AE100 A 503 4

PROF. NARDO

Final Exam

Rm 116

A.M. Jan 9

AG

100

$\overline{F} = F_x\overline{i} + F_y\overline{j} + F_z\overline{k}$

$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$

Equilib. $\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$

$\alpha^2 = \beta^2 + \gamma^2 - 2\beta\gamma\cos A$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$\frac{1}{F} = \frac{\cos\theta_x}{F_x} = \frac{\cos\theta_y}{F_y} = \frac{\cos\theta_z}{F_z}$

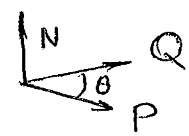
$\overline{F} = F\overline{\lambda} \qquad \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\overline{R}_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$

Transmissibility

Vector product $\overline{P}\times\overline{Q} = |P||Q|\sin\theta = \overline{N}$

$P\times Q = -Q\times P$



$P\times(Q_1 + Q_2) = P\times Q_1 + P\times Q_2$

$$V = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

$i\times i = 0 \quad i\times j = k$

$j\times j = 0 \quad j\times k = i$

$k\times k = 0 \quad k\times i = j$

$M_o = \overline{r}\times\overline{F}$ $\qquad F$ and $F' \equiv$ iff $F = F'$ & $M_o = M_o'$

<u>Varignon's Theorem</u> – $r\times(F_1 + F_2 + \dots + F_n) = r\times R$

where R is Resultant of $\sum_{i=1}^{n} F_i$ about 1 pt. concurrent.

$P\cdot Q = PQ\cos\theta \qquad P\cdot(Q_1 + Q_2) = P\cdot Q_1 + P\cdot Q_2$

$P\cdot Q = P_xQ_x + P_yQ_y + P_zQ_z$

$i\cdot i = 1 \quad i\cdot j = 0$

$j\cdot j = 1 \quad j\cdot k = 0$

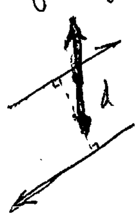
$k\cdot k = 1 \quad k\cdot i = 0$

$$S\cdot(P\times Q) = P\cdot(Q\times S) = Q(S\times P) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

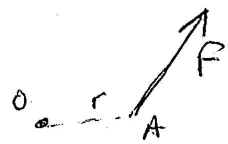
$M_{OL} = \lambda\cdot M_o$

$\lambda = \frac{\overline{F}}{|F|}$

Moment of Couple



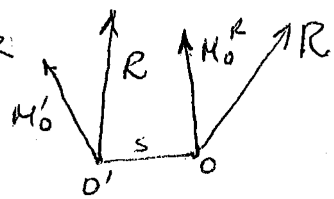
$F \cdot d = M_{couple}$

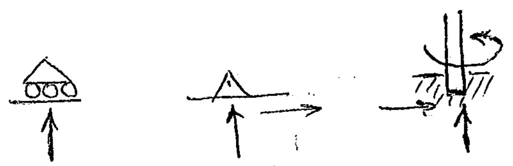


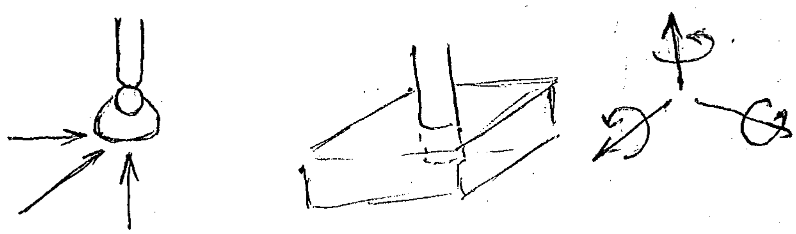
$M_O = \overline{F} + \overline{r} \times \overline{F}$

$M_a = r \times F$

$M^R_{O'} = M^R_O + s \times R$







Vectors - expressing magnitude, direction and obey parallelogram law.

free, bound, sliding vectors, equal vectors, sliding vectors

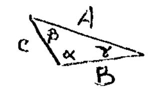
$\overline{F} = |F_x| i + |F_y| j + |F_z| k \qquad |F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$\text{Resultant}_x = \Sigma F_x \quad ; \quad \text{Resultant}_y = \Sigma F_y$

Equilibrium:

$$\Sigma F_x = \Sigma F_y = \Sigma F_z = \Sigma M_x = \Sigma M_y = \Sigma M_z = 0$$

Law of Sines $\dfrac{A}{\sin\alpha} = \dfrac{B}{\sin\beta} = \dfrac{C}{\sin\gamma}$ Law of Cosines $A^2 = B^2 + C^2 - 2BC\cos\alpha$

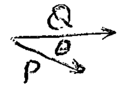


$F_x = F\cos\theta_x \;,\; F_y = F\cos\theta_y \;,\; F_z = F\cos\theta_z \quad \| \text{ unit vector } \lambda = \dfrac{F}{|F|} \|$

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

<u>Principle of transmissibility</u>

$P \times Q$



$\overline{V} = \overline{P} \times \overline{Q} = |P||Q|\sin\theta$ and $\overline{V} \perp P;\ \overline{V} \perp Q$

$P \times Q = -Q \times P$

$V = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$

$i \times j = k \quad i \times i = 0$
$j \times k = i \quad j \times j = 0$
$k \times i = j \quad k \times k = 0$

$M_0 = r \times F$

2 forces equal if they are equal & produce same moment about a given pt.

Varignon's theorem - $r \times (F_1 + F_2 + F_3 + \dots) = r \times F_1 + r \times F_2 + r \times \dots$

$\overline{P} \cdot \overline{Q} = |P||Q|\cos\theta$

$i \cdot i = 1 \quad j \cdot j = 1 \quad k \cdot k = 1$
$i \cdot j = 0 \quad j \cdot k = 0 \quad k \cdot i = 0$

$= P_x Q_x + P_y Q_y + P_z Q_z$

$\cos\theta = (P_x Q_x + P_y Q_y + P_z Q_z)/|P||Q|$

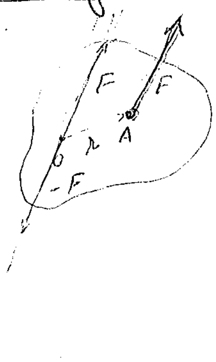
projection of a vector on an axis $= P\cos\theta$

$S \cdot (P \times Q) = Q \cdot (S \times P) = P \cdot (Q \times S)$ Volume of a parallelepiped $= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$

$M_{OL} = \lambda \cdot M$ where λ is the unit vector of the line OL

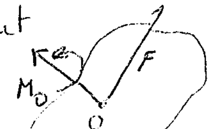
<u>Moment of a couple</u> $F \cdot d = M$

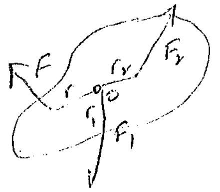
M_0



$M_0 = r \times F$ but

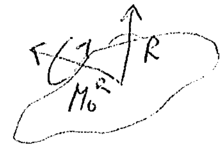
=



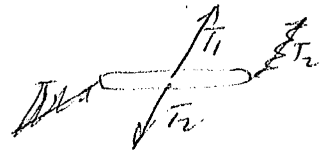


$R = F + F_1 + F_2$

$M_o^R = \Sigma(r \times F)$

= 

if $M_o^R \perp R$ we can

Equilibrium of 2 force body 

$T_1 = T_2$

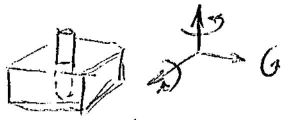
have same line of action

but $\overline{T_1} = -\overline{T_2}$

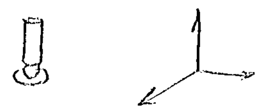
Chapter 4

Equilibrium of 3 force body

line of action must be concurrent or parallel.



Fixed support

Ball & Socket 

Chapt 6 trusses – support tension or compression only are not continuous through joints

Method of Sections, Method of Joints [First obtain outside reactions

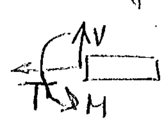
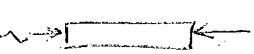
$m + 3 = 2n$

members joints

Frames carry transverse loads. go through joints. use 6 equations of equilibrium taking frame apart.

Machine load at grippers act 

Chapter 7 internal forces

 Multiforce member two force member 

Chapter 8 friction

$f_m = \mu N$ $\mu = \mu_s$ or μ_k depending on motion of object.

if $\tan\theta = \mu_s$ θ is angle of repose if $\tan\theta < \mu_s$ no sliding

$\tan\theta > \mu_s$ we have motion

f_m acts in direction opposite to motion.

Kinematics

$v = \frac{ds}{dt} \quad a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$

let $a = f(t)$ then $\boxed{v = \int f(t)\,dt + C_1 \quad \text{and} \quad s = \int\left[\int f(t)\,dt + C_1\right]dt + C_2}$

let $a = f(s)$ then $v\frac{dv}{ds} = a$ then $\boxed{\frac{v^2}{2} = \int f(s)\,ds + C_1}$

let $a = f(v)$ then $\frac{dv}{dt} = f(v)$ and $\boxed{t = \int \frac{dv}{f(v)} + C_1}$

<u>if</u> $a = 0$ for $t = 1, 2, 3 \ldots \infty$ $t \ni R$ then v is constant

$s = s_0 + vt$

<u>if</u> a = constant then $v = v_0 + at$ and $s = s_0 + v_0 t + \frac{1}{2}at^2$

or if $a = v\frac{dv}{ds}$ then $v^2 = v_0^2 + 2a(s - s_0)$

$s_{B/A}$ = distance of B with respect to A $\quad s_{B/A} = s_B - s_A$

$v_{B/A}$ = velocity of B with respect to A $\quad v_{B/A} = v_B - v_A$

$a_{B/A}$ = acceleration of B with respect to A $\quad a_{B/A} = a_B - a_A$

<u>dependent motions</u> – pulleys ropes length is constant.

Curvilinear motion – if $\bar{v} = \frac{d\bar{r}}{dt}$, $v = \frac{ds}{dt}$; $\bar{a} = \frac{d\bar{v}}{dt}$

$$\frac{d(\bar{P}+\bar{Q})}{du} = \frac{d\bar{P}}{du} + \frac{d\bar{Q}}{du}$$

$$\frac{d(f\bar{P})}{du} = \frac{df}{du}P + f\frac{d\bar{P}}{du}$$

$$\frac{d(\bar{P}\cdot\bar{Q})}{du} = \frac{d\bar{P}}{du}\cdot\bar{Q} + \bar{P}\cdot\frac{d\bar{Q}}{du}$$

$$\frac{d(\bar{P}\times\bar{Q})}{du} = \frac{d\bar{P}}{du}\times\bar{Q} + \bar{P}\times\frac{d\bar{Q}}{du}$$

$$\frac{dr}{dt} = \frac{d}{dt} r_x i + \frac{d}{dt} r_y j + \frac{d}{dt} r_z k$$ but $\frac{d}{dt} i, j, \text{ or } k$ is 0 since unit vectors remain constant

<u>Rectangular coordinates</u>

if $r = xi + yj + zk$; $v = \dot{x}i + \dot{y}j + \dot{z}k$; $a = \ddot{x}i + \ddot{y}j + \ddot{z}k$

or $v_x = \dot{x}$ $\quad v_y = \dot{y}$ $\quad v_z = \dot{z}$

$a_x = \ddot{x}$ $\quad a_y = \ddot{y}$ $\quad a_z = \ddot{z}$ $\qquad$ if $S_x = x$, $S_y = y$, $S_z = z$

for projectile problems $a_x = 0$, $a_z = 0$ $\quad a_y = -g$

if $v_x = v_0 \cos\theta$; $v_y = -gt + v_0 \sin\theta$; $\begin{cases} x = S_x = v_0 \cos\theta\, t \\ y = S_y = -\frac{gt^2}{2} + v_0 \sin\theta\, t \end{cases}$ if $t_0 = 0$, $S_0 = 0$

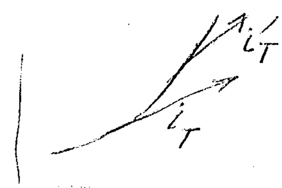
Relative motion

$r_{B/A} = r_B - r_A$

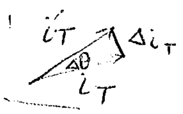
$v_{B/A} = v_B - v_A$

$a_{B/A} = a_B - a_A$

<u>Tangential & Normal</u>



if $\frac{d\bar{i}_T}{d\theta} = i_n$ when $|\bar{i}_n| = 1$, $\bar{i}_n \perp i_T$



let $\boxed{\bar{v} = v\,\bar{i}_T}$

$$\bar{a} = \frac{dv}{dt} \bar{i}_T + v \frac{d\bar{i}_T}{d\theta} \underbrace{\frac{d\theta}{ds}}_{\frac{1}{\rho}} \underbrace{\frac{ds}{dt}}_{v}$$

but since $\rho\Delta\theta = \Delta s$, $\rho\, d\theta = ds$

$$\boxed{\bar{a} = \frac{dv}{dt} \bar{i}_T + \frac{v^2}{\rho} \bar{i}_n}$$

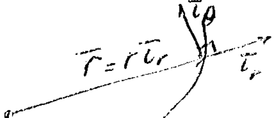
$a_T = \frac{dv}{dt}$; $a_n = \frac{v^2}{\rho}$

a_n is in direction of radius of curvature

$\bar{i}_T \times \bar{i}_N = \bar{i}_B$ $\quad$ where $|\bar{i}_B| = 1$ $\quad \bar{i}_B \perp \bar{i}_N$, $\bar{i}_B \perp \bar{i}_T$

a_N & a_T are contained in the osculating plane formed by $\bar{i}_T$ & $\bar{i}_N$

Radial & Transverse Component. $\quad \frac{d\bar{i}_\theta}{d\theta} = -\bar{i}_r \quad \frac{d\bar{i}_r}{d\theta} = \bar{i}_\theta$



$$\boxed{\bar{r} = r\,\bar{i}_r}$$

$$\bar{v} = \frac{dr}{dt} \bar{i}_r + r \frac{d\bar{i}_r}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{dt} \bar{i}_r + r\frac{d\theta}{dt} \bar{i}_\theta$$

$$\boxed{\bar{v} = \dot{r}\,\bar{i}_r + r\dot{\theta}\,\bar{i}_\theta}$$

$$\bar{a} = \frac{d\dot{r}}{dt}\bar{\imath}_r + \dot{r}\frac{d\bar{\imath}_r}{d\theta}\cdot\frac{d\theta}{dt} + \left[\frac{dr}{dt}\dot{\theta} + r\frac{d\dot{\theta}}{dt}\right]\bar{\imath}_\theta$$

$$+ \left[r\dot{\theta}\right]\frac{d\bar{\imath}_\theta}{d\theta}\cdot\frac{d\theta}{dt}$$

$$= \ddot{r}\bar{\imath}_r + \dot{r}\dot{\theta}\bar{\imath}_\theta + \dot{r}\dot{\theta}\bar{\imath}_\theta + r\ddot{\theta}\bar{\imath}_\theta + r\dot{\theta}^2(-\bar{\imath}_r)$$

$$\boxed{\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\bar{\imath}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\bar{\imath}_\theta}$$

Then $v_r = \dot{r}$; $v_\theta = r\dot{\theta}$ — if r = constant, $\dot{r} = \ddot{r} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2$; $a_r = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

Cylindrical Coordinates

$$\boxed{\begin{aligned} r &= R\bar{\imath}_R + zk \\ v &= \dot{R}\bar{\imath}_R + R\dot{\theta}\bar{\imath}_\theta + \dot{z}k \\ a &= \left(\ddot{R} - R\dot{\theta}^2\right)\bar{\imath}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\bar{\imath}_\theta + \ddot{z}k \end{aligned}}$$

(12) Dynamic equilibrium

$\Sigma F_x = ma_x$; $\Sigma F_y = ma_y$; $\Sigma F_z = ma_z$

Dalembert Principle - external forces on a system = effective forces on various particles of system.

Motion of Mass center.

The mass center of system of particles acts as if the entire mass of the system & all the external forces were concentrated at that pt.

Rectangular Components

$\Sigma F_x = m\ddot{x}$ $\quad \Sigma F_y = m\ddot{y}$ $\quad \Sigma F_z = m\ddot{z}$

Tangential & Normal

$\Sigma F_T = m\frac{dv}{dt}$ $\quad \Sigma F_N = m\frac{v^2}{\rho}$

Radial & Transverse

$\Sigma F_R = m\left(\ddot{r} + r\dot{\theta}^2\right)$ $\quad \Sigma F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$

Under central force field only force acting on particle

if F_r $\therefore$ $F_\theta = 0$ but $m \neq 0$ $\therefore$ $a_\theta = 0$ or $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\dot{h}$ (B) where $\boxed{h = r^2\dot{\theta}}$ (A) h is a constant

Areal velocity $= \frac{dA}{dt} = \dot{A}$ $\qquad \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}h$

Therefore areal velocity is a constant.

From (A) we can get $\frac{dr}{dt}, \frac{d^2r}{dt}$ if $u = \frac{1}{r}$ we can substitute into (B)

and we get $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$ but under central field newton's Gravitation law $= GMmu^2$

or $\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}$ which can be solved for

or $\frac{1}{r} = u = \frac{GM}{h^2} + C\cos\theta$ $\qquad$ the ratio $\frac{Ch^2}{GM} = \epsilon$ eccentricity

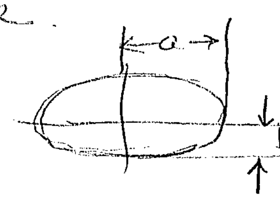
$h = r_0 v_0$ $\qquad \bar{v}_0 = \dot{r}_0\,\bar{i}_r + r_0\dot{\theta}_0\,\bar{i}_\theta$ but $\dot{r}_0 = 0$ $\therefore$ $\bar{v}_0 = r_0\dot{\theta}_0\,\bar{i}_\theta$

or $v_0 = r_0\left(\frac{d\theta}{dt}\right)_0$

but since $h = r_0^2\dot{\theta}_0 = r_0\left(r_0\left(\frac{d\theta}{dt}\right)_0\right) = r_0 v_0$

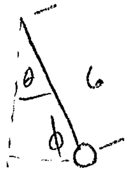
$V_{escape} = \sqrt{\frac{2GM}{r_0}}$ $\qquad V_{circular} = \frac{1}{\sqrt{2}}V_{escape}$

$T_{period} = \frac{2\pi ab}{h}$ $\qquad a = \frac{1}{2}(r_0 + r)$ $\quad b = \sqrt{r_0 r}$



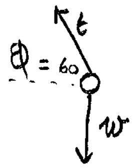
Keplers laws of planetary motion

1. Each planet describes an ellipse with the sun located at one of its foci
2. radius vector drawn from sun to planet sweeps equal areas in equal times.
3. $\frac{(T_1)^2}{(T_2)^2} = \frac{(a_1)^3}{(a_2)^3}$



constant v_0 which means $a_t = 0$

$w = 5$ if $\theta = 30°$ find v_0, tension



$t \sin\phi - w = m a_y \qquad a_y = 0$

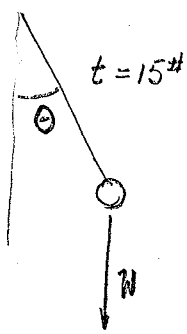
$t \cos\phi = \frac{v^2}{\rho}\frac{w}{g} \qquad$ where $\rho = l \sin\theta$

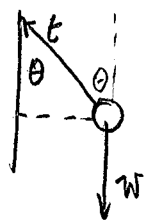
$t \sin\phi = w \qquad t = \frac{w}{\sin\phi}$

$w \cot\phi = \frac{v^2}{\rho}\frac{w}{g}\,;\ \frac{w}{g}v^2 = w l \cot\phi \sin\theta$

$w = 5 \quad l = 6 \quad \phi = 60 \quad \theta = 30$

$$\boxed{\begin{array}{l} v = \sqrt{g\, l \cot\phi \sin\theta} \\ t = \frac{w}{\sin\phi} \end{array}}$$





$t\cos\theta - W = 0$

$t \sin\theta = \frac{W}{g}\frac{v^2}{\rho}$

$\frac{t\sin\theta}{t\cos\theta} = \frac{W v^2/g\rho}{W}$

$\tan\theta = \frac{v^2}{g\rho}$

$t = 15 \quad W = 5$

$\cos\theta = .333$

$70.5°$

$(6(.943))\, g\, (2.8\cancel{2}5) = v^2$

$l \sin\theta = \rho = 6(.943)$

$6(32)(3) =$

30
20
$600 \sim 24.$

$\boxed{22.69 \text{ ft/sec}}$

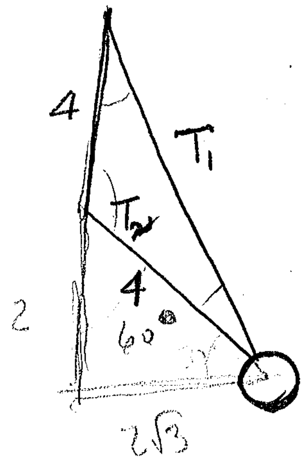
$\cos\theta = \frac{W}{t} = \frac{5}{15} = .333 \qquad \theta = 70.5°$

$15(.943) = \frac{5}{32.2}\left(\frac{225}{l(.943)}\right) = l = \frac{5(\cancel{225})^{15}}{(32.2)(.943)^2} = \frac{75}{(32.2)(.943)^2}$

2.62

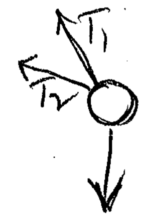
$l = 2.62$

$\frac{75}{32.2}$



$$\frac{T_2}{\sin 30} = \frac{T_1}{\sin 60}$$

$$T_2 \sin 60 = T_1 \sin 30^\circ$$



$$T_1 \sin 60^\circ + T_2 \sin 30^\circ = W$$

$$T_1 \cos 60^\circ + T_2 \cos 30^\circ = \frac{mv^2}{R}$$

$$T_2 \frac{\sin^2 60}{\sin 30} + T_2 \sin 30^\circ = W$$

$$T_1 = T_2 \frac{\sin 60}{\sin 30}$$

$$T_2 \frac{\sin 60 \cos 60^\circ}{\sin 30^\circ} + T_2 \cos 30^\circ =$$

$$\frac{T_2 \sin 120^\circ}{\cancel{2 \sin 30^\circ}} + T_2 \cos 30^\circ = \longrightarrow \frac{W}{g}\frac{v^2}{R}$$

$$T_2(.866) + T_2(.866) = \frac{W}{g}\frac{v^2}{R}$$

$$T_2(1.732) = \longrightarrow$$

$$T_2 = \frac{W v^2}{g} \frac{3}{R(\sqrt{3})}$$

$$T_1 = \frac{3Wv^2}{gR\sqrt{3}}\left[\frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}}\right] = \frac{3Wv^2}{gR}$$

$$T_2 = \frac{3Wv^2}{gR\sqrt{3}}$$

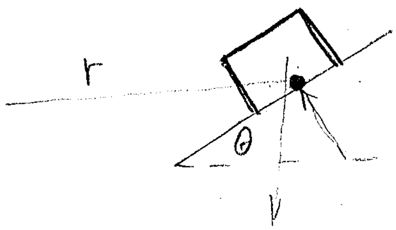
$$\frac{3\cancel{W}v^2}{gR}\left[\frac{\sqrt{3}}{2}\right] + \frac{3\cancel{W}v^2}{gR\sqrt{3}}\left[\frac{1}{2}\right] = \cancel{W}$$

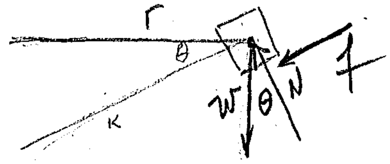
$$R = 2\sqrt{3}$$

$$\frac{3v^2\cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}\,g\,2} + \frac{\cancel{3}v^2}{2\cancel{\sqrt{3}}\,g\,\cancel{\sqrt{3}}\,2} = 1$$

$$\frac{3v^2}{4g} + \frac{v^2}{4g} = 1 \qquad v^2 = g$$

$$v = 5$$





$$W\cos\theta = N$$

$$W\sin\theta - \mu N = \left[\frac{W}{g}\frac{v^2}{R}\right] \Big/ \cos\theta$$

$$\frac{F}{K} = \cos\theta \qquad K = \frac{F}{\cos\theta}$$

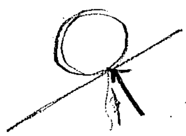
$$W\sin\theta\cos\theta - \mu N\cos\theta = \frac{Wv^2}{gR}$$

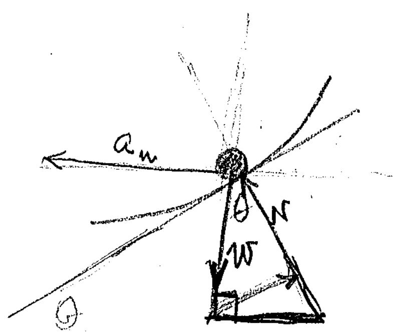
$$\cancel{N\sin\theta - \mu N\cos\theta = \frac{W}{g}\frac{v^2}{gR} = \frac{Nv^2}{gR\cos\theta}}$$

$$\sqrt{gR\sin\theta\cos\theta - \mu gR\cos^2\theta} = v_{max}$$

$$\sqrt{\frac{gR\sin 2\theta}{2} - \mu gR\cos^2\theta} = v_{max}$$

$$\sqrt{gR\left[\frac{\sin 2\theta}{2} - \mu\cos^2\theta\right]} = v_{max}$$





$$\cancel{W\cos\theta = N}$$
$$\cancel{W\sin\theta = 1}$$

$$\frac{N}{W} = \cos\theta$$

$$W = \frac{N}{\cos\theta}$$

$$W^2 = N^2 + K^2$$

$$K^2 = \sqrt{W^2 - N^2}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{m\ell^2u^2} \qquad F = \cancel{\frac{GMmu^2}{mh^2u^2}}$$

$$K = W\tan\theta$$
$$K = N\sin\theta$$
$$W\tan\theta = N\sin\theta$$
$$W = N\cos\theta$$
$$N\sin\theta = \frac{mv^2}{R}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2} \qquad c_1\cos\theta + c_2\sin\theta = u$$

$$\frac{GM}{h^2} + c_1\cos\theta + c_2\sin\theta = u$$

$$(D^2+1)u = \frac{GM}{h^2}$$

$$\frac{W}{g}\frac{v^2}{R} = \frac{K^2x^2}{gR} = \frac{K^2x^2}{gx}$$

$$\frac{k^2}{g}$$

$$\tan\theta = \frac{\frac{W}{g}\frac{v^2}{R}}{W} = \frac{v^2}{gR}$$

let

$$\frac{dy}{dx} = \frac{K^2x^2}{gx} = c^2x$$

$$h - y = \frac{2h}{b^2}\left[\frac{b^2}{2} - \frac{x^2}{2}\right]$$

$$\int_0^h dy = \int_0^b c^2x\,dx$$

$$h = c^2\left[\frac{b^2}{2}\right] \quad c^2 = \frac{2h}{b^2}$$

$$v_0 = \sqrt{\frac{2GMr}{r_0(r+r_0)}} = \sqrt{\frac{(2\times 1.408\times 10^{16})(4360\times\cancel{5280})}{(29{,}960\times\cancel{5280})(34{,}320\times 5280)}}$$

$$= \sqrt{\frac{(2.816\times 10^{16})(4360)}{(29{,}960)(34{,}320\times 5280)}}$$

$$2.26\times 10^7 = 22.6\times 10^6$$

$$\frac{3.\times 10^{16}(4000)}{(3\times 10^4)(3\times 10^4)(5\times 10^3)} = \frac{(3\times 10^{16})(4\times 10^3)}{(9\times 10^8)(5\times 10^3)} = \frac{12\times 10^{19}}{45\times 10^{11}} = \frac{120\times 10^{18}}{45\times 10^{11}} = 2.-\times 10^7$$

$$4.76\times 10^3 = \boxed{3245 \text{ mph}}$$

$$V_0 = \sqrt{\frac{(2\times 1.408\times 10^{16})(29{,}960\times\cancel{5280})}{(4360\times\cancel{5280})(34320\times 5280)}} = \frac{(2.816\times 10^{16})(29960)}{(4360)(34320\times 5280)}$$

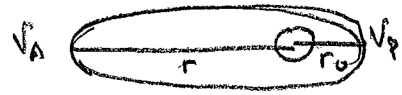
$$1.068^{9} \qquad 10.68\times 10^8$$

$$\frac{(3\times 10^{16})\cancel{(3\times 10^4)}}{(4\times 10^3)\cancel{(3\times 10^4)}(5\times 10^3)} = \frac{3\times 10^{16}}{20\times 10^6} = \frac{30\times 10^{15}}{20\times 10^6} = 1.-\times 10^9$$

$$3.268\times 10^4 = 22300 \text{ mph.}$$

$$V_{cir} = \sqrt{\frac{GM}{r_0}}$$

$$\boxed{V_A = \sqrt{\frac{2GM\, r_0}{(r+r_0)\, r}} \qquad V_P = \sqrt{\frac{2GMr}{(r+r_0)\, r_0}}}$$



$r_0 = R + h_0$

$r = R + h$

$GM = gR^2$

$\boxed{29{,}960 \times 4360}$

$$\frac{1}{r} = \frac{GM}{r_0^2 v_0^2} + C\cos\theta$$

$$\frac{1}{r} = \frac{GM}{r_0^2 v_0^2} + C \qquad \frac{1}{r} - \frac{GM}{r_0^2 v_0^2} = C$$

$$\frac{1}{r} = \frac{GM}{r_0^2 v_0^2} - C$$

$$\frac{1}{r_0} = \frac{GM}{r_0^2 v_0^2} - C = \frac{2GM}{r_0^2 v_0^2} - \frac{1}{r}$$

$$\frac{1}{r_0} + \frac{1}{r} = \frac{2GM}{r_0^2 v_0^2}$$

$$\left(\frac{r+r_0}{r_0 r}\right) \qquad v_0^2 = \frac{2GM(r_0 r)}{r_0^2 (r+r_0)} = \frac{2GMr}{r r_0 + r_0^2}$$

$$v_0 = \sqrt{\frac{2GMr}{r r_0 + r_0^2}}$$

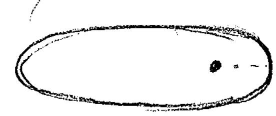
if $r = 29{,}960$ $r_0 = 4360$

if $r = 4360$ $r_0 = 29{,}960$

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \quad \text{where } \theta = 0. \quad \frac{1}{r} = \frac{GM}{h^2} + C$$

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \quad \text{where } \theta = \pi \quad \frac{1}{r} = \frac{GM}{h^2} - C$$

$$\frac{2}{r} = \frac{2GM}{h^2} \qquad \frac{1}{r} = \frac{GM}{h^2} \qquad h^2 = r_0 v_0$$



① let $r = r_0$

② let $r = r$

$\frac{1}{h} = \frac{GM}{r^2 v^2}$ $\quad v^2 = \frac{GM}{r} = \frac{1.408\times10^{16}\ ft^3/sec^2}{h} = \frac{1.408\times10^{16}}{}$

let $h = 4360$

$4360 \times \sqrt{280}$ $\quad \frac{GM}{29960\times\sqrt{280}} = 1.408\times10^{16}$

$r\dot{\theta} = v_\theta$

$r = r_0 \cos\theta$ $\quad r^2 \frac{d\theta}{dt} = h$

$\cancel{\frac{dr}{dt} = -r_0 \sin\theta \frac{d\theta}{dt} + \frac{dr_0}{dt}\cos\theta}$ $\quad \frac{d\theta}{dt} = \frac{h}{r^2}$

$\cancel{= -\frac{r_0 \sin\theta}{r^2}}$

$h = r_0 v_0$

$(r_0^2 \cos^2\theta)\frac{d\theta}{dt} = h$

$r_0^2 \cos^2\theta \frac{d\theta}{dt} = r_0 v_0$

27-7
17-7 KC

$v = r\frac{d\theta}{dt} = \frac{v_0}{\cos^2\theta}$

$r^2\frac{d\theta}{dt} = h$ $\quad h = r_0 v_0$ $\quad r = \frac{r_0}{\cos n\theta}$

$\frac{r_0}{\cos^2 n\theta}\frac{d\theta}{dt} = v_0$

$r = r_0/\cos n\theta$

$V = \frac{dr}{dt}\bar{\iota}_r + r\frac{d\theta}{dt}\bar{\iota}_\theta$ $\quad \frac{dr}{dt} = \frac{d}{dt}\left(\frac{r_0}{\cos n\theta}\right)$

$=$

$\frac{\cos n\theta \frac{dr_0}{dt} + r_0 n \sin n\theta \frac{d\theta}{dt}}{\cos^2 n\theta}$

$r = r\bar{\iota}_r$ $\quad r = \frac{r_0}{\cos n\theta}\bar{\iota}_r$

$\frac{d}{dt}\left[\frac{r_0}{\cos n\theta}\right]\bar{\iota}_r + \frac{r_0}{\cos n\theta}\frac{d\theta}{dt}\bar{\iota}_\theta$ $\quad r_0\frac{d\theta}{dt} = r_0 v_0$

$\frac{d}{dt}\ r_0(\cos n\theta)^{-1}$ $\quad \frac{dr_0}{dt}\cdot(\cos n\theta)^{-1} + r_0(\cos n\theta)^{-2} + n \sin n\theta \frac{d\theta}{dt}$

$$v_\theta = \frac{r_0 n \sin n\theta}{\cos^2 n\theta} \frac{d\theta}{dt} = r_0 v_0$$

$r = r_0$ when $\theta = 0$

$r^2\dot{\theta} = h = r_0 v_0$

$$v_r = \frac{r_0}{\cos n\theta} \qquad \frac{\cos n\theta \frac{dr_0}{dt} - r_0 n \sin n\theta \, d\theta/dt}{\cos^2 n\theta} =$$

$$\frac{r_0}{\cos n\theta} \frac{d\theta}{dt} = v_\theta \qquad r_0 \frac{d\theta}{dt} = v_0 \qquad v_0$$

$$v_\theta = \frac{r\,d\theta}{dt} = v_0 \cos n\theta$$

$v_r = n v_0 \sin n\theta$

$$\frac{dr}{dt} = v_r \qquad r = r_0 \text{ when } v = v_0$$

$$r_0 \frac{r_0^2}{\cos^2 n\theta} n \sin n\theta \, d\theta/dt =$$

$$\frac{r_0^2}{\cos^2 n\theta} \frac{d\theta}{dt} = r^2 \frac{d\theta}{dt} = r_0 v_0$$

$$\frac{n \sin n\theta \, \cancel{r_0} v_0}{\cancel{r_0}} = v_r \qquad v_r = 0 \qquad v_\theta = v_0 \cos$$

$v_0 \cos\theta$

$v_0 \sin\theta$

$$g = \frac{GM}{r^2} \; ; \; \frac{(GM)_E}{81.3} = (GM)_M$$

$$\frac{(GM)_E}{(81.3)(1{,}080 \times 5280)^2} = \frac{1.408 \times 10^{16}}{(81.3)(1{,}080)^2(5280)^2}$$

5.325 ft/sec^2

$$(1000)(1 \times 10^6)(25 \times 10^6)$$

$$\frac{25 \times 10^{14}}{.25 \times 10^{16}}$$

$r = 10(1+\cos\frac{1}{2}\pi t)$ $\quad \theta = \frac{1}{2}\pi t$ $\quad F_r = ma_r \,;\, F_\theta = ma_\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\dot{r} = 10\left(-\frac{1}{2}\pi \sin\frac{1}{2}\pi t\right) \quad \dot{\theta} = \frac{1}{2}\pi$

$\ddot{r} = \frac{-5\pi^2}{2}\cos\frac{1}{2}\pi t \quad \ddot{\theta} = 0$

$a_r = -\frac{5}{2}\pi^2\cos\frac{1}{2}\pi t - \frac{10\pi^2}{4}\left(1+\cos\frac{1}{2}\pi t\right)$

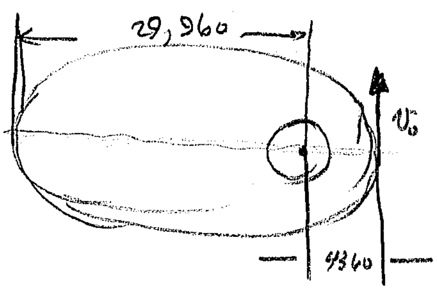
$a_\theta = 10\left(1+\cos\frac{1}{2}\pi t\right)0 + 2\left(-5\pi\sin\frac{1}{2}\pi t\right)\frac{1}{2}\pi$

$F_r = \frac{2}{32.2}\left[-\frac{5\pi^2}{2}\cos\frac{1}{2}\pi t - \frac{10\pi^2}{4}\left(1+\cos\frac{1}{2}\pi t\right)\right]$

$F_\theta = \frac{2}{32.2}\left[\cancel{10\pi}\left(-5\pi^2\sin\frac{1}{2}\pi t\right)\right.$

when $t = 1$; $F_r = \frac{2}{32.2}\left[-\frac{10\pi^2}{4}\right] = -\frac{5\pi^2}{32.2}$

; $F_\theta = \frac{2}{32.2}\left[-5\pi^2\right] = -\frac{10\pi^2}{32.2}$



26400
19920
343 20
when $\theta = 0$

$\frac{1}{r_0} = \frac{GM}{h^2} + C$

$\frac{1}{r} = \frac{GM}{h^2} - C$

$\frac{1}{r_0} - \frac{GM}{h^2} = C$

$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta$

$\frac{1}{r} = \frac{GM}{h^2} - \left(\frac{1}{r_0} - \frac{GM}{h^2}\right)$

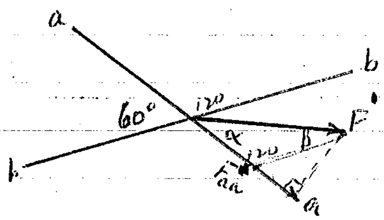
$\frac{1}{r} + \frac{1}{r_0} = \frac{2GM}{h^2}$

$\frac{r+r_0}{rr_0} = \frac{2GM}{r_0^2 v_0^2}$

$v_0 = \sqrt{\frac{2GM(r)}{r_0(r+r_0)}}$

Cesar Levy 713640 AE100A Prof. Nardo.

2.10 The force F of magnitude 800# is to be resolved into 2 components along lines a-a, b-b. Determine $\angle\alpha$ knowing that component of F along a-a is 500#



~~$F_{aa} = 500$ $F = 800$ $\alpha = \text{arc} \sin 5/8 = 51°$~~

~~$F_{bb} = 800 \sin 51° = 711.600\#$~~

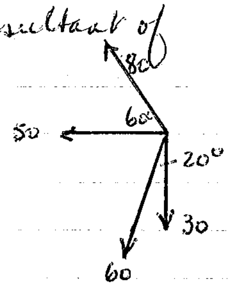
$\therefore$

$$\frac{F'}{\sin 120°} = \frac{F''}{\sin\beta} = \frac{800}{.866} = \frac{500}{\sin\beta}$$

$\beta = 32.8$ $\therefore$ $\alpha = 27.2°$ **good**

10

2.22 Find resultant of

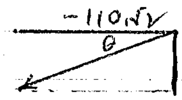


$\Sigma F_y = 80 \sin 60° - 30 - 60 \cos 20°$

$80(.866) - 30 - 60(.940) = -16.12$ **-17.1**

$\Sigma F_x = -(80 \cos 60° + 50 + 60 \cos 70°)$

$-(40 + 50 + 20.52) = -110.52$



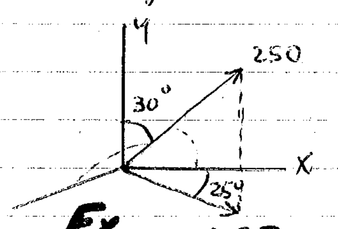
$\tan\theta = .146$ $\theta = 8.4°$ **8.8**

Resultant 111.8# at angle of 188.4° from +x axis **O.K.**

Close

10

2.50 Determine x, y, z components of the 250# force (b) the angles θ_x, θ_y, θ_z that the force forms with coordinates



$\theta_x = \angle$ between ~~plane~~ F & x axis = 63°

$\theta_y = \angle$ between F & y axis = 30°

$\theta_z = \angle$ between ~~plane~~ F & z axis = 78°

How Did You Get These?

$\cos\theta_x = \frac{F_x}{F} = .452$

$\theta_x = 63.1°$

$\cos\theta_z = \frac{F_z}{F} = .211$

$\theta_z = 77.8°$

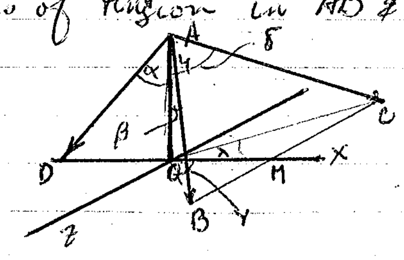
$F_y = 250 \cos 30° = 216.5\#$

$F_x = 250 \cos 60° \cos 25° = 113.3\#$

$F_z = 250 \cos 60° \sin 25° = 52.9\#$

θ_y Can Be Seen

2.64 Knowing that tension in AC is 7000 lbs determine req values of tension in AB & AD $\ni$ R of 3 forces applied at A is vertical



$\therefore \Sigma F_x = \Sigma F_z = 0$

Given $AO = 60'$ $BM = 15'$

$DO = 45'$ $CM = 30'$

$OM = 20'$

9

From given distances & $F_{AD_z} = 0$ we can obtain $\measuredangle\alpha = 37^\circ$

$\measuredangle\beta = 22.5^\circ$

$\measuredangle\gamma = 37^\circ$

$\measuredangle\delta = 31^\circ$

$\measuredangle\lambda = 56.5^\circ$

$\therefore \quad F_{AD_y} = -F_{AD}\cos 37^\circ$

$F_{AD_x} = -F_{AD}\sin 37^\circ$

$F_{AC_x} = F_{AC}\sin 31^\circ \cos 56.5^\circ$

$F_{AC_y} = -F_{AC}\cos 31^\circ$

$F_{AC_z} = -F_{AC}\sin 31^\circ \sin 56.5^\circ$

$F_{AB_x} = F_{AB}\sin 22.5^\circ \cos 37^\circ$

$F_{AB_y} = -F_{AB}\cos 22.5^\circ$

$F_{AB_z} = F_{AB}\sin 22.5^\circ \sin 37^\circ$

$\therefore \quad F_{AD_z} = 0 \quad F_{AD_y} = -F_{AD}(.8) \quad F_{AD_x} = -F_{AD}(.6)$

$F_{AC_x} = 1995^{\#} \quad F_{AC_y} = -6000^{\#} \quad F_{AC_z} = -3010^{\#}$

$F_{AB_x} = F_{AB}(.306) \quad F_{AB_y} = -F_{AB}(.924) \quad F_{AB_z} = F_{AB}(.23)$

$F_{AB} = 13090^{\#}$ ✓ $\quad F_{AD} = 9925^{\#}$ ✓

close

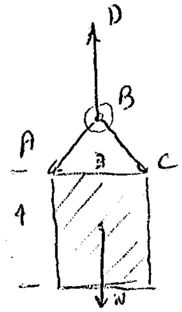
$9\frac{1}{2}$

For a general and more understandable method look at Lopez, Puglise or Trencellito homeworks

$\frac{38.5}{40}$

Cesar May
AE100

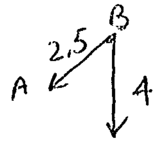
2.44



Free Body Diagram

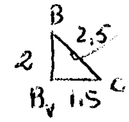
ABC = 5 ft.

