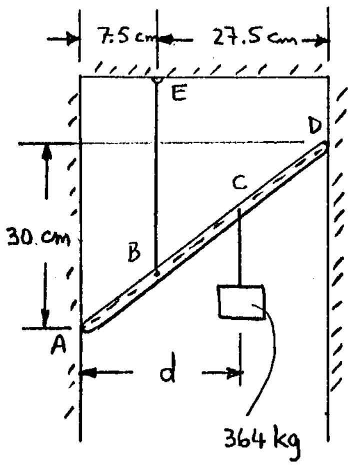
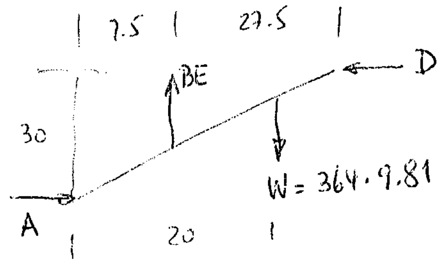
ת.ז. ________



מוט קל AD נתלה מכבל BE ותומך במסה של 364kg ב-C.
הקצוות A ו-D של המוט נוגעים בקירות מאונכות וחלקות.

א) מצאו המתח בכבל BE והתגובות ב A ו-D, אם d=20cm.

ב) מצאו את המרחק המקסימלי, d, שתוכלו להשתמש בו, אם ערך המקסימלי המותר לתגובה (רוחבית) ב-A הוא 2225N



$D = A$

$BE = 364(9.81) = 3571$

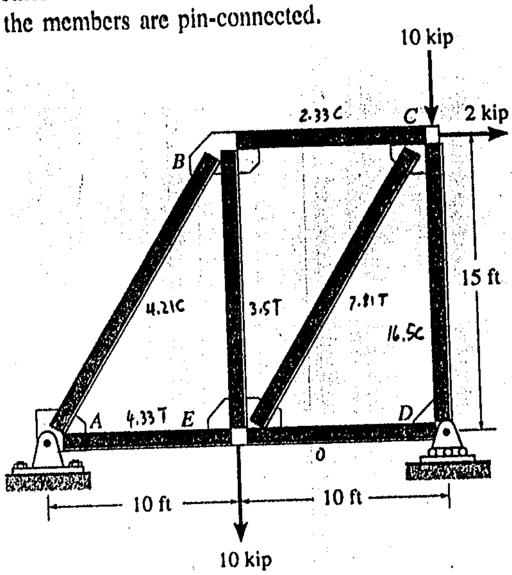
$D \cdot 30 = W \cdot (12.5)$

$A = D = W \cdot \frac{12.5}{30} = 364(9.81)\left(\frac{12.5}{30}\right) = 1488\ N$

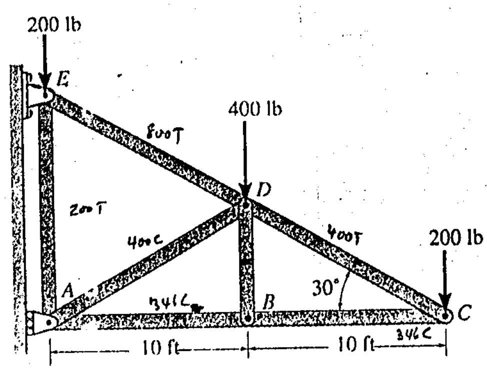
$d = \frac{D \cdot 30}{W} = \frac{2225 \cdot 30}{364(9.81)} = 18.69\,cm$

$d = d' + 7.5\,cm = 26.19\,cm$

6–3. Determine the force in each member of the truss and indicate whether the members are in tension or compression. Assume that all the members are pin-connected.

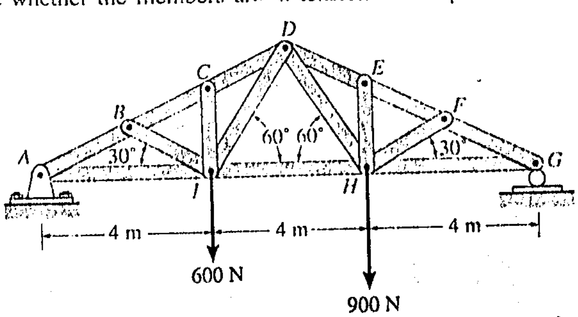


6–11. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



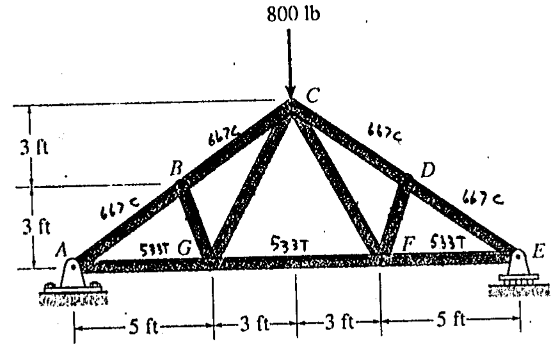
Prob. 6–11

***6–12.** Determine the force in each member of the truss and indicate whether the members are in tension or compression.



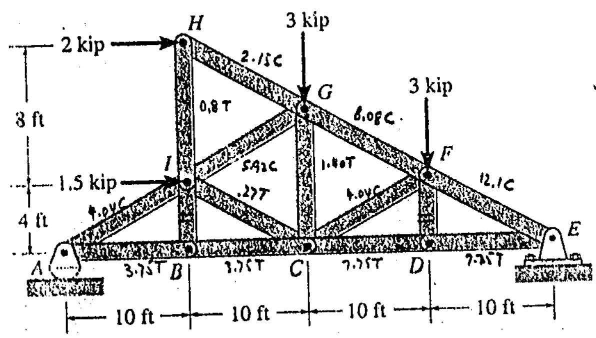
Prob. 6–12

6–13. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



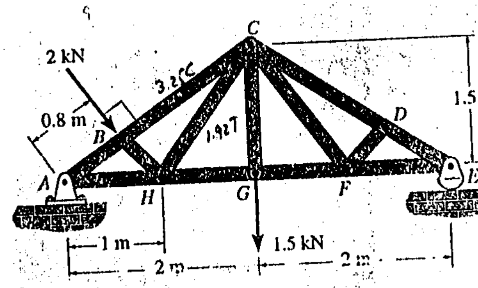
Prob. 6–13

6–18. Determine the force in each member of the truss and indicate whether the members are in tension or compression.

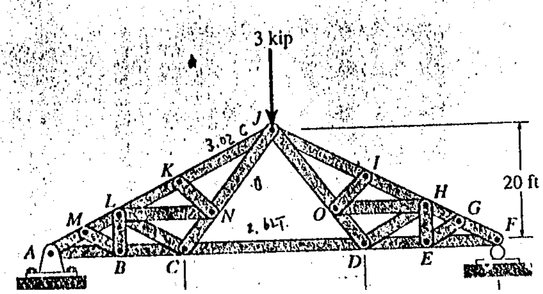


Prob. 6–18

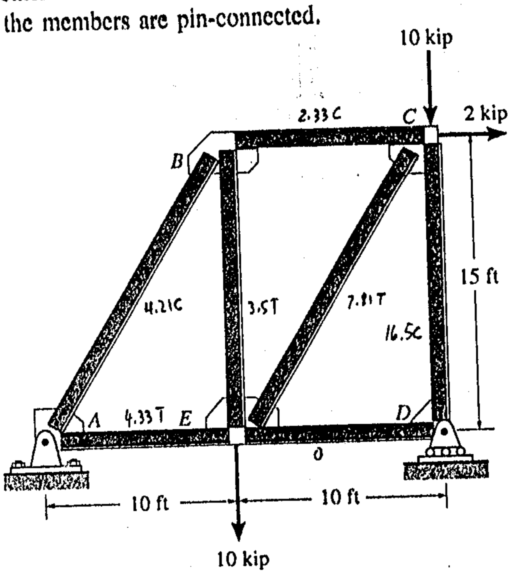
6–29. Determine the force developed in members BC and CH of the roof truss and indicate whether the members are in tension or compression.



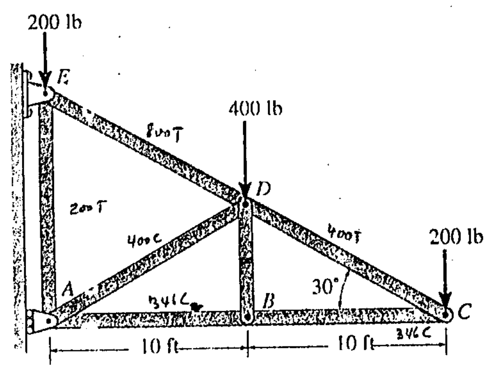
6–37. Determine the force in members KJ, JN, and CD, and indicate whether the members are in tension or compression.



6–3. Determine the force in each member of the truss and indicate whether the members are in tension or compression. Assume that all the members are pin-connected.

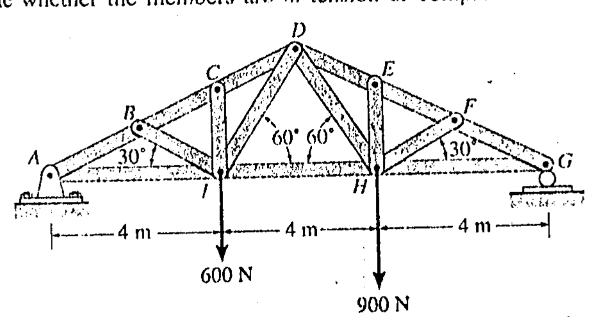


6–11. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



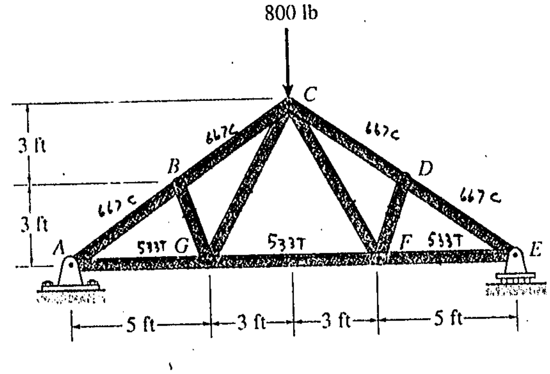
Prob. 6–11

*6–12. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



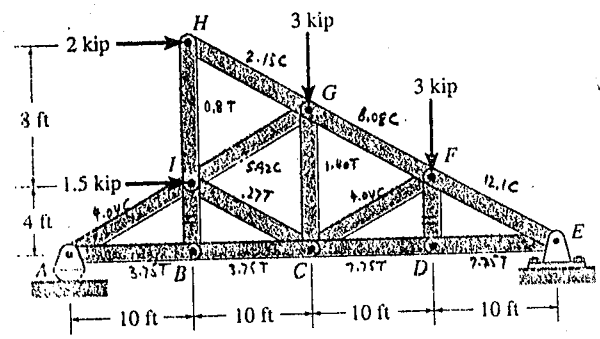
Prob. 6–12

6–13. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



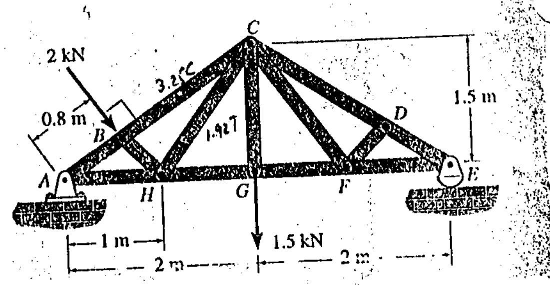
Prob. 6–13

6–18. Determine the force in each member of the truss and indicate whether the members are in tension or compression.

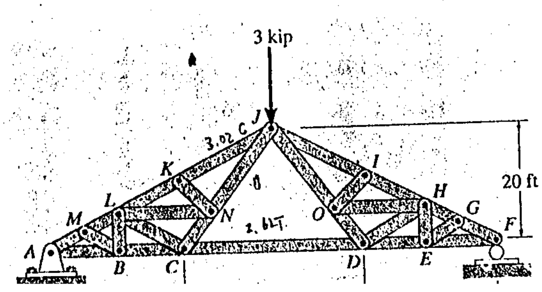


Prob. 6–18

6–29. Determine the force developed in members BC and CH of the roof truss and indicate whether the members are in tension or compression.



6–37. Determine the force in members KJ, JN, and CD, and indicate whether the members are in tension or compression.



שם: ____________________

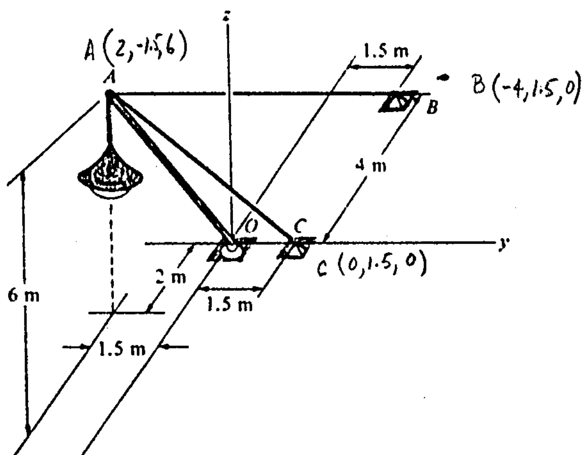
ת.ז. ____________________

האור שנמצא בתרשים נתמך בנקודה A בשני כבלים AB ו-AC ובמוט OA. מסת האור שווה ל-15kg.

א) כתבו ביטויים בשביל הכוחות הקרטזיים $\underline{F_{AB}}$ ו- $\underline{F_{AC}}$

ב) מצאו את הזווית בין שני הכוחות האלה.

ג) מצאו את המומנט M_o (מומנט מסביב 0) שבא מהכוח F_{AC}



Prob. 3–47

$\underline{r}_{B/A} = -6i + 3j - 6k \qquad r_{B/A} = 9 \qquad \underline{u}_{B/A} = -.67\underline{i} + .33j - .67\underline{k}$ (5)

$\underline{r}_{C/A} = -2i + 3j - 6k \qquad r_{C/A} = 7 \qquad u_{C/A} = -.29\underline{i} + .43\underline{j} - .86\underline{k}$ (5)

$$\underline{F_{AB}} = F_{AB}(-.67\underline{i} + .33\underline{j} - .67\underline{k}) = F_{AB}\underline{u}_{B/A} = \left(-\frac{2}{3}\underline{i} + \frac{1}{3}\underline{j} - \frac{2}{3}k\right)F_{AB}$$

$$\underline{F_{AC}} = F_{AC}(-.29\underline{i} + .43\underline{j} - .86\underline{k}) = F_{AC}\underline{u}_{C/A} = \left(-\frac{2}{7}i + \frac{3}{7}\underline{j} - \frac{6}{7}\underline{k}\right)F_{A}$$

$$\cos\theta = \underline{u}_{B/A} \cdot \underline{u}_{C/A} = \left(-\tfrac{2}{3}\left(-\frac{2}{7}\right) + \frac{1}{3}\left(\frac{3}{7}\right) - \frac{2}{3}\left(-\frac{6}{7}\right)\right)$$

$$= \frac{19}{21} \quad (5) \quad .9048 \qquad 25.2^\circ = \theta$$

$$\underline{r}_{c/o} = (0i + 1.5\underline{j} + 0\underline{k})$$

$$\underline{r}_{c/o} \times r = \underline{M}_o = \begin{vmatrix} i & j & k \\ 0 & 1.5 & 0 \\ -.27F_{AC} & .43F_{AC} & -.86F_{AC} \end{vmatrix} = -.86F_{AC}(1.5)i + .27F_{AC}(1.5)\underline{k}$$

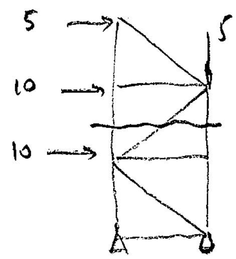
$$= \left(-\frac{6}{7}\cdot\frac{3}{2}\,i + \frac{2}{7}\cdot\frac{3}{2}\,\underline{k}\right)F_{AC}$$

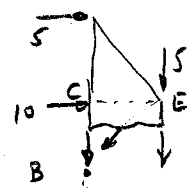
$$\underline{M_o} = \frac{-18i + 6k}{14}\,F_{AC}. \quad (5)$$

$$= (-1.29\underline{i} + .43\underline{k})F_{AC}$$

058798 $73.33\ \bar{x}$ $19.73\ \sigma$ 80.8 10.79

(1)



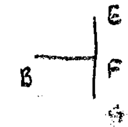


$\Sigma M_E = -5\cdot 4 + F_{CB}\cdot 4 = 0$ 4 $\underline{F_{CB} = 5\ kN}$ 2

$\Sigma M_B = 0 = -5\cdot 8 + 10\cdot 4 - 5\cdot 4 - F_{EF}\cdot 4 = 0$ 8 $\underline{F_{EF} = -25\ kN}$ 2

$\Sigma F_y = 0 = +5 + 10 - F_{BE}\cdot \frac{1}{\sqrt{2}} = 0$ 4 $\underline{F_{BE} = 15\sqrt{2}\ kN}$ 2

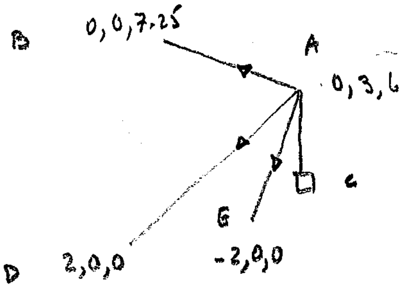
$\underline{BF = 0}$



2

(24) $\cdot \frac{4}{3} = 32$

(2)



$\underline{r}_{D/A} = 2\underline{i} - 3\underline{j} - 6\underline{k}$ $\quad r_{D/A} = 7$; $\underline{u}_{D/A} = \frac{2}{7}\underline{i} - \frac{3}{7}\underline{j} - \frac{6}{7}\underline{k}$ 4

$\underline{r}_{E/A} = -2\underline{i} - 3\underline{j} - 6\underline{k}$ $\quad r_{E/A} = 7$; $\underline{u}_{E/A} = -\frac{2}{7}\underline{i} - \frac{3}{7}\underline{j} - \frac{6}{7}\underline{k}$ 4

$\underline{r}_{B/A} = 0\underline{i} - 3\underline{j} + 1.25\underline{k}$ $\quad r_{B/A} = 3.25$; $\underline{u}_{B/A} = 0\underline{i} - \frac{3}{3.25}\underline{j} + \frac{1.25}{3.25}\underline{k}$ 4

$300 \times 9.81 = 2943\ N$

$\Sigma \underline{F} = \underline{F}_{D/A} + \underline{F}_{E/A} + \underline{F}_{B/A} - 2943\underline{k} = \underline{0}$

$\Sigma F_x = \frac{2}{7}F_{DA} - \frac{2}{7}F_{EA} + 0 = 0$ 4 $\underline{F_{DA} = F_{EA}}$ 2

$\Sigma F_y = 0 \quad -\frac{3}{7}F_{DA} - \frac{3}{7}F_{EA} - \frac{3}{3.25}F_{BA} = 0$ 4 $\Rightarrow -\frac{6}{7}F_{DA} - \frac{3}{3.25}F_{BA} = 0 \qquad F_{BA} = -\frac{3.25}{3}\cdot\frac{6}{7}F_{DA}$

$= -.92857\,F_{DA}$

5

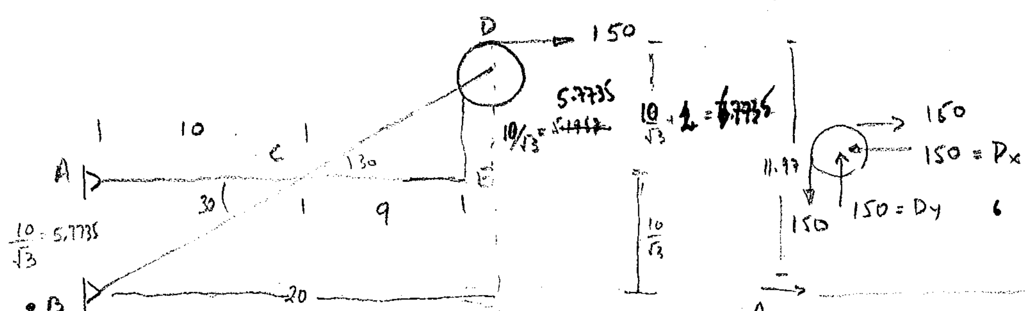
$\Sigma F_z = 0 \quad -\frac{6}{7}F_{DA} - \frac{6}{7}F_{EA} + \frac{1.25}{3.25}F_{BA} - 2943 = 0 \Rightarrow -\frac{12}{7}F_{DA} + \frac{1.25}{3.25}\left(-\frac{3.25}{3}\cdot\frac{6}{7}F_{DA}\right) - 2943 = 0$

$-2.07\,F_{DA} - 2943 = 0$

$\underline{F_{DA} = -1420.8\ N = F_{EA}}$ 2

$\underline{F_{BA} = +1319.3\ N}$ 2

(31)



4 FBD ○ 2 Dx Dy

5 FBD ACE

6 FBD BCD

ΣM_A = 6 Bx

~~ΣM_B 6 Ax~~

ΣM_C ~~Ax = Cx~~ 8 ~~Ax~~ By 2

ΣF_{y} ⇒ Cx 3 + 2

ΣF_y ~~Ay = ...~~ 3 By 2

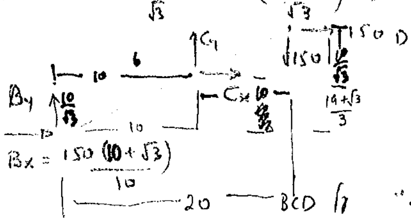
~~Ay = Cy − 150 3 Ay 2~~

$\Rightarrow A_x = C_x \quad A_y = C_y - 150$ 5

חשוב לסמן את הכוחות

$\circlearrowleft \Sigma M_A = 0 \quad B_x\cdot\frac{10}{\sqrt{3}} - 150\left(\frac{10+\sqrt{3}}{\sqrt{3}}\right) = 0$ 4 $\quad \underline{B_x = \frac{150(10+\sqrt{3})}{10} = 176}$ 2

~~$\circlearrowleft \Sigma M_B = 0 \quad A_x\frac{10}{\sqrt{3}} - 150\left(\frac{20+\sqrt{3}}{\sqrt{3}}\right) = 0$ 4 $\quad A_x = -\frac{150(19+\sqrt{3})}{10} = 326$~~ 2



$\Sigma M_C = 0 \Rightarrow -B_y(10) + B_x\left(\frac{10}{\sqrt{3}}\right) - 150\left(\frac{10}{\sqrt{3}}\right) - 150\left(\frac{10}{\sqrt{3}}\right) = 0 \quad \underline{B_y = -135}$ 2

$B_x + C_x + 150 = 0 \quad \underline{C_x = -326}$ 3+2 2

~~$A_x = C_x = -150 - 150(9+\sqrt{3}) = \boxed{-326 = A_x}$~~ $\boxed{C_x = -34}$

$B_y + C_y - 150 = 0 \quad C_y = 285$ 3+2

~~$\Sigma M_B = 0 \Rightarrow C_y = \left(+150(20) + C_x\frac{10}{\sqrt{3}} + 150\left(\frac{20}{\sqrt{3}}\right)\right)/10$~~ $\boxed{150 = C_y}$

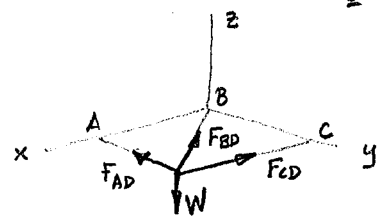
~~$B_y + C_y - 150 = 0$~~ 4 $\boxed{B_y = -150}$ 2

A(8,0,0) $\qquad \underline{r}_{A/D} = \underline{r}_A - \underline{r}_D = (5\underline{i} - 2\underline{j} + 2\underline{k})\,m \qquad r_{A/D} = 5.745\,m \qquad \underline{u}_{A/D} = .870\underline{i} - .348\underline{j} + .348\underline{k}$

B(0,0,0) $\qquad \underline{r}_{B/D} = \underline{r}_B - \underline{r}_D = (-3\underline{i} - 2\underline{j} + 2\underline{k})\,m \qquad r_{B/D} = 4.123\,m \qquad \underline{u}_{B/D} = -.728\underline{i} - .485\underline{j} + .485\underline{k}$

C(0,7,0) $\qquad \underline{r}_{C/D} = \underline{r}_C - \underline{r}_D = (-3\underline{i} + 5\underline{j} + 2\underline{k})\,m \qquad r_{C/D} = 6.164\,m \qquad \underline{u}_{C/D} = -.487\underline{i} + .811\underline{j} + .324\underline{k}$

$$\underline{F}_{AD} = F_{AD}\,\underline{u}_{A/D} = F_{AD}(.870\underline{i} - .348\underline{j} + .348\underline{k})$$
$$\underline{F}_{BD} = F_{BD}\,\underline{u}_{B/D} = F_{BD}(-.728\underline{i} - .485\underline{j} + .485\underline{k})$$
$$\underline{F}_{CD} = F_{CD}\,\underline{u}_{C/D} = F_{CD}(-.487\underline{i} + .811\underline{j} + .324\underline{k})$$
$$\underline{W} = -20\underline{k}$$



$$\Sigma F_x = 0 \Rightarrow \left.\begin{array}{l} .870 F_{AD} - .728 F_{BD} - .487 F_{CD} = 0 \\ -.348 F_{AD} - .485 F_{BD} + .811 F_{CD} = 0 \\ +.348 F_{AD} + .485 F_{BD} + .324 F_{CD} - 20 = 0 \end{array}\right\}$$

פותר לפי מחשבון לקבל

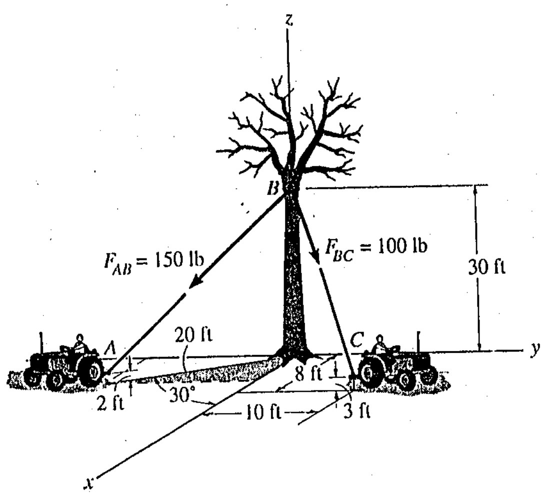
$F_{AD} = 21.57\,N$

$F_{BD} = 13.99\,N$

$F_{CD} = 17.62\,N$

דטרמיננטה = $-.7665$

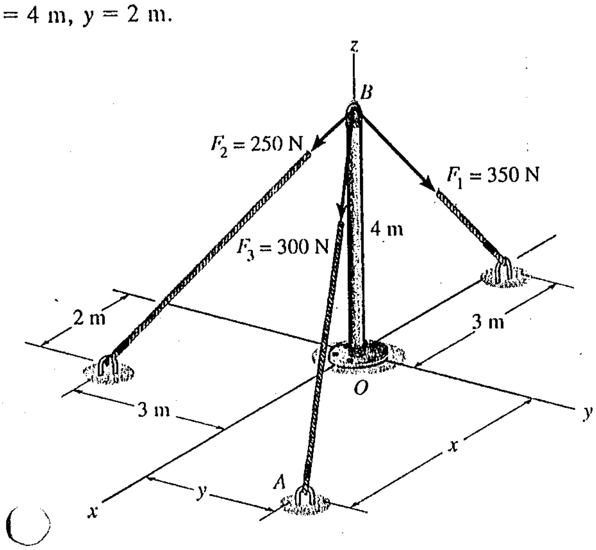
2–89. Two tractors pull on the tree with the forces shown. Represent each force as a Cartesian vector and then determine the magnitude and coordinate direction angles of the resultant force.



$\underline{F}_1 = 75.5\underline{i} - 43.6\underline{j} - 122\underline{k}$

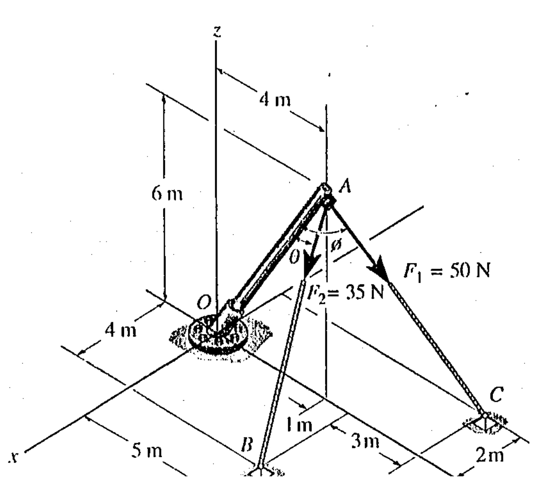
Prob. 2–89

***2–96.** The pole is held in place by three cables. If the force of each cable acting on the pole is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Set $x = 4$ m, $y = 2$ m.



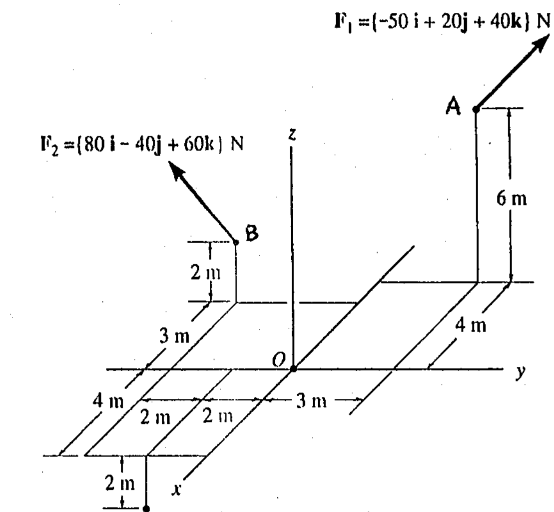
Prob. 2–96

2–113. The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis OA of the pole.



point O. Express the result as a Cartesian vector.

***4–24.** Determine the resultant moment of the forces about point P. Express the result as a Cartesian vector.

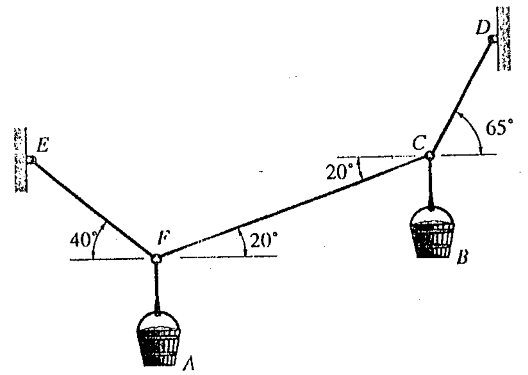


Probs. 4–23/4–24

$\underline{r}_{B/O} = -3\underline{i} - 4\underline{j} + 2\underline{k}$

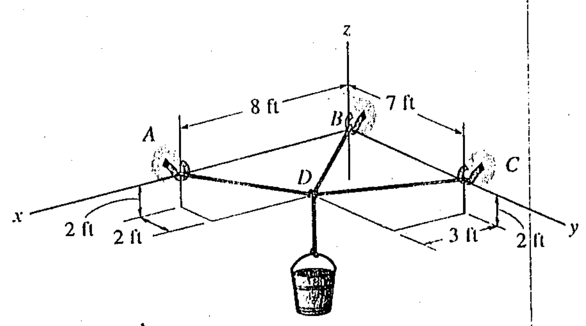
$\underline{M}_O = \underline{r}_{B/O} \times \underline{F}_2$

3–51. The cords suspend the two buckets in the equilibrium position shown. Determine the weight of bucket B. Bucket A has a weight of 60 lb.



Prob. 3–51

3–53. The bucket has a weight of 20 lb. Determine the tension developed in each cord for equilibrium.



$F_{DA} = 21.5$ N

$F_2 = 200\ N$

$$F_1 = F_3 \frac{\cos 45^\circ}{\cos 30^\circ} = F_3 \frac{(.707)}{.866} = F_3\ (.8164)$$

$$400 = F_3\ (.8164) \sin 30 + F_3 \sin 45^\circ$$
$$= F_3\ .4082 + .7071\ F_3 = 1.1153\ F_3$$
$$F_3 = 358.65\ lb$$

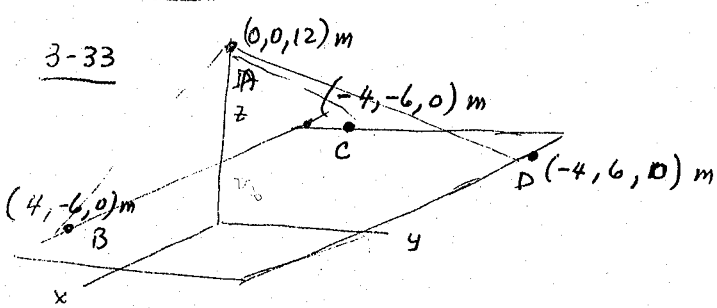
$$F_1 = F_3\ (.8164) = 292.8\ lb$$

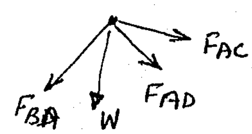
rods support a weight whose mass is 150 kg
3 ~~cables in tension~~
~~due to an upward load W of 1472N~~
find forces in ~~cables~~ the rods.

SESSION #6

REVIEW

3-33





$$\bar{r}_{AC} = \bar{r}_C - \bar{r}_A = (-4\bar{i} - 6\bar{j} + 12\bar{k})\ m$$
$$r_{AC} = \sqrt{4^2 + 6^2 + (12)^2} = 14\ m$$
$$\bar{r}_{AD} = \bar{r}_D - \bar{r}_A = (-4\bar{i} + 6\bar{j} + 12\bar{k})\ m$$
$$r_{AD} = 14\ m$$
$$\bar{r}_{AB} = \bar{r}_B - \bar{r}_A = (4\bar{i} - 6\bar{j} + 12\bar{k})\ m$$
$$r_{AB} = 14 m$$

$$\bar{u}_{AC} = -.286\bar{i} - .429\bar{j} + .857\bar{k}\ ; \quad \bar{F}_{AC} = F_{AC}\bar{u}_{AC}$$

$$\bar{u}_{AD} = -.286\bar{i} + .429\bar{j} + .857\bar{k}\ ; \quad \bar{F}_{AD} = F_{AD}\bar{u}_{AD}$$

$$\bar{u}_{AB} = +.286\bar{i} - .429\bar{j} - .857\bar{k}\ ; \quad \bar{F}_{AB} = F_{AB}\bar{u}_{AB}$$

$$\Sigma\vec{F} = \vec{0} \Rightarrow \bar{F}_{AB} + \bar{F}_{AD} + \bar{F}_{AC} + \bar{W} = \bar{0}$$

$$\Sigma F_x = -.286\,F_{AC} - .286\,F_{AD} + .286\,F_{AB} = 0$$
$$\Sigma F_y = -.429\,F_{AC} + .429\,F_{AD} - .429\,F_{AB} = 0$$
$$\Sigma F_z = -.857\,F_{AC} - .857\,F_{AD} - .857\,F_{AB} - W = 0$$

$-F_{AC} - F_{AD} + F_{AB} = 0$ or $F_{AB} = F_{AC} + F_{AD}$

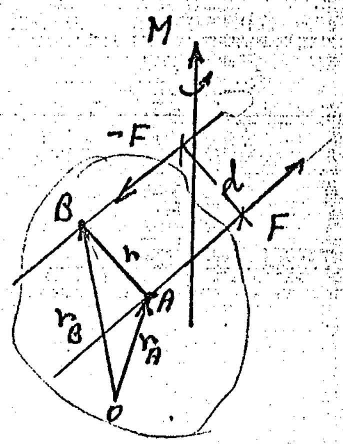
$-F_{AC} + F_{AD} - F_{AB} = 0$ or $F_{AD} = F_{AC} + F_{AB} = F_{AC} + F_{AC} + F_{AD} = 2F_{AC} + F_{AD}$.

THUS $\boxed{F_{AC} = 0} \Rightarrow \underline{F_{AB} = F_{AD}}$

$$-\cancel{F_{AC}} - F_{AD} - F_{AB} - \frac{W}{.857} = 0 \quad \text{or} \quad -2F_{AD} - \frac{W}{.857} = 0 \quad \text{or} \quad F_{AD} = -\frac{W}{2\cdot(.857)} = F_{AB}$$

$$\text{Thus} \quad F_{AD} = F_{AB} = -\frac{W}{2} = -\frac{150kg \cdot 9.81}{2(.857)} = \frac{-735.75\,N}{.857} = -858.52\,N$$

צמדים



הגדרת הצמד במרחב דומה לצמד שבמישור:

$$\vec{M} = \vec{r}_A \times F + \vec{r}_B \times (-F) = (r_A - r_B) \times F$$

$$= \vec{r} \times \vec{F}$$

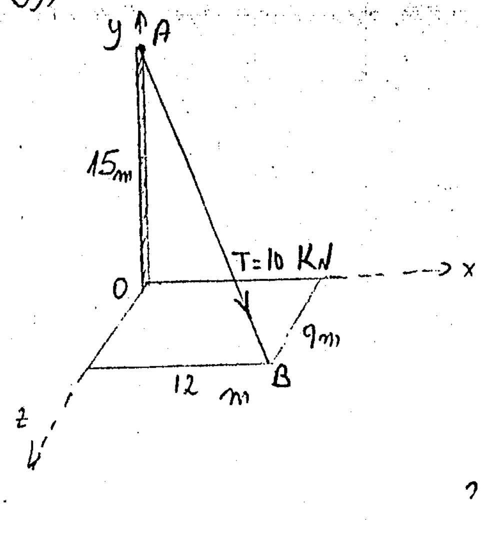
עוצמתו של הצמד תהיה $M = Fd$

* עלינו לזכור שהמומנט שבצמד הינו וקטור חופשי מומנט רגיל הינו וקטור מחליק.

* גם במקרה התלת ממדי ניתן להמיר כח בכח שווה ומקביל + צמד.

