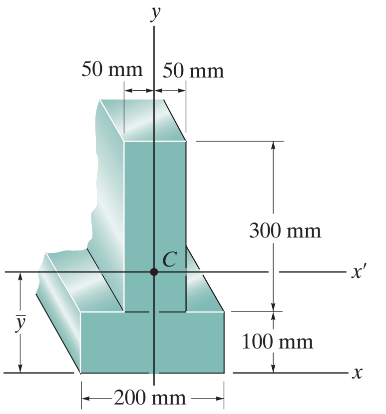


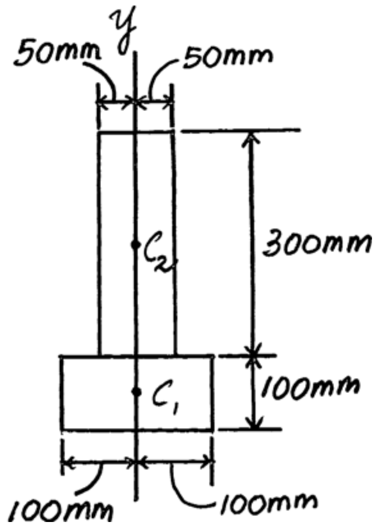


$$I_x = \left[\frac{1}{12}(0.2)(0.1)^3 + (0.2)(0.1)(0.05)^2\right]$$

$$+\left[\frac{1}{12}(0.1)(0.3)^3 + (0.1)(0.3)(0.25)^2\right] = 2.17(10^{-3})\ \text{m}^4 \qquad \textbf{Ans}$$

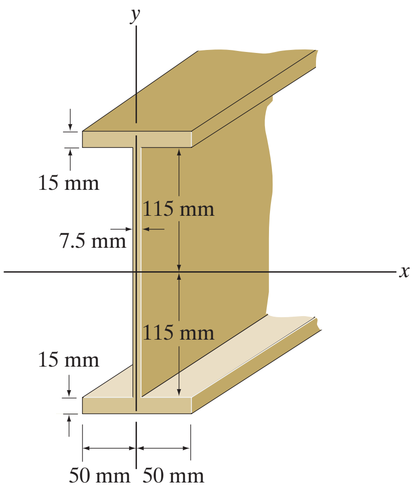
*__10–40.__ Determine the moment of inertia of the beam's cross-sectional area about the y axis.





$$I_y = \frac{1}{12}(100)(200)^3 + \frac{1}{12}(300)(100)^3 = 91.7(10)^6 \text{ mm}^4 \quad \text{Ans}$$

•10–41. Determine the moment of inertia of the beam's cross-sectional area about the x axis.



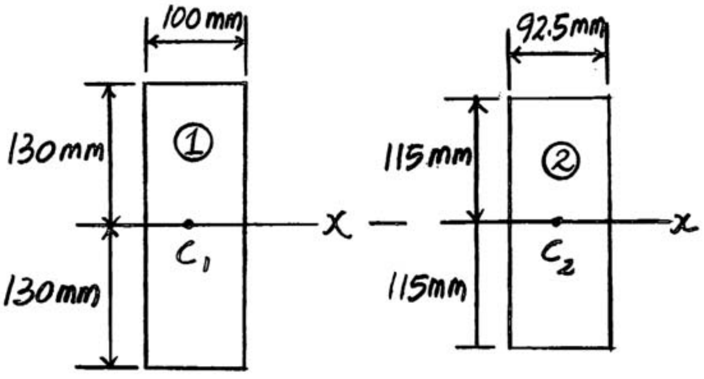
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. *a*. Here, segment (2) is a hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$

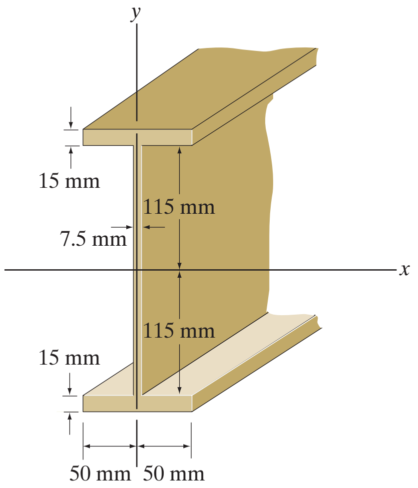
$$= \frac{1}{12}(100)(260^3) - \frac{1}{12}(92.5)(230^3)$$

$$= 52.7(10^6)\ \text{mm}^4$$ **Ans.**



(a)

10–42. Determine the moment of inertia of the beam's cross-sectional area about the *y* axis.



Composite Parts: The composite cross - sectional area of the beam can be subdivided into two similar segments (2) and one segment (1) as shown in Fig. *a*. The location of the centroid of each segment is also indicated.

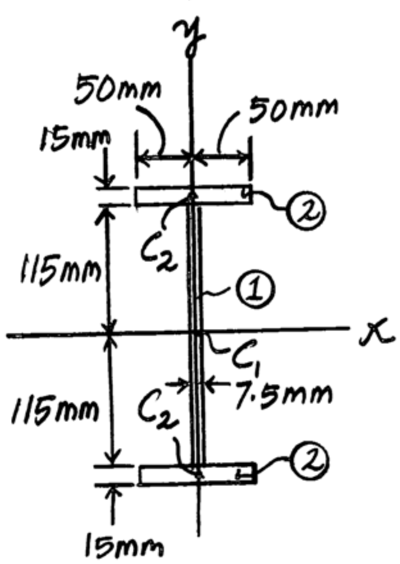
Moment of Inertia: Since the *y* axis passes through the centroid of each segment,

$$I_y = \Sigma\left(I_y\right)_i$$

$$= 2\left[\frac{1}{12}(15)(100^3)\right] + \frac{1}{12}(230)(7.5^3)$$

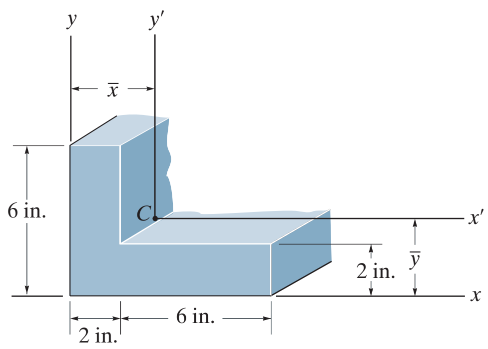
$$= 2.51(10^6)\ \text{mm}^4$$

Ans.



(a)

10–43. Locate the centroid $\bar{y}$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{x'}$ about the x' centroidal axis.

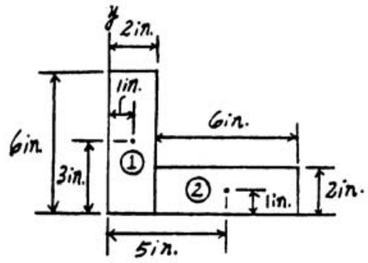


Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (in²)	$\tilde{y}$ (in.)	$\tilde{y}A$ (in³)
1	6(2)	3	36.0
2	6(2)	1	12.0
Σ	24.0		48.0

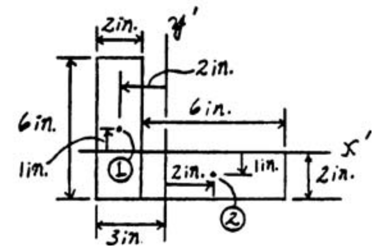
Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{48.0}{24.0} = 2.00 \text{ in.} \qquad \textbf{Ans}$$



Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

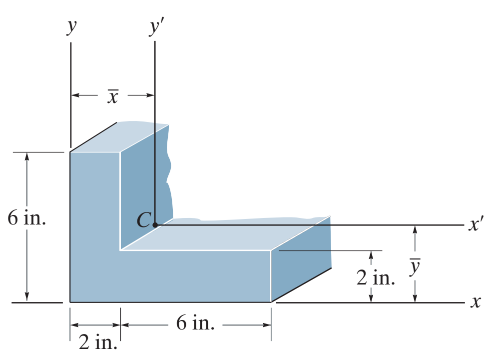
Segment	A_i (in²)	$(d_y)_i$ (in.)	$(\bar{I}_{x'})_i$ (in⁴)	$(Ad_y^2)_i$ (in⁴)	$(I_{x'})_i$ (in⁴)
1	2(6)	1	$\frac{1}{12}(2)(6^3)$	12.0	48.0
2	6(2)	1	$\frac{1}{12}(6)(2^3)$	12.0	16.0



Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 64.0 \text{ in}^4 \qquad \textbf{Ans}$$

***10–44.** Locate the centroid $\bar{x}$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{y'}$ about the y' centroidal axis.



Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (in²)	$\tilde{x}$ (in.)	$\tilde{x}A$ (in³)
1	6(2)	1	12.0
2	6(2)	5	60.0
Σ	24.0		72.0

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{72.0}{24.0} = 3.00 \text{ in.} \qquad \textbf{Ans}$$

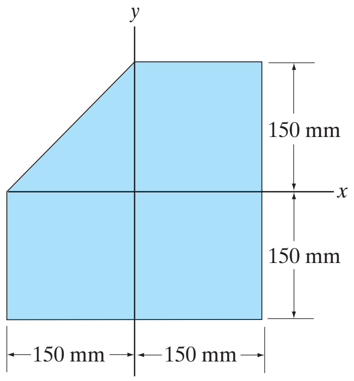
Moment of Inertia : The moment of inertia about the y' axis for each segmentcan be determined using the parallel – axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

Segment	A_i (in²)	$(d_x)_i$ (in.)	$(\bar{I}_{y'})_i$ (in⁴)	$(Ad_x^2)_i$ (in⁴)	$(I_{y'})_i$ (in⁴)
1	6(2)	2	$\frac{1}{12}(6)(2^3)$	48.0	52.0
2	2(6)	2	$\frac{1}{12}(2)(6^3)$	48.0	84.0

Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 136 \text{ in}^4 \qquad \textbf{Ans}$$

•**10–45.** Determine the moment of inertia of the composite area about the x axis.



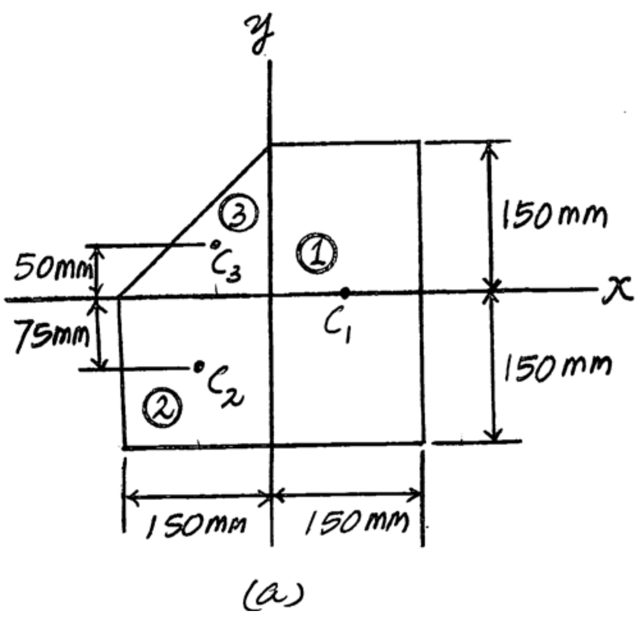
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

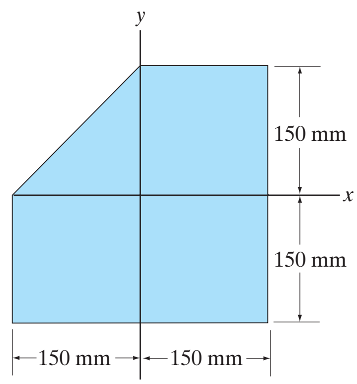
$$= \left[\frac{1}{12}(150)(300^3) + 300(150)(0)^2\right] + \left[\frac{1}{12}(150)(150^3) + 150(150)(75)^2\right] + \left[\frac{1}{36}(150)(150^3) + \frac{1}{2}(150)(150)(50)^2\right]$$

$$= 548(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

10–46. Determine the moment of inertia of the composite area about the y axis.



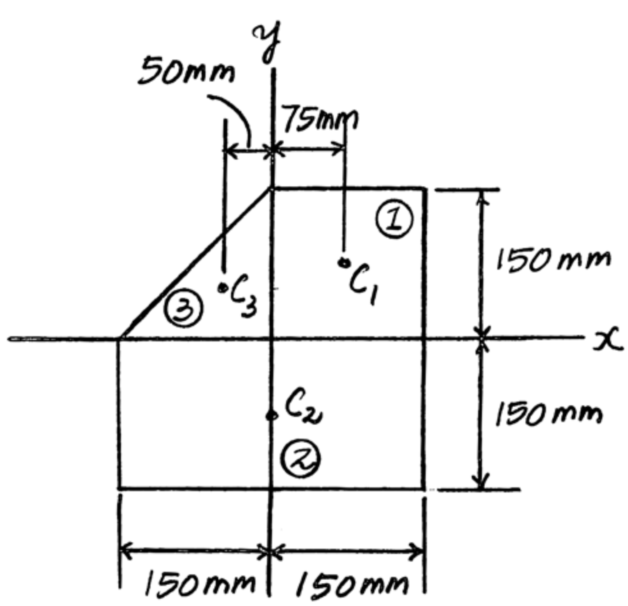
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. a. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

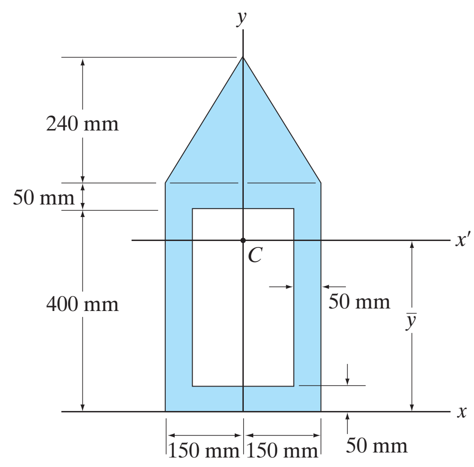
$$= \left[\frac{1}{12}(150)(150^3) + 150(150)(75)^2\right] + \left[\frac{1}{12}(150)(300^3) + 15(300)(0)^2\right] + \left[\frac{1}{36}(150)(150^3) + \frac{1}{2}(150)(150)(50)^2\right]$$

$$= 548(10^6)\ \text{mm}^4$$ **Ans.**



(a)

10–47. Determine the moment of inertia of the composite area about the centroidal y axis.



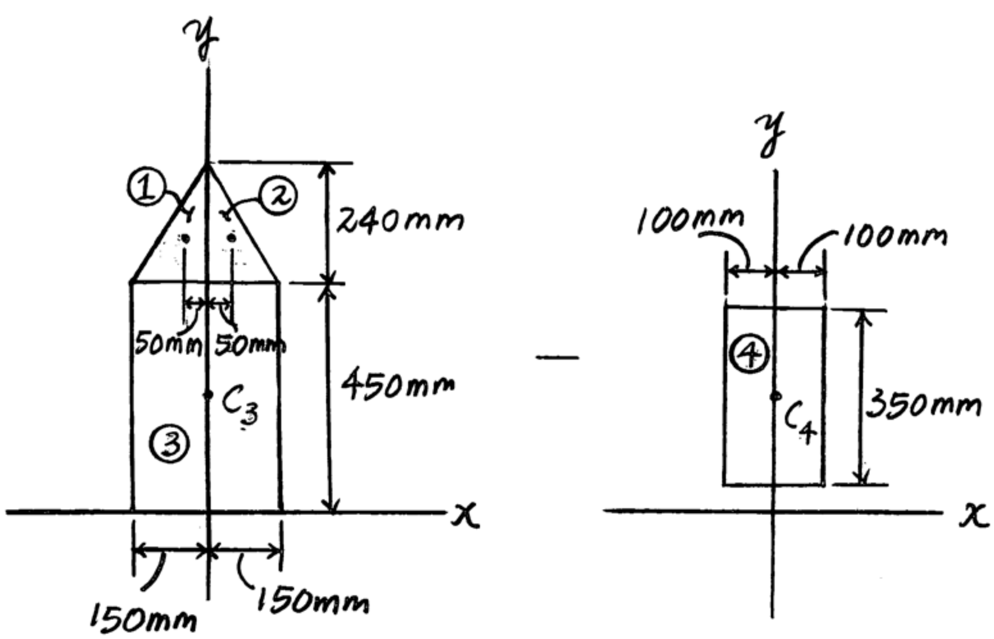
Composite Parts: The composite area can be subdivided into four segments as shown in Fig. *a*. Since segment (4) is a hole, it contributes a negative moment of inertia. The location of the centroid for each segment is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

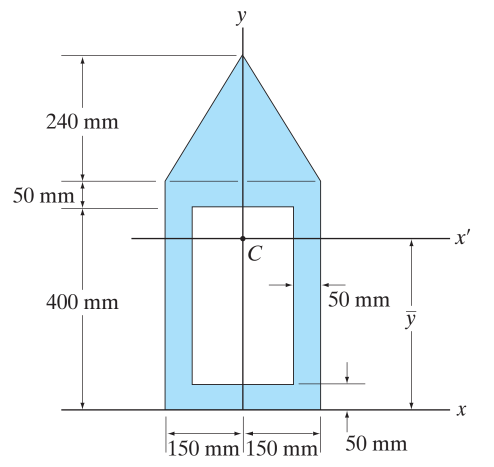
$$= \left[2\left(\frac{1}{36}(240)(150^3)\right) + 2\left(\frac{1}{2}(240)(150)\right)(50)^2\right] + \left[\frac{1}{12}(450)(300^3) + 450(300)(0)^2\right] + \left[-\frac{1}{12}(350)(200^3) + (-350(200))(0)^2\right]$$

$$= 914(10^6)\ \text{mm}^4$$ **Ans.**



(a)

***10–48.** Locate the centroid $\bar{y}$ of the composite area, then determine the moment of inertia of this area about the x' axis.



Composite Parts: The composite area can be subdivided into three segments as shown in Figs. *a* and *b*. Since segment (3) is a hole, it should be considered a negative part.

Centroid: The perpendicular distances measured from the centroid of each segment to the *x* axis are indicated in Fig. *a*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{530\left(\frac{1}{2}(300)(240)\right) + 225(300(450)) + 225(-200(350))}{\frac{1}{2}(300)(240) + 300(450) - 200(350)} = \frac{33.705(10^6)}{101(10^3)} = 333.71 \text{ mm} = 334 \text{ mm} \qquad \textbf{Ans.}$$

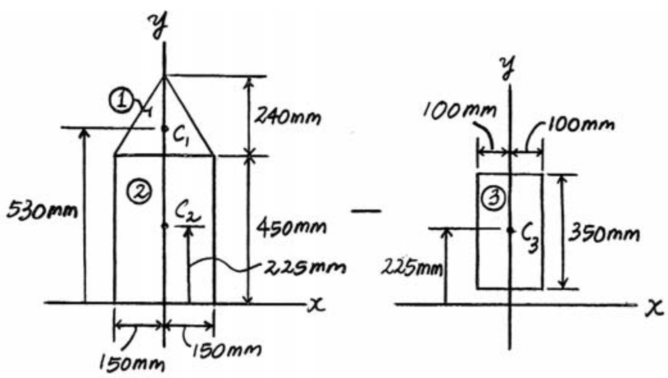
Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each segment to the x' axis is indicated in Fig. *b*.

$$I_{x'} = \bar{I}_{x'} + A(d_{x'})^2$$

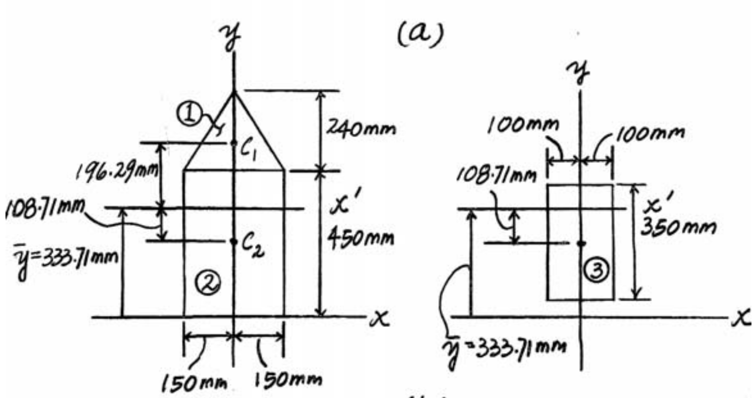
$$= \left[\frac{1}{36}(300)(240^3) + \frac{1}{2}(300)(240)(196.29)^2\right] + \left[\frac{1}{12}(300)(450^3) + 300(450)(108.71)^2\right]$$

$$+ \left[-\frac{1}{12}(200)(350^3) + (-200(350))(108.71)^2\right]$$

$$= 3.83(10^9) \text{ mm}^4 \qquad \textbf{Ans.}$$

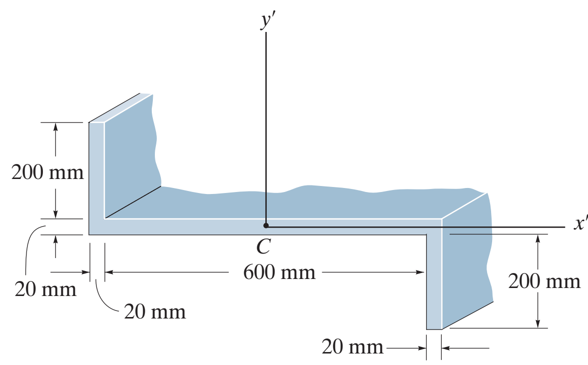


(a)



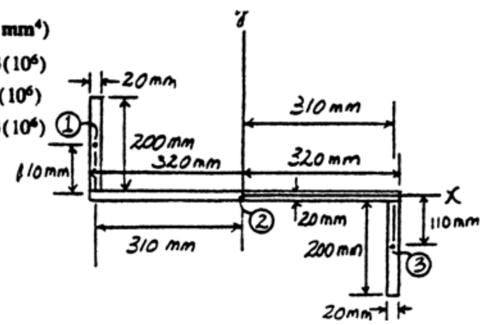
(b)

•10–49. Determine the moment of inertia $I_{x'}$ of the section. The origin of coordinates is at the centroid C.



Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

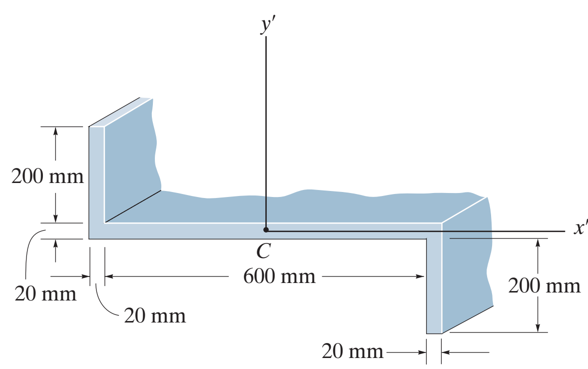
Segment	A_i (mm²)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm⁴)	$(Ad_y^2)_i$ (mm⁴)	$(I_{x'})_i$ (mm⁴)
1	200(20)	110	$\frac{1}{12}(20)(200^3)$	$48.4(10^6)$	$61.733(10^6)$
2	640(20)	0	$\frac{1}{12}(640)(20^3)$	0	$0.427(10^6)$
3	200(20)	110	$\frac{1}{12}(20)(200^3)$	$48.4(10^6)$	$61.733(10^6)$



Thus,

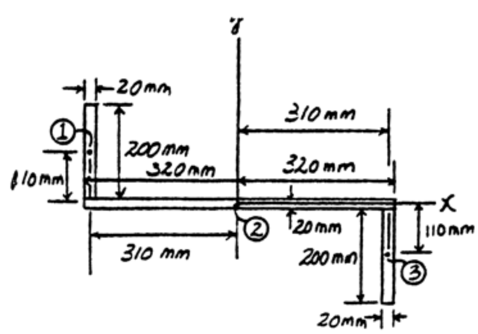
$$I_{x'} = \Sigma(I_{x'})_i = 123.89(10^6)\ \text{mm}^4 = 124(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

10–50. Determine the moment of inertia $I_{y'}$ of the section. The origin of coordinates is at the centroid C.



Moment of Inertia : The moment of inertia about the y' axis for each segment can be determined using the parallel – axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

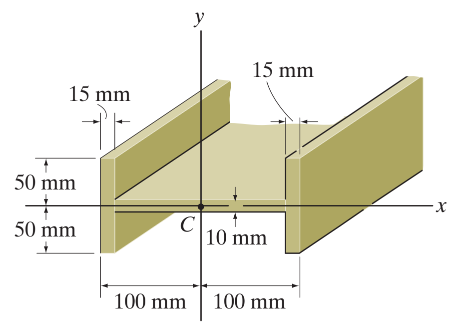
Segment	A_i (mm²)	$(d_x)_i$ (mm)	$(\bar{I}_{y'})_i$ (mm⁴)	$(Ad_x^2)_i$ (mm⁴)	$(I_{y'})_i$ (mm⁴)
1	200(20)	310	$\frac{1}{12}(200)(20^3)$	$384.4(10^6)$	$384.53(10^6)$
2	640(20)	0	$\frac{1}{12}(20)(640^3)$	0	$436.91(10^6)$
3	200(20)	310	$\frac{1}{12}(200)(20^3)$	$384.4(10^6)$	$384.53(10^6)$



Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 1.206(10^9)\ \text{mm}^4 = 1.21(10^9)\ \text{mm}^4 \qquad \textbf{Ans}$$

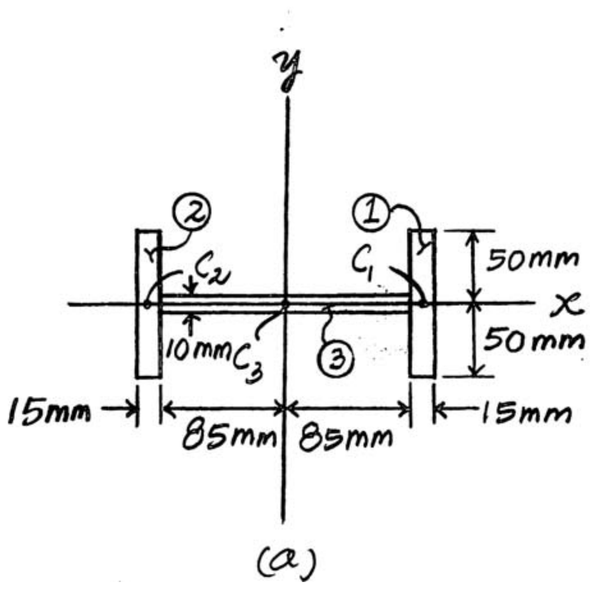
10–51. Determine the beam's moment of inertia I_x about the centroidal x axis.