W = 1500



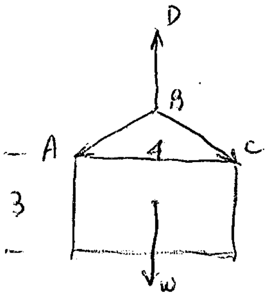
Then $\frac{W}{4} = \frac{F}{2.5}$ $F = 938$ lb.

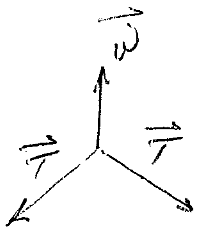
10

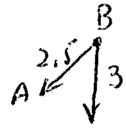


a little more explanation

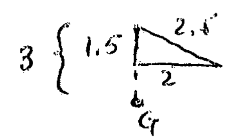
O.K.







Then $\frac{W}{3} = \frac{F}{2.5}$ $F = 1250$ lb



2.68

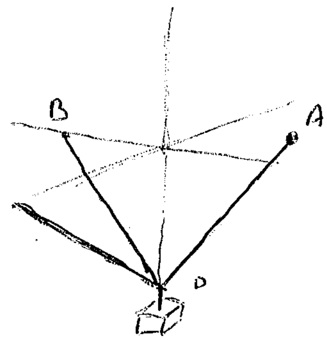
$A(2,0,-3)$

$B(-4.5,0,0)$

$C(0,0,8)$

$D(0,-6,0)$

$W = -262j$



$\vec{T}_{DA} + \vec{T}_{DB} + \vec{T}_{DC} + \vec{W} = 0$

$|T_{DA}|\lambda_A + |T_{DB}|\lambda_B + |T_{DC}|\lambda_C - 262j = 0$

$\lambda_{DA} = \frac{2i + 6j - 3k}{7}$ $\lambda_{DC} = \frac{6j + 8k}{10}$

$\lambda_{DB} = \frac{-4.5i + 6j}{7.5}$

then let $\frac{|T_{DA}|}{7} = \alpha$ $\frac{|T_{DB}|}{7.5} = \beta$ $\frac{|T_{DC}|}{10} = \gamma$ then $2\alpha - 4.5\beta = 0$

$6\alpha + 6\beta + 6\gamma - 262 = 0$

$-3\alpha + 8\gamma = 0$

$\alpha = \frac{8}{3}\gamma$ $\alpha = 2.25\beta$

$6\alpha + \frac{6}{2.25}\alpha + \frac{3}{8}6\gamma = 262$; $6\alpha + 2.56\alpha + 2.25\alpha = 262$

$10.81\alpha = 262$ $\alpha = 24.25$

$\gamma = 9.1$

10

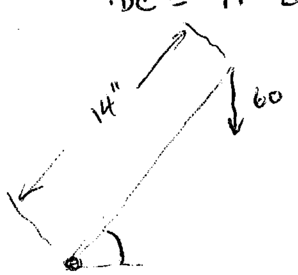
$T_{DA} = 169.5$ #

$T_{DB} = 80.75$ #

$T_{DC} = 91$ #

Excellent

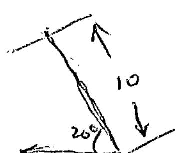
3.4



$F \cdot d = 60\,(14\cos 40^\circ) = 643$ in lb

b. 643 in lb $= F(10\cos 20^\circ)$ $F = 68.4$ #

10

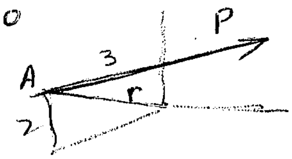


c. if $M_o = 643 = F \cdot d$ & $d \perp F$ then if

$d = 10$ and $M_o = 6.43 \times 10^2$ $F = 64.3$ $\angle 20^\circ$

good

3.20



if $P = 700^{\#}$ then $\lambda_{AB} = \frac{6i + 2j - 3k}{7}$

$P = |P|\lambda_{AB} = 700\left(\frac{6i + 2j - 3k}{7}\right)$ $\quad r = 2j + 3k$

$\underset{A}{r} \times P = \begin{vmatrix} i & j & k \\ 0 & 2 & 3 \\ 600 & 200 & -300 \end{vmatrix} = -1200i + 1800j - 1200k$

$r_B = 6i + 4j$ $\quad r_B \times P = \begin{vmatrix} i & j & k \\ 6 & 4 & 0 \\ 600 & 200 & -300 \end{vmatrix} = r_A \times P$

good 10

3.34 if P, Q, S are coplanar $\quad P \times Q \rightarrow N \perp$ plane of P & Q

if S is in plane of P, Q then $N \cdot S = 0$ since $N \cdot S = |N||S| \cos 90°$

$P \times Q = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & -5 \end{vmatrix} = -7i + 11j - 5k$

$P = i + 2j + 3k$
$Q = 2i - j - 5k$
$S = S_x i + 3j + k$

then $(P \times Q) \cdot S = (-7i + 11j - 5k)(S_x i + 3j + k) = 0$

or $-7S_x + 33 - 5 = 0$ $\quad S_x = 4$

So $\boxed{S = 4i + 3j + k}$

Also if $P \times Q = N$ then $S \times Q = T$ where

$\mu N = T$

10

$P \times Q = N = -7i + 11j - 5k$

$S \times Q = T = 14i - 2j - 5S_x j + 6k + S_x k$

$\therefore -7\mu i + 11\mu j - 5\mu k = 14i - (2 + 5S_x)j + (6 + S_x)k$

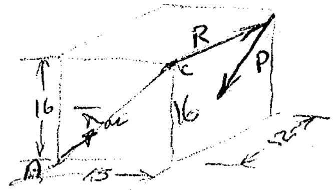
or $\left.\begin{aligned} -7\mu &= 14 \\ -11\mu &= (2 + 5S_x) \\ -5\mu &= 6 + S_x \end{aligned}\right\} \quad S_x = 4$

$\Rightarrow \mu = -2$

$S = 4i + 3j + k$

Cesar diaz AE 100 H

3·40



$P = -40j + 30k \qquad R = -12k$

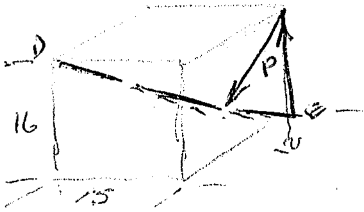
$$R \times P = \begin{vmatrix} i & j & k \\ 0 & 0 & -12 \\ 0 & -40 & 30 \end{vmatrix} = -480i$$

$$\lambda_{AC} = \frac{15i + 16j}{22} \qquad M_{AC} = \lambda_{AC} \cdot (R \times P)$$

$$M_{AC} = \frac{15}{22}(-480) = -327 \text{ in. lb.}$$

9

B



$P = -40j + 30k \qquad R = 16j$

$$\lambda_{DE} = \frac{15i - 16j - 12k}{25} \qquad M_{DE} = \lambda_{DE} \cdot (R \times P)$$

$$M_{DE} = \left(\frac{15i - 16j - 12k}{25}\right) \cdot \begin{vmatrix} i & j & k \\ 0 & 16 & 0 \\ 0 & -40 & 30 \end{vmatrix} = \frac{3}{5}(480) = 288$$

o.k.

52

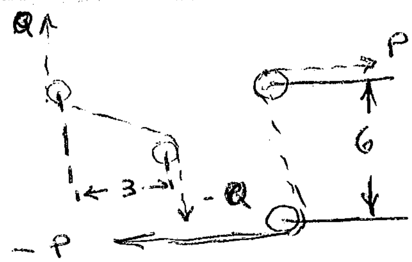
Moment due to $Q = Q \cdot d = (3 + 2r)\,40$

Moment due to $P = P \cdot d = (6 + 2r)\,16$

$M_T = 300$

$\therefore \quad 120 + 80r + 96 + 32r = 300$

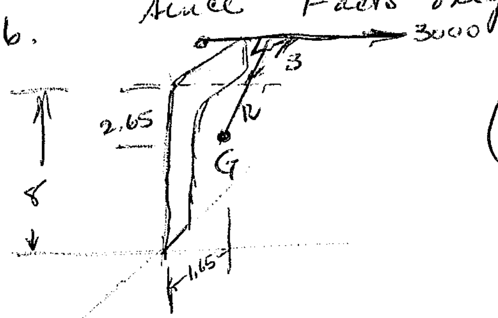
$r = .75'' \qquad D_{PEG} = 2r = 1.50''$



good

10

66. Accel. Facts only in x direction $\quad F = 3000\,i$



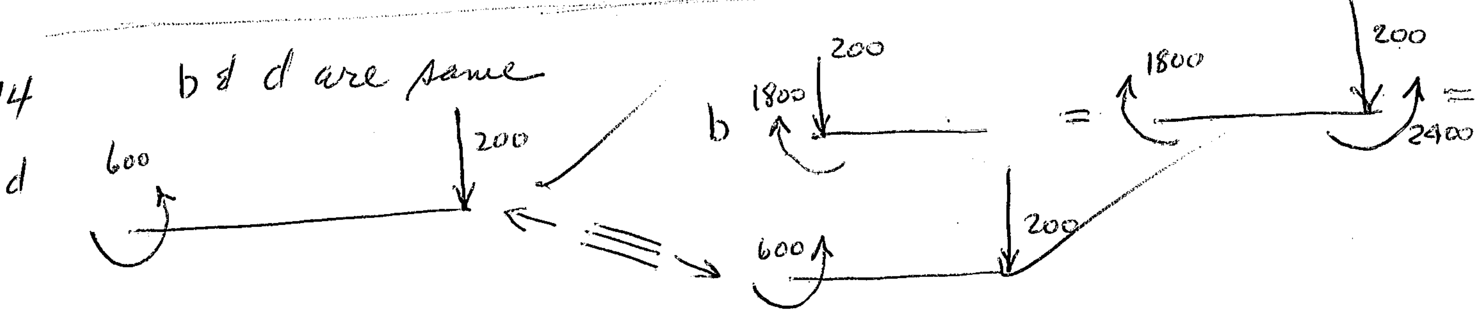
$R_G = 2.65j - 1.35k$

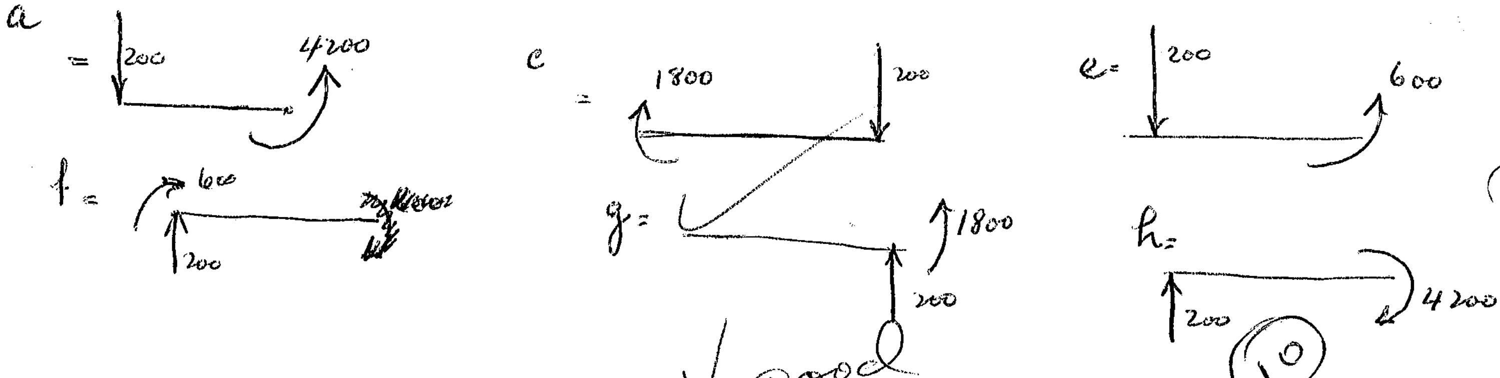
$$(r_G \times F) = \begin{vmatrix} i & j & k \\ 0 & 2.65 & -1.35 \\ 3000 & 0 & 0 \end{vmatrix} = -4050j - 7960k$$

10

good

74 b & d are same





very good

10

82. $100^{\#} = P$, 30

$P = -50j + 86.6i$; $30 = Q$ $Q = 30j$

$R = \Sigma(P+Q) = 86.6i - 20j$ $|R| = 88.8^{\#}$ at an ∡ $347°$

$\Sigma M_{AB} = \Sigma(r \times F)$; $(r \times P) = 800 \sin 30 k$ ⟨here $P = 100\cos 30 i - 100 \sin 30 j$, $r = -8i$⟩

$(r \times Q) = (-3i \times 30j) = -90k$

$\Sigma(r \times F) = -310k + 100k$ ⟨100 from moment at pt 6" below B⟩

$M_{AB} = (a \times R) = (ai) \times (86.6i - 20j) = +20a = -210$

$a = 10.50"$ from B toward A.

10

$\Sigma M_{BC} = \Sigma M_{AB} = -210 = (-Sj) \times R = +86.6S$

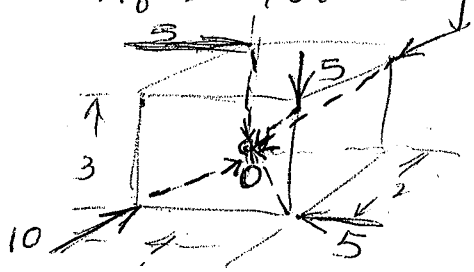
$S = 2.42"$ below B.

good

102a $R = \Sigma F = -10k + 10k + 5i - 5i - 5j = -\cancel{5j}$

$M_O^R = \Sigma(r \times F) = (-10k \times (-2k)) + -5i\,(-4i - 2k) +$
$5i(-3j) + 10k(-4i - 3j) - 5j(-4i - 3j - 2k)$

$M_O^R = 40i - 50j - 35\cancel{k}$

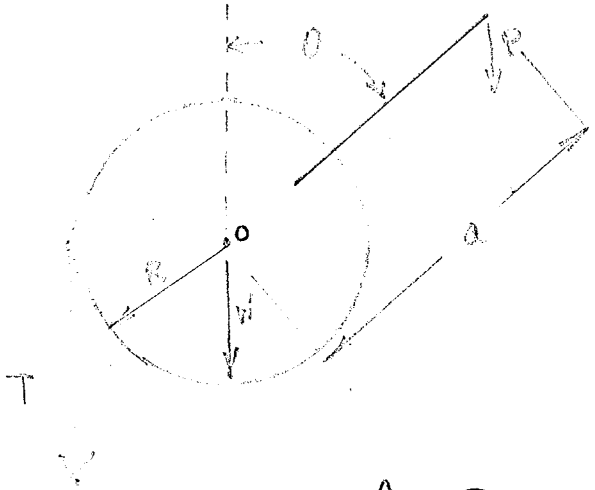


10

59/60

AE100A PROF. NARDO C. LEVY

4-6



a $\quad \sum M_O = |T|R - (P\sin\theta)a = 0$

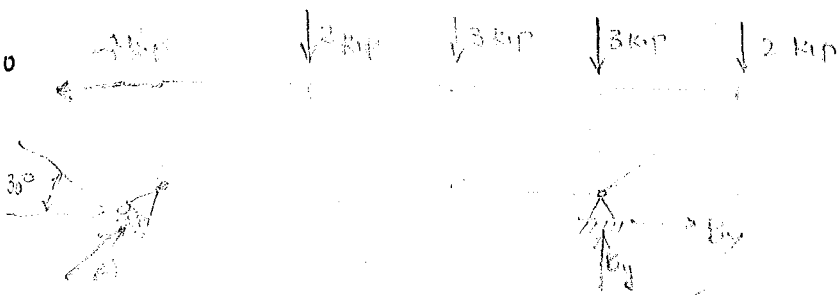
$\therefore \quad T = \dfrac{Pa}{R}\sin\theta$

if $P = 40$, $a = 12\text{ in}$, $R = 4\text{ in.}$, $\theta = 75^\circ$, $\sin\theta = .966$

b $\quad |T| = \dfrac{(40)(12)}{(4)}(.966) = 120(.966) = 116^{\#}$ in $-y$ dir.

10

4-10



$\sum F_x = \sum F_y = 0$

$\sum M_A = 0$

① $\sum F_x = B_x + A\cos 60^\circ - 4 = 0$

② $\sum F_y = B_y + A\sin 60^\circ - 10 = 0$

③ $\sum M_A = 4(10) - 2(16) - 3(32) - 3(48) - 2(64) + B_y(48) = 0$

$B_y = \dfrac{360\text{ kip-ft}}{48\text{ ft}} = 7.5\text{ kips} \uparrow$ From ③

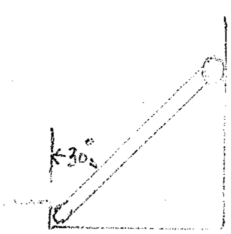
$A = \dfrac{2.5}{.866} = 2.89\text{ kips} \measuredangle 60^\circ$ From ②

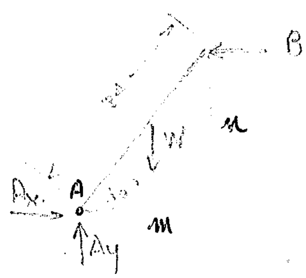
10

$B_x = 4 - A(\cos 60^\circ) = 2.56\text{ kips} \rightarrow$ From ①

4-24

ⓐ





$m = 12''$ $\quad w = 15^{\#}$

$n = 12\sqrt{3}''$

$\sum F_x = \sum F_y = 0 \quad \sum M_A = 0$

$\sum F_x = A_x - B = 0 \quad A_x = B$

$\sum F_y = A_y - w = 0 \quad A_y = w$

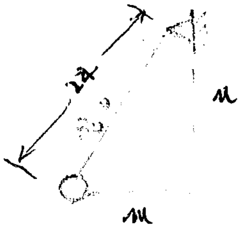
$\sum M_A = w(6) - B(12\sqrt{3}) = 0$

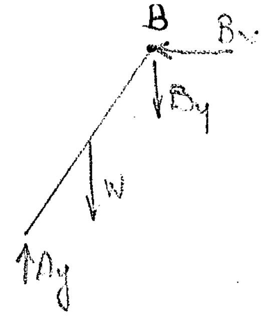
$B = 4.35^{\#} \quad A_x = 4.35^{\#}$

$A_y = 15^{\#}$

$\theta = \arctan \dfrac{15}{4.35} = 73.83^\circ$

(b)





$A_y = W + B_y$ $\qquad \Sigma F_y = 0$

$B_x = 0$ $\qquad \Sigma F_x = 0$

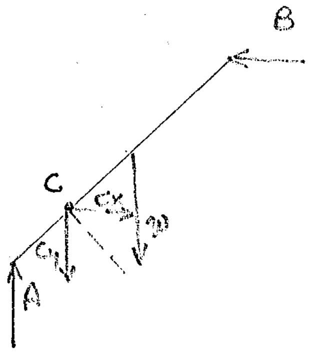
$A_y(12) - W(6) = 0$ $\qquad \Sigma M_B = 0$

$A_y = 7.5$

$B_y = -7.5$ $\quad \therefore$ B_y is in opp dir. to that shown

$A_y = B_y = 7.5$ lb. ↑

(c)



$\Sigma F_x = B - C_x = 0$

$\Sigma F_y = A - C_y - W = 0$

(3) $\Sigma M_C = A(3) + W(3) = 9\sqrt{3}\,B$

C from (3)

$C_y = C\sin 30°$

$C_x = C\cos 30°$

C is 6 ft from A on AB

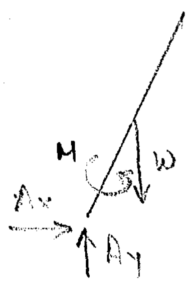
$3(C_y + W) + 3W = 9\sqrt{3}\,C_x$

$7.5^{\#}$ at $30°$ $= C$

$A = +C_y + W = 3.75 + 15 = 18.75^{\#}$ ↑

$B = C\cos 30° = 6.5^{\#}$ ←

(d)



$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y - W = 0$

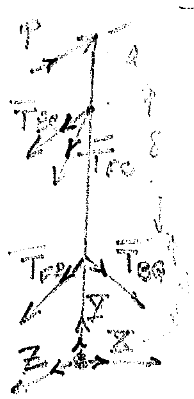
$\Sigma M_A = W(6) - M = 0$

$A_y = 15$ lb ↑

$M = 90$ lb-ft.

10

4-64



$\Sigma \bar{F} = 0$ $\qquad \bar{T}_{EC} + \bar{T}_{FC} + \bar{T}_{FB} + \bar{T}_{BG} + \bar{X} + \bar{Y} + \bar{Z} + \bar{P} =$

$\Sigma M_A^R = 0$ $\qquad (\bar{r}_1 \times \bar{T}_{FB}) + (\bar{r}_2 \times \bar{T}_{BG}) + (\bar{r}_3 \times \bar{T}_{FC}) + (\bar{r}_4 \times \bar{T}_{CE}) + (\bar{r}_5 \times$ $= ($

$r_1 = 4j = r_2 \qquad r_3 = 12j = r_4 \qquad r_5 = 16j$

(1) $\bar{T}_{EC} = T_{EC}\left(\frac{-i-j}{\sqrt{2}}\right)$ (2) $\bar{T}_{FC} = T_{EC}\left(\frac{-j+k}{\sqrt{2}}\right)$

(3) $\bar{T}_{FB} = T_{FB}\left(\frac{-4j+12k}{12.65}\right)$ (4) $\bar{T}_{BG} = T_{FB}\left(\frac{i-j}{\sqrt{2}}\right)$

(5) $\bar{X} = Xi$ (6) $\bar{Y} = Yj$ (7) $\bar{Z} = Zk$ (8) $\bar{P} = -Pk$

(A) $\Sigma F_i = -\frac{T_{EC}}{\sqrt{2}} + \frac{T_{FB}}{\sqrt{2}} + X = 0$

(B) $\Sigma F_j = \frac{-2T_{EC}}{\sqrt{2}} - \frac{4T_{FB}}{12.65} - \frac{T_{FB}}{\sqrt{2}} + Y = 0$

(C) $\Sigma F_k = \frac{T_{EC}}{\sqrt{2}} + \frac{12T_{FB}}{12.65} + Z - P = 0$

$$\Sigma M_A^R = 4j \times T_{FB}\left(\frac{-4j+12k}{12.65}\right) + 4j \times T_{FB}\left(\frac{i-j}{\sqrt{2}}\right) + 12j \times T_{EC}\left(\frac{-j+k}{\sqrt{2}}\right)$$
$$+ 12j \times T_{EC}\left(\frac{-i-j}{\sqrt{2}}\right) + 16j \times -P_R = 0$$

(D) $\Sigma M_{A_i}^R = \frac{48 T_{FB}}{12.65} + \frac{12}{\sqrt{2}} T_{EC} - 16P = 0$; (E) $\Sigma M_{A_j}^R = 0$

(F) $\Sigma M_{A_k}^R = -\frac{4}{\sqrt{2}} T_{FB} + \frac{12}{\sqrt{2}} T_{EC} = 0$

(10) From (F) $T_{EC} = \frac{1}{3} T_{FB}$ $T_{EC} = 725^{\#}$

From (D) & (10) $.95\, T_{FB} + .707\, T_{FB} = 3600$ $T_{FB} = 2175^{\#}$

From (A) $-\frac{725}{\sqrt{2}} + \frac{2175}{\sqrt{2}} = -X$ $X = -1025^{\#}$

From (B) $-\frac{1450}{\sqrt{2}} - \frac{4(2175)}{12.65} - \frac{2175}{\sqrt{2}} = -Y$ $Y = 3253^{\#}$

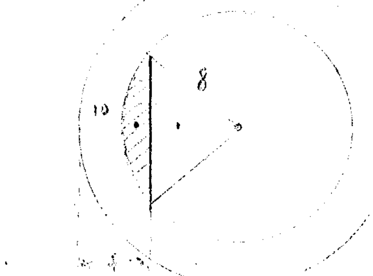
From (C) $\frac{-725}{\sqrt{2}} + \frac{12(2175)}{12.65} + (-900) = -Z$ $Z = -1677$

$\therefore$ $T_{ECF} = 725^{\#}$ $T_{FBG} = 2175^{\#}$ 10

$$M_A = -1025\bar{i} + 3253\bar{j} - 1677\bar{k}$$

40/40

8 C. LEVY AE100



A_1 Area of circle with rad. 10 = $\pi(100)$ = 314.

A_2 Area of circle with rad. 8 = 64π = 201.

A_3 Area of sector = $\alpha(8^2) = 46.4$ $\alpha = \arccos\frac{6}{8} = 41.25^\circ$

A_4 Area of triangle = $12\sqrt{7} = 31.7$

$\bar{y} = 0$

$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A}$

$\Sigma A = A_1 - A_2 + A_3 - A_4$

$\bar{X}_{A_1} = 0$

$\bar{X}_{A_2} = 0$

$\alpha = .723$ rad

$\bar{X}_{A_3} = \frac{2r\sin\alpha}{3\alpha} = \frac{16(.663)}{3(.723)} = 4.9$

$A\bar{X}_{A_3} = (4.9)(46.4) = 227$

$\bar{X}_{A_4} = 4$

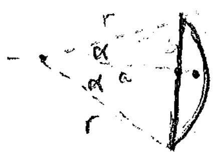
$A\bar{x}_{A_4} = 4(31.7) = 127$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{A_3\bar{x}_{A_3} - A_4\bar{x}_{A_4} - 0 - 0}{A_1 - A_2 + A_3 - A_4} = \frac{100}{113 + 14.7} = \frac{100}{127.7}$$

$\bar{X}$ = .784" to left of origin

10

18



$\bar{y} = 0$

$a = r\cos\alpha$
$b = r\sin\alpha$

$\bar{X}_{sec} = \frac{r\sin\alpha}{\alpha}$

$L_{SEC} = 2r\alpha$

$\bar{X}_{LINE} = r\cos\alpha$

$L_{LINE} = 2r\sin\alpha$

10

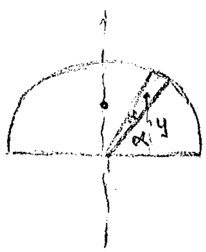
$\frac{\Sigma \bar{x}L}{\Sigma L} = \bar{x}$

$\Sigma \bar{x}L = 2r^2\sin\alpha + 2r^2\sin\alpha\cos\alpha$

$\Sigma L = 2r\alpha + 2r\sin\alpha$

$$\bar{x} = \frac{2r^2\sin\alpha + 2r^2\sin\alpha\cos\alpha}{2r(\alpha + \sin\alpha)} = \frac{r(\sin\alpha + \sin\alpha\cos\alpha)}{\alpha + \sin\alpha}$$

36



$\bar{y} = \frac{\int y\,dA}{A}$

$A = \frac{\pi r^2}{2}$

$dA = \frac{1}{2}r^2\,d\alpha$

$y = \frac{2r}{3}\sin\alpha$

$\left(\frac{2r}{3}\sin\alpha\right)\left(\frac{1}{2}r^2\,d\alpha\right) = y\,dA$

$$\bar{y} = \frac{2\int_0^{\frac{\pi}{2}} \frac{r^3}{3}\sin\alpha\,d\alpha}{\frac{\pi r^2}{2}} = \frac{2r^3}{3}\cdot\frac{2}{\pi r^2} = \frac{4r}{3\pi}$$

since areas are symmetrical about y axis $\bar{x} = 0$

10

30/30

44

$$\bar{y} = \frac{\int \hat{y}\,dA}{A} \qquad \frac{y}{2} = \hat{y} \qquad dA = y\,dx$$

$$\hat{y}\,dA = \frac{y^2}{2}dx = \frac{a^2}{2}\sin^2\frac{\pi x}{L}\,dx$$

$$\int dA = A = \int y\,dx$$

$$= \int a\sin\frac{\pi x}{L}\,dx = \frac{La}{\pi}\left(-\cos\frac{\pi x}{L}\right)\Big]_0^{L/2} = \frac{La}{\pi}$$

$$\int \hat{y}\,dA = \frac{a^2}{2}\int \sin^2\frac{\pi x}{L}\,dx \quad ; \quad \sin^2\frac{\pi x}{L} = \frac{1}{2} - \frac{\cos\left(\frac{2\pi x}{L}\right)}{2}$$

$$\left(\frac{1}{2}x - \frac{L}{4\pi}\sin\frac{2\pi x}{L}\right)\Big]_0^{L/2} = \frac{L}{4} \qquad \int \hat{y}\,dA = \frac{a^2}{2}\frac{L}{4}$$

$$\bar{y} = \frac{\frac{a^2 L\pi}{8\pi}}{\frac{8La}{8\pi}} = \frac{a^2\pi L}{8aL} = \frac{a\pi}{8}$$

$$\bar{x} = \frac{\int x\,dA}{A} \qquad \int x\,dA = \int xy\,dx = \int xa\sin\frac{\pi x}{L}\,dx$$

$$a\int x\sin\frac{\pi x}{L}\,dx \qquad \text{let } x = u \quad \sin\frac{\pi x}{L}dx \quad dx = du \quad -\frac{L}{\pi}\cos\frac{\pi x}{L} = V$$

10

$$\therefore \int x\,dA = \frac{aL^2}{\pi^2} \qquad \frac{aL^2}{\pi^2} = a\Big[_0^{L/2} -\frac{Lx}{\pi}\cos\frac{\pi x}{L} + \frac{L^2}{\pi^2}\sin\frac{\pi x}{L}$$

$$\bar{x} = \frac{\frac{aL^2}{\pi^2}}{\frac{\pi La}{\pi^2}} = \frac{aL^2}{La\pi} = \frac{L}{\pi}$$

100. $V_{BRASS} = \pi r_1^2 h - \pi r_2^2 h \qquad r_1 = 1.5'' \quad r_2 = 1'' \quad h = 1.5'')$

$V_{BRASS} = \pi(1.25)(1.5) = 5.9$ in cu. $\qquad \rho V = W = \frac{530\text{lb}}{1728\text{cui}} \cdot 5.9\text{cui} = 1.805$ lb.

$\bar{y}\,W_{BRASS} = (.75\text{in})(1.805) = 3.160$ lb in

$V_{AL} = \pi r_T^2 h_T \qquad r_T = 1'' \quad h_T = 1.5'' \qquad \rho V_{AL_T} = \frac{170}{1728} \cdot 4.72 = .464$ lb

$\bar{y}\,W_{AL_T} = (.464)(.75\text{in}) = .812$ lb in

$V_{AL} = \pi r_B^2 h_B \qquad r_B = 1.5'' \quad h_B = 1'' \qquad \rho V_{AL_B} = \frac{170}{1728} \cdot 7.08 = .696$ lb

$\bar{y}\,W_{AL_B} = (.5)(.696) = .348$ lb in

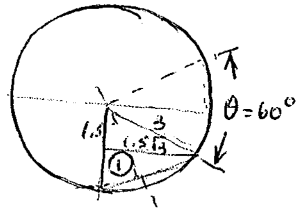
$$\bar{Y} = \frac{\Sigma \bar{y} W}{\Sigma W} = \frac{3.160 + 1.160}{1.805 + 1.160} = \frac{4.320}{2.965} = 1.455$$

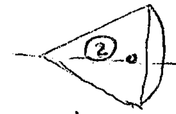
from bottom of aluminum portion

10

Cesar Henry
AG 100

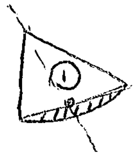
53.





$\bar{x}$ of this sector $= \frac{4r}{3}\left(\frac{\sin^3\alpha}{2\alpha - \sin 2\alpha}\right)$

but we want $\bar{X}$ of this section ①



; so rotating $\bar{x}$ will give it to us

let $\bar{X}$ of ② be r then $\bar{X}$ of ① $= r\cos(\text{rotated angle})$

in our case $\bar{X}$ ① $= \bar{X}$ ② ~~sin~~ $60° = \bar{X}① = \frac{\bar{X}②}{2} = \frac{4r}{6}\left(\frac{\sin^3\alpha}{2\alpha - \sin 2\alpha}\right)$

V = Volume of △① + volume of sector.

Volume of △① $= 2\pi \bar{X}_{\triangle①} A_{\triangle①}$ $\qquad \bar{X} = .5\sqrt{3}$

$A = \frac{1}{2}(1.5)(1.5\sqrt{3})$

Volume of △① $= 2\pi(.5)(\sqrt{3})(.75)(1.5\sqrt{3})$

$= \pi(.75)(1.5)(3) = 10.6$ cu ft.

10

Volume of sector $= 2\pi \bar{X}_{sector} A_{sec}$

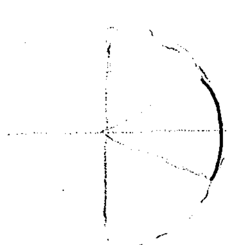
$= 2\pi(2)\left(\frac{1}{8}\right)\left(\frac{1}{\frac{\pi}{3} - \frac{\sqrt{3}}{2}}\right)\left[\frac{\pi}{6}(9) - \frac{9}{4}\sqrt{3}\right]$ $\qquad \bar{X}_{sector} = \frac{4r}{6}\left(\frac{\sin^3\alpha}{2\alpha - \sin 2\alpha}\right)$

$A_{sector} = \alpha r^2 - \frac{1}{2}r^2\sin 2\alpha$

$= 6.84$ cu ft.

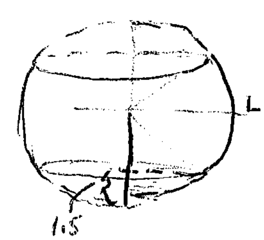
$V_{TOTAL} = 10.6 + 6.84 = 17.44$ cu ft.

54.



what I plan to do in this problem is rotate a piece of wire equidestant from the center and which is divided into 2 by the x axis, such that the surface area of the whole circle minus the surface area due to this wire - the whole difference divided by 2 gives me the surface area ~~rotated~~ obtained by the rotating the $\frac{1}{2}$ wire under the surface of the liquid

good



$S_L = 2\pi\bar{x}L$ $\qquad L = r\theta \qquad \theta = \frac{\pi}{3} \qquad r = 3 \qquad L = \pi$

$\bar{x} = \frac{r\sin\alpha}{\alpha} = \frac{r(1/2)}{\pi/6}$ $\qquad \theta = 2\alpha$

or

$\bar{x} = \frac{r\sin\alpha}{\alpha}$ $\qquad L = 2\alpha r$

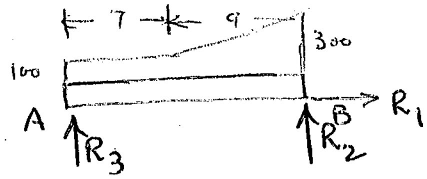
$S_L = 4\pi r^2\sin\alpha$ $\qquad \alpha = \pi/6 \quad \sin\alpha = 1/2 \qquad S_L = 2\pi r^2$

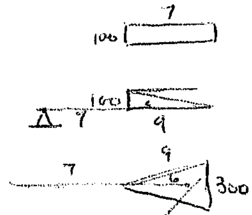
$S_{sphere} = 4\pi r^2$

$S_{under\ liquid} = \dfrac{S_{sphere} - S_L}{2} = \pi r^2 = 9\pi = 28.3\ ft^2$

10

66



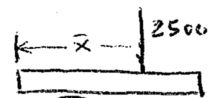


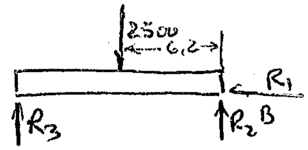
$\bar{x} = 3.5 \quad \bar{x}A = (3.5)(700) = 2450$

$\bar{x} = 10 \quad \bar{x}A = 4500 \quad A = 450$

$\bar{x} = 13 \quad \bar{x}A = 13(1350) = 17550$

$\bar{x} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{24500}{2500} = 9.8$ ft from right





$\Sigma F_x \quad R_1 = 0$

$\Sigma F_y \quad R_2 + R_3 = 2500$

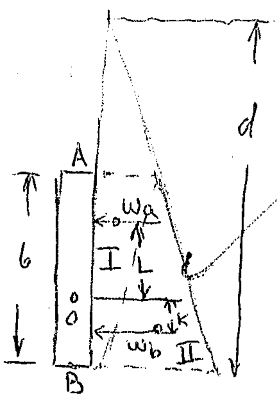
$\Sigma M_B = R_3(16) - 2500(6.2) \qquad R_3 = \dfrac{2500(6.2)}{16}$

$R_3 = 968^{\#}$

$\therefore R_2 = 1532^{\#}$

16

78



$p_A = \gamma(d-6) \qquad w_a = \gamma(d-6)(36\ in^2) \qquad w = pA$

$p_B = \gamma(d) \qquad w_b = \gamma(d)(36\ in^2)$

$\Sigma M_o = 0 \qquad w_a(L) = w_b(K)$

$L = 1.5''$
centroid of Δ I
$= 4''$ from B
and hinge is $2.5''$ up
$\therefore L = 4 - 2.5 = 1.5$

$\therefore \gamma(d-6)(36)(1.5'') = \gamma d(36)(.5'')$

$(d-6)(1.5) = \dfrac{d}{2}$

$(d-6)3 = d$

$-18'' = -2d$

$9'' = d$

$K = .5''$
centroid of Δ II
$= 2''$ from B
hinge $= 2.5''$ from
$\therefore K = 2.5 - 2 = .5$

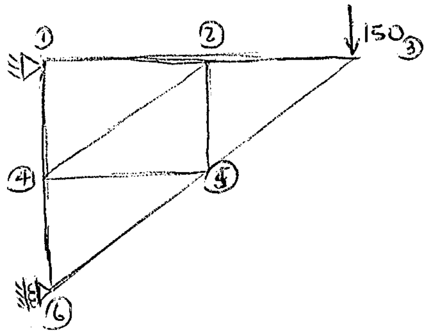
Therefore from free surface to bottom
$= 9''$.

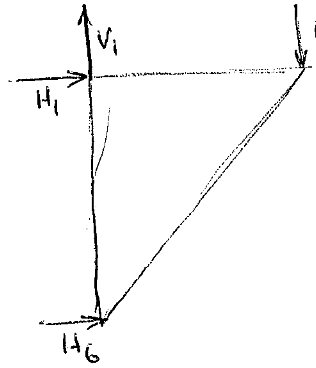
10

very good

8

Cesar Leong
PROFESSOR NARDO





$\Sigma F_x = 0 \quad H_1 + H_6 = 0$

$\Sigma F_y = 0 \quad V_1 - 150 = 0$

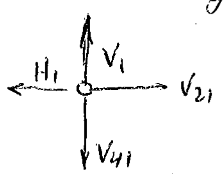
$\Sigma M_1 = 0 \quad H_6(60) - 150(80) = 0$

$H_1 = -H_6$

$V_1 = 150^{\#}$

$H_6 = 200^{\#}\ ; \ H_1 = -200^{\#}$

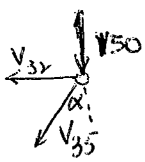
Isolate pin 1



$V_{21} - H_1 = 0 \qquad V_{21} = H_1 = -200^{\#}$

$V_{41} - V_1 = 0 \qquad V_{41} = V_1 = 150^{\#}$

Isolate pin 3



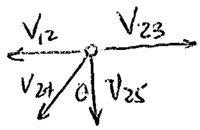
$-150 - V_{35}\cos\alpha = 0$

$V_{35}(3/5) = -150 \qquad V_{35} = -250^{\#}$

$-V_{32} - V_{35}\sin\alpha = 0$

$(-)V_{32} = V_{35}\sin\alpha = -250(4/5) = (-)200^{\#}$

Isolate pin 2

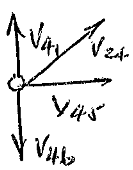


$V_{23} - V_{12} - V_{24}\sin\theta = 0 \qquad$ since $V_{23} = -200^{\#} \quad V_{12} = -200^{\#}$

$-V_{25} - V_{24}\cos\theta = 0 \qquad V_{24}\sin\theta = 0$ or $V_{24} = 0$ since $\sin\theta \neq 0$

$V_{24} = 0$ O.K. $\quad \therefore \quad V_{25} = 0$ also

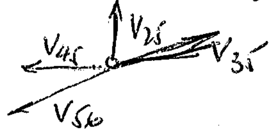
Isolate pin 4



$V_{41} = V_{46} \qquad V_{41} = 150^{\#} = V_{46}$

$V_{45} = 0$

Isolate pin 5



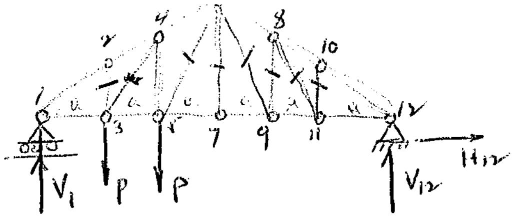
since $V_{45} = 0 = V_{25}$

then $V_{35} = V_{56}$ (since they have same line of action)

therefore $V_{56} = -250^{\#}$

10

13

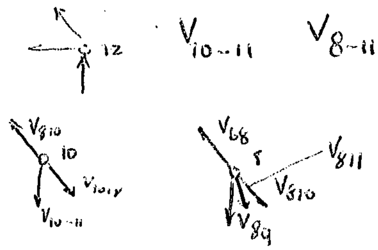


$H_{12} = 0$

$V_1 + V_{12} = 2P$

$\Sigma H_1 = Pa + P(2a) - 6aV_{12} = 0$

$3aP = 6aV_{12}$ $\quad \frac{P}{2} = V_{12} \quad V_1 = \frac{3P}{2}$



V_{10-11} V_{8-11} V_{6-8} V_{8-9} V_{6-7}

V_{6-5} V_{6-9} V_{3-4}

9

These are the zero members

V_{10-11} ✓ $\quad V_{6-5}$ X No

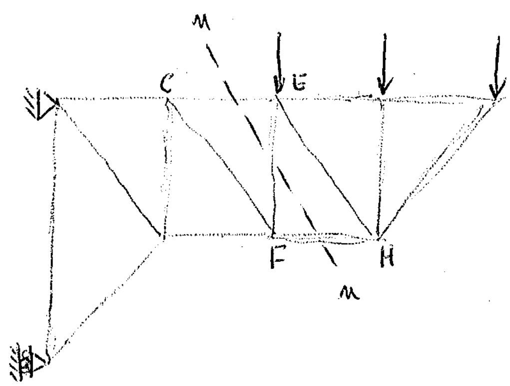
V_{8-11} ✓ $\quad V_{6-9}$ ✓

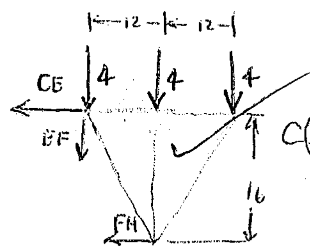
V_{6-8} X No $\quad V_{3-4}$ ✓

V_{8-9} ✓ $\quad$ 2 too many

V_{6-7} ✓

42





good

$\Sigma M_E = 4(12) + 4(24) + FH(16) = 0$

$FH = -\frac{4(36)}{16} = -9$ kips

$\Sigma F_x = -CE - FH = 0$

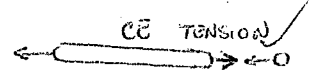
$CE = -FH \quad CE = 9$ kips

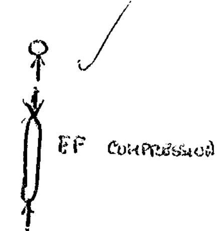
$\Sigma F_y = -4 - 4 - 4 - EF = 0$

-12 kips $= EF$

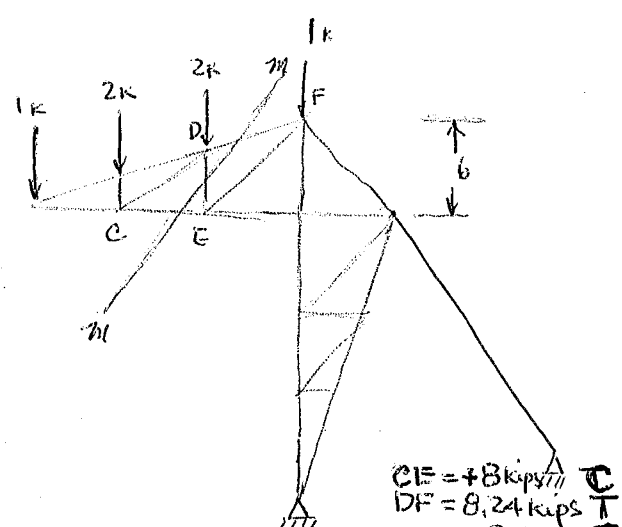
10

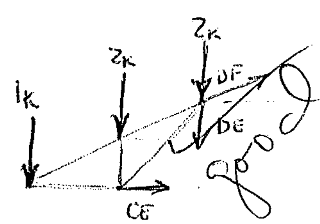
then





46





good

$\Sigma M_D = 16(1) + 8(2) + CE(4) = 0$

$CE = -\frac{32}{4} = -8$ kips

10

$\Sigma F_y = 0 \quad -5K - DE + DF\left(\frac{1}{\sqrt{17}}\right) = 0$

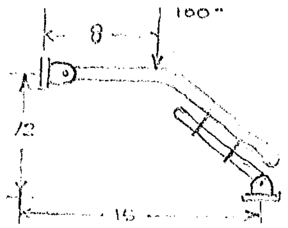
$\Sigma F_x = 0 \quad DF\left(\frac{4}{\sqrt{17}}\right) + CE = 0$

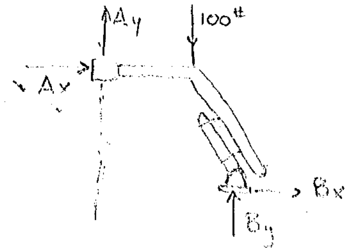
$DF = \frac{8(\sqrt{17})}{4} = 2\sqrt{17} = 8.24$ kips

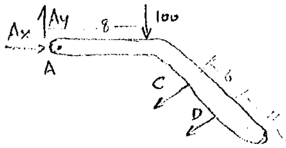
$DE = -5 + DF\left(\frac{1}{\sqrt{17}}\right) = -5 + 2 = -3$ kips

CE = +8 kips C
DF = 8.24 kips T
DE = +3 kips C

Brian Lewey





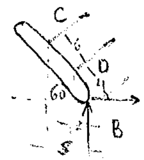


① $\Sigma M_A = -100(8) - C\left(\frac{\sqrt{3}}{2}\right)(12-5\sqrt{3}) - \frac{C}{2}(10) - D\frac{\sqrt{3}}{2}(12-2\sqrt{3}) - \frac{D}{2}(13) = 0$

② $A_x - \frac{C\sqrt{3}}{2} - \frac{D\sqrt{3}}{2} = 0$

③ $A_y - \frac{C}{2} - \frac{D}{2} - 100 = 0$

(8)



① $\Sigma M_B = -\frac{C\sqrt{3}}{2}(5\sqrt{3}) - \frac{C}{2}(5) - \frac{D\sqrt{3}}{2}(2\sqrt{3}) - \frac{D}{2}(2) = 0$

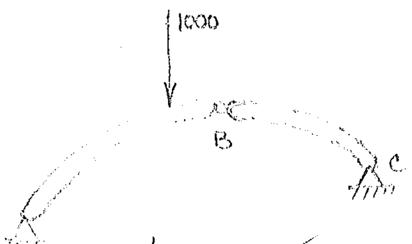
② $F_x = B_x + \frac{D\sqrt{3}}{2} + \frac{C\sqrt{3}}{2} = 0$

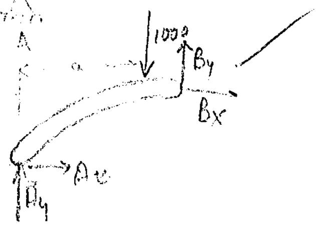
③ $F_y = \frac{C}{2} + \frac{D}{2} + B_y = 0$

How Did You Get these? from the book?