דוגמא

הצינור המתואר בשרטוט נפעל לכח מאמץ $T = 10KN$. חשב את המומנט שיוצר הכח T סביב הציר z



יש דרכים שונות לפתרון הבעיה:

דרך א

נחשב את המומנט שיוצר T סביב הנקודה O ולאחר מכן נטילו על z

1. נחשב את רכיבי T ונרשום אותו כוקטור

$$l = \frac{12}{\sqrt{9^2+12^2+15^2}} = 0.566 \qquad m = \frac{-15}{AB} = -0.707 \qquad n = \frac{9}{AB} = 0.424$$

$$T = 10(0.566\,i - 0.707\,j + 0.424\,k)\ KN$$

נבחר כ $\vec{r}$ את $\vec{OA}$

$$\vec{r} = 15\,j \ \ m$$

פתרון תרגיל 7 [illegible]

$$\underline{r}_{B/A} = 12\underline{i} - 15\underline{j} + 9\underline{k}$$

$$\underline{u}_{AB} = \frac{1}{\sqrt{144+225+81}}\left(12\underline{i} - 15\underline{j} + 9\underline{k}\right) = \frac{\sqrt{2}}{30}\left(12\underline{i} - 15\underline{j} + 9\underline{k}\right)$$

$$= \frac{\sqrt{2}}{10}\left(4\underline{i} - 5\underline{j} + 3\underline{k}\right)$$

$$\underline{T} = T\underline{u}_{AB} = \sqrt{2}\left(4\underline{i} - 5\underline{j} + 3\underline{k}\right)[KN]$$

$$r_{A/O} = 15\underline{j}$$

נחשב מומנט סביב O -

$$\underline{M}_O = \underline{r}_{A/O} \times \underline{T} = \sqrt{2}\begin{bmatrix} i & j & k \\ 0 & 15 & 0 \\ 4 & -5 & 3 \end{bmatrix} = \sqrt{2}\left(45\underline{i} - 60\underline{k}\right) \quad KNm$$

$$= 15\sqrt{2}\left(3\underline{i} - 4\underline{k}\right) \quad KNm$$

$$M_z = \underline{M}_A \cdot \underline{k} = -60\sqrt{2} \ KNm \cong -84.9 \ KN\,m$$

דרך ב

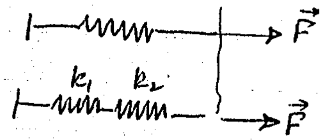
$$\underline{M}_z = \underline{u}_z \cdot \underline{r}_{A/O} \times \underline{F} = \sqrt{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 15 & 0 \\ 4 & -5 & 3 \end{vmatrix} = \sqrt{2}(-60) = 84.9 \ kN\text{-}m$$

$$\underline{M}_B = \underline{r}_{B/O} \times \underline{T} = \left(12\underline{i} + 9\underline{k}\right) \times \sqrt{2}\left(4\underline{i} - 5\underline{j} + 3\underline{k}\right) = \sqrt{2}\left|\begin{matrix} i & j & k \\ 12 & 0 & 9 \\ 4 & -5 & 3 \end{matrix}\right|\begin{matrix} i & j \\ 12 & 0 \\ 4 & -5 \end{matrix}$$

$$= \sqrt{2}\left(36\underline{j} - 60\underline{k} - 36\underline{j} + 45\underline{i}\right) = \sqrt{2}\left(45\underline{i} - 60\underline{k}\right)\,!$$

$$M_z = \underline{u}_z \cdot \underline{r}_{B/O} \times \underline{T} = \underline{k} \cdot (\sqrt{2})\left(45\underline{i} - 60\underline{k}\right) = -60\sqrt{2} \ kN\text{-}m = -84.9 \ kN\text{-}m$$

PROBLEM 3-15



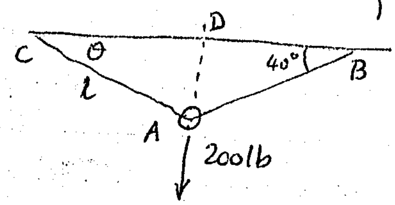
$\vec{F} = k_T(x_1 + x_2)$

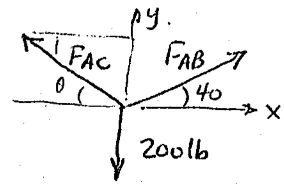
same $\vec{F}$ acts on k_1 & also on k_2

$\vec{F} = k_1 x_1$ & $\vec{F} = k_2 x_2$ $\quad \frac{\vec{F}}{k_1} = x_1 \quad \frac{\vec{F}}{k_2} = x_2 \quad \vec{F}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = x_1 + x_2 = \frac{\vec{F}}{k}$

THUS $\underline{\underline{\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2}}}$

PROBLEM 3-13 Find length l





$F_{AC} = 160 lb.$

$$\Sigma F_x = F_{AB}\cos 40^\circ - F_{AC}\cos\theta = 0$$
$$\Sigma F_y = F_{AB}\sin 40^\circ + F_{AC}\sin\theta - 200\,lb = 0$$

$F_{AB} = F_{AC}\cos\theta / \cos 40^\circ \Rightarrow \quad F_{AC}\left[\cos\theta \frac{\sin 40^\circ}{\cos 40^\circ} + \sin\theta\right] = 200\,lb$

$F_{AC}\left[\sin(\theta + 40^\circ)\right] = 200\,lb \cdot \cos 40^\circ$

$\sin(\theta + 40^\circ) = 200\,lb \cdot \cos 40^\circ / 160\,lb = .9576 \qquad \theta + 40^\circ = 133.25^\circ$

$\theta = +33.25^\circ$

Now since $AB = 2\,ft$ $\quad \perp = AD = AB\sin 40^\circ = 2\,ft \sin 40^\circ = 1.286\,ft$

$AD = AC\sin 33.25^\circ = 2.35\,ft$

or

$\frac{AC}{\sin 40^\circ} = \frac{AB}{\sin 33.25^\circ}$ by law of sines

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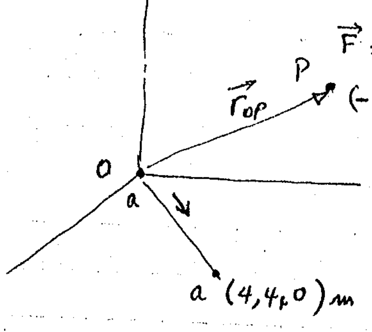
4-53

find proj of momet due to $\vec{F}$ about a,a axis

$\vec{F} = (30\vec{i} + 40\vec{j} + 20\vec{k})N$



Find $M_O \Rightarrow \vec{r}_{op}, \vec{F}$

$\vec{r}_{OP} = \bar{r}_P - \bar{r}_O = (-2\vec{i} + 3\vec{j} + 2\vec{k})\,m$

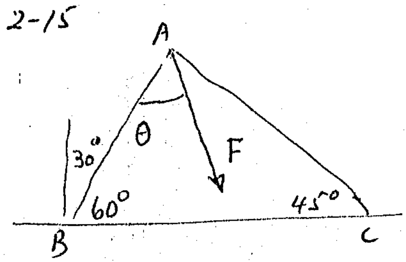
$M_O = \bar{r}_{op} \times \bar{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & 2 \\ 30 & 40 & 20 \end{vmatrix} = -20\vec{i} + \vec{j}(100) + \vec{k}$ N-m

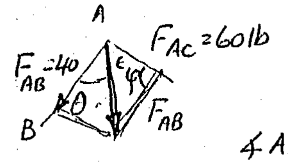
Find $\vec{u}_{aa}$

$\bar{r}_{aa} = 4\vec{i} + 4\vec{j} + 0\vec{k} \qquad \bar{u}_{aa} = .707\vec{i} + .707\vec{j} + 0\vec{k}$

also $\quad \bar{u}\cdot M_o = \bar{u}\cdot(\bar{r}\times\bar{F}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

2-15





$\measuredangle ABC = 60°$
$\measuredangle ACB = 45°$
$\measuredangle BAC = 75°$

$\varphi = [360° - 2(75°)]/2 = 105°$

$F = \sqrt{(F_{AB})^2 + (F_{AC})^2 - 2F_{AB}F_{AC}\cos\varphi} = \cancel{20\text{ lb}}.$ 80.26 lb

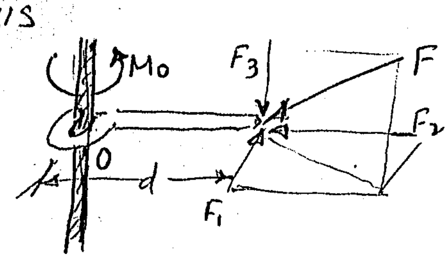
$\dfrac{F}{\sin 105°} = \dfrac{F_{AB}}{\sin\epsilon}$ or $\sin\epsilon = \dfrac{F_{AB}}{F}\sin 105° = .4814 \quad \epsilon = 28.78°$

$\theta = \measuredangle BAC - 28.78° = \underline{46.22°}$

SESSION # 7

- $\Sigma\vec{F} = \vec{0}$ IS A NECESSARY CONDITION BUT NOT SUFFICIENT FOR EQUILIBRIUM
  תנאי הכרחי — תנאי שלא יספיק
- FURTHER RESTRICTION ON ~~...~~ NON CONCURRENT ~~...~~ FORCES MUST ALSO BE MADE
  צריך הגבלות אחרות כאשר הכוחות לא שותפי נקודה
- NON CONCURRENCY GIVES RISE TO CONCEPT OF A MOMENT
  - WILL DEFINE CONCEPT
  - HOW TO OBTAIN IT
  - EQUIVALENT SYSTEM TO ONE COMPRISED OF FORCES & MOMENTS
- LOOK AT THE SIMPLEST CASE FIRST - MOMENT DUE TO COPLANAR FORCES

A FORCE CAN CAUSE TURNING ABOUT AN AXIS IF ITS LINE OF ACTION IS NOT THROUGH THAT AXIS OR DOES NOT ACT PARALLEL TO THE AXIS

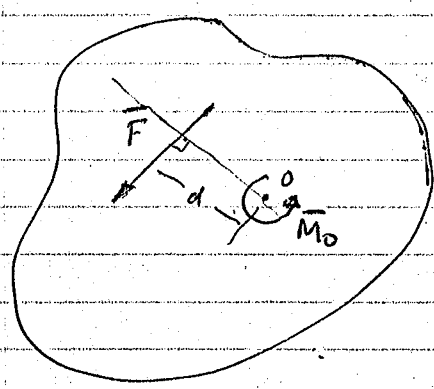


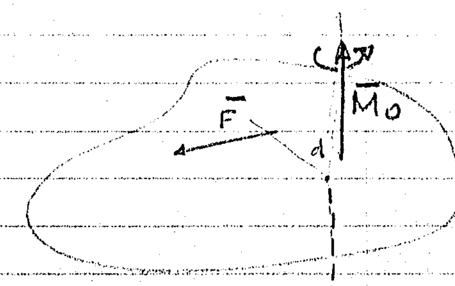
F_2 has line of action through O
F_3 ∥ to axis
F_1 only contributes.

USE BOOK ON TABLE, TO SHOW THIS

סיבוב שנגרם על ידי F נקרא מומנט גם כן
ציר הסיבוב ⊥ ל F
והציר עובר את המישור
שכולל F ו d בנקודה O.

- ROTATIONAL EFFECT OF $\vec{F_1}$ IS CALLED THE MOMENT $\vec{M_o}$. NOTE THAT THE AXIS OF ROTATION IS ⊥ TO F_1 & TO d. ALSO AXIS INTERSECTS THE PLANE THAT CONTAINS F_1 & d AT O.
- $\vec{M_o}$ IS A VECTOR QUANTITY
- magnitude of M_o is Fd, F is magnitude of Force and d is ⊥ distance from O to the line of action of F
- d is the moment arm Units are force·distance (N-m) (lb-ft)
- Direction is using כלל יד ימינית RH rule ⊥ to d & F SHOW THE ( ).
  PUT FINGERS IN DIRECTION OF moment arm, sweep toward +ive direction of force THUMB gives direction.





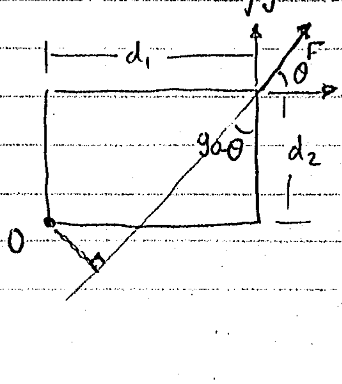
מומנט הוא וקטור
החלקה לאורך
ציר המומנט
שכל נועד
לציר שעובר את
הנקודה O.

- MOMENT CAN BE CONSIDERED A SLIDING VECTOR ALONG THE MOMENT AXIS BUT IT IS BOUND TO THE AXIS PASSING THRU O.
  נועד

מומנט הכוח
שווה לסך הכול
של המומנטים הבאים
מרכיבי הכוח
מסביב אותה נקודה

VARIGNON'S THEOREM – MOMENT OF A FORCE ABOUT A POINT IS EQUAL TO THE SUM OF THE MOMENTS OF THE FORCE'S COMPONENTS ABOUT ~~THE~~ PT

$$\vec{M_o} = (F_x\bar{i} + F_y\bar{j} + F_z\bar{k})(d \text{ to each component})$$



$M_o = F_y d_1 \circlearrowleft + F_x d_2 \circlearrowright$

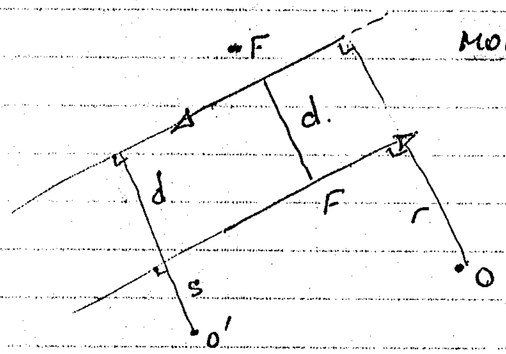
$M_o = F_y d_1 \circlearrowleft - F_x d_2 \circlearrowleft = (F_y d_1 - F_x d_2) \circlearrowleft$

COUPLES
זוג כוחות

- A COUPLE – 2 FORCES, //, SAME MAG. OPPOSITE DIRECTIONS SEPARATED BY A $\perp$ DISTANCE d.

- $\sum \vec{F}$ OF COUPLE $= \vec{F} - \vec{F} = \vec{0}$

  ONLY A MOMENT IS GENERATED (נוצר) magnitude is Fd



MOMENT OF A COUPLE IS A FREE VECTOR <u>NOT</u> TIED TO ANY POINT.

$$\circlearrowleft + \vec{M}_O = (r+d)(+F)\circlearrowleft + rF\circlearrowright$$
$$= -(r+d)F\circlearrowright + rF\circlearrowright = -Fd\circlearrowright = Fd\circlearrowleft$$

$$\circlearrowleft + M_{O'} = (d+s)F\circlearrowleft + F\cdot s\circlearrowright = -Fd\circlearrowright = Fd\circlearrowleft$$

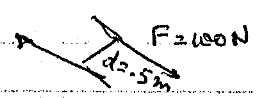
thus the same moment is obtained about any point, thus

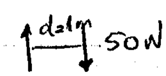
$M = Fd$ depends only on $\perp$ distance & <u>not</u> on location

- EQUIVALENT COUPLES – COUPLES THAT PRODUCE THE SAME MOMENT

  MOMENT OF COUPLE A = MOMENT OF COUPLE B

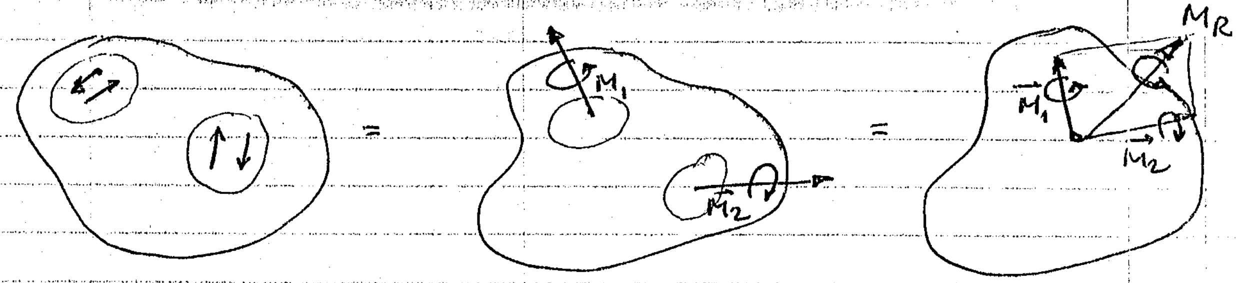
  FORCES THAT PRODUCE THE COUPLE A MUST LIE IN SAME PLANE OR PARALLEL PLANE TO THOSE FORCE THAT PRODUCE COUPLE B





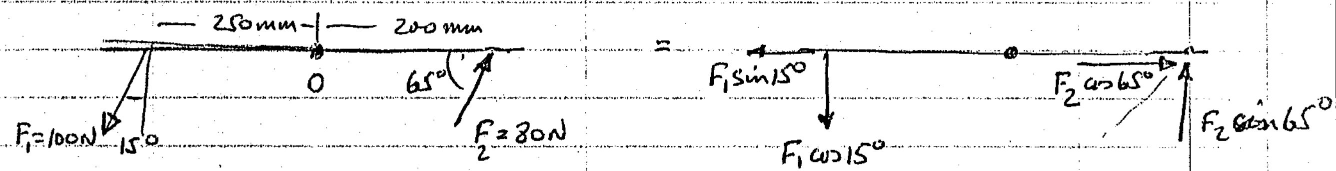
- RESULTANT COUPLES - שקול זוגות כוחות

  CAN BE ADDED VECTORIALLY : SINCE EACH IS A FREE VECTOR AND THE RESULTANT IS ALSO A FREE VECTOR.



SINCE FREE VECTORS CAN BE APPLIED ANYWHERE - APPLY IT TO A PT P ON THE BODY WHICH IS CONVENIENT

Problem
4-3



$F_1 \sin 15°$ & $F_2 \cos 65°$ give no moment.

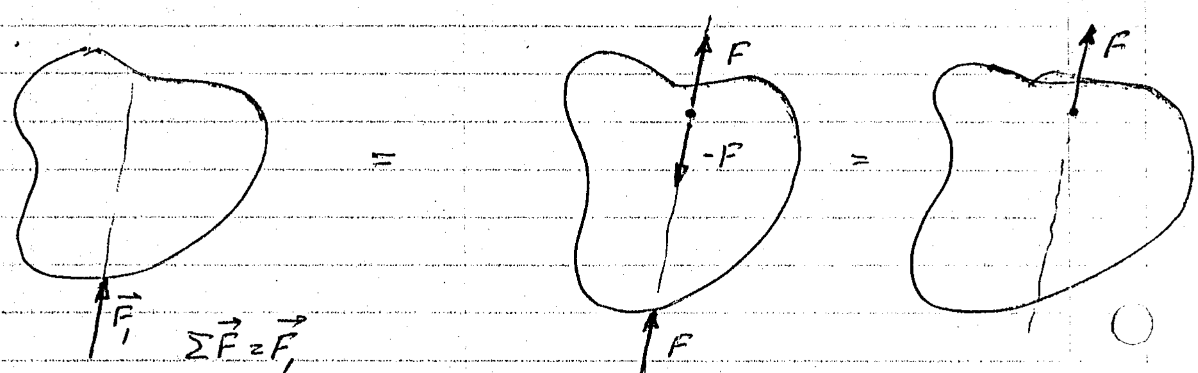
$$\circlearrowright (F_1 \cos 15°)\, 250\,mm + \circlearrowright ((F_2 \sin 65°)\, 200\,mm = (96.59)(250) + (\overset{72.50}{\cancel{33.81}})(200)$$

$$= 24148.15\ Nmm + \overset{14500.93}{\cancel{6761.89}}\ Nmm = \overset{38649.08}{\cancel{30910.04}}\ Nmm = \overset{38.65}{\cancel{30.91}}\ Nm$$

PRINCIPLE OF TRANSMISSIBILITY   עקרון העברת כוח

EXTERNAL EFFECTS ON A RIGID BODY REMAIN UNCHANGED WHEN A FORCE ACTING AT A GIVEN POINT ON THE BODY, IS APPLIED TO ANOTHER POINT LYING ON THE LINE OF ACTION OF THE FORCE.
(THE FORCE IS A SLIDING VECTOR)

השפעות חיצוניות על גוף קשיח לא משתנות כאשר כוח שפועל בנקודה מסוימת על הגוף מיושם בנקודה אחרת שנחה על קו הפעולה של הכוח.

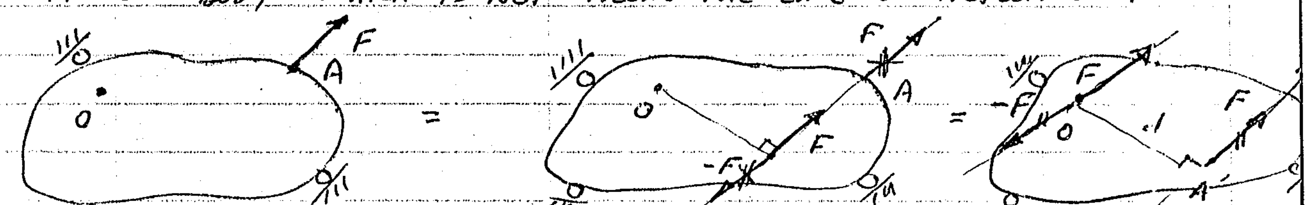


NO MOMENTS ARE GENERATED

WE SAY THE FORCE HAS BEEN TRANSMITTED ALONG ITS LINE OF ACTION
אומרים שהכוח הועבר דרך הקו הפעולה שלו.

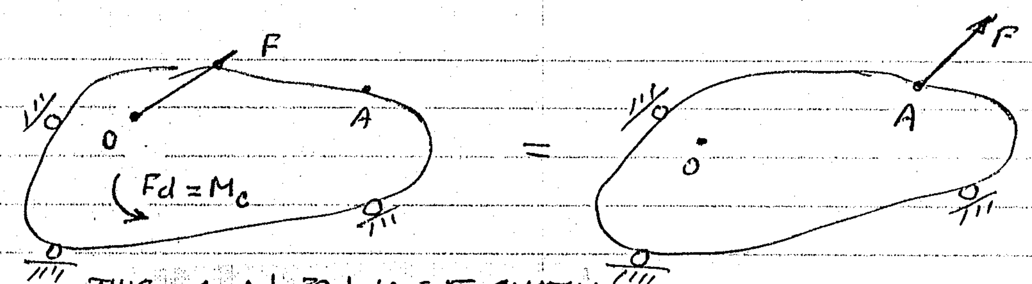
RESOLUTION OF A FORCE INTO A FORCE & COUPLE

- GIVEN A BODY WITH A FORCE ACTING ON IT AT A PT "A" WANT TO FIND WHAT HAPPENS IF WE MOVE FORCE TO ANOTHER PT ON BODY WHICH IS NOT ALONG THE LINE OF ACTION OF F



TRANSMISSIBILITY

-F, F form a couple



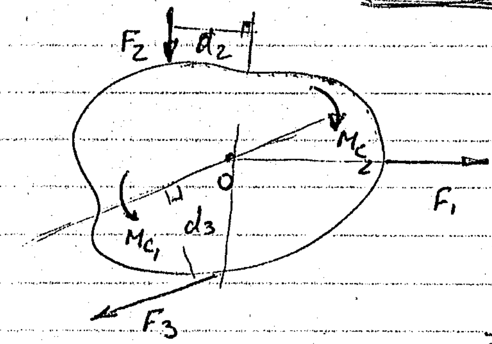
מערכת אקוויוולנטית THIS IS AN EQUIVALENT SYSTEM
TO THE ORIGINAL SYSTEM $\qquad$ M_c CAN BE APPLIED ANY WHERE USUALLY A
WILL PRODUCE SAME REACTIONS AT SUPPORTS AS ORIGINAL SYS

NOTE $\quad$ DIRECTION & MAGNITUDE OF F IS UNCHANGED
DIRECTION & MAGNITUDE OF M_c CAN BE OBTAINED FROM ORIGINAL SY
ABOUT O

סיכום הדבר

- IF A FORCE IS MOVED TO A PT "O" ALONG ITS LINE OF ACTION
  SIMPLY MOVE FORCE (PRINCIPLE OF TRANSMISSIBILITY)
- IF PT "O" IS NOT ON LINE OF ACTION FIND EQUIVALENT SYSTEM.
  MAGNITUDE & DIRECTION OF MOMENT IS MOMENT ABOUT "O".

כל כוחות שוקפי מישור אבל לא שוקפי נקודה
FOR A SET OF COPLANAR (NON COLLINEAR) FORCES & COUPLES



1. MOVE THE FORCES SO THAT THEY ARE COLLINEAR & ACCOUNT FOR THE COUPLE MOMENTS THEY PRODUCE ie F_3d_3, F_2d_2
2. ADD UP THE FORCES $\sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_R$
3. ADD UP THE MOMENTS

$$\vec{M}_{R_o} = \sum\vec{M}_o = \cancel{\vec{M}_1} + \vec{M}_2 + \vec{M}_3 + \vec{M}_{c_1} + \vec{M}_{c_2}$$

$\vec{M}_{c_1}$ & $\vec{M}_{c_2}$ are free vectors & can be moved to O
$\cancel{\vec{M}_1}, \vec{M}_2, \vec{M}_3$ depend on the $\perp$ distance between O & the forces $\cancel{F_1}, F_2, F_3$

$|\vec{F}_R|$ IS INDEPENDENT OF "O" & so is direction of $\vec{F}_R$

$\vec{M}_{R_o}$ will be $\perp$ $\vec{F}_R$ in a coplanar case

ובסכום לקביעת גודלו וקו פעולתו של השקול נדרשות שלוש משוואות

$$R_x = \Sigma F_x$$

$$R_y = \Sigma F_y$$

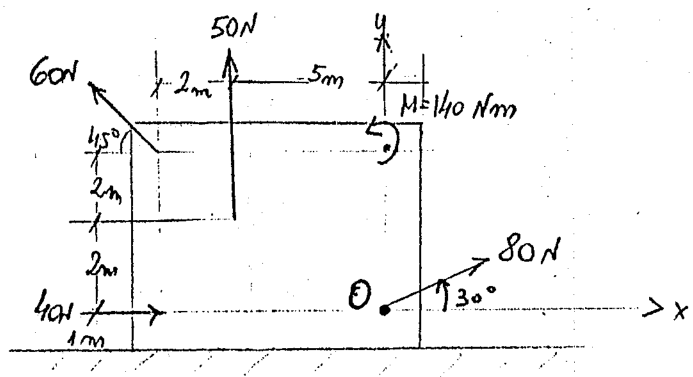
$$R d = \Sigma M_o$$

* במקרה הפרטי בו קוי הפעולה של כל הכוחות נחתכים בנקודה אחת מתמלאת המשוואה השלישית באופן טריוויאלי.

* עובדת היות שקול הכוחות 0 - אינה בהכרח גוררת שהמומנט יהיה 0 - כמו בצמד למשל. גם המשפט ההפוך נכון למשל בכוחות הנחתכים בנקודה אחת.

דוגמא

חשב את השקול וכוונו



$$R_x = \Sigma F_x = 40 + 80\cos 30° - 60\cos 45° = 66.9 \text{ N}$$

$$R_y = 50 + 80\sin 30° + 60\sin 45° = 132.4 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = 148.3 \text{ N}$$

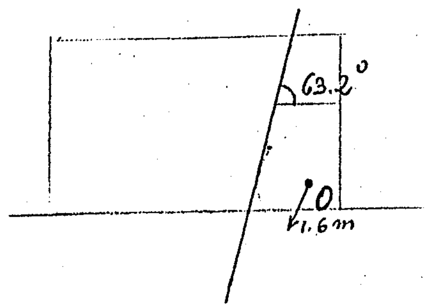
$$\theta = \tan^{-1}\frac{R_y}{R_x} = 63.2°$$

כעת נבקש למצוא את קו הפעולה של הכוח R. לשם כך נבצע מומנטים סביב O.

$$148.3d = 140 - 50\times 5 + 60\cos 45°\times 4 - 60\sin 45°\times 7 = -237.28 \text{ N-m}$$

$$d = -1.6 \text{ m}$$

הסימן השלילי פירושו ש-R צריך לייצר מומנט עם כוון השעון כלומר
עליו להמצא שמאלה ל-O במרחק 1.6m וכוון קו הפעולה θ



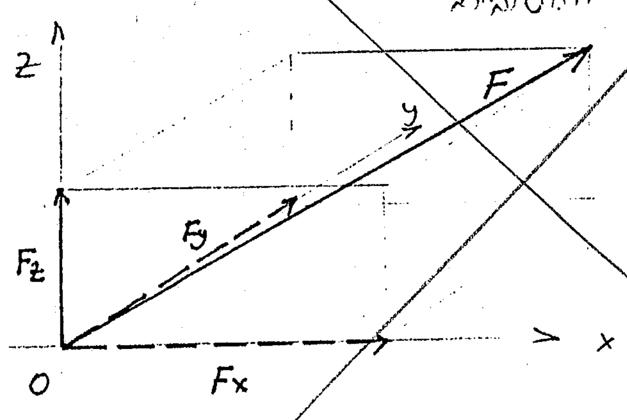
הערות:

1. הזוג M אינו תורם דבר לשקול R אך משפיע על מקומו
2. ב. בחירת -O כראשית הצירים וכמרכז לחשוב המומנטים נעשתה כדי שערכסומים כוחות (מ80 , מ40) יעברו דרכה ולא יתרמו למומנט.

חלק ב' מערכת תלת ממדית

2.6 כוחות קרטזיים

במעבר לתצורה תלת ממדית נשתמש בסימון וקטורי אשר יקל עלינו בהרבה את החישובים



כח F הפועל ב-O ניתן לפירוק במערכות xyz כדלקמן:

$$F_x = F\cos\theta_x$$

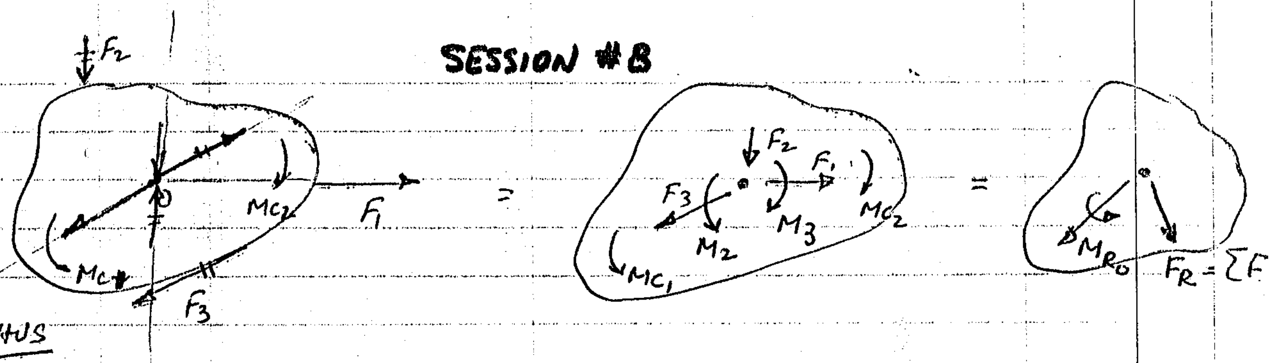
$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

בכתיב וקטורי:

SESSION #8



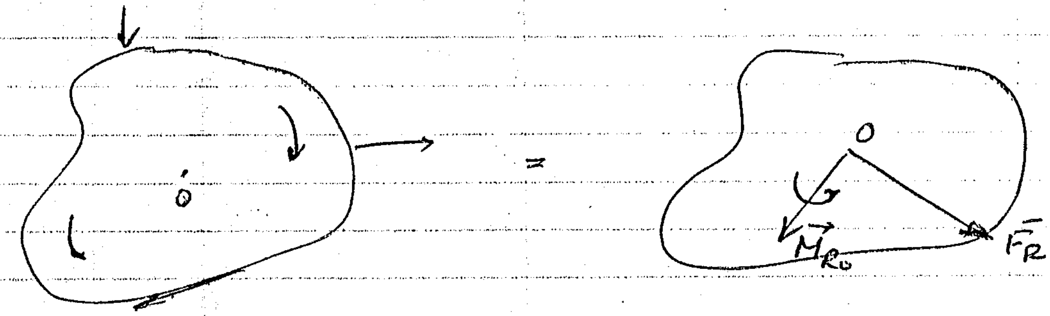
THUS

$$\vec{F}_R = F_{R_x}\bar{\imath} + F_{R_y}\bar{\jmath} = \sum\left(F_{1x}\bar{\imath} + F_{1y}\bar{\jmath}\right) + \left(F_{2x}\bar{\imath} + F_{2y}\bar{\jmath}\right) + \dots$$

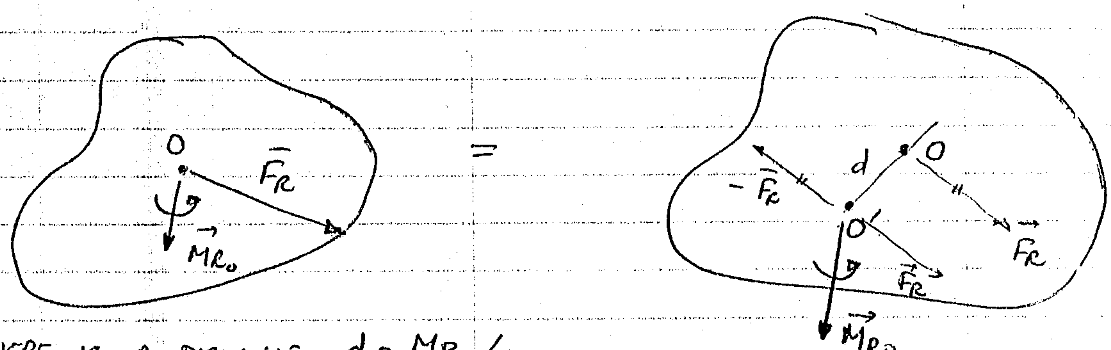
$$\left.\begin{aligned} F_{R_x} &= \sum F_x \\ F_{R_y} &= \sum F_y \end{aligned}\right\} \text{Just as before.}$$

$$\& \quad \vec{M}_{R_0} = \sum \vec{M}_0 = \sum M_{C_i}$$

WE HAVE REDURED THE SYSTEM OF FORCES & COUPLES TO AN EQUIVALENT SYSTEM OF ONE FORCE & 1 COUPLE

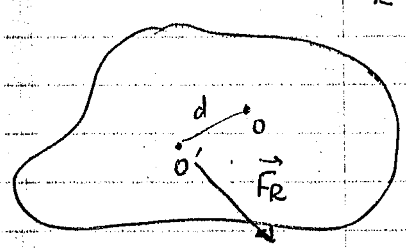


WE CAN REDUCE THIS EVEN FURTHER $\qquad \bar{M}_{R_0}$ is $\perp \bar{F}_R$



THERE IS A DISTANCE $d = M_{R_0}/F_R$

=



where $F_R \cdot d = M_{R_0}$ but has opposite direction to M_{R_0}

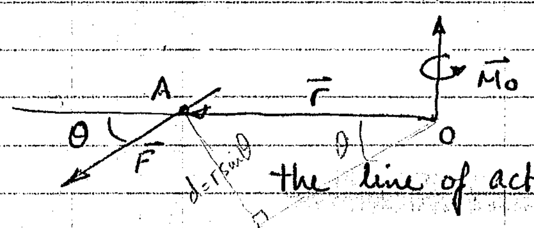
thus THE MOMENT M_{R_0} cancels with $F_R \cdot d$.

CAN BE DONE IN COPLANAR SYSTEMS SINCE $\bar{F}_R$ is $\perp$ $\bar{M}_{R_O}$

[redacted]

- ~~EXAM SEPT 24 1 PAGE BRING CALCULATOR~~
- ~~MR LAMBERTI WILL PROCTOR~~
- ~~COVER CH 1 – 3 SECT 4.7, 4.8, 4.9~~
- ~~3 OR 4 PROBLEMS 1 HR & 15 MIN~~
- ~~SEE ME NOW IF YOU CAN'T MAKE IT~~
- ~~I WILL BE IN MY OFFICE monday until 8PM · TUES TIL' 1PM.~~

MOMENT OF A VECTOR ABOUT A POINT O



$\vec{M}_O = \vec{r} \times \vec{F}$

$\vec{r}$ IS vector from O to any pt on the line of action of $\bar{F}$

$\vec{r} = \vec{r}_A - \vec{r}_O$

$M_o = |\vec{M}_o| = rF\sin\theta$ $\qquad r\sin\theta = d$ $\quad\perp$ distance between O & Line of Actio[n]

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \bar{i}\left(r_y F_z - r_z F_y\right) - \bar{j}\left(r_x F_z - r_z F_x\right) + \bar{k}\left(r_x F_y - r_y F_x\right)$$

direction of $\vec{M}_o$ is by the rh rule of the cross product

$\vec{M}_O$ is $\perp$ to $\vec{r}$ & $\vec{F}$

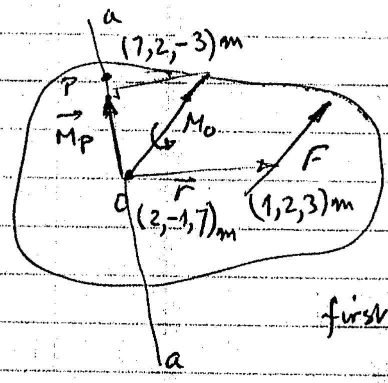
PRINCIPLE OF MOMENTS (VARIGNION'S THEOREM FOR NON CONCURRENT FORCES)

IF $\bar{F} = \bar{F}_1 + \bar{F}_2$ $\qquad \vec{M} = \vec{r} \times \bar{F} = \vec{r} \times \bar{F}_1 + \vec{r} \times \bar{F}_2 = \vec{r} \times (\bar{F}_1 + \bar{F}_2)$

MOMENT OF A FORCE ABOUT A SPECIFIED AXIS

LIKE FINDING THE PROJECTION OF A MOMENT ONTO A SPECIFIC AXIS

SUPPOSE WE WANT THE MOMENT ABOUT A POINT "O"



$\vec{M}_o = \vec{r} \times \vec{F}$

suppose we want to find the projection along the line aa that passes through "O"

first find unit vector along aa $\vec{u}_a$

now $M_P = |\vec{M}_P| = \vec{M}_o \cdot \vec{u}_a$ magnitude of proje[ction]

now $\vec{M}_P = M_P \vec{u}_a$ PROJECTION OF $\vec{M}_o$ along aa

$$\vec{M}_P = M_P \vec{u}_a = (\vec{M}_o \cdot \vec{u}_a)\vec{u}_a = (\vec{u}_a \cdot \vec{M}_o)\vec{u}_a$$

$$M_P = \vec{u}_a \cdot \vec{M}_o = \vec{u}_a \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}.$$

$$= u_{ax}(r_y F_z - r_z F_y) - u_{ay}(r_x F_z - r_z F_x) + u_{az}(r_x F_y - r_y F_z)$$

THE PT "O" IS AN ARBITRARY PT ON THE AXIS aa

Problem suppose $\bar{F} = -(5\bar{i} + 3\bar{j} + 2\bar{k})$ kN

$$\vec{r} = \bar{r}_A - \bar{r}_o = (1\bar{i} + 2\bar{j} + 3\bar{k})\,m - (2\bar{i} - \bar{j} + 7\bar{k})\,m$$
$$= (-1\bar{i} + 3\bar{j} - 4\bar{k})\,m$$

along aa $\bar{r}_{aa} = \bar{r}_P - \bar{r}_o = (7\bar{i} + 2\bar{j} - 3\bar{k})\,m - (2\bar{i} - \bar{j} + 7\bar{k})\,m$

$= (5\bar{i} + 3\bar{j} - 10\bar{k})\,m.$ $r_{aa} = \sqrt{5^2 + 3^2 + (\ )^2} = 11.$

$$\bar{u}_a = \frac{\bar{r}_{aa}}{r_{aa}} = .4319\bar{i} + .2592\bar{j} - .8503\bar{k}$$

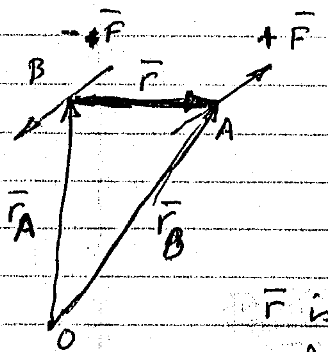
$$\underset{kN\text{-}m}{M_P} = \begin{vmatrix} .4319 & +.2592 & -.8503 \\ -1 & 3 & -4 \\ -5 & -3 & -2 \end{vmatrix} = -(.4319)[3 \cdot 2 - 3(-4)] + .2592[-1(2) - 5(-4)] + .8503[-1(3) - 5(3)]$$

$$= -.4319(18) + .2592(18) + .8503(-18) =$$
$$-7.7742 + 4.6656 - 15.3054 = -18.414 \text{ kN-m}$$

$$\vec{M}_P = M_P \vec{u}_a = -18.414\,\text{kNm}(.4319\bar{i} + .2592\bar{j} - .8503\bar{k}) = -7.95\bar{i} - 4.77\bar{j} + 15.66\bar{k}$$

kN-m

SECT 4.10 MOMENT OF A COUPLE



$\bar{M}_o = \bar{r}_A \times \bar{F} + \bar{r}_B \times (+\bar{F})$

$= (\bar{r}_B - \bar{r}_A) \times \bar{F} \quad = \bar{r} \times \bar{F}$

$\bar{r}_B = \bar{r} + \bar{r}_A \quad \therefore \quad \bar{r}_B - \bar{r}_A = \bar{r}$

$\bar{r}$ is the ~~distance~~ vector from a pt on the line of action of one force to the line of action of the other force

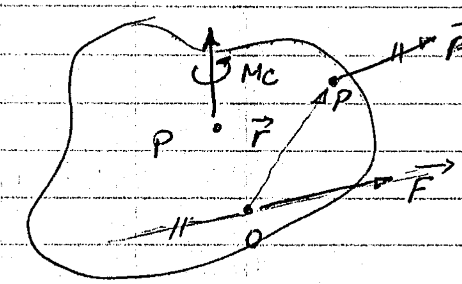
$\bar{M}_o$ is dependent on $\bar{r}$ & not $\bar{r}_A$ or $\bar{r}_B$ thus $\bar{M}_o$ is a free vector

SESSION # 9

EXAM

SESSION #10

MOVING A FORCE IN SPACE TO ANOTHER POINT

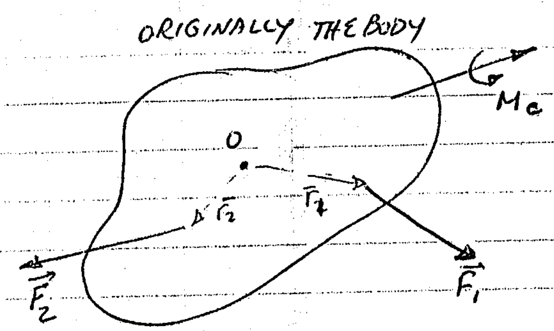


couple is $M_c = \vec{r} \times \vec{F}$ acts at any Pt

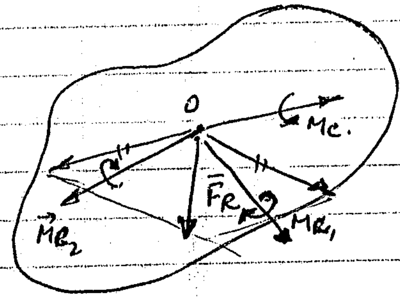
$\vec{r} = \vec{r}_P - \vec{r}_O$

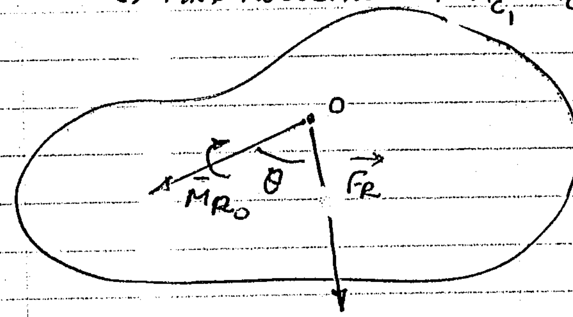
FOR A COUPLE OF FORCES

ORIGINALLY THE BODY



1. FIND POSITION VECTOR FROM O TO EACH LINE OF ACTION
2. MOVE VECTORS TO O & ADD TO BODY moment of couple $M_{c_1} = \bar{r}_1 \times \bar{F}_1$, $M_{c_2} = \bar{r}_2 \times \bar{F}_2$
3. FIND RESULTANT OF $\bar{F}_1$ & $\bar{F}_2$
4. MOVE $\bar{M}_c$ to O (free vector)
5. FIND RESULTANT OF $\bar{M}_{c_1} + \bar{M}_{c_2} + \bar{M}_{c_3} = M_{R_O}$





F_R & $\bar{M}_{R_O}$ WILL IN GENERAL NOT BE $\perp$ TO EACH OTHER i.e. $\theta \neq 90°$

HOWEVER $\bar{M}_{R_O}$ CAN BE RESOLVED INTO A COMPONENT $\bar{M}_{\parallel}$ & $\bar{M}_{\perp}$
$\bar{M}_{\parallel}$ is parallel to $\bar{F}_R$ & $\bar{M}_{\perp}$ is $\perp$ to $\bar{F}_R$

OPEN BOOK TO PAGE 130 - EXPLAIN HOW TO ELIMINATE $\bar{M}_{\perp}$

① $\bar{M}_{\parallel}$ is obtained as follows : $\bar{u}_R = \dfrac{\bar{F}_R}{F_R}$ now $M_{\parallel} = |M_{\parallel}| = \bar{M}_{R_O} \cdot \bar{u}_R$

and $\bar{M}_{\parallel} = M_{\parallel}\, \bar{u}_R = (\bar{M}_{R_O} \cdot \bar{u}_R)\, \bar{u}_R$

② $\bar{M}_{\perp} = \bar{M}_{R_O} - \bar{M}_{\parallel}$ is $\perp$ to $\bar{F}_R$ (why?) parallelogram law.