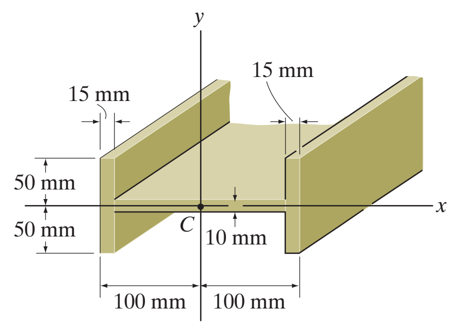
Composite Parts: The composite cross - sectioned area of the beam can be subdivided into three segments as shown in Fig. *a*. The locations of the centroid for each segment is also indicated.

Moment of Inertia: Since the centroid of each segment is located about the x axis then

$$I_x = \frac{1}{12}(15)(100^3) + \frac{1}{12}(15)(100^3) + \frac{1}{12}(170)(10^3)$$
$$= 2.51(10^6)\text{ mm}^4$$ **Ans.**



***10–52.** Determine the beam's moment of inertia I_y about the centroidal y axis.



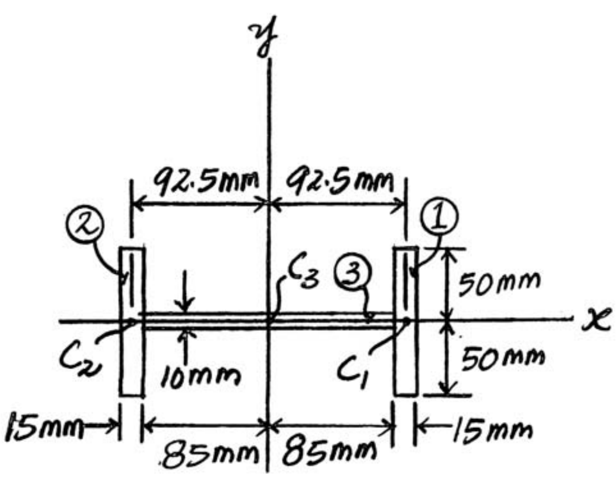
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

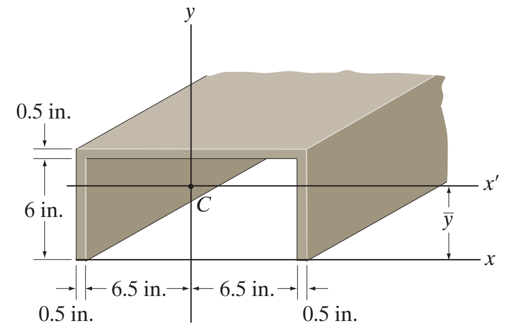
$$= \left[\frac{1}{12}(100)(15^3) + 100(15)(92.5)^2\right] + \left[\frac{1}{12}(100)(15^3) + 100(15)(92.5)^2\right] + \left[\frac{1}{12}(10)(170^3) + 170(10)(0)^2\right]$$

$$= 29.8(10^6)\ \text{mm}^4 \quad \textbf{Ans.}$$



(a)

•**10–53.** Locate the centroid $\bar{y}$ of the channel's cross-sectional area, then determine the moment of inertia of the area about the centroidal x' axis.



Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Figs. *a* and *b*.

Centroid: The perpendicular distances measured from the centroid of each type of segment to the x axis are also indicated in Fig. *a*. Thus,

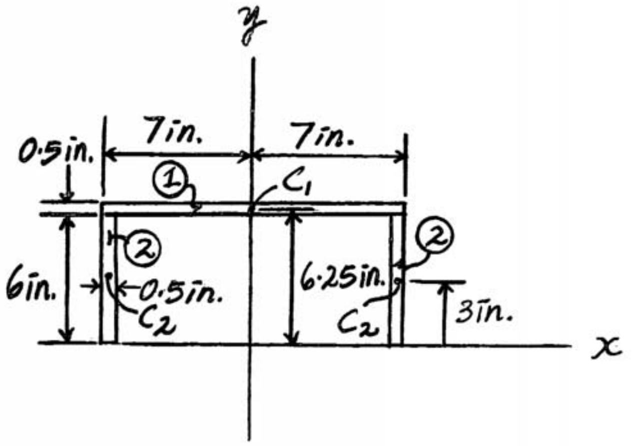
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{6.25(14(0.5)) + (3)(2)(6)(0.5)}{14(0.5) + (2)(6)(0.5)} = \frac{61.75}{13} = 4.75 \text{ in.} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each type of segment to the x' axis is indicated in Fig. *b*.

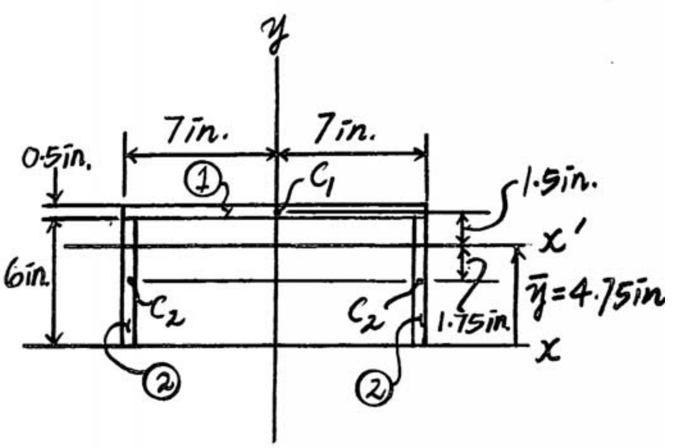
$$I_{x'} = \bar{I}_{x'} + A(d_{y'})^2$$

$$= \left[\frac{1}{12}(14)(0.5^3) + 14(0.5)(1.5)^2\right] + \left[2\left(\frac{1}{12}(0.5)(6^3)\right) + 2(6)(0.5)(1.75)^2\right]$$

$$= 15.896 + 36.375 = 52.3 \text{ in}^4 \qquad \textbf{Ans.}$$

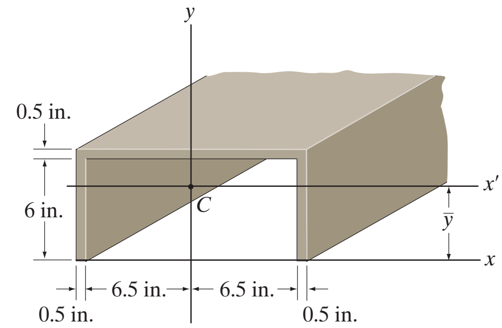


(a)



(b)

10–54. Determine the moment of inertia of the area of the channel about the y axis.



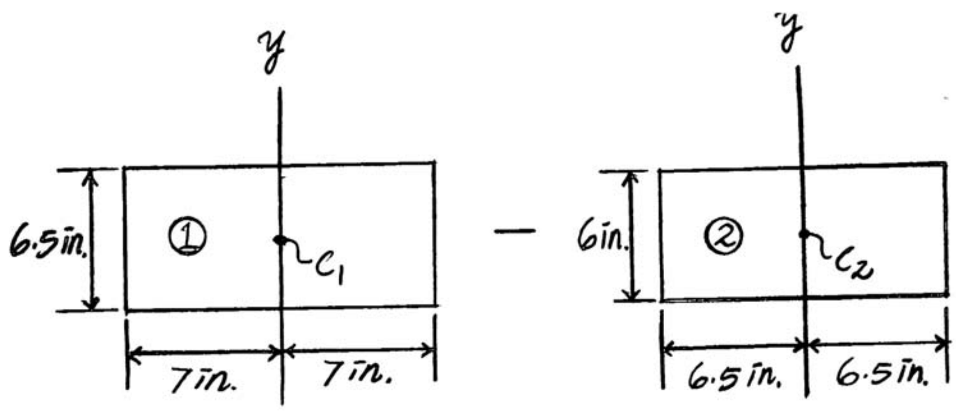
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. *a*. Here, segment (2) is a hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$

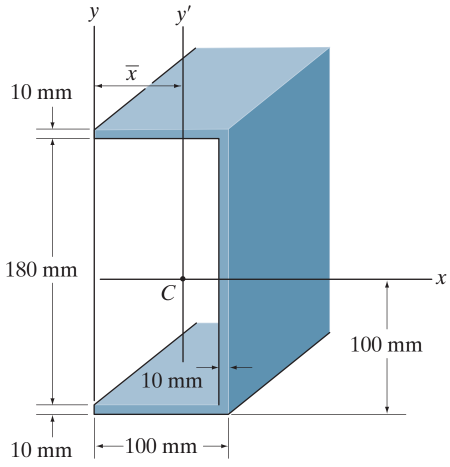
$$= \frac{1}{12}(6.5)(14^3) - \frac{1}{12}(6)(13^3)$$

$$= 388 \text{ in}^4$$ **Ans.**



(a)

10–55. Determine the moment of inertia of the cross-sectional area about the x axis.



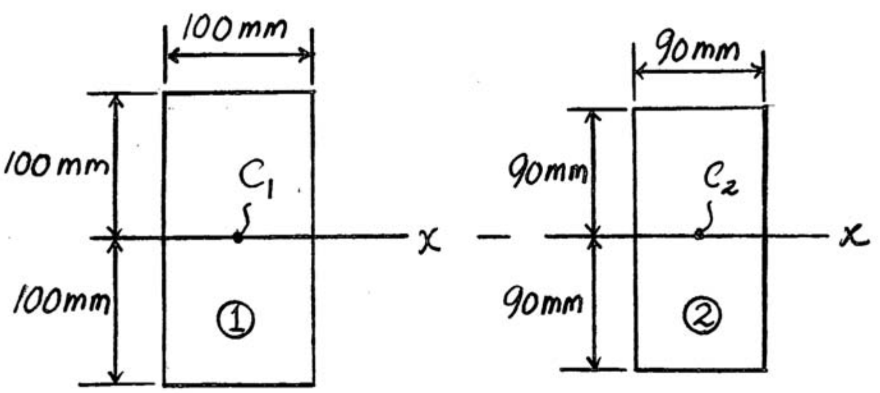
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. *a*. Here, segment (2) is a hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$

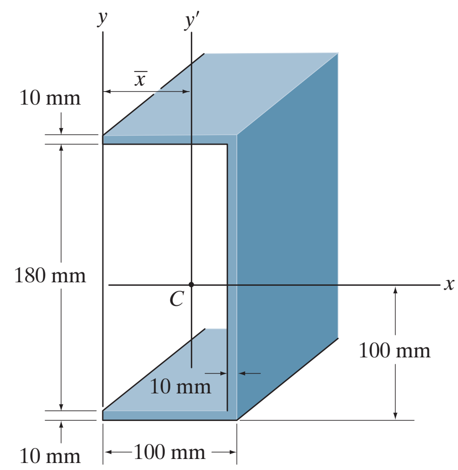
$$= \frac{1}{12}(100)(200^3) - \frac{1}{12}(90)(180^3)$$

$$= 22.9(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

***10–56.** Locate the centroid $\bar{x}$ of the beam's cross-sectional area, and then determine the moment of inertia of the area about the centroidal y' axis.



Composite Parts: The composite cross-sectional area of the beam can be subdivided into segments as shown in Fig. *a*.

Centroid: The perpendicular distance measured from the centroid of each type of segment to the y axis is also indicated in Fig. *a*. Thus,

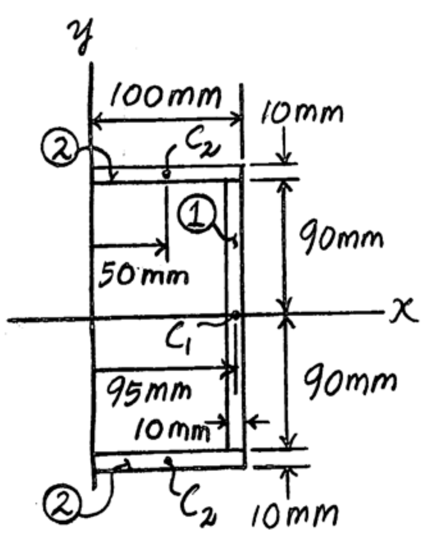
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{95(10(180)) + 50(2(100)(10))}{10(180) + 2(100)(10)} = \frac{271(10^3)}{3.8(10^3)} = 71.32 \text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia of each segment about the y' axis can be determined using the parallel-axis theorem. The perpendicular distance measured from the centroid of each type of segment to the y' axis is indicated in Fig. *b*.

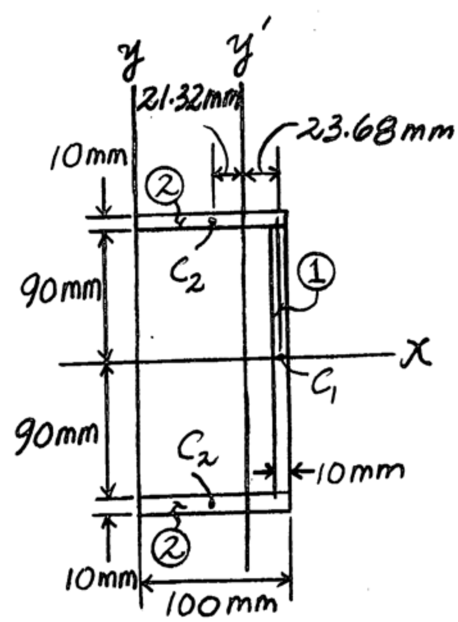
$$I_{y'} = \bar{I}_{y'} + A(d_{x'})^2$$

$$= \left[\frac{1}{12}(180)(10^3) + 180(10)(23.68)^2\right] + \left[2\left(\frac{1}{12}(10)(100^3)\right) + 2(100)(10)(21.32)^2\right]$$

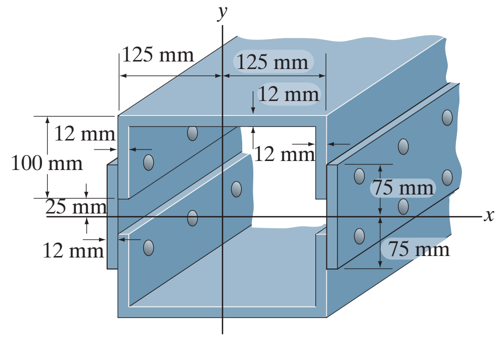
$$= 3.60(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$



(a)



•**10–57.** Determine the moment of inertia of the beam's cross-sectional area about the x axis.



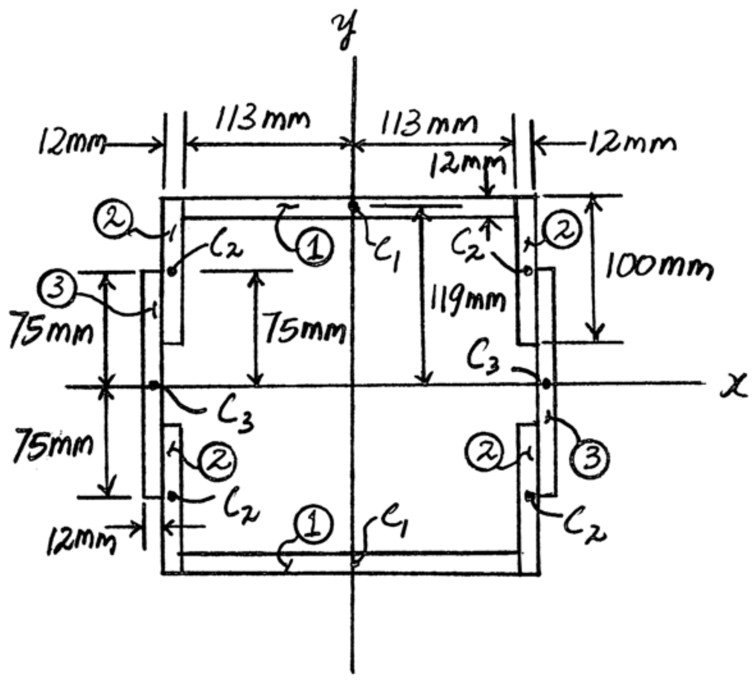
Composite Parts: The composite area can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

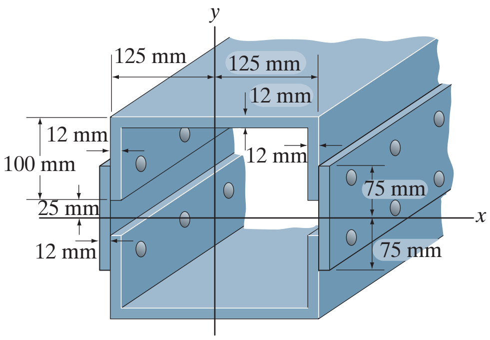
$$= \left[2\left(\frac{1}{12}(226)(12^3)\right) + 2(226)(12)(119)^2\right] + \left[4\left(\frac{1}{12}(12)(100^3)\right) + 4(12)(100)(75)^2\right] + \left[2\left(\frac{1}{12}(12)(150^3)\right) + 2(12)(150)(0)^2\right]$$

$$= 114.62(10^6)\,\text{mm}^4 = 115(10^6)\,\text{mm}^4$$ **Ans.**



(a)

10–58. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



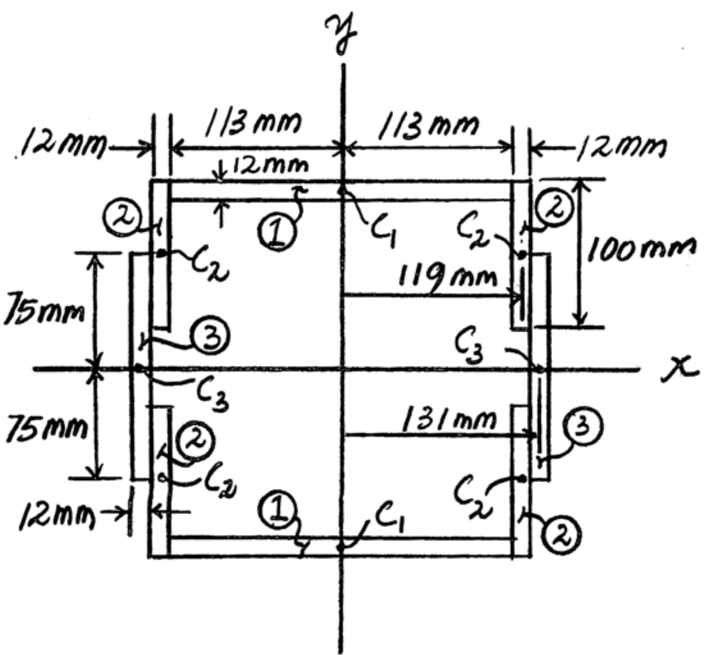
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

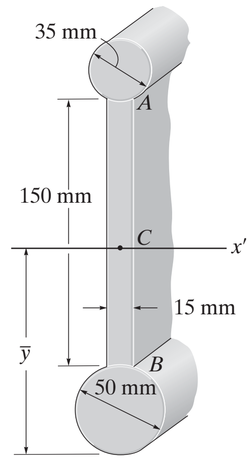
$$I_y = \bar{I}_{x'} + A(d_x)^2$$

$$= \left[2\left(\frac{1}{12}(12)(226^3)\right) + 2(226)(12)(0)^2\right] + \left[4\left(\frac{1}{12}(100)(12^3)\right) + 4(100)(12)(119)^2\right] + \left[2\left(\frac{1}{12}(150)(12^3)\right) + 2(150)(12)(131)^2\right]$$

$$= 152.94(10^6)\text{ mm}^4 = 153(10^6)\text{ mm}^4$$ **Ans.**

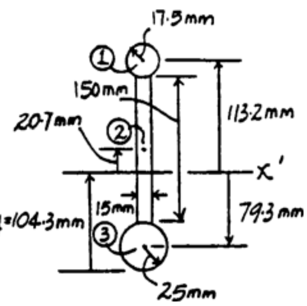


10–59. Determine the moment of inertia of the beam's cross-sectional area with respect to the x' axis passing through the centroid C of the cross section. $\bar{y} = 104.3$ mm.



Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

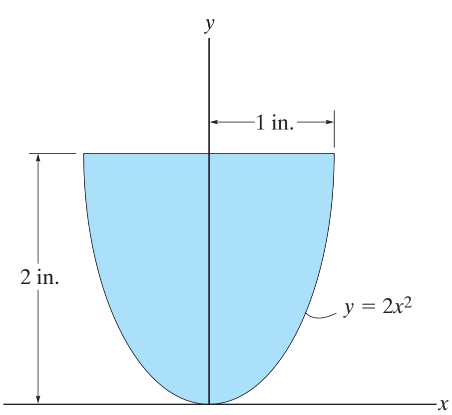
Segment	A_i (mm²)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm⁴)	$(Ad_y^2)_i$ (mm⁴)	$(I_{x'})_i$ (mm⁴)
1	$\pi(17.5^2)$	113.2	$\frac{\pi}{4}(17.5^4)$	$12.329(10^6)$	$12.402(10^6)$
2	$15(150)$	20.7	$\frac{1}{12}(15)(150^3)$	$0.964(10^6)$	$5.183(10^6)$
3	$\pi(25^2)$	79.3	$\frac{\pi}{4}(25^4)$	$12.347(10^6)$	$12.654(10^6)$



Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 30.24\left(10^6\right)\ \text{mm}^4 = 30.2\left(10^6\right)\ \text{mm}^4 \quad \textbf{Ans}$$

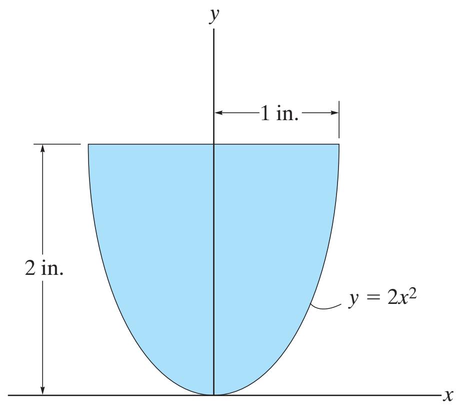
***10–60.** Determine the product of inertia of the parabolic area with respect to the x and y axes.



Due to symmetry about y axis

$I_{xy} = 0$ **Ans**

•10–61. Determine the product of inertia I_{xy} of the right half of the parabolic area in Prob. 10–60, bounded by the lines $y = 2$ in. and $x = 0$.



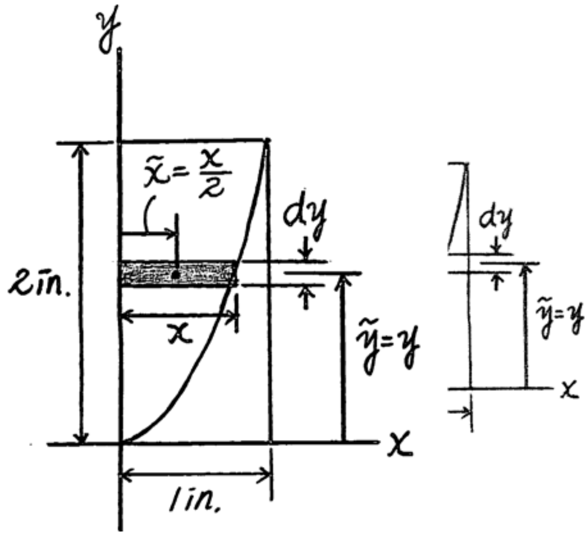
$\tilde{x} = \frac{x}{2}$

$\tilde{y} = y$

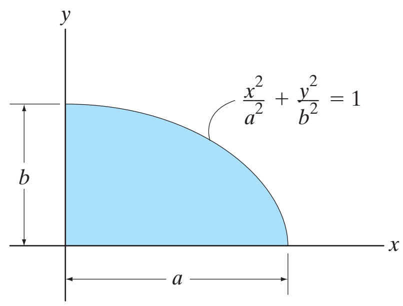
$dA = x\,dy$

$I_{xy} = \int_A \tilde{x}\tilde{y}\,dA = \int_A \left(\frac{x}{2}\right)(y)(x\,dy)$

$= \int_0^2 \frac{1}{2}\left(\frac{1}{2}y^2\right)dy = \frac{1}{12}y^3\Big]_0^2 = 0.667 \text{ in}^4$ **Ans**



10–62. Determine the product of inertia of the quarter elliptical area with respect to the x and y axes.