$C = 29.8^{\#}$ $D = -74.6^{\#}$ $B_y = 22.4^{\#}$ $B_x = 38.8^{\#}$ $A_y = 77.6^{\#}$ $A_x = -38.8^{\#}$

No, from the equations



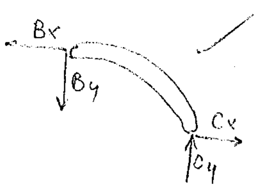


$B_y + A_y - 1000 = 0$

$B_x = -A_x$

$\Sigma M_A = -1000(25) + B_y(30) - 18B_x = 0$

(9 1/2)



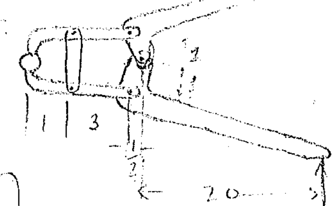
$C_x = B_x$

$C_y = B_y$

$\Sigma M_C = +B_x(X) + B_y(20) = 0$

$B_x = -833.3^{\#}$ $B_y = \cancel{333.3^{\#}}$ $\cancel{C_x = -833.3^{\#}}$ $C_y = 333.3^{\#}$ $A_x = \cancel{+833.3^{\#}}$ $A_y = \cancel{666.7^{\#}}$

$A_y + B_y = 1000$ ∴ $A_y = 1000 - B_y = 1000 - 333.3 = 666.7$



$\Sigma F_y = 0$

$B_y + 125 = C_y$

$\Sigma F_x = 0$

$C_x = B_x$

$\Sigma M_B = 0$

$C_y/2 + C_x - 2500 = 0$

$\Sigma F_x = 0$

$B_x = 0 = C_x$

$\Sigma F_y = 0$

$B_y = A_y + K_y$

$\Sigma M_B = 0$

$A_y(3) + K_y(4) = 0$

$\frac{C_y}{2} = 2500$

$C_y = 5000^{\#}$

$B_y = 4875^{\#}$

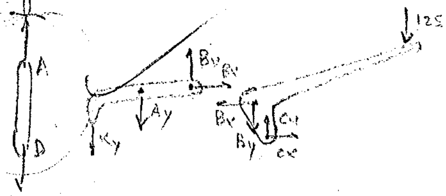
$-(3A_y + 3K_y = (4875)3)$

$+3A_y + 4K_y = 0$

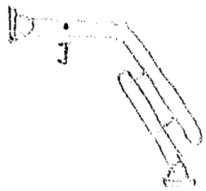
$+K_y + 14625 = 0$

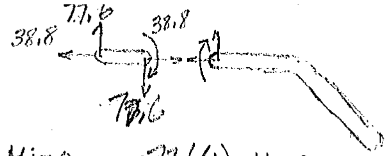
$K_y = -14625^{\#}$

(10)



1.4



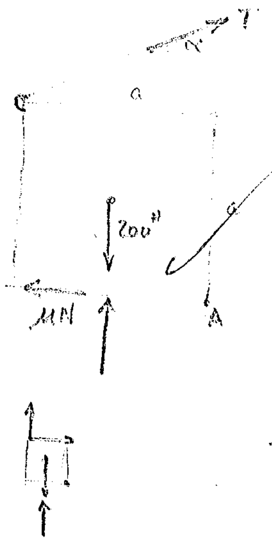


$\Sigma M_J = 0$ on JA part $-77.6(4) + M = 0$

$M = 310$ in-lb

10

8.12



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$T\cos 30^\circ = \mu N$

$T\sin 30^\circ + N - 200 = 0$

$T(.866) = (.3)N$

$T(.866)/(.3) = N$

10

$\frac{T}{2} + 2.89T = 200$

$T = \frac{200}{3.39} = 59.1^{\#}$

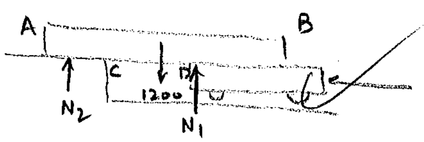
$N = 170.5$

$\Sigma M_A = -T\cos 30^\circ(a) - T\sin 30^\circ(+a) + 200(a/2) \overset{?}{>} 0$ $(r_1 \times T > r_2 \times W$ for tip$)$

$-(.866)59.1 - .5(59.1) + 100 > 0$

Yes since $+(1.366)(59.1) < 100$ ∴ it will not tip over but will slide

8.20



$N_1 + N_2 = 1200$

$\Sigma M_D = -8N_2 + 3600 = 0$

$N_2 = 450^{\#}$

$N_1 = 750^{\#}$

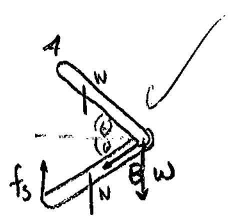
$\mu_s N_2 = (.3)(450) = 135^{\#}$

$\mu_s N_1 = (.3)(750) = 225^{\#}$

10

Since μN_1 will not cause motion since D is above A only $F = \mu N_2$ will cause motion ∴ $P = \mu N_2 = 135^{\#}$

8.28



$\theta = 10^\circ$

$B = \frac{W}{\sin 10^\circ} = \frac{W}{.173}$

$B_x = B\cos\theta = \frac{W}{.173}(.985) = 5.65W$

$\Sigma F_y = 0$ $f_s = 2W = \mu_s N$ $N = 5.65W$

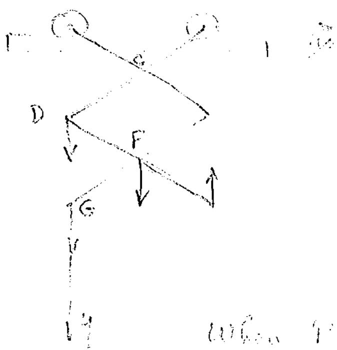
$\mu_s(5.65W) = 2W$

$\mu_s = .355$

10

Each member is L long

C. Levy AE 100
Prof. Nardo.



$y_C = \frac{L}{2}\sin\theta$ $\delta y_C = \frac{L}{2}\cos\theta\,\delta\theta$

$y_D = L\sin\theta$ $\delta y_D = L\cos\theta\,\delta\theta$

$y_F = \frac{3L}{2}\sin\theta$ $\delta y_F = \frac{3L}{2}\cos\theta\,\delta\theta$

$y_G = y_P = 2L\sin\theta$ $\delta y_G = \delta y_P = 2L\cos\theta\,\delta\theta$

When P is applied the machine moves up & out

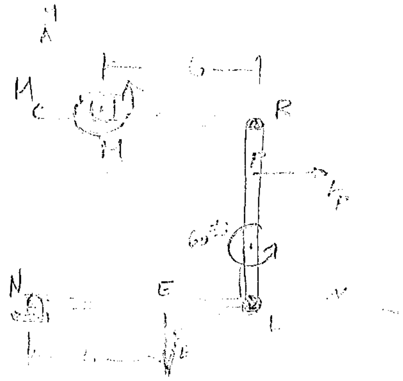
$\therefore \delta U = 0 = -F_D\,\delta y_D - F_C\,\delta y_C - F_G\,\delta y_G - F_F\,\delta y_F + P\,\delta y_G$

Cancelling similar $\cos\theta\,\delta\theta$ from all terms

$P(2L) = 30^{\#}L + 40^{\#}(L/2) + 15(\,\cdot\,) + \ldots$

P = ... upward.

10



member NL has vertical translation only, $\therefore$

HR and NL moves down.

take member NL.

$\delta y = 9\,\delta\theta$

for equilibrium

HR moves down $9\,\delta\theta$ also

$\therefore \delta\alpha = \frac{3}{2}\delta\theta$

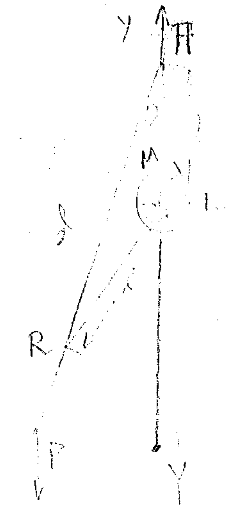
thus

$-F_E\,\delta y_E + M\,\delta\alpha = 0 = \delta U$

$F_E = 120^{\#}$; $\delta y_E = 6\,\delta\theta$; $\delta\alpha = \frac{3}{2}\delta\theta$

$M = \frac{F_E\,\delta y_E}{\delta\alpha} = \frac{120(6)\,\delta\theta}{1.5\,\delta\theta} = 480^{\#}\text{in}$

10



$y_P = d\cos\theta$ $\delta y_P = -d\sin\theta\,\delta\theta$

$-P\,\delta y_P - M\,\delta\alpha = \delta U = 0$

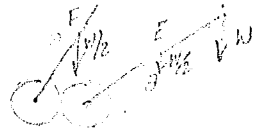
M moves in a $\delta\alpha = 2\delta\theta$ ...

The ... = 2.5 ...

$\therefore$

$+P\,d\sin\theta = 2M$ $M = \frac{P}{2}d\sin\theta$

10



$$W y_D + \frac{W}{2} y_E + W/2\, y_F = V$$

$$y_D = l \sin\theta$$

$$y_E = \frac{l}{2} \sin\theta$$

$$y_F = l/2 \cos\theta$$

$$Wl \sin\theta + \frac{W}{2}\frac{l}{2}\sin\theta + \frac{Wl}{4}\cos\theta = V$$

$$\frac{5Wl\sin\theta}{4} + \frac{Wl}{4}\cos\theta = V$$

$$\frac{dV}{d\theta} = 0 \qquad \frac{5Wl\cos\theta}{4} - \frac{Wl\sin\theta}{4} = 0 \qquad 5\cos\theta = \sin\theta$$

by trial

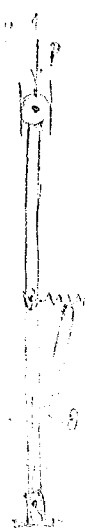
$$\theta = 78^\circ 40'$$
$$\theta = 258^\circ 40'$$

$$\frac{d^2V}{d\theta^2} = -\frac{5Wl\sin\theta}{4} - \frac{Wl\cos\theta}{4} \quad \text{at } \theta = 78^\circ 40'$$

10

$$\frac{d^2V}{d\theta^2} < 0 \text{ unstable}$$

at $\theta = 258^\circ 40'$ $\sin\theta < 0$, $\cos\theta < 0$ $\therefore \frac{d^2V}{d\theta^2} > 0$ stable



$$P y_P - \frac{1}{2} K(\Delta x)^2 = V$$

$$\Delta y = 2l - 2l\cos\theta$$

$$\Delta x = l\sin\theta$$

P

$$P[2l - 2l\cos\theta] - \tfrac{1}{2} K [l^2 \sin^2\theta] = V$$

$$\frac{dV}{d\theta} = 0 \; ; \; 2lP\sin\theta - Kl^2 \sin\theta\cos\theta = 0$$

$$\frac{Kl}{2} = P, \; P$$

$$\boxed{\max P = \frac{Kl}{2}}$$

but it is not stable for $P = \frac{Kl}{2}$

$\therefore$ for stable $P < \frac{Kl}{2}$

$$\frac{d^2V}{d\theta^2} = 2Pl\cos\theta - \tfrac{1}{2} K l^2 (2\cos 2\theta)$$

for $P = \frac{Kl\cos\theta}{2}$ $\quad \frac{d^2V}{d\theta^2} = 2l\left(\frac{Kl\cos\theta}{2}\right)\cos\theta - Kl^2[\cos^2\theta - \sin^2\theta]$

$$\frac{d^2V}{d\theta^2} = Kl^2\sin^2\theta \quad \text{for any } \theta \quad \frac{d^2V}{d\theta^2} > 0 \quad \text{always stable}$$

PROF. NARDO 1
12|9|69 AE100

$a = 9 - 3t^2 \qquad t=0 \quad v=0 \quad s=-3\,ft.$

$\frac{dv}{dt} = 9 - 3t^2 \qquad v = 9t - t^3 + v_0 \qquad at\ t=0 \quad v = v_0 = 0$

$V = 9t - t^3$

$\frac{ds}{dt} = 9t - t^3 \qquad s = \frac{9t^2}{2} - \frac{t^4}{4} + S_0 \qquad at\ t=0 \quad s = S_0 = -3$

$S = \frac{9t^2}{2} - \frac{t^4}{4} - 3$

(a)

$V = 9t - t^3 \qquad (3-t)(3+t)\,t = 0$

$0 = 9t - t^3 \qquad t = 3, -3, 0$

$0 = (9 - t^2)(t) \qquad$ (a) $t = 3\,sec$

(b) $s = \frac{9t^2}{2} - \frac{t^4}{4} - 3 \qquad s(4) = 9(8) - 64 - 3 = 5\,ft$ (b1)

$V = 9t - t^3 \qquad v(4) = 36 - 64 = -28\,ft/sec$ (b2)

(c) $S = \frac{9t^2}{2} - \frac{t^4}{4} - 3 \qquad s(4) - s(0) = [s(1) - s(0)] + [s(2) - s(1)] + [s(3) - s(2)] + [s(4) - s(3)]$

$= 4.25 + 9.75 + 6.25 + 12.25$

$= 32.5\,ft.$ (c)

$s(0) = -3\,ft.$

$s(1) = 1.25\,ft$

$s(2) = 11\,ft.$

$s(3) = 17.25\,ft$

$s(4) = 5$

10

10. $a = 25 - 3s^2 \qquad v_0 = 0 \quad t=0 \quad s=0$

$a = \frac{v\,dv}{ds} \qquad \frac{v^2}{2} = 25s - s^3 + v_0 \qquad$ or $\frac{v^2}{2} = 25s - s^3$

10

$v = \sqrt{50s - 2s^3} = \sqrt{84} = 9.17\,ft/sec$ (a)

$v = 0 = \sqrt{50s - 2s^3} = 50s - 2s^3 = s(50 - 2s^2) = 0 \qquad s = 0 \quad s = +5\,ft$ (b)

$\frac{dv}{ds} = 0 \quad$ but $a = v\frac{dv}{ds} = 0 = 25 - 3s^2 \qquad 25 = 3s^2 \quad 8.33 = s^2$

$s = 2.87\,ft.$ (c)

$a = -20v \qquad t = 0 \qquad v = 200$ ft/sec

$$a = \frac{dv}{dt} = -20v$$

$$\frac{dv}{v} = -20\,dt \qquad \frac{v}{v_0} = 1$$

$$\ln v/v_0 = -20t \qquad v = v_0 = 200$$

$$\ln\left(\frac{v}{200}\right) = -20t$$

$$\frac{v}{200} = e^{-20t} \qquad v = 200\,e^{-20t}$$

$$s - s_0 = \int 200\,e^{-20t}\,dt = -10\,e^{-20t}$$

$$v = 0 = \frac{200}{e^{+20t}} \quad \text{at rest} \quad t = \infty \quad (b)$$

$$\int ds = s = \int_0^\infty 200\,e^{-20t} = \frac{-10}{e^{-20t}}\Big]_0^\infty = 10\text{ ft} \quad (a)$$

10

$t = ?$ $\ni$ $v = 2$ ft/sec

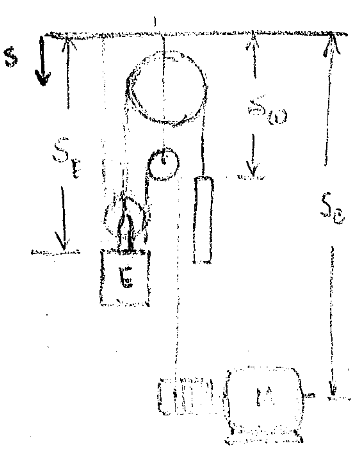
$$\ln\left(\frac{1}{100}\right) = -20t$$

$$\ln 1 - \ln 100 = -20t \qquad \ln 100 = \frac{\log 100}{\log e} = \frac{2}{.434} = 4.62$$

$$-4.62 = -20t$$

$$.231\text{ sec} = t \quad (c)$$

34



$S_E + S_W$ = constant

$V_E + V_W = 0$

$v_E = -12$ ft/sec $\therefore$ $V_W = 12$ ft/sec $\downarrow$ (b)

$V_{W/E} = V_W - V_E = 12 - (-12) = 24$ ft/sec $\downarrow$ (d)

$2S_E + S_C$ = constant

$2V_E + V_C = 0$

$2(-12) + V_C = 0$

$v_C = 24$ ft/sec $\downarrow$ (a)

10

$V_{C/E} = V_C - V_E = 24 - (-12) = 36$ ft/sec $\downarrow$ (c)

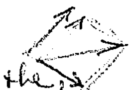
AE100 Prof. Nardo. Get Vector Mechanics for Engineers ... Rm. 309

Mechanics – try to predict motion of a body when it is under forces or stresses

Concepts

Space | Particle
Time | Rigid body – 0 relative displacement between particles in body.
Mass
Force

Principles

1. Newton's laws of Motion 1 Body in rectilinear motion will stay in motion unless acted upon
   2 $\vec{F} = m\vec{a}$ $\vec{a}$ in direct of force. (vector)
   3 Action–Reaction.
4. Assume Parallelogram law for addition of forces 
5. Transmissibility  you can move the force along the body with out changing its motion if force is kept in direction of motion

6. Newton's Gravitational Law.

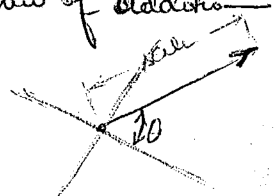
$$F = \frac{GMm}{r^2} \quad (\text{scalar})$$

M = earth
m = particle on surface
R = radius of Earth.

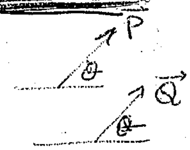
$$\frac{GM}{R^2} = g \qquad F = mg$$

Vectors

Mathematical Exp. has magnitude & direction. Satisfies parallelogram law of Addition



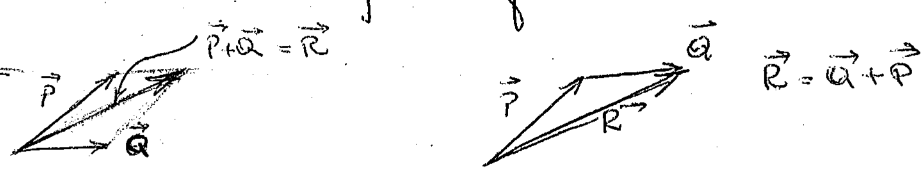
Equal vectors have equal magnitude & direction.



$\vec{P} = \vec{Q}$ if θ & scale are =

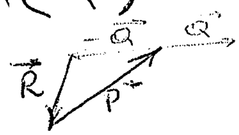
Free Vectors – can be moved without changing its magnitude & direction
Bound Vectors – cannot be changed
Sliding Vectors – can be moved along line of action

Addition of Vectors



Subtraction of 2 Vectors

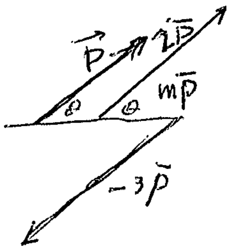
$$\vec{R} = \vec{P} + (-\vec{Q})$$



$x = 2\ln|1+z| + C$

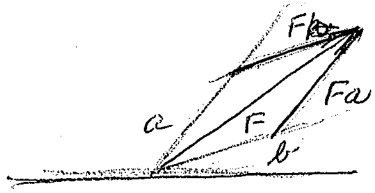
$m\vec{P}$ m is a scalar

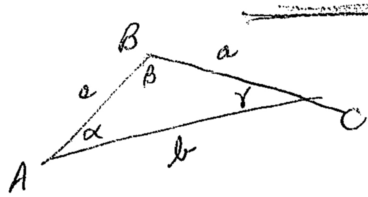
m affects magnitude only.



Force is a vector – obeys parallelogram law.

Resolution of Forces



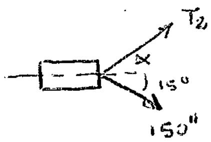


$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

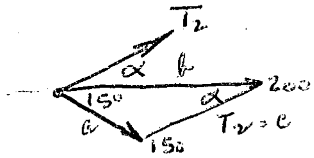
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

| HW. Chapt. 2.10, 2.22, 2.50, 2.64 |

2.8



find T_2 & α for $R = 200$#



$c^2 = a^2 + b^2 - 2ab\cos\gamma$

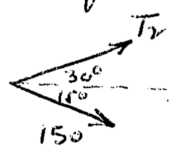
let $c^2 = (T_2)^2$

$T_2^2 = (150)^2 + (200)^2 - 2(150)(200)\cos 15°$

$T_2^2 = 4540$ $\quad T_2 = 67.4$# $\quad \sin 15° = .259$

$$\frac{\sin\alpha}{150°} = \frac{\sin 15°}{67.4} \qquad \alpha = 35.2°$$

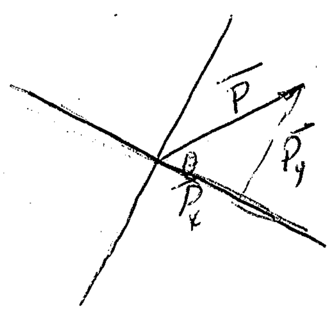
2.9 If $\alpha = 30°$ find T_2



$T_2 / \sin 15° = 150 / \sin 30°$

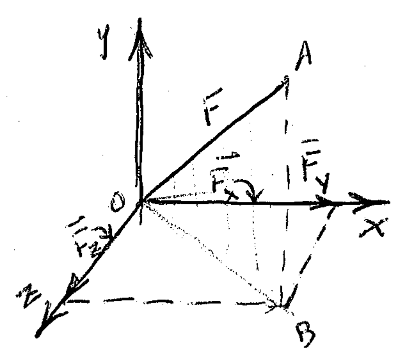
$T_2 = 77.7$#

Rectangular Components



$|\vec{P_x}| = |\vec{P}|\cos\theta$

$|\vec{P_y}| = |\vec{P}|\sin\theta$



$\vec{F_y}$ ∥ y axis

$\vec{F_x}$ ∥ x axis

$\vec{F_z}$ ∥ z axis

$F = \vec{F_x} + \vec{F_y} + \vec{F_z}$ ①

i, j, k unit vector

$$\vec{F} = |F_x|\vec{i} + |F_y|\vec{j} + |F_z|\vec{k} \quad ②$$

③

if $\cos\theta_x, \cos\theta_y, \cos\theta_z$ $\quad |F_x| = |F|\cos\theta_x \quad |F_y| = |F|\cos\theta_y \quad |F_z| = |F|\cos\theta_z$

$$\vec{F} = |\vec{F}|\underbrace{(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k})}_{\text{unit vector } \vec{\lambda}} \quad ①$$

directional cosines are from vector to <u>positive</u> x, y, z axis

e.g. $30°$ $120°$ cos here is $\cos 120°$

$$\bar{\lambda} = \frac{\bar{F}}{|\bar{F}|} \quad ⑤$$ $\bar{\lambda}$ is along line of action of F

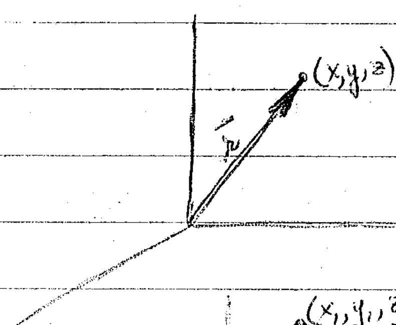
$$\vec{F} = \vec{F_x}\cdot\vec{i} + \vec{F_y}\cdot\vec{j} + \vec{F_z}\cdot\vec{k} \quad \text{where } \vec{F_x}\cdot\vec{i} = |F_x||i|\cos\theta_x$$

where $|i| = 1$

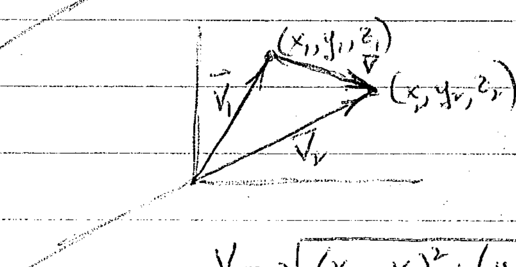
$$|F|^2 = |F|^2(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z)$$

$$(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z) = 1 \quad ⑥$$

radius vector



$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$



$$\bar{V} = V\bar{\lambda} \qquad V_1 = x_1 i + j y_1 + k z_1$$

$$\bar{V}_1 + \bar{V} = \bar{V}_2 \qquad V_2 = x_2 i + y_2 j + z_2 k$$

$$\bar{V} = \bar{V}_2 - \bar{V}_1$$

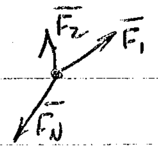
$$\bar{V} = (-x_1 + x_2)i + (-y_1 + y_2)j + (-z_1 + z_2)k$$

$$V = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\bar{\lambda} = \frac{\bar{V}}{|V|} = \frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Particle in Equilibrium in Space

Def: particle is in Equilibrium when resultant of all forces acting on such is 0.



$\sum_i^N \overline{F_i} = \overline{R} = 0$

$\overline{F_1} = |\overline{F_{1_x}}|\,\overline{i} + |\overline{F_{1_y}}|\,\overline{j} + |\overline{F_{1_z}}|\,\overline{k}$

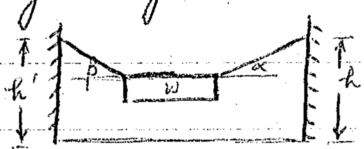
$\vdots$

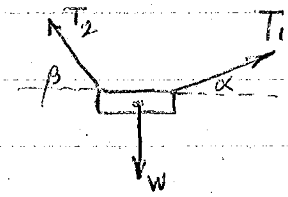
$\overline{F_N} = |\overline{F_{N_x}}|\,\overline{i} + |\overline{F_{N_y}}|\,j + |\overline{F_{N_z}}|\,k$

$\therefore \sum\left[\left(\sum_{i=1}^N |\overline{F_{i_x}}|\right)\overline{i} + \left(\sum_{i=1}^N |\overline{F_{i_y}}|\right)\overline{j} + \left(\sum_{i=1}^N |\overline{F_{i_z}}|\right)\overline{k}\right] = 0$

$\therefore \sum_i |\overline{F_{i_x}}| = 0 \quad \sum_i |\overline{F_{i_y}}| = 0 \quad \sum_i |\overline{F_{i_z}}| = 0$

Free Body Diagram





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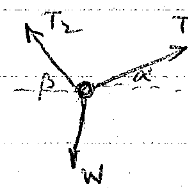
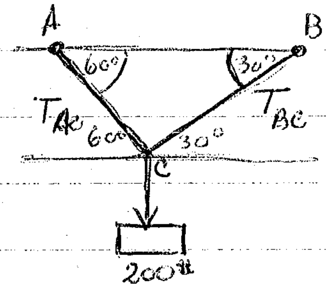
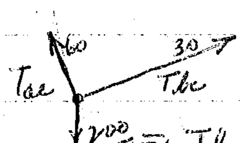




Find T_{ac} & T_{bc}



$\sum|F_x| = T_{bc}\cos 30° - T_{ac}\cos 60° = 0$

$\sum|F_y| = T_{ac}\sin 60° + T_{bc}\sin 30° - 200 = 0$

$.866\,T_{bc} = .5\,T_{ac}$

$.866\,T_{ac} + .5\,T_{bc} - 200 = 0$

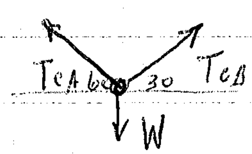
$\dfrac{T_{ac}}{T_{bc}} = 1.732$

$.866(1.732)T_{bc} + .5\,T_{bc} = 200$

$(.866 + .708 + .028 + .002)\,T_{bc} + .5\,T_{bc} = 200$

$(1.602 + .5 = 2.102)\,T_{bc} = 200 \qquad T_{bc} = \dfrac{200}{2.102} = 95.$

VECTORS



$\overrightarrow{T_{CA}} + \overrightarrow{T_{CB}} + \overrightarrow{W} = 0$

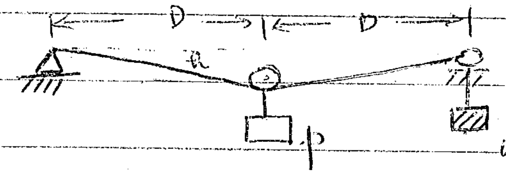
$|\overline{T_{CA}}|(\cos 120°\, i + \cos 30°\, j) +$

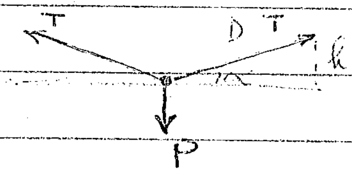
$|T_{CB}|(\cos 30°\, i + \cos 60°\, j) - 200\, j = 0$

$T_{CA}\cos 120 + T_{CB}\cos 30^\circ = 0$ $(-\cos 60^\circ)$

$T_{CA}\cos 30 + T_{CB}\cos 60^\circ - 200 = 0$ $(\sin 60^\circ)$ $(\sin 30^\circ)$

2.47





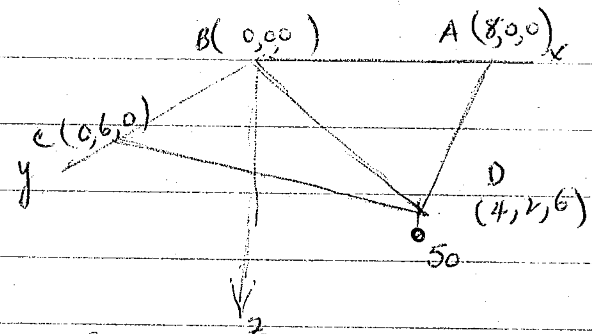
$\Sigma F_x = 0 = -T\cos\alpha + T\cos\alpha$

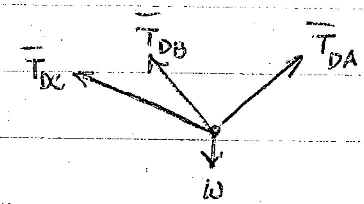
$\Sigma F_y = 0 = +2T\sin\alpha - P$

$2T\left(\dfrac{h}{\sqrt{h^2+d^2}}\right) = P$

$w = \dfrac{P}{2}\sqrt{1+(d/h)^2}$

2.70





R = 0

$\overline{T}_{DA} + \overline{T}_{DB} + \overline{T}_{DC} + \overline{w} = 0$

$|T_{DA}|\overline{\lambda}_{DA} + |T_{DB}|\overline{\lambda}_{DB} + |T_{DC}|\overline{\lambda}_{DC} + 50\overline{k} = 0$

A 8,0,0

B 0,0,0

C 0,6,0

D 4,2,6

$\overline{\lambda}_{DA} = \dfrac{(x_A - x_D)i + (y_A - y_D)j + (z_A - z_D)k}{\sqrt{(x_A-x_D)^2 + (y_A-y_D)^2 + (z_A-z_D)^2}}$

$\lambda_{DA}\ \dfrac{(8-4)i + (0-2)j + (0-6)k}{\sqrt{16+4+36}} = \dfrac{4i - 2j - 6k}{\sqrt{56}}$

$\lambda_{DB} = \dfrac{-4i - 2j - 6k}{\sqrt{56}}$ $\quad\lambda_{DC} = \dfrac{-4i + 4j - 6k}{\sqrt{68}}$

$|T_{DA}|\left(\dfrac{4i - 2j - 6k}{\sqrt{56}}\right) + T_{DB}\left(\dfrac{-4i - 2j - 6k}{\sqrt{56}}\right) + T_{DC}\left(\dfrac{-4i + 4j - 6k}{\sqrt{68}}\right) + 50k = 0$

if $\alpha = \frac{T_{DA}}{\sqrt{56}}$ $\beta = \frac{T_{DB}}{\sqrt{56}}$ $\gamma = \frac{T_{DC}}{\sqrt{68}}$

$4\alpha - 4\beta - 4\gamma = 0$ $\alpha = \beta + \gamma$ $\alpha + \beta = 2\gamma$ $\gamma + \beta + \gamma = \frac{\sqrt{0}}{6}$

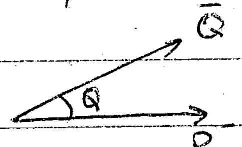
$-2\alpha - 2\beta + 4\gamma = 0$ $\alpha = \frac{2\sqrt{}}{6}$ $\beta = \frac{2\sqrt{}}{18}$ $\gamma = \frac{2\sqrt{}}{9}$

$-6\alpha - 6\beta - 6\gamma + \sqrt{0} = 0$

$T_{DA} = \frac{2\sqrt{}}{6}\sqrt{56}$ $T_{DB} = \frac{2\sqrt{}}{18}\sqrt{56}$ $T_{DC} = \frac{2\sqrt{}}{9}\sqrt{68}$

Vector Algebra

Dot Scalar Product

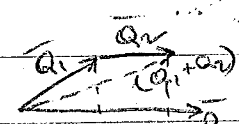


$\bar{P} \cdot \bar{Q} = |\bar{P}||\bar{Q}| \cos Q$

$\bar{P} \cdot \bar{Q} = \bar{Q} \cdot \bar{P}$

$\bar{P} \cdot \bar{P} = |P|^2$ $\bar{P} \cdot \bar{\lambda} = |P| \cos Q$

Distributive law $\bar{P} \cdot (\bar{Q}_1 + \bar{Q}_2) = \bar{P} \cdot \bar{Q}_1 + \bar{P} \cdot \bar{Q}_2$



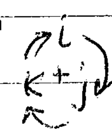
$\bar{P} \cdot \bar{Q}$ if $\bar{P} = P_x i + P_y j + P_z k$ $\bar{Q} = Q_x i + Q_y j + Q_z k$

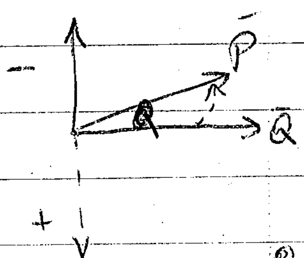
$\therefore \bar{P} \cdot \bar{Q} = P_x Q_x + P_y Q_y + P_z Q_z = PQ \cos Q$

$$\cos Q = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{|P||Q|}$$

Cross Product

$\bar{P} \times \bar{Q} = n|P||Q| \sin Q$

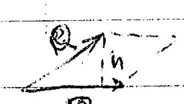




$\bar{P} \times \bar{Q} = -(\bar{Q} \times \bar{P})$

$i \times j = k$ $j \times k = i$ $i \times k = -j$

$i \times i = 0 = j \times j = k \times k = 0$

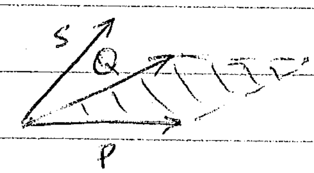


$h = Q \sin Q$ $\therefore Ph =$ Area of parallelogram

$(P\times Q)\times R \neq P\times(Q\times R)$

Mixed Scalar Triple Product

$S\cdot(Q\times P)$ a scalar Volume



$P\times Q$ = area of base

Volume $= S\cdot(P\times Q) = Q\cdot(S\times P)$

$P\cdot(Q\times S)$

$$S\cdot(P\times Q) = (P\times Q)\cdot S$$

$$S\cdot(P\times Q) = -(Q\times P)\cdot S$$

$$\bar P\times(\bar Q_1+\bar Q_2) \overset{?}{=} \bar P\times\bar Q_1 + \bar P\times\bar Q_2$$

let V → $\bar V = \bar P\times(\bar Q_1+\bar Q_2) - \bar P\times\bar Q_1 - \bar P\times\bar Q_2$

$$\bar V\cdot\bar V = (V\cdot P)\times(Q_1+Q_2) - V\cdot(P\times Q_1) - V\cdot(P\times Q_2)$$

$$\bar V^2 = (V\times P)\cdot(Q_1+Q_2) - (V\times P)\cdot Q_1 - (V\times P)\cdot Q_2$$

$\bar V^2 = (V\times P)\cdot[0] = 0$ ∴ distributive law holds for vectors.

H.W. Rd to 107
2.44, 2.68
3.4, 3.20, 3.34

$$(\bar P\times\bar Q) = (P_x\bar i + P_y\bar j + P_z\bar k)\times(Q_x\bar i + Q_y\bar j + Q_z\bar k)$$

$$(P_xQ_y\bar k - P_xQ_z\bar j) + (-P_yQ_x\bar k + P_yQ_z\bar i) + (P_zQ_x\bar j - P_zQ_y\bar i)$$

$$(P_xQ_y - P_yQ_x)\bar k + (P_zQ_x - P_xQ_z)\bar j + (P_yQ_z - P_zQ_y)\bar i$$

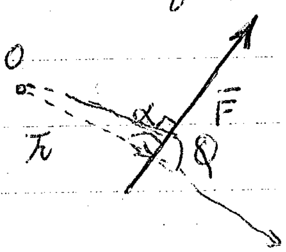
or

$$(\bar P\times\bar Q) = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

$$\bar S\cdot(\bar P\times\bar Q) = S_x(P_yQ_z - P_zQ_y) + S_y(P_zQ_x - P_xQ_z) + S_z(P_xQ_y - P_y\ldots$$

$$S\cdot(P\times Q) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Moment of a Force about a Point

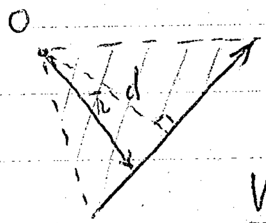


Def $\bar{M}_o$ (moment about o) $= \bar{r} \times \bar{F}$

r is the vector to the line of action of force

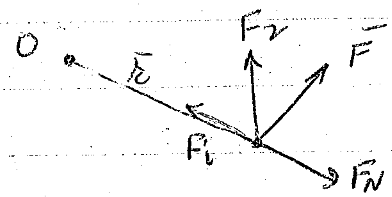
$|\bar{M}_o| = |\bar{r}||\bar{F}| \sin Q = \bar{F}r \sin \alpha$

if $d = r \sin Q$ & $d \perp F$ then $M = Fd$



$|\bar{M}_o| = 2$(area of triangle)

Varignon's Theorem - the moment about a given pt. of resultant of several concurrent forces is = to Σ of moments of various forces about that pt.



$\bar{r} \times R$ where $\bar{R} = \sum_{i=1}^{n} \bar{F}$

$\bar{r} \times \bar{R} = \bar{r} \times (\bar{F}_1 + \bar{F}_2 + \dots \bar{F}_n)$

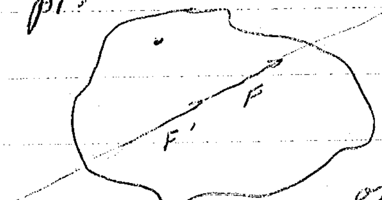
$= \bar{r} \times \bar{F}_1 + \bar{r} \times \bar{F}_2 + \dots + \bar{r} \times \bar{F}_n$

Equivalent Forces

2 Forces, F & F', are equivalent if they have the same effect on a rigid body.

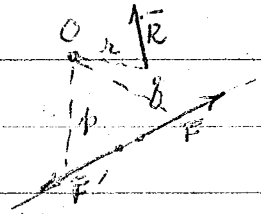
Theorem: 2 Forces, F & F', are equivalent IFF* ① F & F' are equal ② and if they give same moment about a pt 'O'.

F and F' have to be colinear & would give same moment about any pt.