Now to eliminate $\bar{M}_{\perp}$ find $M_{\perp} = |\bar{M}_{\perp}|$ $|\bar{F}_R| = F_R$ and $d = M_{\perp}/F_R$.

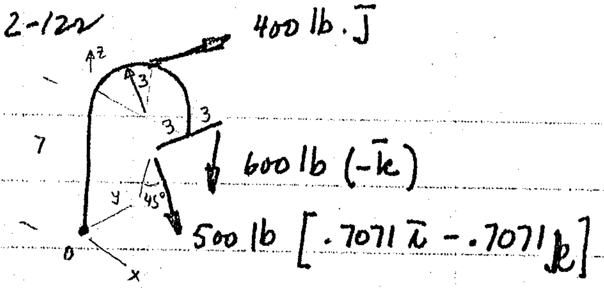
now move $\bar{F}_R$ to a pt d units away from O so that it causes a moment of equal magnitude to $M_{\perp}$ but of opposite direction

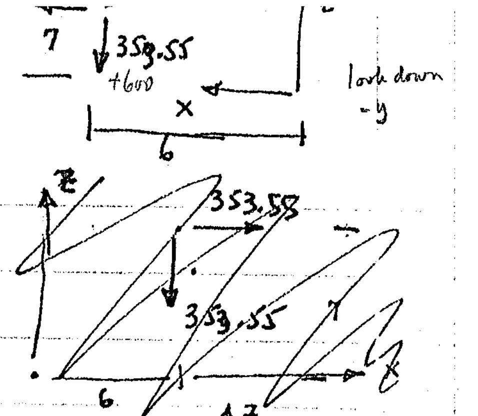
③ This is an equivalent force system where the Force $\bar{F}_R$ and $\bar{M}_{\parallel}$ are collinear (since $\bar{M}_{\parallel}$ is a free vector) $\equiv$ WRENCH.
מפתח לברגים

④ Any system of forces & moment can be reduced to a wrench.

Problem 2-122

הפעולה המשוגבת שבאה
משלושה הכוחות האלה
גורמת לכוח שקול ומומנט שקול

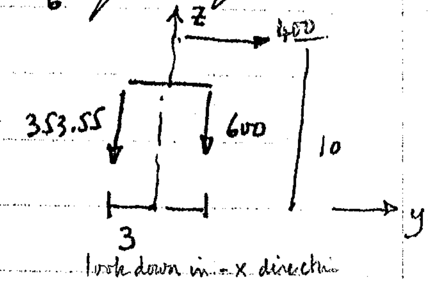


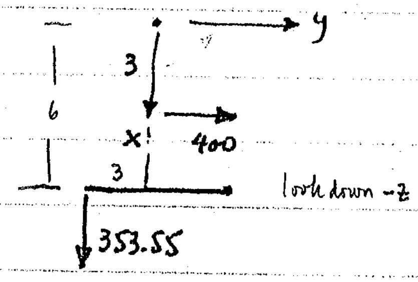


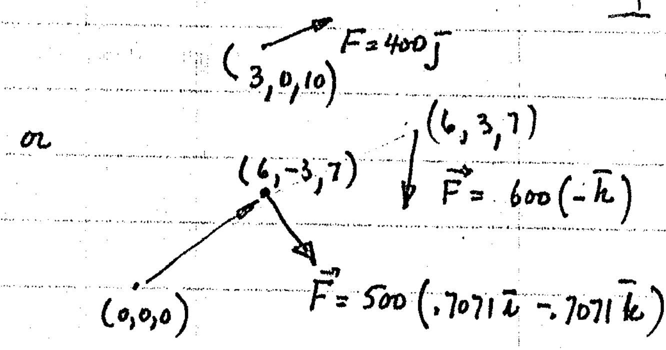
$$M_y = 6(353.55) + 7(353.55) + 600(6)$$

$$M_x = 353.55(3) - 600(3) - 400(10)$$

$$M_z = 353.55(3) + 400(3) = 753.55\times 3 = 2260.75$$







$$\Sigma \vec{M}_O = \begin{vmatrix} \bar{\imath} & \bar{J} & k \\ 6 & -3 & 7 \\ 353.55 & 0 & -353.55 \end{vmatrix} + \begin{vmatrix} \bar{\imath} & \bar{J} & k \\ 6 & +3 & 7 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} \bar{\imath} & \bar{J} & \bar{k} \\ 3 & 0 & 10 \\ 0 & 400 & 0 \end{vmatrix}$$

$$M_R = \Sigma M = \bar{\imath}\left[+3(353.55) - 3(600) - 400(10)\right] + \bar{J}\left[+6(353.55) + 7(353.55) + 6(600)\right] + k\left[3(353.55) + 3(400)\right]$$

$$= \bar{\imath}[-4739.35] + \bar{J}[8196.15] + \bar{k}[2260.65]$$

$$\underline{M_R = 9733.9 \text{ lb-ft}}$$

Also $\Sigma \bar{F} = 353.55\bar{\imath} - 353.55\bar{k} - 600\bar{k} + 400\bar{J} = 353.55\bar{\imath} + 400\bar{J} - 953.55\bar{k}$

$$\underline{F_R = 1092.82 \text{ lb-ft}}$$

$$\underline{u}_R = \frac{F_R}{F_R} = .324\,\underline{i} + .366\,\underline{j} - .873\,\underline{k}$$

$$\underline{M}_{\parallel} = (\underline{M}_R \cdot \underline{u}_R)\underline{u}_R = \left[(.324)(-4739.35) + (.366)(8196.15) + (-.873)(2260.65)\right]\underline{u}_R = -509.31(.324\,\underline{i} + .366\,\underline{j} - .873\,\underline{k})$$

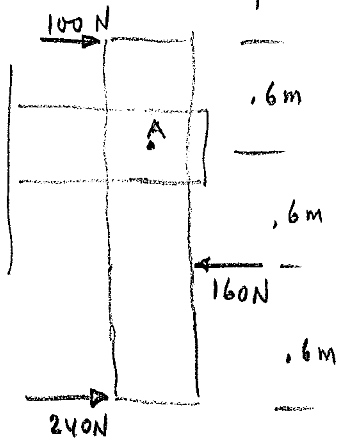
$$= (-165.02\,\underline{i} - 186.41\,\underline{j} + 444.62\,\underline{k}) \text{ lb-ft.}$$

$$\underline{M}_{\perp} = \underline{M}_R - \underline{M}_{\parallel} = (-4574.33\,\underline{i} + 8382.56\,\underline{j} - 2074.24\,\underline{k}) \text{ lb-ft.} \qquad M_{\perp} = 9772.12 \text{ lb-ft}$$

$$d = \frac{M_\perp}{F_R} = 8.94 \text{ ft}$$

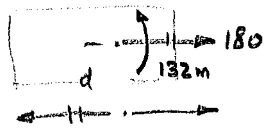
[illegible]

בכוח שקול יחידי מצוא את המרחק d מתחת נקודה A לקו הפעולה שלו



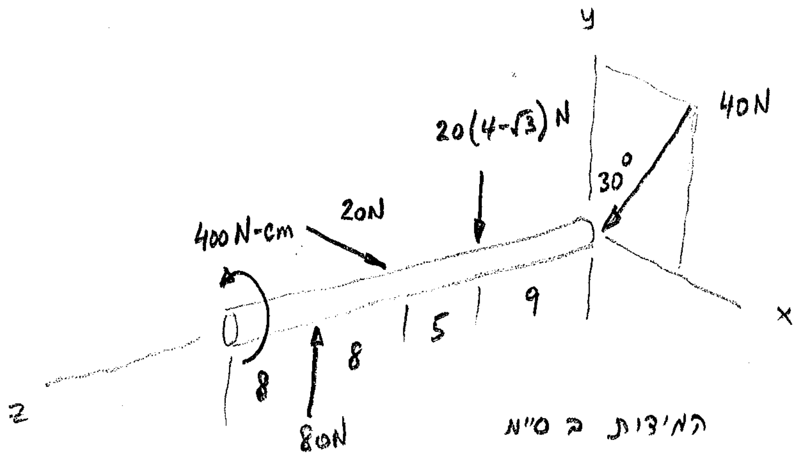
$$\Sigma \underline{F} = 100 - 160 + 240 = 180\,N$$

$$+\circlearrowleft\ \Sigma \underline{M}_A = 100(.6) + 160(.6) - 240(1.2) = -132\ N\text{-}m$$



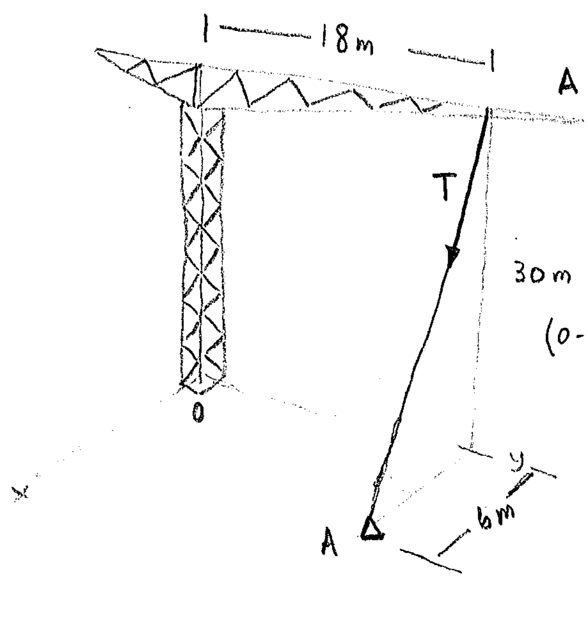
$$d = \frac{132}{180} = 0.733\,m$$

תרגילי בית



המידות בס"מ

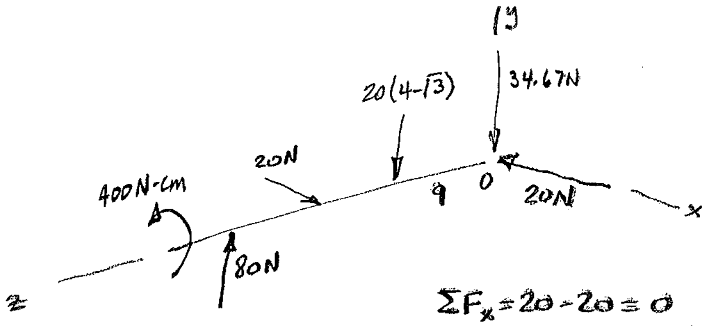
- מצוא כוח שקול ומומנט שקול
- מצוא מפתח לברגים



<u>עגורן</u> להרים משקל A

מפתח T=21kN בכבל.

- מצוא המומנט שמיוצר בכוח T מסביב בסיס העגורן (ב-O)



$\Sigma F_x = 20 - 20 = 0$

$\Sigma F_y = -34.64 - 20(4-\sqrt{3}) + 80 = -34.64 + 34.64 = 0$

$\Sigma \underline{M}_0 = 20(4-\sqrt{3})9\underline{i} + 20(14)\underline{j} - 80(22)\underline{i} + 400\underline{k}$ N-m

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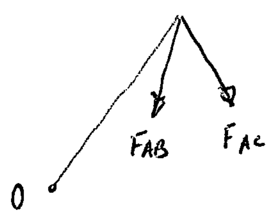
2-113



מצאנו הכוחות

$\underline{F}_{AB} = 19.22\underline{i} + 4.80\underline{j} - 28.84\underline{k}$ N

$\underline{F}_{AC}\ -13.35\underline{i} + 26.75\underline{j} - 40.10\underline{k}$ N

$\underline{F}_R = \underline{F}_{AB} + \underline{F}_{AC} = -5.87\underline{i} + 31.55\underline{j} - 68.94\underline{k}$

מומנט מסביב O

$\underline{r}_{A/0} = (+4\underline{j} + 6\underline{k})$ m

לכן

$$(\underline{r}_{A/0} \times \underline{F}_R) = \left| \begin{matrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 6 \\ -5.87 & 31.55 & -68.94 \end{matrix} \right| \begin{matrix} i & j \\ 0 & 4 \\ -5.87 & 31.55 \end{matrix}$$

$= \underline{i}\left(4(-68.94) - 6(31.55)\right) - \underline{j}\left(0(-68.94) - 6(-5.87)\right) + \underline{k}\left(0 \cdot 31.55 - 4(-5.87)\right)$

$\underline{M}_0 = -465.06\,\underline{i} - 35.22\underline{j} + \underline{k}\,23.48$ N-m

$\underline{M}_0 \perp \underline{F}_R$

נמצא את $\underline{M}_\parallel$ ו- $\underline{M}_\perp$: $\underline{M}_\parallel$ קודם

$F_R = \sqrt{(5.87)^2 + (31.55)^2 + (68.94)^2} = 76.04$ N

$\underline{u}_R = \frac{\underline{F}_R}{F_R} = -0.077\underline{i} + .415\underline{j} - .9066\underline{k}$

לכן

$M_\parallel = \underline{M}_0 \cdot \underline{u}_R = 14.48$ N-m

ו

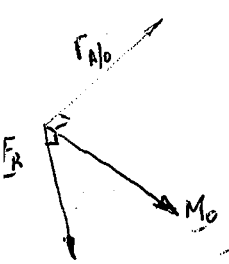
$\underline{M}_\parallel = M_\parallel \underline{u}_R = 14.48(-0.077\underline{i} + .415\underline{j} - .907\underline{k})$

$= (-1.12\underline{i} + 6.01\underline{j} - 13.13\underline{k})$ N-m

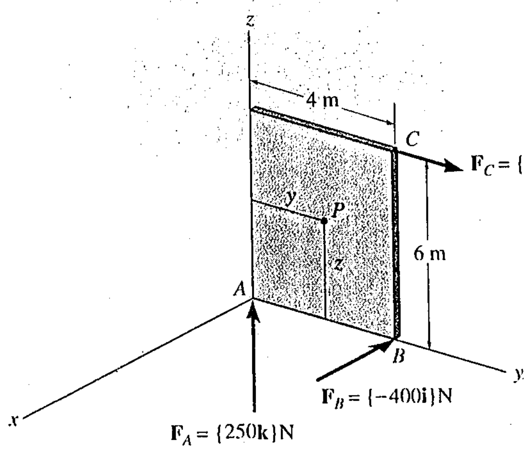
$\underline{M}_\perp = \underline{M}_0 - \underline{M}_\parallel = (-654.36\underline{i} - 35.22\underline{j} + 23.48\underline{k}) - (-1.12\underline{i} + 6.01\underline{j} - 13.13\underline{k})$

$= (-653.24\underline{i} - 41.23\underline{j} + 36.61\underline{k})$ N-m

כדי להסיר את $\underline{M}_\perp$

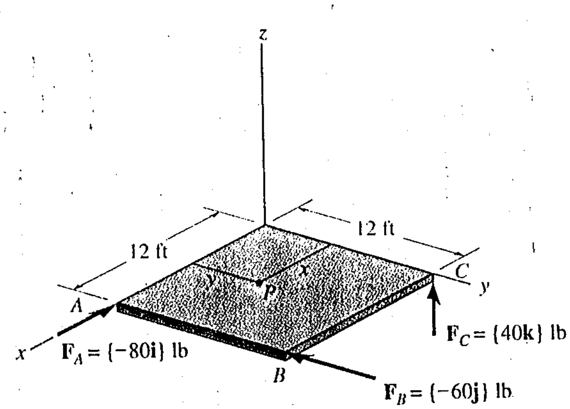


4–121. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.



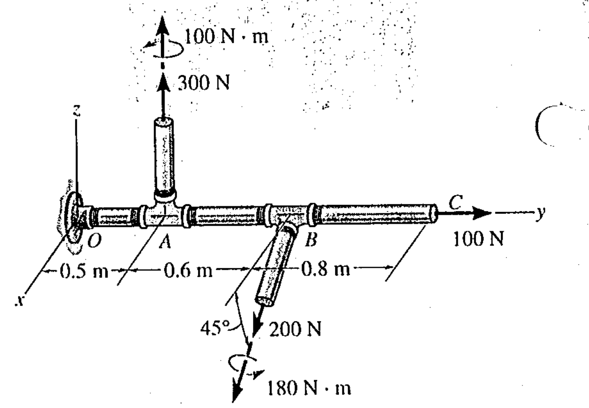
Prob. 4–121

4–122. Replace the three forces acting on the plate by a wrench. Specify the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



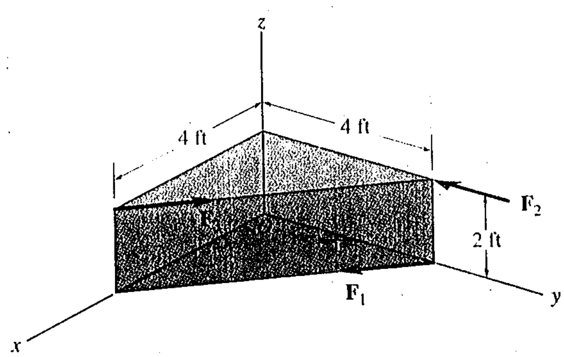
Prob. 4–122

4–123. Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



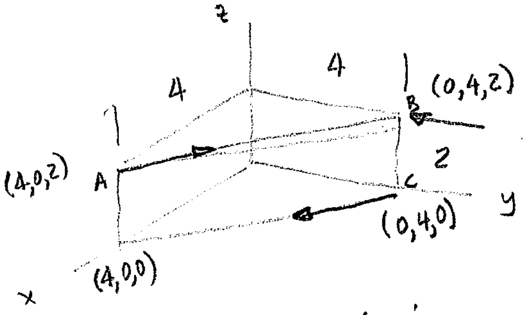
Prob. 4–123

***4–124.** The three forces acting on the block each have a magnitude of 10 lb. Replace this system by a wrench and specify the point where the wrench intersects the z axis measured from point O.



Prob. 4–124

4-124



$\underline{u}_3 = \dfrac{-4i + 4j + 0k}{4\sqrt{2}} = -\dfrac{i}{\sqrt{2}} + \dfrac{j}{\sqrt{2}}$

$\underline{u}_1 = \dfrac{4i - 4j + 0k}{4\sqrt{2}} = \dfrac{i}{\sqrt{2}} - \dfrac{j}{\sqrt{2}}$

$\underline{u}_2 = -\underline{j}$

$r_{A/O} = 4i + 0j + 2k$

$r_{B/O} = 0i + 4j + 2k$

$r_{C/O} = 0i + 4j + 0k$

$F_R = F_1 + F_2 + F_3 = 10\left(\dfrac{i - j}{\sqrt{2}}\right) + (-10\underline{j}) + 10\left(\dfrac{-i + j}{\sqrt{2}}\right) = -10\underline{j}$

$F_R = 10 \quad u_R = -j$

$\sum M_O = r_{A/O} \times F_3 + r_{C/O} \times F_1 + r_{B/O} \times F_2$

$\begin{vmatrix} i & j & k \\ 4 & 0 & 2 \\ -\frac{10}{\sqrt{2}} & \frac{10}{\sqrt{2}} & 0 \end{vmatrix} \begin{matrix} i & j \\ 4 & 0 \\ -\frac{10}{\sqrt{2}} & \frac{10}{\sqrt{2}} \end{matrix} \quad \begin{vmatrix} i & j & k \\ 0 & 4 & 0 \\ \frac{10}{\sqrt{2}} & -\frac{10}{\sqrt{2}} & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 4 \\ \frac{10}{\sqrt{2}} & -\frac{10}{\sqrt{2}} \end{matrix} \quad \begin{vmatrix} i & j & k \\ 0 & 4 & 2 \\ 0 & -10 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 4 \\ 0 & -10 \end{matrix}$

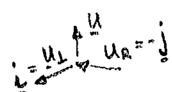
$M_O = -\dfrac{20}{\sqrt{2}}j + \cancel{\dfrac{40}{\sqrt{2}}k} - \dfrac{20}{\sqrt{2}}i - \cancel{\dfrac{40}{\sqrt{2}}k} + 20i = +20i - \dfrac{20}{\sqrt{2}}i - \dfrac{20}{\sqrt{2}}j$

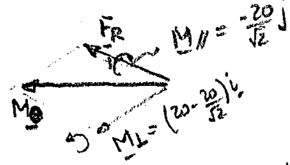
$M_\parallel = M_O \cdot u_R = \left[\left(20 - \dfrac{20}{\sqrt{2}}\right)i - \dfrac{20}{\sqrt{2}}j\right] \cdot (-j) = \dfrac{20}{\sqrt{2}}$ $\qquad u_R = u_\parallel$

$\underline{M}_\parallel = M_\parallel u_R = -\dfrac{20}{\sqrt{2}}j$

$\underline{M}_\perp = \underline{M}_O - \underline{M}_\parallel = \left(20 - \dfrac{20}{\sqrt{2}}\right)i \quad \Rightarrow \quad u_\perp = i$

$\underline{M}_\perp = M_\perp u_\perp = \left(20 - \dfrac{20}{\sqrt{2}}\right)i$



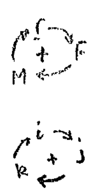


$d = F \times M$

$u_3 = u_\parallel \times u_\perp$
$= (-j) \times i = -(-k) = k$

$d = \dfrac{M_\perp}{F_R} = \dfrac{20 - 20/\sqrt{2}}{10} = 2 - \sqrt{2} = 2 - 1.414 = .586$

$M = r \times F$



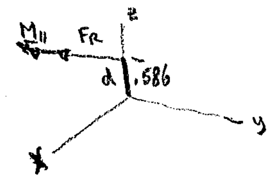
$\underline{M}_\perp = d\,\underline{u}_3 \times F_R$

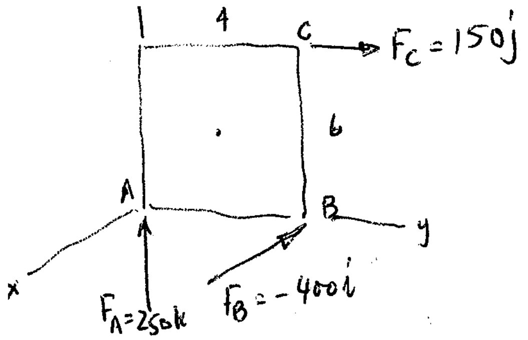
$\left(20 - \dfrac{20}{\sqrt{2}}\right)i = d\,u_3 \times (-10j)$

$= d\,k \times (-10j)$

$\left(20 - \dfrac{20}{\sqrt{2}}\right)i = 10d\,i$

$\underline{d = .586\ m}$





החלף את הכוחות האלו שפועלים על הטבלה במפתח לבריגים. תפרט את השקול והמומנט של המפתח ומיקום הפעולתי P(y,z) שבו קו הפעולה חותך את הטבלה

$$\underline{F}_R = -400\underline{i} + 150\underline{j} + 250\underline{k} \quad N \qquad F_R = 494.975N$$

$$\underline{M}_A = (6k \times 150j) + 4j \times (-400\underline{i})$$
$$= (-900\,\underline{i} + 1600\underline{k})\,Nm$$

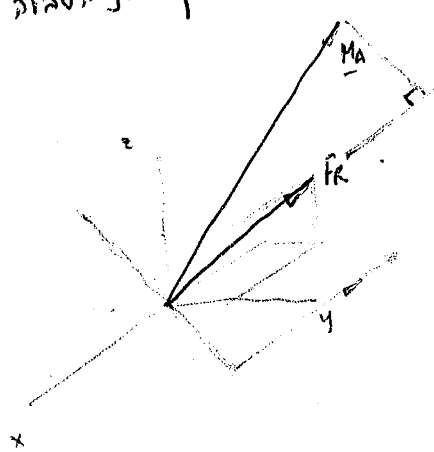
$$\underline{F}_R \cdot \underline{M}_A = 36\times10^4 + 40\times10^4 \neq 0 \Rightarrow \underline{F}_R \not\parallel \underline{M}_A$$

$$\underline{u}_R = \frac{\underline{F}_B}{F_R} = -.808\underline{i} + .303\underline{j} + .505\underline{k}$$

$$M_{\parallel} = \underline{M}_A \cdot \underline{u}_R = -900(-.808) + 1600(.505) = 1535.2 \ N\text{-}m$$

$$\underline{M}_{\parallel} = M_{\parallel}\underline{u}_R = (1535.2)(-.808\underline{i} + .303\underline{j} + .505\underline{k}) \quad N\text{-}m \qquad M_{\parallel} = 1535\,Nm$$
$$= -1240.4\underline{i} + 465.2\underline{j} + 775.3\underline{k}$$

$$\underline{M}_{\perp} = \underline{M}_A - \underline{M}_{\parallel} = (-900\underline{i} + 1600\underline{k}) - (-1240.4\underline{i} + 465.2\underline{j} + 775.3\underline{k})$$
$$= 340.4\underline{i} - 465.2\underline{j} + 824.7\underline{k} \quad N\text{-}m$$



$$M_{\perp} = 1006.2 \ N\text{-}m$$
$$\underline{u}_{\perp} = \underline{M}_{\perp}/M_{\perp} = .338\underline{i} - .462\underline{j} + .820\underline{k}$$
$$d = \frac{M_{\perp}}{F_R} = \frac{1006.2}{494.975} = 2.03m$$

$$\underline{u}_R \times \underline{u}_{\perp} = \underline{u}_b$$
$$\begin{bmatrix} i & j & k \\ -.808 & .303 & .505 \\ .338 & -.462 & .820 \end{bmatrix}$$

$$P(y,z) =$$

$$\cancel{\underline{d} = d\,\underline{u}_R = -.686\underline{i} + .938\underline{j} + 1.665\underline{k}}$$
$$\underline{d} = d\,\underline{u}_b$$

u_R (0,y,z)

$-.686, +.938, -1.665$

$$\underline{r}_{P/d} = +.686\underline{i} + (y + .938)\underline{j} + (z + 1.665)\underline{k}$$
$$r_{P/D} = \sqrt{(.686)^2 + (y+.938)^2 + (z+1.665)^2}$$
$$+.686/r_{PD} = -.808 \qquad (y+.938)/r_{PD} = .303$$

$$\cancel{r_{PD} = \frac{.686}{-.808} = -.849}$$
$$\cancel{y = (.303)(-.849) + .938 = +.68}$$
$$\cancel{z = .505(-.849) + 1.665 = 2.09}$$

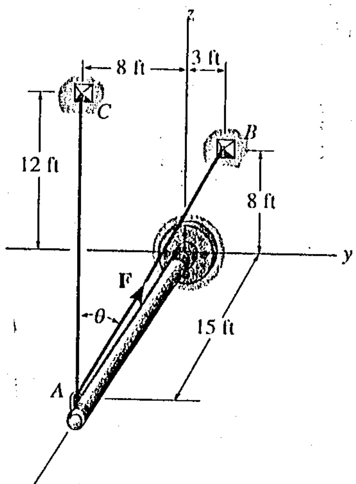
$$\cancel{\underline{r}_{B/D} = (-6\underline{i} + 4\underline{j})}$$

$$\cancel{\underline{M}_D = \underline{r}_{B/D} \times \underline{F}_B = \begin{vmatrix} i & j & k \\ -6 & 4 & 0 \\ -400 & 0 & 0 \end{vmatrix} = +1600\underline{k}}$$

$$\cancel{\underline{F}_R = -400\underline{i} + 150\underline{j} + 250\underline{k}}$$
$$\cancel{\underline{u}_R = -.808\underline{i} + .303\underline{j} + .505\underline{k}}$$
$$\cancel{M_{\parallel} = \underline{M}_D \cdot \underline{u}_R = 808 \ Nm}$$
$$\cancel{\underline{M}_{\parallel} = M_{\parallel}\underline{u}_R = 808(-.808\underline{i} + .303\underline{j} + .505\underline{k})}$$
$$\cancel{= -652.9\underline{i} + 244.8\underline{j} + 408.0\underline{k} \quad N\text{-}m}$$
$$\cancel{\underline{M}_{\perp} = \underline{M}_D - \underline{M}_{\parallel} = 652.9\underline{i} - 244.8\underline{j} + 1192\underline{k} = 1381 \ N\text{-}m}$$

2–110. Determine the angle θ between cables AB and AC.

2–111. If $\mathbf{F}$ has a magnitude of 55 lb, determine the magnitude of its projected component acting along the x axis and along cable AC.

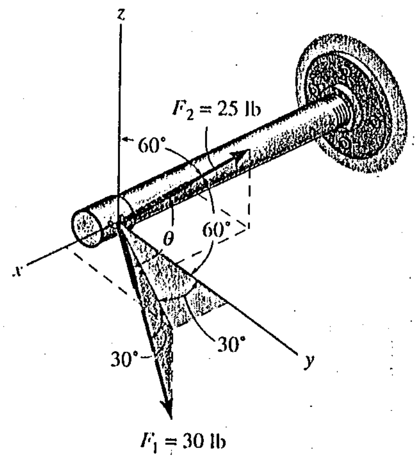


Probs. 2–110/2–111

***2–112.** Determine the angles θ and ϕ between the axis OA of the pole and each cable, AB and AC.

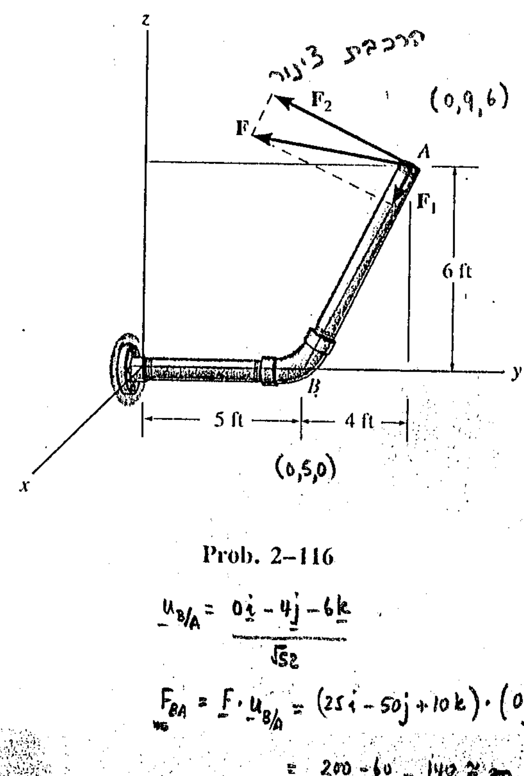
2–114. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.

2–115. Determine the angle θ between the two cables attached to the pipe.



Probs. 2–114/2–115

***2–116.** The force $\mathbf{F} = \{25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}\}$ N acts at the end A of the pipe assembly. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of AB and perpendicular to it.



Prob. 2–116

$u_{B/A} = \dfrac{0\underline{i} - 4\underline{j} - 6\underline{k}}{\sqrt{52}}$

$F_{BA} = \underline{F} \cdot u_{B/A} = (25\underline{i} - 50\underline{j} + 10\underline{k}) \cdot \dfrac{(0\underline{i} - 4\underline{j} - 6\underline{k})}{\sqrt{52}}$

$= \dfrac{200 - 60}{\sqrt{52}} = \dfrac{140}{\sqrt{52}} = 19.41\ N$

$\underline{F_1} = F_{BA} = \dfrac{140}{\sqrt{52}} u_{B/A} = \dfrac{140}{52}(0\underline{i} - 4\underline{j} - 6\underline{k})$

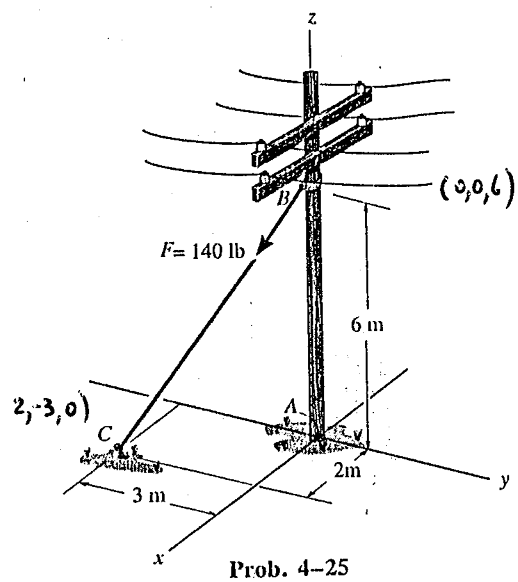
$= \dfrac{-560\underline{j} - 840\underline{k}}{52} = -10.77\underline{j} - 16.15\underline{k}$

$\underline{F_2} = \underline{F} - \underline{F_1} = (25\underline{i} - 50\underline{j} - 10\underline{k}) + \dfrac{560\underline{j} + 840\underline{k}}{52}$

$= 25\underline{i} - \left(50 - \dfrac{560}{52}\right)\underline{j} - \left(10 - \dfrac{840}{52}\right)\underline{k}$

$= 25\underline{i} - (39.23)\underline{j} + 6.15\underline{k}$

4–25. The cable exerts a 140-N force on the telephone pole as shown. Determine the moment of this force at the base A of the pole. Solve the problem two ways, i.e., by using a position vector from A to C, then A to B.



Prob. 4–25

$M_A = 360\underline{i} + 240\underline{j}$

$\underline{r}_{C/B} = 2\underline{i} - 3\underline{j} - 6\underline{k}$

$r_{C/B} = 7$

$u_{CB} = \frac{2}{7}\underline{i} - \frac{3}{7}\underline{j} - \frac{6}{7}\underline{k} \Rightarrow \underline{F} = 40\underline{i} - 60\underline{j} - 120\underline{k}$

$r_{B/A} = 0\underline{i} + 0\underline{j} + 6\underline{k}$

$\begin{vmatrix} i & j & k \\ 0 & 0 & 6 \\ 40 & -60 & -120 \end{vmatrix} = -6(-60\underline{i} - 40\underline{j}) = 360\underline{i} + 240\underline{j}$

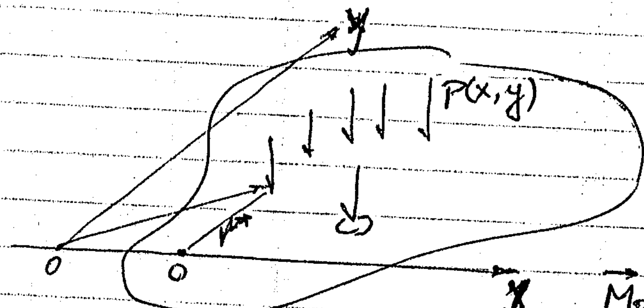
- WRENCH causes translation (due to $\vec{F}_R$) and rotation (due to $\vec{M}_{\parallel}$).
- Axis of the wrench & pt through which it acts are unique.