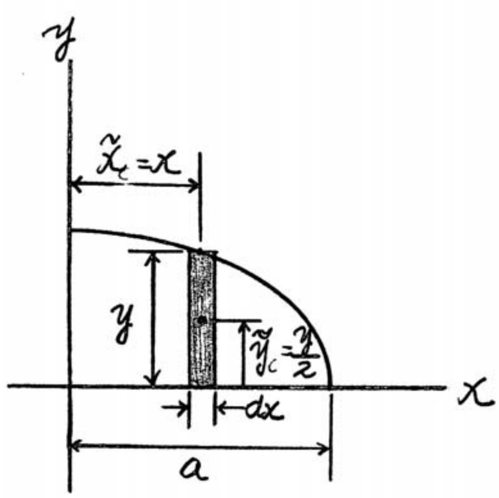


Differential Element: The area of the differenial element parallel to the y axis shown shaded in Fig. a is $dA = y\,dx$. Here, $y = \frac{b}{a}\sqrt{a^2 - x^2}$. Thus, $dA = \frac{b}{a}\sqrt{a^2 - x^2}\,dx$. The coordinates of the centroid of this element are $x_c = x$ and $y_c = \frac{y}{2} = \frac{b}{2a}\sqrt{a^2 - x^2}$. Thus, the product of inertia of this element with respect to the x and y axes is

$$dI_{xy} = d\bar{I}_{x'y'} + dA x_c y_c$$
$$= 0 + \left(\frac{b}{a}\sqrt{a^2 - x^2}\,dx\right)(x)\left(\frac{b}{2a}\sqrt{a^2 - x^2}\right)$$
$$= \frac{b^2}{2a^2}\left(a^2 x - x^3\right)dx$$

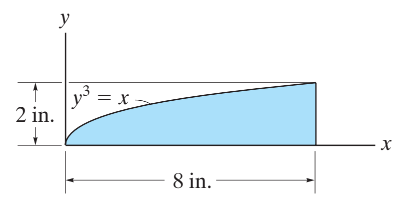
Product of Inertia: Performing the integration,

$$I_{xy} = \int dI_{xy} = \int_0^a \frac{b^2}{2a^2}\left(a^2 x - x^3\right)dx = \frac{b^2}{2a^2}\left(\frac{a^2}{2}x^2 - \frac{x^4}{4}\right)\Bigg|_0^a = \frac{a^2 b^2}{8}$$ **Ans.**



(a)

10–63. Determine the product of inertia for the area with respect to the x and y axes.

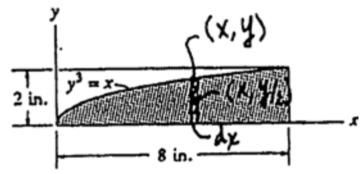


$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \frac{xy^2}{2}\,dx$



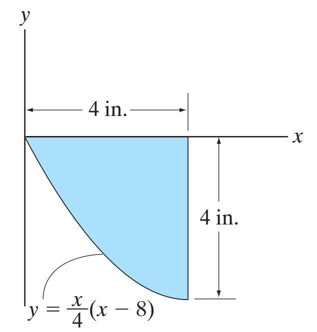
$I_{xy} = \int dI_{xy}$

$= \frac{1}{2}\int_0^8 x^{5/3}\,dx$

$= \frac{1}{2}\left(\frac{3}{8}\right)\left[x^{8/3}\right]_0^8$

$= 48 \text{ in}^4$ **Ans**

***10–64.** Determine the product of inertia of the area with respect to the x and y axes.



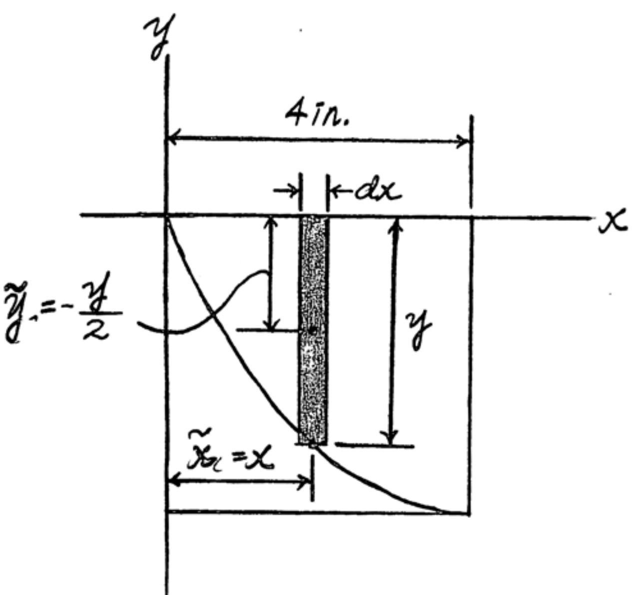
Differential Element: The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = y\,dx = \frac{x}{4}(x-8)\,dx = \left(\frac{x^2}{4} - 2x\right)dx$. The coordinates of the centroid of this element are $\tilde{x} = x$ and $\tilde{y} = -\frac{y}{2} = -\frac{1}{2}\left(\frac{x^2}{4} - 2x\right)$.

Thus, the product of inertia of this element with respect to the x and y axes is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y}$$
$$= 0 + \left(\frac{x^2}{4} - 2x\right)dx\,(x)\left[-\frac{1}{2}\left(\frac{x^2}{4} - 2x\right)\right]$$
$$= \left(-\frac{x^5}{32} - 2x^3 + \frac{x^4}{2}\right)dx$$

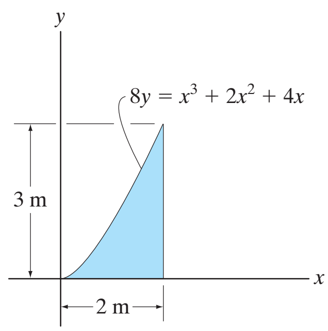
Product of Inertia: Performing the integration,

$$I_{xy} = \int dI_{xy} = \int_0^{4\text{ in.}} \left(-\frac{x^5}{32} - 2x^3 + \frac{x^4}{2}\right)dx = \left[-\frac{x^6}{192} - \frac{x^4}{2} + \frac{x^5}{10}\right]_0^{4\text{ in.}} = -46.9\text{ in}^4 \quad \textbf{Ans.}$$



(a)

•10–65. Determine the product of inertia of the area with respect to the x and y axes.



Differential Element: The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = y\,dx = \frac{1}{8}(x^3 + 2x^2 + 4x)\,dx$. The coordinates of the centroid of this element are $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2}$

Thus, the product of inertia of this element with respect to the x and y axes is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y}$$

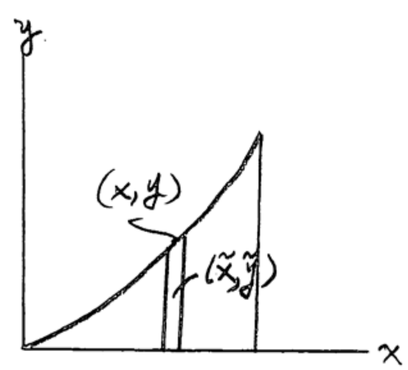
$$= 0 + \left(\frac{1}{8}(x^3 + 2x^2 + 4x)\,dx\right)(x)\left[\frac{1}{16}(x^3 + 2x^2 + 4x)\right]$$

$$= \frac{1}{128}(x^3 + 2x^2 + 4x)^2 x\,dx$$

Product of Inertia: Performing the integration,

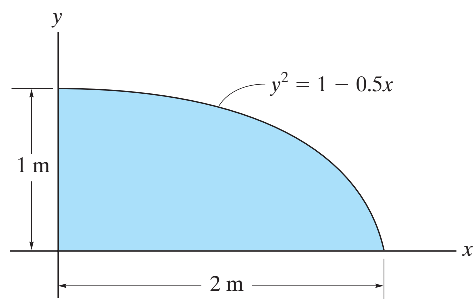
$$I_{xy} = \int dI_{xy} = \int_0^{4\text{ in.}} \frac{1}{128}(x^7 + 4x^6 + 12x^5 + 16x^4 + 16x^3)dx$$

$$= \frac{1}{128}\left[\frac{x^8}{8} + \frac{4x^7}{7} + \frac{12x^6}{6} + \frac{16x^5}{5} + \frac{16x^4}{4}\right]\Bigg|_0^{2\text{ in.}} = 3.12\text{ m}^4$$ **Ans.**



(a)

10–66. Determine the product of inertia for the area with respect to the *x* and *y* axes.

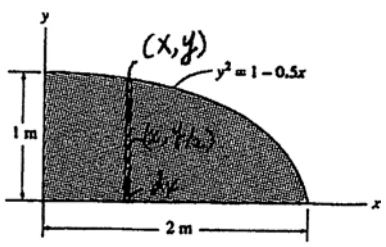


$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \frac{xy^2}{2}\,dx$



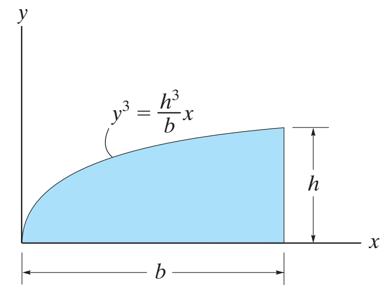
$I_{xy} = \int dI_{xy}$

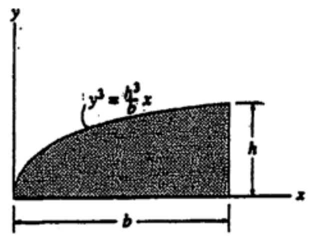
$= \int_0^2 \frac{1}{2}(x - 0.5x^2)\,dx$

$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2$

$= 0.333 \text{ m}^4$ **Ans**

10–67. Determine the product of inertia for the area with respect to the x and y axes.



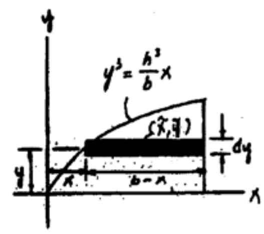


The product of inertia of the element (shaded) is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$

$$= 0 + (b-x)(dy)\left(x + \frac{b-x}{2}\right)(y) = \frac{1}{2}(b^2 - x^2)\, y dy \qquad \text{Where } x^2 = \frac{b^2}{h^6}y^6$$

$$= \frac{1}{2}\left(b^2 y - \frac{b^2}{h^6}y^7\right)dy$$



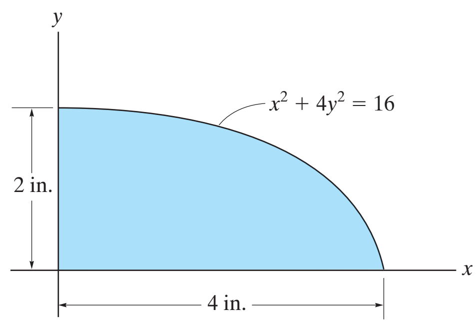
Integrating

$$\int dI_{xy} = I_{xy} = \frac{1}{2}\int_0^h \left(b^2 y - \frac{b^2}{h^6}y^7\right)dy$$

$$= \left[\frac{1}{2}\left(\frac{b^2}{2}y^2 - \frac{b^2}{8h^6}y^8\right)\right]\Bigg|_0^h$$

$$= \frac{3}{16}b^2h^2 \qquad \textbf{Ans}$$

***10–68.** Determine the product of inertia for the area of the ellipse with respect to the x and y axes.



$$I_{xy} = \int_A \tilde{x}\,\tilde{y}\,dA = \int_0^4 \left(\frac{y}{2}\right)(xy)\,dx$$

$$= \frac{1}{2}\int_0^4 y^2 x\,dx$$

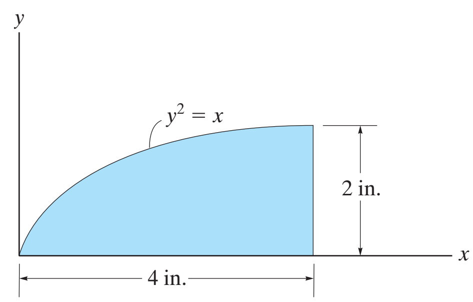
$$= \frac{1}{2}\int_0^4 \frac{1}{4}(16 - x^2)x\,dx$$

$$= \frac{1}{8}\int_0^4 (16x - x^3)\,dx$$

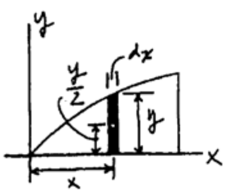
$$= \frac{1}{8}\left[8x^2 - \frac{1}{4}x^4\right]_0^4$$

$$I_{xy} = 8\text{ in}^4 \qquad \textbf{Ans}$$

•10–69. Determine the product of inertia for the parabolic area with respect to the x and y axes.



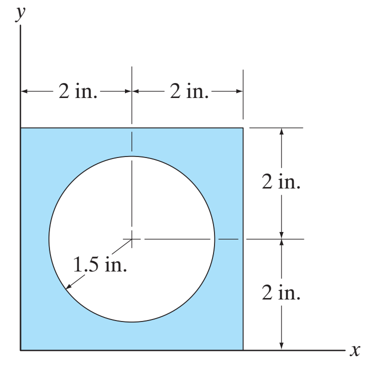
$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\,\tilde{y}$$



$$I_{xy} = 0 + \int_A x\left(\frac{y}{2}\right)y\,dx$$

$$= \frac{1}{2}\int_0^4 x^2\,dx = \frac{1}{6}x^3\Big|_0^4 = 10.6667 = 10.7\text{ in}^4 \qquad \textbf{Ans}$$

10–70. Determine the product of inertia of the composite area with respect to the x and y axes.

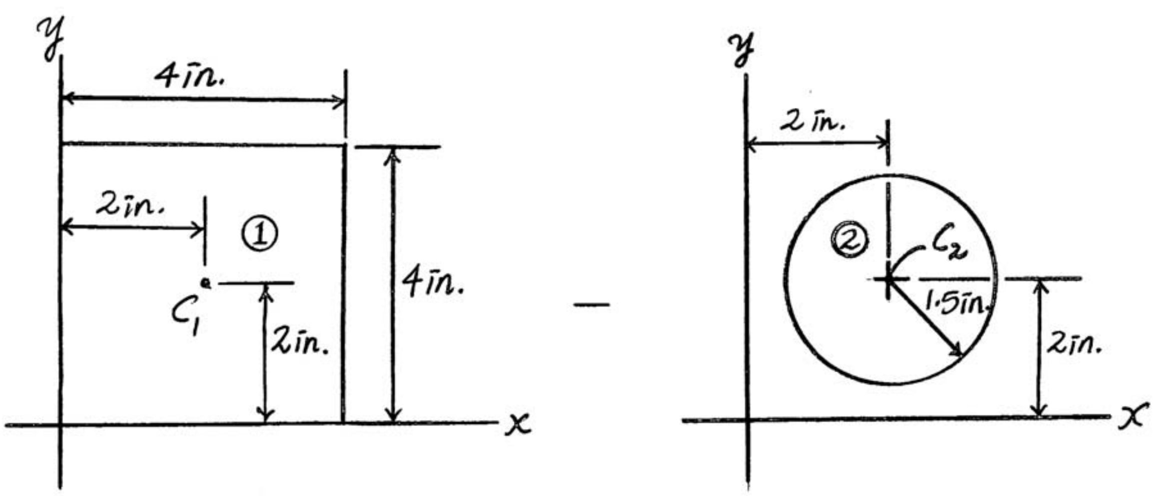


Composite Parts: The composite area can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered a negative area. The perpendicular distances measured from the centroid of each segment to the x and y axes are also indicated.

Product of Inertia: Since the centroidal axes are the axes of symmetry for both segments, then $\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{x'y'}\right)_2 = 0$. The product of inertia of each segment with respect to the x and y axes can be determined using the parallel - axis theorem.

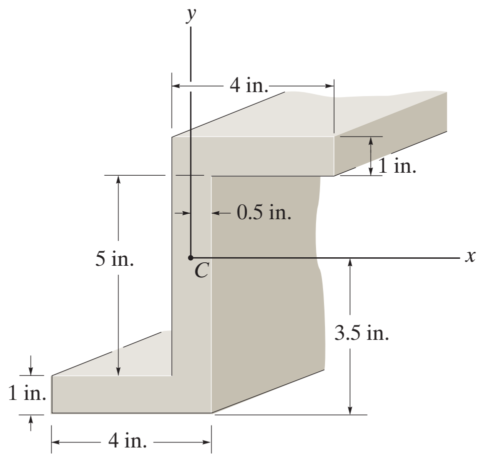
$$I_{xy} = \left(\bar{I}_{x'y'}\right)_i + Ad_x d_y = Ad_x d_y$$

$$= 4(4)(2)(2) + \left(-\pi(1.5^2)\right)(2)(2) = 64 + (-9\pi) = 35.7 \text{ in}^4 \qquad \textbf{Ans.}$$



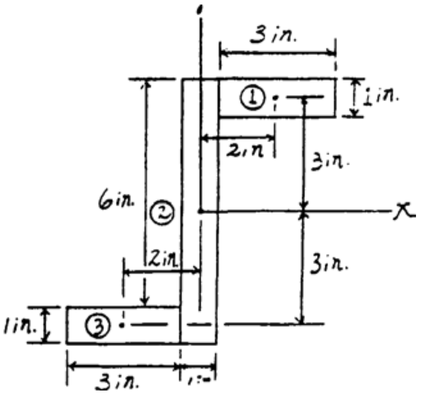
(a)

10–71. Determine the product of inertia of the cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



Product of Inertia : The area for each segment, its centroid and product of inertia with respect to x and y axes are tabulated below.

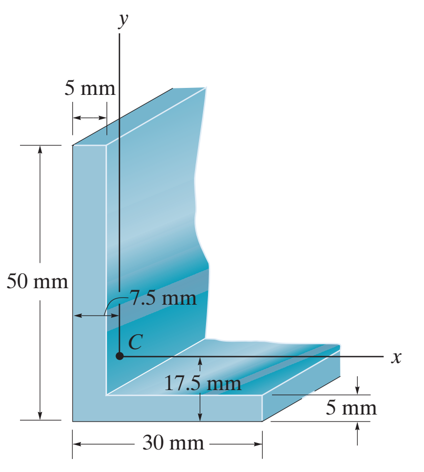
Segment	A_i (in²)	$(d_x)_i$ (in.)	$(d_y)_i$ (in.)	$(I_{xy})_i$ (in⁴)
1	3(1)	2	3	18.0
2	7(1)	0	0	0
3	3(1)	−2	−3	18.0



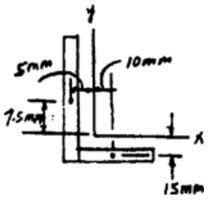
Thus,

$$I_{xy} = \Sigma(I_{xy})_i = 36.0 \text{ in}^4 \qquad \textbf{Ans}$$

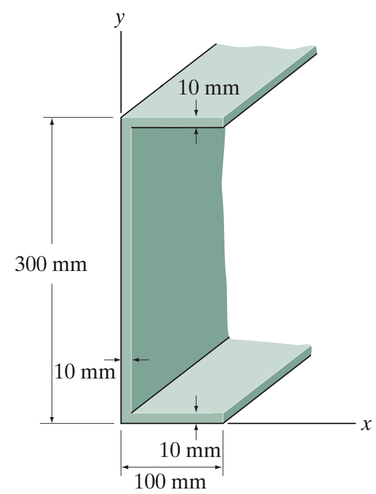
***10–72.** Determine the product of inertia for the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



$$I_{xy} = 25(5)(10)(-15) + 50(5)(-5)(7.5) = -28.1(10^3) \text{ mm}^4 \qquad \textbf{Ans}$$



•10–73. Determine the product of inertia of the beam's cross-sectional area with respect to the x and y axes.



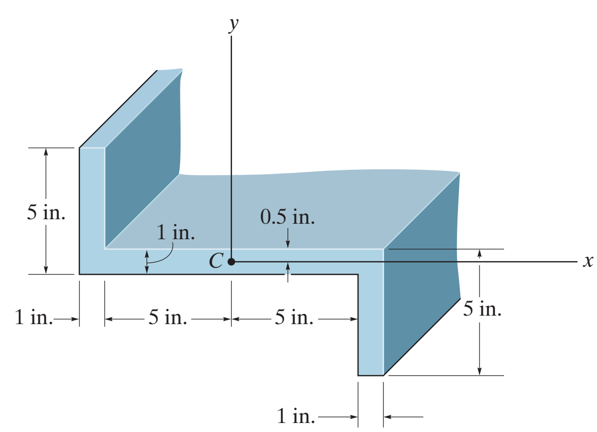
Composite Parts: The composite cross-sectional area of the beam can be subdivided into three segments. The perpendicular distances measured from the centroid of each element to the x and y axes are also indicated.

Product of Inertia: Since the centroidal axes are the axes of all the segments are the axes of symmetry, then $\bar{I}_{x'y'} = 0$. Thus, the product of inertia of each segment with respect to the x and y axes can be determined using the parallel-axis theorem.

$$I_{xy} = \bar{I}_{x'y'} + Ad_x d_y = Ad_x d_y$$

$$= 90(10)(55)(295) + 300(10)(5)(150) + 90(10)(55)(5) = \Sigma I_{xy} = 17.1(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

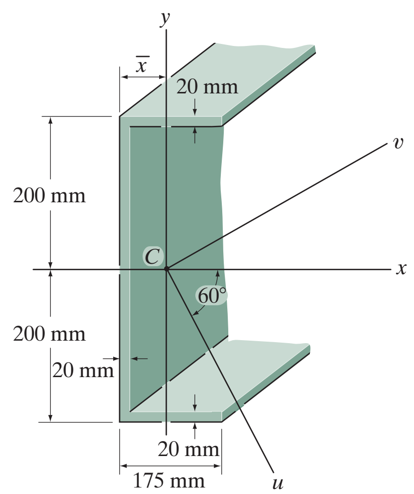
10–74. Determine the product of inertia for the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.

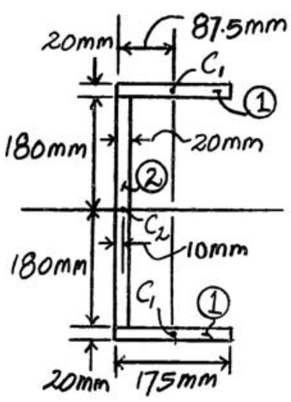


$$I_{xy} = 5(1)(5.5)(-2) + 5(1)(-5.5)(2)$$

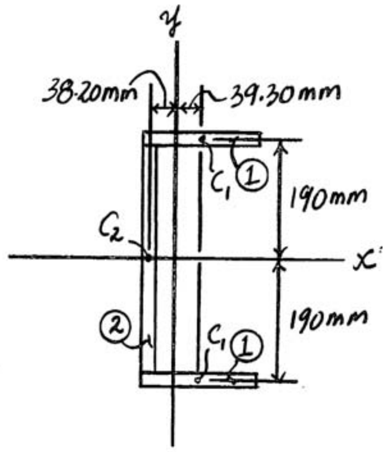
$$= -110\text{ in}^4 \quad \textbf{Ans}$$

10–75. Locate the centroid $\bar{x}$ of the beam's cross-sectional area and then determine the moments of inertia and the product of inertia of this area with respect to the u and v axes. The axes have their origin at the centroid C.





(a)



(b)

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left of the beam's cross-sectional area leads to

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{2[(87.5)(175)(20)] + 10(360)(20)}{2(175)(20) + 360(20)} = 48.204 \text{ mm} = 48.2 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from the centroid of each segment to the x and y axes with the parallel-axis theorem gives

$$I_x = 2\left[\frac{1}{12}(175)(20^3) + 175(20)(190)^2\right] + \frac{1}{12}(20)(360^3)$$

$$= 330.69(10^6)\text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(175^3) + 20(175)(39.30^2)\right] + \left[\frac{1}{12}(360)(20^3) + 360(20)(38.20^2)\right]$$

$$= 39.42(10^6)\text{ mm}^4$$

Since the cross-sectional area is symmetrical about the x axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the u and v Axes: With $\theta = -60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{330.69 + 39.42}{2} + \frac{330.69 - 39.42}{2}\cos(-120°) - 0\sin(-120°)\right](10^6)$$

$$= 112.25(10^6)\text{ mm}^4 = 112(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \left[\frac{330.69 + 39.42}{2} - \frac{330.69 - 39.42}{2}\cos(-120°) + 0\sin(-120°)\right](10^6)$$

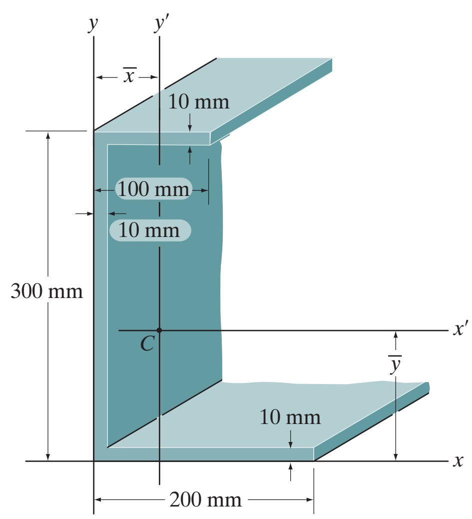
$$= 257.88(10^6)\text{ mm}^4 = 258(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \left[\frac{330.69 - 39.42}{2}\sin(-120°) + 0\cos(-120°)\right](10^6)$$

$$= -126.12(10^6)\text{ mm}^4 = -126(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

***10–76.** Locate the centroid $(\bar{x}, \bar{y})$ of the beam's cross-sectional area, and then determine the product of inertia of this area with respect to the centroidal x' and y' axes.



Composite Parts: The composite cross-sectional area of the beam can be subdivided into three segments as shown in Figs. *a* and *b*.

Centroid: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *a*.