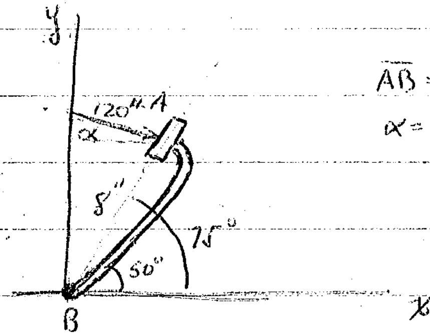


Principle of transmissibility

Hold for a force along 1 line of action

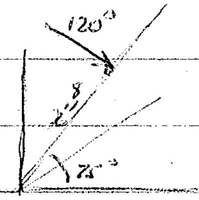


~~&~~ we can add forces in equilibrium to the body without changing its moment. ie if $\bar{F}$ and $\bar{F}'$ are = and opposite and $\bar{R}$ the moment about O is just the distance of r times $\bar{R}$ $(\bar{p}\times\bar{F}' + q\times F + r\times\bar{R} = r\times R$



$\overline{AB} = 8''$ $\quad \bar{M}_B = ?$

$\alpha = 30^\circ$



$\bar{M} \times \bar{F}$

$\bar{r}_x = 8\cos 75^\circ i \qquad r_y = 8\sin 75^\circ j$

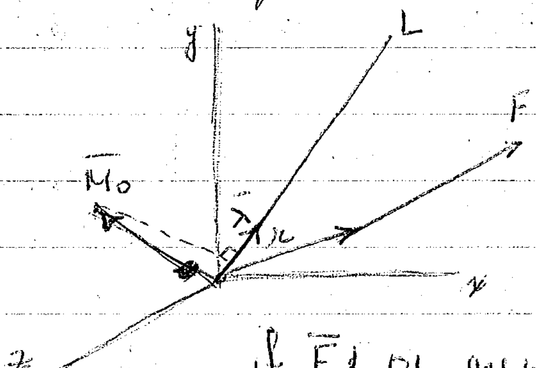
$\bar{F}_x = 120\cos 30^\circ i \qquad F_y = -120\sin 30 j$

$$\bar{M}_B = \bar{r}\times\bar{F} = (8\cos 75^\circ i + 8\sin 75^\circ j)\times(120\cos 30^\circ i - 120\sin 30 j)$$

$$= 960\,(-\cos 75^\circ \sin 30^\circ k - \cancel{8}\sin 75^\circ \cancel{j} \cos 30^\circ k)$$

$$-960\,(\sin 105^\circ)\,\bar{k}$$

Moment of a Force about a Given Axis



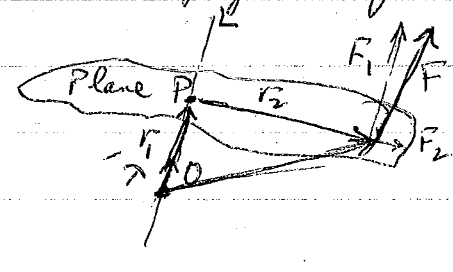
$$\bar{M}_{O_L} = \bar{\lambda}\cdot\bar{M}_o \quad \text{scalar}$$

$$= \bar{\lambda}\cdot(\bar{r}\times\bar{F})$$

$$= \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

if $\bar{F}$ & OL are in plane of black board, then M_o is out of black board. but M_o on OL is $\bar{M}_o\cdot\bar{\lambda} = |M_o||\lambda|\cos 90^\circ = 0$ since $\bar{M}_o \perp OL$.

For more physical feel of M_{OL}



$OL \perp \boxed{P}$

$\bar{F}_1 + \bar{F}_2 = F \qquad F_1 \perp \boxed{P} \quad F_2 \parallel \boxed{P}$

$$M_{OL} = \bar{\lambda}\cdot(\bar{r}\times\bar{F})$$

$$\bar{\lambda}\cdot((\bar{r}_1 + \bar{r}_2)\times(\bar{F}_1 + \bar{F}_2))$$

① coplanar $=0$ ② ③ coplanar $=0$

$$\bar{\lambda}\cdot\left(\bar{r}_1\times\bar{F}_1+\bar{r}_1\times\bar{F}_2+\bar{r}_2\times\bar{F}_2+\bar{r}_2 F_1\right)$$

$\lambda\cdot r_1\times F_2=\lambda\times r_1\cdot F_2$ since $\sin\theta=0$ $O = 0$

$r_1\times F_2$ is in plane P

$\perp$ to OL

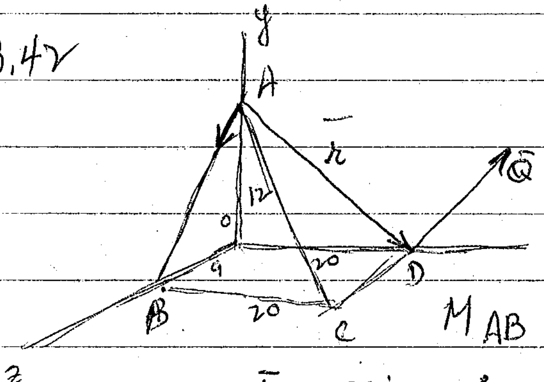
$\therefore\ \lambda\cdot r_1\times F_2$

$\lambda\cdot M=|\lambda||M|\cos 90^\circ$

$=0$

$$M_{OL}=\lambda\cdot(\bar{r}_2\times\bar{F}_2)$$

3.42



Find M about AB for a force $Q=50i+25j+50k$

$$M_{AB}=\bar{\lambda}\cdot(\bar{r}\times\bar{Q})$$

$\bar{r}=20i-12j$ $\qquad \lambda=\frac{-12j+9k}{15}=-.8j+.6k$

$M_{AB}=\lambda\cdot(\bar{r}\times\bar{Q})$ $\qquad \bar{r}\times\bar{Q}=\begin{vmatrix} i & j & k\\ 20 & -12 & 0\\ 50 & 25 & 50\end{vmatrix}$

~~$i|-180|$ $j|-180|$ ...~~

~~$r\times Q=-180i+180j$~~

$i\,|-600|-j\,|1000|+k\,|500+600|$

$-600i-1000j+1100k$ $\qquad r\times Q=-6i-10j+11k$

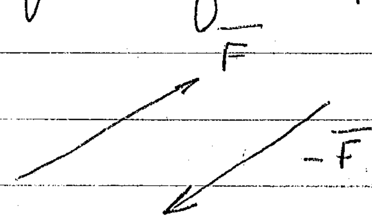
$i\cdot i$
$j\cdot j$
$k\cdot k$

$M_{AB}=$ ~~...~~ $-.8(-1000)+1100(.6)$

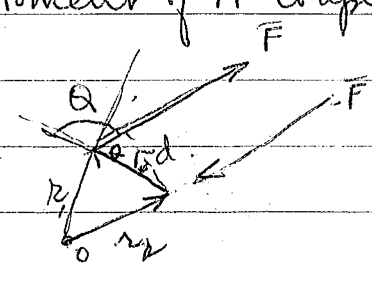
$800+660=1460$ lb-in

Definition of a Couple & a moment of a couple

2 forces $=$, same line of action, opposite sense



Moment of A Couple



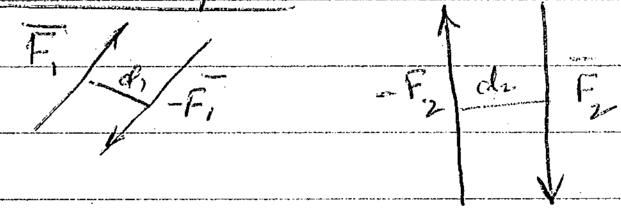
$\bar{r}_2 + \bar{r}_s = r_2$ $\qquad \overline{M_o} = r_1 \times \bar{F} + r_2 \times (-\bar{F})$

$$= (r_1 - r_2) \times F \qquad \text{independent of } O$$

$\bar{M}_o = r \times F \qquad |\bar{M}_o| = F \cdot d$ when $d = r \sin\theta$

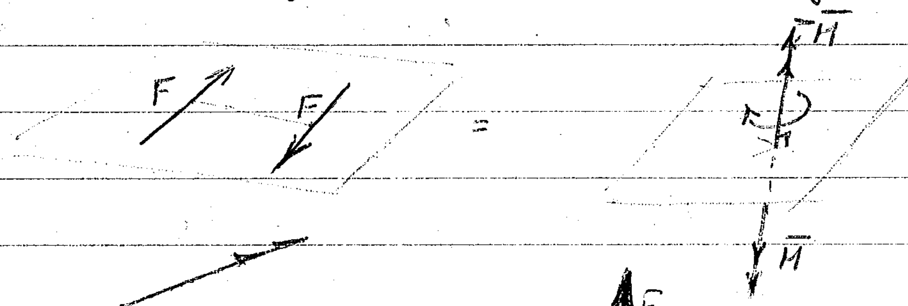
Equivalent Couples

Eg



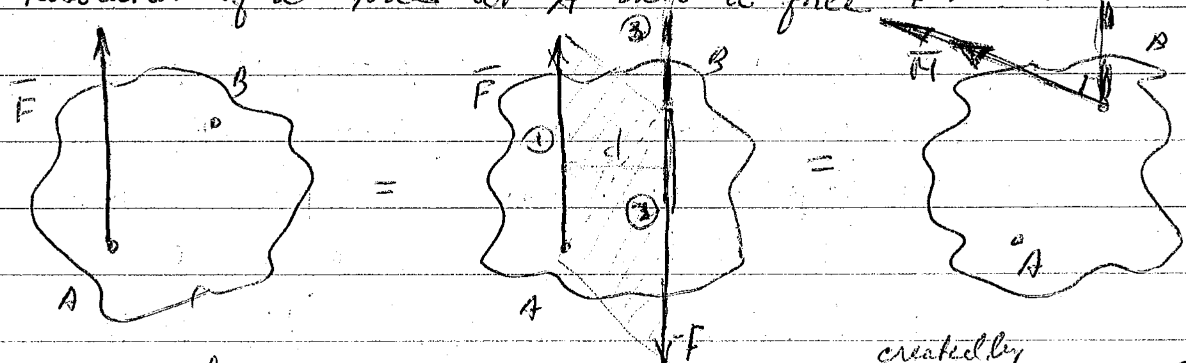
if $F_1 d_1 = F_2 d_2$

Representation of moment of a couple by a vector.



$|\bar{M}| = Fd$

Resolution of a Force at "A" into a force & moment at "B"



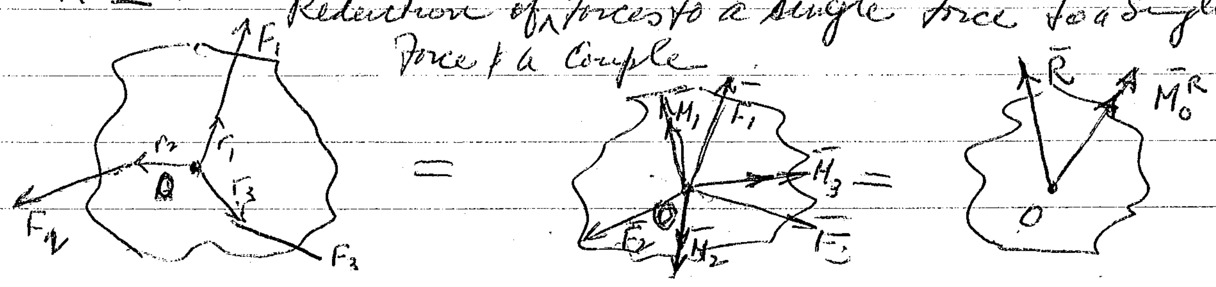
① & ② form a couple

$M \xrightarrow{\text{created by}}$ couple ① ②

$\bar{M} \perp$ to plane of Couple F & $-F$

$\bar{M} \perp \bar{F}$

Reduction of System of Forces to a Single Force to a Single Force & a Couple

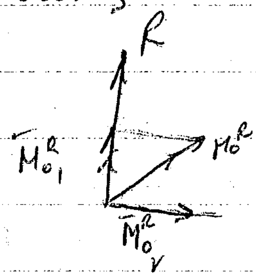


$\overline{M}_o^R = \sum_i \overline{M}_i \qquad \overline{R} = \sum_i \overline{F}_i$

$= \sum_i r_i \times F_i \qquad R \cancel{\perp} \overline{M}_o \qquad \overline{R}$ is not perpend to $\overline{M}_o$

If M_o^R is $\perp$ to R then we can find an O' $\ni$ we can represent the whole system by one single force

When $\overline{M_o}^R$ is not $\perp$ to R then take $\overline{M}_o^R$ & break it up into 2 vectors



$M_{o_1}^R \parallel$ to R $\qquad M_{o_2}^R \cancel{\parallel}$ to R

__Wrench__ - take $M_{o_2}^R$ & form a couple

$\ni$ R & $\overline{M}_{o_1}^R$ will lie on 1 line of action

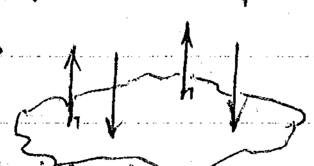
$\overline{R} = M\overline{a} \qquad |\overline{M}_1| = |\overline{T}| = I\alpha$

Cases in which resultant is a single force

(1) Concurrent forces - Forces passing through 1 point then only a resultant is formed and there is no moment

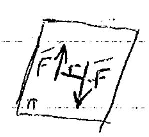
(2) Coplanar forces - all in one plane R is in plane
$\overline{M}_o^R$ is $\perp$ to plane component on R = 0

(3) Parallel forces



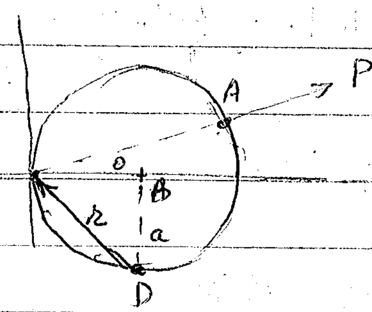
F is $\perp$ to plane M_o is $\parallel$ to plane and M_o component on F = 0

(2) (3) it is possible there can be a couple



$\sum F = 0 \qquad M_o^R = F \times r$

3.87



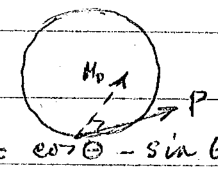
$\overline{M}_D = \overline{r} \times \overline{P}$

$\overline{r} = -ai + aj$

$\overline{P} = P\cos\theta\, i + P\sin\theta\, j$

$\begin{vmatrix} i & j & k \\ -a & a & 0 \\ P\cos\theta & P\sin\theta & 0 \end{vmatrix} = -Pa\sin\theta\, \overline{k} - Pa\cos\theta\, \overline{k}$

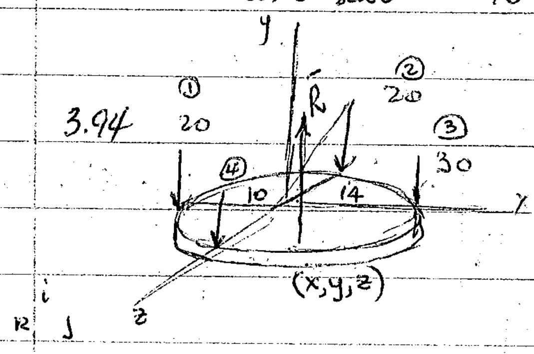
$\overline{M}_D = -Pa(\sin\theta + \cos\theta)\overline{k}$



Let $y = \sin\theta + \cos\theta$ $\qquad y' = \cos\theta - \sin\theta = 0$

$\cos\theta = \sin\theta \qquad 45° = \theta$

3.94



$\overline{R} = -20\overline{j} - 20\overline{j} - 30\overline{j} - 10\overline{j}$

$R = -80j$

Calculate $\overline{M}_o^R = r_1 \times (-20j) + r_2 \times (-20j) + r_3 \times (-30j) + r_4(-10j)$

$r_1 = -14i \qquad r_2 = -14k \qquad r_3 = +14i \qquad r_4 = 14k$

$280k - 280i - 420k + 140i \qquad R = -140k - 140i$

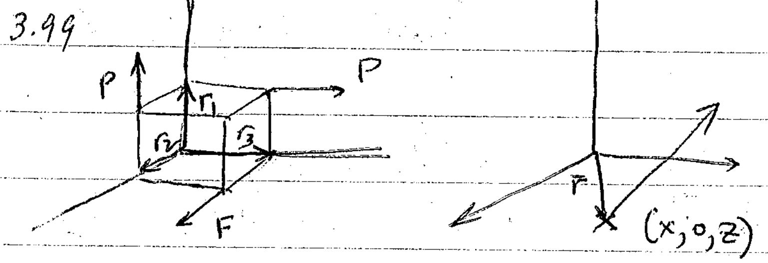
$(\overline{M}_o^R) = (xi + zk) \times (-80j) =$

$-80xk + 80zi =$ $\qquad -80x = -140 \qquad x = \frac{+7}{4}$

$80z = -140 \qquad z = -7/4$

3 (74, 66, 40, 52, 82, 102a)

3.99



$\overline{R} = Pi + Pj + Fk$

$\overline{M}_o^R = \overline{r}_1 \times P\overline{i} + \overline{r}_2 \times P\overline{j} + \overline{r}_3 \times F\overline{k}$

$= aj \times Pi$

$+ ak \times Pj$

$+ ai \times Fk$

$= -aPk - aPi - aFj$

Choose $F_3 \ni \overline{M}_o^R \cdot R = 0$

$(Pai + Faj + Pak) \cdot (Pi + Pj + Fk) = P^2a + FPa + PFa = 0$

$P^2a + 2PFa = 0 \qquad P + 2F = 0 \qquad F = -P/2$

$\therefore\ R = Pi + Pj - \frac{P}{2}k \qquad |R| = \sqrt{\frac{8P^2}{4} + \frac{P^2}{4}} = \frac{P}{2}\sqrt{9} = P(1.5)$

$\cos\theta_x = \frac{R_x}{|R|} = \frac{P}{1.5P} = \frac{1}{1.5};\ \theta_x = 48.2^\circ$

$\cos\theta_y = \frac{R_y}{|R|} = \frac{P}{1.5P} = \frac{1}{1.5};\ \theta_y = 48.2^\circ$

$\cos\theta_z = \frac{R_z}{|R|} = \frac{-P}{3P} = \frac{-1}{3};\ \theta_z = 109.5^\circ$

$\therefore\ \bar{r} \times R = \bar{M}_o^R = -Pai + \frac{Pa}{2}j - Pa\,k$

$(x\bar{i} + zk) \times (P\bar{i} + Pj - P/2\,k) = -(Pa\bar{i} - P/2\,aj + Pak)$

$\cancel{Pxk + \frac{Px}{2}j}$

$$\begin{vmatrix} i & j & k \\ x & 0 & z \\ P & P & -P/2 \end{vmatrix} = i\,|-Pz| - j\left|-\frac{xP}{2} - Pz\right| + k\,|xP|$$

$Pxk - iPz + Pzj + \frac{xP}{2}j$

$x = a \quad z = a$

Read chapter 4 First quiz the 21/10/69

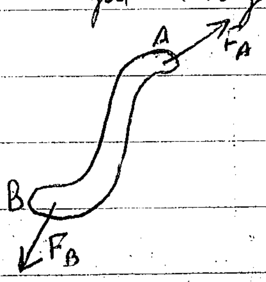
Equil. of Rigid Body

Rigid body is said to be in equil when external forces acting on it form a system of forces equal to 0

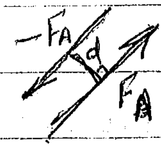
if $\bar{R} = \bar{M}_o^R = 0 \qquad \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$

Rigid Body subject to 2 forces only



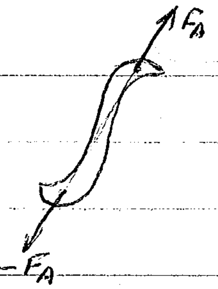
$\bar{R} = 0 \qquad F_A + B_B = 0 \quad F_A = -F_B$



$\bar{M}_o^R = 0$

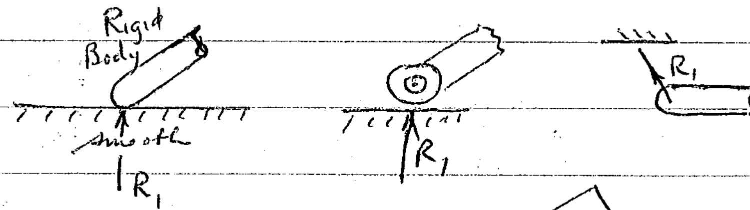
$\therefore\ d = 0$

and F_A & F_B are colinear

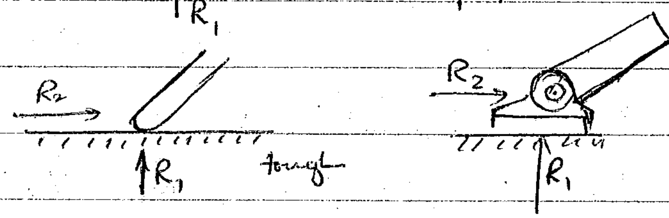


Two Dimensional Problem.

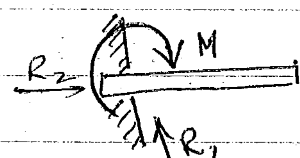
1 unknown



2 unknown



3 unknowns



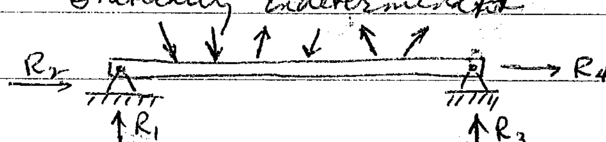
P. 111

$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_o = 0$ **OR** $\sum F_x = 0$

A & B must not be colinear || to any of the axis $\sum M_A = 0$

OR $\sum M_A = 0, \sum M_B = 0, \sum M_C = 0$ (A, B, C $\neq$ collinear) $\sum M_B = 0$

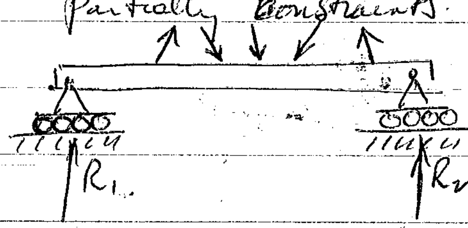
Statically indeterminate



more unknowns than equations

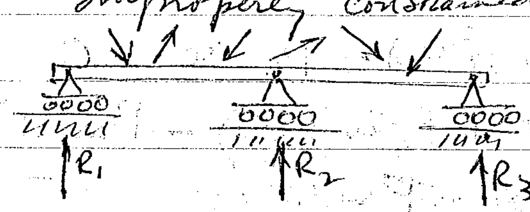
3 eq., 4 unknowns.

Partially constraints



3 eq., 2 unknowns

Improperly constrained

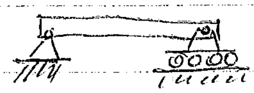


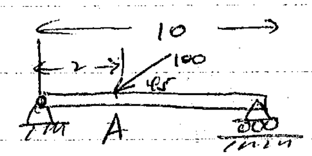
3 unknowns

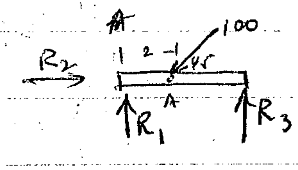
3 eq. but effectively

2 eq.

① watch out for equations where reactions a parallel
② concurrent.







$\Sigma F_x = = \Sigma F_y = 0 \qquad \Sigma M_A = 0$

$\Sigma F_y \quad R_1 + R_3 - 100 \sin 45 = 0$

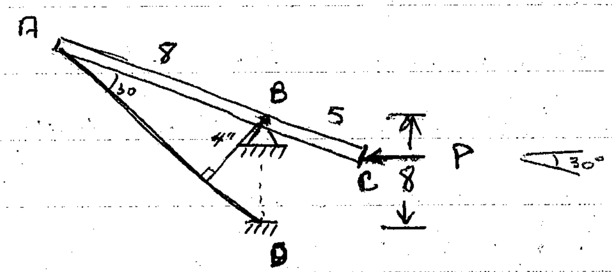
$\Sigma F_x = R_2 - 100 \cos 45^\circ = 0 \qquad R_2 = 70.7$

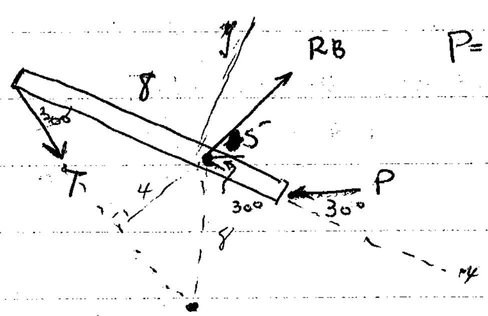
$\Sigma M_A = R_1(2) - R_3(8) = 0 \qquad R_1 = 4R_3$

$5R_3 = 70.7 \qquad R_3 = 14.14 = 14.1\#$

$R_1 = 56.56\# = 56.6\#$

4-2





$P = 400\#$

$\Sigma F_x = 0 \quad T \cos 30^\circ + R_B \cos(\theta + 30) - P \cos 30^\circ = 0$

$\Sigma F_y = 0 \quad -T \sin 30^\circ + R_B \sin(\theta + 30) - P \sin 30^\circ = 0$

$\Sigma M_B = T \sin 30 (8) - 5P \sin 30^\circ = 0 \qquad 4T - 1000 = 0 \qquad T = 250\#$

$\sum F_y = 0; \quad -125 + R_B \sin(\theta+30) - 200 = 0 \qquad R_B \sin(\theta+30) = 32$

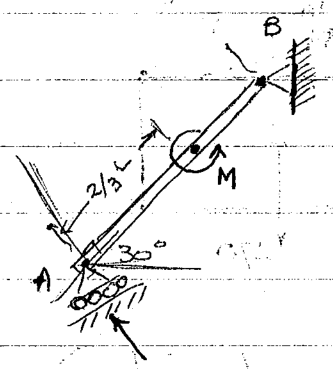
$\sum F_x = 0; \quad +216.5 + R_B \sin(\theta+30) - 346.5 = 0 \qquad R_B \cos(\theta+30) = 130$

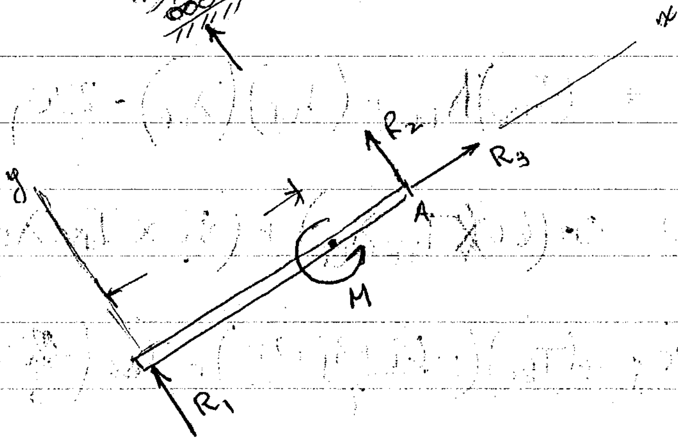
$R_B^2 = (R_B \sin(\theta+30))^2 + (R_B \cos(\theta+30))^2 = \overline{325}^2 + \overline{130}^2$

$R_B =$

$R_B \tan(\theta+30) = 2.5 \qquad \theta = 38.2°$

4-18 (d)

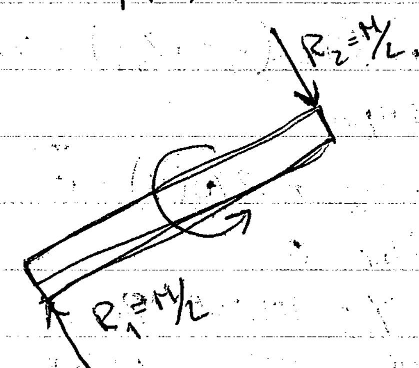




$\sum F_x = 0 \qquad R_3 = 0$

$\sum F_y = 0 \qquad R_1 + R_2 = 0 \qquad R_1 = -R_2$

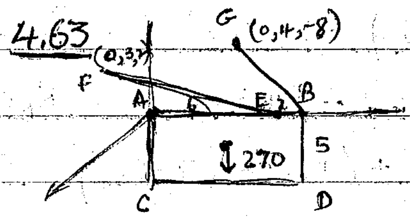
$\sum M_B = 0 \qquad R_1(L) + M = \quad \boxed{R_1 = \frac{M}{L} \quad R_2 = -\frac{M}{L} \quad R_3 = 0}$

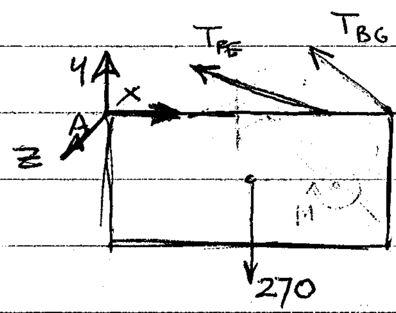


[redacted]

4.6, 10, 24, 64

4.63





$\lambda_{BG} = \frac{\overline{BG}}{|BG|} = \frac{-8i+4j-8k}{\sqrt{144}}$

$= -\frac{2}{3}i + \frac{1}{3}j - \frac{2}{3}k$

$\lambda_{EF} = \frac{-6i+3j+2k}{7}$

$= -\frac{6}{7}i + \frac{3}{7}j + \frac{2}{7}k$

$R = 0 \Rightarrow (T_{BG})\lambda_{BG} + (T_{EF})(\lambda_{EF}) - 270j + Xi + Yj + Zk = 0$

$\bar{M}_0^R = 0 \quad 0 = (6i \times T_{EF}\lambda_{EF}) + (8i \times T_{BG}\lambda_{BG}) + ((4i - \frac{5}{2}j) \times (-270j))$

$0 = -270j + (T_{EF})\left(\frac{-6i+3j+2k}{7}\right) + T_{BG}\left(-\frac{2i}{3} + \frac{j}{3} - \frac{2}{3}k\right) + Xi + Yj + Zk$

$\Sigma i = 0 \quad -\frac{6}{7}T_{EF} - \frac{2}{3}T_{BG} + X = 0$

$\Sigma j = \quad -270j + \frac{3}{7}T_{EF} + \frac{1}{3}T_{BG} + Y = 0$

$\Sigma k = \quad \frac{2}{7}T_{EF} - \frac{2}{3}T_{BG} + Z = 0$

$\boxed{\begin{array}{l} \Sigma M_0^R = 6T_{EF}\left(+\frac{3}{7}k - \frac{2}{7}j\right) + 8T_{BG}\left(\frac{1}{3}k + \frac{2}{3}j\right) - 270(4)k = 0 \\ \Sigma M_i = 0 \quad \Sigma M_j = 0 \quad \Sigma M_k = 0 \end{array}}$

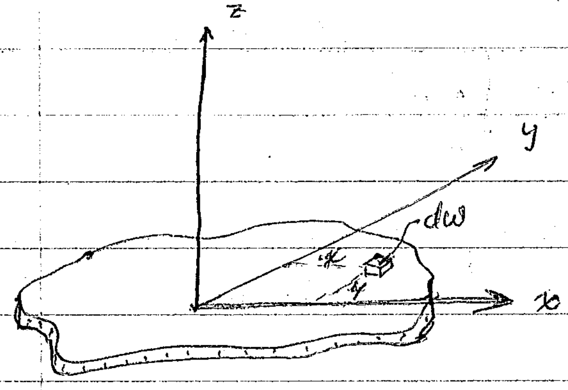
$M_k = \frac{3}{7} \cdot 6T_{EF} + 8T_{BG} \cdot \frac{1}{3} - 270(4) = 0$

$M_j = -\frac{2}{7}6T_{EF} + \frac{2}{3}8T_{BG} = 0$

$X = 338^{\#} \quad Y = 101.2^{\#} \quad Z = -22.5^{\#}$

$T_{EF} = 315^{\#} \quad T_{BG} = 101.3^{\#}$

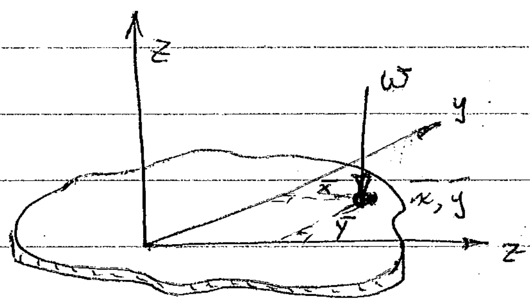
Chap. 5 Distributed Forces; Centroids and Center of Gravity



Center of Gravity

Flat horizontal plate

=



$$W = \int dW \quad (1)$$

$$M_x = \int y\, dW = W\bar{y}$$

$$\boxed{\bar{y} = \frac{\int y\, dW}{W}} \quad (2)$$

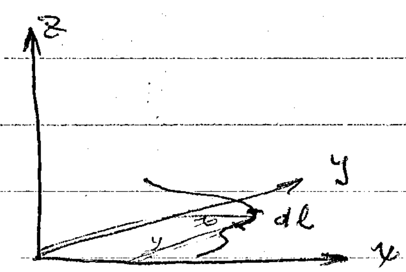
$$M_y = \int x\, dW = W\bar{x} \qquad \boxed{\bar{x} = \frac{\int x\, dW}{W}} \quad (3)$$

If t is uniform and it is a homogeneous plate, then $\rho = \text{constant}$ $\quad dW = \rho\, dV = \rho\, t\, dA$ and t is uniform $\therefore$ $\rho\, dA = dW \implies \rho A = W \qquad M_x = \int_A y\, \rho\, dA$

and $\rho A \bar{y} = M_x$ and $\boxed{\bar{y} = \frac{\rho \int y\, dA}{\rho \int dA}}$

$$\boxed{\bar{x} = \frac{\int_A x\, dA}{A}}$$

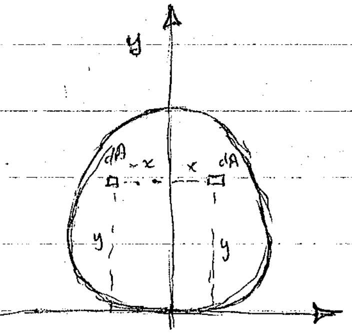
$\bar{x}, \bar{y}$ describe centroid of the area.



uniform cross-section homogeneous

$$\bar{x} = \frac{\int x\, dL}{L}\ ; \quad \bar{y} = \frac{\int y\, dL}{L}$$

Planes area one axis of symmetry

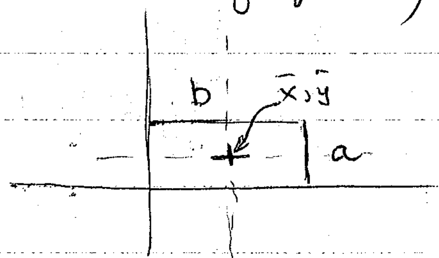


First Moment

$$\bar{x} = \frac{\int x\,dA}{A}$$

since for every $x\,dA$ there's a corresponding $-x\,dA$ $\ni$ $\int x\,dA = 0$ and $\bar{x} = 0$

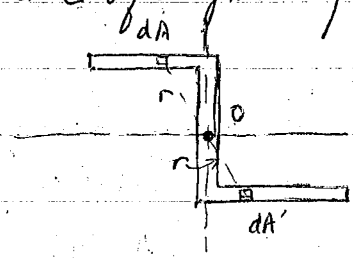
2 axes of symmetry



$\bar{x} = b/2$

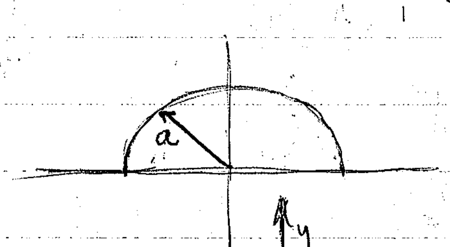
$\bar{y} = \cancel{b}a/2$

Center of Symmetry

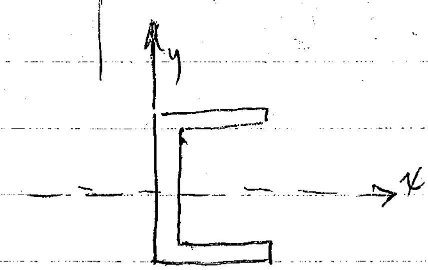


If for every dA at a distance r from O. There must be a dA' and a distance r from O.

$\ni r\,dA = r\,dA'$



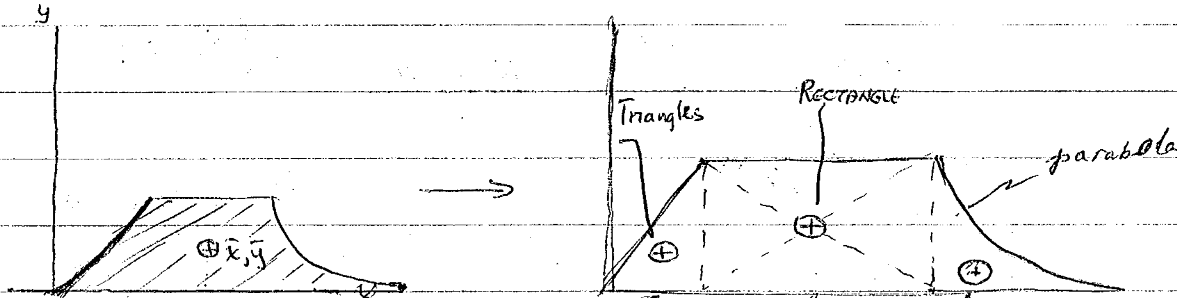
$\bar{x} = 0$ $\quad \bar{y} = \frac{2a}{\pi}$



$\bar{x} = ?$

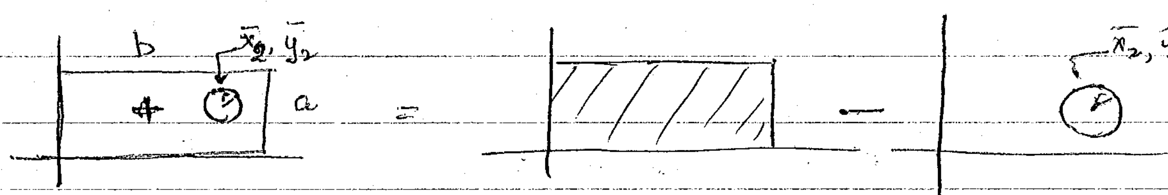
$\bar{y} = 0$

Composite Figures



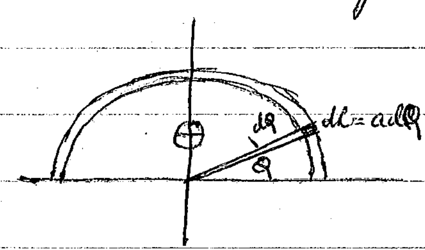
$$M_x = A_{TOT}\,\bar{y} = A_{TRI}\,\bar{y}_{TRI} + A_{RECT}\,\bar{y}_{RECT} + A_{PARABOLIC}\,\bar{y}_{PARABOLA}$$

$$\bar{y} = \frac{\sum_{i=TRIA}^{PARAB} A_i\,\bar{y}_i}{\sum_{i=TRI}^{PAR} A_i} \quad ; \quad \bar{x} = \frac{\sum_{i=TRI}^{RECT} A_i\,\bar{x}_i}{\sum_{i=TRIA}^{PAR} A_i}$$

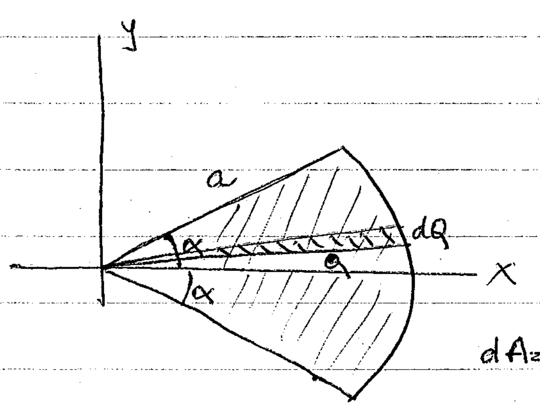


$$\bar{x} = \frac{\sum A_i\,x_i}{\sum A_i} = \frac{A_{REC}\,\bar{X}_{REC} + (-A_{CIRC})\bar{X}_2}{A_{REC} + (-A_{CIRC})}$$

Centroids by integration



$\bar{x} = 0$ $$\bar{y} = \frac{\int y\,dL}{L} = \frac{\int y\,dl}{\int dl} = \frac{a^2\sin\theta\,d\theta}{\pi a} = \frac{-a^2\cos\theta\Big]_0^{\pi}}{\pi a} = \frac{2a}{\pi}$$



$\bar{y} = 0$

$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

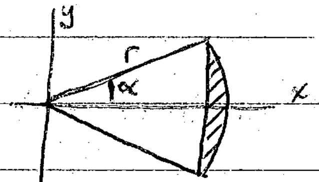
$dA = \frac{1}{2}(a\,d\theta)a$

$x = \frac{2a}{3}\cos\theta \qquad \bar{x} = \dfrac{2\int_0^\alpha \frac{a^3}{3}\cos\theta\, d\theta}{2\int_0^\alpha \frac{a^2}{2}\, d\theta}$

$$\bar{x} = \frac{\frac{2a^3}{3}\sin\theta\Big]_0^\alpha}{2\frac{a^2}{2}\alpha} = \frac{2a\sin\alpha}{3\alpha}$$

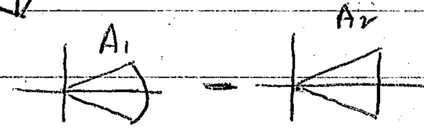
$\therefore\ \bar{x} = \frac{2a}{3\alpha}\sin\alpha\,;\ \bar{y} = 0$

5.14



$\bar{y} = 0 \qquad \bar{x} = ?$

Consider



$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} = \frac{A_1(\bar{x}_1) + A_2(\bar{x}_2)}{A_1 + A_2}$$

$A_1 = \frac{\pi r^2 2\alpha}{2\pi} = r^2\alpha \qquad A_2 = 2\left[\frac{1}{2}r^2\sin\alpha\cos\alpha\right]$

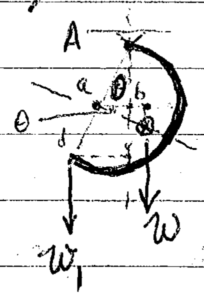
$A_1 + A_2 = r^2\alpha - r^2\sin\alpha\cos\alpha$

$\bar{x}_1 = \frac{2r}{3\alpha}\sin\alpha \qquad \bar{x}_2 = \frac{2}{3}r\cos\alpha$

$$\bar{x} = \frac{\left[\frac{2r\sin\alpha}{3\alpha}\cdot r^2\alpha - r^2\sin\alpha\cos\alpha\cdot\frac{2}{3}r\cos\alpha\right]}{r^2\alpha - r^2\sin\alpha\cos\alpha}$$

$$= \frac{\sin\alpha\left[\frac{2r}{3} - \frac{2}{3}r\cos^2\alpha\right]}{\alpha - \sin\alpha\cos\alpha} = \frac{4r\sin^3\alpha}{3(2\alpha - \sin 2\alpha)}$$

5.24 Wire or rod bent in semicircle



$\sum M_A = 0 \qquad W(\overline{bc}) - W_1(\overline{de}) = 0$

$\overline{bc} = \overline{ac} - \overline{ab} \qquad W = [\overline{ac} - \overline{ab}]$

$W_1\overline{de} = (2r\sin\theta)W_1$

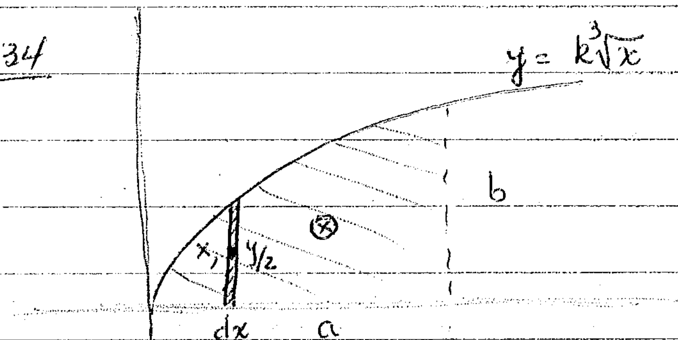
$\cos\theta$

$\overline{ac}$ is the $\bar{y}$ comp of center of moment or $\overline{ac} = \frac{2r}{\pi}\cos\theta$

$\overline{ab} = r\sin\theta \quad \therefore \quad \omega \cdot \left[\frac{2r}{\pi}\cos\theta - r\sin\theta\right] = \omega_1\, 2r\sin\theta$

$\therefore \quad \tan\theta = \dfrac{(2\omega/\pi)}{(2\omega_1 + \omega)}$

5.34



$y = k\sqrt[3]{x}$ at $x = a$ $y = b$

$b = k a^{1/3}$

$\dfrac{b}{a^{1/3}} = k$

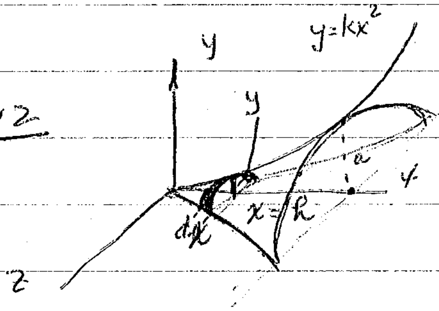
$x, y/2$ centroid of rectangle

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int_0^a (y/2)\,y\,dx}{\int_0^a y\,dx} = \frac{\frac{1}{2}\int_0^a k^2 x^{2/3}\,dx}{\int_0^a k x^{1/3}\,dx}$$

$$\bar{y} = \frac{\frac{k}{2}\; x^{5/3}/5/3\Big]_0^a}{x^{4/3}/4/3\Big]_0^a} \Longrightarrow 2/5\, k a^{1/3} = 2/5\, b$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^a x y\,dx}{\int_0^a y\,dx} = \frac{\int_0^a k x^{4/3}\,dx}{\int_0^k k x^{1/3}\,dx} = \frac{x^{7/3}/7/3\Big]_0^a}{x^{4/3}/4/3\Big]_0^a} = \frac{4}{7}a$$

5.112



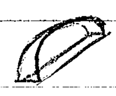
$$\bar{x} = \frac{\int x\,dV}{\int dV} \qquad \bar{y} = \frac{\int y\,dV}{\int dV}$$

$\bar{z} = 0$

$dV = \dfrac{\pi y^2\,dx}{2}$

x coordinate of centroid =

y coordinate of centroid = 4/3



$$\bar{x} = \frac{\int_0^h x\,\frac{\pi}{2}y^2\,dx}{\int_0^h \frac{\pi}{2}y^2\,dx} \quad \frac{\frac{\pi k^2}{2}\int_0^h x^5\,dx}{\frac{\pi k^2}{2}\int_0^h x^4\,dx} = \frac{\frac{\pi k^2}{2}\,\frac{x^6}{6}\Big]_0^h}{\frac{\pi k^2}{2}\,\frac{x^5}{5}\Big]_0^h}$$

$$\frac{\pi k^2 h^6}{12} \Big/ \frac{\pi k^2 h}{10} = \frac{5h}{6} = \overline{x}$$

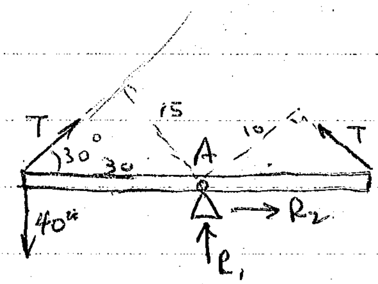
$$\overline{y} = \frac{\int y\,dV}{\int dV} = \frac{\int_0^h \frac{4y}{3\pi}\left(\frac{\pi}{2}y^2\right)dx}{\frac{\pi k^2 h^5}{10}} = \frac{20\,a}{21\pi}$$

$a = kh^2$

4 questions - definitions, look over derivations

Equi of particle, rigid body in 2 dim, rid body in 3 dim

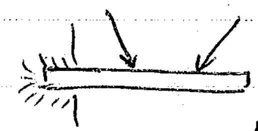
Equiv force system, Moment about line

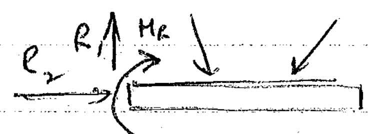


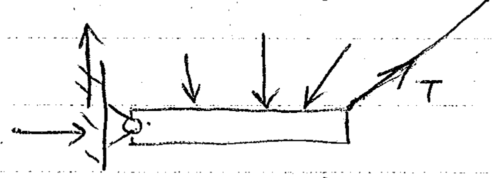
$\sum M_A = 0$

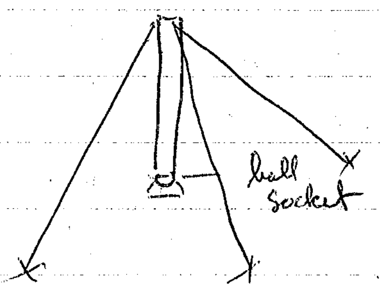
$40(30) - T(15) + T(10) = 0$

$\sum F_y = 0 \quad \sum F_x = 0$



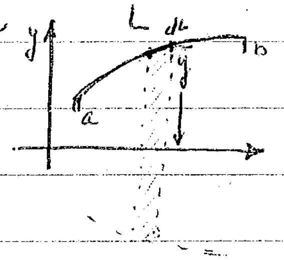






Theorem's Pappus - Guldinus

(1) Surface of Revolution - the area of a surface of revolution about a non-intersecting axis is equal to the length of the generating curve multiplied by the distance traveled by the centroid of the generating curve

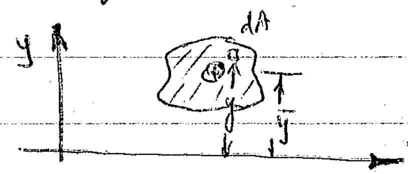


$$\int_A^B dS = \int_A^B 2\pi y \, dL \qquad \bar{y} = \frac{\int y \, dL}{L}$$

$$S = 2\pi \int_A^B y \, dL$$

$$\therefore \boxed{S = 2\pi \bar{y} L}$$

(2) Volume of Revolution - the volume of a body or revolution equals the generating area multiplied by the ~~distance~~ ~~of the area~~ traveled by the centroid of the area.