DISTRIBUTED LOADINGS

SESSION #13

- CAUSED BY WIND, FLUIDS, MATERIAL WEIGHT

INTENSITY OF A LOAD IS A FORCE/UNIT AREA - intensity an infinite number of parallel ~~forces~~ FORCES acting on a differential dx dy



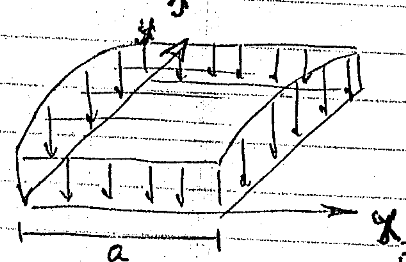
$$p(x,y) = \lim_{\Delta A} \frac{\Delta \vec{F}}{\Delta A} = \frac{d\vec{F}}{dA}$$

$$\vec{F}_R = \sum d\vec{F} = \iint -p(x,y)\,dx\,dy\,\vec{k}$$

$$\vec{M}_{R_o} = \sum \vec{r} \times d\vec{F} = -\iint p(x,y)\vec{k} \times (r_x\vec{i} + r_y\vec{j})\,dx\,dy$$
$$= +\iint [r_x\, p(x,y)\vec{j} - r_y\, p(x,y)\vec{i}]\,dx\,dy$$

- REPLACE THIS FORCE SYSTEM BY AN EQUIVALENT FORCE $\vec{F}_R$ SO THAT ABOUT SOME PT THE SAME OVERALL MOMENT IS PRODUCED

SUPPOSE ① body is symmetric and the loading $p(x,y)$ is a constant in the x direction ie $\int dx\, p(x,y) = W(y) = p(y)a$ [lb/ft] are units



$$\vec{F}_R = \sum d\vec{F} = -\int W(y)\,dy\,\vec{k}$$ EQUAL TO AREA UNDER

$$\vec{M}_{R_o} = +\int r_x\, W(y)\,dy\,\vec{j} - \int r_y\, W(y)\,dy\,\vec{i}$$

$$-\vec{F}_R \times \vec{r} = (\bar{x}\vec{i} + \bar{y}\vec{j}) \times \left[-\int W(y)\,dy\,\vec{k}\right]$$
$$= \bar{x}\int W(y)\,dy\,\vec{j} - \bar{y}\int W(y)\,dy\,\vec{i} = \vec{M}_{R_o}$$

thus $\bar{x}\int W(y)\,dy = +\int r_x\, W(y)\,dy$ or $\bar{x} = \dfrac{+\int r_x\, W(y)\,dy}{\int W(y)\,dy}$ $\quad \bar{y} = r_y$

area under loading curve

$$\bar{x} = \frac{\iint x\, p(x,y)\,dx\,dy}{\iint p(x,y)\,dx\,dy}$$ for $p(x,y) = P(y)$ only

$$= \frac{\int x\,dx \int P(y)\,dy}{\int dx \int P(y)\,dy} = \frac{x^2}{2}\Big|_0^a \Big/ x\Big|_0^a = \frac{a}{2}$$

$W(y) = \int p(x,y)\,dx = \int P(y)\,dx = P(y)$

LOADING unit [lb/ft]

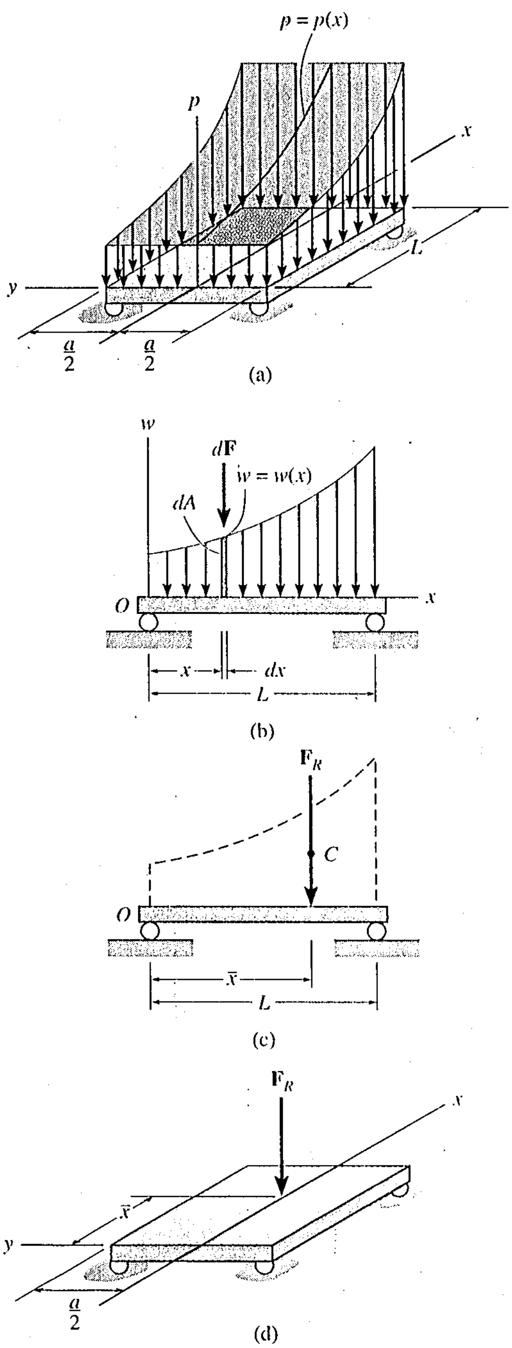


Fig. 4–51

Magnitude of Resultant Force. From Eq. 4–19 ($F_R = \Sigma F$), the magnitude of $\mathbf{F}_R$ is equivalent to the sum of all the forces in the system. In this case integration must be used, since there is an infinite number of parallel forces $d\mathbf{F}$ acting along the plate, Fig. 4–51*b*. Since $d\mathbf{F}$ is acting on an element of length dx, and $w(x)$ is a force per unit length, then at the location x, $dF = w(x)\ dx = dA$. In other words, the magnitude of $d\mathbf{F}$ is determined from the colored differential *area* dA under the loading curve. For the entire plate length,

$$+\downarrow F_R = \Sigma F; \qquad \boxed{F_R = \int_L w(x)\ dx = \int_A dA = A} \qquad (4\text{–}21)$$

Hence, the magnitude of the resultant force is equal to the total area under the loading diagram $w = w(x)$.

Location of Resultant Force. Applying Eq. 4–20 ($M_{R_O} = \Sigma M_O$), the location $\bar{x}$ of the line of action of $\mathbf{F}_R$ can be determined by equating the moments of the force resultant and the force distribution about point O (the y axis). Since $d\mathbf{F}$ produces a moment of $x\ dF = x\ w(x)\ dx$ about O, Fig. 4–51*b*, then for the entire plate, Fig. 4–51*c*,

$$\curvearrowleft + M_{R_O} = \Sigma M_O; \qquad \bar{x}F_R = \int_L x\ w(x)\ dx$$

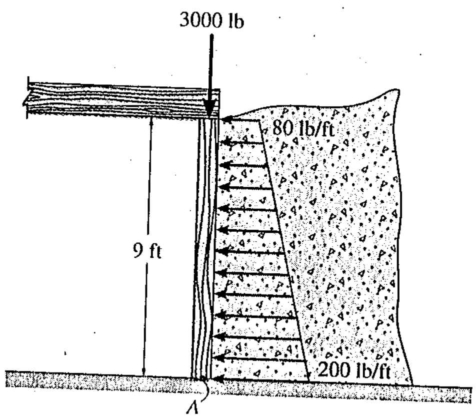
Solving for $\bar{x}$, using Eq. 4–21, we can write

$$\boxed{\bar{x} = \frac{\displaystyle\int_L x\ w(x)\ dx}{\displaystyle\int_L w(x)\ dx} = \frac{\displaystyle\int_A x\ dA}{\displaystyle\int_A dA}} \qquad (4\text{–}22)$$

This equation represents the x coordinate for the geometric center or *centroid* of the *area* under the distributed-loading diagram $w(x)$. *Therefore, the resultant force has a line of action which passes through the centroid C (geometric center) of the area defined by the distributed-loading diagram $w(x)$*, Fig. 4–51*c*.

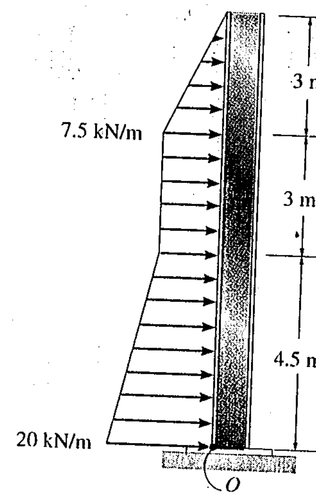
Once $\bar{x}$ is determined, $\mathbf{F}_R$ by symmetry passes through point ($\bar{x}$, 0) on the surface of the plate, Fig. 4–51*d*. If we now consider the three-dimensional pressure loading $p(x)$, Fig. 4–51*a*, we can therefore conclude that *the resultant force has a magnitude equal to the volume under the distributed-loading curve $p = p(x)$ and a line of action which passes through the centroid (geometric center) of this volume*. Detailed treatment of the integration techniques for computing the centroids of volumes or areas is given in Chapter 9. In many cases, however, the distributed-loading diagram is in the shape of a rectangle, triangle, or other simple geometric form. The centroids for such common shapes do not have to be determined from Eq. 4–22; rather, they can be obtained directly from the tabulation given on the inside back cover.

4–130. The column is used to support the floor which exerts a force of 3000 lb on the top of the column. The effect of soil pressure along the side of the column is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column measured from its base A.



Prob. 4–130

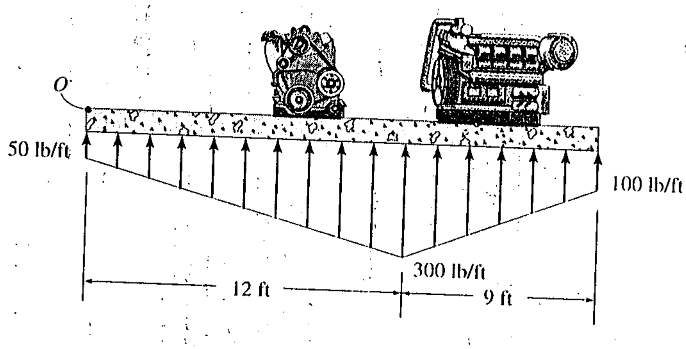
***4–132.** Replace the loading by an equivalent resultant force and couple moment acting at point O.



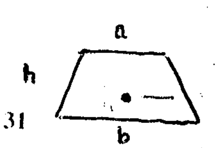
Prob. 4–132

הפצת עומס האדמה מתחת טבלת בניין

4–131. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location measured from point O.



Prob. 4–131

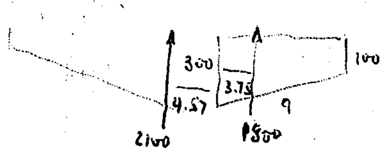


$A = \frac{1}{2} h (a+b)$

$y_c = \frac{1}{3}\left(\frac{2a+b}{a+b}\right) h$

$A = \frac{1}{2} 12 (350) = 2100$ 50

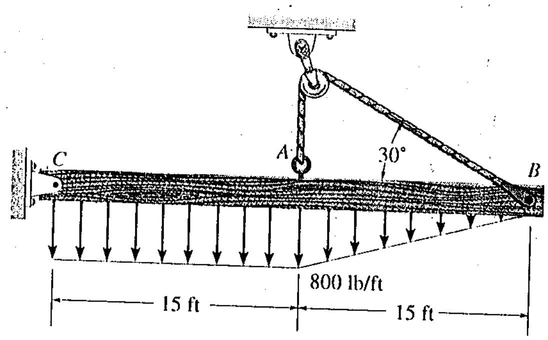
$y_c = \frac{1}{3} \cdot 12 \left(\frac{400}{350}\right) = 4.57$



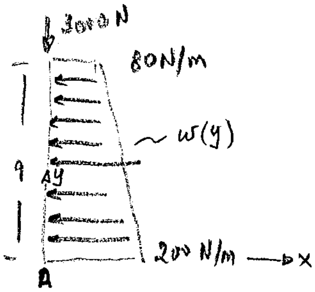
$y_c = \left(\frac{500}{400}\right) 3$

$A = \frac{1}{2} \cdot 9 (400) = 1800$

4–133. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam measured from the pin at C.



Prob. 4–133



$$w(y) = \frac{80-200}{9}y + 200$$

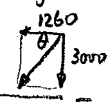
$\frac{80-200}{9} =$ שיפוע של עומס המפורס

$$F_x = \int_0^9 w(y)\,dy = \frac{80-200}{9}\frac{y^2}{2}\Big|_0^9 + 200y\Big|_0^9 = -60(9) + 200(9) = 1260\ N$$

$$M_A = \int_0^9 y\,w(y)\,dy = \frac{80-200}{9}\frac{y^3}{3}\Big|_0^9 + 200\frac{y^2}{2}\Big|_0^9 = -120(27) + 200(40.5) = 4860\ N\ +\circlearrowleft$$

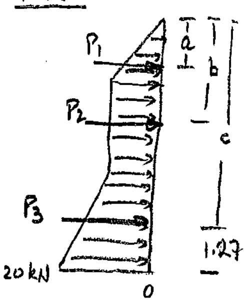
$$d = \frac{M_A}{F_x} = \frac{4860}{1260} = 3.857\ m$$

$$F_R = (F_x^2 + F_y^2)^{1/2} = ((1260)^2 + (3000)^2)^{1/2} = 3254\ N$$



$\tan\theta = \frac{-3000}{-1260} = +2.381 \qquad \theta = 67.22^\circ$

4-132



$P_1 = \frac{7.5\times 3}{2} = 11.25\,kN \qquad a = 2\,m$

$P_2 = 7.5\times 3 = 22.5\,kN \qquad b = 4.5\,m$

$P_3 = \frac{(20+7.5)}{2}\cdot\cancel{4.5} = \overset{61.88}{\cancel{41.25}}\,kN \qquad c = 9 - \frac{\cancel{3}}{\cancel{3}}\left(\frac{2\cdot 7.5+20}{27.5}\right) = 9 - \frac{35}{27.5} = 7.73$

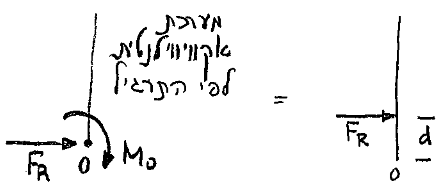
$F_R = P_1 + P_2 + P_3 = \sum_{i=1}^{3} P_i = \overset{95.63}{\cancel{75}}\,kN$

$\circlearrowright M_o = P_1(9-a) + P_2(9-b) + P_3(9-c) = 11.25(7) + 22.5(4.5) + \overset{61.88}{\cancel{41.25}}(1.27) = \overset{258.6}{\cancel{232.5}}\ kN\text{-}m \circlearrowright$

לטרפז

$(A+B)\frac{H}{2} =$ שטח

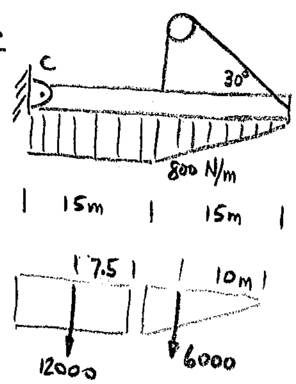
$\frac{2A+B}{A+B}\frac{H}{3} = \bar{x}$

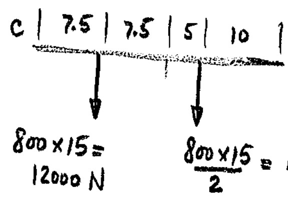


אם רציתי מערכת אקוויולנטית בלי מומנט.

$$d = \frac{M_O}{F_R} = \frac{232.5}{75} = 3.1\,m$$

4-133



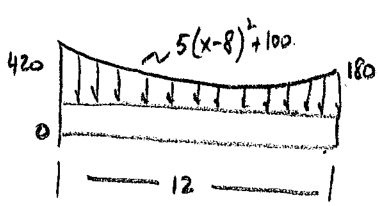


$800\times 15 = 12000\,N \qquad \frac{800\times 15}{2} = 6000\,N$

$F_R = 12000 + 6000 = 18000\,N \downarrow$

$M_C = 12000(7.5) + 6000(20) = 210000\ N\text{-}m$

$d = \frac{M_C}{F_R} = \frac{210\times 10^3}{18\times 10^3} = 11.67\,m$: C מסיבה



תרגיל אחרון

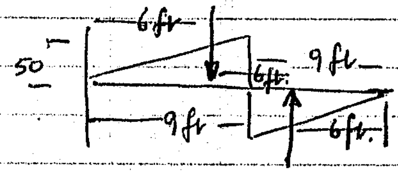
$$F_R = \int_0^{12} w(x)\,dx = \int_0^{12}\left[5(x-8)^2 + 100\right]dx = 5\frac{(x-8)^3}{3} + 100x\Big|_0^{12} = 1306.67 + 853.33 = \boxed{2160}\ N \downarrow$$

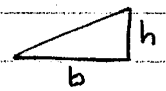
$$M_o = \int_0^{12} x\,w(x)\,dx = \int_0^{12}(5x^3 - 80x^2 + 420x)\,dx = \frac{5x^4}{4} - \frac{80}{3}x^3 + \frac{420x^2}{2}\Big|_0^{12} = 10080\ N\text{-}m$$

$$d = \frac{M_o}{F_R} = 4.67\,m$$

Do Problem 4-89

SESSION # 11





$\int dA = \frac{1}{2} bh.$

$\int x\,dA = \int x \cdot y\,dx = \int x \cdot \frac{x}{b} h\,d$

$= \frac{h}{b} \int x^2 dx = \frac{x^3 h}{3b}\Big|_0^b = \frac{b^2}{3}$

$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{1}{\cancel{3}} b^2 h}{\frac{b\cancel{3}h}{2}} = \frac{2b}{3}$

$\bar{x} = 2/3$ base of the triangle

$F_R = \int dA = \frac{bh}{2} = \frac{9\text{ft} \cdot 50\text{ lb/ft}}{2} = 225\text{ lb}.$

$\Sigma F = \downarrow 225\text{lb} + \uparrow 225\text{lb} = 0$

$\Sigma M = \text{couple} = 225\text{lb} \cdot 6\text{ft} = 1350\text{ lbft}$ ↺

EQUILIBRIUM OF A RIGID BODY

$\Sigma \vec{F} = 0$ and $\Sigma \vec{M}_0 = 0$ to prevent translation & rotation of a body

- First study equilibrium in 2-D
  - FREE BODY DIAGRAM – Show body & all forces acting on it
  - Supports also produce forces. What are they – forces that result from resistance to movement or rotation

העומס בשל מההתנגדות
לתנוצה או לסביבה

Pg 152 #4, #8, #9

הכוחות שהעומסים
האלה מציגים את
שיקול כוח מפורס שקיים
במקום המגע בין הגוף
לבין הסמך.
כאשר שטח המגע קטן,
יכולים להשתמש בכוח
מרוכז

- Reality – these forces represent the resultant of the distributed load that exists where the contact occurs. If the contact area is small ~~the~~ with respect ~~the~~ to the ~~length~~ area of the support, then the concentrated load is a good representation

- FBD
  - Isolate the body
  - Identify all the forces, moments, support reactions that are external
  - Indicate the dimensions

בודדים את הגוף
מציגים את כל הכוחות, מומנטים,
תגובות החיצוניים.
מציינים את המידות

ΣM_O is sum of all moments due to force components about an axis $\perp$ to x-y plane passing through pt. O.

- STATIC EQUILIBRIUM means $\vec{F_R} = \vec{0}$ and $\vec{M}_{R_O} = \vec{0}$

$$\vec{F_R} = \Sigma F_x \vec{i} + \Sigma F_y \vec{j} = \vec{0} = 0\vec{i} + 0\vec{j} \quad \text{thus} \quad \Sigma F_x = 0 \quad \Sigma F_y = 0$$

אפשר גם לעבוד בסט אחר אך ורק שהנקודות A ו-B לא נחות על קו אנכי לציר-y

ALTERNATIVE SETS ARE $\Sigma F_y = 0$, $\Sigma M_A = 0$ $\Sigma M_B = 0$
- A, B do not lie on line $\perp$ to y axis
-

$$\Sigma M_A,\ \Sigma M_B,\ \Sigma M_C = 0$$

- A,B,C do not lie on the same line.

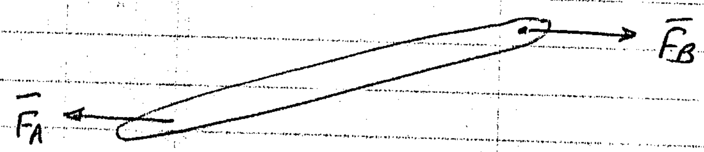
או אם A,B,C לא נחות על אותו קו

SIMPLIFICATIONS מצבים מיוחדים

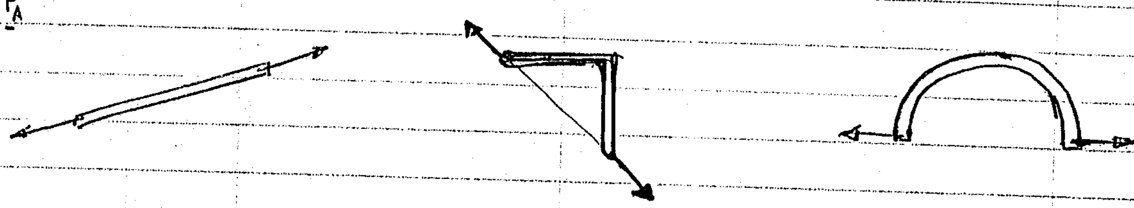
איברים שני כוחות
- FORCES APPLIED AT TWO POINTS ONLY — TWO FORCE MEMBERS

כדי שיהיה שיווי משקל הכוח F_A ו-F_B שווים בגודל אבל בכיוון הפוך
- FORCE EQUILIB. REQUIRES

$\overline{F_A} = \overline{F_B}$ same magnitude opposite direction



שיווי משקל של מומנטים צריך שקו הפעולה של F_A ו-F_B יהיה אותו קו
- MOMENT EQUIL REQUIRES LINE OF ACTION OF $\overline{F_A}$ & $\overline{F_B}$ BE SAME.



אם לגוף יש שלוש כוחות הכוחות צריכים להיות שותפי מישור וגם שותפי נקודה או שותפי קו פעולה אחד

- IF A BODY IS SUBJECTED TO 3 FORCES THAT ARE COPLANAR
  - FORCES MUST BE CONCURRENT $(\Sigma M_O) = 0$ SATISFIED
  - OR " " " PARALLEL

must only satisfy $\Sigma \vec{F} = \vec{0}$

Gautam did 5.22, 5.27, 5.17, 5.12 (set ups), 5.1, 5.2

Table 5–1 . Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

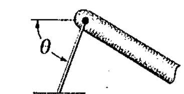
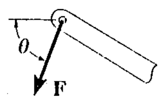
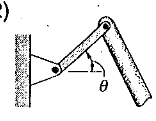
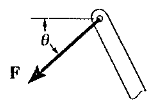
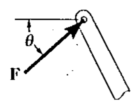
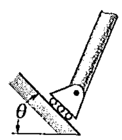
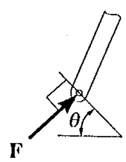
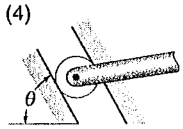
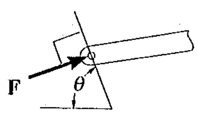
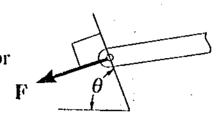
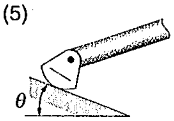
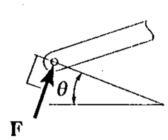
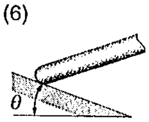
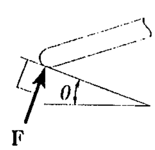
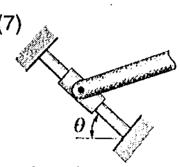
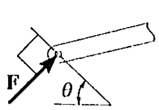
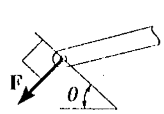
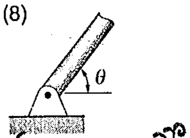
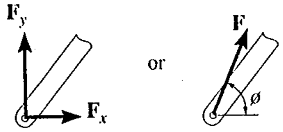
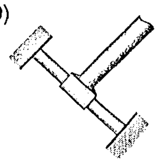
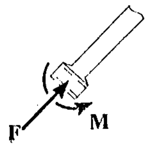
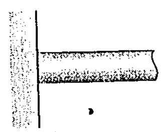
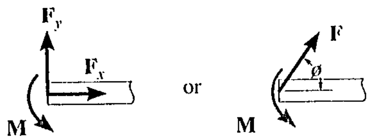
Types of Connection	*Reaction*	*Number of Unknowns*
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2) חוליה, קשר  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3) מגלה  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) פרוזדור ~~חלק~~ חלק  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

Table 5–1 (Contd.)

Types of Connection	*Reaction*	*Number of Unknowns*
(8)  ציר סיכה חלקית smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

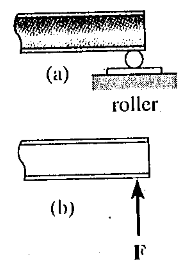


Fig. 5–2

Support Reactions. Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of support between bodies subjected to coplanar force systems.

The principles involved for determining these reactions can be illustrated by considering three ways in which a horizontal member, such as a beam, is commonly supported at its end. The first method of support consists of a *roller* or cylinder, Fig. 5–2*a*. Since this type of support only prevents the beam from translating in the vertical direction, it is necessary that the roller exerts a force on the beam in this direction, Fig. 5–2*b*.

The beam can be supported in a more restrictive manner by using a *pin* as shown in Fig. 5–3*a*. The pin passes through holes in the beam and two leaves which are fixed to the ground. Here the pin will prevent translation of the beam in *any direction* ϕ, Fig. 5–3*b*, and so it must exert a force **F** on the beam in this direction. For purposes of analysis, it is generally easier to represent this effect by its two components $\mathbf{F}_x$ and $\mathbf{F}_y$, Fig. 5–3*c*.

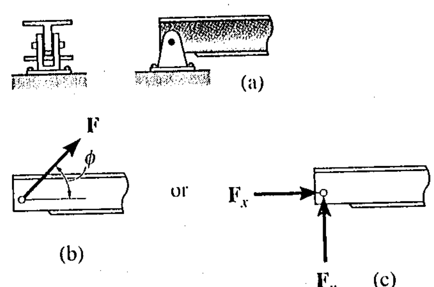


Fig. 5–3

SAMPLE FREE-BODY DIAGRAMS

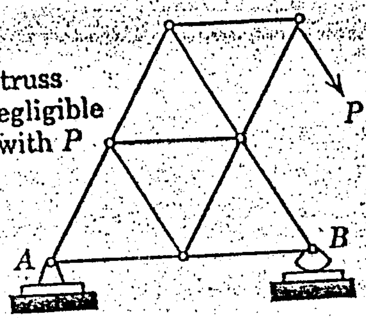
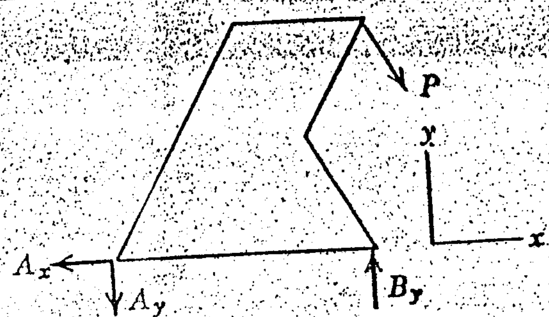
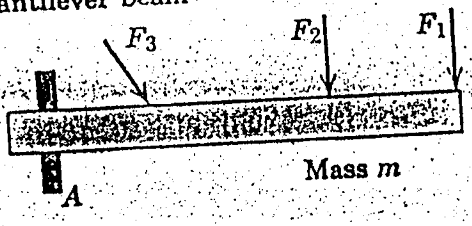
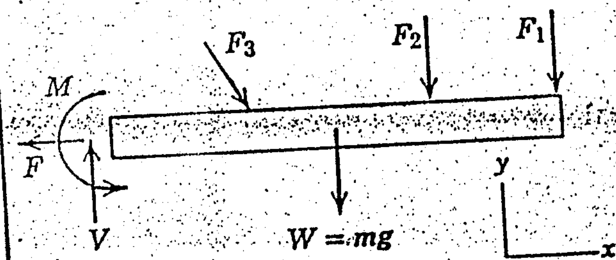
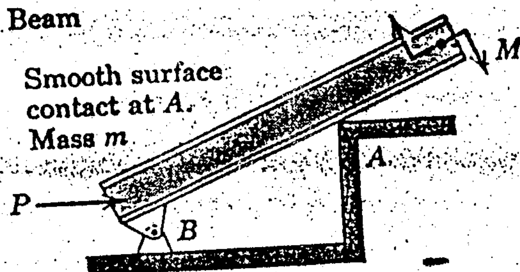
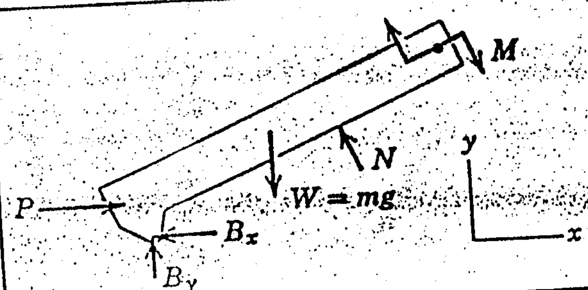
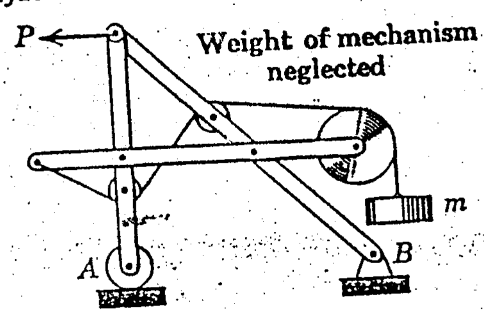
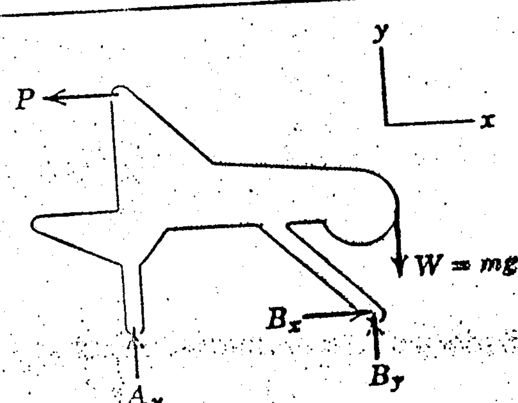
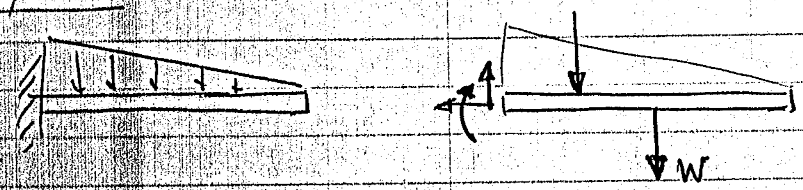
Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

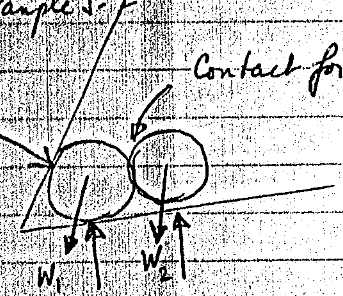
Figure 3/2

Example 5-1

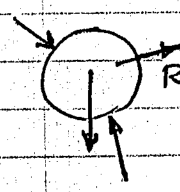


Example 5-2

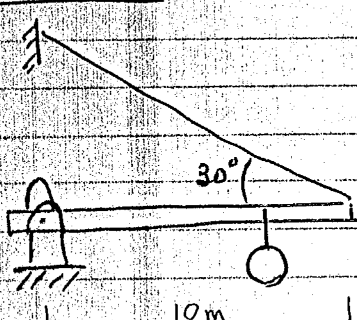
כוחות נגיעה

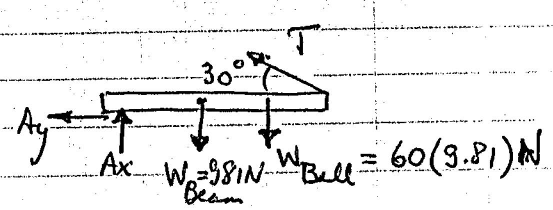


Contact forces here are neglected as they are internal w.r.t the FBD of both balls



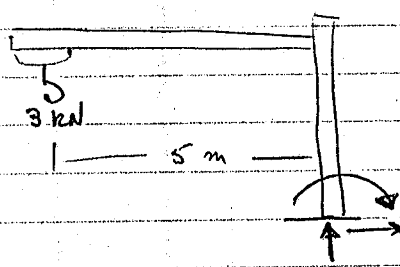
Problem 5-1

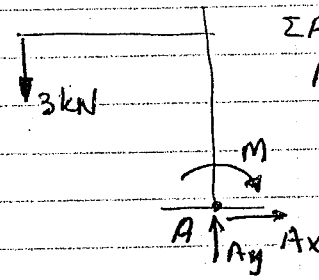




Problem 5-4

צינור





$\Sigma F_x = 0 \quad \Sigma F_y = 0$

$A_x = 0 \quad A_y = 3\,kN$

$M_A = 15\,kN\text{-}m$

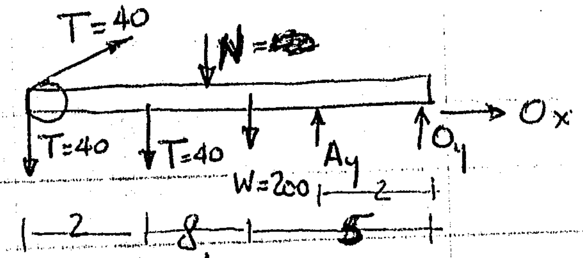
- Equilibrium is $\Sigma F_x, \Sigma F_y, \Sigma M_o = 0$ for a body whose forces are coplanar,

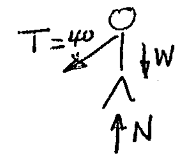
3 eqs give at most 3 unknowns.

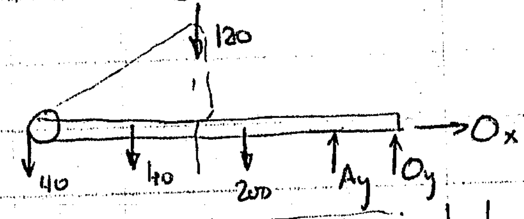
ראינו שמערכת כלשהי
של כוחות ומומנטים
מערכת כזאת ניתן
להחליף בכוח השקול
ושקול המומנט בנקודה
מסויימת

- in Chapter 4 have shown that any system of forces can be reduced to a resultant force $\vec{F_R}$ and a resultant moment about some point $\vec{M_{R_O}}$.

3-21







$280 = A_y + O_y$

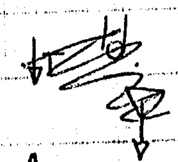
$O_x = 0$

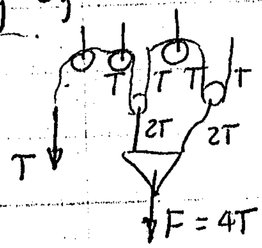
$\Sigma M_o = A_y(2) - 200(5) - 120(6)$
$+ 40(8) - 40(10) = 0$

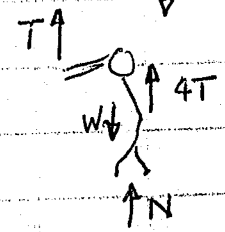
$A_y = \frac{2440}{2} = 1220 \text{ lb}$

$O_y = -940$

3/18







$N + 5T = W$

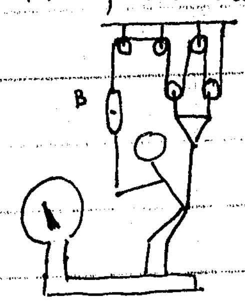
$67 + 5(19) = 162 \text{ lb}.$

14/April 28/April

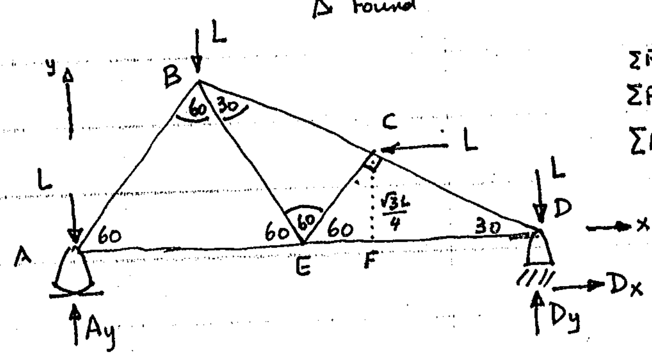
\$ ~~491~~ : 40×12
616
52×10 + 24×4

Sixt
14 313 p/NT

A former student of mechanics wants to weigh himself but has access to a scale A with a max. capacity of 100 lb & a small 20 lb spring dynamometer B.
With the rig shown, he finds that when he pulls with a force of that registers 19 lb, the scale reads 67 lb. What is his weight



A Found



$\Sigma F_y = 0 \quad A_y + D_y = 3L$

$\Sigma F_x = 0 \quad D_x = L$

$\Sigma M_D = 0 \quad A_y \cdot 2l + L \cdot 2l - L \cdot \frac{3l}{2} - L\frac{\sqrt{3}l}{2} = 0$

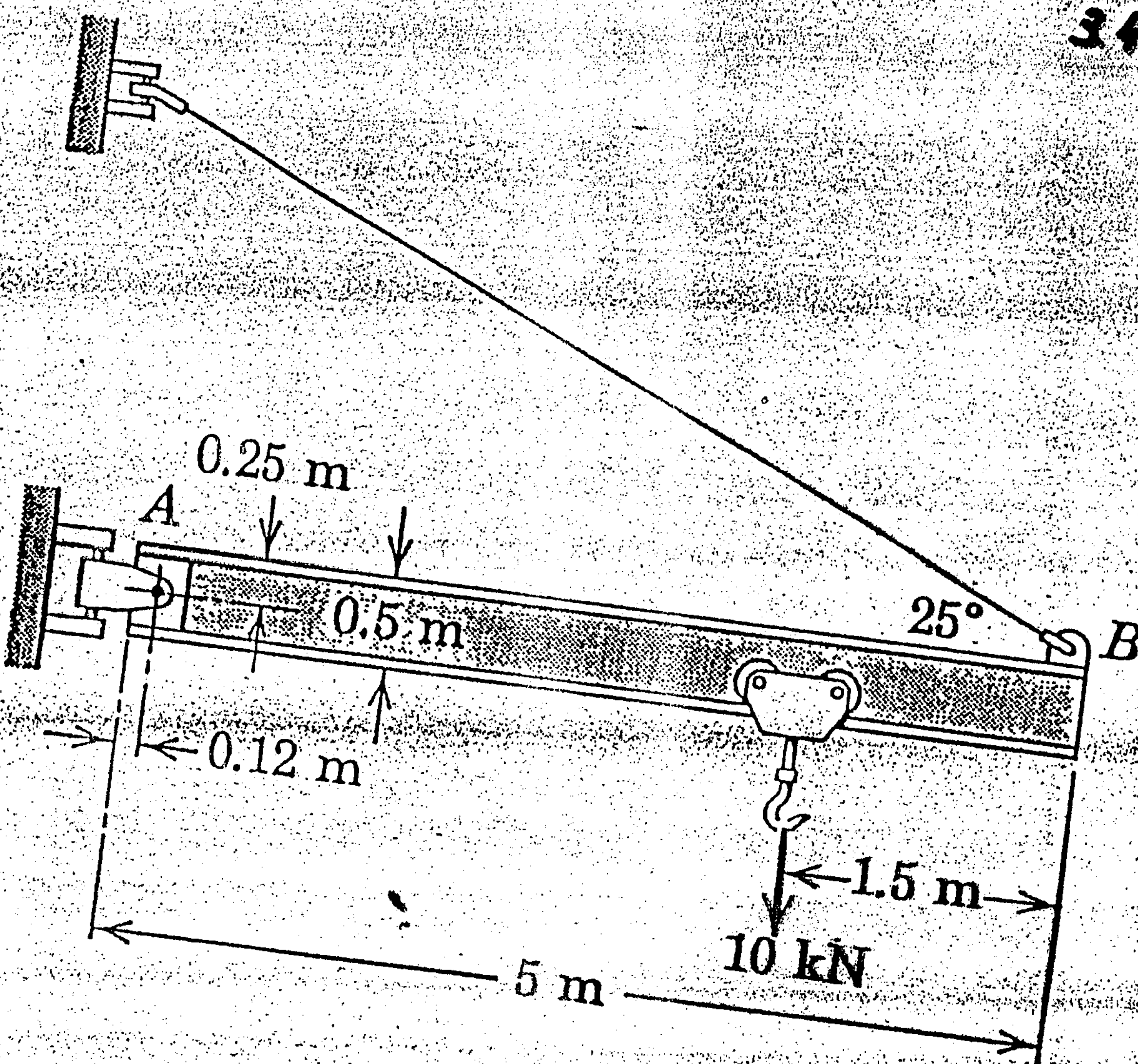
$A_y \cdot 2 - \frac{2L}{2} \cdot \frac{\sqrt{3}}{4} L = 0$

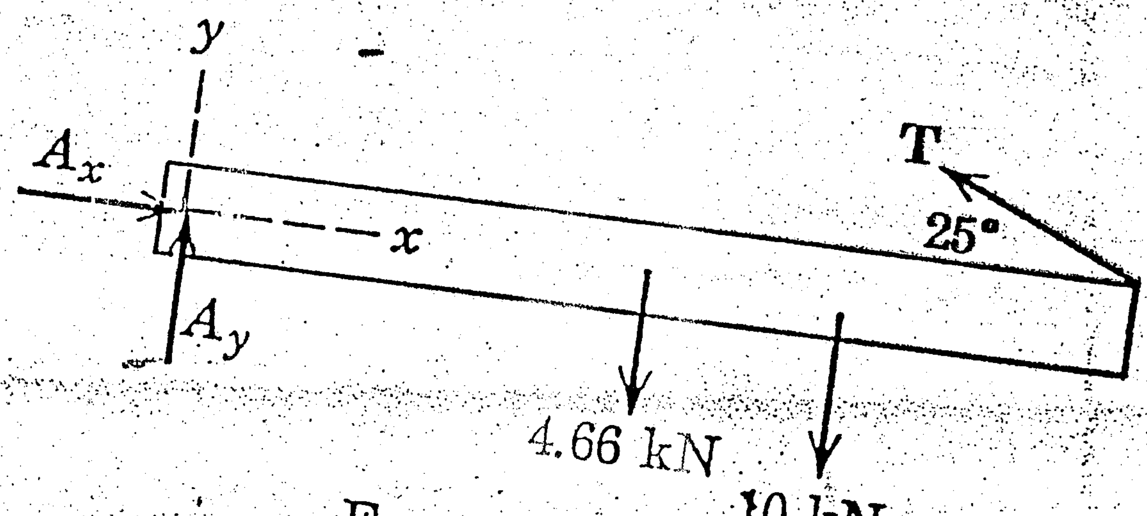
$A_y = \frac{(3.5 + .433)L}{2} = \frac{3.933}{2}L = 1.967L$

$D_y = 3L - A_y = 1.033L$

ED = l

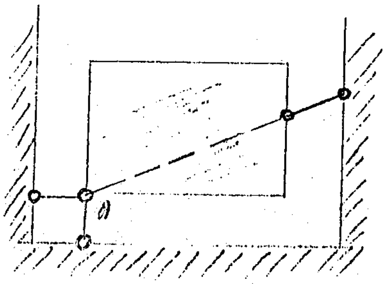
3-4 fpl





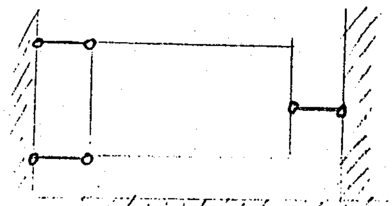
Free–Body Diagram

מצב ב'

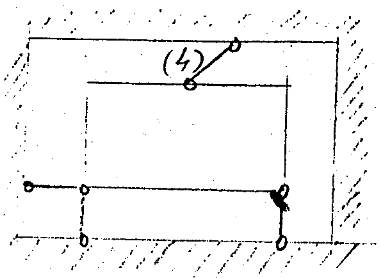


במקרה זה הכח שיפעיל המוט הימני
יעבור דרך הנקודה A. לכן לא תהיה
כל מגבלת תנועה התחלתית בנדנוד
לגוף המרוסן. זהו מצב של הגבלה בלתי מושלמת.
הריתקויות נותנות את $M_0=0$ האפס לכן אין התנגדות לסיבוב סביב A.