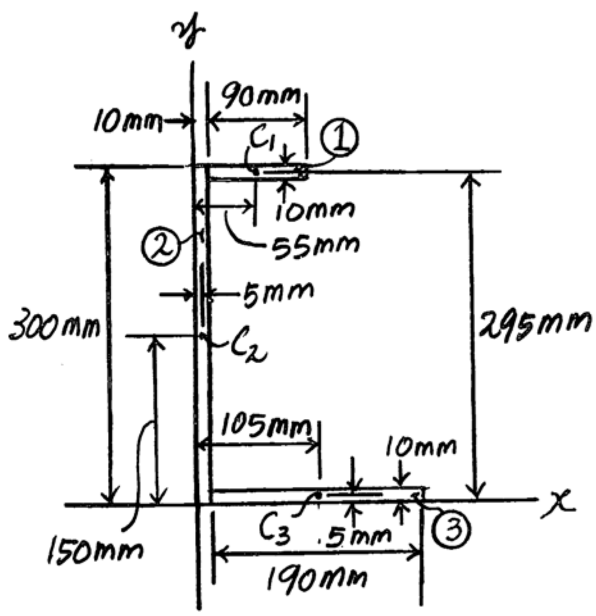
$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{55(90(10)) + 5(300(10)) + 105(190(10))}{90(10) + 300(10) + 190(10)} = \frac{264(10^3)}{5.8(10^3)} = 45.52 \text{ mm} = 45.5 \text{ mm} \quad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{295(90(10)) + 150(300(10)) + 5(190(10))}{90(10) + 300(10) + 190(10)} = \frac{725(10^3)}{5.8(10^3)} = 125 \text{ mm} \quad \textbf{Ans.}$$

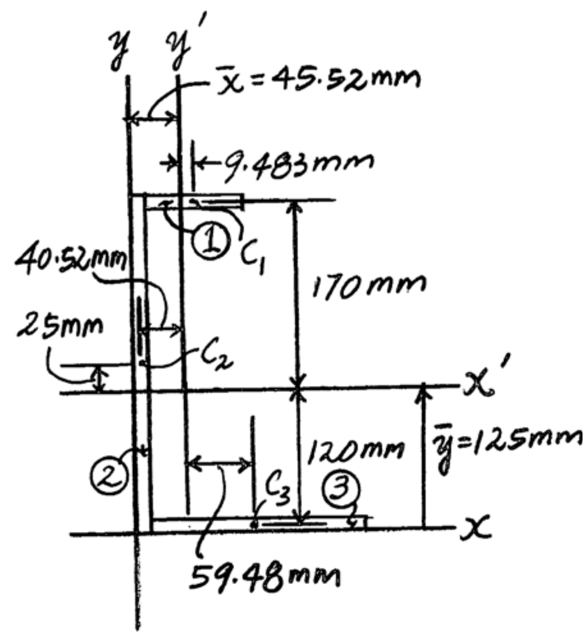
Product of Inertia: Since the centroidal axes are the axes of all the segments are the axes of symmetry, then $\bar{I}_{x'y'} = 0$. Thus, the product of inertia of each segment with respect to the x' and y' axes can be determined using the parallel - axis theorem. The perpendicular distances measured from the centroid of each segment to the x' and y' axes are indicated in Fig. *b*.

$$I_{x'y'} = \bar{I}_{x'y'} + Ad_{x'}d_{y'} = Adx'dy'$$

$$= 90(10)(9.483)(170) + 300(10)(-40.52)(25) + 190(10)(59.48)(-120) = \Sigma I_{x'y'} = -15.15(10^6) \text{ mm}^4 \quad \textbf{Ans.}$$

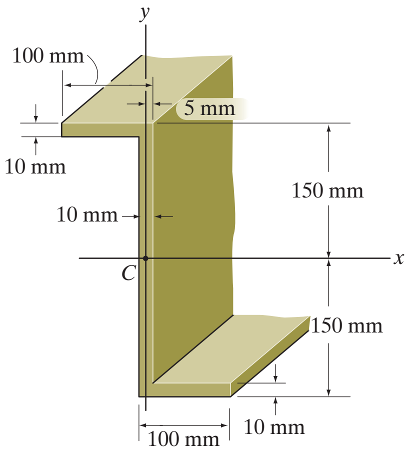


(a)



(b)

•**10–77.** Determine the product of inertia of the beam's cross-sectional area with respect to the centroidal x and y axes.

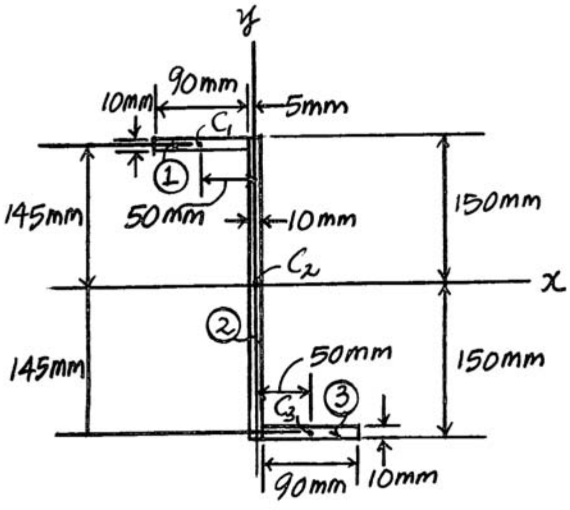


Composite Parts: The composite cross - sectional area of the beam can be subdivided into three segments as shown in Fig. *a*. The perpendicular distances measured from the centroid of each segment to the x and y axes are also indicated.

Product of Inertia: The product of inertia of segment (2) is equal to zero, $\left(\bar{I}_{x'y'}\right)_2 = 0$ since the x and y axes are axes of symmetry. Also, the centroidal axes of segments (1) and (3) are axes of symmetry. Thus, $\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{x'y'}\right)_3 = 0$. Using the parallel - axis theorem, the product of inertia of the two segments can be determined from

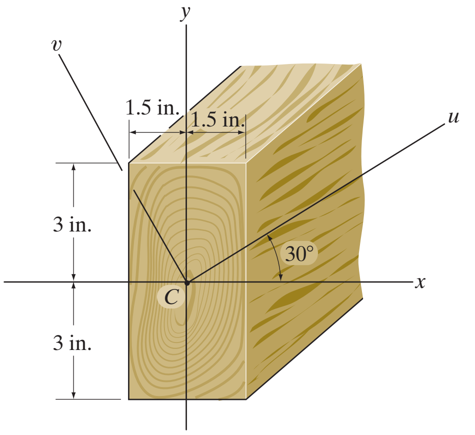
$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y = A d_x d_y$$

$$= 90(10)(-50)(145) + 90(10)(50)(-145) = -13.05(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

10–78. Determine the moments of inertia and the product of inertia of the beam's cross-sectional area with respect to the u and v axes.



Moment and Product of Inertia with Respect to the x and y Axes: Since the rectangular beam cross - sectional area is symmetrical about the x and y axes, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4 \qquad I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

Moment and Product of Inertia with Respect to the u and v Axes: With $\theta = 30°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{54 + 13.5}{2} + \frac{54 - 13.5}{2}\cos 60° - 0 \sin 60°$$

$$= 43.9 \text{ in}^4$$ **Ans.**

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{54 + 13.5}{2} - \frac{54 - 13.5}{2}\cos 60° + 0 \sin 60°$$

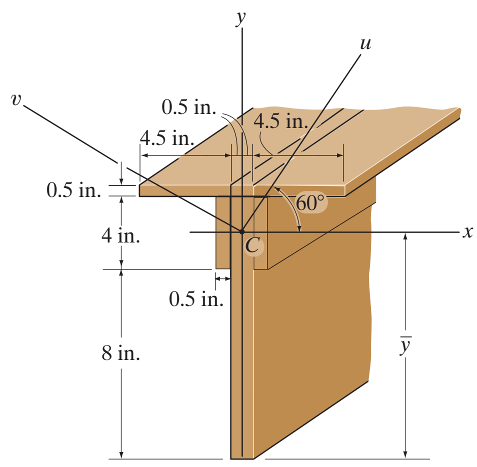
$$= 23.6 \text{ in}^4$$ **Ans.**

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{54 - 13.5}{2}\sin 60° + 0 \cos 60°$$

$$= 17.5 \text{ in}^4$$ **Ans.**

10–79. Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia and the product of inertia of this area with respect to the u and v axes.



Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma y_C A}{\Sigma A} = \frac{12.25(10)(0.5) + 2[10(4)(0.5)] + 6(12)(1)}{10(0.5) + 2(4)(0.5) + 12(1)} = 8.25 \text{ in.} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = \left[\frac{1}{12}(10)(0.5^3) + 10(0.5)(4)^2\right] + 2\left[\frac{1}{12}(0.5)(4^3) + 0.5(4)(1.75)^2\right] + \left[\frac{1}{12}(1)(12^3) + 1(12)(2.25)^2\right]$$

$$= 302.44 \text{ in}^4$$

$$I_y = \frac{1}{12}(0.5)(10^3) + 2\left[\frac{1}{12}(4)(0.5^3) + 4(0.5)(0.75)^2\right] + \frac{1}{12}(12)(1^3)$$

$$= 45 \text{ in}^4$$

Since the cross-sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the *u* and *v* Axes: With $\theta = 60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{302.44 + 45}{2} + \frac{302.44 - 45}{2}\cos 120° - 0 \sin 120°$$

$$= 109.36 \text{ in}^4 = 109 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

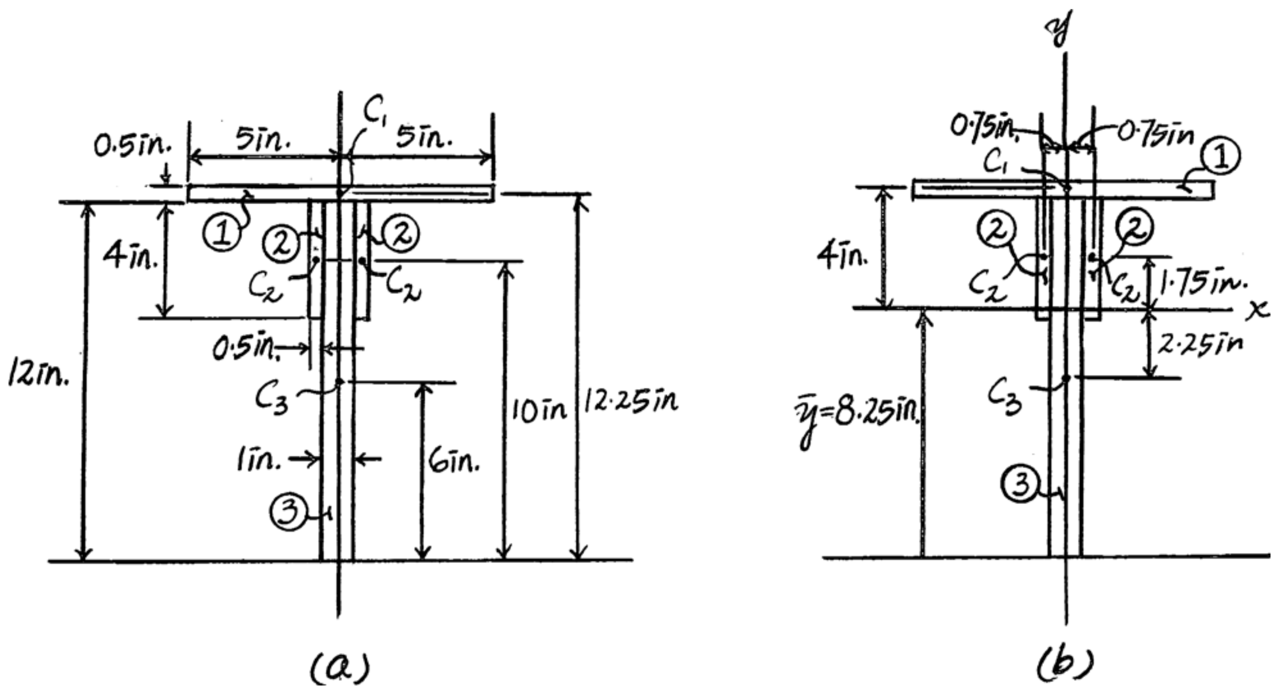
$$= \frac{302.44 + 45}{2} - \frac{302.44 - 45}{2}\cos 120° + 0 \sin 120°$$

$$= 238.08 \text{ in}^4 = 238 \text{ in}^4 \qquad \textbf{Ans.}$$

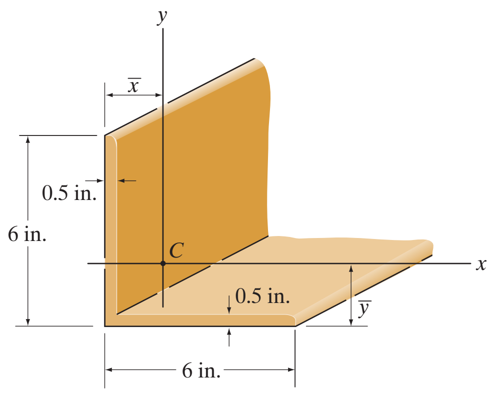
$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{302.44 - 45}{2}\sin 120° + 0 \cos 120°$$

$$= 111.47 \text{ in}^4 = 111 \text{ in}^4 \qquad \textbf{Ans.}$$


0.5in.
5in.
5in.
C_1
4in.
12in.
0.5in.
1in.
6in.
10in
12.25in
C_2
C_3
(a)
0.75in.
0.75in
C_1
4in.
1.75in.
2.25in
$\bar{y}$=8.25in.
C_3
x
y
(b)

***10–80.** Locate the centroid $\bar{x}$ and $\bar{y}$ of the cross-sectional area and then determine the orientation of the principal axes, which have their origin at the centroid C of the area. Also, find the principal moments of inertia.



Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left and bottom of the beam's cross - sectional area are indicated in Fig. *a*. Thus,

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.25(6)(0.5) + 3.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{3(6)(0.5) + 0.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the x **and** y **Axes:** The perpendicular distances measured from the centroid of each segment to the x and y axes are indicated in Fig. *b*.

$$I_x = \left[\frac{1}{12}(5.5)(0.5^3) + 5.5(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(6^3) + 0.5(6)(1.315^2)\right]$$
$$= 19.908 \text{ in}^4$$

$$I_y = \left[\frac{1}{12}(6)(0.5^3) + 6(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(5.5^3) + 0.5(5.5)(1.565^2)\right]$$
$$= 19.908 \text{ in}^4$$

$$I_{xy} = 6(0.5)(-1.435)(1.315) + 5.5(0.5)(1.565)(-1.435)$$
$$= -11.837 \text{ in}^4$$

Principal Moment of Inertia:

$$I_{\substack{\max \\ \min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}$$

$$= \frac{19.908 + 19.908}{2} \pm \sqrt{\left(\frac{19.908 - 19.908}{2}\right)^2 + (-11.837)^2}$$

$$= 19.908 \pm 11.837$$

$$I_{\max} = 31.7 \text{ in}^4 \qquad I_{\min} = 8.07 \text{ in}^4 \qquad \textbf{Ans.}$$

Orientation of Principal Axes:

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-(-11.837)}{(19.08 - 19.08)/2} = \infty$$

$$2\theta_p = 90° \text{ and } -90°$$

$$\theta_p = 45° \text{ and } -45° \qquad \textbf{Ans.}$$

Substituting $\theta = \theta_p = 45°$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left(\frac{19.908 + 19.908}{2}\right) + \left(\frac{19.908 - 19.908}{2}\right)\cos 90° - (-11.837)\sin 90°$$

$$= 31.7\ \text{in}^4 = I_{max}$$

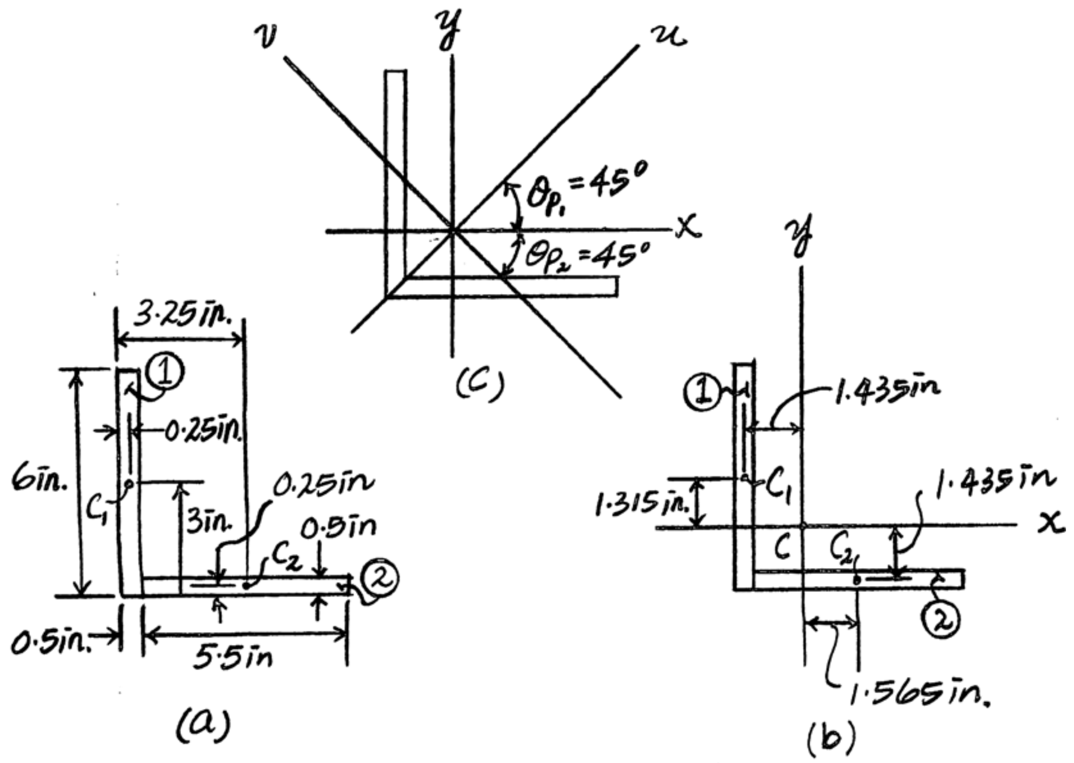
This shows that I_{max} corresponds to the principal axis orientated at

$I_{max} = 31.7\ \text{in}^4$ $\quad (\theta_p)_1 = 45°$ **Ans.**

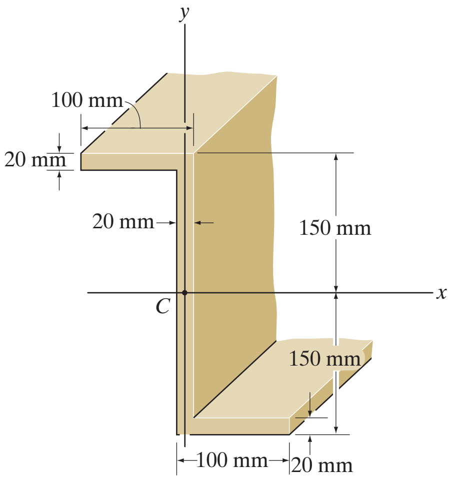
and I_{min} corresponds to the principal axis orientated at

$I_{min} = 8.07\ \text{in}^4$ $\quad (\theta_p)_2 = -45°$ **Ans.**

The orientation of the principal axes is shown in Fig. *c*.



•10–81. Determine the orientation of the principal axes, which have their origin at centroid C of the beam's cross-sectional area. Also, find the principal moments of inertia.



Moment and Product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from each subdivided segment to the x and y axes are indicated in Fig. a. Applying the parallel - axis theorem,

$$I_x = 2\left[\frac{1}{12}(80)(20^3) + 80(20)(140^2)\right] + \frac{1}{12}(20)(300^3) = 107.83(10^6)\text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(80^3) + 20(80)(50^2)\right] + \frac{1}{12}(300)(20^3) = 9.907(10^6)\text{ mm}^4$$

$$I_{xy} = 80(20)(-50)(140) + 80(20)(50)(-140) = -22.4(10^6)\text{ mm}^4$$

Principal Moment of Inertia:

$$I_{\substack{\max \\ \min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}$$

$$= \left[\frac{107.83 + 9.907}{2} \pm \sqrt{\left(\frac{107.83 - 9.907}{2}\right)^2 + (-22.4)^2}\right](10^6)$$

$$= 58.867 \pm 53.841$$

$$I_{\max} = 112.71(10^6) = 113(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_{\min} = 5.026(10^6) = 5.03(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

Orientation of Principal Axes:

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-(-22.4)(10^6)}{(107.83 - 9.907)(10^6)/2} = 0.4575$$

$$2\theta_p = 24.58° \text{ and } -155.42°$$

$$\theta_p = 12.29° \text{ and } -77.71°$$

Substituting $\theta = \theta_p = 12.29°$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{107.83 + 9.907}{2} + \left(\frac{107.83 - 9.907}{2}\right)\cos 24.58° - (-22.4)\sin 24.58°$$

$$= 112.71(10^6)\text{ mm}^4 = I_{\max}$$

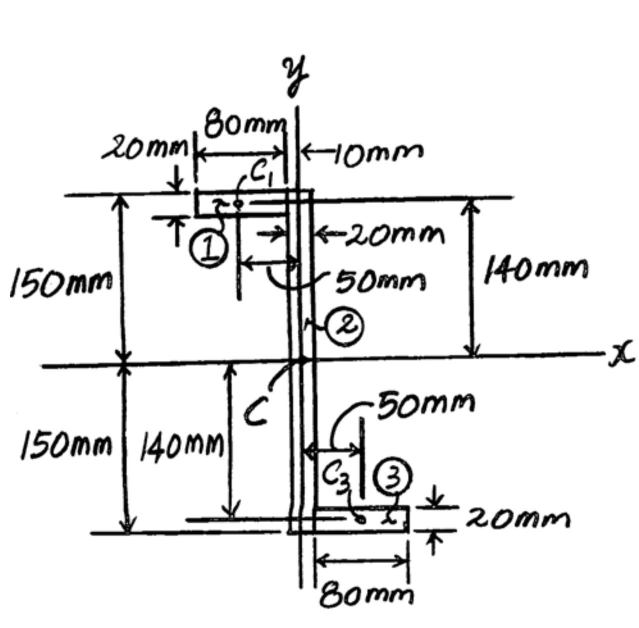
This shows that $I_{\max}$ corresponds to the principal axis orientated at

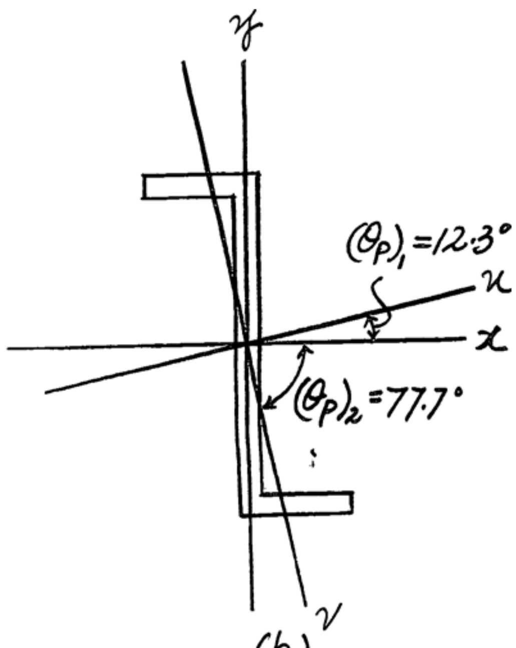
$I_{\max} = 113(10^6)\text{ mm}^4$ $\qquad (\theta_p)_1 = 12.3°$ **Ans.**

and $I_{\min}$ corresponds to the principal axis orientated at

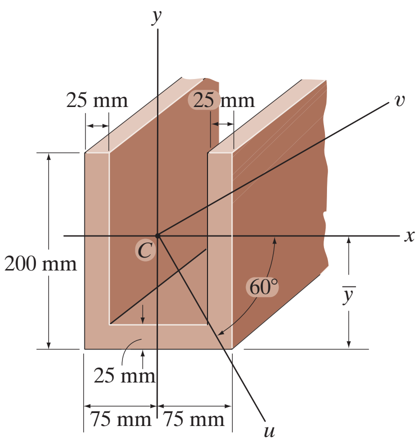
$I_{\min} = 5.03(10^6)\text{ mm}^4$ $\qquad (\theta_p)_2 = -77.7°$ **Ans.**

The orientation of the principal axes are shown in Fig. b.


y
80mm
20mm
10mm
C_1
1
20mm
50mm
140mm
150mm
2
x
C
50mm
150mm
140mm
C_3
3
20mm
80mm
(a)


y
$(\theta_P)_1 = 12.3°$
u
x
$(\theta_P)_2 = 77.7°$
v
(b)

10–82. Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia of this area and the product of inertia with respect to the u and v axes. The axes have their origin at the centroid C.



Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma y_C A}{\Sigma A} = \frac{2[100(200)(25)] + 12.5(2.5)(100)}{2(200)(25) + 25(100)} = 82.5 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = 2\left[\frac{1}{12}(25)(200^3) + 25(200)(17.5)^2\right] + \left[\frac{1}{12}(100)(25^3) + 100(25)(70)^2\right]$$

$$= 48.78(10^6) \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(200)(25^3) + 200(25)(62.5)^2\right] + \frac{1}{12}(25)(100^3)$$

$$= 41.67(10^6) \text{ mm}^4$$

Since the cross-sectional area is symmetrical about the y axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the *u* and *v* Axes: With $\theta = -60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{48.78 + 41.67}{2} + \left(\frac{48.78 - 41.67}{2}\right)\cos(-120°) - 0\sin(-120°)\right](10^6)$$

$$= 43.4(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

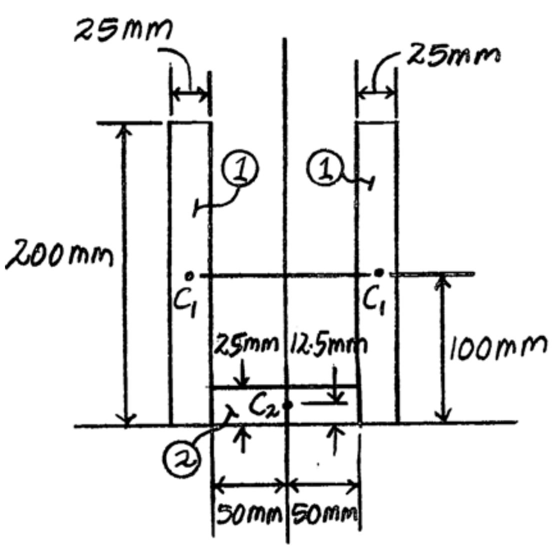
$$= \left[\frac{48.78 + 41.67}{2} - \left(\frac{48.78 - 41.67}{2}\right)\cos(-120°) + 0\sin(-120°)\right](10^6)$$

$$= 47.0(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

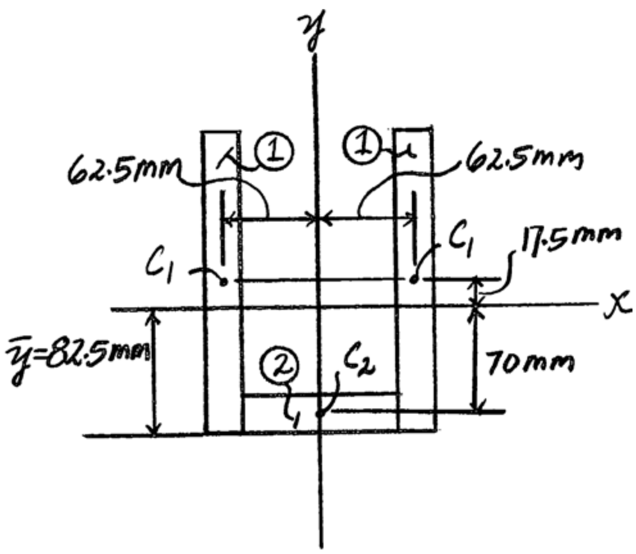
$$= \left(\frac{48.78 - 41.67}{2}\right)\sin(-120°) + 0\cos(-120°)$$

$$= -3.08(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$



25mm
25mm
200mm
1
1
C_1
C_1
25mm
12.5mm
100mm
C_2
2
50mm
50mm

(a)



y
62.5mm
1
1
62.5mm
C_1
C_1
17.5mm
x
$\bar{y}$=82.5mm
2
C_2
70mm

(b)

10–83. Solve Prob. 10–75 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{2[(87.5)(175)(20)] + 10(360)(20)}{2(175)(20) + 360(20)} = 48.204 \text{ mm} = 48.2 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = 2\left[\frac{1}{12}(175)(20^3) + 175(20)(190)^2\right] + \frac{1}{12}(20)(360^3)$$

$$= 330.69(10^6) \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(175^3) + 20(175)(39.30^2)\right] + \left[\frac{1}{12}(360)(20^3) + 360(20)(38.20^2)\right]$$

$$= 39.42(10^6) \text{ mm}^4$$

Since the cross-sectional area is symmetrical about the *x* axis, $I_{xy} = 0$.

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \left(\frac{330.69 + 39.42}{2}\right)(10^6) \text{ mm}^4 = 185.06(10^6) \text{ mm}^4$$

The coordinates of the reference point *A* are $[330.69, 0](10^6) \text{ mm}^4$. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

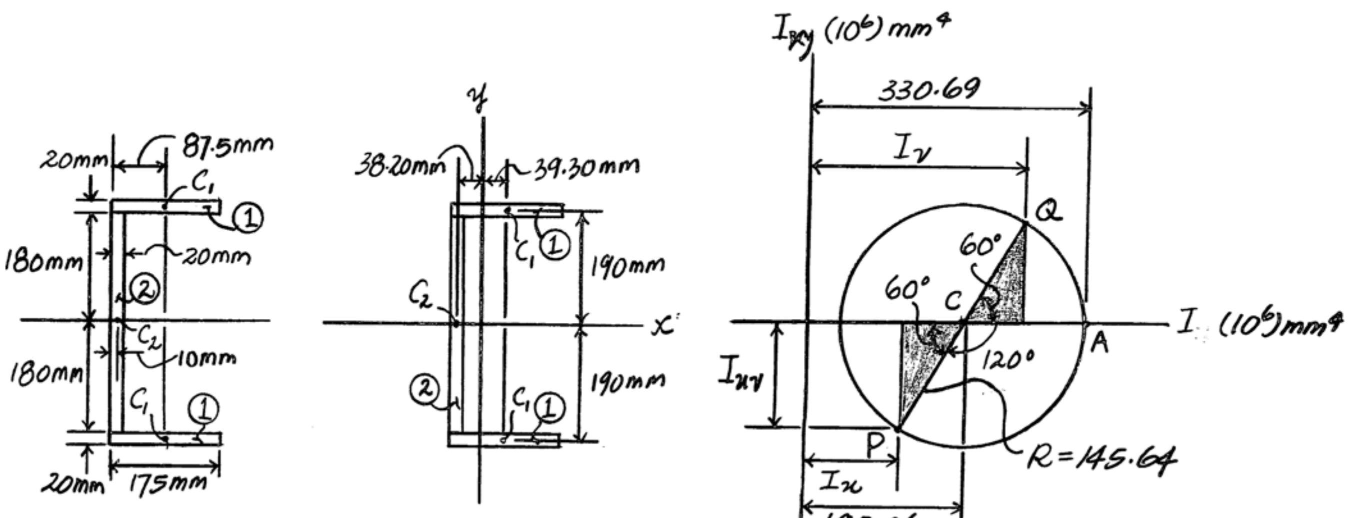
$$R = CA = (330.69 - 185.06)(10^6) = 145.64(10^6) \text{ mm}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

$$I_u = (185.06 - 145.64 \cos 60°)(10^6) = 112(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = (185.06 + 145.64 \cos 60°)(10^6) = 258(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = (-145.64 \sin 60°)(10^6) = -126(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$



(a) (b) (c)

***10–84.** Solve Prob. 10–78 using Mohr's circle.

Moment and Product of Inertia with Respect to the x and y Axes: Since the rectangular beam cross - sectional area is symmetrical about the x and y axes, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4 \qquad I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

Construction of Mohr's Circle: The center C of the circle lies along the u axis at a distance

$$I_{avg} = \frac{I_x + I_y}{2} = \frac{54 + 13.5}{2} = 33.75 \text{ in}^4$$

The coordinates of the reference point A are $(54, 0)$ in^4. The circle can be constructed as shown in Fig. *a*. The radius of the circle is

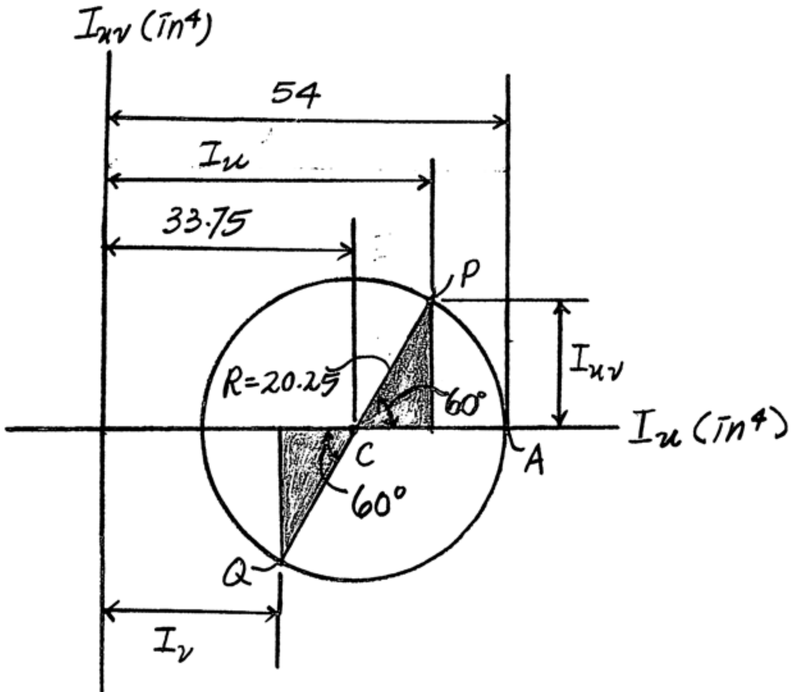
$$R = CA = 54 - 33.75 = 20.25 \text{ in}^4$$

Moment and Product of Inertia with Respect to the u and v Axes: By referring to the geometry of the circle,

$$I_u = 33.75 + 20.25\cos 60° = 43.9 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_v = 33.75 - 20.25\cos 60° = 23.6 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = 20.25\sin 60° = 17.5 \text{ in}^4 \qquad \textbf{Ans.}$$



(a)

•10–85. Solve Prob. 10–79 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{12.25(10)(0.5) + 2[10(4)(0.5)] + 6(12)(1)}{10(0.5) + 2(4)(0.5) + 12(1)} = 8.25 \text{ in.} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = \left[\frac{1}{12}(10)(0.5^3) + 10(0.5)(4)^2\right] + 2\left[\frac{1}{12}(0.5)(4^3) + 0.5(4)(1.75)^2\right] + \left[\frac{1}{12}(1)(12^3) + 1(12)(2.25)^2\right]$$

$$= 302.44 \text{ in}^4$$

$$I_y = \frac{1}{12}(0.5)(10^3) + 2\left[\frac{1}{12}(4)(0.5^3) + 4(0.5)(0.75)^2\right] + \frac{1}{12}(12)(1^3)$$

$$= 45 \text{ in}^4$$

Since the cross-sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{avg} = \frac{I_x + I_y}{2} = \frac{302.44 + 45}{2} = 173.72 \text{ in}^4$$

The coordinates of the reference point *A* are (302.44, 0) in^4. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

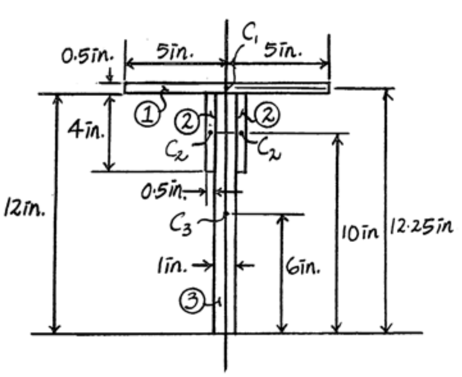
$$R = CA = (302.44 - 173.72) = 128.72 \text{ in}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

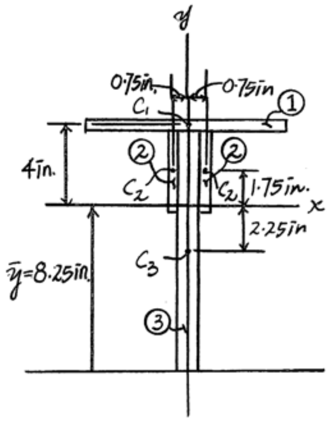
$$I_u = 173.72 - 128.72 \cos 60° = 109 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_v = 173.72 + 128.72 \cos 60° = 238 \text{ in}^4 \qquad \textbf{Ans.}$$

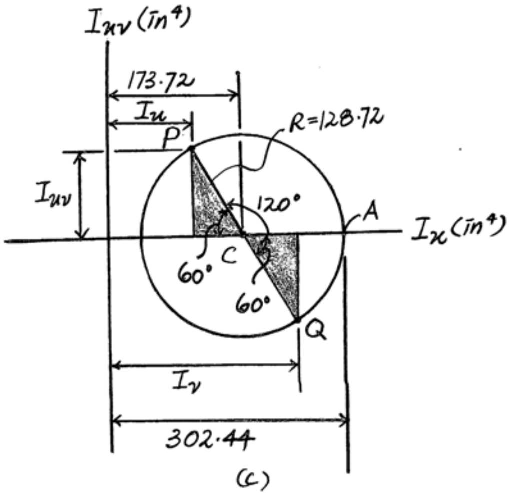
$$I_{uv} = 128.72 \sin 60° = 111 \text{ in}^4 \qquad \textbf{Ans.}$$



(a)



(b)



(c)

10–86. Solve Prob. 10–80 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the left and bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.25(6)(0.5) + 3.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{3(6)(0.5) + 0.25(5.5)(0.5)}{6(0.5) + 5.5(0.5)} = 1.685 \text{ in.} = 1.68 \text{ in} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*.

$$I_x = \left[\frac{1}{12}(5.5)(0.5^3) + 5.5(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(6^3) + 0.5(6)(1.315^2)\right]$$

$$= 19.908 \text{ in}^4$$

$$I_y = \left[\frac{1}{12}(6)(0.5^3) + 6(0.5)(1.435^2)\right] + \left[\frac{1}{12}(0.5)(5.5^3) + 0.5(5.5)(1.565^2)\right]$$

$$= 19.908 \text{ in}^4$$

$$I_{xy} = 6(0.5)(-1.435)(1.315) + 5.5(0.5)(1.565)(-1.435)$$

$$= -11.837 \text{ in}^4$$

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{avg} = \frac{I_x + I_y}{2} = \frac{19.908 + 19.908}{2} \text{ in}^4 = 19.908 \text{ in}^4$$

The coordinates of the reference point *A* are (19.908, −11.837) in^4. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

$$R = CA = 11.837 \text{ in}^4$$

Principal Moment of Inertia: By referring to the geometry of the circle,

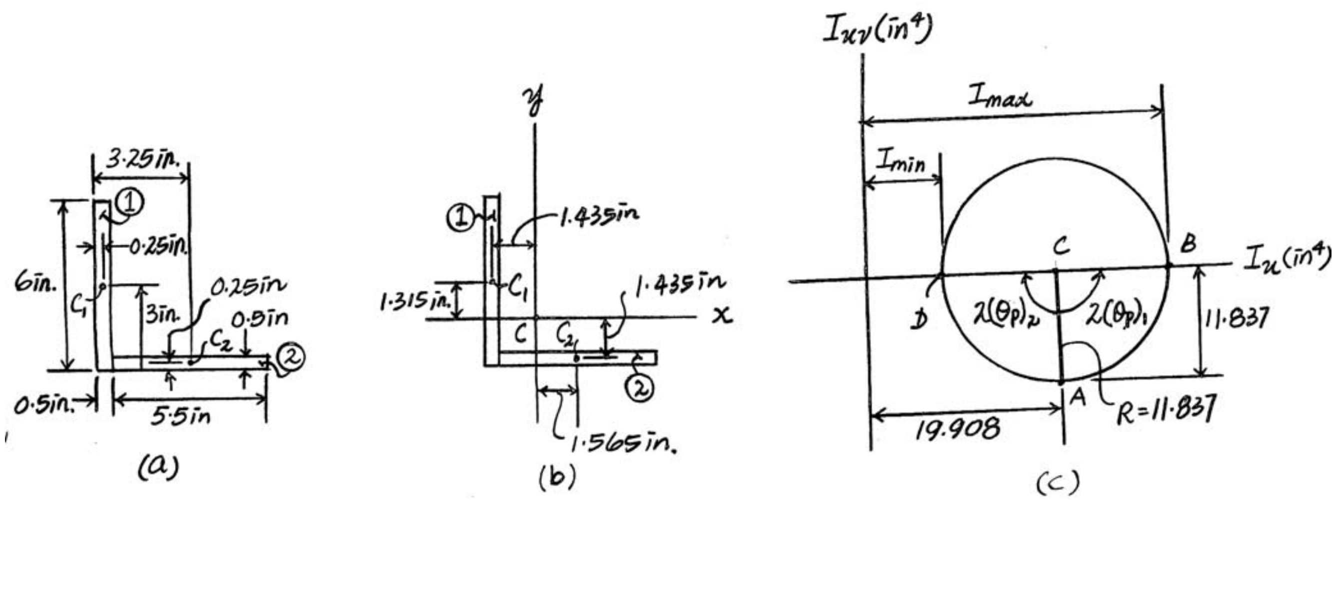
$$I_{max} = 19.908 + 11.837 = 31.7 \text{ in}^4 \qquad \textbf{Ans.}$$

$$I_{min} = 19.908 - 11.837 = 8.07 \text{ in}^4 \qquad \textbf{Ans.}$$

Orientation of Principal Axes: Here $(\theta_p)_1$ and $(\theta_p)_2$ are the orientation of the principal axes about which I_{max} and I_{min} occur, respectively. By observing the geometry of the circle,

$2(\theta_p)_1 = 90°$ $\quad (\theta_p)_1 = 45°$ (counterclockwise) **Ans.**

$2(\theta_p)_2 = 90°$ $\quad (\theta_p)_2 = 45°$ (clockwise) **Ans.**



10–87. Solve Prob. 10–81 using Mohr's circle.

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from each subdivided segment to the *x* and *y* axes are indicated in Fig. *a*. Applying the parallel - axis theorem,

$$I_x = \left[\frac{1}{12}(80)(20^3) + 80(20)(140^2)\right] + \frac{1}{12}(20)(300^3) = 107.83(10^6)\ \text{mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(80^3) + 20(80)(50^2)\right] + \frac{1}{12}(300)(20^3) = 9.907(10^6)\ \text{mm}^4$$

$$I_{xy} = 80(20)(-50)(140) + 80(20)(50)(-140) = -22.4(10^6)\ \text{mm}^4$$

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance of

$$I_{avg} = \frac{I_x + I_y}{2} = \left(\frac{107.83 + 9.907}{2}\right)(10^6) = 58.867(10^6)\ \text{mm}^4$$

The coordinates of the reference point *A* are $(107.83,\ -22.4)\ \text{mm}^4$. The circle can be constructed as shown in Fig. *b*. The radius of the circle is

$$R = CA = \left(\sqrt{(107.83 - 58.867)^2 + (-22.4)^2}\right)(10^6) = 53.84(10^6)\ \text{mm}^4$$

Principal Moment of Inertia: By referring to the geometry of the circle, we obtain

$$I_{max} = (53.84 + 53.84)(10^6) = 112.70(10^6) = 113(10^6)\ \text{mm}^4$$ **Ans.**

$$I_{min} = (53.84 - 53.84)(10^6) = 5.026(10^6) = 5.03(10^6)\ \text{mm}^4$$ **Ans.**

Orientation of Principal Axes: Here $(\theta_p)_1$ and $(\theta_p)_2$ are the orientation of the principle axes about which I_{max} and I_{min} occur. From the geometry of the circle,

$$\tan 2(\theta_p)_1 = \frac{22.4}{107.83 - 58.867}$$

$$2(\theta_p)_1 = 24.58°$$

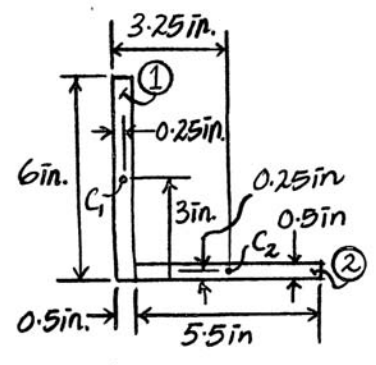
$$(\theta_p)_1 = 12.29° = 12.3°\ \text{(counterclockwise)}$$ **Ans.**

Thus,

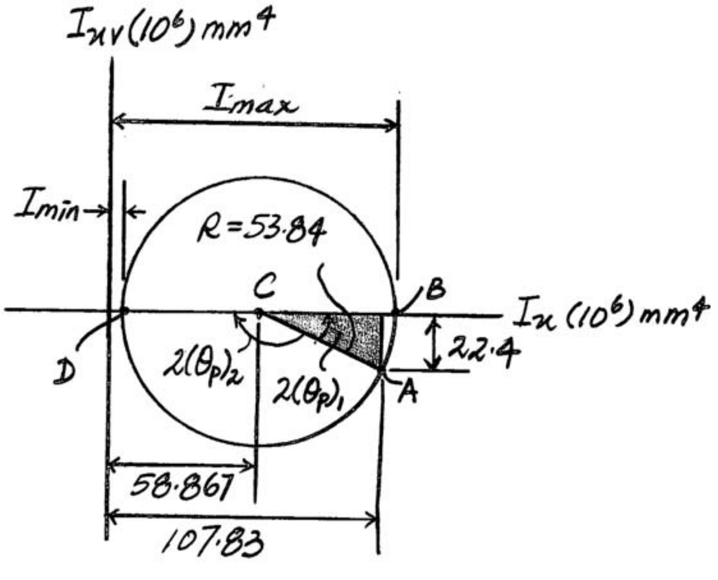
$$2(\theta_p)_2 = 180° - 2(\theta_p)_1 = 155.42°$$

$$(\theta_p)_2 = 77.7°\ \text{(clockwise)}$$ **Ans.**

The orientation of the principle axes are shown in Fig. *c*.



(a)



(b)

*__10–88.__ Solve Prob. 10–82 using Mohr's circle.

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross-sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{2[100(200)(25)] + 12.5(2.5)(100)}{2(200)(25) + 25(100)} = 82.5 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel-axis theorem,

$$I_x = 2\left[\frac{1}{12}(25)(200^3) + 25(200)(17.5)^2\right] + \left[\frac{1}{12}(100)(25^3) + 100(25)(70)^2\right]$$

$$= 48.78(10^6) \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(200)(25^3) + 200(25)(62.5)^2\right] + \frac{1}{12}(25)(100^3)$$

$$= 41.67(10^6) \text{ mm}^4$$

Since the cross-sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.

Construction of Mohr's Circle: The center *C* of the circle lies along the *u* axis at a distance

$$I_{avg} = \frac{I_x + I_y}{2} = \left(\frac{48.78 + 41.67}{2}\right)(10^6) \text{ mm}^4 = 45.22(10^6) \text{ mm}^4$$

The coordinates of the reference point *A* are $[48.78, 0](10^6) \text{ mm}^4$. The circle can be constructed as shown in Fig. *a*. The radius of the circle is

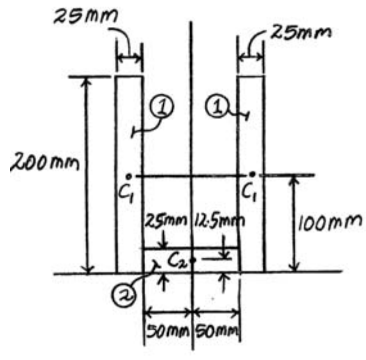
$$R = CA = (48.78 - 45.22)(10^6) = 3.56(10^6) \text{ mm}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

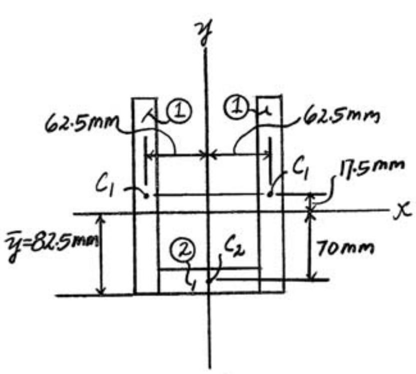
$$I_u = (45.22 - 3.56\cos 60°)(10^6) = 43.4(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = (45.22 + 3.56\cos 60°)(10^6) = 47.0(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

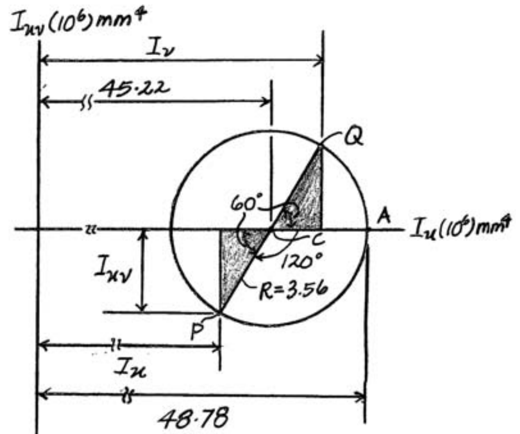
$$I_{uv} = -3.56\sin 60° = -3.08(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$



(a)

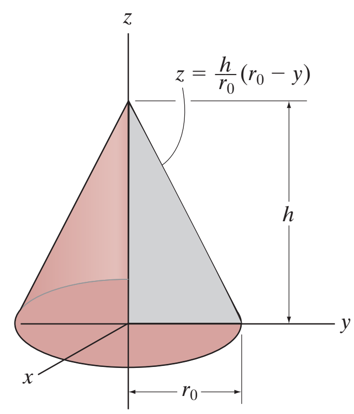


(b)



(c)

•10–89. Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \frac{r_o}{h}z$.

Thus, $dm = \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is $dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2$

$= \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$.

Mass: The mass of the cone can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_o^2 h$$

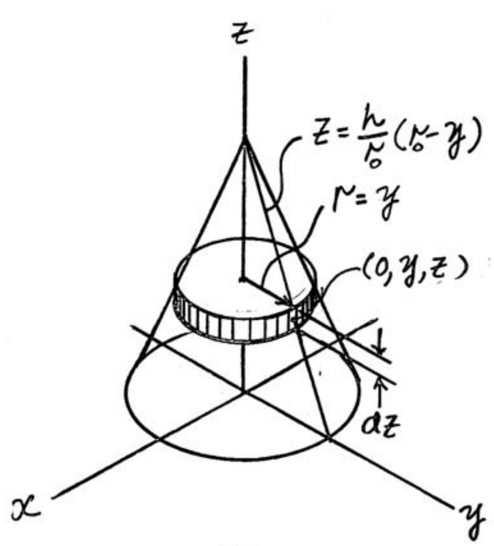
Mass Moment of Inertia: Integrating dI_z,

$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_o^4 h$$

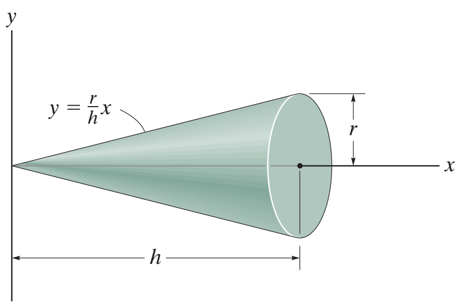
From the result of the mass, we obtain $\rho\pi r_o^2 h = 3m$. Thus, I_z can be written as

$$I_z = \frac{1}{10}\left(\rho\pi r_o^2 h\right)r_o^2 = \frac{1}{10}(3m)r_o^2 = \frac{3}{10}mr_o^2$$ **Ans.**

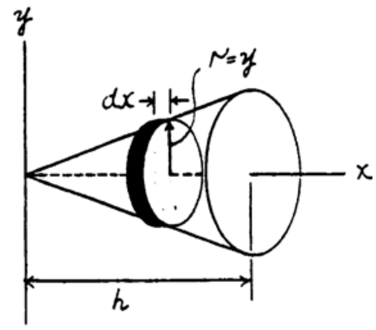


(a)

10–90. Determine the mass moment of inertia I_x of the right circular cone and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



Differential Disk Element : The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2 dx = \rho\pi\left(\frac{r^2}{h^2}x^2\right)dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\left[\rho\pi\left(\frac{r^2}{h^2}x^2\right)dx\right]\left(\frac{r^2}{h^2}x^2\right) = \frac{\rho\pi r^4}{2h^4}x^4 dx$.