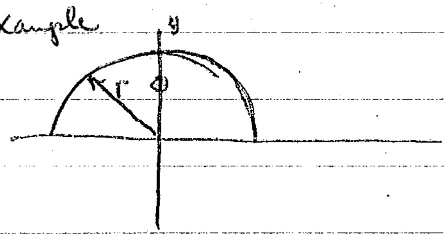


$$\int_A dV = \int_A 2\pi y \, dA \qquad \bar{y} = \frac{\int_A y \, dA}{A}$$

$$V = 2\pi \int_A y \, dA = 2\pi \bar{y} A$$

$$\therefore \boxed{V = 2\pi \bar{y} A}$$

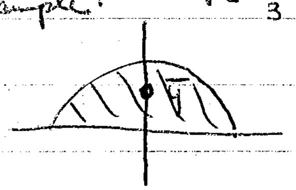
Example



$$\bar{y} = \frac{2r}{\pi} \quad \text{and} \quad L = \pi r$$

$$S = 2\pi \bar{y} L = 2\pi \left(\frac{2r}{\pi}\right)(\pi r) = 4\pi r^2$$

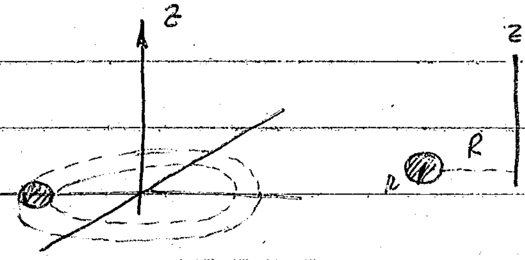
Example: $V = \frac{4}{3}\pi r^3$



$$V = 2\pi \bar{y} A \qquad A = \frac{\pi r^2}{2} \qquad V = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi r^3 = 2\pi \bar{y} \left(\frac{\pi r^2}{2}\right) = \quad \bar{y} = \frac{4r}{3\pi}$$

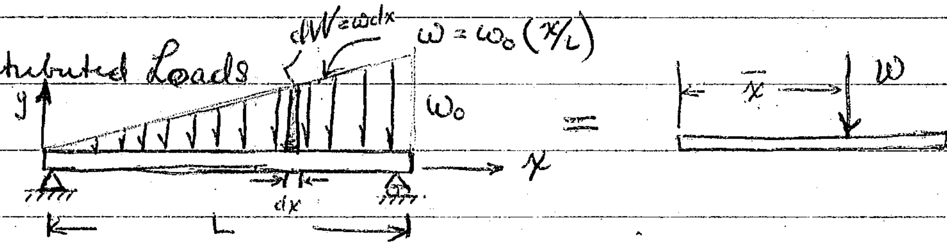
5-52



$$S = 2\pi \bar{y} h$$
$$h = 2\pi r$$
$$2\pi R = \bar{y}\, 2\pi$$
$$4\pi^2 R r = S$$

$$V = 2\pi \bar{y} A = (\pi r^2)(2\pi R) = 2\pi^2 R r^2$$

Distributed Loads



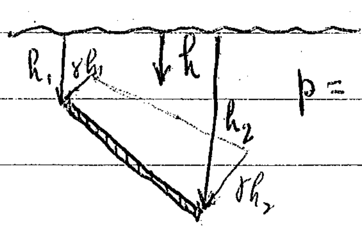
$$W = \int_0^L \omega\, dx \qquad W \bar{x} = \int_0^L (\omega dx)\, x$$ area under load curve

$$\bar{x} = \frac{\int_0^L \omega x\, dx}{\int_0^L \omega\, dx} = \frac{\int_0^L \omega_0 \frac{x^2}{L}\, dx}{\int_0^L \omega_0 \frac{x}{L}\, dx} = \frac{x^3/3}{x^2/2} = \frac{2x}{3}\Big]_0^L = \frac{2L}{3}$$

$$\boxed{\frac{\omega_0 L}{2} \text{ total load} = W}$$

treat the distributed load as an area $\ni$ the load must pass through its center of gravity.

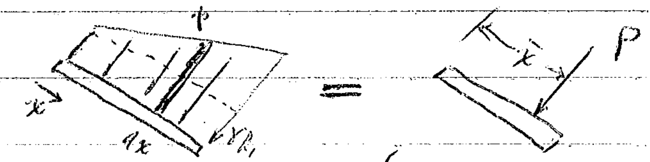
Submerged Bodies



$p = \gamma h$

h = distance from free surface
γ = density

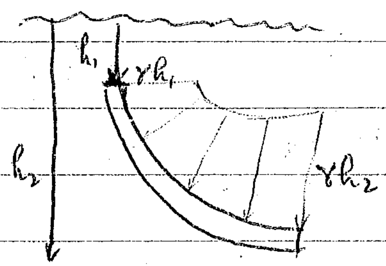
if completely submerged total effect = 0
if thickness is very small.



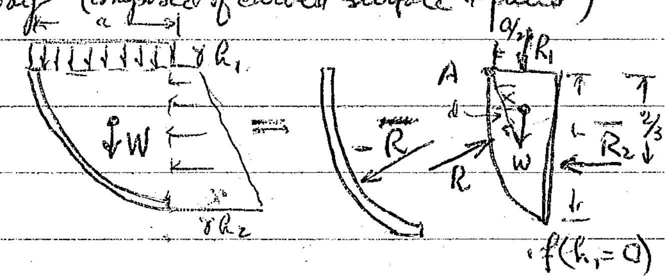
same effect on body.

$$P = \int p\,dx \qquad P\bar{x} = \int (p\,dx)\,x$$

$$\bar{x} = \frac{(\gamma h_1)\,h\cdot\frac{h}{2} + \gamma\frac{(h_2-h_1)}{2}\,h\cdot\frac{2}{3}h}{\gamma h_1 h + \frac{1}{2}\gamma(h_2-h_1)\,h}$$



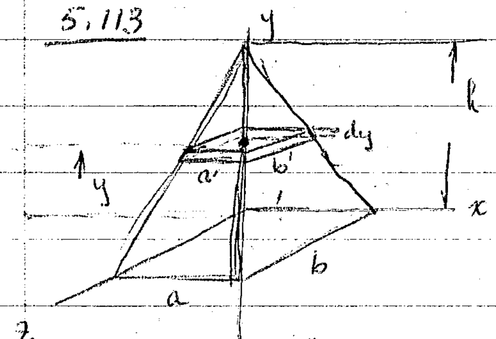
Free-Body (composed of curved surface + fluid)



$$\left.\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma M_A &= 0\end{aligned}\right\}\begin{array}{l}R_x\\R_y\\d\end{array}$$

Problems due next week. 5.8, 18, 36, 44, 100 (Thursday) || 5.53, 54, 66, 78 (A week from 10/28)

5.113



$$\frac{a'}{a} = \frac{h-y}{h};\quad \frac{b'}{b} = \frac{h-y}{y}$$

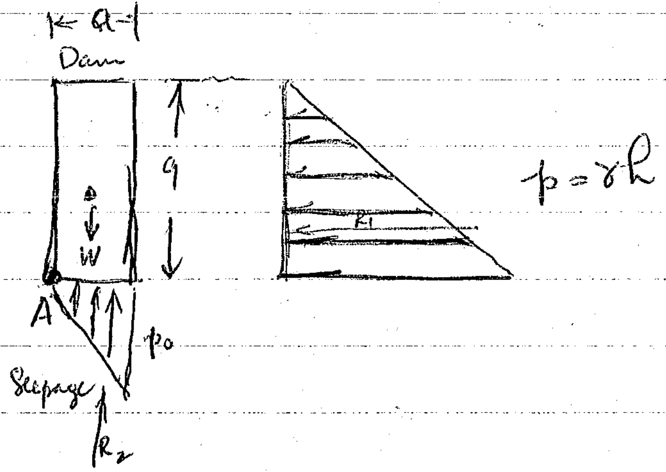
$$\bar{y} = \frac{\int y\,dV}{\int dV} \qquad \bar{y} = \frac{\int_0^h y\,a'b'\,dy}{\int_0^h a'b'\,dy}$$

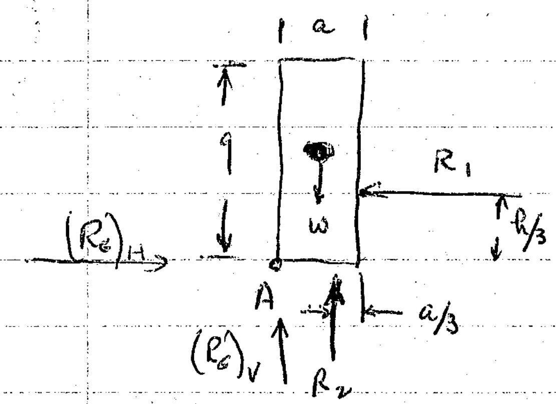
$$\bar{y} = \frac{\int_0^h y\left(ab\left(\frac{h-y}{h}\right)^2 dy\right)}{\int_0^h ab\left(\frac{h-y}{h}\right)^2 dy} = h/4$$

$$\bar{x} = \frac{\int x\,dV}{\int dV} = \frac{\int_0^h \frac{a'}{2}\,a'b'\,dy}{\int_0^h a'b'\,dy} = \frac{\int_0^h \frac{ba^2}{2}\left(\frac{h-y}{h}\right)^3}{\int ab\left(\frac{h-y}{h}\right)^2} = 3/8\,a$$

$\bar{z} = \frac{3}{8} b$

5.86





$M_A = 0$ unit depth

$$w\frac{a}{2} - R_1(3) - R_2\left(\frac{2a}{3}\right) = 0$$

ρ V

$$w = (150\ lb/ft^3)[9a \cdot 1] = 1350a$$

$$p_o = 62.4\ lb/cu\ ft^3 (9) = 562\ \#/ft^2$$

$$R_1 = \text{average press} \times \text{Area} = \left(\frac{562}{2}\right)(9 \cdot 1) = 2530\#$$

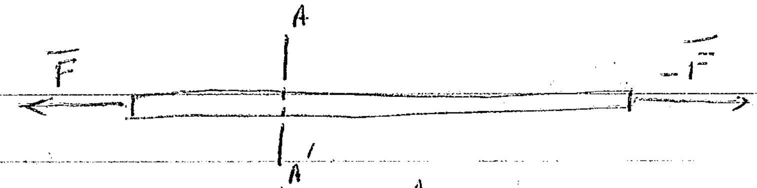
$$R_2 = \left(\frac{562}{2}\right) \times a \cdot 1 = 281a\#$$

$$\therefore \frac{(1350a)(a)}{2} - (2530)(3) - (281)\frac{2a^2}{3} = 0$$

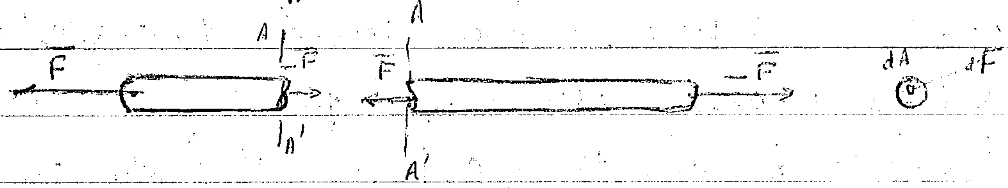
$a^2 = 15.5$ $a = 3.94$ ft.

Pg 236–237 Internal Forces in Structures

Straight 1 dimensional element Two Force element

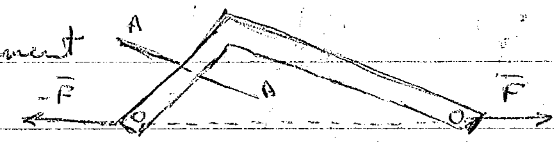


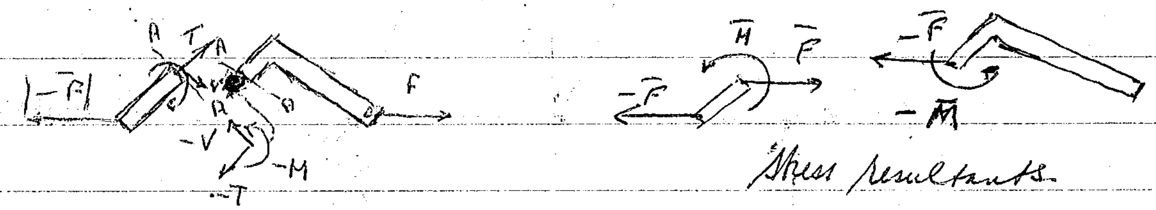
if equilibrium wanted



$$|\bar{F}| = \int_A dF$$

2 force element

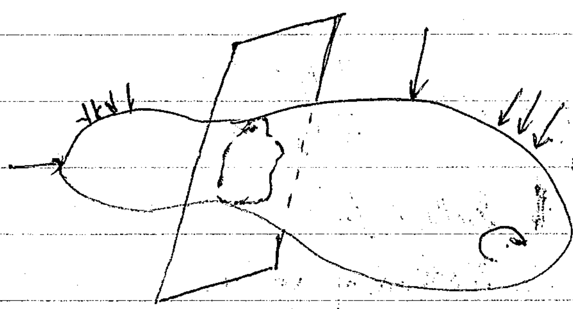


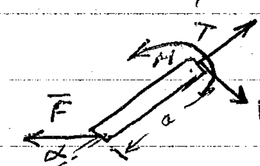


Stress resultants.

Read Chapter 6 187-201; 205-235

T axial forces,
V - shear forces.

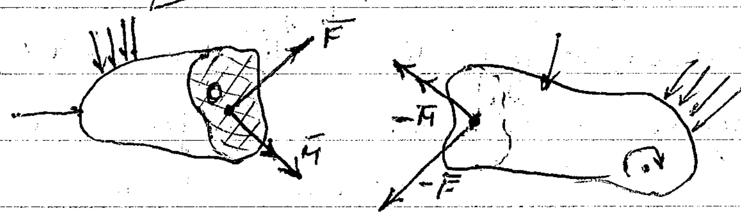


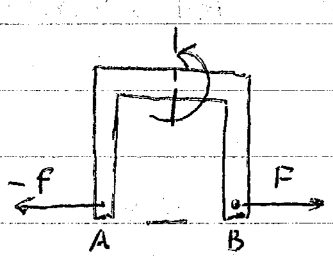


$$\Sigma F_x = -F\cos\alpha + T = 0$$

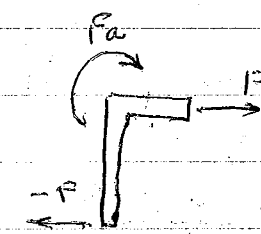
$$\Sigma F_y = +F\sin\alpha - V = 0$$

$$\Sigma M_F = M - Va = 0$$

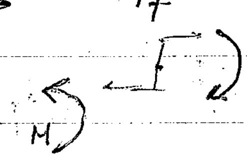




T = F
V = 0
M = Fa



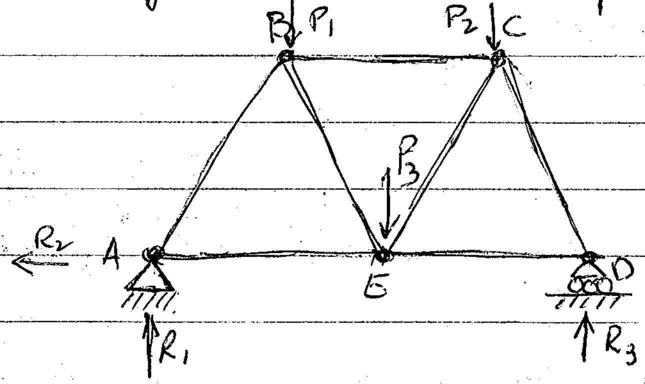
if

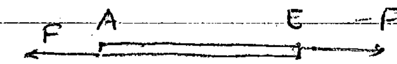


axial force - symmetric
shear force - antisymmetric
moment - symmetric

Trusses, Frames and Machines

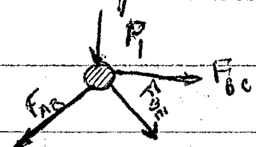
Trusses – a plane structure, assembly of straight one dimensional bars and each one dimensional element has a perfect pin at each end. No bar is continuous thru a joint. All loads are applied at joint. Members are 2 force members. NOTE: CAN TAKE COMPRESSION





Simple Truss

Isolate pin



$\Sigma F_x = 0$

$\Sigma F_y = 0$

Method of Joints – Isolate pins

Necessary Criterion for Method of Joints

let n equals no. of joints therefore $2n$ equations

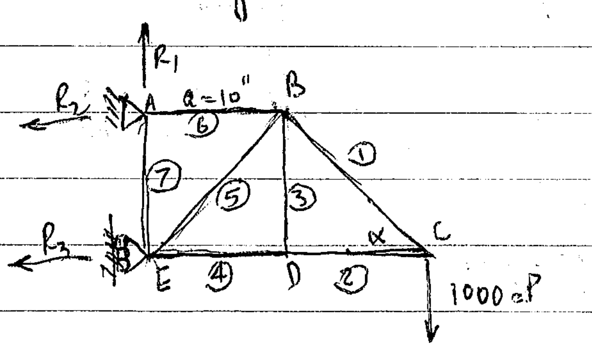
if m equals no. of bars therefore m unknowns

plus 3 reactions

or $m + 3$ unknowns

Necessary $2n = m+3$

6.



Calculate reactions

Assume truss is free body

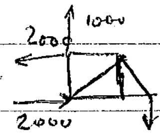
$\Sigma F_x = 0$ $\qquad \alpha = 45^\circ$

$-R_2 - R_3 = 0 \qquad R_3 = -R_2$

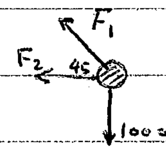
$\Sigma F_y = 0 \qquad R_1 - 1000 = 0 \qquad R_1 = 1000^{\#}$

$\Sigma M_A = 0 \qquad -R_3(10) - 1000(20) = 0 \qquad R_3 = -2000^{\#}$

$R_2 = +2000^{\#}$



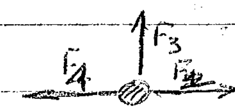
Isolate pin @ C



$\Sigma F_x = 0 : -F_2 - F_1 \cos 45 = 0 \qquad F_2 = -1000^{\#}$

$\Sigma F_y = F_1 \sin 45 - 1000 = 0 \qquad F_1 = \frac{1000}{.707} = 1414^{\#}$

Isolate pin @ D

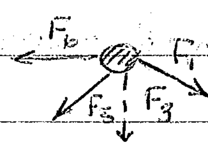


$F_3 = 0 \qquad \Sigma F_y = 0$

$F_2 = F_4 \qquad \Sigma F_x = 0$

$F_4 = -1000^{\#}$

Isolate pin @ B



$\Sigma F_x = 0$

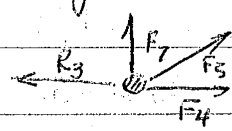
$-F_6 - F_5 \cos 45 + F_1 \cos 45 =$

$-F_6 - F_5 \frac{\sqrt{2}}{2} + 1000 = 0$

$\Sigma F_y = 0 \qquad -F_5 \sin 45 - F_1 \sin 45 = 0 \qquad F_1 = -F_5 \qquad F_5 = -1414^{\#}$

$F_6 = +2000^{\#}$

Isolate Joint @ E



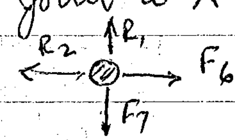
$\Sigma F_x = 0$

$= -R_3 + F_5 \cos 45 + F_4 = 0$

$= 2000 - 1000 - 1000 = 0$ ✓

$\Sigma F_y = 0 \qquad F_7 + F_5 \sin 45^\circ = 0 \qquad F_7 = -F_5 \sin 45^\circ \qquad F_7 = 1000^{\#}$

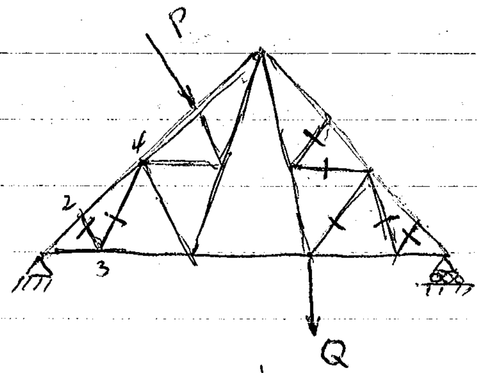
Joint @ A



$F_7 = R_1$

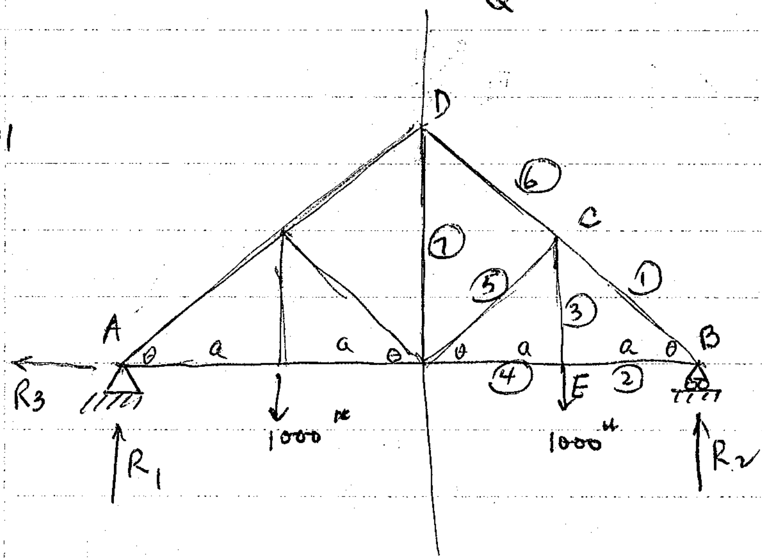
$R_2 = F_6$

6.14



members crossed are 0 force members

6.11



$R_3 = 0$

$R_1 = R_2 = 1000$

By inspection

$\Sigma F_x = 0 \qquad -R_3 = 0 \qquad R_3 = 0$

$\Sigma F_y = 0 \qquad R_1 + R_2 - 2000 = 0$

$\Sigma M_A = 0 \qquad -1000a - 1000(3a) + R_2 4a = 0$

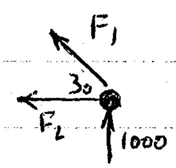
$-4000a = -4aR_2 \qquad R_2 = 1000^{\#}$

$R_1 = 1000^{\#}$

Due
Hw. 11/11/69

6.8	6.13
6.42	6.46

Isolate Joint B



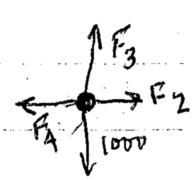
$-F_2 - F_1 \cos 30^\circ = 0$

$+F_1 \sin 30 + 1000 = 0$

$\boxed{F_1 = -2000^{\#}}$

$\boxed{F_2 = 1732^{\#}}$

Isolate Joint E

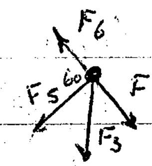


$\boxed{F_3 = 1000^{\#}}$

$F_2 - F_4 = 0$

$\boxed{F_4 = F_2 = 1732^{\#}}$

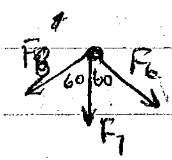
Joint C



$$\Sigma F_x = -F_6 \cos 30^\circ - F_5 \cos 30^\circ + F_1 \cos 30^\circ = 0$$
$$-F_6 - F_5 + F_1 = 0$$
$$-F_6 - F_5 - 2000 = 0$$

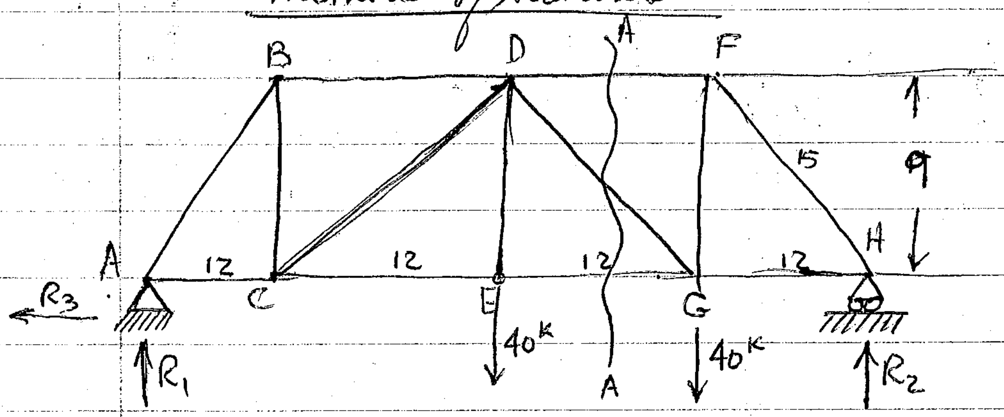
$$\Sigma F_y = F_6 \sin 30^\circ - F_5 \sin 30^\circ - F_3 - F_1 \sin 30^\circ = 0$$
$$\frac{F_6}{2} - \frac{F_5}{2} - 1000 + 1000 = 0$$
$$F_6 - F_5 = 0 \qquad F_5 = F_6$$
$$-2F_6 - 2000 = 0 \qquad \boxed{F_6 = -1000^{\#}} \qquad \boxed{F_5 = -1000^{\#}}$$

Joint D



$$F_6 \cos 30^\circ - F_8 \cos 30^\circ = 0$$
$$-F_6 \cos 60^\circ - F_8 \cos 60^\circ - F_7 = 0$$
$$\boxed{F_6 = F_8 = -1000^{\#}}$$
$$-\frac{1000}{2} - \frac{1000}{2} - F_7 = 0 \qquad \boxed{F_7 = +1000^{\#}}$$

Method of Sections



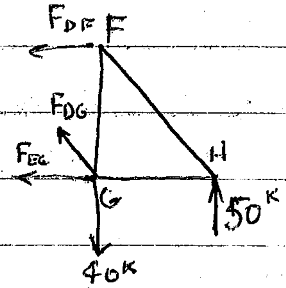
Find force in member DG

$R_3 = 0$

$R_1 + R_2 = 80^K$

$$R_3 = 0 \quad R_1 = 30^K \quad R_2 = 50^K$$

$$\Sigma M_A = -40(24) - 40(36) + R_2(48)$$
$$= -40(60) + R_2(48)$$
$$\frac{40(60)}{48} = 50\text{ K} = R_2$$
$$30^K = R_1$$



$\Sigma F_y = 0$

$F_{DG}\left(\frac{3}{5}\right) - 40^K + 50^K = 0$

$F_{DG} = \frac{5}{3}\left(-10^K\right) = -16\frac{2}{3}^K$

Comp

$\Sigma M_G = 0 \quad F_{DF}(9) + 50(12) = 0$

$F_{DF} = 50\,\frac{12}{9}\frac{4}{3}$

$= -66.7^K$ Compress

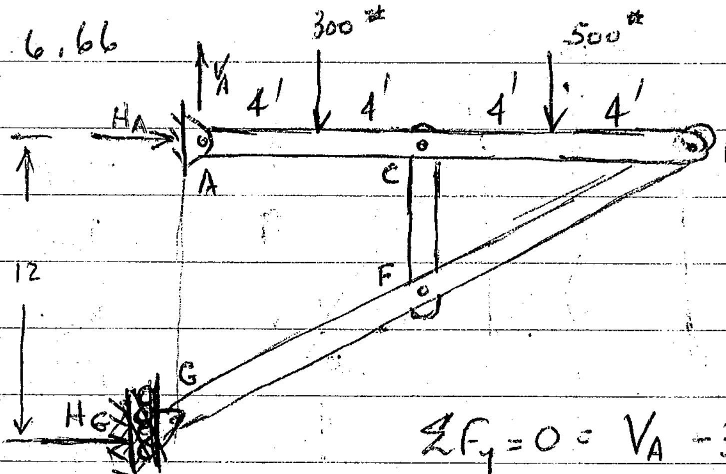
$\Sigma F_x = 0 \quad -F_{EG} - F_{DG}\left(\frac{4}{5}\right) - F_{DF} = 0$

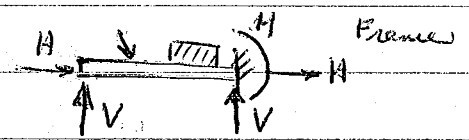
$+F_{EG} = \left(16\frac{2}{3}\right)\left(\frac{4}{5}\right) + (50)\left(\frac{4}{3}\right)$

$\frac{5}{3}(10)\left(\frac{4}{5}\right) \quad + \quad 66.7$

$F_{EG} = 13\frac{1}{3} + 66\frac{2}{3} = 80^K$

Frames





Can go through a joint
can carry transverse force

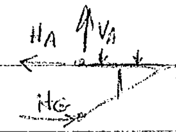
$\Sigma F_y = 0 = V_A - 300 - 500 \qquad V_A = 800\#$

$\Sigma F_x = 0 \quad H_A + H_G = 0$

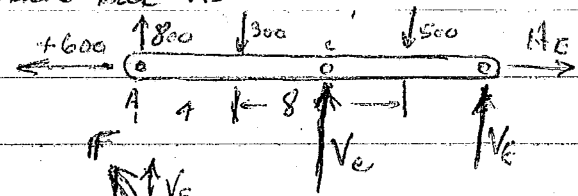
$\Sigma M_A = 0 \quad -4(300) - 500(12) + 12H_G = 0$

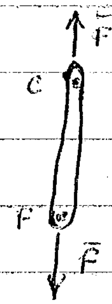
$-1200 - 6000 + 12H_G = 0$

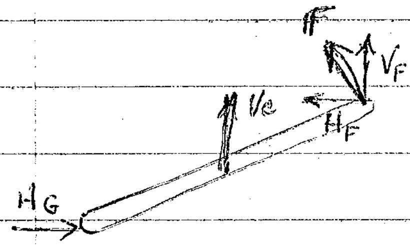
$H_G = 600\# \quad H_A = -600\#$



Isolate bar AE







$H_E = 600$

$V_c + V_E + 800 - 800 = 0$

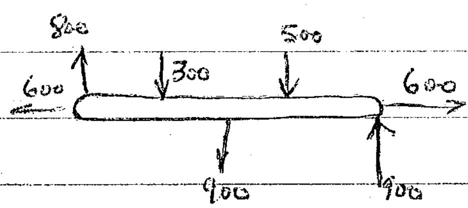
$4(500) - V_c(8) + 300(12) - 800(16) = 0$

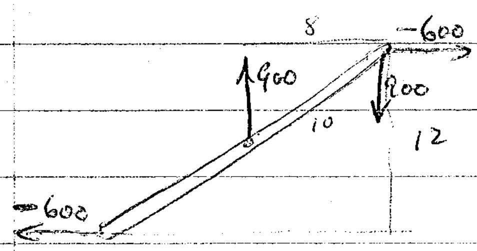
$2000 - V_c 8 + 3600 - 12800 = 0$

$8 V_c = 7200$

$V_c = -900^{\#}$

$V_E = +900^{\#}$

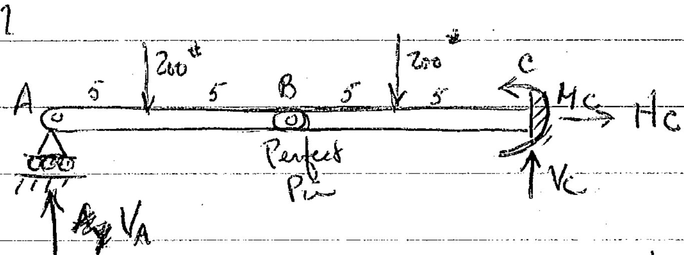




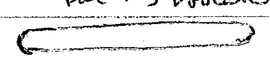
$-72 = -72$

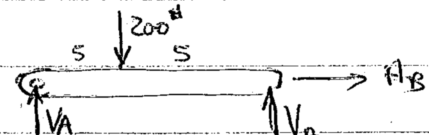
$12(600) \overset{?}{=} 900(8)$ yes.

6.11



Bar AB isolate



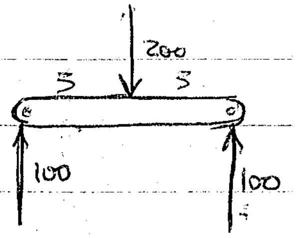


$\Sigma F_x = 0 \quad H_B = 0$

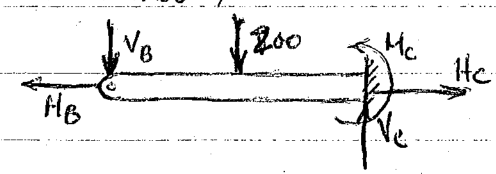
$\Sigma F_y = V_A + V_B - 200 = 0$

$\Sigma M_B = 0 \qquad V_A(-10) + 200(5) = 0 \qquad V_A = 100^{\#}$

$V_B = 100^{\#}$



Consider bar bc



$H_B = H_c = 0 \qquad \Sigma F_x = 0 = -H_B + H_c$

$H_c = 0$

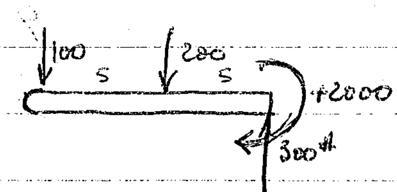
$\Sigma M_{pt\ C} = 0 \qquad V_B(+10) + 200(5) + M_c = 0$

$1000 + 1000 = -M_c$

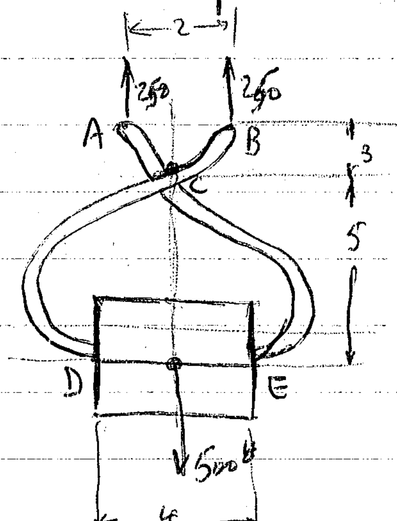
$-2000^{ft\text{-}lb} = M_c$

$\Sigma F_y = 0 \qquad -V_B - 200 + V_c = 0$

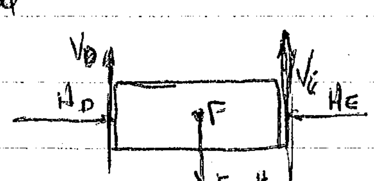
$V_c = 300^{\#}$



Q.105



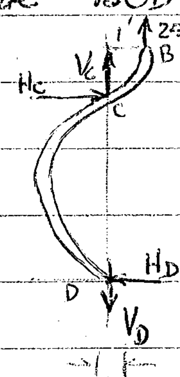
Isolate load



$V_D + V_E - 500 = 0 \qquad H_D = H_E$

$+V_D(2) + (-V_E)(2) = 0 \qquad V_D = V_E \qquad V_D = 250^{\#} = V_E = 250^{\#}$

Isolate BCD



$\Sigma F_y = \cancel{\ldots} = 0$

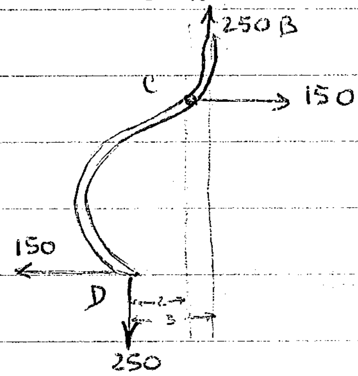
$V_c + 250 - V_D = 0$

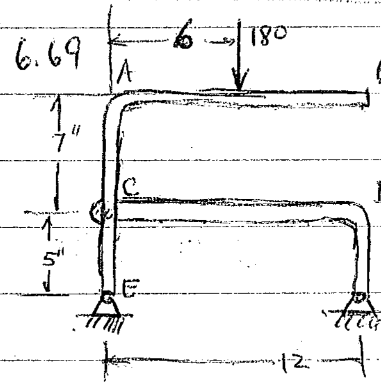
$V_c = -250 + 250 = 0$

$\Sigma M_D = 0 = \quad +H_c(5) - \cancel{V_c}(2) - 250(3) = 0$

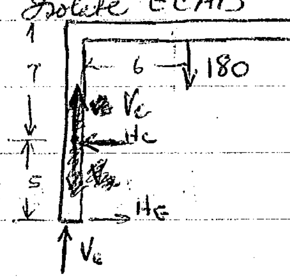
$H_c = 150^{\#}$

$\Sigma F_x = 0 \quad H_c - H_D = 0 \qquad H_D = 150^{\#}$

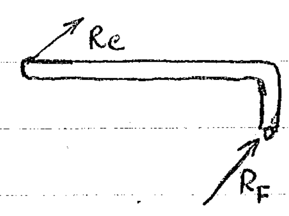


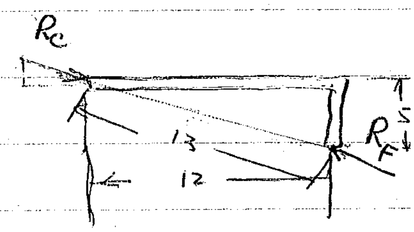


Isolate ECAB



Isolate CDF





$V_c = R_c \frac{5}{13}$

$H_c = \frac{12}{13} R_c \quad \text{or} \quad \frac{V_c}{H_c} = \frac{5}{12}$

$\Sigma F_y = 0 \quad -180 + V_C + V_E = 0$

$\Sigma F_x = 0 \quad -H_C + H_E = 0$

$\Sigma M_E = 0 = -180(6) + H_C(5) = 0$

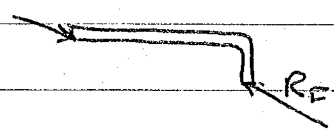
$+\frac{180(6)}{5} = +216^{\#} \qquad V_C = \frac{5}{12}$

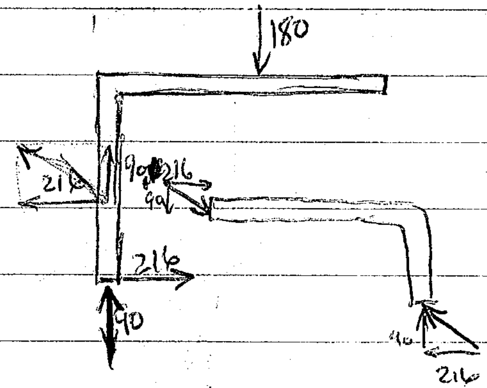
$H_E = + H_C = 216^{\#}$

$0 = -180 + \left(\frac{5}{12}\right) H_C + V_E$

$-180 + \frac{5}{12}(216) + V_E = 0$

$-180 + 90 + V_E = 0 \qquad V_E = 90$



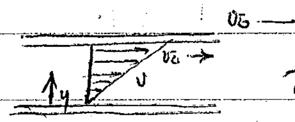


due 11/18 6-70 6.81 6.112 7.4 8.12 8.20 8.28

Friction

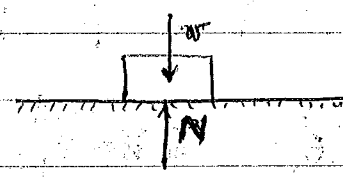
1. Fluid friction

2. Dry Friction (Coulomb)



$\tau = \mu \dfrac{dv}{dy} \quad \dfrac{\#}{in^2}$

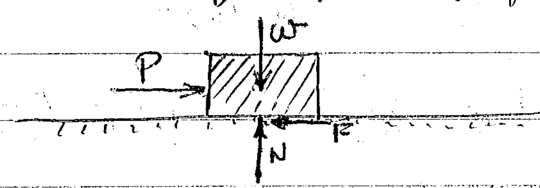
DRY FRICTION



Rough Surface
No motion

$-W + N = 0 \qquad W = N$

Apply a small horizontal force P (compared to W.