מצב ג'



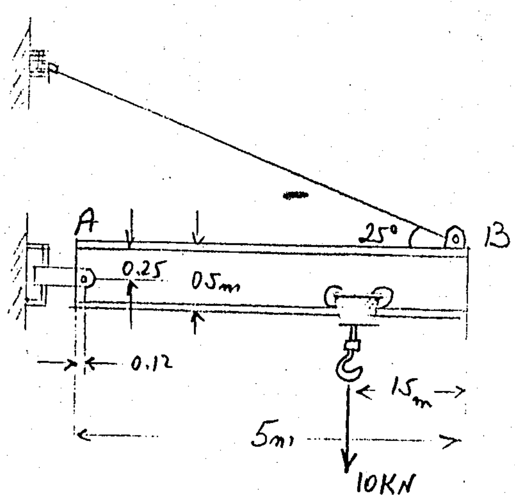
אין התנגדות לכח בכיוון y!
(מתקיימות רק 2 מ.ש.מ.)



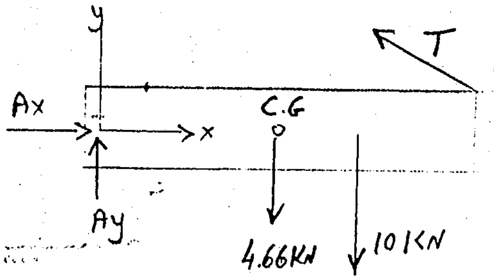
מצב ד'

מצב של ריתום יתר במוט (4)
מבנה בלתי מסוים סטטית פנימית.

דוגמא (3/2)



חשב את כח המתיחה בכבל
והכוחות הפועלים על הפין A
במנוף המתואר. מסת הקורה
95kg/m (שקף 3.4)



1. דיאגרמת גוף חופשי

ישנם שלושה נעלמים: A_y, A_x, T

2. משקל הקורה הנמצא במרכז הכובד

$mg = 95 \times 5 \times 9.81 = 4.66$ KN

ב. פתרון מ. שווי המשקל

נתחיל במשוואת מומנטים סביב A שבה יהיה נעלם אחד T

$$\Sigma M_A = 0 \qquad (T\cos 25^\circ)\,0.25 + (T\sin 25^\circ)(5-0.12) - 10(5-1.5-0.12) +$$

$$-\,4.66 \times (2.5-0.12) = 0$$

$$T = 19.61\ KN$$

ב) שווי משקל בכיוונים x ו y

$$\Sigma F_x = 0 \qquad A_x - 19.61\cos 25^\circ = 0 \qquad A_x = 17.77\ KN$$

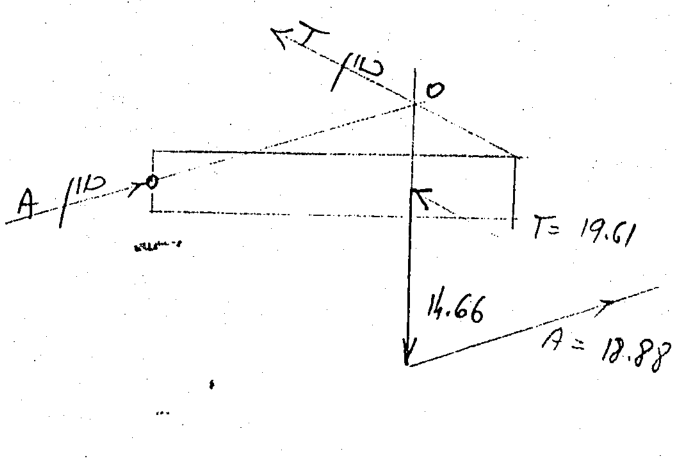
$$\Sigma F_y = 0 \qquad A_y + 19.61\sin 25^\circ - 4.66 - 10 = 0 \qquad A_y = 6.37$$

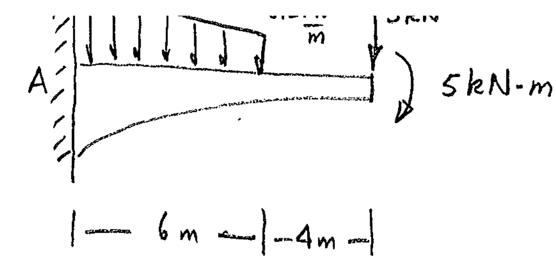
$$A = \sqrt{A_x^2 + A_y^2} \qquad A = 18.88\ KN$$

ג. <u>פתרון גרפי</u>:

1. מצאו גרפית את שקול 4.66; 10 (R)

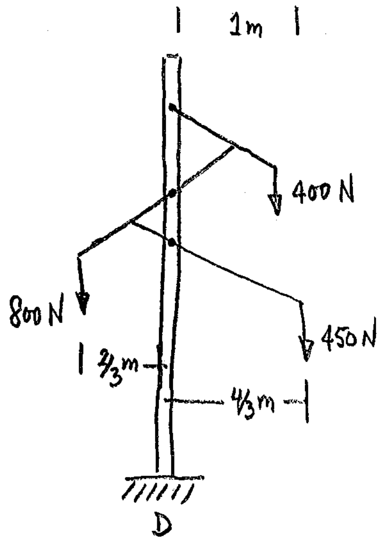
2. שלושת הכוחות T ו-R ו-A חייבים לעבור דרך נקודה אחת ולסגור פוליגון. לכן כיוון הכח A הוא מ-A דרך O



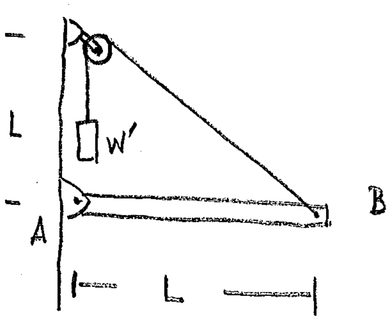


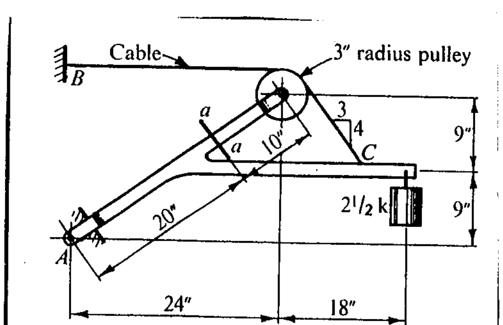
מצאו את התגובות ב-D

$M_D \cong 0.47$ kN-m

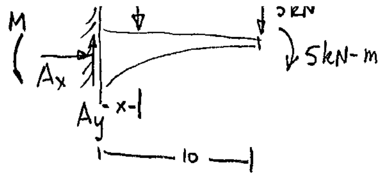


למוט יש משקל W. מצאו את המשקל 'W כדי שיהיה שיווי משקל





PROB. 2-13

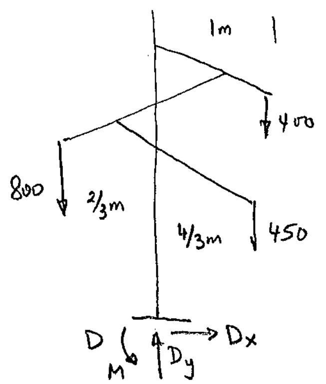


$$x = \left(\frac{2(0.5)+0.8}{0.5+0.8}\right)\frac{6}{3} = \frac{3.6}{1.3} = 2.77\ \text{m}$$

$\Sigma F_x = A_x = 0 \qquad \underline{A_x = 0}$

$\Sigma F_y = A_y - Q - 3 = 0 \qquad \underline{A_y = Q+3 = 6.9\ \text{kN}}$

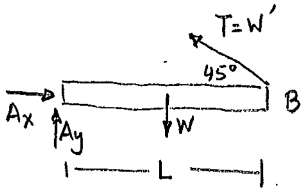
$+\circlearrowright \Sigma M_A = -M + Q\cdot x + 3\cdot 10 + 5 = 0 \qquad M = (3.9)(2.77) + 30 + 5 = 45.8\ \text{kN-m}$



$\Sigma F_x = \underline{D_x = 0}$

$\Sigma F_y = -400 - 800 - 450 + D_y = 0 \qquad \underline{D_y = 1650\text{N}}$

$+\circlearrowright \Sigma M = -800(2/3) + 400(1) + 450(4/3) - M = 0 \qquad M = 467\ \text{N-m}$



$\Sigma F_x = A_x - W'\cos 45 = 0$

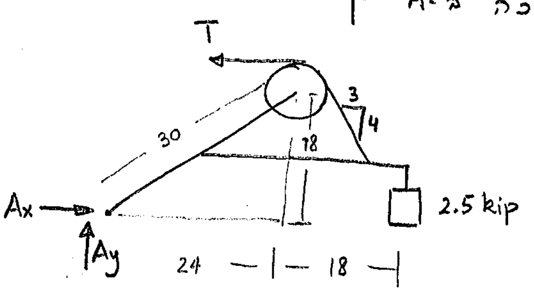
$\Sigma F_y = A_y - W + W'\sin 45^\circ$

$+\circlearrowright \Sigma M_A = W\cdot L/2 - W'\sin 45\cdot L = 0 \qquad \underline{W' = \frac{W}{2\sin 45} = \frac{W}{\sqrt{2}}}$

$\underline{A_x} = W'\cos 45 = \frac{W}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \underline{\frac{W}{2}}$

$\underline{A_y} = W - W'\sin 45 = \underline{W/2}$

לפי הנתונים אי אפשר לפתור התרגיל הרביעי. אבל בסיכה ב-A כן אפשר

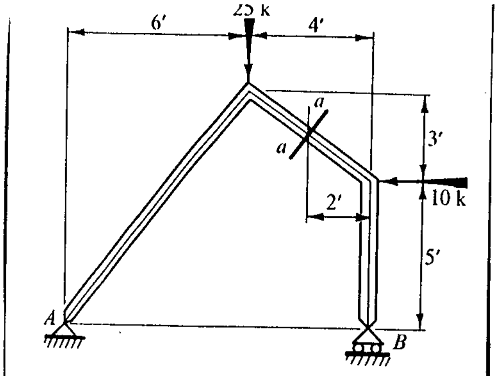


$\Sigma F_x = -T + A_x = 0$

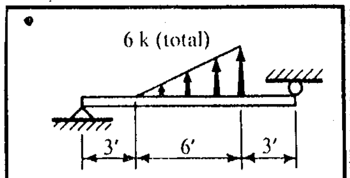
$\Sigma F_y = A_y - 2500 = 0 \qquad A_y = 2500$

$+\circlearrowright \Sigma M_A = -T(21) + 2500(42) = 0 \qquad T = 5000$

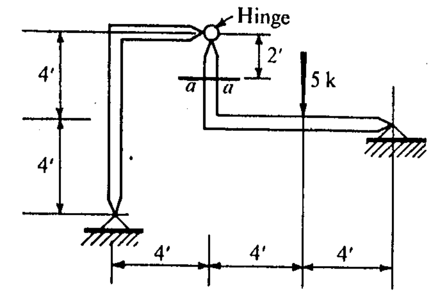
$A_x = T = 5000$



PROB. 2-10

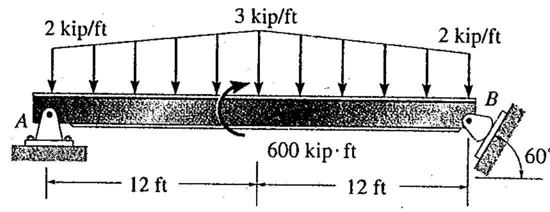


PROB. 2-5

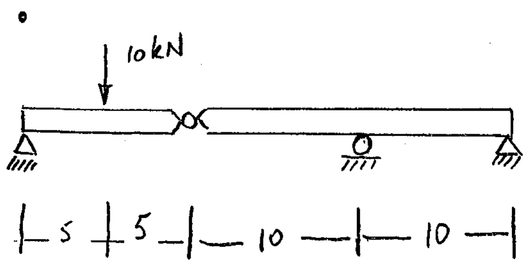


PROB. 2-8

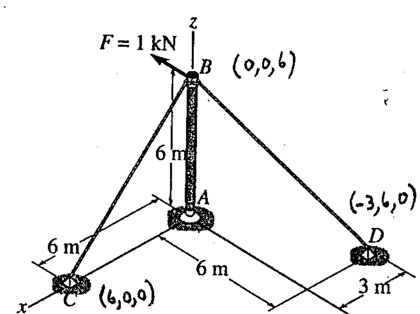
5–29. Determine the support reactions on the beam.



Prob. 5–29



Determine the tension in cables *BC* and *BD* and the reactions at the ball-and-socket joint *A* for the mast shown in Fig. 4–28*a*.



(a)

SOLUTION (*VECTOR ANALYSIS*)

Free-Body Diagram. There are five unknown force magnitudes shown on the free-body diagram, Fig. 4–28*b*.

Equations of Equilibrium. Expressing each force in Cartesian vector form, we have

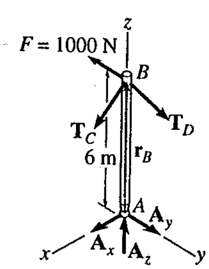
$$\mathbf{F} = \{-1000\mathbf{j}\}\ \text{N}$$

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{T}_C = 0.707T_C\mathbf{i} - 0.707T_C\mathbf{k}$$

$\underline{r}_{C/B} = 6i - 6k$ $\quad$ $\underline{u}_{C/B} = \frac{1}{\sqrt{2}}\underline{i} - \frac{1}{\sqrt{2}}\underline{k}$

$$\mathbf{T}_D = T_D\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = -0.333T_D\mathbf{i} + 0.667T_D\mathbf{j} - 0.667T_D\mathbf{k}$$



(b)

Applying the force equation of equilibrium gives

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F} + \mathbf{F}_A + \mathbf{T}_C + \mathbf{T}_D = \mathbf{0}$$

$$(A_x + 0.707T_C - 0.333T_D)\mathbf{i} + (-1000 + A_y + 0.667T_D)\mathbf{j} + (A_z - 0.707T_C - 0.667T_D)\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad A_x + 0.707T_C - 0.333T_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad A_y + 0.667T_D - 1000 = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad A_z - 0.707T_C - 0.667T_D = 0 \qquad (3)$$

Summing moments about point A, we have

$$\Sigma\mathbf{M}_A = \mathbf{0}; \qquad \mathbf{r}_B \times (\mathbf{F} + \mathbf{T}_C + \mathbf{T}_D) = \mathbf{0}$$

$$6\mathbf{k} \times (-1000\mathbf{j} + 0.707T_C\mathbf{i} - 0.707T_C\mathbf{k} - 0.333T_D\mathbf{i} + 0.667T_D\mathbf{j} - 0.667T_D\mathbf{k}) = \mathbf{0}$$

Evaluating the cross product and combining terms yields

$$(-4T_D + 6000)\mathbf{i} + (4.24T_C - 2T_D)\mathbf{j} = \mathbf{0}$$

$$\Sigma M_x = 0; \qquad -4T_D + 6000 = 0 \qquad (4)$$

$$\Sigma M_y = 0; \qquad 4.24T_C - 2T_D = 0 \qquad (5)$$

The moment equation about the z axis, $\Sigma M_z = 0$, is automatically satisfied.

$T_c = 707\text{ N}$ $\quad$ $T_D = 1500\text{ N}$

$A_x = 0\text{ N}$ $\quad$ $A_y = 0\text{ N}$ $\quad$ $A_z = 1500\text{N}$

מפני שהמוט המאונך AB הוא אבר של
שני כוחות (כי כל הכוחות פועלים או
ב-A או ב-B) רק מבדיקת מצב
הכוחות אפשר לומר ש- $A_x=A_y=0$

SESSION # 12

EQUILIBRIUM IN 3-D — Draw FBD as usual

- Support Reactions
  developed when translation or rotation is restricted

pg 180 take about (6), (8), (5)

<u>Equilibruim equations</u>

For <u>non concurrent</u> forces $\sum \vec{F} = \vec{0}$

$\sum \vec{M}_0 = \vec{0}$

Results in 6 equations

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$$

$$\sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$$

sum of the moment components ~~along~~ ∥ to the x, y, z axes passing through O.

O can be on or off the body.

at most 6 unknowns can be found.

1. Draw FBD for the body
2. Indicate all forces acting on the body
3. Indicate all dimensions
4. If it is difficult to determine moments arms or forces use vector analysis $\sum \vec{F} = \vec{0}$, $\sum \vec{M}_0 = 0$
5. <u>Not necessary for axes for force sum</u> be the same as that <u>for moment summation.</u>
6. Choose direction of moment axes so that as many ~~unknowns~~ ~~can be eliminated~~ lines of actions of unknowns intersect it. Moment due to these unknowns will be zero.
7. Can choose any set of 3 non orthogonal axes for force or moment summation as long as they are <u>not parallel to each other</u>

allowed to rotate freely about *any* axis, no couple moment is resisted by a ball-and-socket joint.

It should be noted that the *single* bearing supports (5) and (7), the *single* pin (8), and the *single* hinge (9) are shown to support both force and couple-moment components. If, however, these supports are used in conjunction with *other* bearings, pins, or hinges to hold the body in equilibrium, and provided the physical body maintains its *rigidity* when loaded and the supports are *properly aligned* when connected to the body, then the *force reactions* at these supports may *alone* be adequate for supporting the body. In other words, the couple moments become redundant and may be neglected on the free-body diagram. The reason for this will be clear after studying the examples which follow, but essentially the couple moments will not be developed at these supports since the rotation of the body is prevented by the reactions developed at the other supports and not by the supporting couple moments.

Table 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

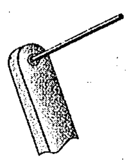
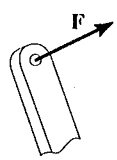
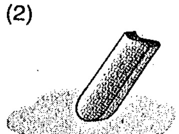
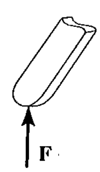
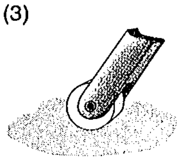
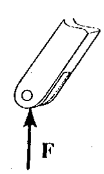
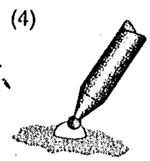
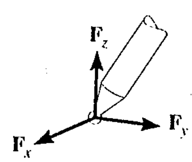
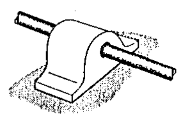
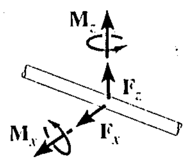
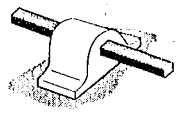
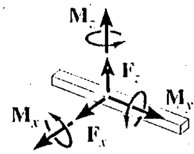
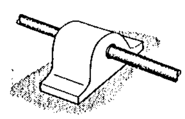
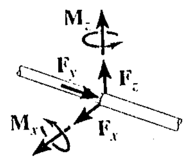
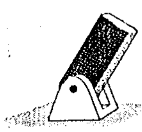
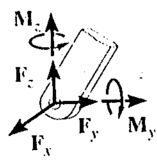
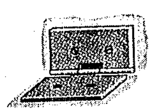
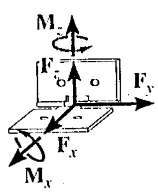
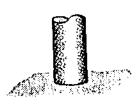
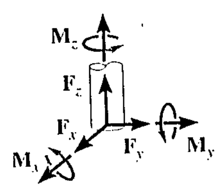
Types of Connection	*Reaction*	*Number of Unknowns*
(1)  cable		One unknown. The reaction is a force which acts away from the member in the direction of the cable.
(2)  סמך שטח חלק smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. Rough surface שטח מחוספס גורם לעוצבות - כוח החיכוך מקביל לשטח.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4) כדור ותושבת  ball and socket		Three unknowns. The reactions are three rectangular force components.

Table 5–2 (Contd.)

Types of Connection	*Reaction*	*Number of Unknowns*
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft.
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

:CHANICAL ACTION OF FORCES IN THREE–DIMENSIONAL ANALYSIS

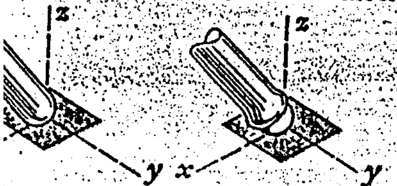
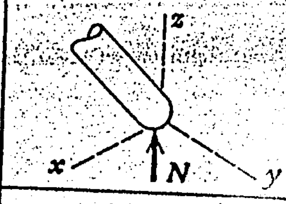
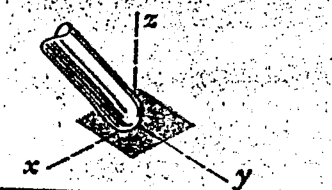
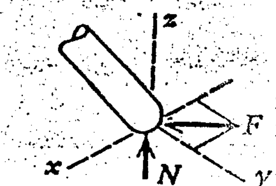
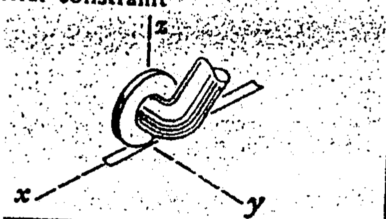
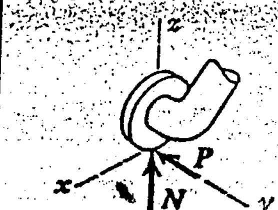
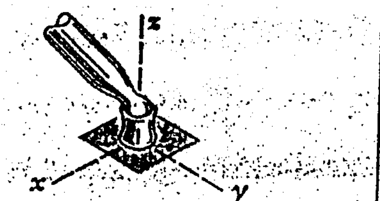
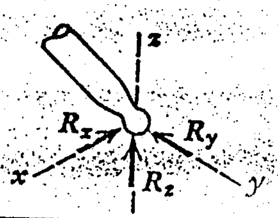
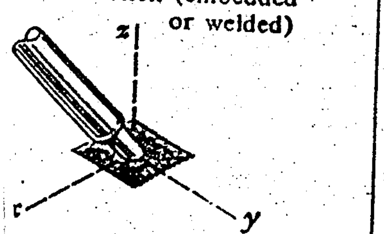
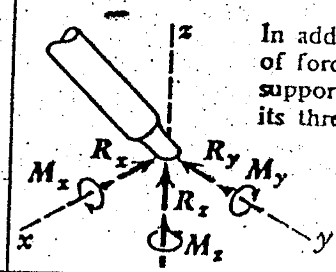
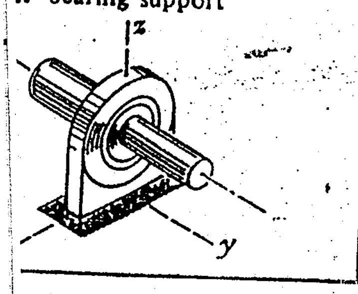
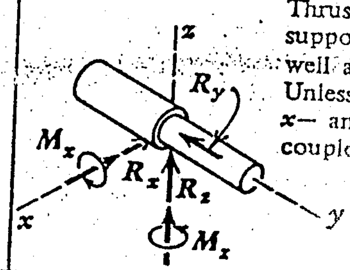
ı of Contact and Force Origin	Action on Body to be Isolated
Member in contact with smooth ;urface, or ball–supported member 	 Force must be normal to the surface and directed toward the member.
Iember in contact with rough ırface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
oller or wheel support with teral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
ll–and–socket joint 	 A ball–and–socket joint free to pivot about the center of the ball can support a force R with all three components.
:d connection (embedded or welded) 	 In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
ıt–bearing support 	 Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Unless bearing is pivoted about x– and z–axes, it can support couples M_x and M_z.

Figure 3/8

3.6 4p6

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS

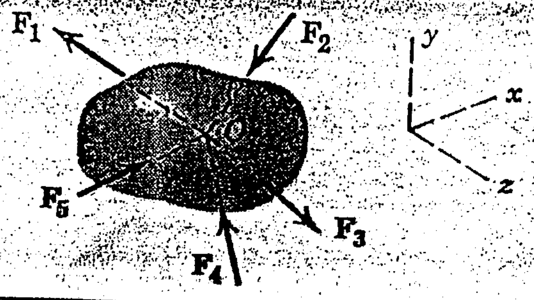
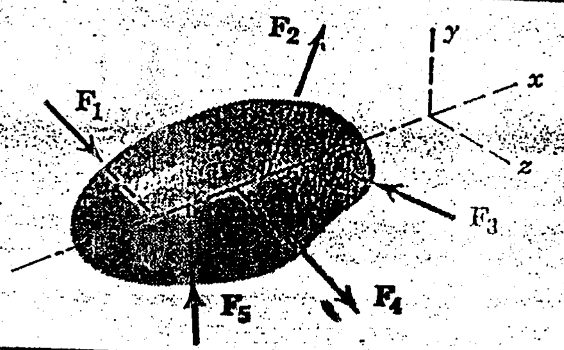
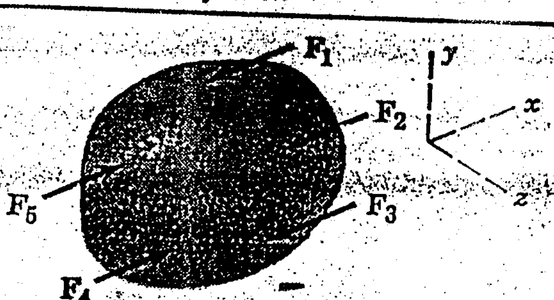
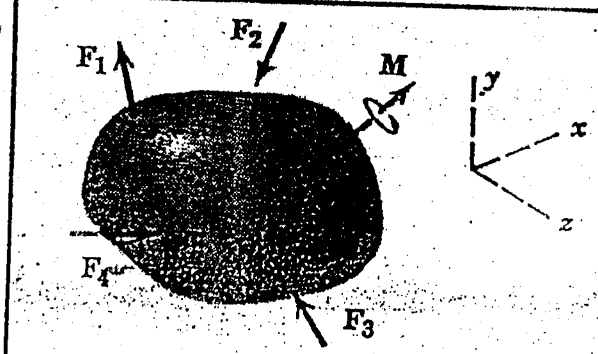
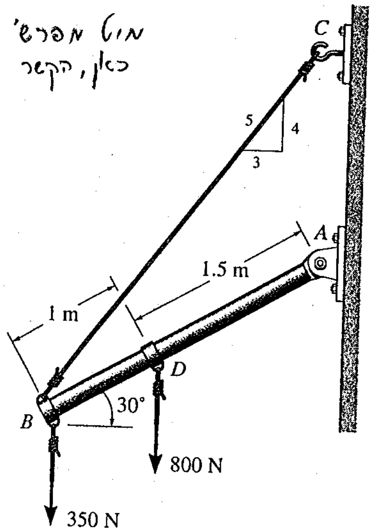
System	Free-Body Diagram	Independent Equations
urrent point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
urrent a line Concurrent w/a line		All forces pass through one line // to x axis $\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
el Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
l General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Figure 3/9

size of the collars at D and B and the thickness of the boom, determine the horizontal and vertical components of force at pin A and the force in cable CB.

boom – מנוף מפרשי
collar – כולן, הקשר

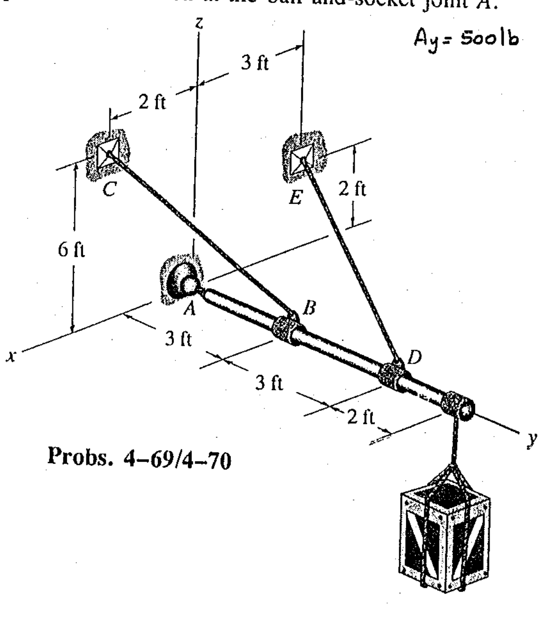


4–69. The boom supports a load having a weight of $W = 850$ lb. Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in cables BC and DE.

$T_{DE} = 721$ lb $A_z = 1.21$ kip

4–70. Cable BC or DE can support a maximum tension of 700 lb before it breaks. Determine the greatest weight W that can be suspended from the end of the boom. Also, determine the x, y, z components of reaction at the ball-and-socket joint A.

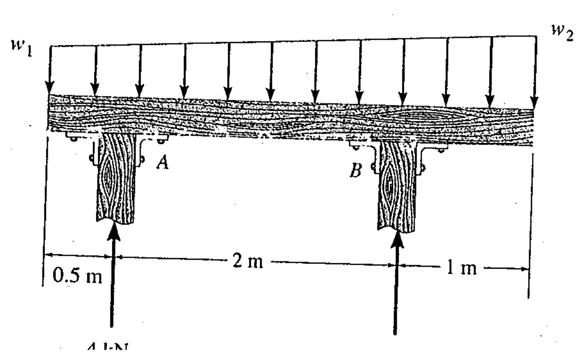
$A_y = 500$ lb



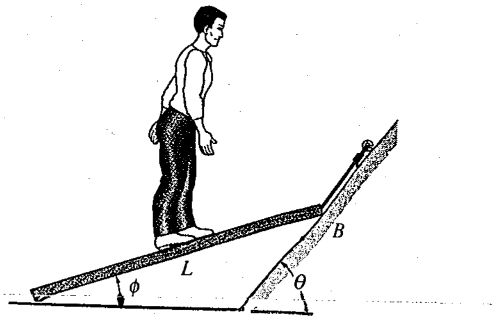
Probs. 4–69/4–70

5–50. Determine the intensity w_1 and w_2 of the trapezoidal loading if the supports at A and B exert forces of 4 kN and 4.5 kN, respectively, on the beam.

$w_2 = 1.63$ kN/m



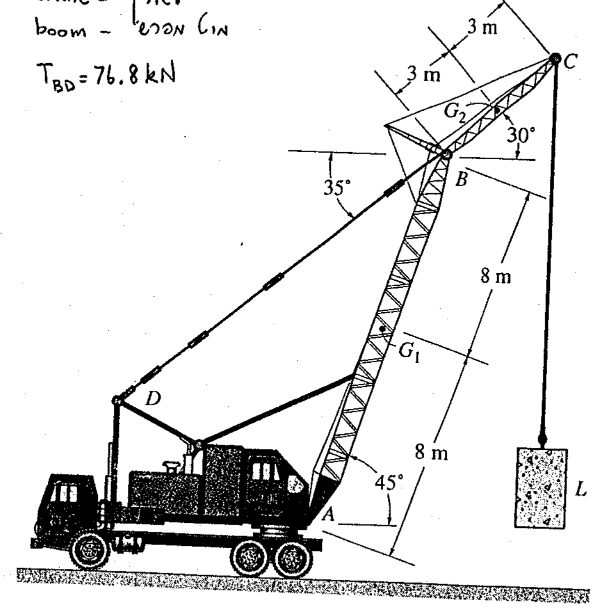
***5–48.** The man has a weight W and stands at the center of the plank. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.