Total Mass : Performing the integration, we have

$$m = \int_m dm = \int_0^h \rho\pi\left(\frac{r^2}{h^2}x^2\right)dx = \frac{\rho\pi r^2}{h^2}\left(\frac{x^3}{3}\right)\Bigg|_0^h = \frac{1}{3}\rho\pi r^2 h$$

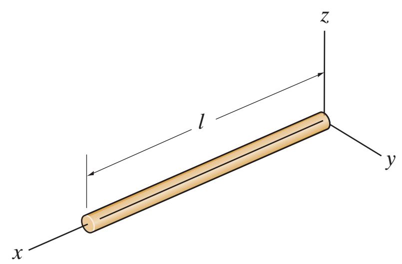
Mass Moment of Inertia : Performing the integration, we have

$$I_x = \int dI_x = \int_0^h \frac{\rho\pi r^4}{2h^4}x^4 dx = \frac{\rho\pi r^4}{2h^4}\left(\frac{x^5}{5}\right)\Bigg|_0^h = \frac{1}{10}\rho\pi r^4 h$$

The mass moment of inertia expressed in terms of the total mass is

$$I_x = \frac{3}{10}\left(\frac{1}{3}\rho\pi r^2 h\right)r^2 = \frac{3}{10}mr^2 \qquad \textbf{Ans}$$

10–91. Determine the mass moment of inertia I_y of the slender rod. The rod is made of material having a variable density $\rho = \rho_0(1 + x/l)$, where ρ_0 is constant. The cross-sectional area of the rod is A. Express the result in terms of the mass m of the rod.



Differential Element: The mass of the differential element shown shaded in Fig. *a* is $dm = \rho dV = \rho_o\left(1+\frac{x}{l}\right)(Adx) = \rho_o A\left(1+\frac{x}{l}\right)dx$, where dA is the cross - sectional area of the rod. The mass moment of inertia of this element about the z axis is $dI_z = r^2 dm$. Here, $r = x$. Thus, $dI_z = \rho_o A\left[x^2\left(1+\frac{x}{l}\right)\right]dx = \rho_o A\left(x^2+\frac{x^3}{l}\right)dx$.

Mass: The mass of the rod can be determined by integrating dm. Thus,

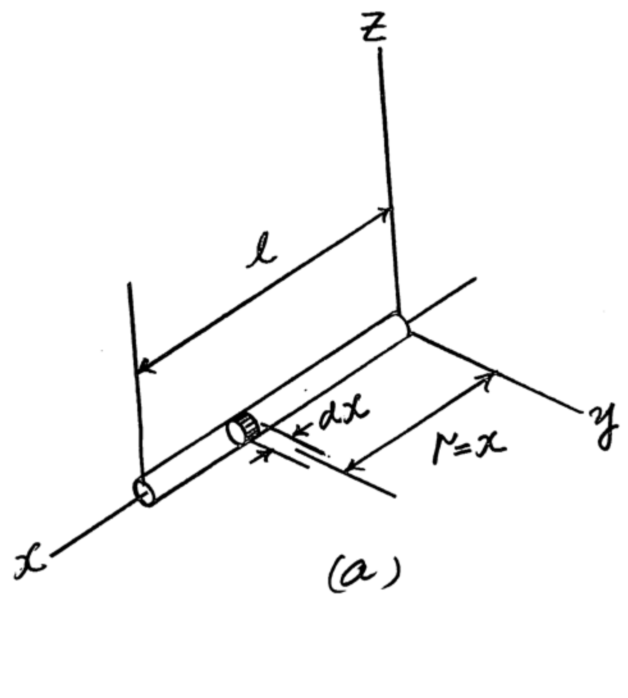
$$m = \int dm = \int_0^l \rho_o A\left[\left(1+\frac{x}{l}\right)\right]dx = \rho_o A\left(x+\frac{x^2}{2l}\right)\bigg|_0^l = \frac{3}{2}\rho_o Al$$

Mass Moment of Inertia: Integrating dI_z,

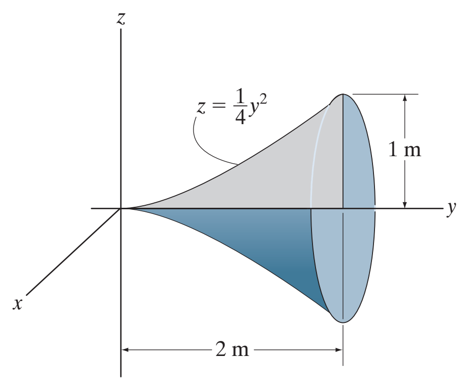
$$I_z = \int dI_z = \int_0^l \rho_o A\left(x^2+\frac{x^3}{l}\right)dx = \rho_o A\left(\frac{x^3}{3}+\frac{x^4}{4l}\right)\bigg|_0^l = \frac{7}{12}\rho_o Al^3$$

From the result of the mass, we obtain $\rho_o Al = \frac{2}{3}m$. Thus, I_z can be written as

$$I_z = \frac{7}{12}(\rho_o Al)l^2 = \frac{7}{12}\left(\frac{2}{3}m\right)l^2 = \frac{7}{18}ml^2$$ **Ans.**



***10–92.** Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the solid.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^2$. Thus, $dm = \rho\pi\left(\frac{1}{4}y^2\right)^2 dy = \frac{\rho\pi}{16}y^4 dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2$ $= \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(\frac{1}{4}y^2\right)^4 dy = \frac{\rho\pi}{512}y^8 dy.$

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{2\text{ m}} \frac{\rho\pi}{16}y^4 dy = \frac{\rho\pi}{16}\left(\frac{y^5}{5}\right)\Big|_0^{2\text{ m}} = \frac{2}{5}\rho\pi$$

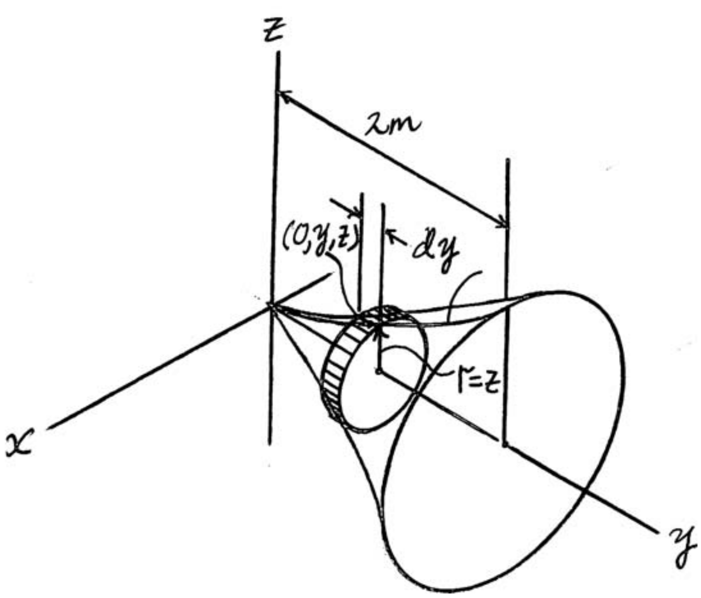
Mass Moment of Inertia: Integrating dI_y,

$$I_y = \int dI_y = \int_0^{2\text{m}} \frac{\rho\pi}{512}y^8 dy$$

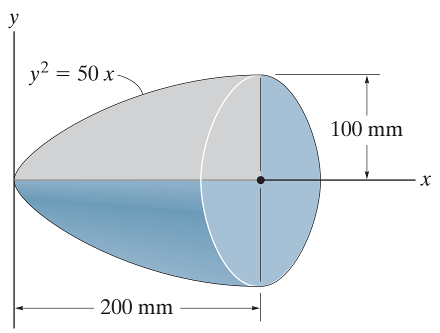
$$= \frac{\rho\pi}{512}\left(\frac{y^9}{9}\right)\Big|_0^{2\text{m}} = \frac{\pi\rho}{9}$$

From the result of the mass, we obtain $\pi\rho = \frac{5m}{2}$. Thus, I_y can be written as

$$I_y = \frac{1}{9}\left(\frac{5m}{2}\right) = \frac{5}{18}m$$ **Ans.**



•10–93. The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \text{Mg/m}^3$.



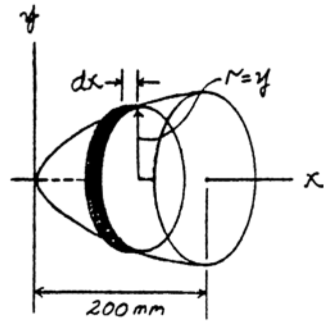
Differential Disk Element : The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2 dx = \rho\pi(50x)\,dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}[\rho\pi(50x)\,dx](50x) = \frac{\rho\pi}{2}(2500x^2)\,dx$.

Total Mass : Performing the integration, we have

$$m = \int_m dm = \int_0^{200\text{mm}} \rho\pi(50x)\,dx = \rho\pi\left(25x^2\right)\Big|_0^{200\text{mm}} = 1\left(10^6\right)\rho\pi$$

Mass Moment of Inertia : Performing the integration, we have

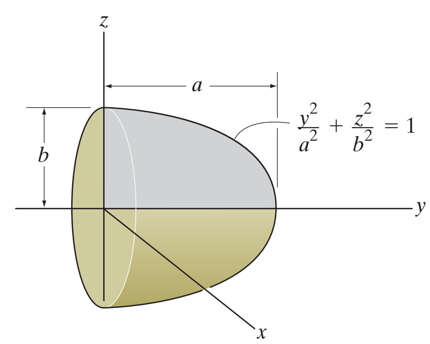
$$I_x = \int dI_x = \int_0^{200\text{mm}} \frac{\rho\pi}{2}\left(2500x^2\right)dx = \frac{\rho\pi}{2}\left(\frac{2500x^3}{3}\right)\Big|_0^{200\text{mm}} = 3.333\left(10^9\right)\rho\pi$$



The radius of gyration is

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{3.333(10^9)\rho\pi}{1(10^6)\rho\pi}} = 57.7\ \text{mm} \qquad \textbf{Ans}$$

10–94. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The density of the material is ρ. Express the result in terms of the mass m of the semi-ellipsoid.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = b\sqrt{1+\frac{y^2}{a^2}}$.

Thus, $dm = \rho\pi\left(b\sqrt{1+\frac{y^2}{a^2}}\right)^2 dz = \rho\pi b^2\left(1-\frac{y^2}{a^2}\right)dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2$

$= \frac{1}{2}(\rho\pi r^2 dy)r^2 = \frac{1}{2}\rho\pi r^4 dy = \frac{1}{2}\rho\pi\left(b\sqrt{1+\frac{y^2}{a^2}}\right)^4 dy = \frac{1}{2}\rho\pi b^4\left(1-\frac{y^2}{a^2}\right)^2 dy = \frac{1}{2}\rho\pi b^4\left(1+\frac{y^4}{a^4}-\frac{2y^2}{a^2}\right)dy.$

Mass: The mass of the semi - ellipsoid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^a \rho\pi b^2\left(1-\frac{y^2}{a^2}\right)dy = \rho\pi b^2\left(y-\frac{y^3}{3a^2}\right)\Big|_0^a = \frac{2}{3}\rho\pi ab^2$$

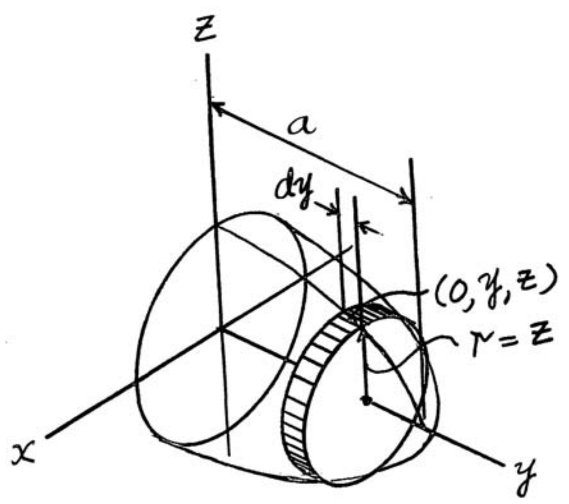
Mass Moment of Inertia: Integrating dI_y,

$$I_y = \int dI_y = \int_0^a \frac{1}{2}\rho\pi b^4\left(1+\frac{y^4}{a^4}-\frac{2y^2}{a^2}\right)dy$$

$$= \frac{1}{2}\rho\pi b^4\left(y+\frac{y^5}{5a^4}-\frac{2y^3}{3a^2}\right)\Big|_0^a = \frac{4}{15}\rho\pi ab^4$$

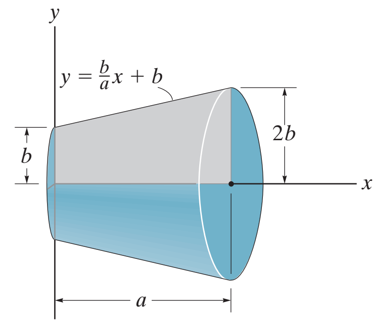
From the result of the mass, we obtain $\rho\pi ab^2 = \frac{3m}{2}$. Thus, I_y can be written as

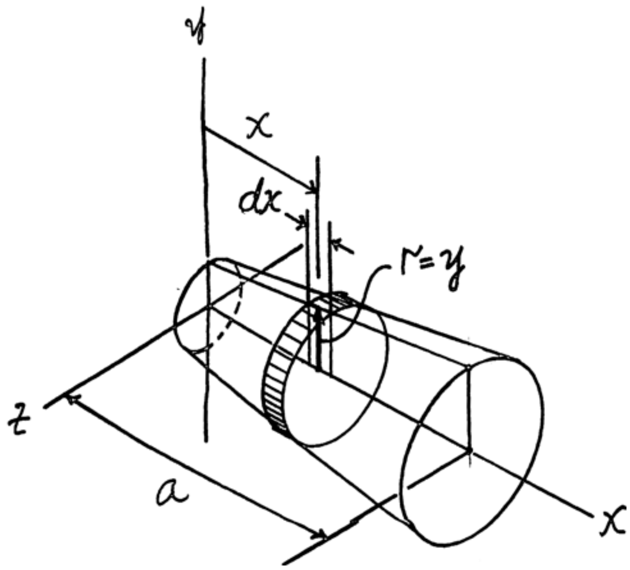
$$I_y = \frac{4}{15}\left(\rho\pi ab^2\right)b^2 = \frac{4}{15}\left(\frac{3m}{2}\right)b^2 = \frac{2}{5}mb^2$$ **Ans.**



(a)

10–95. The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The material has a constant density ρ.





$$dm = \rho dV = \rho\pi y^2 dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dm y^2 = \frac{1}{2}\rho\pi y^4 dx$$

$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

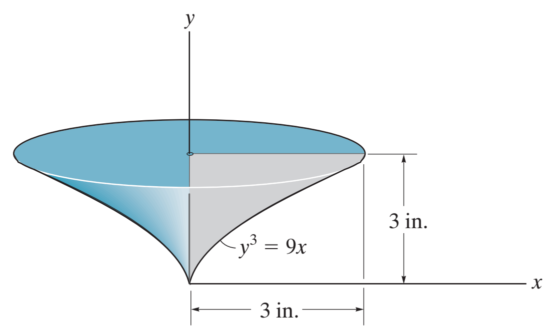
$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

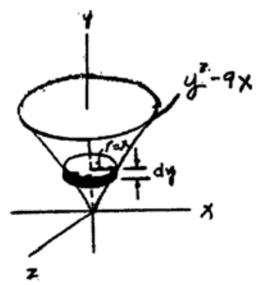
$$= \frac{31}{10}\rho\pi a b^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi a b^2$$

$$I_x = \frac{93}{70}mb^2 \quad \textbf{Ans}$$

***10–96.** The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is $\gamma = 380\ \text{lb/ft}^3$.





The moment of inertia of the solid : The mass of the disk element $dm = \rho\pi x^2\,dy = \frac{1}{81}\rho\pi y^6\,dy$.

$$dI_y = \frac{1}{2}dm\,x^2$$

$$= \frac{1}{2}(\rho\pi x^2 dy)x^2$$

$$= \frac{1}{2}\rho\pi x^4 dy = \frac{1}{2(9^4)}\rho\pi y^{12} dy$$

$$I_y = \int dI_y = \frac{1}{2(9^4)}\rho\pi\int_0^3 y^{12} dy$$

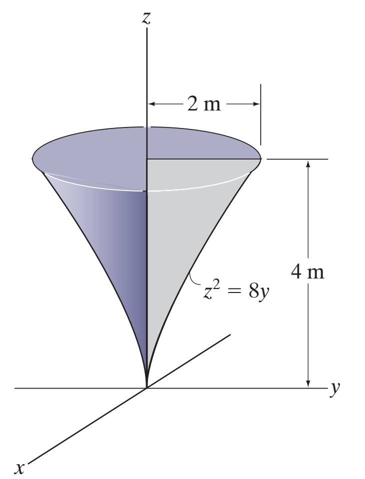
$$= 29.362\rho$$

The mass of the solid :

$$m = \int_m dm = \frac{1}{81}\rho\pi\int_0^3 y^6 dy = 12.118\rho$$

$$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.362\rho}{12.118\rho}} = 1.56\ \text{in.} \qquad \textbf{Ans}$$

•10–97. Determine the mass moment of inertia I_z of the solid formed by revolving the shaded area around the z axis. The density of the material is $\rho = 7.85\ \text{Mg/m}^3$.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. The mass moment of inertia of this element about the z axis is $dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz$. Here, $r = z = \frac{z^2}{8}$.

Thus, $dI_z = \frac{1}{2}\rho\pi\left(\frac{z^2}{8}\right) = \frac{\rho\pi}{8192}z^8 dz$.

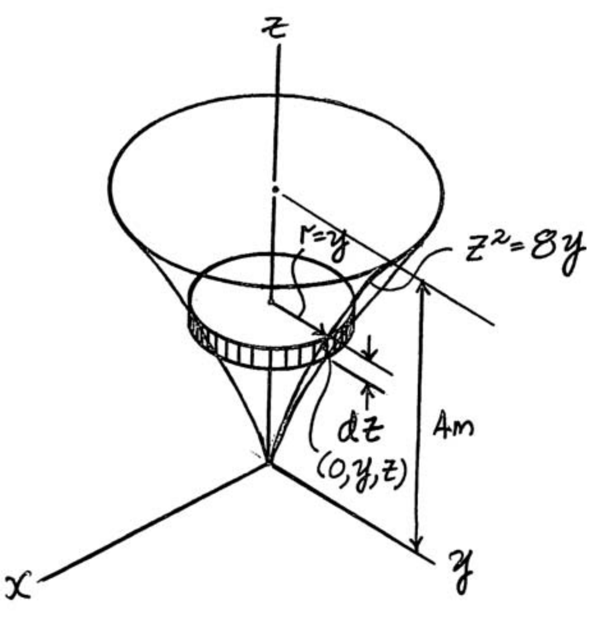
Mass Moment of Inertia: Integrating dI_z,

$$I_z = \int dI_z = \int_0^{4\text{ m}} \frac{\rho\pi}{8192}z^8 dz$$

$$= \frac{\rho\pi}{8192}\left(\frac{y^9}{9}\right)\Bigg|_0^{4\text{ m}} = \frac{32}{9}\pi\rho$$

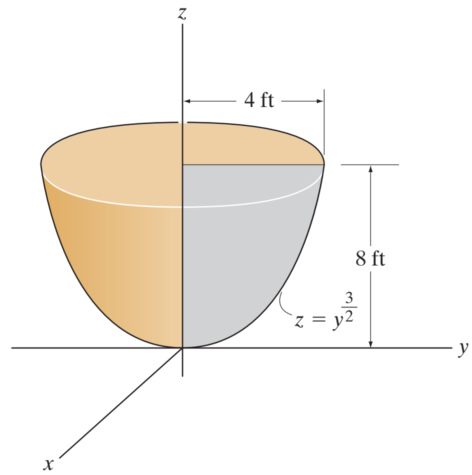
Substituting $\rho = 7.85(10^3)\ \text{kg/m}$ into I_z,

$$I_z = \frac{32}{9}\pi\rho\left[7.85(10^3)\right] = 87.7(10^3)\ \text{kg}\cdot\text{m}^2$$ **Ans.**



(a)

10–98. Determine the mass moment of inertia I_z of the solid formed by revolving the shaded area around the z axis. The solid is made of a homogeneous material that weighs 400 lb.



Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \left(\frac{\gamma}{g}\right)dV = \left(\frac{\gamma}{g}\right)\pi r^2 dz$. Here, $r = y = z^{2/3}$.

Thus, $dm = \left(\frac{\gamma}{g}\right)\pi\left(z^{2/3}\right)^2 dz = \left(\frac{\gamma}{g}\right)\pi z^{4/3} dz$. The mass moment of inertia of this element about the z axis is $dI_y = \frac{1}{2} dm r^2$

$$= \frac{1}{2}\left[\left(\frac{\gamma}{g}\right)\pi r^2 dz\right] r^2 = \frac{1}{2}\left(\frac{\gamma}{g}\right)\pi r^4 dz = \frac{1}{2}\left(\frac{\gamma}{g}\right)\pi\left(z^{2/3}\right)^4 dz = \frac{\pi}{2}\left(\frac{\gamma}{g}\right) z^{8/3} dz.$$

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{8\text{ ft}} \frac{\gamma}{g}\pi z^{4/3} dz = \frac{\pi\gamma}{g}\left(\frac{3}{7} z^{7/3}\right)\Bigg|_0^{8\text{ ft}} = \frac{384\pi}{7g}\gamma$$

The mass of the solid is $m = \frac{400}{g}$ slug. Thus,

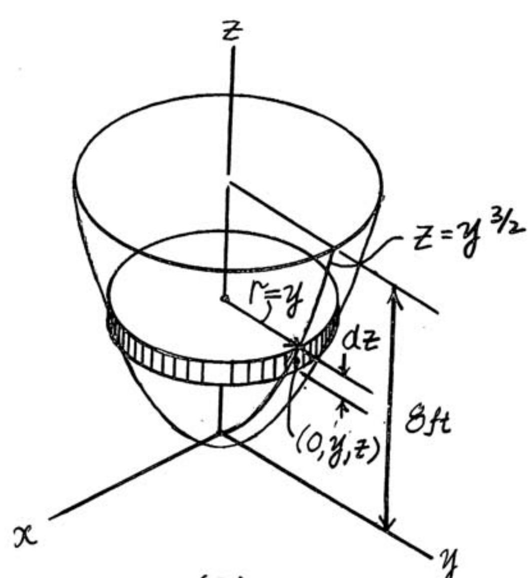
$$\frac{400}{g} = \frac{384\pi}{7g}\gamma \qquad \gamma = 2.321 \text{ lb/ft}^3$$

Mass Moment of Inertia: Integrating dI_y,

$$I_z = \int dI_z = \int_0^{8\text{ ft}} \frac{\pi}{2}\left(\frac{\gamma}{g}\right) z^{8/3} dz = \frac{\pi}{2}\left(\frac{\gamma}{g}\right)\left(\frac{3}{11} z^{11/3}\right)\Bigg|_0^{8\text{ ft}} = \frac{877.36\gamma}{g}$$

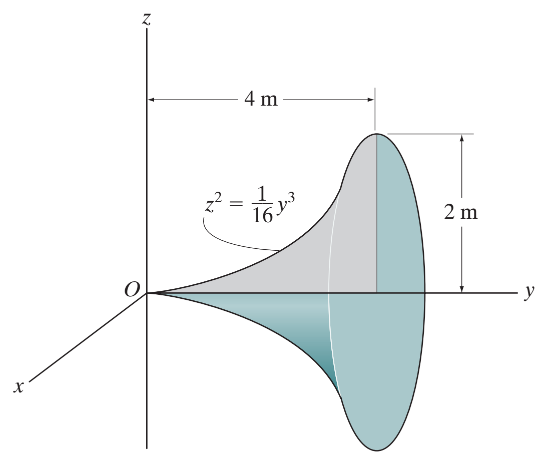
Substituting $\gamma = 2.321 \text{ lb/ft}^3$ and $g = 32.2 \text{ ft/s}^2$ into I_z,

$$I_z = \frac{877.36(2.321)}{32.2} = 63.2 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans.}$$



(a)

10–99. Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The total mass of the solid is 1500 kg.



Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^{3/2}$.