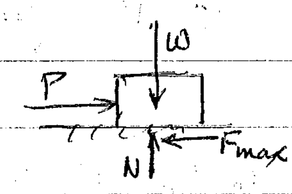
$\Sigma F_y = 0 \qquad N = W$

$\Sigma F_x = 0 \qquad P = F$

No motion

$F_{max} = \mu_s N$; μ_s - coefficient of static friction $\qquad \mu_K < \mu_S$

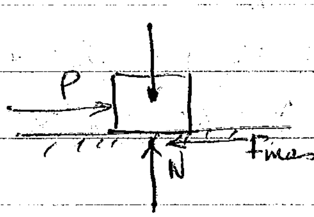
μ_K - coefficient of kinetic friction.



$\Sigma F_y = 0 \qquad W = N$

$\Sigma F_x = 0 \qquad P = F_{max} = \mu_s N$

impending motion



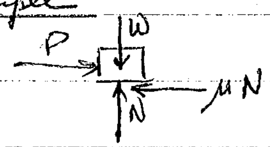
If $P > F_{max}$ motion is to right.

$\left(\text{Unbalanced force} = P - \mu_K N\right) = \dfrac{W}{g} \dfrac{d^2x}{dt^2}$

μ depends on materials used and the surface

(If the area of contact is of no matter)

Example



$W = 300^{\#}$ $\mu_s = .4$ $\mu_k = .3$

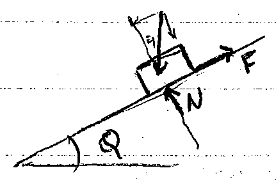
what is P? for impending motion

$\Sigma F_y = 0$ $W = N = 300^{\#}$

$\Sigma F_x = 0$ $P = \mu_s N = .4(300^{\#}) = 120^{\#}$

what is P of block in motion

$\Sigma F_x = 0$ $P = \mu_k N = .3(300^{\#}) = 90^{\#}$



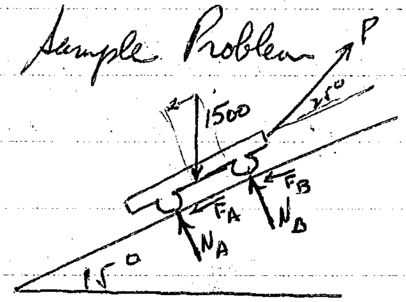
$Q = ?$ for impending motion

$N = W \cos Q$ $W \sin Q = F = \mu_s N$

$\frac{F}{N} = \tan Q$ $\frac{F}{N} = \tan Q$ ~~$\arctan^{-1} \frac{1}{\mu_s W} = Q$~~

~~$Q = \arctan^{-1} \frac{1}{\mu_s W}$~~ $\boxed{\mu_s = \tan Q}$ Q = angle of repose

P. 281 Sample Problem



$\mu_s = .40$

$\mu_k = .30$

(a) $P = ?$ for impending motion up incline

$\Sigma F_x = 0$ $P \cos 25° - F_A - F_B - 1500 \sin 15 = 0$

$P \cos 25° - \mu_s(N_A + N_B) = 1500 \sin 15$

~~$P \cos 25° - \mu_s (1500 \cos 15°) = 1500 \sin 15$~~

$\Sigma F_y = 0$ $P \sin 25° - 1500 \cos 15° + N_A + N_B = 0$

$N_A + N_B = 1500 \cos 15° - P \sin 25°$

$$P\cos 25^\circ - .4\left(1500\cos 15^\circ - P\sin 25^\circ\right) = 1500\left(\sin 15^\circ\right)$$

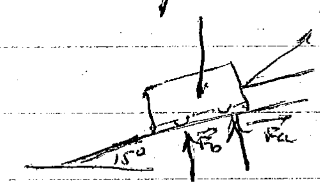
$$P\left(\cos 25^\circ + .4\sin 25^\circ\right) = 1500\left(\sin 15^\circ + .4\cos 15^\circ\right)$$

$$P = \frac{1500\left(\sin 15^\circ + .4\cos 15^\circ\right)}{\left(\cos 25^\circ + .4\sin 25^\circ\right)} = 900^{\#} \text{ (a)}$$

part b. P_2? for uniform motion upincline once motion has started?

$$\mu_k \rightarrow \mu_s \qquad P = \frac{1500\left(\sin 15^\circ + .3\cos 15^\circ\right)}{\left(\cos 25^\circ + .3\sin 25^\circ\right)} = 795^{\#}$$

part c. force needed to keep sled from sliding

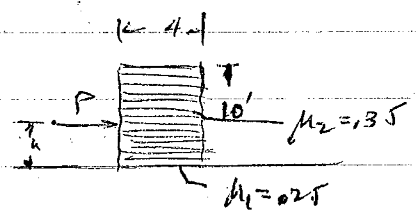


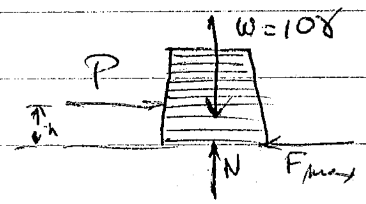
$$P\cos 25^\circ + \underbrace{F_A + F_B}_{\mu_s(N_A + N_B)} - 1500\sin 15^\circ = 0$$

$$N_A + N_B - 1500\cos 15^\circ + P\sin 25^\circ = 0$$

$$P = \frac{1500\left(\sin 15^\circ - .4\cos 15^\circ\right)}{\left(\cos 25^\circ - .4\sin 25^\circ\right)} = -258^{\#}$$

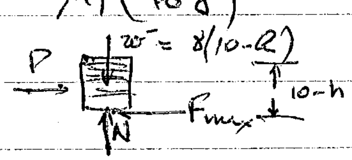
8.18





$$P = F_{max} \qquad N = \omega = 10\gamma$$

$$P = \mu_1 N = \mu_1(10\gamma)$$



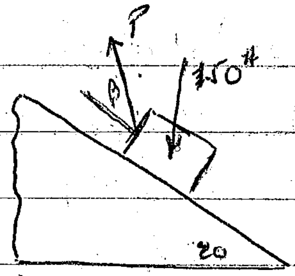
$$N = \omega' = (10 - h)\gamma$$

$$P = F_{max} = \mu_2 N = \mu_2(10 - h)\gamma$$

$$P = 10\mu_1\gamma = \mu_2(10 - h)\gamma$$

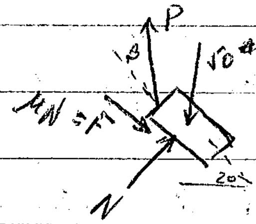
$$2.5 = 3.5 - \mu_2 h \qquad h = 2.86'$$

8.4



$\mu_s = .25$

find P min



① $\sum F_x = 0 \quad -P\cos\beta + 50\sin 20° + \mu N = 0$

② $\sum F_y = 0 \quad +P\sin\beta + N - 50\cos 20° = 0$

Solve for N and Put in 1 $\quad N = -P\sin\beta + 50\cos 20°$

① $-P\cos\beta + 50\sin 20° + \mu\left[50\cos 20° - P\sin\beta\right] = 0$

$+P(\cos\beta + \mu\sin\beta) = +50(\mu\cos 20° + \sin 20°) = C$

$$P = \frac{C}{[\cos\beta + \mu\sin\beta]} \quad ③$$

$$\frac{dP}{d\beta} = 0 \quad \frac{-C\left[-\sin\beta + \mu\cos\beta\right]}{[\cos\beta + \mu\sin\beta]^2} = 0$$

$-C(-\sin\beta + \mu\cos\beta) = 0 \quad C \neq 0 \quad \therefore \quad -\sin\beta = -\mu\cos\beta$

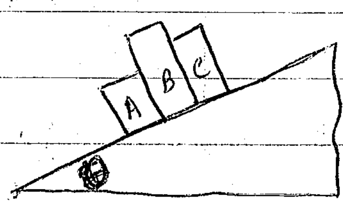
$\tan\beta = \mu$

$\beta = 14°$

$$P_{min} = \frac{50[.25\cos 20° + \sin 20°]}{\cos 14° + .25\sin 14°}$$

$P_{min} = 28^{\#}$

for impending motion down plane replace μ by $(-\mu)$ and P min from ③ $P_{min} = 5.23^{\#}$



Given $W = 10$ for a, b, c

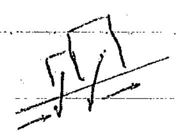
$\mu_s = .3$ for a + c

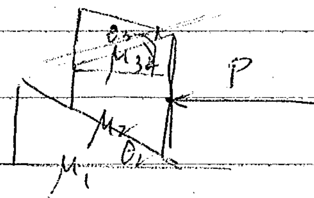
$\mu_s = .1$ for b

$\tan\theta = .25$

C stays put since $\mu_s > \tan\theta$

then   if





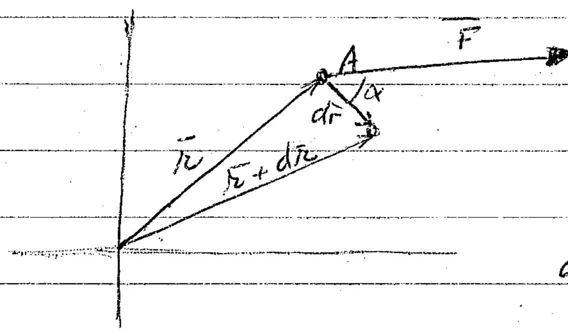
will P move a, b, c

$$\left.\begin{array}{l} \text{if } \mu_3 < \tan\theta_3 \\ \text{if } \mu_2 < \tan\theta_2 \\ \text{if } \mu_1 N_T < P \end{array}\right\} \text{all blocks will slide}$$

TEST DEC 2

Virtual Work; potential Energy; Stability

U = Work



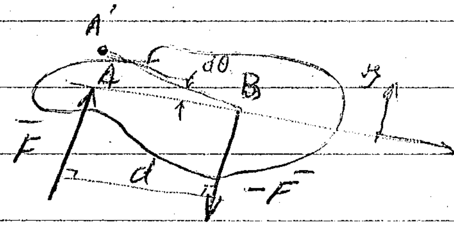
Definition of Work

$dU = \bar{F} \cdot d\bar{r}$

or

$du = F\cos\alpha \, dr$ scalar

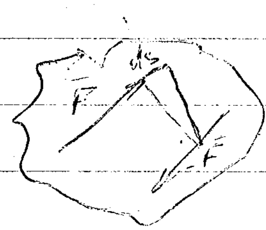
$$du \begin{array}{c} > \\ = \\ < \end{array} 0 \qquad \begin{array}{l} \alpha < \pi/2 \\ \alpha = \frac{\pi}{2} \\ \alpha > \frac{\pi}{2} \end{array}$$



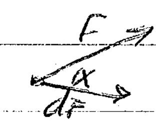
for translation - $dU = 0$

for rotation - $F(d)\,d\theta = du$

$du = M\,d\theta$

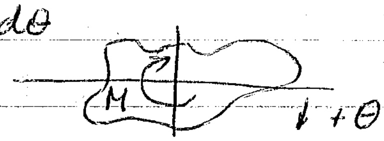


potential [illegible]

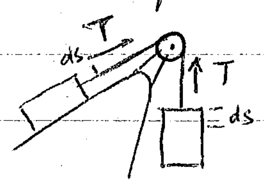


$du = \bar{F} \cdot d\bar{r} = F\cos\alpha \, dr$

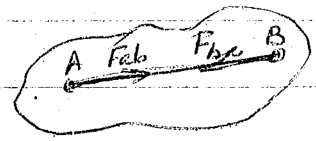
$du = M\,d\theta$



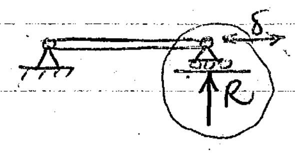
Internal forces do no work



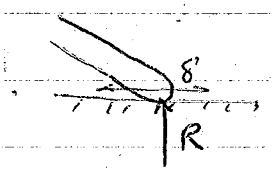
$$+T\,ds - T\,ds = 0$$



Reactions at smooth surfaces do no work

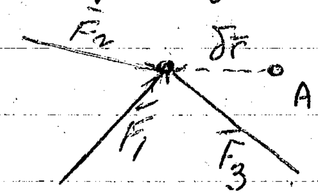


does no work



Principle of virtual work

Equilibrium



a small imaginary displacement = virtual displacement

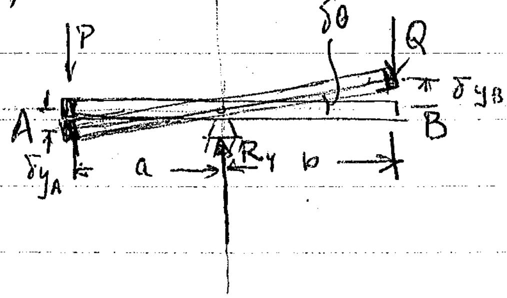
$$dU = F_1 \cdot \delta \bar{r} + F_2 \cdot \delta \bar{r} + F_3 \cdot \delta \bar{r} + \dots$$
$$= (F_1 + F_2 + F_3 + \dots) \cdot \delta \bar{r}$$
$$= \left(\sum_{i=1}^{n} F_i\right) \cdot \delta \bar{r}$$
$$= R \cdot \delta \bar{r}$$

principle of virtual work (for particle)

NECESSARY & SUFFICIENT

If a particle is in equilibrium the total virtual work of all the forces acting on that particle for any virtual displacement $= 0$

$R_x = 0$



1 degree of freedom system

$$\delta U = 0$$
$$P\delta y_a - Q\delta y_B = 0$$

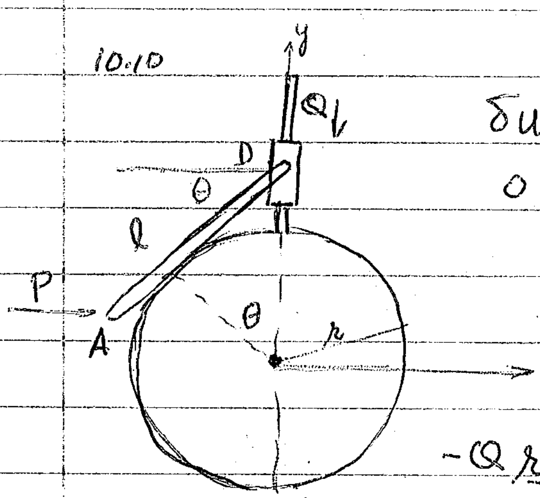
$\delta y_a = a\,\delta\theta$ $\qquad P a\,\delta\theta - Q b\,\delta\theta = 0$

$\delta y_B = b\,\delta\theta$ $\qquad (Pa - Qb)\,\delta\theta = 0 \quad$ if $\delta\theta \neq 0$

$(Pa - Qb) = 0 \qquad Pa = Qb \qquad P = Q\frac{b}{a}$

$\delta u = 0$

10.10



$\delta u = 0$

$0 = -Q\,\delta y_B + P\,\delta x_A$

$y_B = \frac{r}{\cos\theta} \qquad \delta y_B = \frac{+r\sin\theta\,\delta\theta}{\cos^2\theta}$

$x_A = -l\cos\theta \qquad \delta x_A = -l\sin\theta\,\delta\theta$

$-Q\frac{r\sin\theta\,\delta\theta}{\cos^2\theta} + Pl\sin\theta\,\delta\theta = 0$

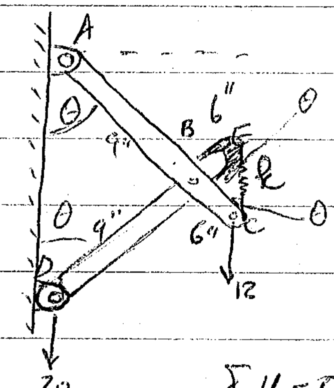
$-\frac{Qr\sin\theta}{\cos^2\theta} = -Pl\sin\theta$

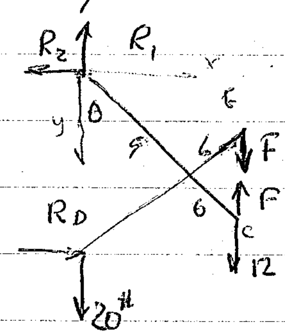
$-Qr = -Pl\cos^2\theta$

$+Q = \frac{Pl\cos^2\theta}{r}$

$k = 15^{\#}/in$

10.23





R_1, R_2, R_D do no work since A does not move and roller can move only up & down

$\delta u = 0$

$20\,\delta y_D + 12\,\delta y_C - F\,\delta y_C + F\,\delta y_E = 0$

$y_D = 18\cos\theta; \quad \frac{\delta y_D}{\delta\theta} = -18\sin\theta \qquad y_E = 3\cos\theta$

$y_C = 15\cos\theta; \quad \frac{\delta y_C}{\delta\theta} = -15\sin\theta \qquad \frac{\delta y_E}{\delta\theta} = -3\sin\theta$

$B_y = -B_x\left(\frac{2}{5}\right)$

$-1B_x(30) = +25000$

$B_x = \frac{2500}{3}$

rest length

$F = k\delta = k[\,y_C - y_G - 5\,]$

$20[-18\sin\theta] + 12(-15\sin\theta) + F(\delta y_G - \delta y_C) = 0$

$-540\sin\theta\,\delta\theta + k[12\cos\theta - 5][+12\sin\theta\,\delta\theta] = 0$

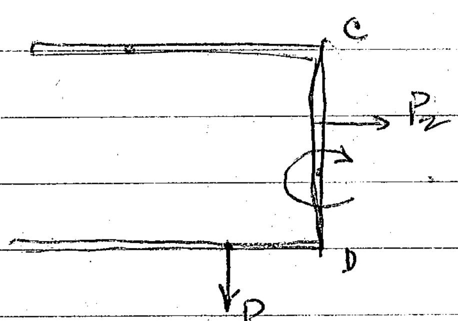
$-540 + \cancel{k[\ldots]}$

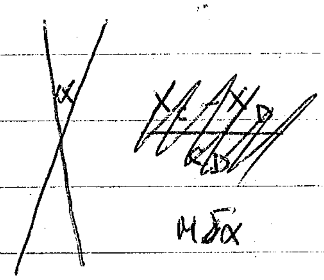
$15(12)(12\cos\theta - 5) = 0$

$\cos\theta = 2/3 \qquad \theta =$

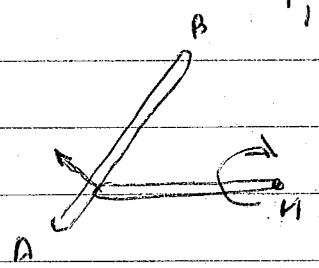
$AD = 18\cos\theta = 12$ inches

10.4, 6, 12, 44, 61.

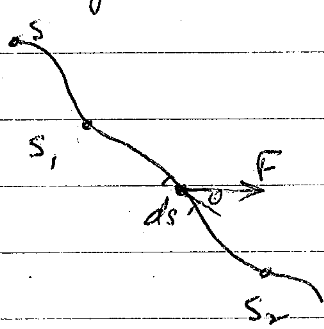




$M\delta\alpha$



Work of a Force during a Finite Displacement



$dU = \bar{F}\cdot d\bar{s}$

$U_{1-2} = \int_{s_1}^{s_2} F\cdot ds$ Vector

$dU = (F\cos\alpha)\,ds$

$U_{12} = \int_{s_1}^{s_2} F\cos\alpha\,ds$

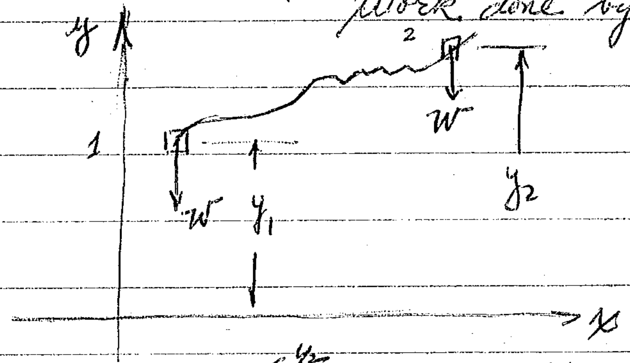
Test Cancelled to
December 4, 1969

For couple or moment

$dU = M\,d\theta$

$$U_{1-2} = \int_{\theta_1}^{\theta_2} M\,d\theta$$

Work done by Mass of Weight W

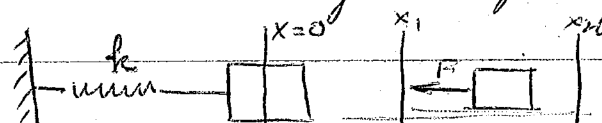


the negative sign since
W is in $-y$ direct

$$U_{1-2} = -\int_{y_1}^{y_2} W\,dy = -Wy\Big|_{y_1}^{y_2} = -W(y_2 - y_1)$$

$\boxed{U_{1-2} = W(y_1 - y_2)}$ work is done on it

Work done by a Spring



the negative sign since
F of spring is in $-x$ direct

$$U_{1-2} = -\int_{x_1}^{x_2} F\,dx = -\int_{x_1}^{x_2} kx(dx) = -\frac{kx^2}{2}\Bigg]_{x_1}^{x_2}$$

$$U_{1-2} = \frac{k}{2}(x_1^2 - x_2^2)$$

When the work done does not depend on path then Wy, $\frac{1}{2}kx^2$ (the functions) are given name of potential energy.

V - potential energy

Mass $\quad U_{1-2} = V_{g_1} - V_{g_2} = Wy_1 - Wy_2$

Spring $\quad U_{1-2} = V_{e_1} - V_{e_2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

In computing V datum can be selected for convenience

∴ Extend concept to any force system provide

$\int du = -\int dV$ ← perfect differential

$U_{1-2} = V_1 - V_2$

Any system for which the path makes no difference is a conservative force.

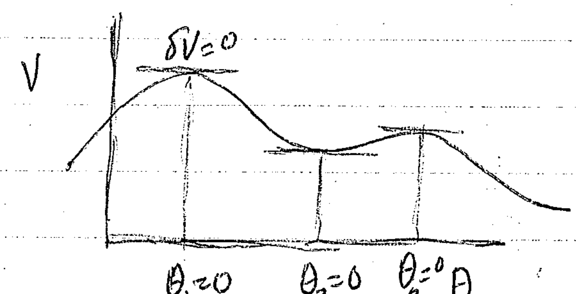
Principle of Virtual Displacement

$\delta U = 0$

for Conservative force field $\delta U = -\delta V$

∴ $\delta V = 0$ static equilibrium

Loads in virtual work remain constant.

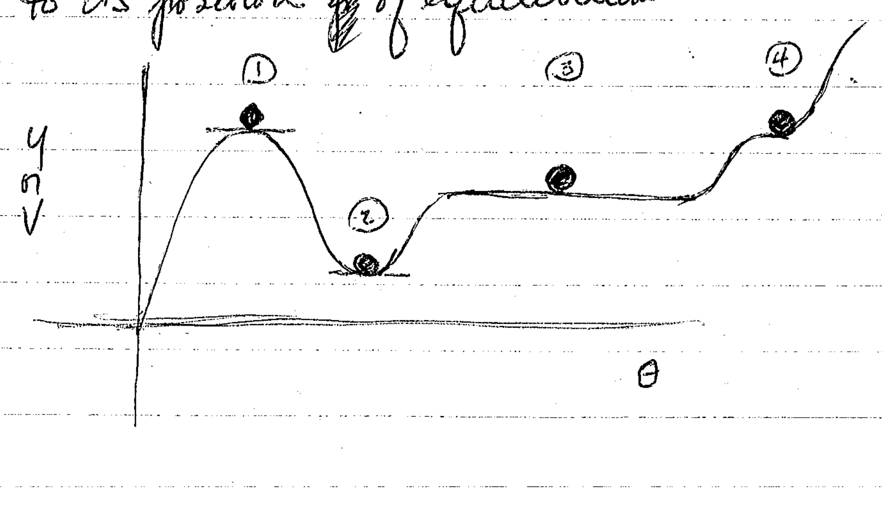


$$\delta V = \frac{dV}{d\theta}\,\delta\theta$$

$$\frac{dV}{d\theta} = 0 \quad \text{if } \delta V = 0$$

Stability of Equilibrium

Stability – after disturbance it tends to return to its original position of equilibrium



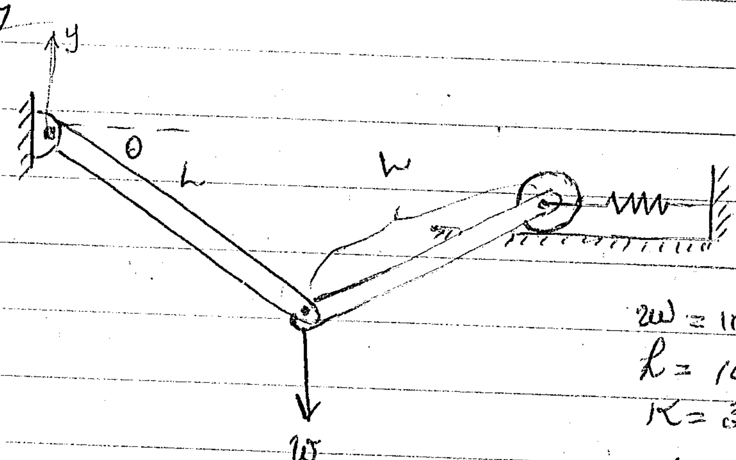
POSITION 1 ① unstable $\frac{d}{d\theta}\left(\frac{dV}{d\theta}\right) < 0$ or $\frac{d^2V}{d\theta^2} < 0$

POSITION 2 ② stable $\frac{d^2V}{d\theta^2} > 0$

POSITION 3 ③ neutral $\frac{d^2V}{d\theta^2} = 0$

POSITION 4 ④ unstable $\frac{d^2V}{d\theta^2} = 0$ but $\frac{d^3V}{d\theta^3} < 0$

10-47



$2W = 100^{\#}$

$l = 10''$

$K = 30^{\#}/in$

unstretched $\theta = 0°$

$\delta V = 0 \qquad \frac{dV}{d\theta} = 0$

$V = Wy$

$V = W y_B + \frac{1}{2} K \delta^2 \;\Big\|\; y_B = -l \sin\theta$

$\delta = 2L - X_c \qquad X_c = 2l\cos\theta$

$\delta = 2l(1-\cos\theta)$

$V = -W(l\sin\theta) + \frac{1}{2}k\left[(2l)^2(1-\cos\theta)^2\right]$

$\frac{dV}{d\theta} = -Wl\cos\theta + \cancel{\tfrac{1}{2}}K(2l)^2\cancel{(2)}(1-\cos\theta)(\sin\theta)$

for equilibrium $\frac{dV}{d\theta} = 0 = -Wl\cos\theta + 4kl^2(1-\cos\theta)\sin\theta$

$0 = \frac{dV}{d\theta} = -\cancel{100}(\cancel{10})\cos\theta + \cancel{4}(\cancel{30})(\cancel{100})(1-\cos\theta)\sin\theta$

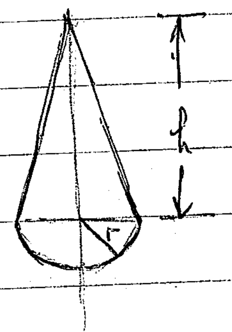
$= -1 + 12(1-\cos\theta)\tan\theta$

$\tan\theta - \sin\theta = 1/12 \qquad \theta = 30.7°$

stable   to check $\cos - \cos^2\theta$

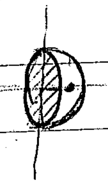
$$\frac{d^2V}{d\theta^2} = +Wl\sin\theta + 4kl^2\left[(1-2\cos\theta)\cos\theta + \sin\theta(\sin\theta)\right]$$

$$Wl\sin\theta + 4kl^2\left(\left[2\sin^2\theta\right]\right)$$



what is h_{max} for static stability

Given: hemisphere ①

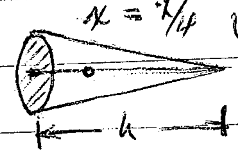


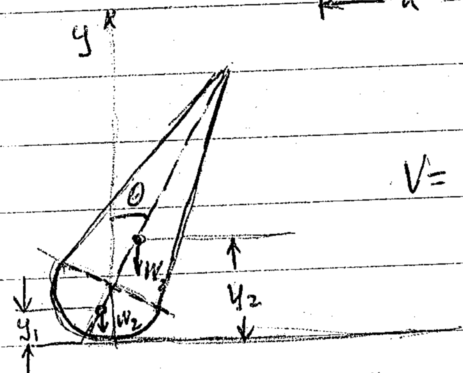
$\bar{x} = \frac{3}{8} r$

and Volume $= \frac{2\pi}{3} r^3$

Cone body ②

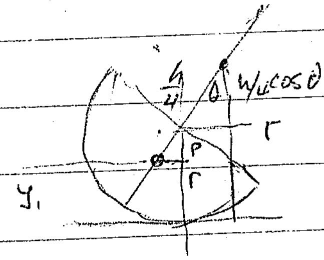
$\bar{x} = h/4$ $V = \frac{1}{2}\pi r^2 h$





$$V = W_1 y_1 + W_2 y_2$$

$$V = \rho\left(\frac{2\pi}{3} r^3\right)\left(r - \frac{3}{8} r\cos\theta\right) + \rho\left(\frac{\pi}{3} r^2 h\right)\left(r + \frac{h}{4}\cos\theta\right)$$



$y_1 = r - P$

$P = \frac{3}{8} r\cos\theta$

$$\frac{dV}{d\theta} = \frac{2\pi}{3}\rho r^3\left(\frac{3}{8} r\sin\theta\right) + \rho\frac{\pi}{3} r^2 h\left(-\frac{h}{4}\sin\theta\right)$$

$$\frac{dV}{d\theta} = 0 \text{ at } \theta = 0^\circ$$

$$\frac{d^2V}{d\theta^2} = \frac{1}{4}\rho\pi r^4\cos\theta - \rho\frac{\pi}{3}r^2 h\left(+\frac{h}{4}\cos\theta\right)$$

for neutral stability $\frac{d^2V}{d\theta^2} = 0$

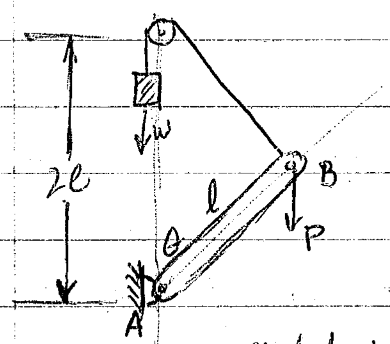
$$\frac{1}{4}\pi\rho r^4 - \rho\frac{\pi}{12}r^2h^2 = 0$$

$$3r^4 = r^2h^2$$

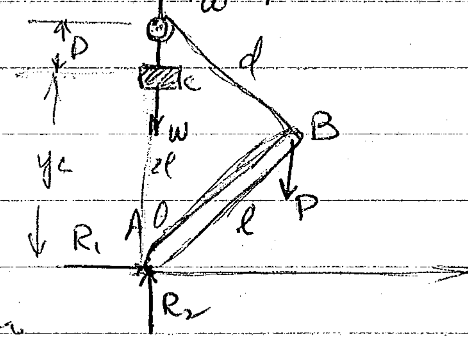
$$3r^2 = h^2$$

$h \leq r\sqrt{3}$ neutral stability

10.36



Determine P/w for which $\theta = 0°$ is stable



neglect weight of bar

$$V = Py_B + Wy_c \qquad y_B = l\cos\theta$$

$D = 2l - y_c$ $\qquad d + 2l - y_c = K$ length of cable

$$d^2 = l^2 + (2l)^2 - 2(l)(2l)\cos\theta$$

$$= 5l^2 - 4l^2\cos\theta$$

$$d = l\sqrt{5 - 4\cos\theta}$$

$$V = P(l\cos\theta) + W(d + 2l - K)$$

$$P(l\cos\theta) + w\left(l\sqrt{5-4\cos\theta} + 2l - K\right) = V$$

$$\frac{dV}{d\theta} = -Pl\sin\theta + W\left(\frac{l}{2}\cdot\frac{4\sin\theta}{\sqrt{5-4\cos\theta}}\right)$$

$$-Pl\sin\theta + 2Wl\frac{\sin\theta}{\sqrt{5-4\cos\theta}}$$

$\theta = 0$ is an equil position

$$\frac{d^2\theta}{d\theta^2} = -Pl\cos\theta + 2Wl\left[\frac{\sqrt{5-4\cos\theta}(\cos\theta) - \sin\theta\,\frac{4\sin\theta}{2\sqrt{5-4\cos\theta}}}{5-4\cos\theta}\right]$$

$$\frac{d^2V}{d\theta^2} - Pl\cos\theta + \frac{2wl\left[(5-4\cos\theta)(\cos\theta) - 2\sin^2\theta\right]}{(5-4\cos\theta)^{3/2}}$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0} = -Pl + 2wl$$

at neutral stability $-Pl + 2wl = 0$

$P = 2w$, $P/w = 2$ max. for equilibrium

if $P > 2w$ the system is unstable

at $\theta = 180^\circ$

$$\frac{d^2V}{d\theta^2} = Pl + 2wl(1/3)(-1/3)$$

set $\frac{d^2V}{d\theta^2} = 0$ $\quad P = 2/3\,W$ if $P < 2/3\,W$ unstableness sets in.

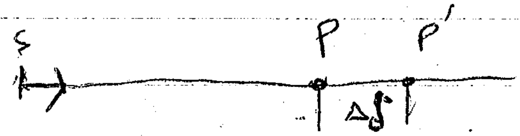
$$+Pl - \frac{2wl[9]}{27} = \quad Pl = \frac{2wl}{3} \qquad P = \frac{2w}{3}$$

Dynamics

1. Kinetics - Relation of F, M, Motion
2. Kinematics - Geometry

Kinematics of Particles

(a) Rectilinear Motion



$s = s(t)$

Define velocity - the time rate of change of s with t $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = v$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \qquad \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{ds} \cdot \frac{ds}{dt} \; : \; a = v\frac{dv}{ds}$$

Three Cases

① $a = a(t)$ ② $a = a(s)$ ③ $a = a(v)$

to find $s = s(t)$

① $a = \frac{dv}{dt}$ $\quad a(t) = \frac{dv}{dt}$ $\quad \int_{v_0}^{v} dv = \int_0^t a(t)\,dt$

Assumption — initially $t=0$

initially at $t=0$ $v = v_0$ $\quad \boxed{v - v_0 = \int_0^t a(t)\,dt}$ yields $v(t)$

$v(t) = ds/dt$ $\quad \int_{s_0}^{s} ds = \int_0^t v(t)\,dt$

assume at $t=0$ $s = s_0$ $\quad \boxed{s - s_0 = \int_0^t v(t)\,dt}$ yields $s = s(t)$

② $a(s) = v\,dv/ds$ $\quad \int_{s_0}^{s} a(s)\,ds = \int_{v_0}^{v} v\,dv$

v as a function of s $\quad \boxed{\int_{s_0}^{s} a(s)\,ds = \frac{v^2}{2} - \frac{v_0^2}{2}}$

$v(s) = ds/dt$ $\quad dt = \frac{ds}{v(s)}$ $\quad \int_0^t dt = \int_{s_0}^{s} \frac{ds}{v(s)}$

$$\boxed{t = \int_{s_0}^{s} \frac{ds}{v(s)}}$$

③ $a = a(v)$ $\quad a(v) = \frac{dv}{dt}$

$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{a(v)}$ $\quad \boxed{t = \int_{v_0}^{v} \frac{dv}{a(v)}}$

$v(t) = \frac{ds}{dt}$ $\qquad \int_{s_0}^{s} ds = \int_0^t v(t)\,dt$

$$\boxed{s - s_0 = \int_0^t v(t)\,dt \qquad s = s(t)}$$

Special Cases

$v = v_0 =$ constant

$v = v_0 = ds/dt \qquad \int_0^t v_0\,dt = \int_{s_0}^{s} ds \qquad v_0 t = s - s_0$

$$\boxed{v_0 t + s_0 = s}$$

$a = a_0 =$ constant $\qquad a = a_0 = dv/dt$

$\int_{v_0}^{v} dv = \int_0^t a_0\,dt$

$v - v_0 = a_0 t \qquad \boxed{v = v_0 + a_0 t} \checkmark$

$v = ds/dt \qquad \int_0^t (v_0 + a_0 t)\,dt = \int_{s_0}^{s} ds$

$$\boxed{s_0 + v_0 t + \tfrac{1}{2} a_0 t^2 = s} \checkmark\checkmark$$

$a_0 = dv/dt \qquad dv = a_0\,dt \qquad v = a_0 t + A$

at $v(0) = v_0 \qquad v_0 = A \qquad \boxed{v = a_0 t + v_0} \checkmark$

at $s(0) = s_0 \qquad \tfrac{1}{2} a_0 t^2 + v_0 t + B = s \Rightarrow B = s_0$

$\therefore \boxed{\tfrac{1}{2} a_0 t^2 + v_0 t + s_0 = s \quad \checkmark\checkmark}$

s

$a = kt^3 \qquad s(0) = s_0$

$v(0) = v_0$

$a = \frac{dv}{dt} = kt^3$ $\quad \int_{v_0}^{v} dv = \int_0^t kt^3\,dt$

$v = \frac{kt^4}{4} + A$

$v_0 = v$ at $v(0)$

$v = v_0 + \frac{kt^4}{4}$

$$\boxed{s = v_0 t + \frac{kt^5}{20} + s_0}$$

$a = 100s \quad s_0 = 50 \quad v_0 = 25 \quad s(t) = s$

$a = v\frac{dv}{ds}$

$\int 100s\,ds = \int v\,dv$

$\frac{100s^2}{2} = \frac{v^2}{2} + A \qquad \frac{100(50)^2}{2} = \frac{(25)^2}{2} + A$

$(50)^3 = \frac{(25)^2}{2} + A \qquad 125000 = 312.5 + A \qquad A = 124,687.5$

$\frac{50s^2}{2} = \frac{v^2}{2} + 124,687.5$

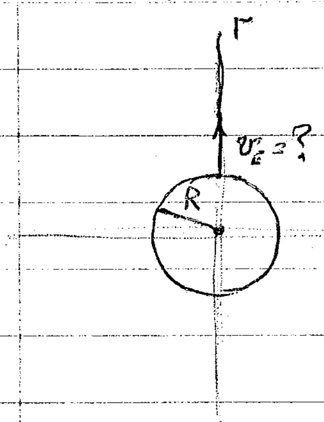
$v^2 = 100s^2 - (124,687.5)2$

$v = \sqrt{2(50s^2 - 124,687.5)}$

$\frac{ds}{dt} = \sqrt{100s^2 - 249,375.0}$

$\int dt = \int \frac{ds}{\sqrt{100s^2 - 249,375}} + B$

$t = 10\int \frac{ds}{\sqrt{s^2 - 249\tfrac{3}{8}}}$

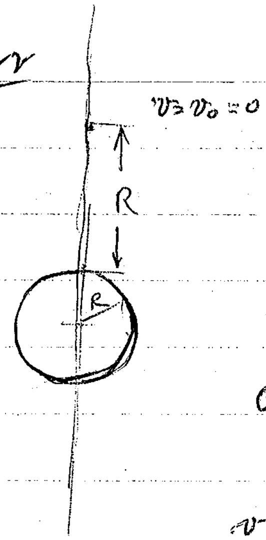


$a = -g\frac{R^2}{r^2} \qquad v\frac{dv}{dr} = -g\frac{R^2}{r^2}$

$\frac{v^2}{2}\Big|_{v_E}^{0} = +g\frac{R^2}{r}\Big|_{R}^{\infty}$

$-\frac{v_E^2}{2} = -gR \qquad v_E = \sqrt{2gR}$

11.22



(a) $v = ?$ @ earths surface

(b) cal time of fall

$R = 3960$ mil $\quad g = 32.2$ ft/sec^2

$$a = -g\frac{R^2}{r^2}$$

$$a = v\frac{dv}{dr} = -g\frac{R^2}{r^2}$$

$$v\,dv = -g\frac{R^2}{r^2}\,dr$$

$$\frac{v^2}{2}\Bigg]_{v_{ER}}^{0} = +g\frac{R^2}{r}\Bigg]_{R}^{2R} \qquad -\frac{v_{ER}^2}{2} = \frac{gR^2}{2R} - \frac{gR^2}{R}$$

$$-\frac{v_{ER}^2}{2} = -\frac{gR^2}{2R} = -\frac{gR}{2}$$

$$+v = \sqrt{gR}$$

time of fall

$$v = dr/dt \qquad dt = \frac{dr}{\sqrt{\frac{2gR^2}{r} - gR}} \qquad \text{let } z^2 = r$$

$$2z\,dz = dr$$

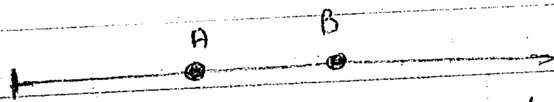
$$dt = \frac{2z\,dz}{\sqrt{\frac{2gR^2}{z^2} - gR}} \Rightarrow dt = \frac{2z^2\,dz}{\sqrt{2gR^2 - gRz^2}}$$

$$dt = \frac{2z^2\,dz}{\sqrt{gR}\sqrt{2R - z^2}} \qquad dt\Big|_0^t = \frac{2}{\sqrt{gR}}\left\{-\frac{z}{2}\sqrt{2R - z^2} + R\sin^{-1}\frac{z}{\sqrt{2R}}\right\}_{\sqrt{2R}}^{\sqrt{R}}$$

$$= \sqrt{\frac{R}{2}}\left(1 + \frac{\pi}{2}\right)$$

$= 2075$ seconds $= 34.5$ min

Motion of Several Particles



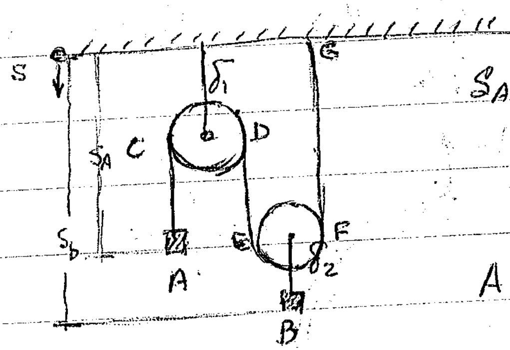
definition

for same
rel of $s_{B/A} = s_B - s_A$

both respect to A $\qquad s_B = s_A + s_{B/A}$

$v_B = v_A + v_{B/A}$ definition

$a_B = a_A + a_{B/A}$ definition



$S_A + 2S_B = \text{Const.}$ ∴ one degree of freedom

length of rope is constant $= L$

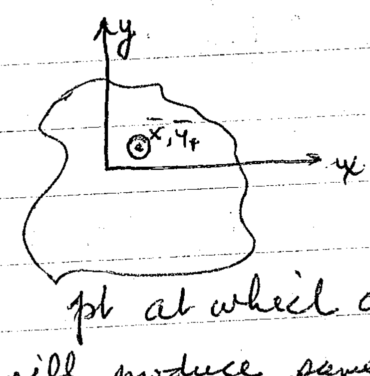
$AC + CD + DE + EF + FG = L$

$AC = S_A - \delta_1 \qquad DE = \delta_1 + \delta_2 + S_B$

$S_A + 2S_B = \text{Constant} \qquad S_A - \delta_1 + CD + S_B - \delta_1 - \delta_2 + EF + S_B - \delta_2 = L$

Def.