4–33. The crane lifts a 400-kg load L. If the primary boom AB has a mass of 1.20 Mg and a center of mass at G_1, whereas the secondary boom BC has a mass of 0.6 Mg and a center of mass at G_2, determine the tension in the cable BD and the horizontal and vertical components of reaction at the pin A.

crane – עגורן
boom – מנוף מפרשי

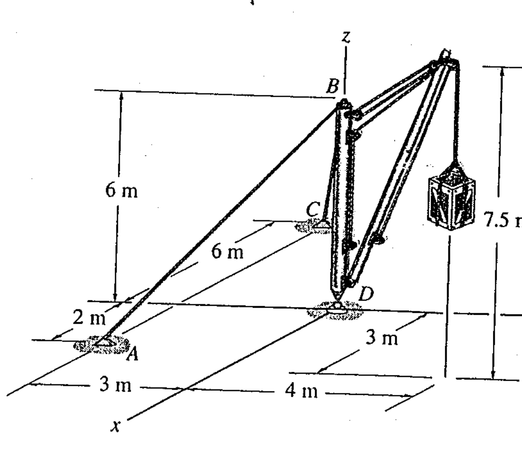
$T_{BD} = 76.8$ kN



Prob. 4–33

***5–96.** The stiff-leg derrick used on ships is supported by a ball-and-socket joint at D and two cables BA and BC. The cables are attached to a smooth collar ring at B, which allows rotation of the derrick about the z axis. If the derrick supports a crate having a mass of 100 kg, determine the tension in the supporting cables and the x, y, z components of reaction at D.

stiff leg derrick עגורן מגדל קפיצי



$$F_{DE} = F_{DE} \frac{\{\ \ \}}{\sqrt{(-3)^2+(-6)^2+2^2}} = F_{DE}\{\ldots\}$$

$$\bar{F}_{BC} = F_{BC}\frac{\{2i-3j+6k\}}{\sqrt{2^2+3^2+6^2}} = F_{BC}\{.286\,i - .429\,j + .857\,k\}$$

$$\Sigma M_x = 0 \quad .286\,F_{DE}\,6 + .8571\,F_{BC}\,3 - 850 \cdot 8 = 0$$

$$\Sigma M_z = 0 \quad .429\,F_{DE}\cdot 6 - .286\,F_{BC}\cdot 3 = 0$$

$$F_{DE} = 721\ \text{lb} \qquad F_{BC} = 2164\ \text{lb}$$

$$\Sigma F_x = 0 \quad A_x + .286\,(2164) - .429\,(721) = 0$$
$$A_x = -309\ \text{lb}$$

$$\Sigma F_y = 0 \quad A_y - .857\,(721) - .429\,(2164) = 0$$
$$A_y = 1545\ \text{lb}$$

$$\Sigma F_z = 0 \quad A_z - 850 + .286\,(721) + .857\,(2164) = 0$$
$$A_z = -1211\ \text{lb}$$

לפי מה שמצאנו למעלה

4-70

$$\underline{F_{DE}} = F_{DE}\{-.429\,\underline{i} - .857\,\underline{j} + .286\,\underline{k}\} = \left(-\tfrac{3}{7}\underline{i} - \tfrac{6}{7}\underline{j} + \tfrac{2}{7}\underline{k}\right)F_{DE}$$

$$\underline{F_{BC}} = F_{BC}\{.286\,\underline{i} - .429\,\underline{j} + .857\,\underline{k}\} = \left(\tfrac{2}{7}\underline{i} - \tfrac{3}{7}\underline{j} + \tfrac{6}{7}\underline{k}\right)F_{BC}$$

$$\underline{W} = -W\underline{k}$$

$$\Sigma M_{x_A} = .286\,F_{DE}\cdot 6 + .857\,F_{BC}\cdot 3 - W\cdot 8 = 0$$

$$\Sigma M_{z_A} = .429\,F_{DE}\cdot 6 - .286\,F_{BC}\cdot 3 = 0 \implies F_{BC} = 3F_{DE} \implies$$

לכן F_{BC} צריך להיות
את ה-700 lb

ו- $F_{DE} = \frac{1}{3}F_{BC} = \underline{233.3\ \text{lb}}$

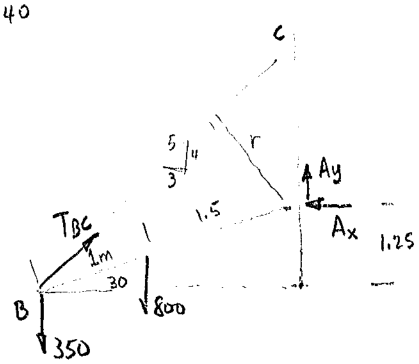
ו- $W = \underline{275\ \text{lb}}$

$$\Sigma F_x = 0 \quad A_x + .286\,F_{BC} - .429\,F_{DE} = 0$$
$$\underline{A_x = -100\ \text{lb}}$$

$$\Sigma F_y = 0 \quad A_y - .857\,F_{DE} - .429\,F_{BC} = 0$$
$$\underline{A_y = 500\ \text{lb}}$$

$$\Sigma F_z = 0 \quad A_z - W + .286\,F_{DE} + .857\,F_{BC} = 0$$
$$\underline{A_z = -392\ \text{lb}}$$

4-40



$$\Sigma M_A = T_{BC}\cdot r - 350\,(2.5\cos 30) - 800\,(1.5\cos 30) = 0$$

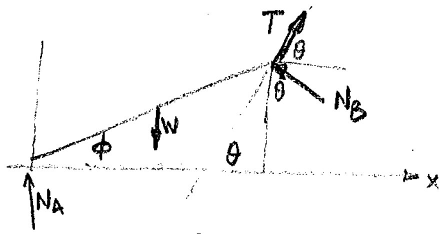
זווית $\measuredangle CBA = \tan^{-1}(4/3) - 30^\circ = 23.13^\circ$

$$r = 2.5\sin 23.13^\circ = .9821$$

$$\Sigma M_A = 0 \implies T_{BC} = \frac{[350\,(2.5) + 800\,(1.5)]\cos 30^\circ}{r} = 1830\ \text{N}$$

$$\Sigma F_x = 0 = -A_x + T_{BC}\cos 53.13^\circ = -A_x + 1830\cdot\tfrac{3}{5} = 0$$
$$A_x = 1098\ \text{N}$$

$$\Sigma F_y = 0 = -350 - 800 + T_{BC}\sin 53.13^\circ + A_y = 0$$
$$A_y = -314\ \text{N}$$

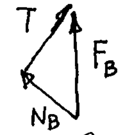


$\Sigma F = 0 = N_A \underline{j} - W\underline{j} + F_B\,\underline{j} = \underline{0}$

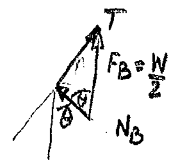
$F_B = -N_A + W$

F_B הוא השקול של הרכיבים N_B ו-T

$\Sigma M_A \circlearrowleft + = 0 \qquad W\cdot \frac{L}{2}\cos\phi - F_B \cdot L\cos\phi = 0 \qquad F_B = \frac{W}{2}$

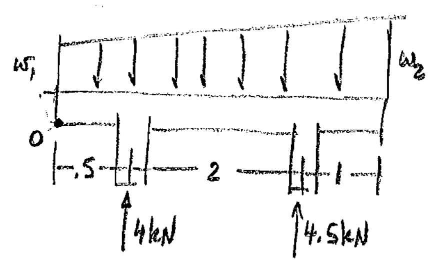


F_B השקול של N_B ו-T



$T = F_B \sin\theta = \frac{W}{2}\sin\theta$

5-50



$w(x) = \frac{w_2 - w_1}{3.5}x + w_1$

$\Sigma F_y = 0 = 4 + 4.5 - \int_0^{3.5} w(x)dx = 8.5 - \left[\frac{w_2-w_1}{3.5}\frac{x^2}{2} + w_1 x\right]_0^{3.5} = 0$

$8.5 - \left\{1.75(w_2 - w_1) + 3.5w_1\right\} = 0$

$8.5 - 1.75(w_2 + w_1) = 0 \implies w_2 + w_1 = \frac{8.5}{1.75} = 4.857$

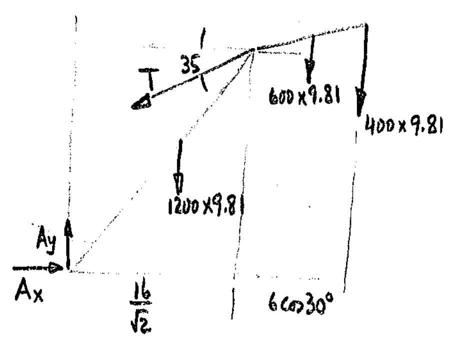
$\Sigma M_o \circlearrowleft + = 4(0.5) + 4.5(2.5) - \int_0^{3.5} w(x)\cdot x\,dx = 0$

$= 13.25 - \left[\frac{w_2 - w_1}{3.5}\frac{x^3}{3} + w_1\frac{x^2}{2}\right]_0^{3.5} = 0$

$= 13.25 - \left[(w_2 - w_1)\,4.08\bar{3} + w_1(6.125)\right] = 13.25 - (4.083w_2 + 2.042w_1) = 0$

$w_1 = 3.224\frac{kN}{m} \qquad w_2 = 1.633\frac{kN}{m}$ לפי המחשבון :

4-33



$\Sigma M_A \circlearrowleft + = 0 \quad 1200\times 9.81\times\frac{8}{\sqrt{2}} + 600\times 9.81\left(\frac{16}{\sqrt{2}} + 3\cos 30^\circ\right) + 400\times 9.81\left(\frac{16}{\sqrt{2}} + 6\cos 30^\circ\right)$

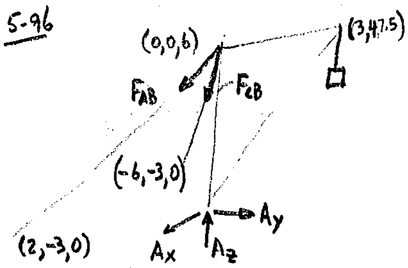
$+ T\sin 35^\circ\cdot\frac{16}{\sqrt{2}} - T\cos 35^\circ\cdot\frac{16}{\sqrt{2}} = 0$

$\underline{T = 76{,}758\ N}$

$\Sigma F_y = 0 \quad A_y - T\sin 35^\circ - 2200\times 9.81 \quad A_y = T\sin 35^\circ + 2200\times 9.81 = 65609\,N$

$\Sigma F_x = 0 \quad A_x - T\cos 35^\circ = 0 \quad A_x = T\cos 35^\circ = \underline{62877\ N}$

5-96



$\Sigma \underline{F} = \underline{0} = \underline{F}_{AB} + \underline{F}_{CB} - W\underline{k} + (A_x\underline{i} + A_y\underline{j} + A_z\underline{k}) = \underline{0}$

$\underline{F}_{AB} = F_{AB}(2\underline{i} - 3\underline{j} - 6\underline{k})/7 \qquad \underline{F}_{BB} = F_{CB}(-6\underline{i} - 3\underline{j} - 6\underline{k})/9$

$\Sigma F_x = 0: \ \frac{2}{7}F_{AB} - \frac{6}{9}F_{CB} + A_x = 0$

$\Sigma F_y = 0: \ -\frac{3}{7}F_{AB} - \frac{3}{9}F_{CB} + A_y = 0$

$\Sigma F_z = 0: \ -6/7\,F_{AB} - 6/9\,F_{CB} + A_z - W = 0$

$\Sigma \underline{M}_A = \underline{0}: \ 6\underline{k}\times F_{AB}(2\underline{i} - 3\underline{j} - 6\underline{k})/7 + 6\underline{k}\times F_{CB}(-6\underline{i} - 3\underline{j} - 6\underline{k})/9 + (3\underline{i} + 4\underline{j} + 7.5\underline{k})\times(-981\underline{k}) = 0$

$\frac{12}{7}F_{AB}\underline{j} + \frac{18}{7}F_{AB}\underline{i} - \frac{36}{9}F_{BC}\underline{j} + \frac{18}{9}F_{BC}\underline{i} + 2943\underline{j} - 3924\underline{i} = 0$

$\left.\begin{aligned}\frac{12}{7}F_{AB} - 4F_{BC} + 2943 = 0\\ \frac{18}{7}F_{AB} + 2F_{BC} - 3924 = 0\end{aligned}\right\} \longrightarrow \quad F_{AB} = 715.3\ N$

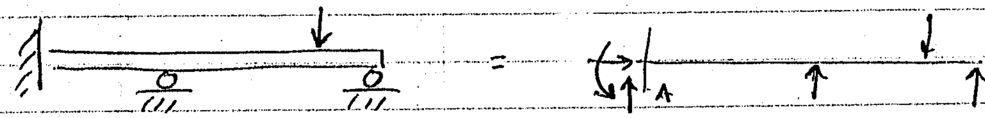
$F_{BC} = 1042.3\ N$

$A_x = 490.5\ N$ מעולה, נקבל

$A_y = +654\ N$

- If unknowns are determined from eqs of equilib - statically determinate
- if more unknowns than equations   indeterminate
  - must use ~~of~~ other means

Degree of indeterminacy = # of unknowns − # of available equations



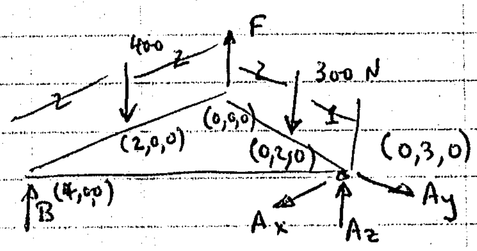
5 unknowns   3 eqs $\Sigma F_x, \Sigma F_y$, ΣM_A

indeterminate of degree 2

Read sections 5.6 on indeterminate

Do 5-53, 5-45

triangle is supported by 2 balls @ B + C and a ball/socket at A



$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y = 0$

$\Sigma F_z = B - 400 + F - 300 + A_z = 0.$

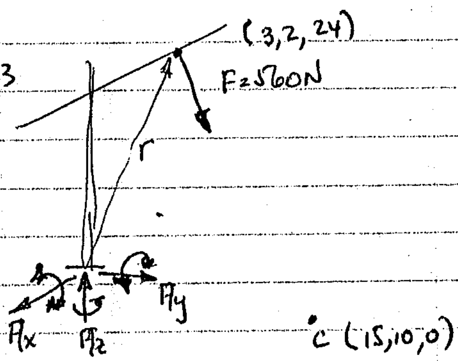
$\Sigma \bar{M}_A = \vec{0} = \Sigma M_{Ax}\bar{i} + \Sigma M_{Ay}\bar{j} + \Sigma M_{Az}\bar{j}$

$(300 \cdot 1\ \bar{i}) - 3F\bar{i} + 400(3)\bar{i} + (400 \cdot 2\bar{j}) - B(3)\bar{i} - B(4)\bar{j}$

$\Sigma M_{A.x} = 300 - 3F + 1200 - 3B = 0$ $F = 300$ N

$\Sigma M_{Ay} = 800 - 4B = 0$ $B = 200$ N $A_z = 200$ N.

5-53



$\vec{r} = (15-3, 10-2, 0-24)$

$= (12, 8, -24) = 4(3, 2, -6)$

$r = 4 \cdot 7 = 28m.$

$\bar{u} = .429\bar{i} + .286\bar{j} - .857\bar{k}$

$\bar{F} = F\bar{u} = (240\bar{i} + 160\bar{j} - 480\bar{k})N$

$\Sigma\vec{F} = 0$ $\Sigma F_x = A_x + 240 = 0$ $A_x = -240$ N

$\Sigma F_y = A_y + 160 = 0$ $A_y = -160$ N

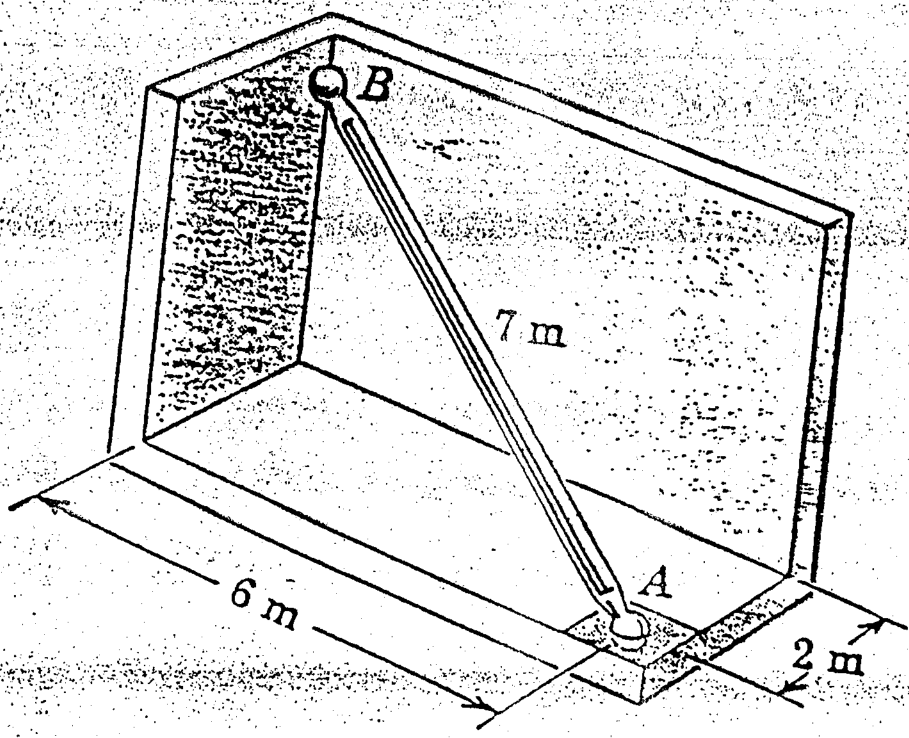
$\Sigma F_z = A_z - 480 = 0$ $A_z = 480$ N

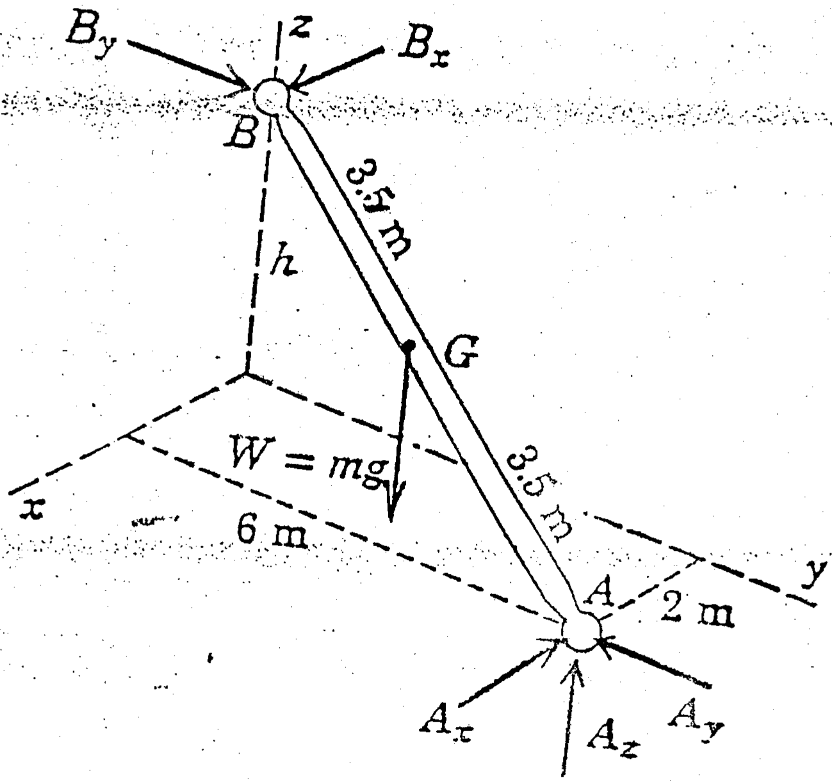
$$\bar{M}_A = \vec{r} \times \bar{F} = \begin{vmatrix} i & j & k \\ 3 & 2 & 24 \\ 240 & 160 & 480 \end{vmatrix} = \bar{i}(2 \cdot 480 - 160 \cdot 24) - \bar{j}(3 \cdot 480 - 240 \cdot 24) + k(3 \cdot 160 - 2 \cdot 240)$$

$= i(-18 \cdot 160) - j(-9 \cdot 480) + k(0)$

$M_{Ax}\underline{i} + M_{Ay}\underline{j} + M_{Az}\underline{k} + \underline{M}_A = \underline{0} \Rightarrow (M_{Ax} - 18 \cdot 160)\underline{i} + (M_{Ay} + 9 \cdot 480)\underline{j} + (M_{Az} + 0)\underline{k} = \underline{0}$

3.7 fpe

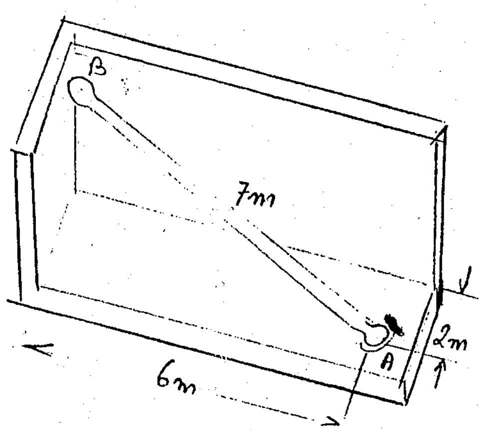




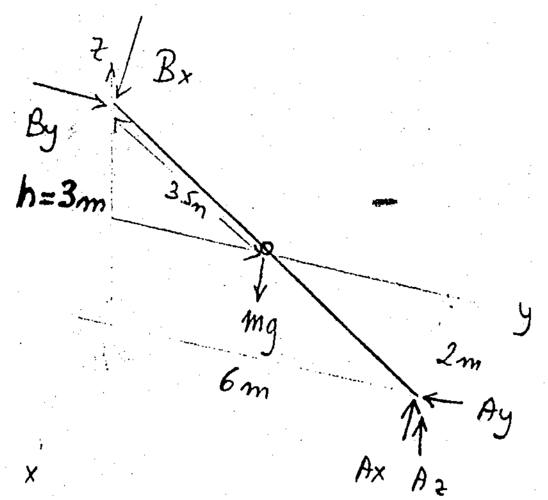
3.4.3 מסוימות סטטית

יש מסוימות ש.המשקל אם על הוא הן הכרחיות ומספיקות לקבוע את ש.המשקל והן בהכרח מספיקות להגדרתם של כל הכוחות הנעלמים. קיומם של כוחות מרובים גורם לאי מסוימות של המבנה. ושנים מדברים בהם הסמכה והנם מתאימים במקרה הקו-מומנט, ובכן נעסוק בהם. (עמ' 106-107)

דוגמא 3/3



נתון המוט שבשרטוט הנשען על קיר חלק ועומד בגומה חלקה בתחתיתו. חשב את תגובות הקיר על המוט. אורך המוט 7m ומסתו 200 ק"ג (ראה שקף 3.7)



א. פתרון

נחשב הכל בשיטה וקטורית. נשתמש ב-A כמרכז מומנטים. לצורך זה יש לחשב רדיוסי וקטורים.

$$r_{G/A} = -1i - 3j + 1.5k \quad [m]$$

$$r_{B/A} = -2i - 6j + 3k \quad [m]$$

$$\sum \vec{M}_A = r_{B/A} \times (iB_x + jB_y) + r_{G/A} \times (-mg\vec{k}) = 0$$

$$mg = 200 \times 9.81 = 1962 \text{ N}$$

$$(-i - 3j + 1.5k) \times (iB_x + jB_y) + (-2i - 6j + 3k) \times (-1962k) = 0$$

$$(-3B_y + 5886)\, i + (3B_x - 1962)\, j - (2B_y + 6B_x)\, k = 0$$

לכן כל רכיב צ"ל - 0

$B_y = 1962\ N$ $\quad B_x = 654\ N$ $\quad B = 2068$

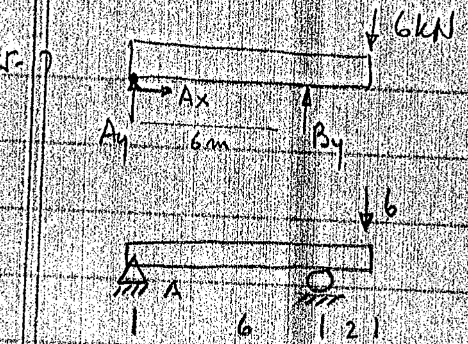
נחשב את הכוחות ב- A משיווי משקל בשלושת הכיוונים:

$$\sum \vec{F} = 0 \qquad (654 - A_x)i + (1962 - A_y)j + (-1962 + A_z)k = 0$$

$A_x = 654\ N \quad A_y = 1962\ N \quad A_z = 1962\ N$

$A = 2851\ N$

5-?



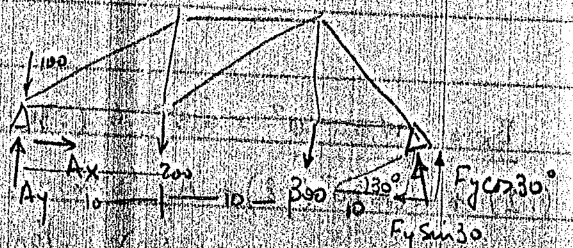
$\Sigma F_x = 0 = A_x = 0$

$\Sigma F_y = A_y + B_y - 6\text{ kN} = 0$

$\Sigma M_A = B_y \cdot 6 - 6 \cdot 8 = 0$ $B_y = 8\text{ kN}$

$A_y = -2\text{ kN}$

5-22



$\Sigma F_x = A_x - F_y \sin 30° = 0$

$\Sigma F_y = A_y - 100 - 200 - 300 + F_y \cos 30° = 0$

$\Sigma M_A = 0$

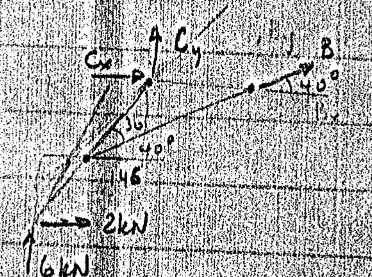
$-200 \cdot 10 - 300 \cdot 20 + F_y \cos 30° \cdot 30 = 0$

$F_y = \dfrac{8000}{30 \cos 30°} = 307.92\text{ lb}$

$A_x = 153.96\text{ lb} = F_y \sin 30°$

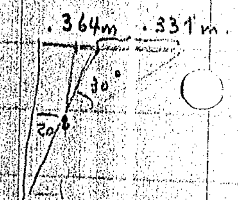
$A_y = 600 - F_y \cos 30° = 333.33\text{ lb}$

5-26



$\Sigma M_B = C_y(.331\text{ m}) - 2\text{kN}(1\text{m}) + 6\text{kN}(.695\text{m}) = 0$

$C_y = -6.56\text{ kN}$



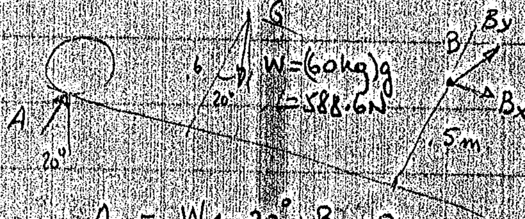
$\Sigma M_C = B \sin 40°(.331) + 2\text{kN}(1\text{m}) - 6\text{kN}(.364\text{m}) = 0$

$B = .865\text{ kN}$

$\Sigma M_A = C_y(.146) - C_x(.4) - 6\text{kN}(.218) + 2(.6) = 0$

$C_x = -2.664\text{ kN}$

5-28



$A_y - W\cos 20^\circ + B_y = 0$

$W\sin 20^\circ + B_x = 0$ $\quad \underline{B_x = -W\sin 20^\circ} = -588.6\,(.342) = -201.31\,N$

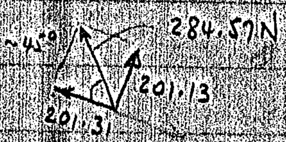
$\Sigma M_B \circlearrowleft = -W\sin 20^\circ\,(.1) + W\cos 20^\circ\,(.8) - A_y\,(1.2m) = 0$

$\boxed{A_y = \dfrac{W\sin 20^\circ(.1) - W\cos 20^\circ(.8)\,Nm}{1.2m}} = \dfrac{20.13N - (553)(.8)}{1.2m}$ (442.48)

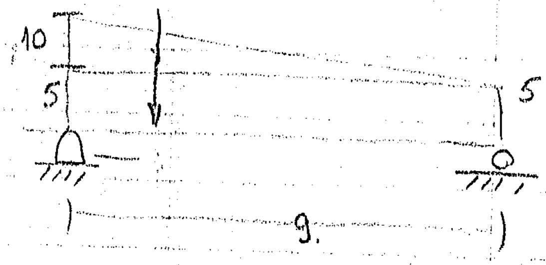
$\boxed{B_y = W\cos 20^\circ - A_y}$ $\quad = \dfrac{422.35\,Nm}{1.2m} = 351.96N$

$B_y = (553.1 - 351.96) = 201.13\,N.$

thus



4-88



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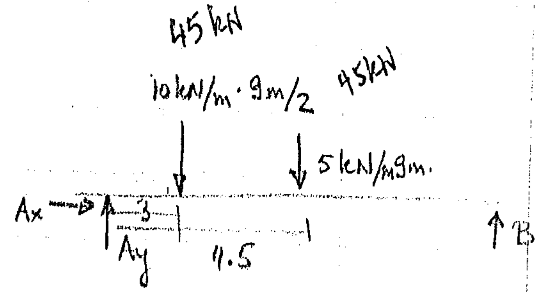


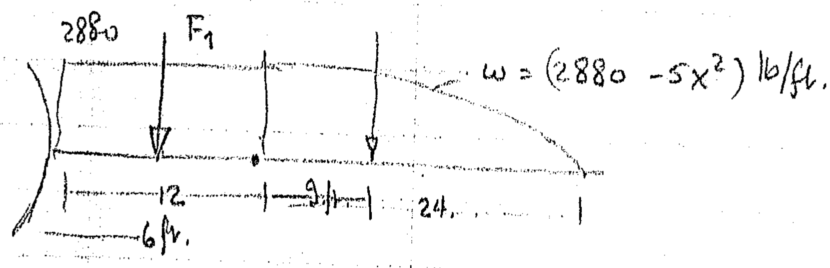
$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y + B - 45 \text{ kN} \quad - 45(\quad) \overset{kN}{=} 0$

$\curvearrowleft + \Sigma M_0 = -45 \cdot 3 - 45(4.5) + B(9) = 0 \qquad B = \cancel{48}^{5}\left(\frac{7.5}{\cancel{9}}\right) = 37.5 \text{ kN}$

$A_y = 90 - B = 90 - 37.5 = 52.5 \text{ kN}.$

4-92



$F_1 = 2880 \text{ lb/ft} \cdot 12 \text{ ft} = 34560 \text{ lb.}$ acting 6 ft from A or B.

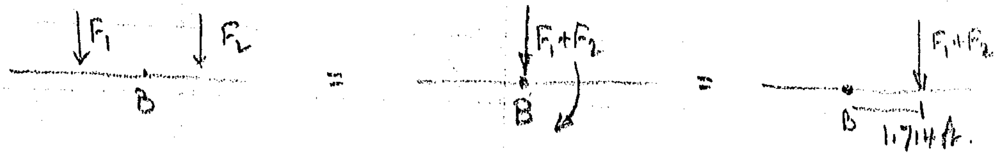
$F_2 = \int_0^{24} w\, dx = \int_0^{24} (2880 - 5x^2)\, dx = \left[2880x - \frac{5x^3}{3}\right]_0^{24} = 46080 \text{ lb.}$

$F_2 \bar{x} = \int w x\, dx = \int (2880x - 5x^3)\, dx = \left[1440x^2 - \frac{5}{4}x^4\right]_0^{24} = 414720 \text{ lb}$

$\bar{x} = 9 \text{ ft.}$

$\Sigma F_y = F_1 + F_2 = 34560 + 46080 = 80640 \text{ lb.}$

$\Sigma M_B = F_1 \cdot 6 \text{ ft} + F_2 \cdot 9 \text{ ft} = 138240 \text{ lb ft.}$



$d = \frac{\Sigma M}{\Sigma F_y} =$

SESSION #13

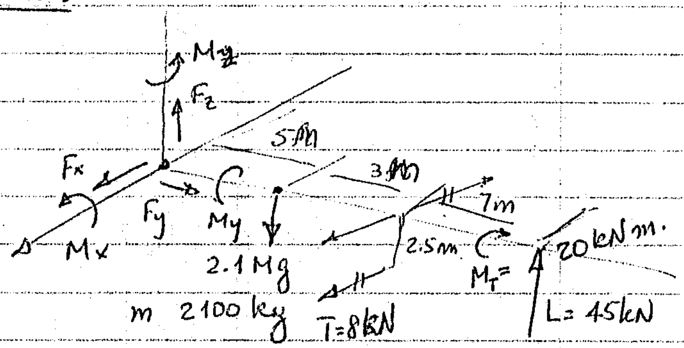
Section 4.12 Review of Ch #4 and #5

Problems 4-34, 4-88, 5-63, 5-67

SESSION #14

Problem 4-92 , 5-22 (If not done) , 5-59 , Go over the exam.

5-59

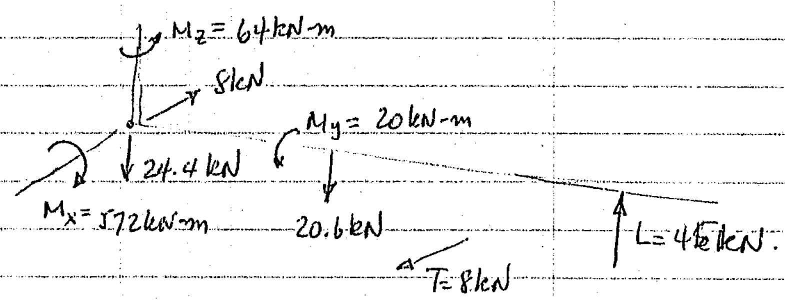


$\Sigma\vec{F} = \vec{0}$ $\qquad \Sigma F_x = 0 = F_x + T = 0 \qquad F_x = -T = -8\,kN$

$\Sigma F_y = 0 = \boxed{F_y = 0}$

$\Sigma F_z = 0 = L + F_z - W = 45\,kN + F_z - 20.6\,kN = 0$

$\boxed{F_z = -24.4\,kN}$



W causes moment about x axis

T causes moment about z axis & about y axis

L causes moment about x axis

$\Sigma M_x = M_x - (20.6\,kN)\,5m + 45\,kN \cdot 15m = M_x - 103 + 675 = 0$

$M_x = -572\ kN\text{-}m$

$\Sigma M_y = M_y - M_T = 0 \qquad M_y = M_T = 8\,kN\,(2.5\,m) = 20\,kN\text{-}m$

$\Sigma M_z = M_z - T \cdot 8m = 0 \qquad M_z = T \cdot 8m = 64\,kN\text{-}m$

SESSION #15

CHAPTER 6

- USE EQUIL EQUATIONS TO ANALYZE STRUCTURES MADE UP OF PIN CONNECTED MEMBERS
- IF STRUCTURE IS IN EQUIL SO MUST EACH MEMBER BE

- TRUSS - STRUCTURE MADE UP OF SLENDER MEMBERS JOINED AT THEIR END POINTS
- - CONNECTION IS MADE VIA A GUSSET PLATE OR PIN JOINTS

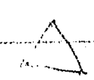
- PLANAR TRUSS - LIE IN A SINGLE PLANE
  - USED TO SUPPORT ROOFS & BRIDGES

- DESIGN OF A TRUSS $\boxed{\text{ASSUMPTIONS}}$
  - MEMBERS ARE JOINED TOGETHER BY SMOOTH PINS
    - GOOD EVEN WITH GUSSET PLATE IF ALL MEMBERS ARE CONCURRENT
  - LOADING APPLIED AT THE JOINTS
    - WEIGHT OF MEMBERS << FORCE SUPPORTED & NEGLECTED
    - IF NOT NEGLIGIBLE - ½ AT EACH END OF MEMBER.

THUS MEMBER IS A TWO FORCE MEMBER

SIMPLE TRUSS
- SIMPLEST FRAME WORK THAT IS RIGID OR STABLE IS A TRIANGLE

- TO CONSTRUCT A SIMPLE TRUSS START W/ TRIANGLE & ADD ON ~~AT LEAST~~ 2 MEMBERS. FOR EACH 2 MEMBERS ADDED JOINT INCREASES BY ONE.



$m+3=2j$

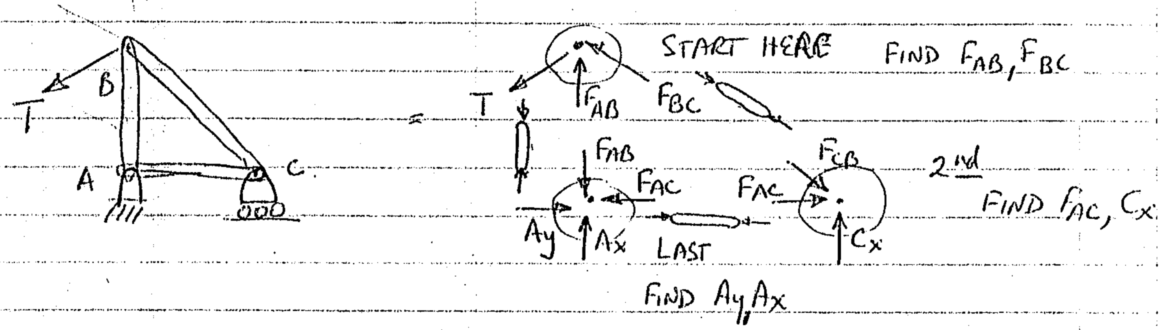
- TWO METHODS TO ATTACK PROBLEM
  - METHOD OF JOINTS & METHOD OF SECTIONS

- JOINTS TRUSS IN EQUIL $\Rightarrow$ JOINT MUST BE IN EQUIL.
- SATISFY $\Sigma\vec{F}=\vec{0}$ ON THE PIN AT EACH JOINT
- SINCE MEMBERS ARE TWO FORCE MEMBERS WHOSE LINE OF ACTION PASSES THROUGH THE JOINT $\Sigma\vec{M}\equiv\vec{0}$

- DRAW FREE BODY DIAGRAM OF EACH JOINT SHOWING FORCES & SUPPORT FORCES
  - LINE OF ACTION FOR EACH MEMBER IS GEOMETRY DEPENDENT
  - ONLY UNKNOWNS ARE MAGNITUDE OF FORCES

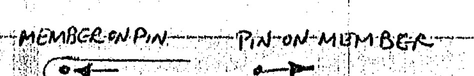
- JOINT ANALYSIS BEGINS AT JOINT HAVING NO MORE THAN 2 UNKNOWNS TO ALLOW USE OF $\Sigma F_x = 0$ $\Sigma F_y = 0$

- ASSUME A DIRECTION FOR FORCES IF YOU CANNOT INTUITIVELY GUESS



- + answer means assumed direction correct, − we reverse sense.

- DRAW FBD
- SHOW ALL FORCES & SUPPORTS - PICK DIRECTIONS, SOLVE FOR REACTIONS @ SUPPORTS WHEN NECESSARY
- ORIENT AXES TO MAKE RESOLUTION EASIEST
- GIVE ANSWER & TELL WHETHER MEMBER IS IN COMPRESSION OR TENSION
- REMEMBER REACTIONS ON THE PIN CORRECT DIRECTION ON MEMBER HAS OPPOSITE DIRECTION



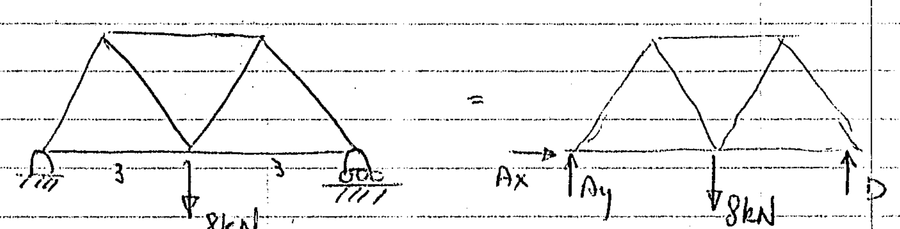
LOAD INTO PIN = COMPRESSION
LOAD AWAY FROM PIN = TENSION

ZERO FORCE MEMBERS

- CAN SIMPLIFY ANALYSIS BY FINDING THOSE MEMBERS NOT CARRYING LOAD
- USE THESE MEMBER TO INCREASE STABILITY OF TRUSS IF LOADING CHANGES

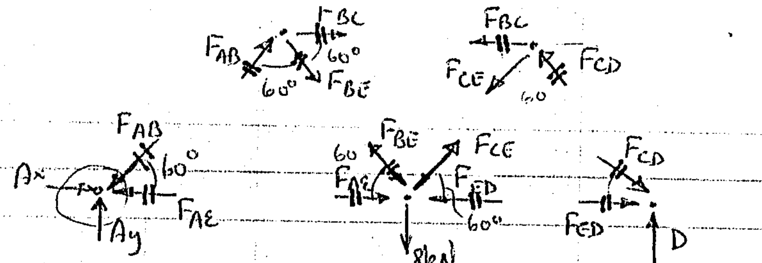
- EX: 2 MEMBER CARRYING NO EXTERNAL LOAD & $\perp$ TO EACH OTHER
SEE PG 208 FIG. 6-12

EX: 6-3 PG 240



$\Sigma F_x = 0$ $A_x = 0$

$\Sigma F_y = 0$ $A_y + D - 8kN = 0$ $\circlearrowleft + \Sigma M_A = -8 \cdot 3 + D \cdot 6 = 0$ $D = 4kN$ $A_y = 4kN$



START here | START here.

$A_y = 4kN$
$D = 4kN$
$A_x = 0$

$\cancel{A_x} - F_{AB}\cos 60^\circ - F_{AE} = 0$

$A_y - F_{AB}\sin 60^\circ = 0$

$F_{AB} = A_y / \sin 60^\circ$ $\quad$ $F_{AE} = -F_{AB}\cos 60^\circ$

$\boxed{F_{AB} = 4.619\ kN \text{ compression}}$ $\quad$ $\boxed{F_{AE} = -2.309\ kN \text{ tension}}$

@D $F_{CD}\cos 60^\circ + F_{ED} = 0$

$-F_{CD}\sin 60^\circ + D = 0$

$F_{ED} = -F_{CD}\cos 60^\circ = -2.309kN$ tension ~~compression~~

$F_{CD} = 4.619\ kN$ compression

@ B $\quad F_{AB}\cos 60^\circ + F_{BC} + F_{BE}\cos 60^\circ = 0$ $\quad$ $F_{BC} = -2F_{AB}\cos 60^\circ = -4.619\ kN$ compression

$F_{AB}\sin 60^\circ - F_{BE}\sin 60^\circ = 0$ $\quad$ $\boxed{F_{BE} = F_{AB} = 4.619\ kN \text{ tension}}$

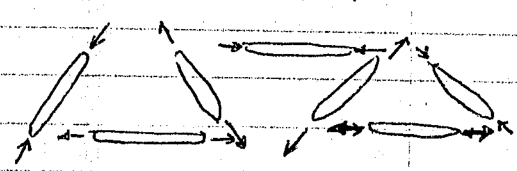
@ C $\quad F_{CD}\sin 60^\circ - F_{CE}\sin 60^\circ = 0$ $\quad$ $\boxed{F_{CD} = F_{CE} = 4.619kN \text{ tension}}$

Check $\quad -F_{BC} - F_{CE}\cos 60^\circ - F_{CD}\cos 60^\circ = 0$ $\quad$ $F_{BC} = -2F_{CE}\cos 60^\circ$
$= -4.619kN$ compression

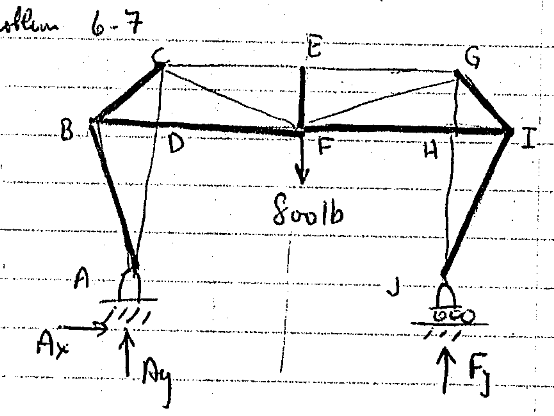
$\therefore$ $F_{AB} = 4.619\ kN$ Comp. $\quad$ $F_{CD} = 4.619\ kN$ comp. $\quad$ $F_{BC} = 4.619\ kN$ Comp

$F_{AE} = 2.309\ kN$ tension $\quad$ $F_{BE} = 4.619\ kN$ tens

$F_{ED} = 2.309\ kN$ tension ~~comp.~~ $\quad$ $F_{CE} = 4.619\ kN$ ten.