Thus, $dm = \rho\pi\left(\frac{1}{4}y^{3/2}\right)^2 dy = \frac{\rho\pi}{16}y^3 dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2$

$= \frac{1}{2}\left(\rho\pi r^2 dy\right)r^2 = \frac{\rho\pi}{2}r^4 dy = \frac{\rho\pi}{2}\left(\frac{1}{4}y^{3/2}\right)^4 dy = \frac{\rho\pi}{512}y^6 dy.$

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{4\text{ m}} \frac{\rho\pi}{16}y^3 dy = \frac{\rho\pi}{16}\left(\frac{y^4}{4}\right)\Bigg|_0^{4\text{ m}} = 4\pi\rho$$

The mass of the solid is $m = 1500$ kg. Thus,

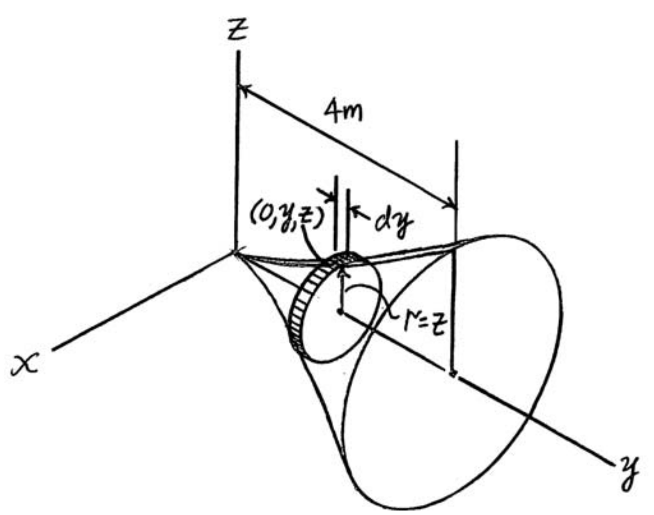
$$1500 = 4\pi\rho \qquad \rho = \frac{375}{\pi}\text{ kg/m}^3$$

Mass Moment of Inertia: Integrating dI_y,

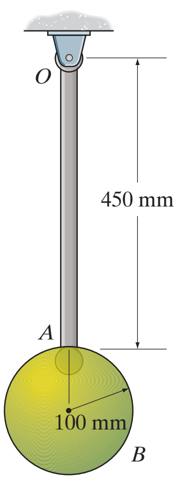
$$I_y = \int dI_y = \int_0^{4\text{ m}} \frac{\rho\pi}{512}y^6 dy = \frac{\rho\pi}{512}\left(\frac{y^7}{7}\right)\Bigg|_0^{4\text{ m}} = \frac{32\pi}{7}\rho$$

Substituting $\rho = \frac{375}{\pi}\text{ kg/m}^3$ into I_y,

$$I_y = \frac{32\pi}{7}\left(\frac{375}{\pi}\right) = 1.71(10^3)\text{ kg}\cdot\text{m}^2$$ **Ans.**



***10–100.** Determine the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through point O. The slender rod has a mass of 10 kg and the sphere has a mass of 15 kg.



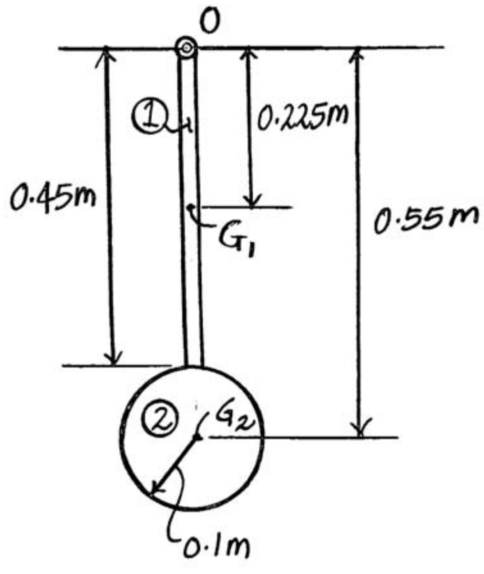
Composite Parts: The pendulum can be subdivided into two segments as shown in Fig. *a*. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Moment of Inertia: The moment of inertia of the slender rod segment (1) and the sphere segment (2) about the axis passing through their center of mass can be computed from $(I_G)_1 = \frac{1}{12}ml^2$ and $(I_G)_2 = \frac{2}{5}mr^2$. The mass moment of inertia of each segment about an axis passing through point O can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

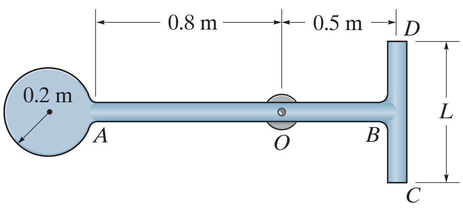
$$= \left[\frac{1}{12}(10)(0.45^2) + 10(0.225^2)\right] + \left[\frac{2}{5}(15)(0.1^2) + 15(0.55^2)\right]$$

$$= 5.27 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

•10–101. The pendulum consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass per unit length of 2 kg/m. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through point O?

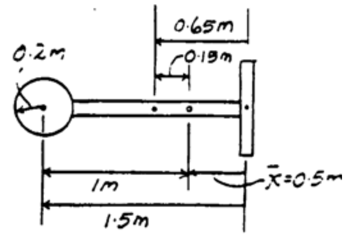


Location of Centroid : This problem requires $\bar{x} = 0.5$ m.

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m}$$

$$0.5 = \frac{1.5(6) + 0.65[1.3(2)] + 0[L(2)]}{6 + 1.3(2) + L(2)}$$

$$L = 6.39 \text{ m} \qquad \textbf{Ans}$$



Mass Moment of Inertia About an Axis Through Point O : The mass moment of inertia of each rod segment and disk about an axis passing through the center of mass can be determine using $(I_G)_i = \frac{1}{12}ml^2$ and $(I_G)_i = \frac{1}{2}mr^2$. Applying Eq. 10 – 15, we have

$$I_O = \Sigma(I_G)_i + m_i d_i^2$$

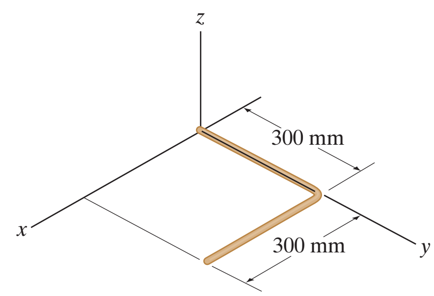
$$= \frac{1}{12}[1.3(2)](1.3^2) + [1.3(2)](0.15^2)$$

$$+ \frac{1}{12}[6.39(2)](6.39^2) + [6.39(2)](0.5^2)$$

$$+ \frac{1}{2}(6)(0.2^2) + 6(1^2)$$

$$= 53.2 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans}$$

10–102. Determine the mass moment of inertia of the 2-kg bent rod about the z axis.



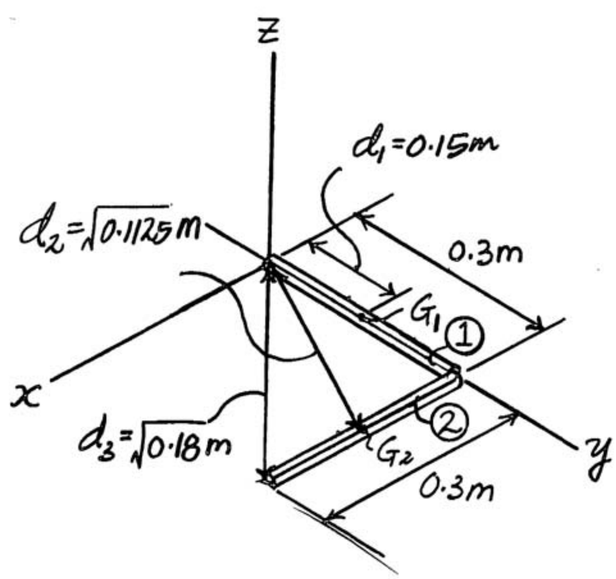
Composite Parts: The bent rod can be subdivided into two segments as shown in Fig. a.

Mass moment of Inertia: Here, the mass for each segment is $m_1 = m_2 = \dfrac{2\text{ kg}}{2} = 1$ kg. The perpendicular distances measured from the centers of mass of segments (1) and (2) are $d_1 = 0.15$ m and $d_2 = \sqrt{0.3^2 + 0.15^2} = \sqrt{0.1125}$ m, respectively. Thus, the mass moment of inertia of each segment about the z axis can be determined using the parallel-axis theorem.

$$I_z = \Sigma (I_z)_G + md^2$$

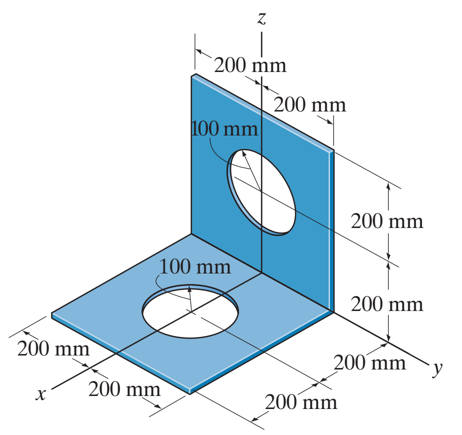
$$= \left[\frac{1}{12}(1)(0.3^2) + 1(0.15^2)\right] + \left[\frac{1}{12}(1)(0.3^2) + 1\left(\sqrt{0.1125}\right)^2\right]$$

$$= 0.150 \text{ kg}\cdot\text{m}^2$$ **Ans.**



(a)

10–103. The thin plate has a mass per unit area of $10\ \text{kg/m}^2$. Determine its mass moment of inertia about the y axis.



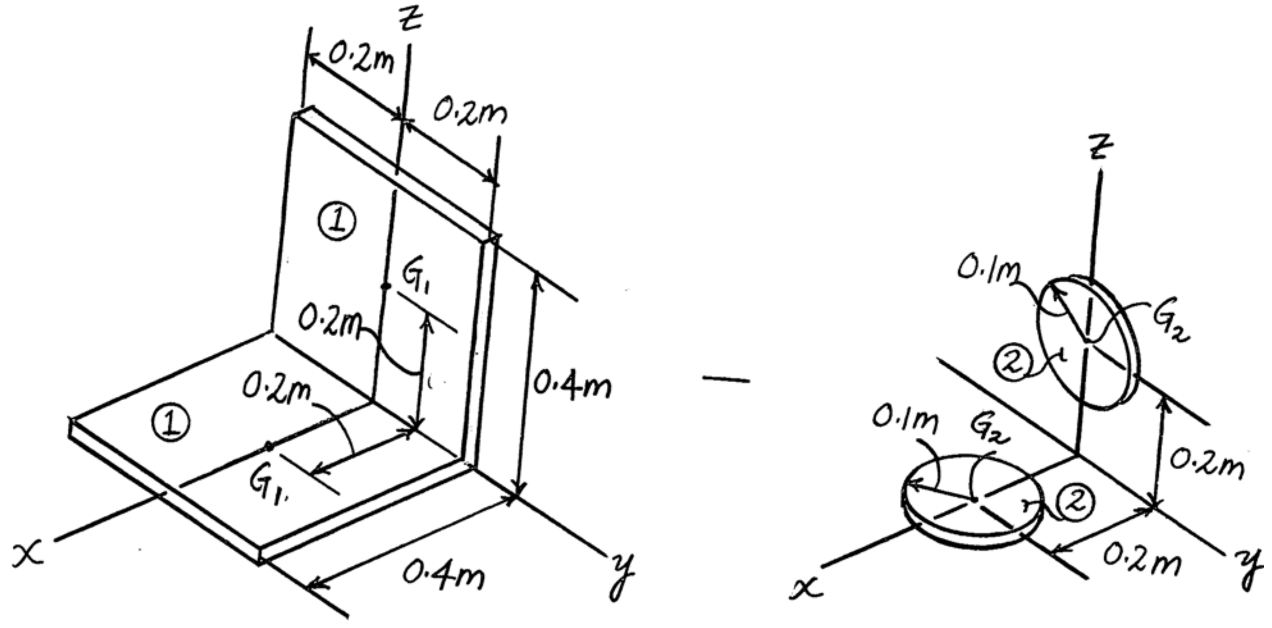
Composite Parts: The thin plate can be subdivided into segments as shown in Fig. *a*. Since the segments labeled (2) are both holes, they should be considered as negative parts.

Mass moment of Inertia: The mass of segments (1) and (2) are $m_1 = 0.4(0.4)(10) = 1.6$ kg and $m_2 = \pi(0.1^2)(10) = 0.1\pi$ kg. The perpendicular distances measured from the centroid of each segment to the y axis are indicated in Fig. *a*. The mass moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem.

$$I_y = \Sigma(I_y)_G + md^2$$

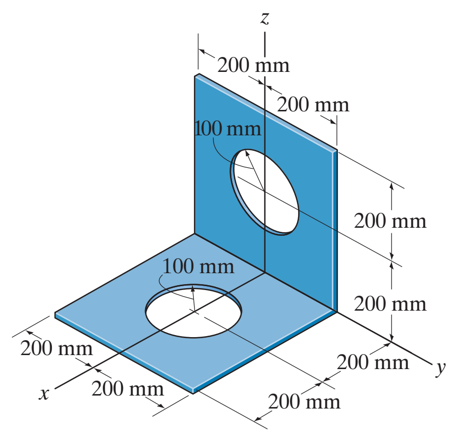
$$= 2\left[\frac{1}{12}(1.6)(0.4^2) + 1.6(0.2^2)\right] - 2\left[\frac{1}{4}(0.1\pi)(0.1^2) + 0.1\pi(0.2^2)\right]$$

$$= 0.144\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

***10–104.** The thin plate has a mass per unit area of $10\ \text{kg/m}^2$. Determine its mass moment of inertia about the z axis.



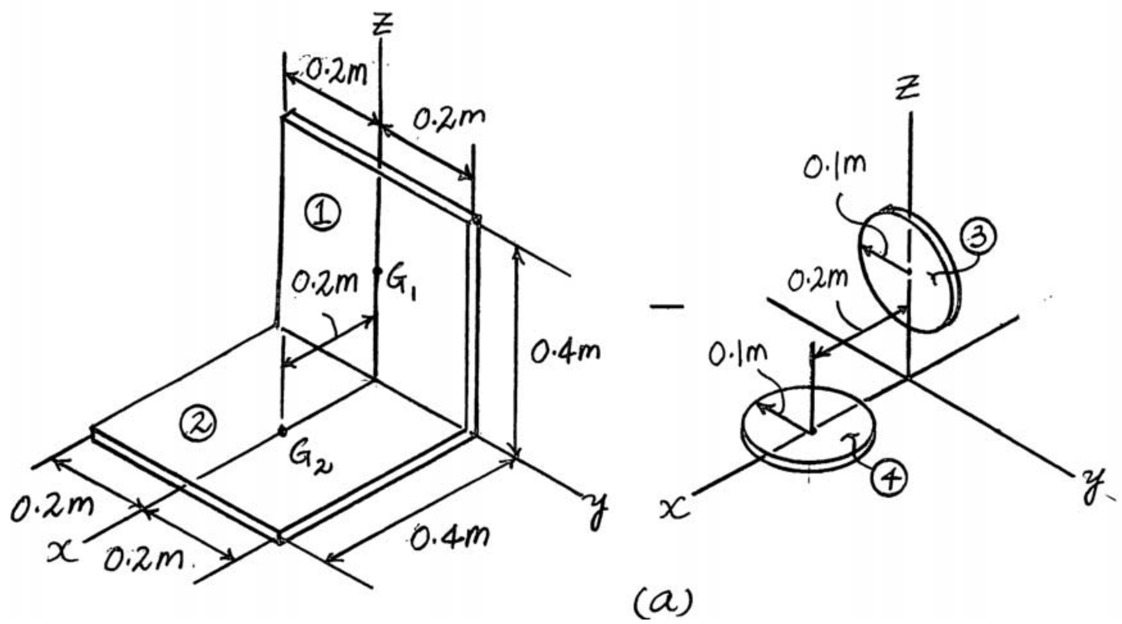
Composite Parts: The thin plate can be subdivided into four segments as shown in Fig. *a*. Since segments (3) and (4) are both holes, they should be considered as negative parts.

Mass moment of Inertia: Here, the mass for segments (1), (2), (3), and (4) are $m_1 = m_2 = 0.4(0.4)(10) = 1.6$ kg and $m_3 = m_4 = \pi(0.1^2)(10) = 0.1\pi$ kg. The mass moment of inertia of each segment about the z axis can be determined using the parallel - axis theorem.

$$I_z = \Sigma(I_z)_G + md^2$$

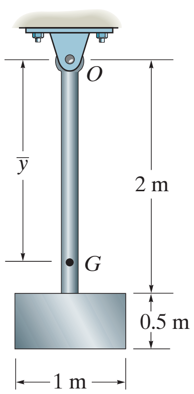
$$= \frac{1}{12}(1.6)(0.4^2) + \left[\frac{1}{12}(1.6)(0.4^2 + 0.4^2) + 1.6(0.2^2)\right] - \frac{1}{4}(0.1\pi)(0.1^2) - \left[\frac{1}{2}(0.1\pi)(0.1^2) + 0.1\pi(0.2^2)\right]$$

$$= 0.113\ \text{kg}\cdot\text{m}^2$$ **Ans.**



(a)

•10–105. The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then find the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.

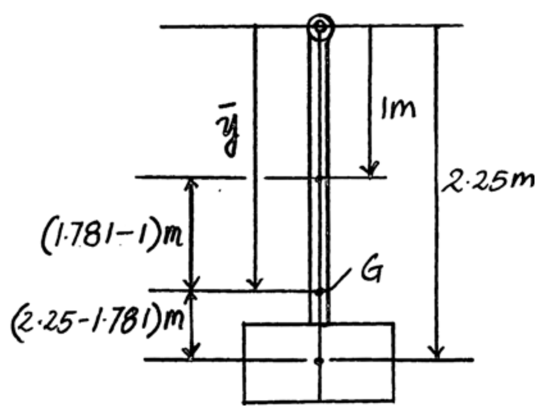


$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{1(3) + 2.25(5)}{3+5} = 1.781 \text{ m} = 1.78 \text{ m} \qquad \text{Ans}$$

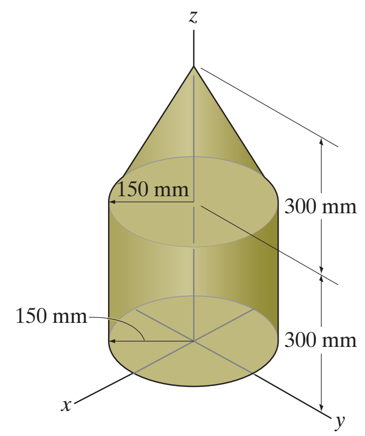
$$I_G = \Sigma \bar{I}_{G'} + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2 \qquad \text{Ans}$$



10–106. The cone and cylinder assembly is made of homogeneous material having a density of 7.85 $\mathrm{Mg/m^3}$. Determine its mass moment of inertia about the z axis.



Composite Parts: The assembly can be subdivided into a circular cone segment (1) and a cylindrical segment (2) as shown in Fig. *a*.

Mass: The mass of each segment is calculated as

$$m_1 = \rho V_1 = \rho\left(\frac{1}{3}\pi r^2 h\right) = 7.85(10^3)\left[\frac{1}{3}\pi(0.15^2)(0.3)\right] = 17.6625\pi \text{ kg}$$

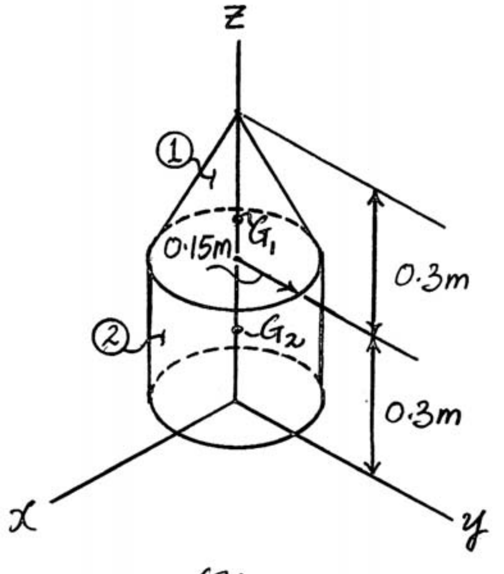
$$m_2 = \rho V_2 = \rho\left(\pi r^2 h\right) = 7.85(10^3)\left[\pi(0.15^2)(0.3)\right] = 52.9875\pi \text{ kg}$$

Mass Moment of Inertia: Since the z axis is parallel to the axis of the cone and cylinder and passes through their center of mass, their mass moment of inertia can be computed from $(I_z)_1 = \frac{3}{10}m_1 r^2$ and $(I_z)_2 = \frac{1}{2}m_2 r^2$. Thus,

$$I_z = (I_z)_1 + (I_z)_2$$

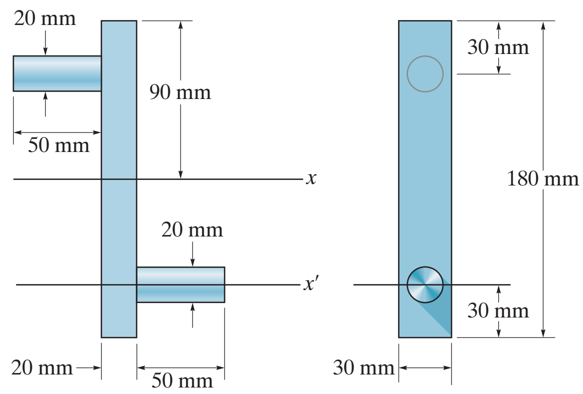
$$= \frac{3}{10}(17.6625\pi)(0.15^2) + \frac{1}{2}(52.9875\pi)(0.15^2)$$

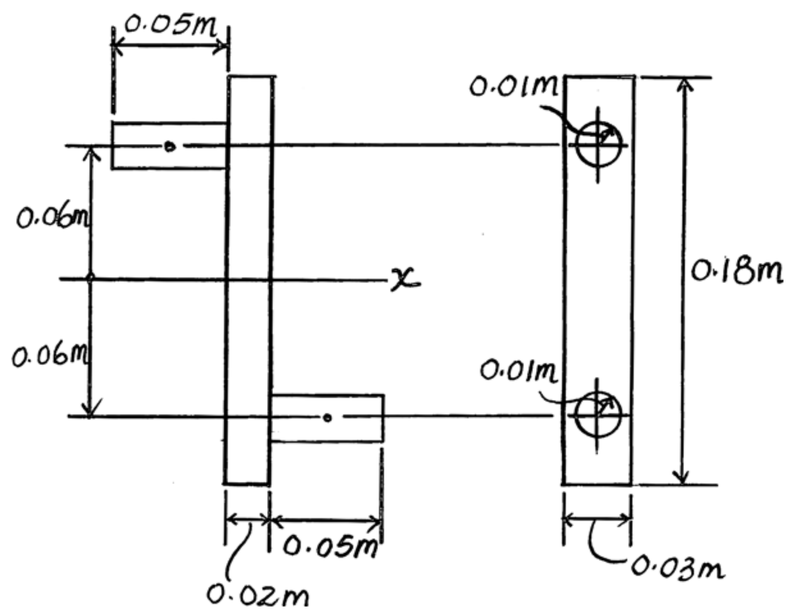
$$= 2.247 \text{ kg}\cdot\text{m}^2 = 2.25 \text{ kg}\cdot\text{m}^2$$ **Ans.**



(a)

10–107. Determine the mass moment of inertia of the overhung crank about the x axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.





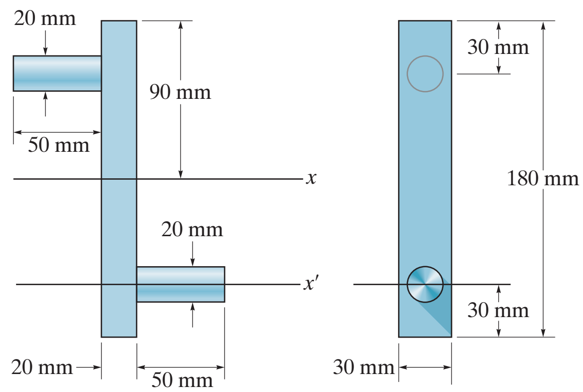
$$m_c = 7.85(10^3)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

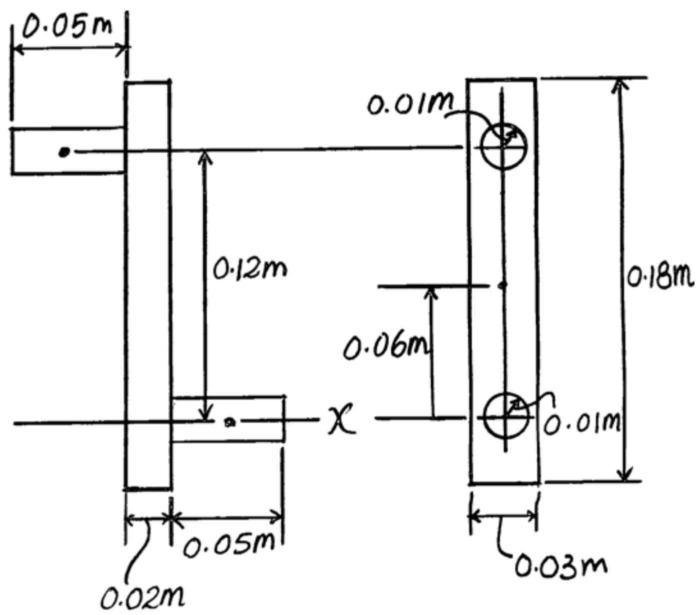
$$m_p = 7.85(10^3)\left((0.03)(0.180)(0.02)\right) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right] + \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right)\right]$$

$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2 \quad \text{Ans}$$

***10–108.** Determine the mass moment of inertia of the overhung crank about the x' axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.





$$m_c = 7.85(10^3)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

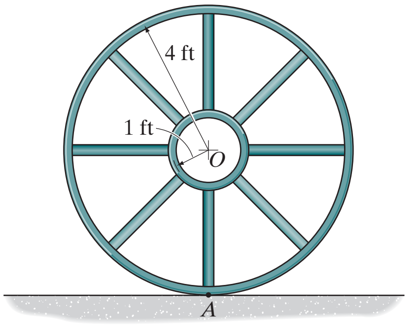
$$m_p = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_{x'} = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2 \quad \textbf{Ans.}$$

•10–109. If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A.