① Centroids, ② Trusses, ③ Frames, ④ Friction, ⑤ Virtual Work - Potential Stab. ⑥ Internal forces & moments

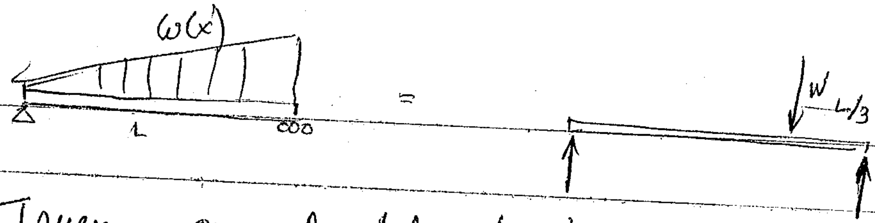


$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} \; ; \; \bar{y} = \frac{\int_A y\,dA}{\int_A dA}$$

pt at which a concentration of line, Area, volume which will produce same moment about the axes.

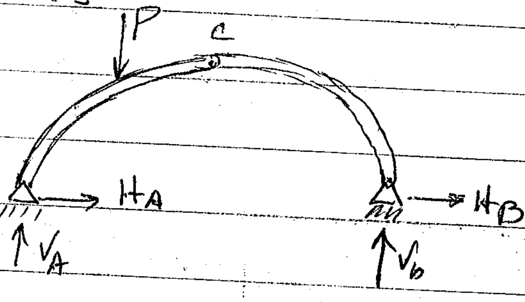
rectangle $\bar{x} = x$, $\bar{y} = y/2$ $\qquad$ triangle $\bar{x} = 1/3$ up from base, $\bar{y} = ?$

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum y_i A_i}{\sum y_i}$$

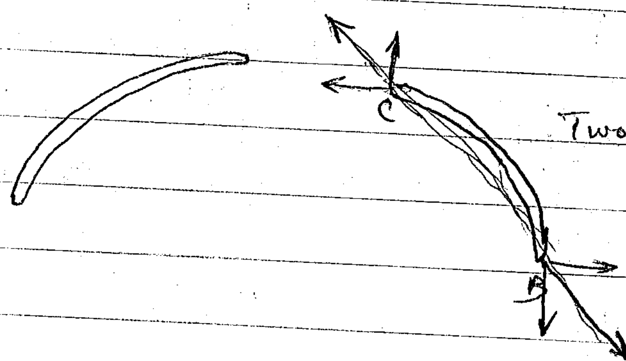


Trusses - can only take tension or compression, perfect pins at each end. (2 force member)

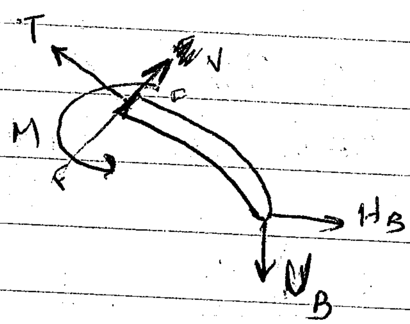
Frame - rigid body that can take transverse loads.



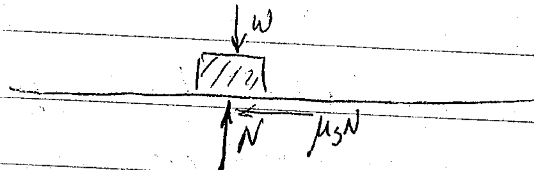
$$V_b + V_A = P$$
$$H_A + H_B = 0$$



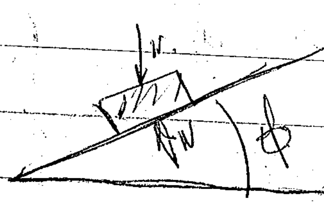
Two force.



$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_B = 0$$

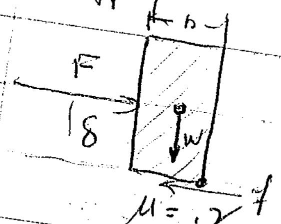


$$f_{max} = \mu_s N$$

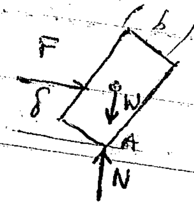


$$\mu_s = \tan\phi$$

Tipping



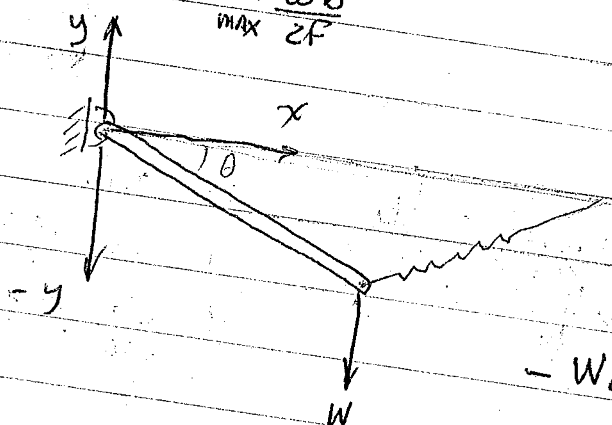
$F < \mu N < \mu W$



$M_A = F\delta - \frac{wb}{2}$

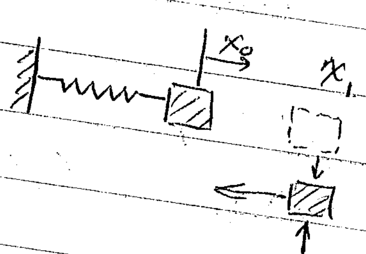
will start to tip when $M_A = 0$ if $F\delta > \frac{wb}{2}$ Incipient tipping

$\delta_{MAX} = \frac{wb}{2F}$



$\delta U = 0$

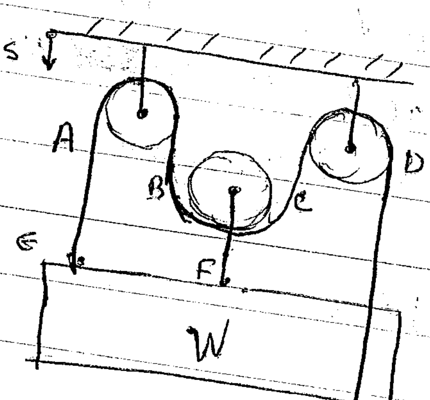
$-W\delta y_c$



$\delta U = -F\delta x_A$

$F = k(x_1 - x_0)$

11.36



$S_E + 2S_F + S_P =$ constant

$v_E + 2v_F + v_P = 0$ ①

$V_E = V_F = 12$ ft/sec

Using ① $3(12) + V_P = 0$

$V_P = -36$ ft/sec $= V_D$

$V_C = -V_D = 36$ ft/sec

$V_E = V_A = -V_B = 12$ ft/sec

$v_{P/W} = v_P - v_W$

$-36 - 12 = -48$ ft/sec

$v_{B/C} = v_B - v_C = -12 - 36$ ft/sec $= -48$ ft/sec

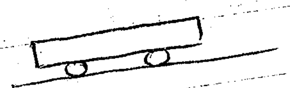
$s=0$ $\quad$ $s=600$, $t=20$ $\quad$ $a = 1.5$ ft/sec^2 unif

$v_0 = ?$

$v(20) = ?$

$s(10) = ?$

11.24



$a = \dfrac{dv}{dt}$ $\quad$ $\cancel{1.5 \int dt = \int dv}$ $\quad$ $\cancel{1.5t = v}$

$dv = a\,dt$

$v = at + C_1$ (1)

$v = \dfrac{ds}{dt} = at + C_1$

$s = \dfrac{at^2}{2} + C_1 t + C_2$ (2) $\quad$ at $t=0$ $\quad s=0$

$\therefore \quad C_2 = 0$

$s(20) = 600 = \dfrac{1.5(20)^2}{2} + 20C_1$ $\qquad C_1 = 15$

$600 = 300 + 20C_1$

$\therefore \quad s = \dfrac{at^2}{2} + 15t$

$v = at + 15$ $\quad$ at $t=0$ $\quad v=15$

at $t=20$ $\quad v=$ [redacted] 45

$s(10) = \dfrac{(1.5)(100)}{2} + 150 = 225$ ft/sec

11.8, 10, 14, 34 $\;\|\;$ 11.82, 11.94

DEPARTMENT OF [illegible] AND APPLIED MECHANICS

AE 100 Examination No. 3 JANUARY 9, 1969

The tangential and normal components of velocity and acceleration are

$$\vec{v} = v\vec{i}_t \qquad , \qquad \vec{a} = \dot{v}\vec{i}_t + \frac{v^2}{\rho}\vec{i}_n$$

The radial and transverse components of velocity and acceleration are

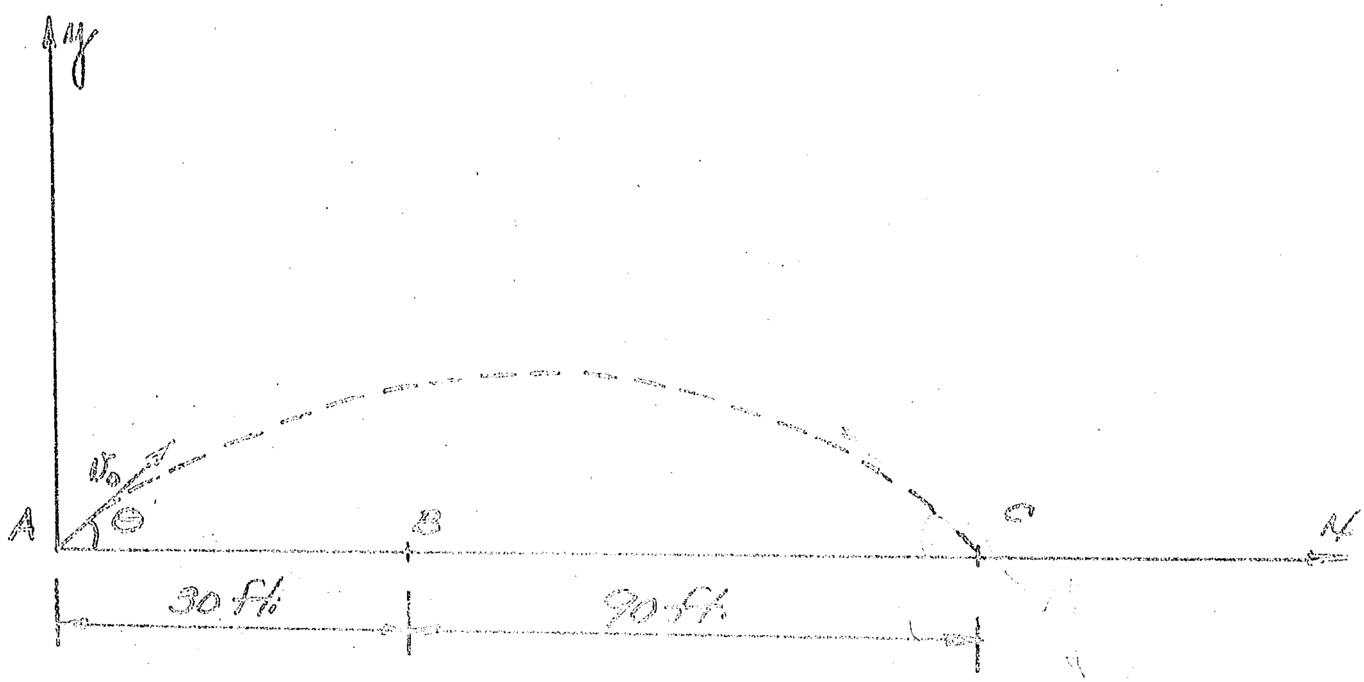
$$\vec{v} = \dot{r}\vec{i}_R + r\dot{\theta}\vec{i}_\theta \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{i}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\vec{i}_\theta$$

<u>Instructions</u>

Answer questions 1, 2 and 3.	3
Answer <u>either</u> question 4 or 5.	1
Answer <u>either</u> question 6 or 7.	1
	5 questions total

1. a) A quarterback, located at pt. A, throws a pass to a receiver who is running straight ahead at pt. B at a speed of 30 ft/sec when the ball is released. If the receiver catches the ball at pt. C, calculate v_0, θ and the time the ball remains in the air. (20)

b) Calculate the velocity of the ball relative to the receiver at pt. C. (ie: an instant before the receiver catches the ball.) (5)



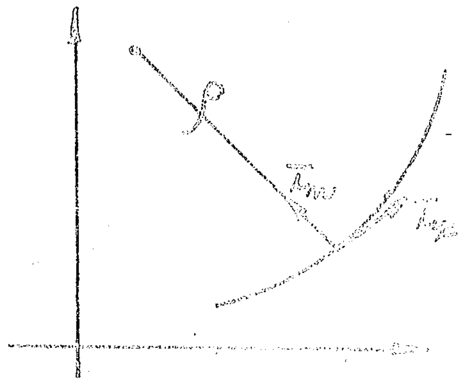
2. a)

Given

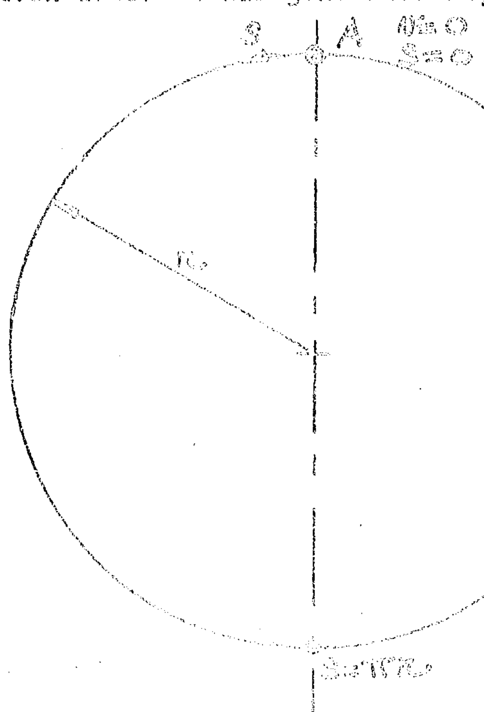
$$\bar{1}_n = \frac{d\bar{1}_t}{d\theta} \quad \text{and} \quad \frac{1}{\rho} = \frac{d\theta}{ds}$$

Show that the acceleration in tangential and normal components (path coordinates) is

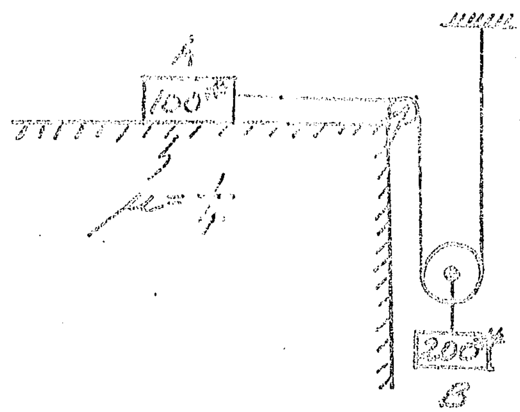
$$\bar{a} = \frac{dv}{dt}\,\bar{1}_t + \frac{v^2}{\rho}\,\bar{1}_n \qquad (5)$$



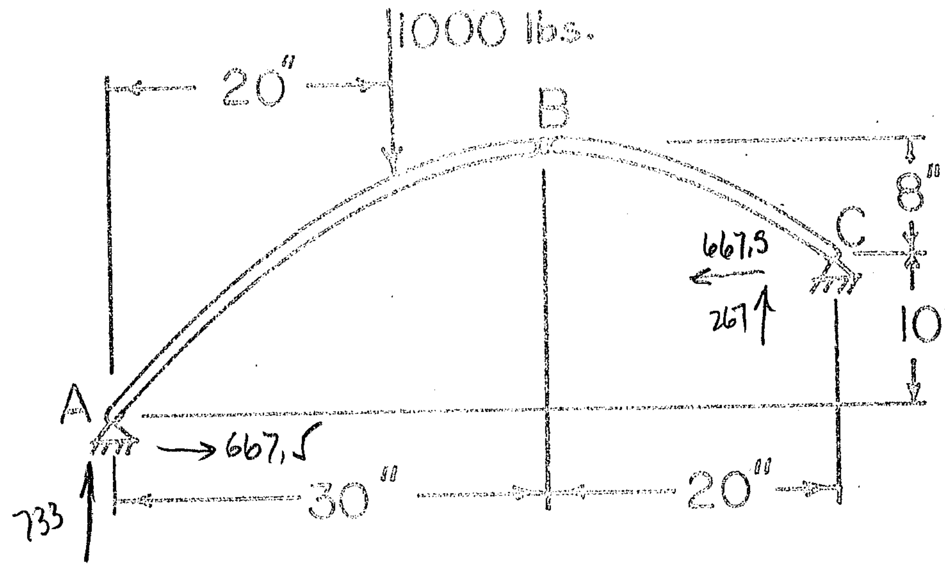
b) A particle, travelling around a circular path of radius r, starts from rest at position A and accelerates such that $a_t = 4s$ ft/sec^2. Find the speed of the particle and the magnitude of its total acceleration after it has gone half way around the circle($s=\pi r$). (15)



3. The two blocks shown start from rest. The coefficient of friction between block A and the plane is 1/4, and the pulleys are weightless and frictionless. Determine the acceleration of each block and the tension in the cord. (20)



4. Compute the reactions at A & C for frame loaded as shown. (20)



$C_x = D_x$

$C_y = D_y$

$C_x(8) + C_y(20) = 0$

$C_x = A_{\cancel{y}}$

$C_y - 1000 - A_y = 0$

$-20(1000) + C_y(30) - C_x(18) = 0$

$\sqrt{29}$, 2, 5

$C_x = \frac{5}{\sqrt{29}} C$

$C_y = \frac{2}{\sqrt{29}} C$

$\frac{5}{\sqrt{29}}(8)C + \frac{2}{\sqrt{29}}(20)C = 0$

$C_x = -C_y\left(\frac{20}{8}\right)$ (20)

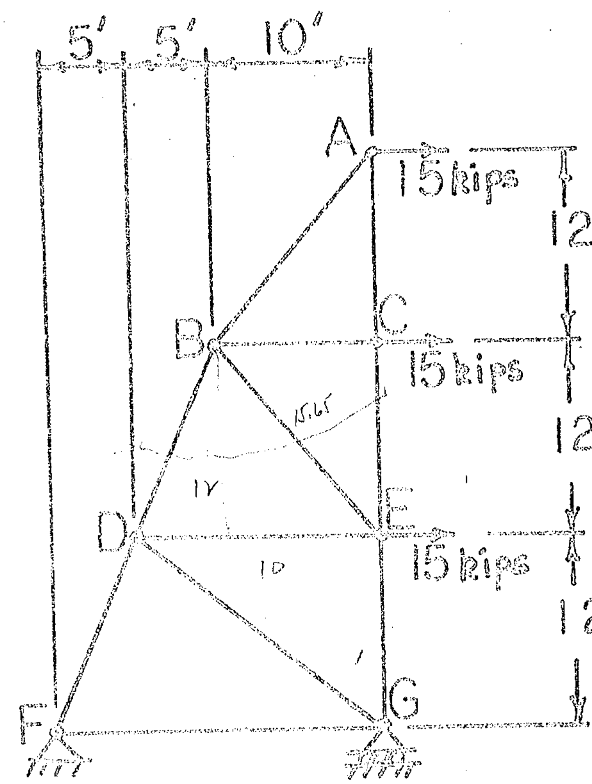
5. Find the forces in Bars BD, BE, CE of the truss shown.

$-20000 + C_y(30) + C_y\left(\frac{20}{8}\right)(18) = 0$

$20000 = C_y(30 + 45)$

$\frac{20000}{75} = C_y$

$2' = C_y$



$75 \overline{)20000}$ 267

$267 = C_y$

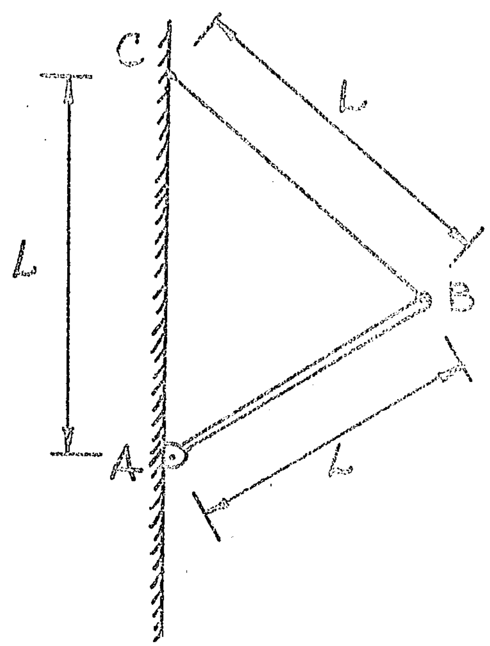
$C_x = -267\left(\frac{5}{2}\right)$

$= -667.5$

CHOOSE EITHER PROBLEM 6 OR 7

6. A uniform bar of length L and weight W is hinged at A and supported at B by cable CB. Find the reaction at A and the tension in the cable

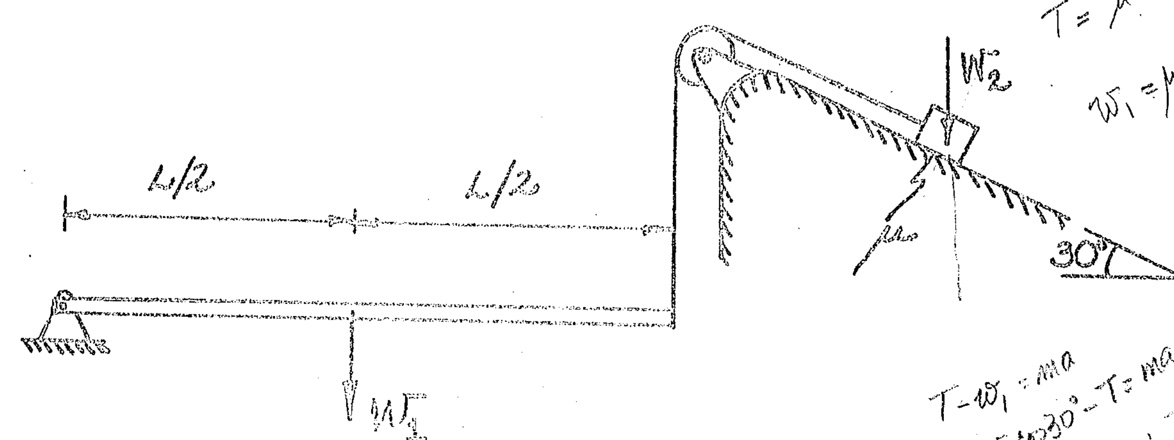
(15)



7. A beam of length L is supported as shown. Find the maximum weight of the beam W_1 (given L, μ, W_2) for equilibrium. (15)

$T = \mu W \cos 30^\circ$

$W_1 = \mu W_2 \cos 30^\circ$



$T - W_1 = ma$

$-\mu W_2 \cos 30^\circ - T = ma$

$a = 0 \therefore T = W_1$

$-\mu W_2 \cos 30^\circ = W_1$ no!

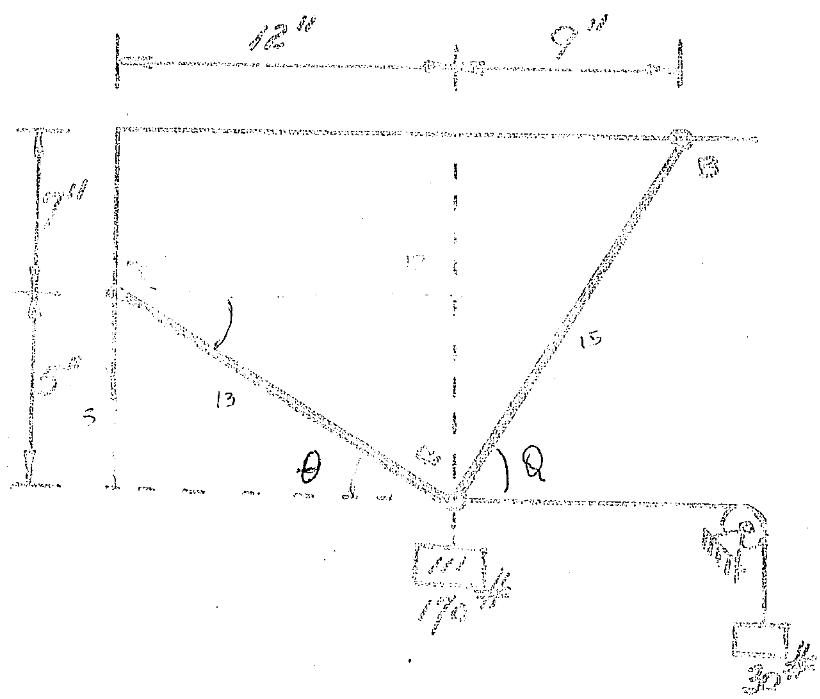
$ma + W_1 = T$

$-\mu W_2 \cos 30^\circ - ma - W_1 = ma$

DEPARTMENT OF AEROSPACE ENGINEERING AND APPLIED MECHANICS

AE 100 A, A2 EXAM NO. 1 OCTOBER 21, 1969

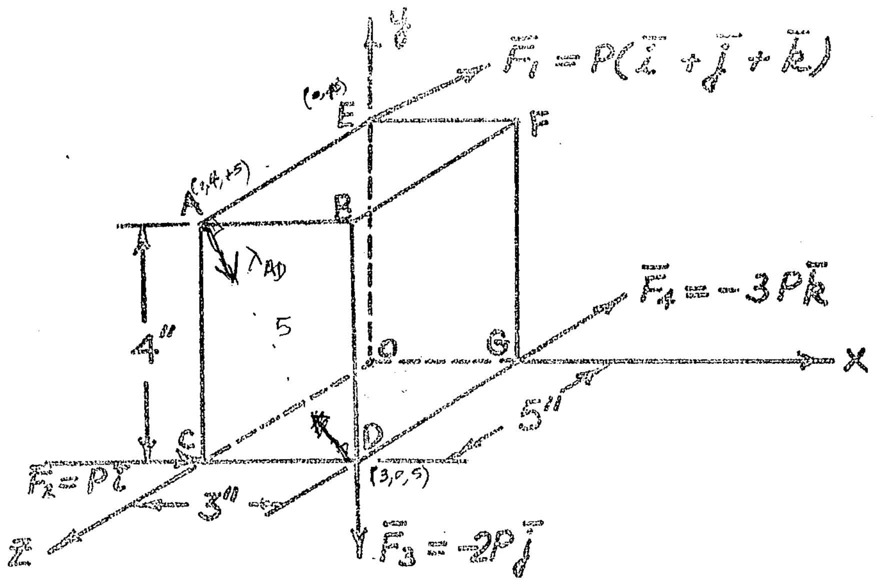
1. (a) Define a Vector (3)
   (b) The system shown below is in equilibrium. Find the tensions in cables AC and BC. (12)



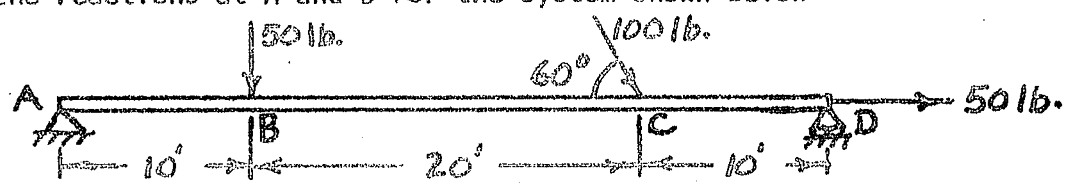
2. (a) Show that the scaler product obeys the distributive law. (6)
   (b) State the Principle of Transmissibility. (3)
   (c) Prove that the moment of a couple is a free Vector. (6)

3. (a) Find the magnitude of the moment of $\bar{F}_1$ about axis AD in the figure shown below. (10)
   (b) Replace the system of forces $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$, and $\bar{F}_4$ by an equivalent system at the origin. (10)

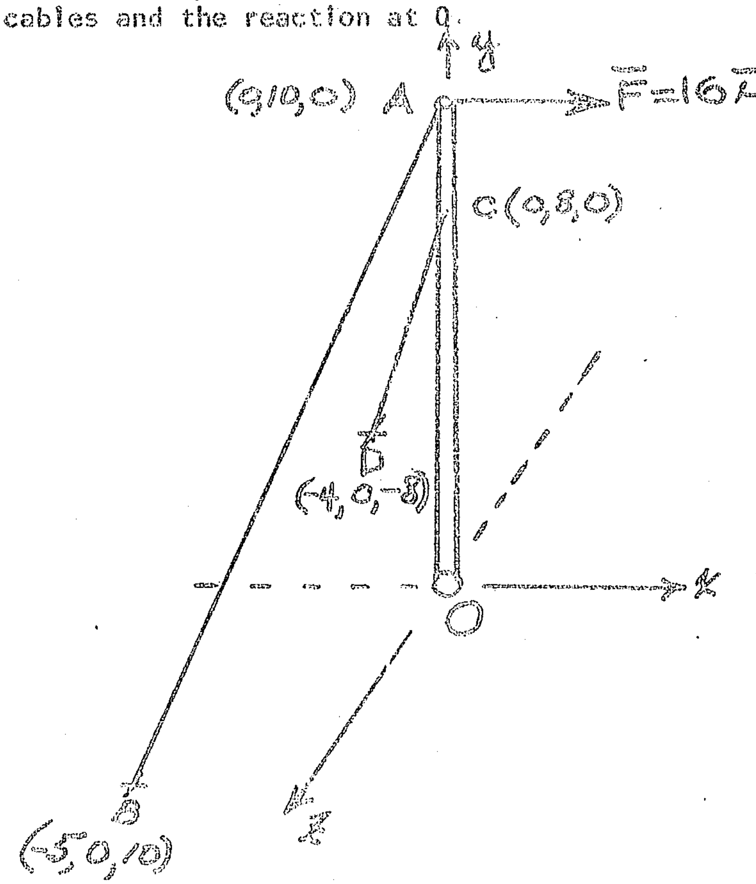
(c) Can the system be reduced to a single force? Explain. (5)



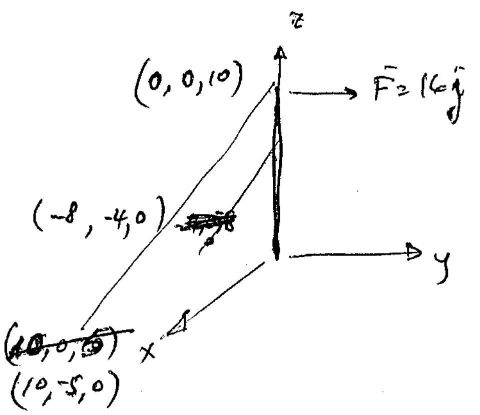
4. Find the reactions at A and D for the system shown below (15)

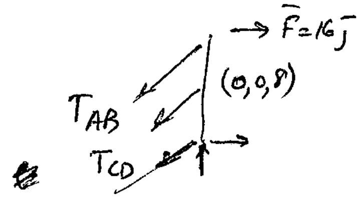


5. The vertical pole shown below is subjected to a horizontal force of 16 lb parallel to the X axis. The cables AB and CD and the ball and socket joint at O hold the pole in equilibrium. Determine the tensile forces in the two cables and the reaction at O. (30)



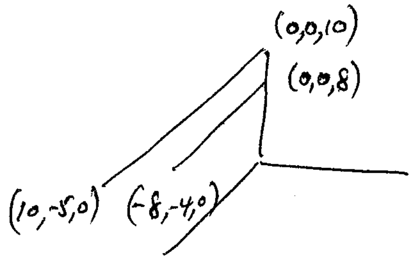
10:46 - 10:57 <u>11 min</u>





$\cancel{\bar{T}_{CD} = \frac{(10, -5, -8)}{17} T_{CD}}$

$\cancel{\bar{T}_{AB}}$



$$\bar{T}_{AB} = \frac{(10, -5, -10)}{15} T_{AB} = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right) T_{AB}$$

$$\bar{T}_{CD} = \frac{(-8, -4, -8)}{12} T_{CD} = \left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right) T_{CD}.$$

$$\Sigma F_x = \frac{2}{3} T_{AB} - \frac{2}{3} T_{CD} + X = 0$$

$$-\frac{1}{3} T_{AB} - \frac{1}{3} T_{CD} + Y + 16 = 0$$

$$-\frac{2}{3} T_{AB} - \frac{2}{3} T_{CD} + Z = 0$$

$$\bar{\Sigma} M_0 = \left(8\bar{k} \times \left(-\frac{2}{3}\bar{i} - \frac{1}{3}\bar{j} - \frac{2}{3}\bar{k}\right) T_{CD}\right) + \left(10\bar{k} \times \left(\frac{2}{3}\bar{i} - \frac{1}{3}\bar{j} - \frac{2}{3}\bar{k}\right) T_{AB}\right) = + 16\bar{j} \times 10\bar{k} = 0$$

$$= \left(-\frac{16}{3}\bar{j} + \frac{8}{3}\bar{i}\right) T_{CD} + \left(\frac{20}{3}\bar{j} + \frac{10}{3}\bar{i}\right) T_{AB} + 160\bar{i} = 0$$

$$-\frac{16}{3} T_{CD} + \frac{20}{3} T_{AB} = 0$$

$$\frac{8}{3} T_{CD} + \frac{10}{3} T_{AB} \mp 160 = 0$$

$$\frac{4}{5} T_{CD} = T_{AB}$$

$$\frac{8}{3} T_{CD} + \frac{\cancel{40}\,8}{\cancel{15}\,3} T_{CD} \mp 160 = 0$$

$$\frac{16}{3} T_{CD} \mp 160 = 0 \qquad T_{CD} = +30\,lb$$

$$T_{AB} = +24\,lb.$$

$-16 + 20 + X = 0 \qquad X = +4\,lb$

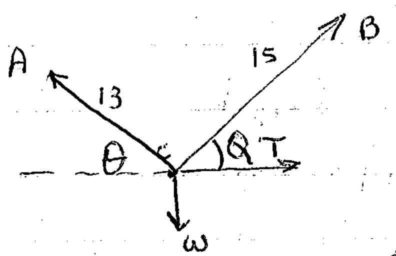
$\mp 8 \mp 10 + Y + 16 = 0 \qquad Y = \cancel{-34\,lb}$ 2lb.

$+16 + 20 + Z = 0 \qquad Z = +36\,lb.$

obeys parallelogram law of addition

1. a A vector is a system that has magnitude and direction

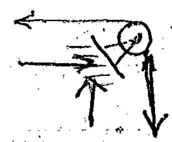
b. ~~T_{AC}~~ =

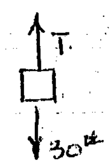


$\omega = 170^{\#}$

$T = 30^{\#}$

14





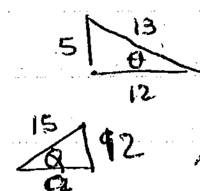
F_y $\qquad$ F_x

$\overline{T}_{AC} = +T_{AC} \sin\theta + T_{AC} \cos\theta$

$\overline{T}_{CB} = T_{BC} \sin Q + T_{BC} \cos Q$

$T = 30 \rightarrow$

$W = 170 \downarrow$



$\sin\theta = 5/13$

$\cos\theta = 12/13$

$\sin Q = 4/5$

$\cos Q = 3/5$

$\Sigma F_y = 0 \qquad T_{AC} \sin\theta + T_{BC} \sin Q - 170 = 0$

$\Sigma F_x = 0 \qquad -T_{AC} \cos\theta + T_{AC} \cos Q + 30 = 0$

$12\left(\frac{5}{13} T_{AC} + \frac{4}{5} T_{BC} = 170\right)$

$5\left(-\frac{12}{13} T_{AC} + \frac{3}{5} T_{BC} = -30\right)$

$\frac{60}{13} T_{AC} + \frac{48}{5} T_{BC} = 170(12)$

$-\frac{60}{13} T_{AC} + \frac{15}{5} T_{BC} = -150$

$\frac{63}{5} T_{BC} = 1890 \qquad T_{BC} = \frac{5(\cancel{1890})}{\cancel{63}} = 150^{\#}$

30

2040
− 150
170(12) 1890

$|\overline{T}_{BC}| = 150 \text{ lb} \qquad |\overline{T}_{AC}| = 130 \text{ lb}$

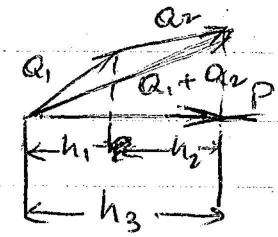
2 ~~$P\cdot(Q_1+Q_2)$~~

~~Let $V = P\cdot(Q_1+Q_2) - P\cdot Q_1 - P\cdot Q_2$~~

~~$\bar{V}\cdot\bar{V} = (\bar{V}\cdot\bar{P})(\bar{Q}_1+\bar{Q}_2) - \bar{V}\cdot P$~~

9

Prove $\bar{P}\cdot(\bar{Q}_1+\bar{Q}_2) = \bar{P}\cdot\bar{Q}_1 + \bar{P}\cdot Q_2$



let $\bar{P} = P_x i + P_y j + P_z k$

$\bar{Q}_1 = Q_{1x} i + Q_{1y} j + Q_{1z} k$

$\bar{Q}_2 = Q_{2x} i + Q_{2y} j + Q_{2z} k$

$\bar{P}\cdot\bar{Q}_1 = P_x Q_{1x} + P_y Q_{1y} + P_z Q_{1z}$ not valid step

$\bar{P}\cdot\bar{Q}_2 = P_x Q_{2x} + P_y Q_{2y} + P_z Q_{2z}$

$\bar{P}\cdot\bar{Q}_1 + \bar{P}\cdot\bar{Q}_2 = P_x(Q_{1x}+Q_{2x}) + P_y(Q_{1y}+Q_{2y}) + P_z(Q_{1z}+Q_{2z})$

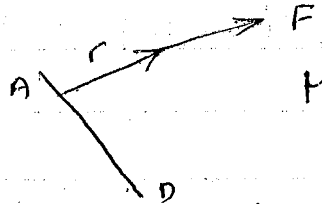
but these are the components of $\bar{P}\cdot(\bar{Q}_1+\bar{Q}_2)$ in the x, y, z direct. $\therefore$ $\bar{P}\cdot(\bar{Q}_1+\bar{Q}_2) = \bar{P}\cdot\bar{Q}_1 + \bar{P}\cdot\bar{Q}_2$

Q.E.D.

b. The principle of transmissibility – a vector can be displaced along its line of action if and only if the motion of the object remains as it was before the vector was shifted.

c. the moment of a couple depends only on the Force and the perpendicular distance between the forces. If the couple is shifted the $\perp$ distance bet. the couples remains the same and so does the Force $\therefore$ shifting the moment of a couple will not change the motion of the object, with respect to where the momet couple was before. Should have been shown mathematically!

3. $\overline{F_1} = P(i+j+k) \qquad \overline{r_1} = -5k + 0j$



$$M_A = (r \times F) = \begin{vmatrix} i & j & k \\ 0 & 0 & -5 \\ P & P & P \end{vmatrix} =$$

$$i\begin{vmatrix} 0 & -5 \\ P & P \end{vmatrix} = 5Pi$$

$$-j\begin{vmatrix} 0 & -5 \\ P & P \end{vmatrix} = -5Pj$$

$$k\begin{vmatrix} 0 & 0 \\ P & P \end{vmatrix} = 0$$

$$M_{AD} = \lambda_{AD} \cdot (\overline{r} \times \overline{F})$$

$$\overline{r} \times \overline{F} = 5P(i-j)$$

$$\lambda_{AD} = \frac{3i - 4j + 0k}{\sqrt{25}} = \frac{3i-4j}{5}$$

20

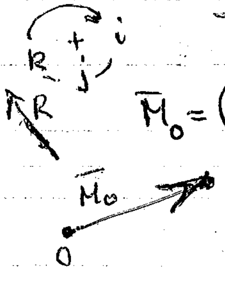
$$M_{AD} = \lambda \cdot (\overline{r} \times \overline{F}) = \left(\frac{3i-4j}{5}\right)\left(5P(i-j)\right) = 3P + 4P = 7P$$

$$P(3i-4j)(i-j)$$

$$\overline{M_{AD}} = 7P$$

10

b. $R = \overline{F_1} + \overline{F_2} + \overline{F_3} + \overline{F_4}$
$= Pi + Pj + Pk + Pi - 2Pj - 3Pk$
$= 2Pi - Pj - 2Pk \qquad \boxed{P(2i - j - 2k) = R}$



$r_1 = 4i \qquad r_2 = 5k \qquad r_3 = 3i + 5k \qquad r_4 = 3i$

$$\overline{M_o} = (\overline{r_1} \times \overline{F_1}) + (\overline{r_2} \times \overline{F_2}) + (\overline{r_3} \times \overline{F_3}) + (\overline{r_4} \times \overline{F_4})$$

$14Pi + 14Pj - 10Pk$

$$\overline{r_1} \times \overline{F_1} = (4i) \times (Pi + Pj + Pk) = 4Pk - 4Pj$$

$$\overline{r_2} \times \overline{F_2} = 5k \times Pi = +5Pj$$

$$\overline{r_3} \times \overline{F_3} = (3i + 5k) \times (-2Pj) = -6Pk + 10Pi$$

$$\overline{r_4} \times \overline{F_4} = 3i \times (-3Pk) = 9Pj$$

wrong

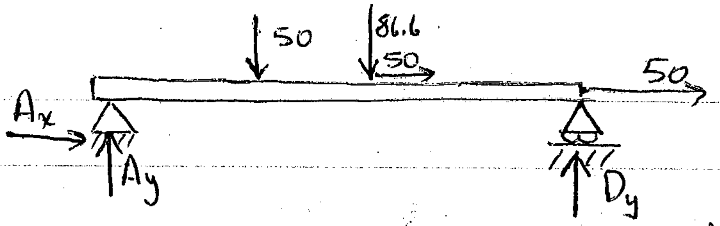
$|V|$ 8

$$\overline{M_o} = 4Pk - 4Pj + 5Pj - 6Pk + 10Pi + 9Pj = 10Pi + 10Pj - 2Pk$$

∴ ~~we can replace the whole syste with $M_o = 10Pi + 4Pj - 7Pk$ and Resultant at~~

we can replace the syste by an equiv syste at O whre $\overline{M_o} = P(10i + 10j - 2k)$

Part C after quest. 4



$\Sigma F_x = 0 \qquad A_x + 50 + 50 = 0 \qquad A_x = 2(-50\,lb) = -100^{\#}$

$\Sigma F_y = 0 \qquad A_y - 50 - 86.6 + D_y = 0$

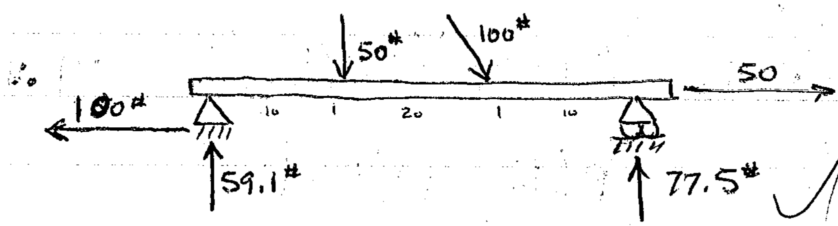
$\Sigma M_A = 0 \qquad 50(10) + 86.6(30) - 40 D_y = 0$

$500 + 2600$

$3100 = 40 D_y$

$\frac{310}{4} = D_y \qquad\qquad A_y - 136.6 + 77.5 = 0$

$77.5^{\#} = D_y \qquad\qquad A_y = 59.1^{\#}$

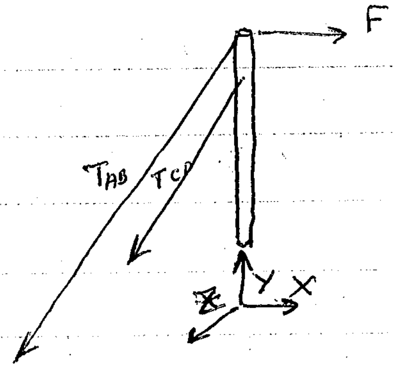


3c. ~~No No the moment about the origin is the single force since the forces are not concurrent they can't provide us with a resultant at the origin so that I can set up a resultant couple system. It doesn't obey Varignon's theorem and therefore cannot be manipulated in such a way that~~

Check if $M_o^R \perp R$ — Yes since $M_A^R = \bar{r} \times \bar{R}$ we can set up a Resultant couple at another point A' $\Rightarrow$ we have a force-moment system at the point, we can the take components of Force along and $\perp$ to M_o' and then manipulate this and find another point A'' where the Moment and the Force Vector lie on the same line of action.

if $M_o^R \not\perp R$ then no if

You did not read question carefully!