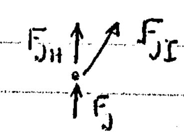
Problem 6-7



BY SYMMETRY

$\Sigma F_x = 0$

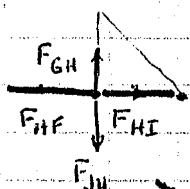
$\Sigma F_y = A_y - 800 + J = 0 \quad A_y = F_J = 400lb$



Since no external forces @J in x dir $\Rightarrow F_{JI} = 0$

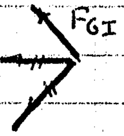
$\therefore F_{JH} + F_J = 0 \quad F_{JH} = -F_J$

$F_{JH} = 400lb$ Comp.

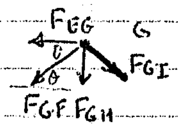


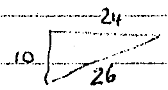
Since no x forces @ H $\Rightarrow$ $F_{HF} = F_{HI} = 0$

$F_{GH} = F_{JH} = -F_J = -400\,lb.$ $\boxed{F_{HG} = 400\,lb \quad Comp.}$



@ I since $F_{HI} = F_{IJ} = 0 \Rightarrow F_{GI} = 0$





$\sin\theta = \frac{10}{26} = \frac{5}{13}$

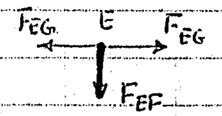
$\cos\theta = \frac{24}{26} = \frac{12}{13}$

960 lb Comp.

$\Sigma F_x = -F_{EG} - F_{GF}\frac{12}{13} = 0$ $\boxed{F_{EG} = -\frac{12}{5}F_J}$

$\Sigma F_y = -F_{GH} - \frac{5}{13}F_{GF} = 0 = F_J - \frac{5}{13}F_{GF}$

tension $1040\,lb = \boxed{\frac{13}{5}F_J = F_{GF}}$



since no forces in y dir @ E $\boxed{F_{EF} = 0}$

SESSION #16

CORRECT ERROR ON METH OF JOINTS — ie. only solve for support reaction first iff necessary.

METHOD OF ~~JOINTS~~ SECTIONS שיטת חתכים

- USED WHEN ONLY A FEW MEMBERS ARE TO BE ANALYZED. MOST DIRECT IN THIS CASE
- PASS AN IMAGINARY SECTION THROUGH TRUSS CUTTING IT INTO 2.
- EACH SECTION IS IN EQUIL.
- 3 EQS OF EQUIL CAN BE APPLIED TO EACH SECTION
- CUT SHOULD PASS THROUGH NO MORE THAN 3 MEMBERS SINCE WE HAVE 3 EQS.
- LINE OF ACTION OF EACH CUT TRUSS IS DETERMINED BY GEOM.
- EQUAL BUT OPPOSITE FORCES ON EACH CUT MEMBER ON OTHER SECTION

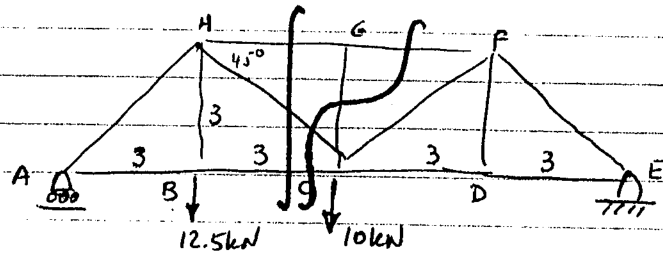
  Pg 213 ~~SEE~~ 6-14
- DEPENDING ON WHICH SECTION YOU CHOOSE TO SOLVE - MAY HAVE TO GET SUPPORT REACTION SEE 6-14a 6-14b
- IF YOU NEED SUPPORT REACTION - CONSIDER WHOLE TRUSS.

- PICK A DIRECTION FOR THE FORCE LIKE IN METHOD OF JOINTS
- SUM MOMENT ABOUT POINT OF INTERSECTION OF LINES OF ACTIONS OF FORCES

PROCEDURE

- FREE BODY DIAGRAM FIRST
- MAKE CUT AS APPROPRIATE (REMEMBER NO MORE THAN 3 MEMBERS)
- IF NECESSARY SOLVE FOR SUPPORT REACTIONS FIRST ON WHOLE TRUSS
- PICK DIRECTION OF FORCES
- APPLY $\Sigma F_x, \Sigma F_y, \Sigma M = 0$
- ARROWS REPRESENT ACTUAL DIRECTION OF FORCES ON THE MEMBER.

PROBLEM 6-17 pg 217



Get BC, HC, CG

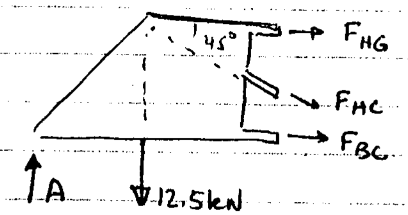
must solve for reactions.

$\Sigma F_x = 0 \qquad E_x = 0$

$\Sigma F_y = 0 \qquad A + E_y - 22.5\text{kN} = 0$

$\circlearrowleft + \Sigma M_E = 0 \qquad 10 \cdot 6 + (12.5)9 - A(12) = 0$

$A = 14.375 \qquad E_y = 8.125\text{kN}$



$\Sigma F_y = A - 12.5\text{kN} - F_{HC}\sin 45° = 0$

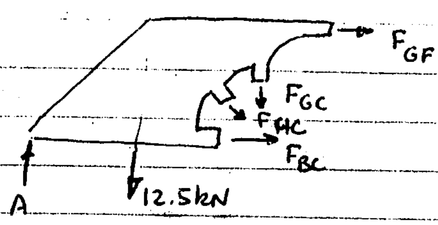
$(14.375 - 12.5) - .7071 F_{HC} = 0 \qquad \boxed{F_{HC} = 2.652 \text{ kN } (T)}$

$\Sigma F_x = 0 \qquad F_{BC} + F_{HG} + F_{HC}\cos 45° = 0$

$\circlearrowleft \Sigma M_H = 0$

MUST MAKE ANOTHER CUT TO DETERMINE F_{GC}

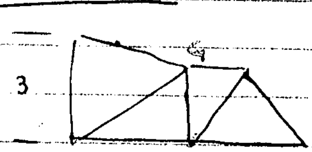
$-A \cdot 3 + F_{BC} \cdot 3 = 0 \qquad \boxed{F_{BC} = A = 14.375 \text{kN } (T)}$

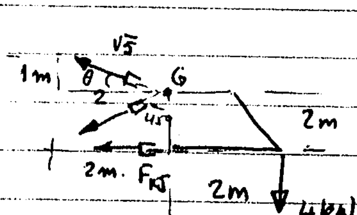


$\Sigma F_y = \underbrace{A - 12.5 - F_{HC}\sin 45°}_{=0} - F_{GC} = 0$

$\Rightarrow \boxed{F_{GC} = 0}$

PROBLEM 6-23 218





$\circlearrowleft + \Sigma M_G = -2 F_{KJ} + 4\text{kN}(2m) = 0 \qquad F_{KJ} = 4\text{kN } (C)$

$$\Sigma F_x = -F_{GF}\cos\theta - F_{GK}\cos\phi - \overset{+4\,kN}{F_{KJ}} = 0$$

$$\Sigma F_y = +F_{GF}\sin\theta - F_{GK}\sin\phi - 4\,kN = 0$$

$\cos\theta = \frac{2}{\sqrt{5}}$ $\sin\theta = \frac{1}{\sqrt{5}}$ $\cos\phi = \frac{1}{\sqrt{2}}$ $\sin\phi = \frac{1}{\sqrt{2}}$

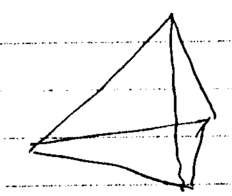
$F_{GF} = +5.963\,kN$ (T)

$F_{GK} = -1.886\,kN$ (C).

SESSION #17

<u>SPACE TRUSS</u> - 3 DIMENSIONAL TRUSS MADE UP OF MEMBERS TO FORM A STABLE STRUCTURE

- SIMPLEST STRUCTURE IS A TETRAHEDRON



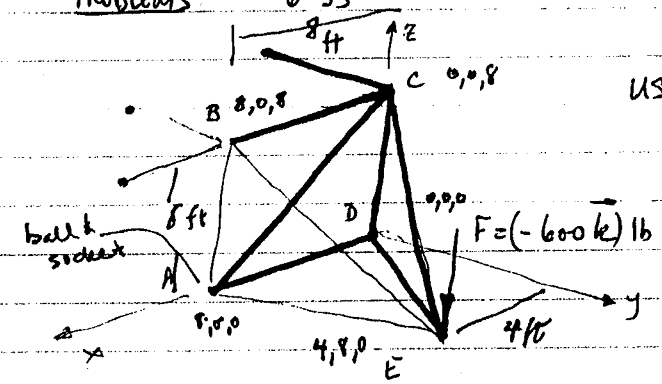
- CAN TREAT EACH MEMBER AS 2-FORCE MEMBER IF
  - JOINTS ARE BALL & SOCKET CONNECTIONS (NO M_x, M_y, M_z)
  - FORCES AT JOINTS
  - WEIGHT OF MEMBER IS ACC'TED FOR (½, ½) AS BEFORE

- CAN USE METHOD OF SECTIONS - IF SOME FORCES ARE REQ'D
  JOINTS - IF ALL FORCES REQUIRED

- SECTIONS USE 6 equations $\Sigma\vec{F} = \vec{0}$, $\Sigma\vec{M} = \vec{0}$

- JOINTS USE 3 eqs $\Sigma\vec{F} = \vec{0}$

<u>PROBLEMS</u> 6-33

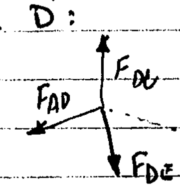


Find forces in each member

USE: method of Joints $\Rightarrow \Sigma F_x = \Sigma F_y = \Sigma F_z = 0$

start @ pt where there are 3 unknowns

@ D:

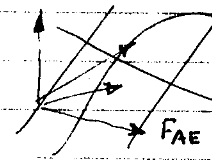


$$\bar{F}_{ED} = F_{ED}\frac{1}{\sqrt{5}}\bar{i} + F_{ED}\frac{2}{\sqrt{5}}\bar{j}$$

$\Sigma F_x = \quad F_{AD} + F_{ED} \frac{1}{\sqrt{5}} = 0 \qquad\qquad \Sigma F_z = F_{CD} = 0$

$\Sigma F_y = \quad F_{ED} \frac{2}{\sqrt{5}} = 0 \qquad\qquad \Rightarrow \quad F_{AD} = F_{ED} = F_{CD} = 0$

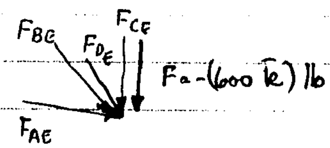
~~@ A~~



$\cancel{\Sigma F_x = F_{AD} - \frac{1}{\sqrt{5}} F_{AE} - F_{AC}\frac{1}{\sqrt{2}} = 0} \qquad \cancel{\Sigma F_z = F_{AB} + F_{AC}\frac{1}{\sqrt{2}} = 0}$

$\cancel{\Sigma F_y = F_{AE} \frac{2}{\sqrt{5}} = 0 \quad \Rightarrow \quad F_{AE} = F_{AB} = 0}$

@ E



@ B (8, 0, 8) @ E (4, 8, 0)

$\bar{r}_{BE} = (-4, 8, -8)$ ft $\quad r_{BE} = 12$ ft

$\bar{F}_{BE} = F_{BE}\left[-\frac{1}{3}\bar{i} + \frac{2}{3}\bar{j} - \frac{2}{3}\bar{k} \right]$

@ C = (0, 0, 8) $\quad \bar{r}_{CE} = (4, 8, -8)$ ft

$\bar{F}_{CE} = F_{CE}\left[\frac{1}{3}\bar{i} + \frac{2}{3}\bar{j} - \frac{2}{3}\bar{k} \right]$

$\bar{F}_{AE} = -\frac{1}{\sqrt{5}} F_{AE}\,\bar{i} + \frac{2}{\sqrt{5}} F_{AE}\,\bar{j}$

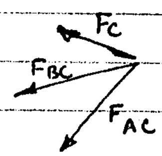
$\Sigma F_x = \quad F_{CE}\frac{1}{3} - F_{BE}\frac{1}{3} - \frac{1}{\sqrt{5}} F_{AE} = 0$

$\Sigma F_y = \frac{2}{3} F_{BE} + \frac{2}{3} F_{CE} + \frac{2}{\sqrt{5}} F_{AE} = 0 \qquad F_{CE} = 0$

$\Sigma F_z = -\frac{2}{3} F_{BE} - \frac{2}{3} F_{CE} - 600 = 0. \qquad F_{BE} = -900$ lb $\qquad \underline{900 \text{ lb } T}$

$F_{AE} = 670.82$ lb $\qquad \underline{670.82 \text{ lb } C}$

@ C

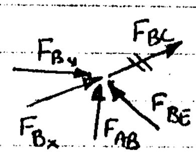


$\Sigma F_x = F_{BC} + F_{AC}\frac{1}{\sqrt{2}} = 0 \qquad F_{BC} = 0$

$\Sigma F_y = -F_C = 0 \qquad F_C = 0$

$\Sigma F_z = -F_{AC}\frac{1}{\sqrt{2}} = 0 \qquad F_{AC} = 0$

@ B.



$\bar{F}_{BE} = F_{BE}\left[\frac{1}{3}\bar{i} - \frac{2}{3}\bar{j} + \frac{2}{3}\bar{k} \right]$

$\Sigma F_x = -F_{Bx} + \frac{1}{3} F_{BE} = 0 \qquad F_{Bx} = -300$ lb $\qquad 300$ lb T

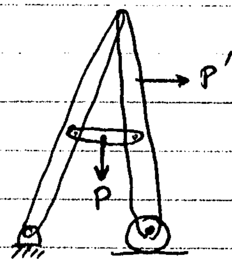
$\Sigma F_y = F_{By} - \frac{2}{3} F_{BE} = 0 \qquad F_{By} = -600$ lb $\qquad 600$ lb T

$\Sigma F_z = F_{AB} + \frac{2}{3} F_{BE} = 0 \qquad F_{AB} = 600$ lb $\qquad 600$ lb C

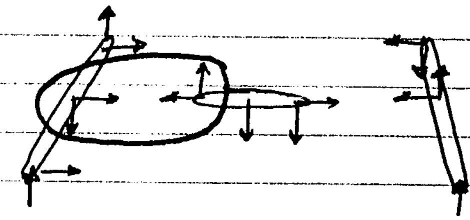
- FRAMES & MACHINES
  - MADE UP OF MEMBERS HAVING MORE THAN 2 DISTINCT FORCES APPLIED TO THEM W/ FORCES NOT APPLIED AT PINS
- FRAMES - STATIONARY - USED TO SUPPORT LOADS
  - MACHINES - INVOLVE MOVING PARTS - TRANSMIT & ALTER LOADS

    כוללים חלקים נעים ומעבירים ומשנים עומסים
- PROPERLY DESIGNED ONES REQUIRE $\Sigma F_x = \Sigma F_y = \Sigma M_0 = 0$ at JOINTS FOR EQUILIBRIUM.

LOOK AT Pg 225 FIG 6-19



- AT EACH PIN THERE ARE 2 COMPONENTS OF REACTION



- REACTION IS EQUAL BUT OPPOSITE ON OTHER MEMBER

$\Sigma F_x, \Sigma F_y, \Sigma M$

- # OF AVAILABLE EQ'S = 3 TIME NO. OF MEMBERS $= 3n$
- FOR STATICALLY DETERMINATE PROBLEM # EQS = # UNKNOWNS
- CAN LOOK AT PART OF BODY (METHOD OF SECTIONS)

  OR
- CAN LOOK AT FBD OF WHOLE

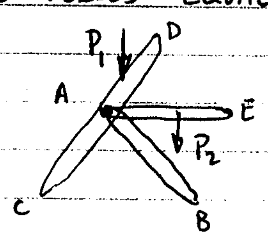
DEPENDS ON CONVENIENCE
WHAT YOU HAVE TO SOLVE FOR

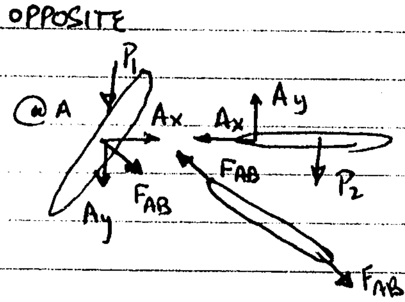
- IF MORE EQS THAN UNKNOWNS — THE EXTRAS ARE NOT UNIQUE & ONLY SERVE TO CHECK RESULTS.

- CAN SOLVE $\Sigma F_x = \Sigma F_y = \Sigma M = 0$ on FBD of whole thing. Then apply same eqs to $2n-1$ members to get rest of unknowns
- Remaining member can be used as check

- IF MORE UNKNOWNS THAN EQS See FIG 620(c) MUST DISMEMBER FRAME OR MACHINE TO SOLVE PROBLEM

- IF MORE THAN 3 MEMBERS JOIN AT A PIN ONLY 2 MEMBERS CAN HAVE FORCES EQUAL & OPPOSITE



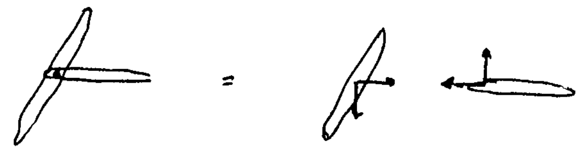


<u>PROCEDURE</u> - IF FORCES ACT ON BODY OTHER THAN AT PIN (FRAME OR MACHINE)

DRAW FBD - IF MORE UNK THAN EQS FOR WHOLE BODY

DISMEMBER

IF DISMEMBER — EQUAL BUT OPPOSITE FORCES ON JOINING MEMBERS



RECOGNIZE: 2 FORCE MEMBERS - FORCES ARE COLLINEAR BUT OPPOSITE IN DIR
NO MATTER WHAT THE SHAPE

EQS: 3 EQS PER MEMBER : 3 EQS FOR WHOLE BODY

FOR MOMENTS - TAKE PT WHERE LINE OF ACTIONS OF AS MANY UNKNOWNS PASS THROUGH

<u>READ &</u>

<u>GO THROUGH</u> EXAMPLES 6-8 TO 6-15

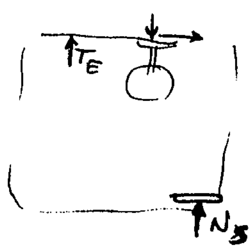
6-8 NOTE <u>BE</u> IS A 2 FORCE MEMBER

NOTE IN FBD 3 UNK D_x, A_x, A_y

6-9 NOTE AB 2 FORCE

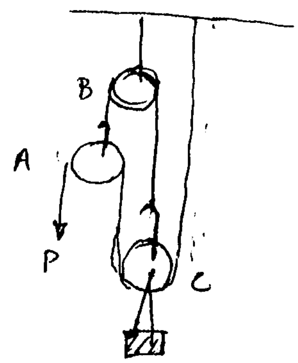
NOTE FBD 4 UNK. A_x, A_y, N_w, P (force of compressed body)

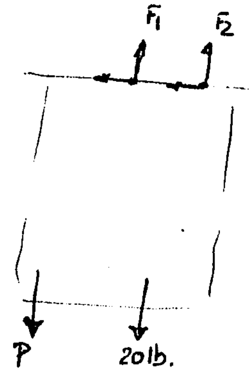
6-14 FBD OF WHOLE INVOLVES 4 UNK



<u>MUST DISMEMBER.</u>

Problem 6-37

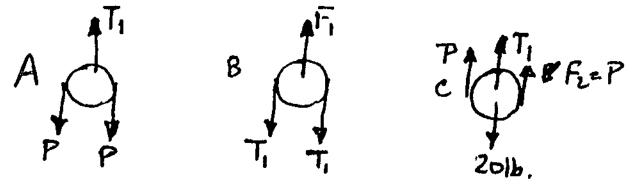




Since no x forces, $\Sigma F_x = 0$

$\therefore$ 2 eqs & 3 unk

must dismember.



$F_2 = P$ $\quad T_1 = 2P$

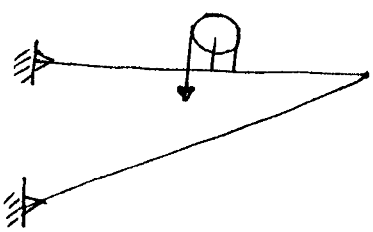
@ C $\quad 4P - 20\,lb = 0 \quad P = 5\,lb$

$T_1 = 10\,lb$

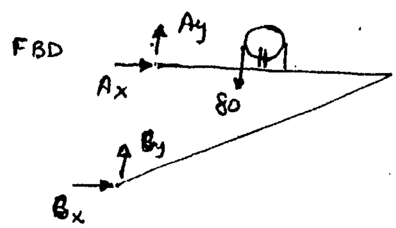
$F_1 = 20\,lb$

$\underline{F_2 = P}$

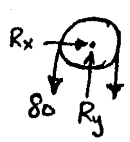
Problem 6-47



FBD

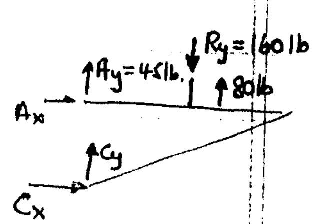


4 unknowns - 3 eqs

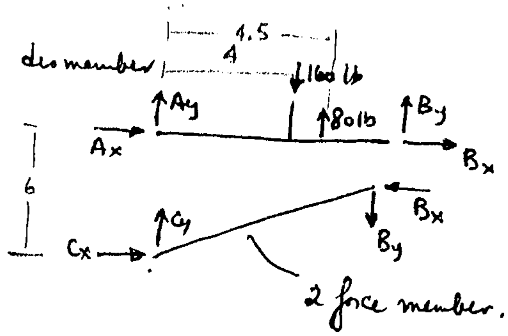


$R_x = 0 \Leftarrow \Sigma F_x = 0$

$\Sigma F_y = 0 \Leftarrow \quad R_y = 160\,lb.$



must dismember



$\Sigma F_x = A_x + B_x = 0$

$\Sigma F_y = A_y - 80 + B_y = 0$

$(+\Sigma M_A = -160 \cdot 4 + 80(4.5) + B_y \cdot 8 = 0$

$B_y = 35\,lb$

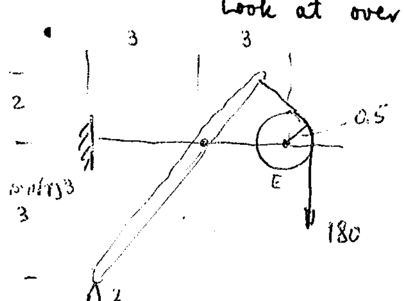
$\boxed{A_y = 45\,lb}$

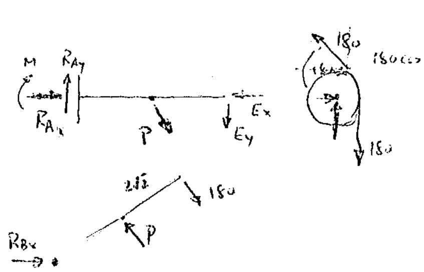
Look at overall truss $(+\Sigma M_C = -A_x \cdot 6 - 160 \cdot 4 + 80 \cdot (4.5) = 0 \quad A_x = -46.67\,lb$

$\Rightarrow B_x = -A_x = 46.67\,lb$

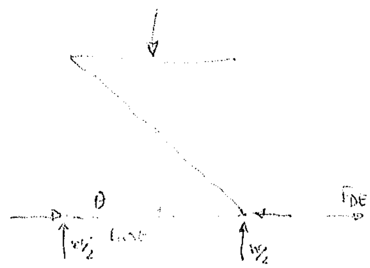
$C_x = B_x = 46.67\,lb$

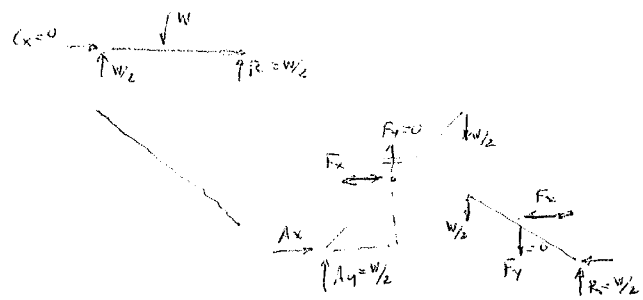
$C_y = B_y = 35\,lb.$





6-86





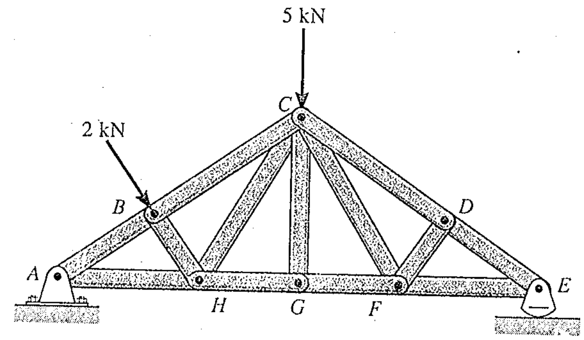
$$\overset{W/2}{A_y} \cdot 2L\cos\theta = A_x \cdot L s\theta$$

$$W L\cos\theta = A_x \cdot L s\theta$$

$$A_x = \frac{W}{\tan\theta} = F_x = F_{DE}$$

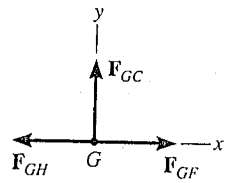
Example 6–4

Using the method of joints, determine all the zero-force members of the *Fink truss* shown in Fig. 6–13*a*.

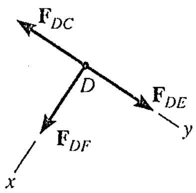


(a)

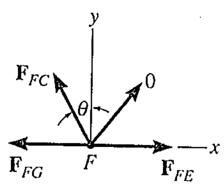
Fig. 6–13



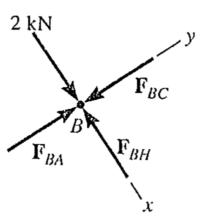
(b)



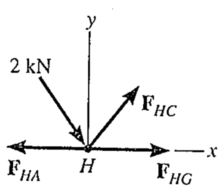
(c)



(d)



(e)



(f)

SOLUTION

Looking for joint geometries that are similar to those outlined in Figs. 6–11 and 6–12, we have

Joint G (Fig. 6–13*b*)

$+\uparrow \Sigma F_y = 0;$ $\qquad F_{GC} = 0$ $\qquad$ ***Ans.***

Realize that we could not conclude that GC is a zero-force member by considering joint C, where there are five unknowns. The fact that GC is a zero-force member means that the 5-kN load at C must be supported by members CB, CH, CF, and CD.

Joint D (Fig. 6–13*c*)

$+\swarrow \Sigma F_x = 0;$ $\qquad F_{DF} = 0$ $\qquad$ ***Ans.***

Joint F (Fig. 6–13*d*)

$+\uparrow \Sigma F_y = 0;$ $\quad F_{FC} \cos\theta = 0$, since $\theta \neq 0;$ $\qquad F_{FC} = 0$ $\qquad$ ***Ans.***

Note that if joint B is analyzed, Fig. 6–13*e*,

$+\searrow \Sigma F_x = 0;$ $\qquad 2 - F_{BH} = 0 \qquad F_{BH} = 2$ kN (C)

Consequently, the numerical value of F_{HC} must satisfy $\Sigma F_y = 0$, Fig. 6–13*f*, and therefore HC is *not* a zero-force member.

PR

6–1. cate wh

6–2. D cate wh

6.4 The Method of Sections

The *method of sections* is used to determine the loadings acting within a body. It is based on the principle that if a body is in equilibrium, then any part of the body is also in equilibrium. To apply this method, one passes an *imaginary section* through the body, thus cutting it into two parts. When a free-body diagram of one of the parts is drawn, the loads acting at the section must be *included* on the free-body diagram. One then applies the equations of equilibrium to the part in order to determine the loading at the section. For example, consider the two truss members shown colored in Fig. 6–14. The internal loads at the section indicated by the dashed line can be obtained using one of the free-body diagrams shown on the right. Clearly, it can be seen that equilibrium requires that the member in tension be subjected to a "pull" **T** at the section, whereas the member in compression is subjected to a "push" **C**.

The method of sections can also be used to "cut" or section several members of an entire truss. If either of the two parts of the truss is isolated as a free-body diagram, we can then apply the equations of equilibrium to that part to determine the member forces at the "cut section." Since only *three* independent equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) can be applied to the isolated part of the truss, one should try to select a section that, in general, passes through not more than *three* members in which the forces are unknown. For example, consider the truss in Fig. 6–15*a*. If the force in member *GC* is to be determined, section *aa* would be appropriate. The free-body diagrams of the two parts are shown in Figs. 6–15*b* and 6–15*c*. In particular, note that the line of action of each cut member force is specified from the *geometry* of the truss, since the force in a member passes along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part—Newton's third law. As noted above, members assumed to be in *tension* (*BC* and *GC*) are subjected to a "pull," whereas the member in *compression* (*GF*) is subjected to a "push."

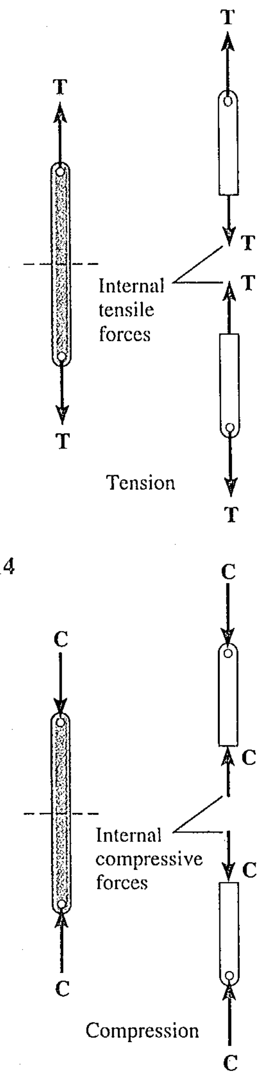
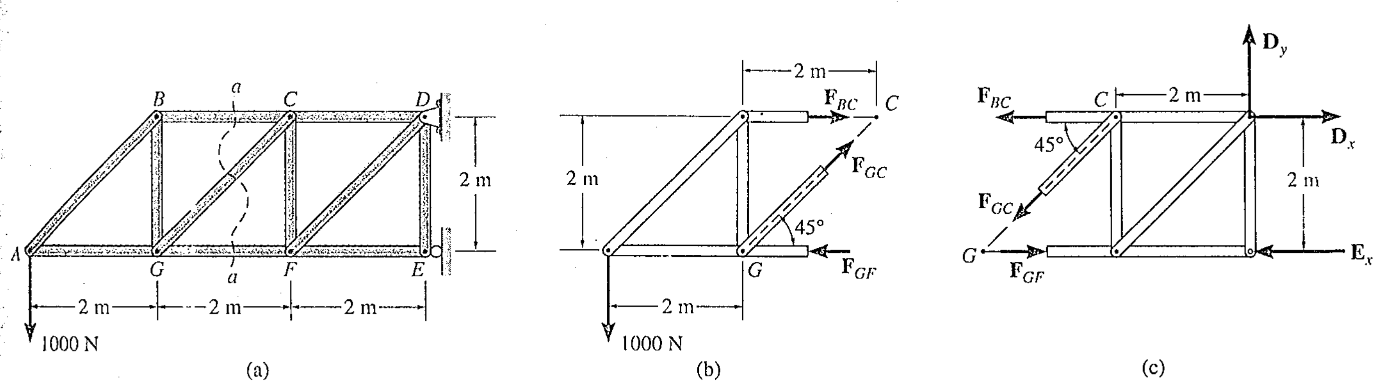
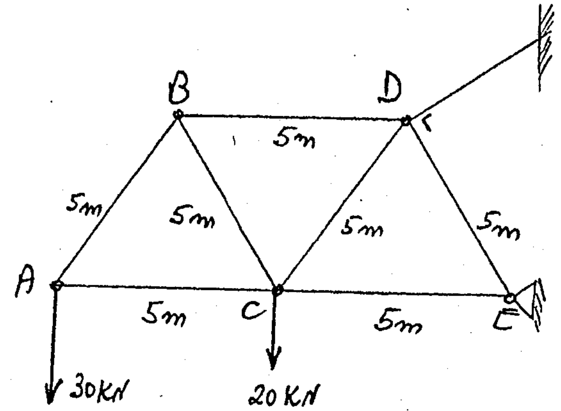


Fig. 6–14

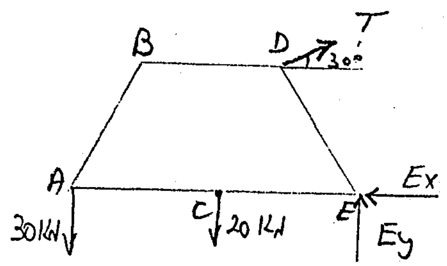
Fig. 6–15





דוגמא (4.1)

חשב את כל הכוחות הפועלים במוטות ובסמכים של המסבך המשני הבא:



שלב א' - קביעת הריאקציות

$$\Sigma M_E = 0 \quad 5T - 5\times 20 - 10\times 30 = 0$$

$$T = 80.\ KN$$

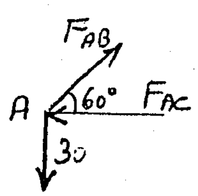
$$\Sigma Fx = 0 \quad T\cos 30 - Ex = 0$$

$$Ex = 69.3\ KN$$

$$\Sigma Fy = 0 \quad T\sin 30 + Ey - 50 = 0$$

$$Ey = 10\ KN$$

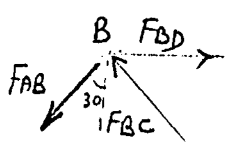
שלב ב' - הכוחות במוטות



נתחיל בצומת A

$$\Sigma Fy = 0 \qquad F_{AB}\sin 60° - 30 = 0 \qquad F_{AB} = 34.64\ KN\ T$$

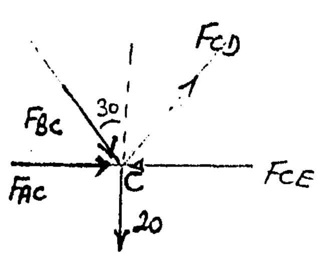
$$\Sigma Fx = 0 \qquad F_{AB}\cos 60° - F_{AC} = 0 \qquad F_{AC} = 17.32\ KN\ C$$



נעבור לצומת B

$$\Sigma Fy = 0 \quad -F_{AB}\cos 30° + F_{BC}\cos 30° = 0 \quad F_{BC} = 34.64\ KN\ C$$

$$\Sigma Fx = 0 \quad -F_{AB}\sin 30 - F_{BC}\sin 30° + F_{BD} = 0 \quad F_{BD} = 34.64\ KN\ T$$

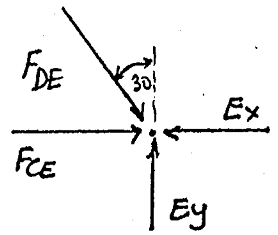


נעבור לצומת C

$$\Sigma Fy = 0 \quad -F_{BC}\cos 30° - 20 + F_{CD}\cos 30° = 0 \quad F_{CD} = 57.74\ KN\ T$$

$$\Sigma Fx = 0 \quad F_{AC} + F_{BC}\sin 30° + F_{CD}\sin 30° - F_{CE} = 0 \quad F_{CE} = 63.51\ KN\ C$$

לבסוף נעבור לצומת E



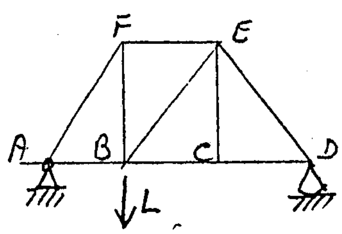
$\Sigma F_y=0 \quad -F_{DE}\cos 30^\circ+E_y=0 \qquad F_{DE}=11.55\,KN\ C$

בדיקת סגירת שיווי משקל ב-D נותנת כמובן 0.

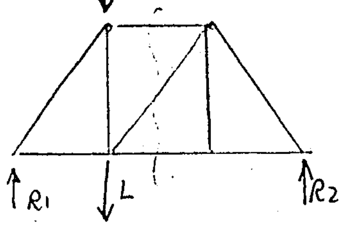
4.4 שיטת החתכים

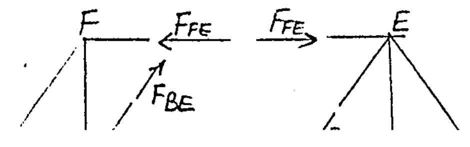
בשיטת הצמתים נצלנו רק שתי מ. שווי משקל של הכוחות כיון שדנו תמיד בכוחות הנחתכים בנקודה. בשיטת החתכים אנו מנצלים גם את משוואת המומנטים כיון שאנו דנים בשווי משקלו של חלק מהמסבך הכולל והוא נפעל בד"כ לכוחות שאינם נחתכים בנקודה. בשיטה זאת ניתן בד"כ לחשב ישירות את הכח הפועל במוט זה או אחר במסבך מבלי להגיע אליו בדרך סיסטמטית כבשיטת הצמתים. אנו חותכים חלק מהמסבך כך שהחתך יעבור דרך מוט זה. ויש לזכור שבזמן החתך יש להקפיד שמס' הנעלמים שנוצרים לא יעלה על שלושה שניתן לחשבם משלושת מש. שווי המשקל.

נדגים את השיטה בעזרת המסבך שבשרטוט. ראשית נחשב את הריאקציות ובשלב שני נדרוש חישוב הכח במוט BE.



זאת נבצע ע"י "חתוך" המסבך לשנים:
כעת נוצרו שלושה כוחות נעלמים הפועלים על כל אחד מחצאי המסבך.
כיון הכחות שסומן ניתן להערכה [illegible] ואין הוא חשוב כרגע ויש להבטיח שעל כל חלק אותו כוח יפעל בכיוונים הפוכים!!!
כעת נעבור לחישוב שלושת הכוחות:

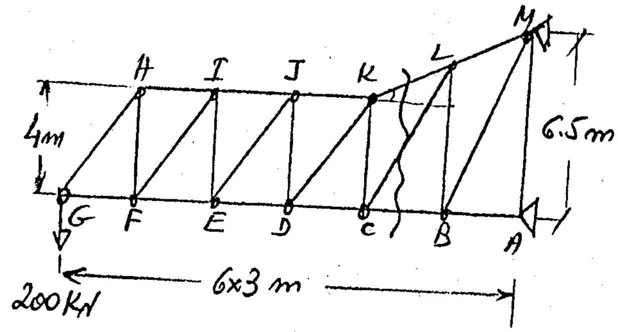




ו. חשב מומנט סביב B נותן ישירות את F_{FE}

ב. $\Sigma F_y=0$ של הגוף נותן ישירות את F_{BE}

ג. $\Sigma F_x=0$ של הגוף נותן ישירות את F_{BC}



דוגמא

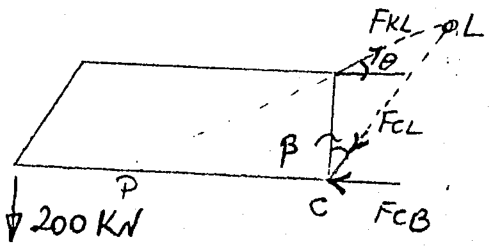
חשב את הכוחות במוטות KL , CL , CB .