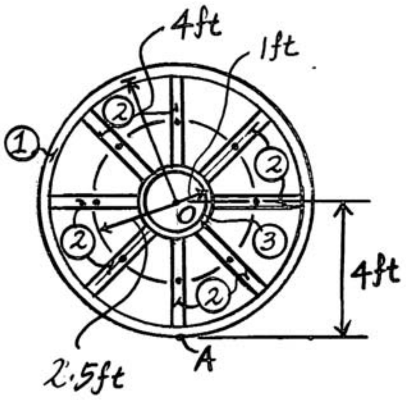
Composite Parts: The wheel can be subdivided into the segments shown in Fig. a. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ft $= 2.5$ ft from point O can be grouped as segment (2).

Mass Moment of Inertia: First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{1}{12}\left(\frac{20}{32.2}\right)(3^2) + \left(\frac{20}{32.2}\right)(2.5^2)\right] + \left(\frac{15}{32.2}\right)(1^2)$$
$$= 84.94 \text{ slug} \cdot \text{ft}^2$$

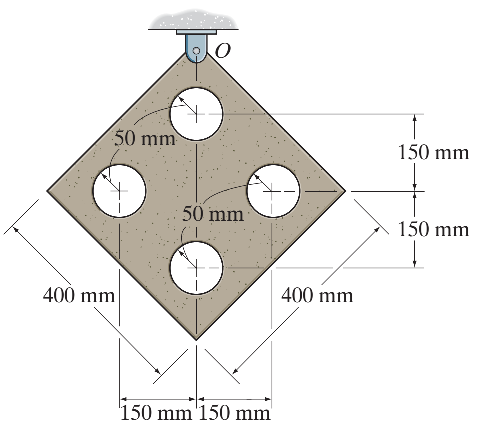
The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A can be found using the parallel - axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404$ slug and $d = 4$ ft. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58 \text{ slug} \cdot \text{ft}^2 = 222 \text{ slug} \cdot \text{ft}^2$$ **Ans.**



(a)

10–110. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



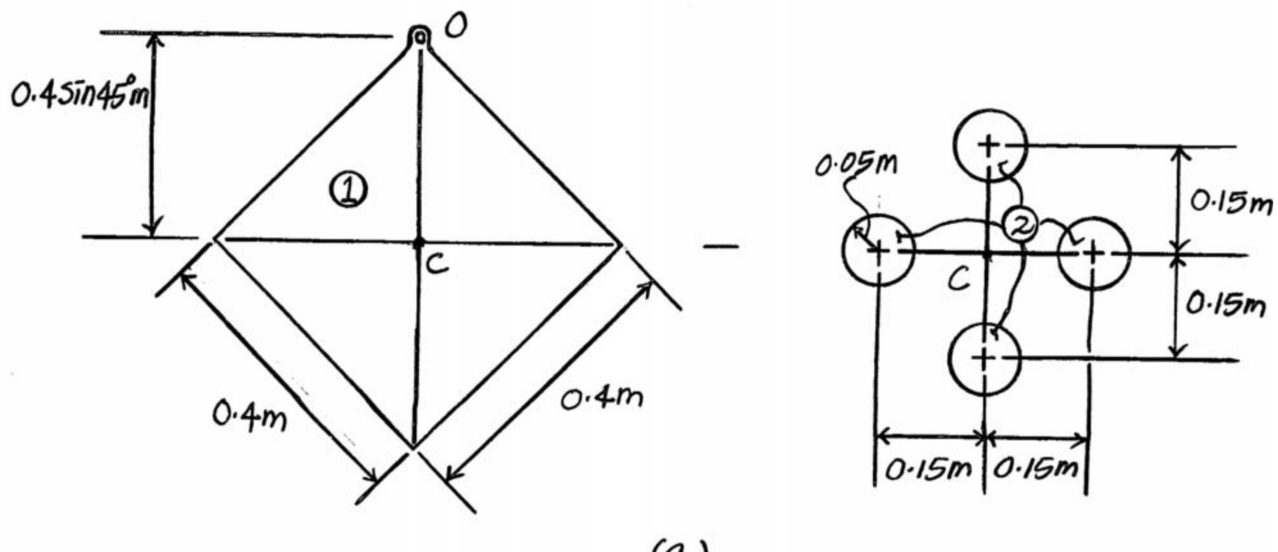
Composite Parts: The plate can be subdivided into the segments shown in Fig. *a*. Here. the four similar holes of which the perpendicular distances measured from their centers of mass to point C are the same and can be grouped as segment (2). This segment should be considered as a negative part.

Mass Moment of Inertia: The mass of segments (1) and (2) are $m_1 = (0.4)(0.4)(20) = 3.2$ kg and $m_2 = \pi(0.05^2)(20) = 0.05\pi$ kg, respectively. The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point C is

$$I_C = \frac{1}{12}(3.2)(0.4^2 + 0.4^2) - 4\left[\frac{1}{2}(0.05\pi)(0.05^2) + 0.05\pi(0.15^2)\right]$$
$$= 0.7041\ \text{kg}\cdot\text{m}^2$$

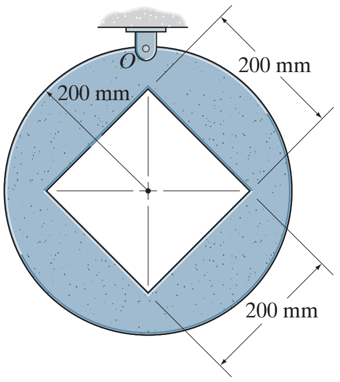
The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point O can be determined using the parallel-axis theorem $I_O = I_C + md^2$, where $m = m_1 - m_2 = 3.2 - 4(0.05\pi) = 2.5717$ kg and $d = 0.4\sin 45$. Thus,

$$I_O = 0.07041 + 2.5717(0.4\sin 45°)^2 = 0.276\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

10–111. Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



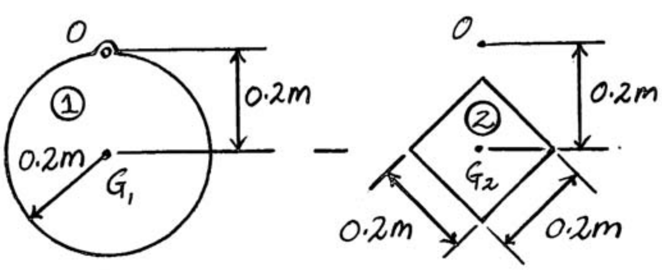
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass moment of Inertia: The masses of segmemts (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the point O for each segment can be determined using the parallel - axis theorem.

$$I_O = \Sigma I_G + md^2$$

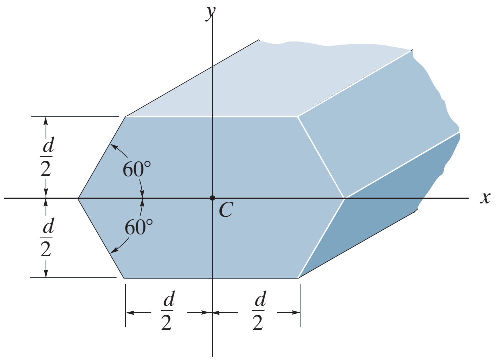
$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



(a)

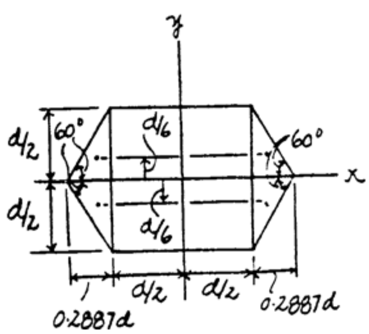
***10–112.** Determine the moment of inertia of the beam's cross-sectional area about the *x* axis which passes through the centroid *C*.



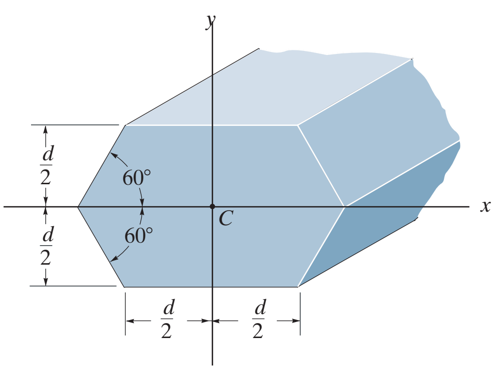
Moment of Inertia : The moment of inertia about the *x* axis for the composite beam's cross section can be determined using the parallel – axis theorem $I_x = \Sigma(\bar{I}_x + Ad_y^2)_i$.

$$I_x = \left[\frac{1}{12}(d)(d^3) + 0\right] + 4\left[\frac{1}{36}(0.2887d)\left(\frac{d}{2}\right)^3 + \frac{1}{2}(0.2887d)\left(\frac{d}{2}\right)\left(\frac{d}{6}\right)^2\right]$$

$= 0.0954d^4$ **Ans**



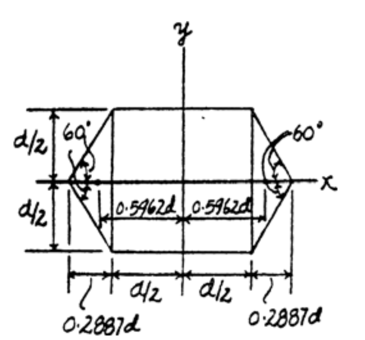
•10–113. Determine the moment of inertia of the beam's cross-sectional area about the *y* axis which passes through the centroid *C*.



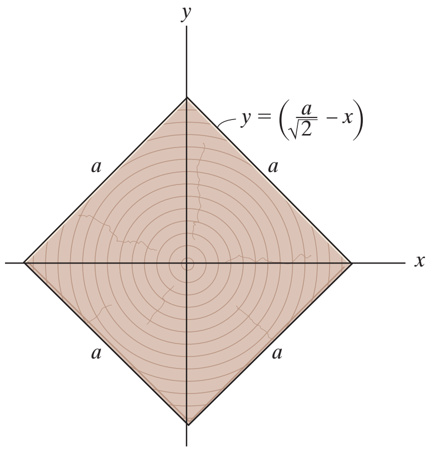
Moment of Inertia : The moment of inertia about yaxis for the composite beam's cross section can be determined using the parallel – axis theorem $I_y = \Sigma(\bar{I}_y + Ad_x^2)_i$.

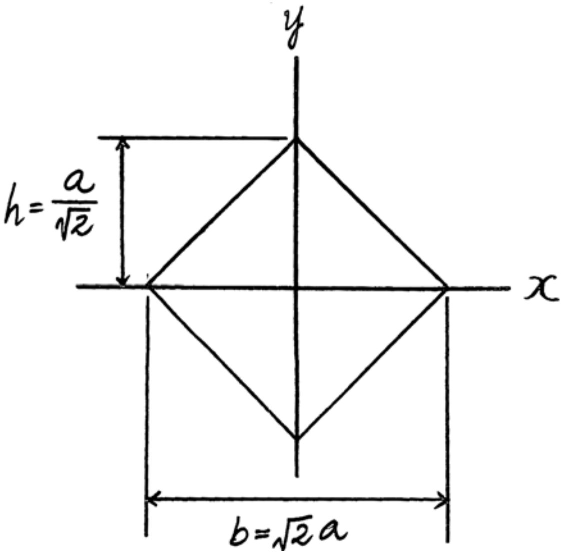
$$I_y = \left[\frac{1}{12}(d)(d^3) + 0\right] + 2\left[\frac{1}{36}(d)(0.2887d)^3 + \frac{1}{2}(d)(0.2887d)(0.5962d)^2\right]$$

$= 0.187d^4$ **Ans**



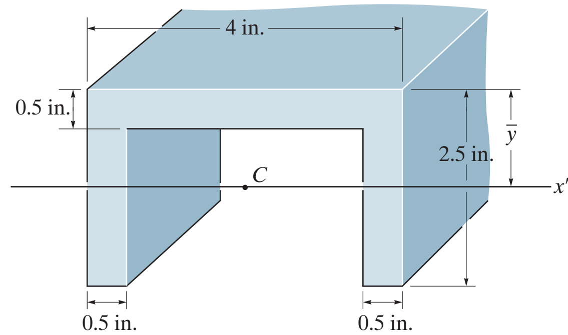
10–114. Determine the moment of inertia of the beam's cross-sectional area about the *x* axis.

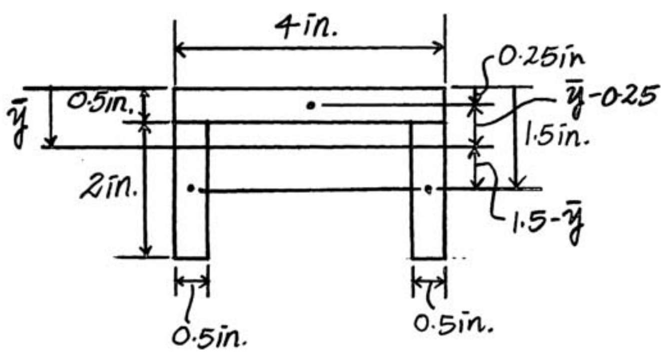




$$I_x = 2\left(\frac{bh^3}{12}\right) = 2\left(\frac{\sqrt{2}\,a\left(\frac{a}{\sqrt{2}}\right)^3}{12}\right) = \frac{1}{12}a^4 \quad \textbf{Ans}$$

10–115. Determine the moment of inertia of the beam's cross-sectional area with respect to the x' axis passing through the centroid C.



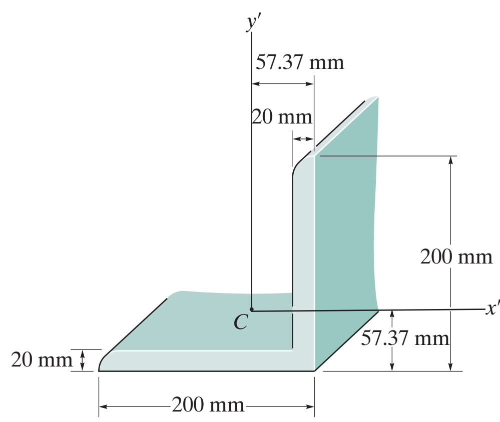


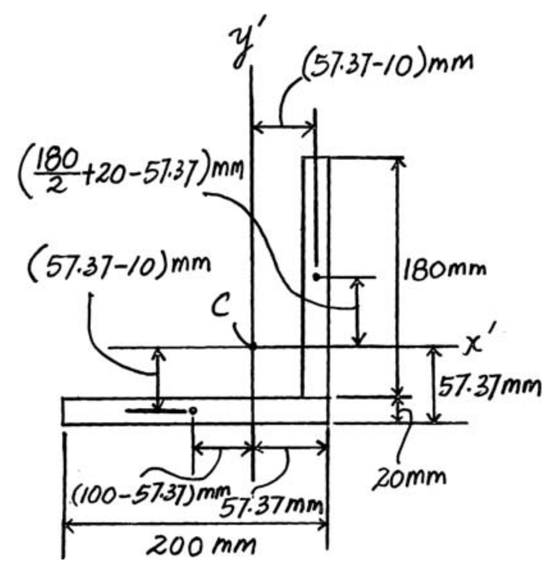
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.25(0.5)(4) + 2[1.5(2)(0.5)]}{0.5(4) + 2(2)(0.5)} = 0.875 \text{ in.}$$

$$I_{x'} = \frac{1}{12}(4)(0.5)^3 + 4(0.5)(0.875 - 0.25)^2 + 2\left[\frac{1}{12}(0.5)(2)^3 + (0.5)(2)(1.5 - 0.875)^2\right.$$

$$= 2.27 \text{ in}^4 \quad \textbf{Ans}$$

***10–116.** Determine the product of inertia for the angle's cross-sectional area with respect to the x' and y' axes having their origin located at the centroid C. Assume all corners to be right angles.



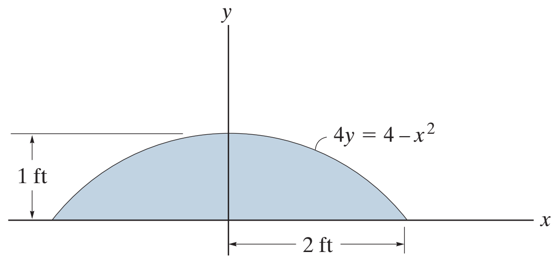


$$I_{xy} = \Sigma \tilde{x}\tilde{y}A$$

$$= \left(\frac{180}{2} + 20 - 57.37\right)(57.37 - 10)(180)(20) + (-(57.37 - 10))(-(100 - 57.37))(200)(20)$$

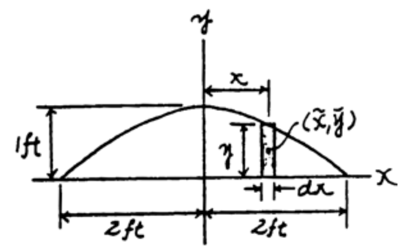
$$= 17.1(10)^6 \text{ mm}^4 \quad \textbf{Ans}$$

•10–117. Determine the moment of inertia of the area about the y axis.



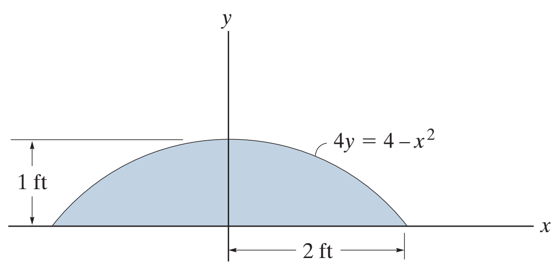
Differential Element : Here, $y = \frac{1}{4}\left(4 - x^2\right)$. The area of the differential element parallel to the y axis is $dA = ydx = \frac{1}{4}\left(4 - x^2\right)dx$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have



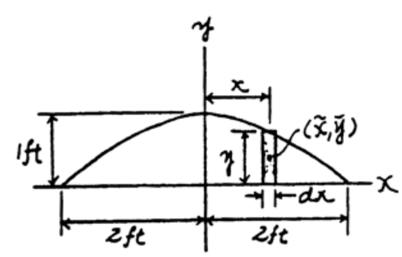
$$I_y = \int_A x^2 dA = \frac{1}{4}\int_{-2\,\text{ft}}^{2\,\text{ft}} x^2\left(4 - x^2\right)dx$$
$$= \frac{1}{4}\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]\Big|_{-2\,\text{ft}}^{2\,\text{ft}}$$
$$= 2.13\ \text{ft}^4 \qquad \textbf{Ans}$$

10–118. Determine the moment of inertia of the area about the x axis.



Differential Element : Here, $y = \frac{1}{4}\left(4 - x^2\right)$. The area of the differential element parallel to the y axis is $dA = ydx$. The moment of inertia of this differential element about the x axis is

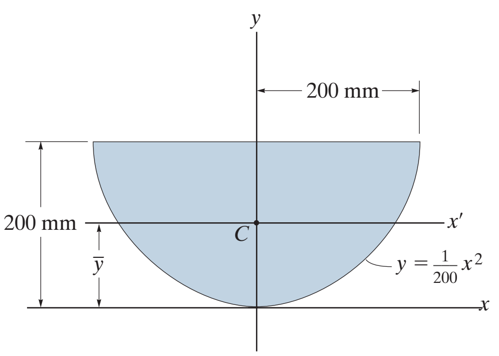
$$dI_x = d\bar{I}_{x'} + dA\,\tilde{y}^2$$
$$= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2$$
$$= \frac{1}{3}\left[\frac{1}{4}\left(4 - x^2\right)\right]^3 dx$$
$$= \frac{1}{192}\left(-x^6 + 12x^4 - 48x^2 + 64\right)dx$$



Moment of inertia : Performing the integration, we have

$$I_x = \int dI_x = \frac{1}{192}\int_{-2\,\text{ft}}^{2\,\text{ft}}\left(-x^6 + 12x^4 - 48x^2 + 64\right)dx$$
$$= \frac{1}{192}\left(-\frac{1}{7}x^7 + \frac{12}{5}x^5 - 16x^3 + 64x\right)\Big|_{-2\,\text{ft}}^{2\,\text{ft}}$$
$$= 0.610\ \text{ft}^4 \qquad \textbf{Ans}$$

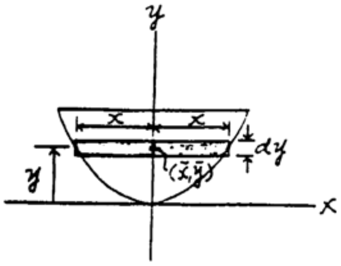
10–119. Determine the moment of inertia of the area about the x axis. Then, using the parallel-axis theorem, find the moment of inertia about the x' axis that passes through the centroid C of the area. $\bar{y} = 120$ mm.



Differential Element : Here, $x = \sqrt{200}y^{\frac{1}{2}}$. The area of the differential element parallel to the x axis is $dA = 2x dy = 2\sqrt{200}y^{\frac{1}{2}}dy$.

Moment of Inertia : Applying Eq. 10–1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^{200\text{mm}} y^2\left(2\sqrt{200}y^{\frac{1}{2}}dy\right)$$
$$= 2\sqrt{200}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Big|_0^{200\text{mm}}$$
$$= 914.29\left(10^6\right)\ \text{mm}^4 = 914\left(10^6\right)\ \text{mm}^4 \qquad \textbf{Ans}$$

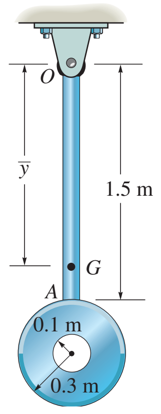


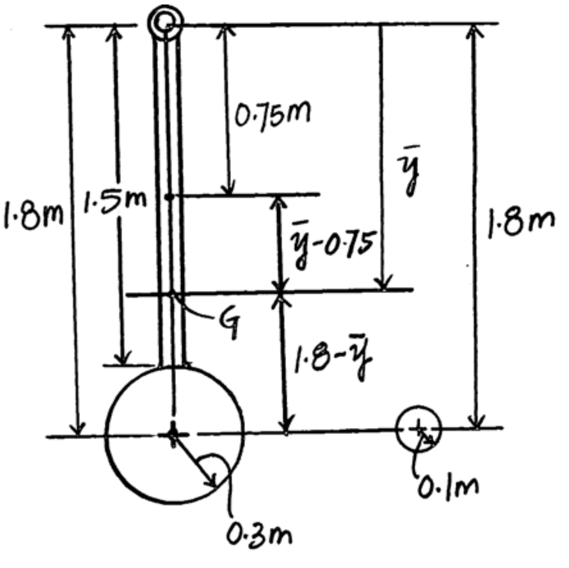
The moment of inertia about the x' axis can be determined using the parallel – axis theorem. The area is $A = \int_A dA = \int_0^{200\text{mm}} 2\sqrt{200}y^{\frac{1}{2}}dy = 53.33\left(10^3\right)\ \text{mm}^2$

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$914.29\left(10^6\right) = \bar{I}_{x'} + 53.33\left(10^3\right)\left(120^2\right)$$

$$\bar{I}_{x'} = 146\left(10^6\right)\ \text{mm}^4 \qquad \textbf{Ans}$$

***10–120.** The pendulum consists of the slender rod *OA*, which has a mass per unit length of 3 kg/m. The thin disk has a mass per unit area of 12 kg/m^2. Determine the distance $\bar{y}$ to the center of mass *G* of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through *G*.



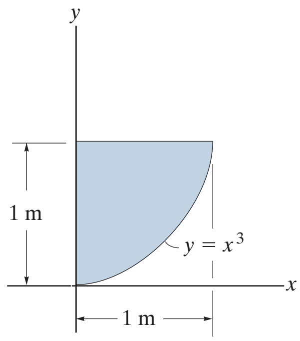


$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.75[1.5(3)] + 1.8[\pi(0.3)^2(12)] - 1.8[\pi(0.1)^2(12)]}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12)}$$

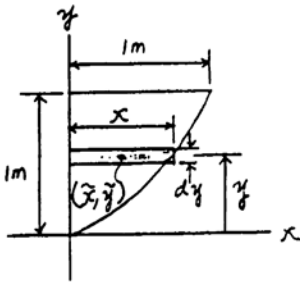
$= 1.1713 \text{ m} = 1.17 \text{ m}$ **Ans**

$$I_G = \frac{1}{12}[1.5(3)](1.5)^2 + [1.5(3)](1.1713 - 0.75)^2 + \frac{1}{2}[\pi(0.3)^2(12)](0.3)^2 + [\pi(0.3)^2(12)](1.8 - 1.1713)^2$$
$$- \frac{1}{2}[\pi(0.1)^2(12)](0.1)^2 - [\pi(0.1)^2(12)](1.8 - 1.1713)^2 = 2.99 \text{ kg} \cdot \text{m}^2$$ **Ans**

•10–121. Determine the product of inertia of the area with respect to the x and y axes.



Differential Element : Here, $x = y^{\frac{1}{3}}$. The area of the differential element parallel to the x axis is $dA = x\,dy = y^{\frac{1}{3}}dy$. The coordinates of the centroid for this element are $\tilde{x} = \frac{x}{2} = \frac{1}{2}y^{\frac{1}{3}}$, $\tilde{y} = y$. Then the product of inertia for this element is



$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y}$$
$$= 0 + \left(y^{\frac{1}{3}}dy\right)\left(\frac{1}{2}y^{\frac{1}{3}}\right)(y)$$
$$= \frac{1}{2}y^{\frac{5}{3}}dy$$

Product of Inertia : Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \int_0^{1\,\text{m}} \frac{1}{2}y^{\frac{5}{3}}dy = \frac{3}{16}y^{\frac{8}{3}}\Big|_0^{1\,\text{m}} = 0.1875\ \text{m}^4 \qquad \textbf{Ans}$$