$\Sigma M_x = 0$
$\Sigma M_y = 0$
$\Sigma M_z = 0$
$\Sigma F_x = 0$
$\Sigma F_z = 0$
$\Sigma F_y = 0$

$$\overline{T}_{CD} = T_{CD}\overline{\lambda}_{CD} \quad ; \quad \overline{T}_{AB} = T\overline{\lambda}_{AB}$$

$$\Sigma \overline{R} = 0 \quad \therefore \quad \overline{T}_{CD} + \overline{T}_{AB} + \overline{F} + \overline{X} + \overline{Y} + \overline{Z} = 0$$

$$\overline{\lambda}_{CD} = \frac{-4i - 8j - 8k}{12} = \frac{-i - 2j - 2k}{3}$$

$$\overline{\lambda}_{AB} = \frac{-5i - 10j + 10k}{15} = \frac{-i - 2j + 2k}{3}$$

$$\Sigma F_x = 0 \quad \therefore \quad 16 - \frac{T_{CD}}{3} - \frac{T_{AB}}{3} + X = 0$$

$$\Sigma F_y = 0 \quad \therefore \quad -\frac{2T_{CD}}{3} - \frac{2T_{AB}}{3} + Y = 0$$

$$\Sigma F_z = 0 \quad \therefore \quad -\frac{2T_{CD}}{3} + \frac{2T_{AB}}{3} + Z = 0$$

$$\Sigma M_A = 0 \qquad \overline{r}_{CD} \times \overline{T}_{CD} + \overline{r}_x \times \overline{X} + \overline{r}_z \times \overline{Z} = 0$$

$$-2j = \overline{r}_{CD} \qquad \overline{r}_x = -10j \qquad \overline{r}_z = -10j$$

$\Sigma M_o = 0$ better

$$\therefore (-2j) \times \left(\frac{T_{CD}}{3}(-i - 2j - 2k)\right) + \cancel{(-10j) \times \left(\frac{T_{AD}}{3}(-i - 2j + 2k)\right)}$$

$$(-10j \times Xi) + (-10j \times Zk) = 0$$

$$-\frac{2T_{CD}}{3}(+k - 2i) + 10Xk - 10Zi = 0$$

$$+\frac{4T_{CD}}{3} - 10Z = 0 \qquad \frac{4T_{CD}}{30} = Z$$

$$-\frac{2T_{CD}}{3} + 10X = 0 \qquad \frac{2T_{CD}}{30} = X$$

sub for $\overline{X}$ & $\overline{Z}$.

$$\Sigma F_x = 16 - \frac{10T_{CD}}{30} - \frac{10T_{AB}}{30} + \frac{2T_{CD}}{30} = 0 \qquad + \frac{8T_{CD}}{30} + \frac{10T_{AB}}{30} = +16$$

$$\Sigma F_z = -\frac{2T_{CD}}{3} + \frac{2T_{AB}}{3} + \frac{4T_{CD}}{30} = 0 \qquad -\frac{16T_{CD}}{30} \; \frac{2T_{CD}}{30} + \frac{2T_{AB}}{30} = 0$$

$$T_{CD} = -T_{AB}$$

$$\therefore \; \frac{-8T_{AB} + 10T_{AB}}{30} = 16$$

$$2T_{AB} = 480$$

$$\boxed{\begin{aligned} T_{AB} &= 240^{\#} \\ T_{CD} &- 240^{\#} \\ \overline{X} &= -16^{\#} \\ \overline{Y} &= 0^{\#} \\ \overline{Z} &= -32^{\#} \end{aligned}}$$

Very good, only small arithmetical error.

$$-\frac{2T_{AB}}{3} - \frac{2T_{CD}}{3} + Y = 0$$

$$-\frac{2}{3}(T_{AB} + T_{CD}) + Y = 0$$

$$\cancel{-\frac{16}{30}T_{CD} + \frac{2T_{AB}}{30} = 0}$$

$$\cancel{\frac{16T_{CD}}{30} + \frac{10T_{AB}}{30} = 32}$$

$$\cancel{T_{AB} = \frac{32(30)}{22}} =$$

~~8TCD = 2TAB~~

28

$T_{AB} = 24$ $\quad X - 2$

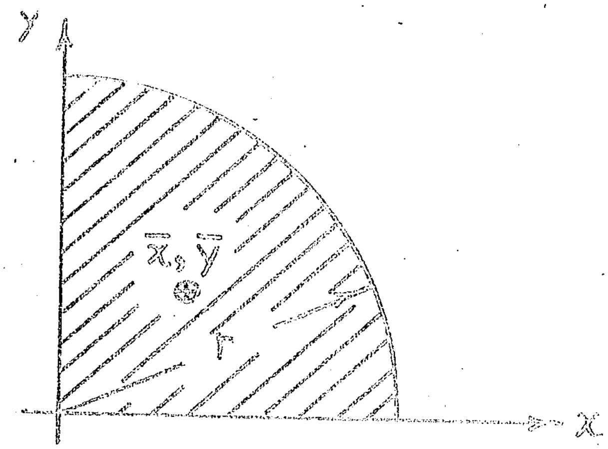
$T_{AC} = 30$ $\quad Y - 36$

$Z - 4$

DEPARTMENT OF AEROSPACE ENGINEERING AND APPLIED MECHANICS

AE 100 EXAM NO. 2 DECEMBER 4, 1969

1\. (a) Show by integration that the area and centroid of the quarter-circle shown below is given by

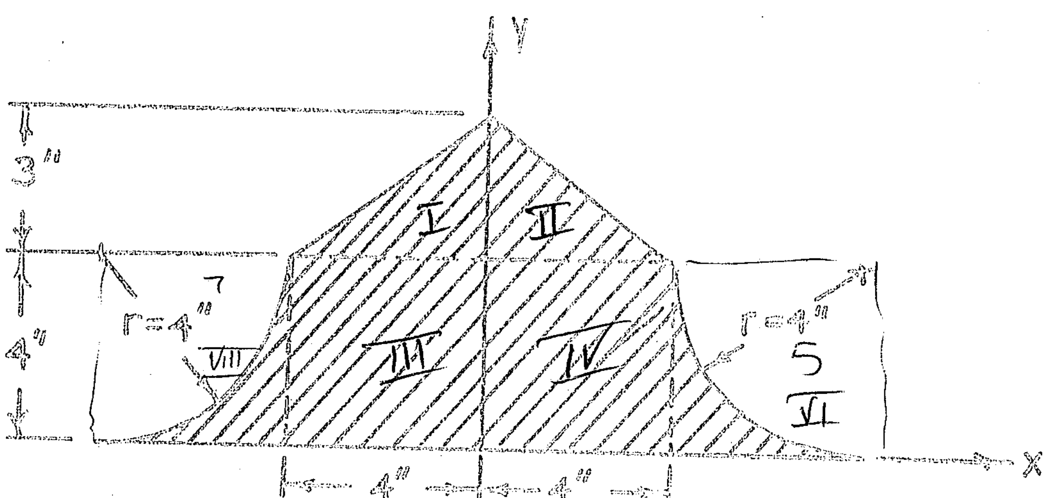


$$A = \frac{\pi r^2}{4}$$

$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

(12)

(b) Find the centroid of the composite area shown.

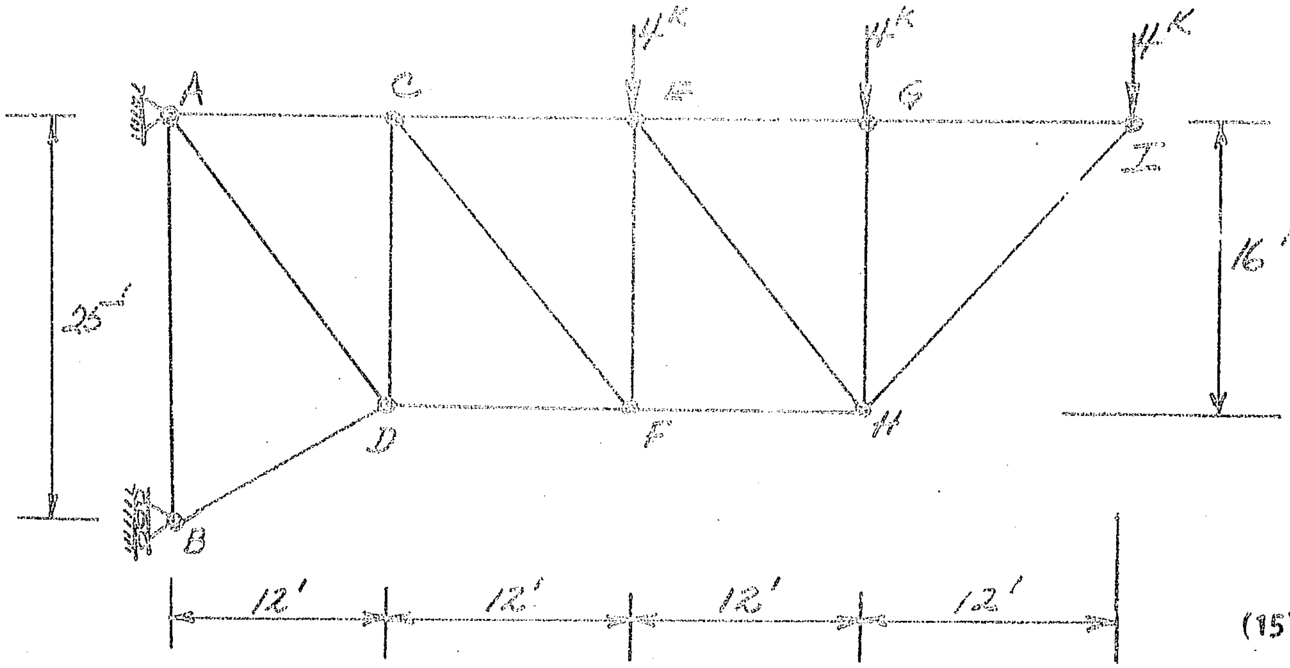


(13)

(c) Use the Theorem of Pappus-Guldinus to obtain the volume of the solid formed when the area of part (b) is rotated 360° about the X-axis.

(5)

2. Find the forces in bars GI, HI, EF and CE in the truss below. State whether the bar is in tension or compression.



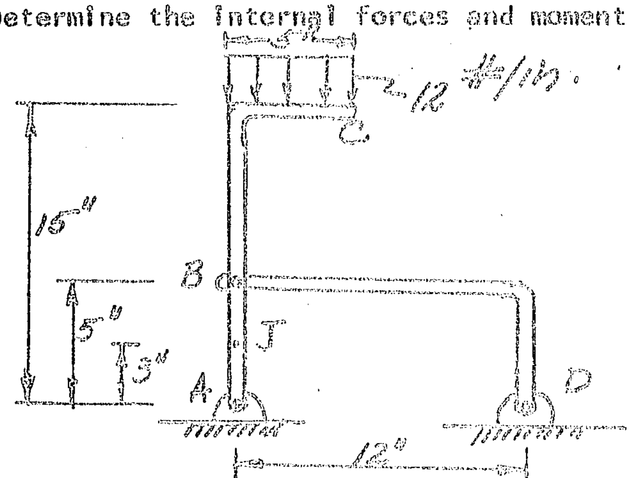
(15)

3. (a) Find the reactions at A and D in the frame shown.

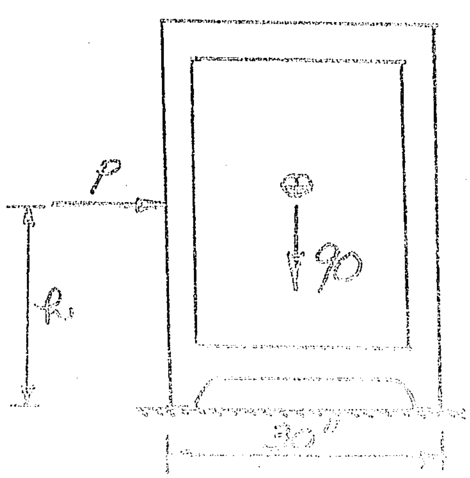
(15)

(b) Determine the internal forces and moment at point J.

(5)



4.

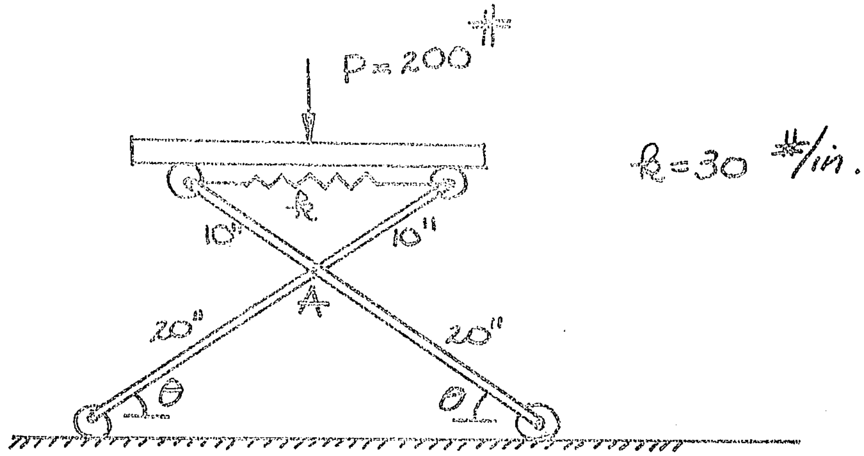


(a) Find the force P required to move the 90 lb cabinet shown to the right. The coefficient of friction μ=0.3.

(b) Find the largest value of h if the cabinet is not to tip over.

(10)

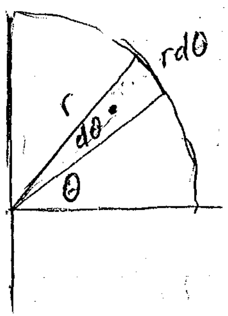
5.



(a) A student designs a chair for his instructor. Find the 2 equilibrium positions of the chair (assuming the instructor weighs 200 lbs.) and determine whether or not they are stable. Assume the Point A is constrained to move in a vertical path. The spring is unstretched when $\theta = \frac{\pi}{2}$. (22)

b) What is meant by a conservative force system? (3)

1.



$\bar{x}_\Delta = \frac{2}{3}r$ $\qquad$ $dA = \frac{1}{2}r^2 d\theta$

$x_\Delta = \frac{2}{3} r\cos\theta$ $\qquad$ $\bar{y} = \frac{2}{3} r \sin\theta$

$$\bar{x} = \frac{\int_0^{\pi/2} x\, dA}{\frac{\pi r^2}{4}} = \frac{\frac{1}{3}r^3 \int_0^{\pi/2} \cos\theta\, d\theta}{\pi r^2/4}$$

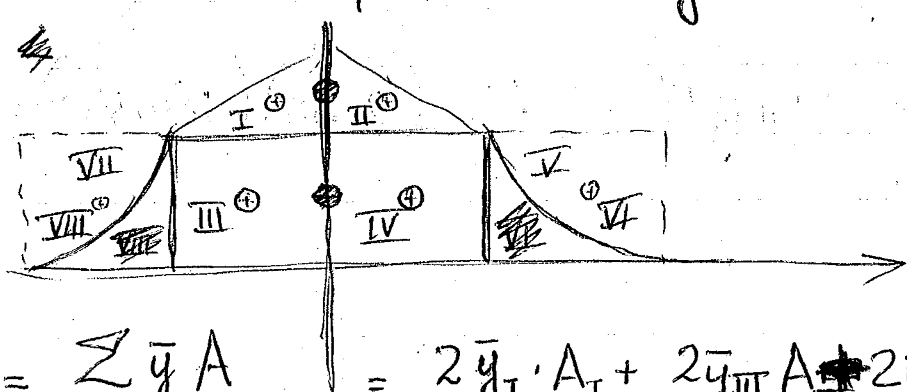
$$\frac{\frac{1}{3}r^3 \left(+\sin\theta\right]_0^{\pi/2}}{\frac{\pi r^2}{4}} = \frac{\frac{1}{3}r^3}{\frac{\pi r^2}{4}} = \frac{4r}{3\pi} = \bar{x}$$

$$\bar{y} = \frac{\int_0^{\pi/2} \frac{2}{3} r\sin\theta \cdot \frac{1}{2} r^2 d\theta}{\pi/4\, r^2} = \frac{\int y\, dA}{A}$$

$$\frac{\frac{1}{3}r^3 \left(-\cos\theta\right)_0^{\pi/2}}{\frac{\pi r^2}{4}} = \frac{\frac{1}{3}r^3}{\frac{\pi r^2}{4}} = \frac{4r}{3\pi} = \bar{y}$$

18

2. $\bar{x} = 0$ since symmetric about y axis



$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{2\bar{y}_I \cdot A_I + 2\bar{y}_{III} A_{III} + 2\bar{y}_V A_V - 2\bar{y}_{VI} A_{VI}}{2A_I + 2A_{III} + 2A_V - 2A_{VI}}$$

$\bar{y}_I = \bar{y}_{II} = 5''$ $\quad$ $\bar{y}_{III} = \bar{y}_{IV} = 2''$

$\bar{y}_V = \bar{y}_{VII} = 2''$ $\quad$ $\bar{y}_{VI} = \bar{y}_{VIII} = 4 - \frac{16}{3\pi}$

$A_I = A_{II} = 6^{\square}$ $\quad$ $A_{III} = A_{IV} = 16^{\square}$

$A_5 = A_7 = 16^{\square}$ $\quad$ $A_6 = A_8 = 4\pi$

$\cancel{\Sigma \bar{y} A = 2(5 \cdot 6) + 2(2 \cdot 16) + 2(2 \cdot 16) - 2\left(4 - \frac{16}{3\pi}\right)(4\pi)}$

$\cancel{\Sigma A = 12 + 64 \pm \frac{\pi}{2} = 77.17}$

after proble 5

$\cancel{\Sigma \bar{y} A = 60 + 64 + 64 - 2\left(\pi - \frac{4}{3}\right)}$

$\cancel{60 + 64 + 64 - 6.28 + 2.67 =}$

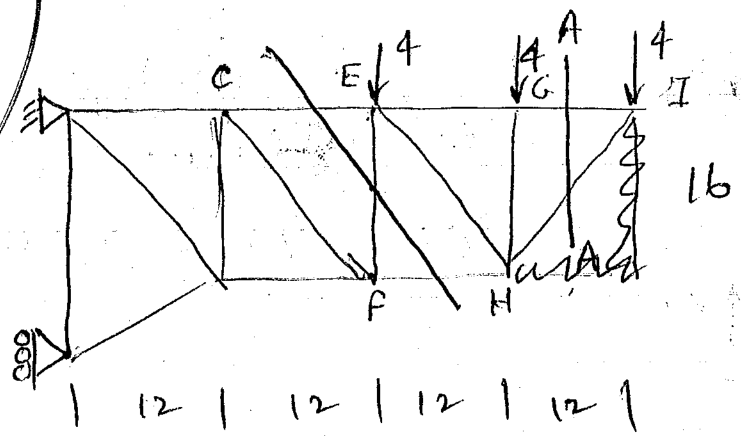
$\cancel{188 + 2.67 - 6.28 =}$ $\quad$ $\cancel{190.68 - 6.28 = 184.40}$

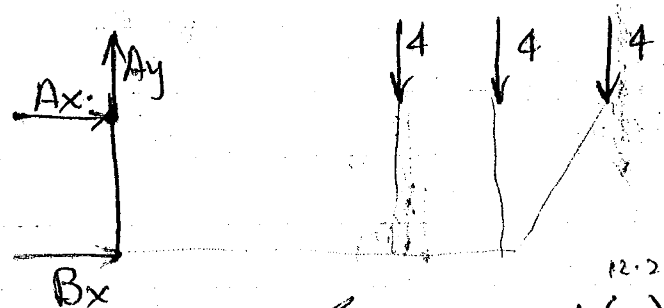
$\cancel{\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{184.40}{77.17} = 2.38''}$

$\cancel{\bar{y} = 2.38''}$

$\cancel{V = 2\pi \bar{y} A = 2\pi (2.38)(184.40) = 2760 \text{ cu in.}}$

(15)





$\Sigma F_y \quad A_y - 12 = 0$

$A_y = 12 \text{ kips}$

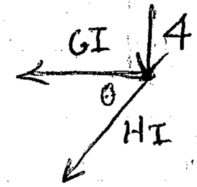
$A_x = -B_x$

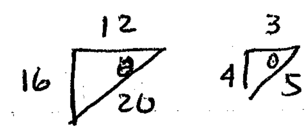
$$\Sigma M_A = -4(\overset{12\cdot 2}{24}) - 4(\overset{12\cdot 3}{36}) - 4(\overset{12\cdot 4}{48}) + B_x(25) = 0$$

$$12(-8-12-16) = -B_x(25)$$

$$12(36) = B_x(25) \qquad B_x = \frac{12}{25}(36) = 17.28 \text{ ki}$$

$A_x = -17.28$ kips





$\cos\theta = 3/5$
$\sin\theta = 4/5$

$\Sigma F_x = HI\cos\theta + GI = 0$

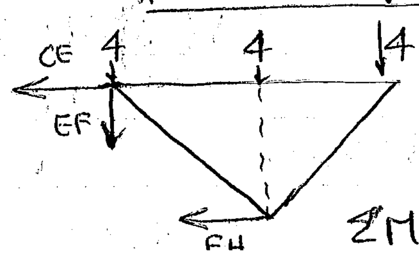
$\Sigma F_y = HI\sin\theta + 4 = 0$

$\frac{HI(4)}{5} = -4 \qquad HI = -5$ kips

$\boxed{HI = 5 \text{ kips in compression}}$

$-5\left(\frac{3}{5}\right) + GI = 0$

$\boxed{GI = 3 \text{kips in tension}}$



$\Sigma F_y \quad EF + 12 = 0$

$\boxed{EF = 12 \text{ kips in compression}}$

$\frac{144}{16} = 9$

$\Sigma M_F = -4(12) - 4(24) - FH(16) = 0$

FH = − 9 kips

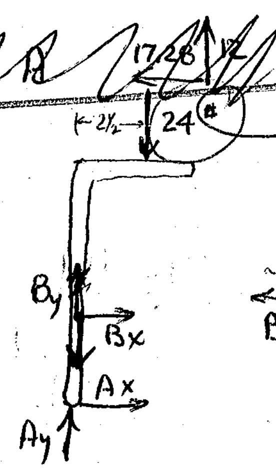
ΣF_y FH + CE = 0

FH = 9 kips compression

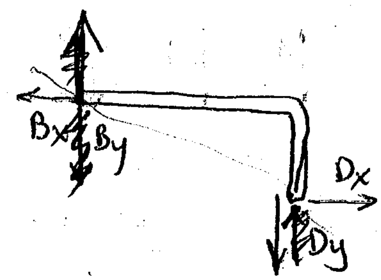
$\boxed{\cancel{CE = + \ \text{kips tension}}}$

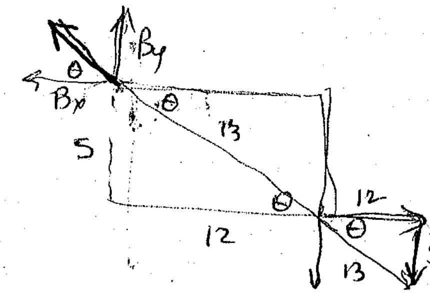
$\boxed{CE = 9K \text{ in tension}}$

3.



60# !!



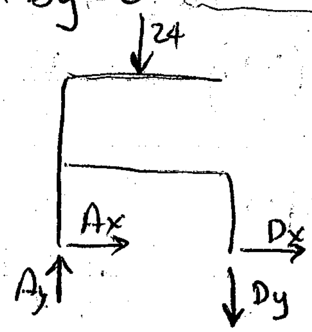


$Ay + By = 24$

$Ax + By = 0$

$By = B\sin\theta = B\frac{5}{13} = Dy$

$Bx = B\cos\theta = \frac{12}{13}B = Dx$



18

$\Sigma F_x \quad Ax + \frac{12}{13}D = 0$

$\Sigma F_y \quad Ay = 24 + \frac{5}{13}D$

$\Sigma M_A = (-60) - \left(\frac{5}{13}D\right)(12) = 0$

$-60 = 5D\left(\frac{12}{13}\right) \quad D = -13$

$-24(2.5) - Dy(12) = 0$

$-60 - \frac{60}{13}D = 0$

$D = -13^{\#}$

$D = 13^{\#}$

$D = +13^{\#}$

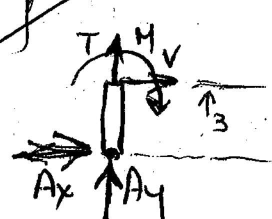
$Ax = +12^{\#}$?

$Ay = 19^{\#}$

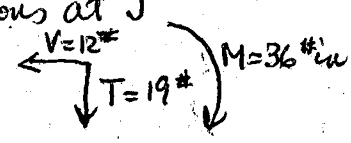
8K

for 24#

Part b



b Reactions at J



$V + Ax = 0 \quad \cancel{V = 12^{\#}} \quad V = -12^{\#}$

$T + Ay = 0 \quad T = -19$

$V(-3) - M = 0$

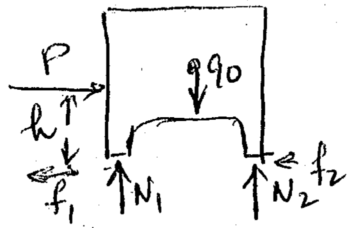
$\cancel{M = 36^{\#}\text{in}} \quad M = 36^{\#}\text{in}$

$-3V - M = 0$

$V = -12 \quad -3V = 36 - M = 0 \quad M = 36^{\#}\text{in}$

?" P

(10) 4.



$90^{\#} = N_1 + N_2$ ✓

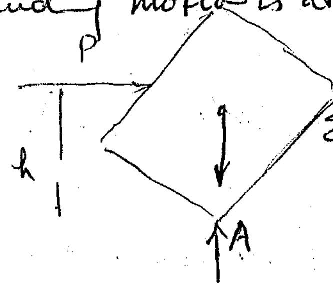
$f_1 + f_2 \leq P$

$f_1 + f_2 = \mu(N_1 + N_2) \leq P$

$.3(90) \leq P$

Impending motion is at $P = 27^{\#}$

$P \geq 27^{\#}$ ✓



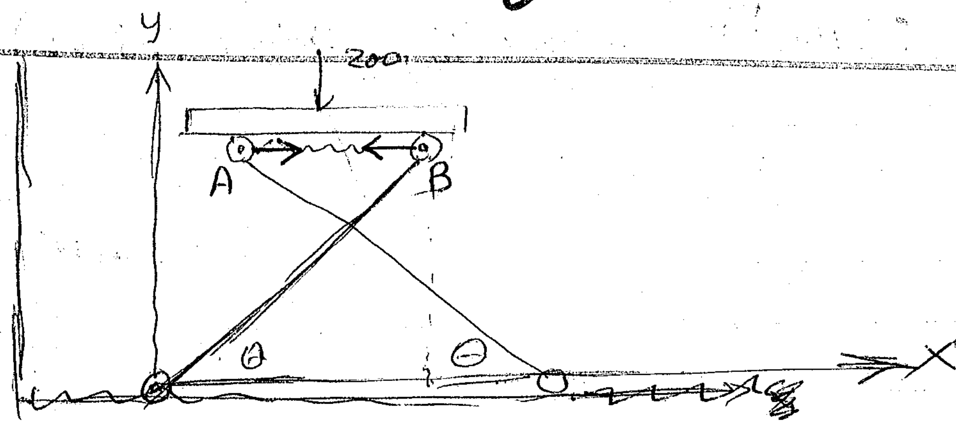
$\Sigma M_A \; -P \cdot h + 90(15) = 0$

$Ph = 90(15)$

$27h = 90(15) = (9.3)(10.5)$

✓ $h = \cancel{50^{\#}}\ 50''$

(17) 5.



$V = V_g + V_E \qquad V_g = 200\, y_P = 200(30 \sin\theta)$

$V_E = \cancel{\tfrac{1}{2}kx^2}\ \tfrac{1}{2}kx^2 \qquad X_B = 30\cos\theta$

This is V_E

$X_A = +(20\cos\theta - 10\cos\theta)$

$= +10\cos\theta$

$V = 200(30)\sin\theta + \cancel{\tfrac{1}{2}k(30\cos\theta)^2} - \tfrac{1}{2}k(10\cos\theta)^2$

$\frac{dV}{d\theta} = 6000\cos\theta - 450(30)\sin 2\theta + 50(30)\sin 2\theta = 0$

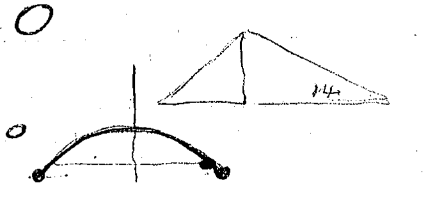
$200\cos\theta - 450\sin 2\theta + 50\sin \cancel{2\theta} = 0$

$200\cos\theta - 800\sin\theta\cos\theta = 0$

$\cos\theta = 0$ is also an equil pos

$200 = 800\sin\theta$

$1/4 = \sin\theta \qquad 14.5^\circ$



if $\frac{d^2V}{d\theta^2} > 0$ stable

$\frac{d^2V}{d\theta^2} < 0$ unstable

~~sin θ~~ $\theta = 14.5^\circ, 165.5^\circ$

$\frac{dV}{d\theta} = 6000\cos\theta - 400(30)\sin 2\theta$

$\theta = 14.5$ ~~90~~

$\cos 2\theta > 0$

$\sin\theta > 0$

$\frac{d^2V}{d\theta^2} = -6000\sin\theta - 800(30)\cos 2\theta$

$-6000(1/4) - \cancel{800}\ 24000(+) = \cancel{\ldots} < 0$

$\boxed{\begin{array}{l}\text{for } \theta = 14.5 \text{ it is unstable} \\ \text{for } \theta = 165.5^\circ \text{ it is stable}\end{array}}$

$\theta = 165.5^\circ \quad -6000(1/4) - 24000(-.8746) > 0$

$\frac{dV}{d\theta} = $ mi

5b) 2 ?

-3

$2(6\cdot 5) = (30)2 = 60$

$4(16\cdot 2) = 128$

$2\left(4-\frac{16}{3\pi}\right)\frac{\pi 16}{4} = 32\pi - \frac{64}{3}$

1b.

	$\bar{y}$	A	$\bar{y}A$
I	5	6	30
II	5	6	30
III	2	16	32
IV	2	16	32
V	2	16	32
VI	$4-\frac{16}{3\pi}$	-4π	$-16\pi + \frac{64}{3}$
VII	2	16	32
VIII	$4-\frac{16}{3\pi}$	-4π	$-16\pi + \frac{64}{3}$

$\Sigma\bar{y}A = 60 + 128 - 32\pi + 128/3 = 188 - 100.5 + 42.7 = 130.2$

$\Sigma A = 12 + 32 + 32 - 8\pi = 76 - 25.1 = 50.9$

$\bar{y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{130.2}{50.9} = 2.56$

c $V = 2\pi\bar{y}A = 2\pi(2.56)(130.2) =$ 2090/2.56 cu in

This is $(\bar{y}A)$

$\Sigma \bar{x} A = \bar{X} \Sigma A$

$\Sigma \bar{y} A = \bar{Y} \Sigma A$

$$\bar{x} = \frac{\int x\, dA}{\int dA} \;;\; \bar{y} = \frac{\int y\, dA}{\int dA}$$

$$\bar{x} = \frac{\int x\, dL}{\int dL} \;;\; \bar{y} = \frac{\int y\, dL}{\int dL}$$

$$\bar{x} = \frac{\int x\, dV}{\int dV} \;;\; \bar{y} = \frac{\int y\, dV}{\int dV}$$

Centroid pt. at which we can concentrate whole load and produce the same effect as if we integrated the function over the whole Area, length or volume.

Pappus-Guldinus

Surface Area $= 2\pi \bar{y} L$

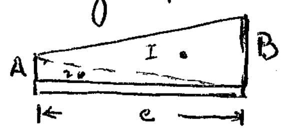
Volume $= 2\pi \bar{y} A$

L is the length of revolved member

A is the area of revolved member

For distributed loads

The distributed load can be replaced by a concentrated load whose magnitude equals the area under the load curve, and its line of action passes through the centroid of that area.



Area I $= [B \cdot c]/2$

Area II $= [A \cdot c]/2$

$\bar{X}_I \text{Area}_I = \frac{B \cdot c^2}{3}$

$\bar{X}_{II} \text{Area}_{II} = \frac{A c^2}{6}$

$\bar{X}_I = \frac{2}{3} c$

$\bar{X}_{II} = \frac{1}{3} c$

$$\sum_{n=I}^{II} \bar{X}_n \text{Area}_n = \frac{2Bc^2 + Ac^2}{6}$$

$$\bar{X} \Sigma \text{Area}_n = \Sigma \bar{X}_n \text{Area}_n$$

$$\bar{X} = \frac{\frac{2Bc^2 + Ac^2}{6}}{\frac{3Bc + 3Ac}{6}} = \frac{2Bc + Ac}{3(B + A)}$$

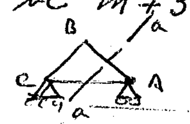
Weight = Volume · density

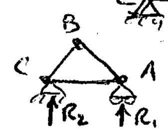
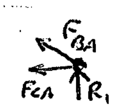
for volume use 1 unit for depth.

Pressure $= \rho h$

Definition of truss – consists of straight members connected at joints; no member is continuous through a joint. Can support loads at joints; produce 2 force members but no couple, can support little lateral loading.

If there are 2n pins there must be m + 3 equations

Method of Sections 

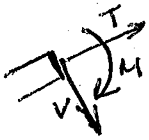
Method of Joints  

Frames are designed to support loads, usually stationary

Machines transmit loads & modify forces.

Can go through a pin.

//



V - shear force.

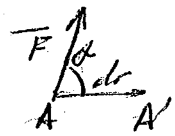
8.1 use μ_k when object already sliding

μ_s up to & max itself $F_{max} = \mu_s N$

$\mu_s = \tan\phi$ where ϕ is the $\measuredangle$ of repose.

10. $dU = \overline{F} \cdot dr$

$= F\,ds\cos\alpha$



if $\alpha = \frac{\pi}{2}$ $dU = 0$ at a smooth pin, frictionless surface

$dU = M\,d\theta$

for certain cases $\sum dU = 0$

Principle of virtual work - total work (virtual) done by a rigid system if the system is in equilibrium is 0 for any virtual displacement of the system. $\delta U = R \cdot \delta s$

$\delta U = -\delta V$

$V = Wy$

$V = \frac{1}{2}Kx^2$

a conservative force is a force whose ~~potential energy~~ work depends only on its initial & final value and is independent of the pa[th]

$dU = -dV$ $\quad \frac{dV}{d\theta} = 0$ $\quad$ if $\frac{d^2V}{d\theta^2} < 0$ it is unstable

$\frac{d^2V}{d\theta^2} > 0$ stable, $\quad \frac{d^2V}{d\theta^2} = 0$ neutral equil

if system in equil. $\cancel{= P\ell\cos\theta - K\ell\cos 2\theta}$

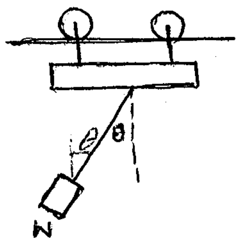
$= \cancel{P\cos\theta - K\ell\cos 2\theta}$

10, 30, 50, 54, 63

Cesar Levy
AE100
Prof Kondo.

7

10.



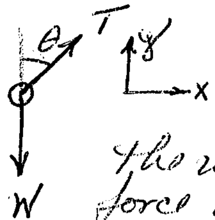
$\Sigma F_y = \Sigma m a_y$

$-W\cos\theta = m a_y$ but $a_y = 0$

$\Sigma F_x = \Sigma m a_x$

$W\sin\theta = m a_x$; $a_x = 2.5$ ft/sec^2 ; $\frac{W}{g} = m$.

$W\sin\theta = \frac{W a_x}{g}$



$\underline{\sin}\,\theta \overset{TAN}{=} \frac{2.5}{32.2} = 4.455°$

the unbalanced force is in the y-direction ∴

$T\sin\theta = m a_x$, $T\cos\theta = W = mg$, divide one by the other

$\frac{T\sin\theta}{T\cos\theta} = \frac{m a_x}{m g} \Rightarrow \tan\theta = \frac{a_x}{g}$

$\theta = TAN^{-1}\left(\frac{2.5}{32.2}\right)$

$\theta \approx 4.5$

30.

$v^2_{final_{Truck}} = v^2_{initial_{Truck}} + 2a_{TRUCK}\, S_{TRUCK}$

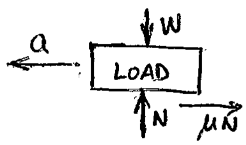
$v_{final} = 0$

$v_{initial} = 88$

$S = 250$

$0 = (88)^2 + 500a$

$a_T = -15.5$ ft/sec^2



$N = W$

$\mu N = -ma$; $.3W = -\frac{W}{g}a$

$-.3g = a_L = -9.66$ ft/sec^2

$a_{L/T} = a_L - a_T$

$= -9.66 - (-15.5) = +5.84$

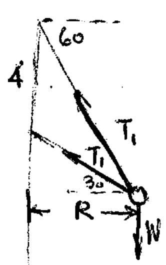
10

$5.84 = v_{L/T}\, d(v_{L/T}) / dS_{L/T}$

$(5.84)\, d(S_{L/T}) = v_{L/T}\, d(v_{L/T})$; $\int_0^{15} 5.84\, ds = \int_0^{v_{L/T}} v\, dv$; $\sqrt{175.4} = v$

$v_{L/T} = 13.26$ ft/sec

50.



① $T_1\sin 60 + T_1\sin 30 = W$

② $T_1\cos 60 + T_1\cos 30 = m a_x$

from ① $T_1 = \frac{W}{1.366}$; from ② $\frac{W}{1.366}(.5 + .866) = \frac{W}{g}\left(\frac{v^2}{R}\right)$

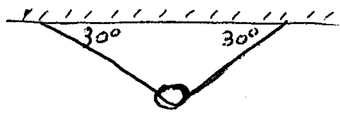
$\Rightarrow gR = v^2$;

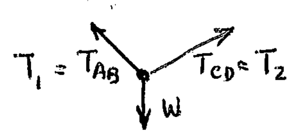
10

from geometry $R = 2\sqrt{3}$; $g = 32.2$

$v = \sqrt{gR} = 10.55$ ft/sec

54.



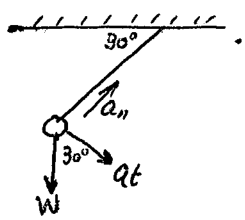


$T_1 \cos 30° = T_2 \cos 30°$; $T_1 = T_2$

$T_1 \sin 30° + T_2 \sin 30° - W = 0$

$2\ T_2 \sin 30° = W$

$T_2 = W = T_1$ $\boxed{T_{CD} = W}$



$T_{CD} - W \sin 30° = \frac{W}{g} \cdot a_n$

$T_{CD} - \frac{W}{2} = 0$ since acceleration is not radial.

$\boxed{T_{CD} = \frac{W}{2}}$

$W \cos 30° = \frac{W}{g}\, at$

$(.866\, g) = a_t$ $\searrow 60°$

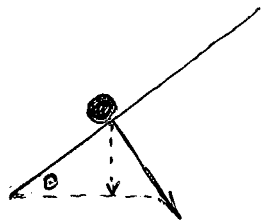
$27.9\ ft/sec^2 = a_t$

10

$v = kx$

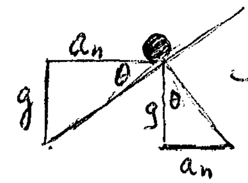
Cesar Levy
AE100
Prof nardo

$V = kx$



① $m = \frac{dy}{dx} = \tan\theta$;

② $\tan\theta = \frac{a_n}{g}$



$$\frac{dy}{dx} = \frac{a_n}{g} = \frac{v^2}{xg} = \frac{k^2x^2}{xg}$$

$$\frac{dy}{dx} = \frac{k^2x}{g} \text{ ; now call } \frac{k^2}{g} = c^2$$

Therefore $\frac{dy}{dx} = c^2x$ or

$dy = c^2 x\,dx$; now integrate

$$\int_y^h dy = c^2\int_x^b x\,dx$$

$$h - y = c^2\left[\frac{b^2}{2} - \frac{x^2}{2}\right]$$

$$h - y = \frac{c^2}{2}\left[b^2 - x^2\right]$$

$9\frac{1}{2}$

O.K.

$$y = \frac{c^2x^2}{2} + K$$

at $x = 0, y = 0 \Rightarrow K = 0$

at $x = b, y = h \Rightarrow c^2 = \frac{2h}{b^2}$

$\therefore y = h\left(\frac{x}{b}\right)^2$