$m=23 \quad j=13$

$23+4 \stackrel{?}{=} 2\times 13$

$27 > 26$

המסבך אינו מסוים סטטית חיצונית בגלל שני הסמכים. נחתך את המסבך ונקבל את המצב הבא:



מומנט סביב L מכוון כי F_{CB} חייב להיות לחיצה ומומנטים סביב הנקודה C מכוון כי F_{KL} חייב להיות מתיחה ומכאן ש- F_{CL} חייב להיות לחיצה כי נבדוק מומנטים סביב P מפגש של F_{KL} ו F_{CB} .

הזרוע BL $\quad BL = 4 + \frac{6.5-4}{2} = 5.25\,m \quad \left(\frac{6.5+4}{2}\right)$

$\Sigma M_L = 0 \qquad 200\times(5\times 3) - F_{CB}\times 5.25 = 0 \qquad F_{CB} = 571\ KN\ C$

נחשב θ $\quad \theta = \tan^{-1}\frac{5}{12} \Rightarrow \cos\theta = \frac{12}{13}$

$\Sigma M_C = 0 \qquad 200\times(4\times 3) - \frac{12}{13}F_{KL}\times 4 = 0 \qquad F_{KL} = 650\ KN\ T$

נחשב PC , β $\quad \frac{\overline{PC}}{4} = \frac{6}{6.5-4} \Rightarrow \overline{PC} = 9.6\ m$

$\bar{\beta} = \tan^{-1}\frac{\overline{CB}}{\overline{BL}} \qquad \beta = 29.7^\circ \qquad \cos\beta = 0.868$

$\Sigma M_P = 0 \qquad 200\,(12-9.6) - F_{CL}\cdot 0.868\times 9.6 = 0 \qquad F_{CL} = 56.7\ KN\ C$

4.5 מסבכים מרחביים

המסבך המרחבי הוא בן זוגו של המסבך המישורי. גם הוא בנוי מוטות המעבירים כחות לאורכם בלבד והמחוברים ביניהם פרקים. היחידה הבסיסית של מסבך זה הוא הטטרהדרון.

תנאי המסוימות: אם מס' הצמתות הוא j אזי מס' המשוואות הוא $3j$. מס' הנעלמים הוא $m+6$ כאשר m הוא מס' המוטות ולכן תנאי המסוימות

$m+6=3j$ (או: $m+2=3j$)

זהו גם תנאי הכרחי ולא מספיק ליציבות המסבך.

במקרה ש: $m+6>3j$

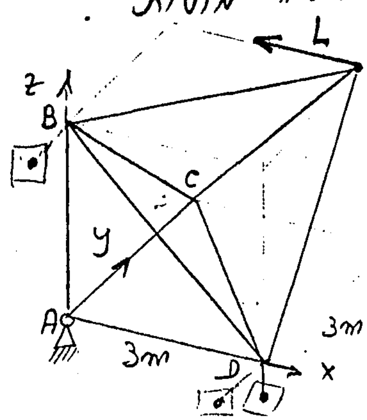
המסבך בלתי מסוים סטטי בדרגה של: $m+6-3j$

במקרה ש: $m+6<3j$ המסבך בלתי קשיח.

שיטות הפתרון כבמסבכים המישוריים:

שיטת הצמתים: בכל צומת 3 משוואות $\Sigma\vec{F}=0$ וניתן לפתור בו צמתים עד שלושה נעלמים.

שיטת החתכים: לרשותנו כל שש המשוואות $\Sigma\vec{F}=0$ $\Sigma\vec{M}=0$ וניתן "לחתוך" גם יותר משלושה מוטות.



דוגמא:

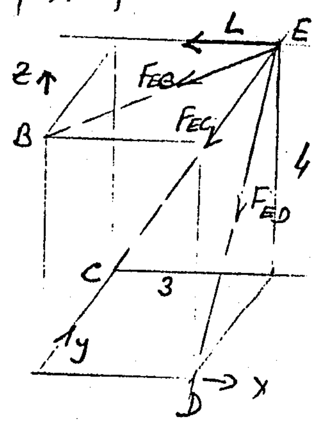
המסבך שבשרטוט נפעל לכח L. חשב את הכוחות במוטות הנפגשים ב-E.

א. המסבך רתום כהלכה. מס' המוטות $m=9$ ומס' הצמתים $j=5$

$$9+6 = 3\times 5 = 15$$

המסבך יציב ומסוים סטטית.

ב. ניתן לחשב את הריאקציות בצמתים A,B,D ברם אין צורך לשאלה שהוצבה.



ג. נסתכל בצומת E יש בה שלושה נעלמים וכח ידוע אחד. נרשום בצורה וקטורית.

$$\vec{L} = -L\,i$$

$$\vec{F}_{EB} = \frac{F_{EB}}{\sqrt{2}}(-i-j)$$

$$\vec{F}_{EC} = \frac{F_{EC}}{5}(-3i-4k)$$

$$\vec{F}_{ED} = \frac{F_{ED}}{5}(-3j-4k)$$

כולם סומנו כמתיחה!!!

3. נרשום שיווי משקל

$$\sum \vec{F}_E = 0 \qquad \vec{L} + \vec{F}_{EB} + \vec{F}_{EC} + \vec{F}_{ED} = 0$$

$$\left(-L - \frac{F_{EB}}{\sqrt{2}} - \frac{3F_{EC}}{5}\right)\vec{i} + \left(-\frac{F_{EB}}{\sqrt{2}} - \frac{3F_{ED}}{5}\right)\vec{j} + \left(-\frac{4F_{EC}}{5} - \frac{4F_{ED}}{5}\right)\vec{k} = 0$$

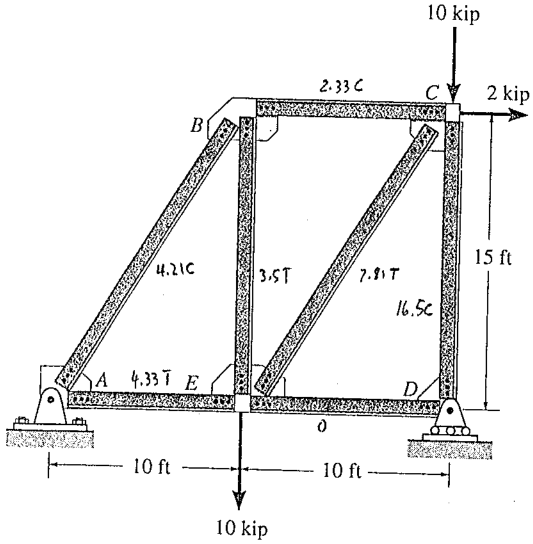
כיוון שכל רכיב חייב להתאפס נקבל

$$\frac{F_{EB}}{\sqrt{2}} + \frac{3F_{EC}}{5} = -L \qquad \frac{F_{EB}}{\sqrt{2}} + \frac{3F_{ED}}{5} = 0 \qquad F_{EC} + F_{ED} = 0$$

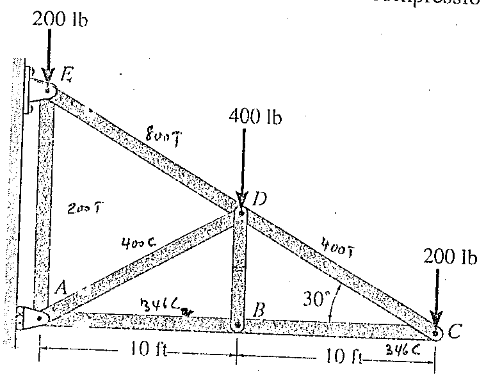
$$F_{EB} = -\frac{L}{\sqrt{2}}\ C \qquad F_{EC} = -\frac{5L}{6}\ C \qquad F_{ED} = \frac{5}{6}L\ \ T$$

פתרון זה התאפשר הודות לעובדה שבצומת E היו שלושה נעלמים בלבד. להמשך הפתרון נזדקק לחשב הריאקציות.

cate whether the members are in tension or compression. Assume that all the members are pin-connected.

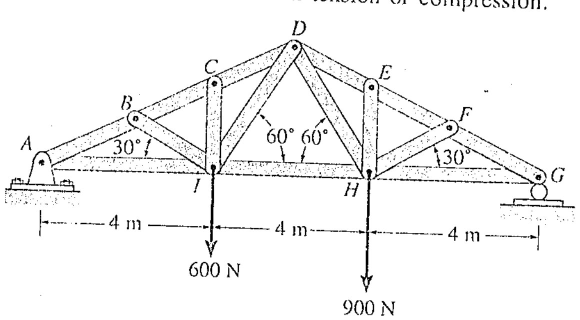


6–11. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



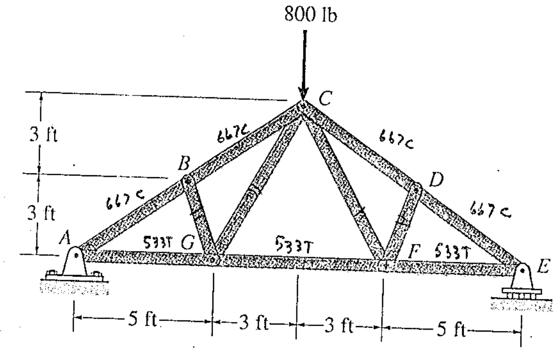
Prob. 6–11

***6–12.** Determine the force in each member of the truss and indicate whether the members are in tension or compression.



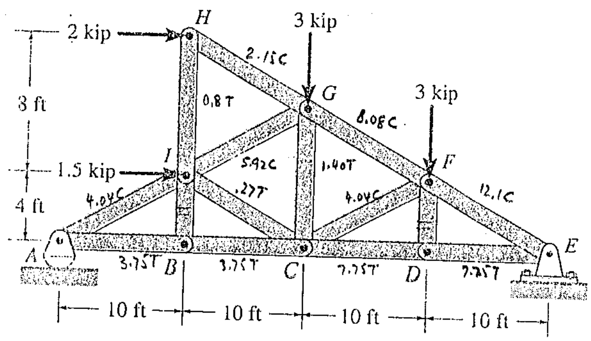
Prob. 6–12

6–13. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



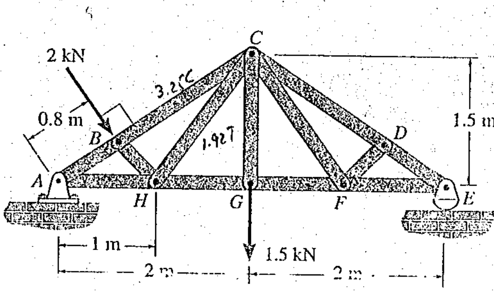
Prob. 6–13

6–18. Determine the force in each member of the truss and indicate whether the members are in tension or compression.

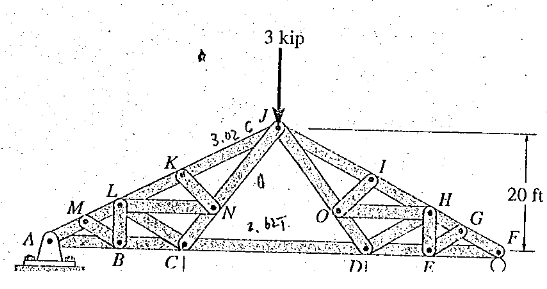


Prob. 6–18

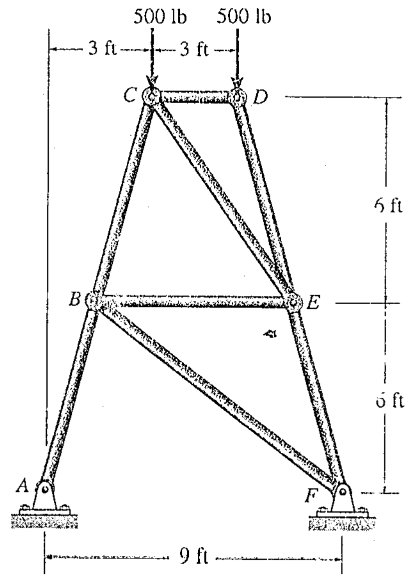
6–29. Determine the force developed in members BC and CH of the roof truss and indicate whether the members are in tension or compression.



6–37. Determine the force in members KJ, JN, and CD, and indicate whether the members are in tension or compression.

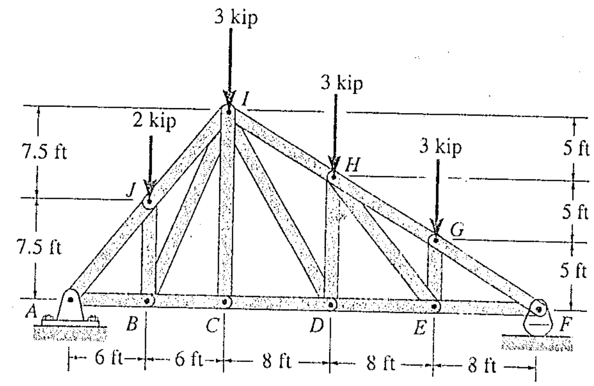


6-20. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



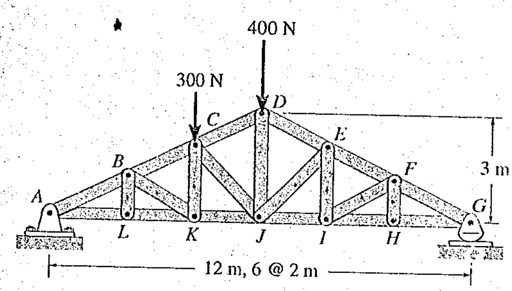
Prob. 6–20

6–19. Determine the force in each member of the roof truss and indicate whether the members are in tension or compression.



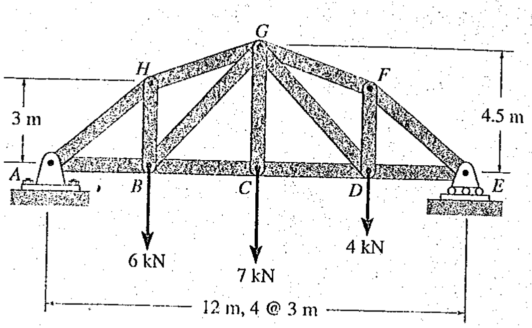
Prob. 6–19

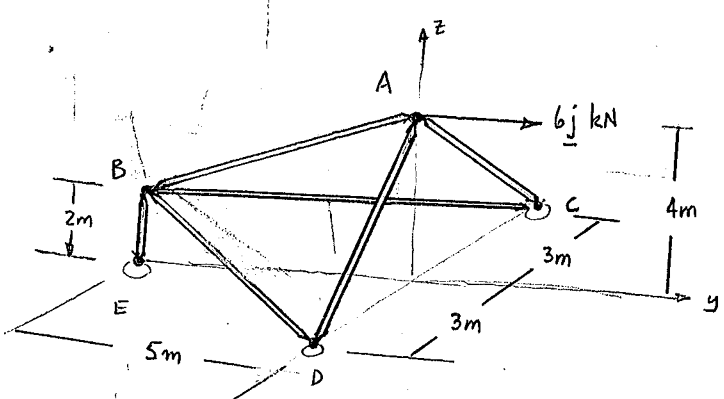
6–31. For the given loading, determine the force in members *CD*, *CJ*, and *KJ* of the *Howe roof truss*. Indicate whether the members are in tension or compression.



Prob. 6–31

***6–32.** Determine the force in members *BG*, *HG*, and *BC* of the truss and indicate whether the members are in tension or compression.





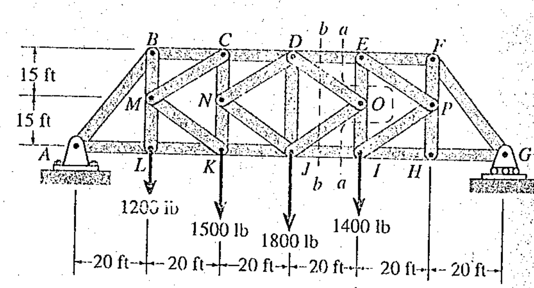
מה הכוחות במוטות המסגרת?

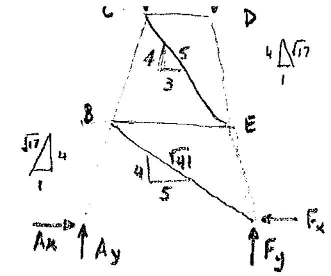
ב-C, D, E יש רק כדור ותושבת

ב-A יש סיכה (פרק)

6–39. Determine the force in members *DE*, *JI*, and *DO* of the *K truss*, and indicate whether the members are in tension or compression. *Hint:* Use sections *aa* and *bb*.

***6–40.** Determine the force in members *CD* and *KJ* of the *K truss*, and indicate whether the members are in tension or compression. *Hint:* Note section *aa* can be used to find the force in members *DE* and *JI*.





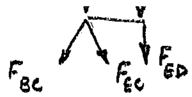
$A_x = F_x$

$A_y + F_y = 1000$

$\Sigma M_A = 500\cdot3 + 500\cdot6 - F_y\cdot9 = 0$

$F_y = 500\,lb$

AB = 515.4 (δ)	CE = 0		
BC = 515.4 (δ)	BF = 0		
CD = 125 (δ)	A_x = 125 →		
DE = 515.4 (δ)	A_y = 500 ↑		
EF = 515.4 (δ)	F_x = 125 ←		
BE = 0	F_y = 500 ↑		



$\Sigma M_c = 0 = 500\cdot3 + F_{ED}\cdot\frac{4\cdot3}{\sqrt{17}} = 0$

$F_{ED} = -500\cdot\frac{3\cdot\sqrt{17}}{4\cdot3} = -515.4\,lb$

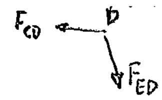
$\Sigma F_x = 0 = -F_{BC}\cdot\frac{1}{\sqrt{17}} + F_{EC}\cdot\frac{3}{5} + F_{ED}\cdot\frac{1}{\sqrt{17}}$

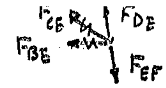
$\Sigma F_y = 0 \quad -F_{BC}\cdot\frac{4}{\sqrt{17}} - F_{EC}\cdot\frac{4}{5} - F_{ED}\cdot\frac{4}{\sqrt{17}} - 1000$

$F_{BC} = -515.4\,lb$

$F_{EC} = 0\,lb$

$F_{CD} = \frac{1}{\sqrt{17}}F_{ED} = -125\,lb$





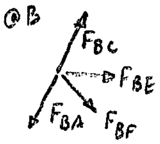
$\Sigma F_x = 0 \quad -F_{BE} - \cancel{F_{CE}}\cdot\frac{3}{5} - F_{DE}\cdot\frac{1}{\sqrt{17}} + F_{EF}\cdot\frac{1}{\sqrt{17}} = 0$

$\Sigma F_y = 0 \quad \cancel{F_{CE}}\cdot\frac{4}{5} + F_{DE}\cdot\frac{4}{\sqrt{17}} - F_{EF}\cdot\frac{4}{\sqrt{17}} = 0$

$F_{BE} = 0\,lb$

$F_{EF} = F_{DE} = -515.4\,lb$

@B



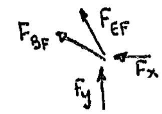
$\Sigma F_x = 0 = F_{BC}\cdot\frac{1}{\sqrt{17}} + \cancel{F_{BE}} + F_{BF}\cdot\frac{5}{\sqrt{41}} - F_{BA}\cdot\frac{1}{\sqrt{17}}$

$\Sigma F_y = 0 \quad F_{BC}\cdot\frac{4}{\sqrt{17}} - F_{BA}\cdot\frac{4}{\sqrt{17}} - F_{BF}\cdot\frac{4}{\sqrt{41}} = 0$

$F_{BA} = -515.4\,lb$

$F_{BF} = 0\,lb$

@F



$\Sigma F_x = 0 \quad -F_{EF}\cdot\frac{1}{\sqrt{17}} - \cancel{F_{BF}}\cdot\frac{5}{\sqrt{41}} - F_x = 0$

$\Sigma F_y = 0 = F_y + \cancel{F_{BF}}\cdot\frac{4}{\sqrt{41}} + F_{EF}\cdot\frac{4}{\sqrt{17}}$

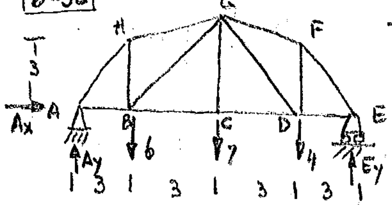
$F_x = 125\,lb$

$F_y = 500\,lb$

$A_y = 1000 - F_y = 500\,lb$

$A_x = F_x = 125\,lb$

6-32



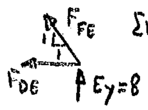
מפני שבכל צומת יש לפחות 3 נעלמים,
צריך למצוא את התגובות החיצוניות..

$+\circlearrowleft \Sigma M_A = 0 = -6\cdot3 - 7\cdot6 - 4\cdot9 + 12\cdot E_y \qquad E_y = 8\,kN$

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow A_y + E_y - 17 = 0 \qquad A_y = 9\,kN$

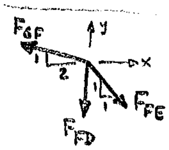
@E



$\Sigma F_y = F_{FE}\cdot\frac{1}{\sqrt{2}} + E_y = 0 \qquad F_{FE} = -11.32\,kN$

$\Sigma F_x = 0 = -F_{FE}\frac{1}{\sqrt{2}} + F_{DE} \Rightarrow F_{DE} = 8\,kN$

@F



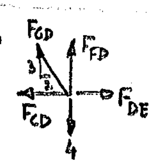
$\Sigma F_y = +F_{GF}\cdot\frac{1}{\sqrt{5}} - F_{FE}\cdot\frac{1}{\sqrt{2}} - F_{FD} = 0$

$\Sigma F_x = -F_{GF}\cdot\frac{2}{\sqrt{5}} + F_{FE}\cdot\frac{1}{\sqrt{2}} = 0$

$F_{GF} = -8.95\,kN = F_{FE}\cdot\frac{\sqrt{5}}{2\sqrt{2}}$

$F_{FD} = 4\,kN$

@D



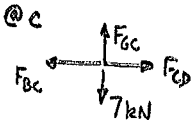
$\Sigma F_y = F_{FD} - 4 - F_{GD}\cdot\frac{3}{\sqrt{13}} = 0$

$\Sigma F_x = F_{DE} - F_{CD} - F_{GD}\cdot\frac{2}{\sqrt{13}} = 0$

$F_{GD} = 0$

$F_{CD} = F_{DE} = 8\,kN$

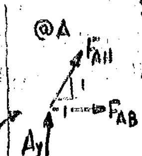
@C



$F_{CD} = F_{BC} = 8\,kN$

$F_{GC} = 7\,kN$

@A

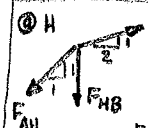


$\Sigma F_y = 0 \Rightarrow$

$F_{AH} = -A_y\cdot\sqrt{2} = -12.73\,kN$

$F_{AB} = -F_{AH}\cdot\frac{1}{\sqrt{2}} = 9\,kN$

@H



$F_{HG}\cdot\frac{2}{\sqrt{5}} - F_{AH}\cdot\frac{1}{\sqrt{2}} = 0$

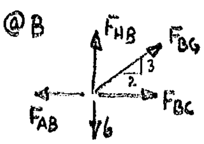
$F_{HG} = -10.06\,kN$

$F_{HG}\cdot\frac{1}{\sqrt{5}} - F_{HB} - F_{AH}\cdot\frac{1}{\sqrt{2}} = 0$

$F_{HB} = 4.5\,kN$

AH = 12.73 (δ)	GF = 8.95 (δ)	$A_x = 0$
AB = 9 (N)	GD = 0	E_y = 8kN ↑
HB = 4.5 (N)	CD = 8 (N)	A_y = 9kN ↑
HG = 10.06 (δ)	FD = 4 (N)	
BG = 1.81 (N)	FE = 11.32 (δ)	
BC = 8 (N)	DE = 8 (N)	

@B



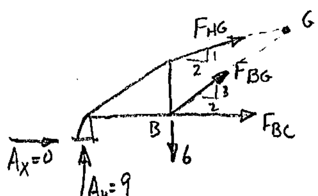
$\Sigma F_y = F_{HB} - 6 + F_{BG}\cdot\frac{3}{\sqrt{13}} = 0$

$\Sigma F_x = F_{BC} + F_{BG}\cdot\frac{2}{\sqrt{13}} - F_{AB} = 0$

$F_{BG} = +1.81\,kN$

כמו שמצאנו ב-C $\quad F_{BC} = 8\,kN$

6-32 לפי שיטת החתכים



עוד פעם צריך למצוא את התגובות ב-A ו-E כמו שעשיתי למעלה. אחרי זה

$+\circlearrowleft \Sigma M_G = 0 = -A_y\cdot6 + 6\cdot3 + F_{BC}\cdot4.5 = 0$

$F_{BC} = 8\,kN$

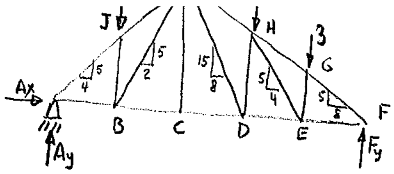
$+\circlearrowleft \Sigma M_B = 0 = -F_{HG}\cdot\frac{2}{\sqrt{5}}\cdot3 - A_y\cdot3 = 0$

$F_{HG} = -10.06\,kN$

$\Sigma F_y = 0 = 9 - 6 + F_{HG}\cdot\frac{1}{\sqrt{5}} + F_{BG}\cdot\frac{3}{\sqrt{13}} = 0$

$F_{BG} = 1.81\,kN$

כמו שמצאנו למעלה



$A_y + F_y = 11$

$A_x = 0$

$\Sigma M_A = 0 = 2\cdot 6 + 3\cdot 12 + 3\cdot 20 + 3\cdot 28 - F_y \cdot 36$

$\underline{F_y = 5.33\ kN} \uparrow$

$A_y = 5.67\ kN \uparrow$

@ A

$\Sigma F_y = 0 \Rightarrow A_y = \frac{5}{\sqrt{41}} F_{AJ}$ $\quad \underline{F_{AJ} = 7.26\ kN\ (\delta)}$

$\Sigma F_x = 0 \Rightarrow F_{AB} = \frac{4}{\sqrt{41}} \cdot F_{AJ}$ $\quad \underline{F_{AB} = 4.54\ kN\ (N)}$

$-F_{EF} + F_{GF} \cdot \frac{8}{\sqrt{89}} = 0 = \Sigma F_x$

$F_{GF} = 10.06\ kN\ (\delta)$

$F_{EF} = 8.53\ kN\ (N)$

@ G

$\Sigma F_y = 0 \Rightarrow F_{GE} = 3\ kN\ (\delta)$

$\Sigma F_x = 0 \Rightarrow F_{GF} = F_{HG} = 10.06\ kN\ (\delta)$

@ E

$\Sigma F_y = F_{HE} \cdot \frac{5}{\sqrt{41}} - F_{GE} = 0$

$\Sigma F_x = F_{EF} - F_{HE} \cdot \frac{4}{\sqrt{41}} - F_{DE} = 0$

$F_{HE} = 3.85\ kN\ (N)$

$F_{DE} = 6.13\ kN\ (N)$

@ J

$F_{JI} = F_{AJ} = 7.26\ kN\ (\delta)$

$\Sigma F_y = 0 \Rightarrow F_{JB} = 2\ kN\ (\delta)$

$\Sigma F_y = -3 + F_{HD} - F_{HI} \cdot \frac{5}{\sqrt{89}} + F_{GH} \cdot \frac{5}{\sqrt{89}} - F_{HE} \cdot \frac{5}{\sqrt{41}} = 0$

$F_{HI} = 7.23\ kN\ (\delta)$

$F_{HD} = 4.5\ kN\ (\delta)$

@ D

$\Sigma F_y = 0 \Rightarrow F_{ID} \cdot \frac{15}{17} = F_{HD}$ $\quad F_{ID} = 5.10\ kN\ (N)$

$\Sigma F_x = 0 \Rightarrow F_{CD} = F_{DE} - F_{ID} \cdot \frac{8}{17} = 3.73\ kN\ (N)$

@ C

$\Sigma F_y = F_{IC} = 0$

$F_{BC} = F_{CD} = 3.73\ kN\ (N)$

@ B

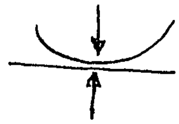
$F_{IB} \cdot \frac{5}{\sqrt{29}} = F_{JB}$ $\quad F_{IB} = 2.16\ kN\ (N)$

$F_{BC} + \frac{2}{\sqrt{29}} F_{IB} = F_{AB} = 4.54\ kN$ ✓

FRICTION

- UP TO NOW ALL SURFACES IN CONTACT ~~WHERE~~ WERE SMOOTH
- INTERACTION OF ~~[illegible]~~ SURFACES WAS CONTACT FORCE $\perp$ to SURFACE

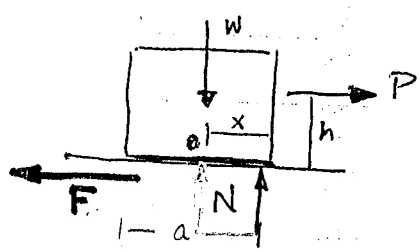
הַפְּעוּלָה הַהֲדָדִית.



- REALITY : SURFACES ARE ROUGH (מחוספס) & BECAUSE OF IT A FORCE **TANGENTIAL** TO SURFACE EXISTS
- DUE TO RESISTANCE OF ONE SURFACE TO MOVEMENT OVER THE OTHER

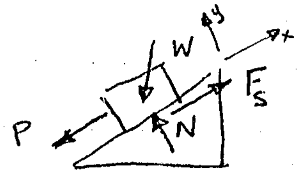
- Friction – a force resisting the movement of a body
- 2 TYPES (שני סוגים) Fluid Friction (חיכוך זרם (נוזל))

  Dry or Coloumb Friction (חיכוך יבש) after C.A. Coloumb
- FRICTION (חיכוך) EXISTS BETWEEN CONTACTING SURFACES OF BODIES THAT ARE NOT LUBRICATED (מְשׁוּמָּן)
- FRICTION CAN CAUSE WEAR (בְּלַאי) ON SURFACES SINCE LAYERS OF MOLECULES CAN BE STRIPPED AWAY (מֵסִיר את שכבות מולקולות)
- FRICTION FORCE ACTS IN DIRECTION OPPOSING THE MOTION OF A BODY (תנועת הגוף)



Sense of motion → motion

Normal Force N is force surface applies to body & acts $\perp$ to surface



& NORMAL FORCE

- EVEN THOUGH AT EACH PT OF CONTACT A FRICTIONAL FORCE ^ ACTS

ניסתכל בהשפעת כללית

- WE LOOK AT EFFECT IN THE GENERAL SENSE (OVERALL EFFECT) F_s, N
- N acts at a distance off the LINE OF ACTION of W to counteract TIPPING EFFECT OF FORCE P (iE A MOMENT ABOUT O).
- DISTANCE $x = Ph/N = Ph/W$

הגוף ~~יבלה ונופל~~ מתהפך

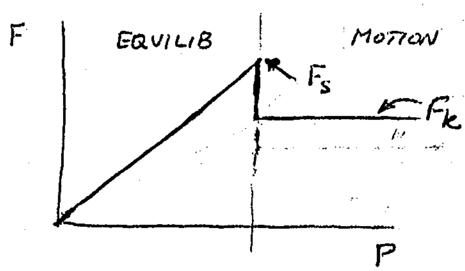
- TIPPING OCCURS WHEN $x = a/2$ OR $P_t \geq \dfrac{Wa}{2h}$ FORCE NEEDED TO CAUSE

אם יש משטח עיקומי כמו בכדור המשטח יתגלגל במקום להטות

- IF A CURVED SURFACE LIKE A BALL ROLLING INSTEAD OF TIPPING CAN OCCUR.

עוד סוג תנועה — תנועה שעומדת להתרחש

- ANOTHER TYPE OF MOTION OF THE BLOCK IS IMPENDING MOTION
  - IF H IS SMALL OR IF SURFACE IS NOT VERY ROUGH
  - CAN INCREASE P UNTIL WE REACH F_s (~~MAXIMUM OR~~ LIMITING STATIC FRICTIONAL FORCE) — כוח חיכוך סטטי גבולי F_s
  - FURTHER INCREASE IN P CAUSES MOTION



דרך ניסיונות

- EXPERIMENTALLY

$$F_s = \mu_s N$$

מקדם לחיכוך סטטי

- μ_s – COEFFICIENT OF STATIC FRICTION – DIMENSIONLESS
- PG 285 SOME VALUES OF μ_s – depends on contact surfaces
- NORMALLY WILL BE DETERMINED BY EXPERIMENT

- MOTION: IF $P > F_s$ MOTION WILL OCCUR & F DUE TO FRICTION DROPS TO A VALUE $< F_s$. THIS VALUE IS F_k

- $P > F_s > F_k$ BLOCK MOVES W/ INCREASING SPEED

הגוף עובר מעל הפסגות של המשטחים המחוספסים

- SINCE BLOCK MOVES IT RIDES OVER PEAKS OF ROUGH SURFACES
- THUS CONTACT AREA IS SMALLER HENCE F_k IS SMALLER THAN F_s

• FROM EXPERIMENTS $\underline{F_k = \mu_k N}$

• NORMALLY $\mu_k \cong 0.75\,\mu_s$

משפטים לחיכוך יבש RULES OF DRY FRICTION

1. F is tangent to surfaces in contact

לא תלוי בגודל שטח המגע
2. $|F_s|$ is independent of area of contact

3. $F_s > F_k$; if one body moves slowly $F_s \cong F_k$

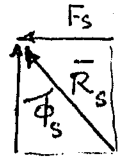
תנועה עומדת להתרחש
4. When slip is about to occur $F_s = \mu_s N$ impending motion

5. When slip is occuring $F_k = \mu_k N$

6. If $F < F_s$ cannot use $F = \mu_s N$ must solve for F from equ

VERGE OF SLIDING $P = F_s$

• F_s & N combine to form a resultant R_s



$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}(\mu_s)$$

זווית של חיכוך סטטי זווית השכיבה
• ϕ_s - angle of static friction or angle of repose

WHEN IN MOTION $F_k < F_s$

F_k & N form a resultant R_k

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}(\mu_k)$$

$\phi_s > \phi_k$ like $F_s > F_k$

SESSION #19

NOT IMPENDING MOTION

• IF $F < \mu_s N$ USE EQUIL EQS TO GET F

• IF YOU DONT KNOW SENSE OF MOTION ASSUME A DIRECTION, F IS OPPOSITE TO IT

- A RIGID BODY IN EQUIL MUST SATISFY

$$\sum \vec{F} = \vec{0}$$

$$\sum \vec{M}_0 = \vec{0}$$

- IF FRICTION IS INCLUDED, MUST SATISFY

$$\sum \vec{F} + \vec{F}_s \text{(due to friction)} = \vec{0}$$

$$\sum \vec{M}_0 = \vec{0}$$

and $F_s = \mu_s N$ אם התנועה עומדת להתרחש
IF IMPENDING MOTION

TYPES OF PROBLEMS

- MUST DETERMINE FRICTIONAL FORCE (F_s) TO CAUSE IMPENDING MOTION OR SLIPPING (F_k)

- GIVEN ALL THE LOADS CHECK TO SEE IF FRICTIONAL FORCE DEVELOPED IS LESS THAN OR EQUAL TO $F_s = \mu_s N$
  - MUST CHECK TO SEE IF TIPPING OR SLIPPING HAPPENS (FIG 8-4)

לחשב את כוח

CALCULATE P_{SLIP} & P_{TIP} & pick smallest value - this is what will happen FIRST

IF $P_{SLIP} = P_{TIP}$ both can happen simultaneously

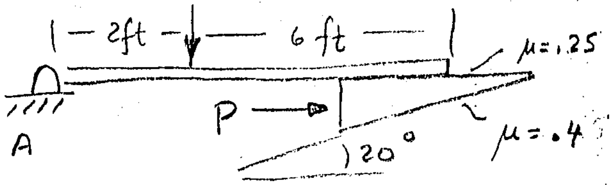
PROCEDURE

- Draw FBD   3 eqs of eq + friction equation (if necessary)

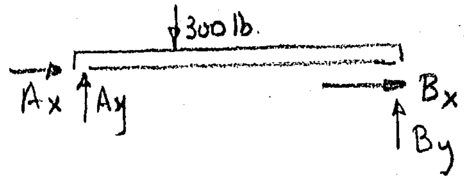
LOOK AT DIFFERENT EXAMPLES 8-1 ON

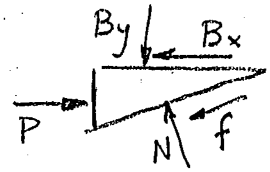
ANNOUNCE EXAM #2

what force is required to move the horiz beam upward



→ motion





$f = \mu N$

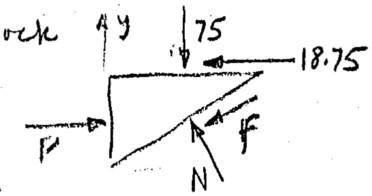
$B_x = \mu B_y$

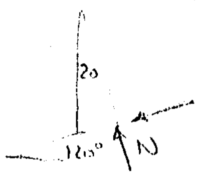
$$\Sigma F_x = A_x + B_x = 0$$

$$\Sigma F_y = A_y + B_y - 300 \text{ lb} = 0$$

$$+\circlearrowleft \Sigma M_A = -2(300) + B_y(8) = 0 \quad B_y = 75 \text{ lb} \quad \underline{B_x = \mu B_y = 18.75 \text{ lb}}$$

for block





① $\Sigma F_x = P - 18.75 - f\cos 20° - N\sin 20°$

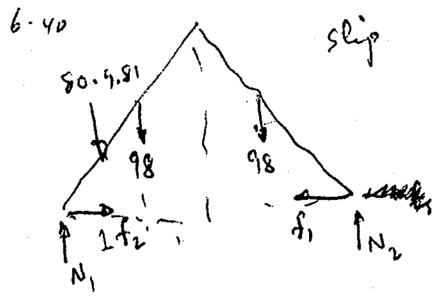
② $\Sigma F_y = -75 - f\sin 20° + N\cos 20° = 0$

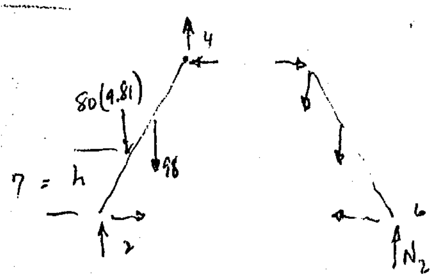
$$N = \frac{B_y}{\cos 20° - \mu \sin 20°} = 87.8 \text{ lb}$$

$$P = 18.75 + N\left[\mu\cos 20° + \sin 20°\right] = \underline{\underline{69.41 \text{ lb}}}$$

6-40

slip

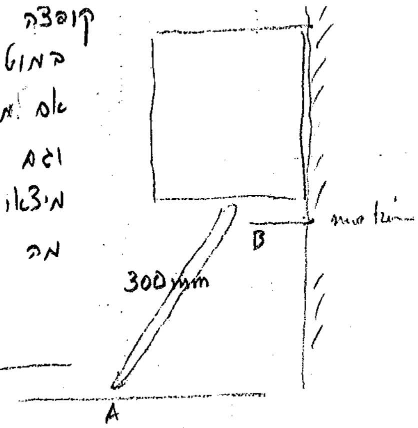


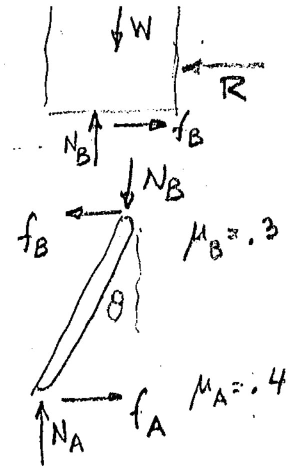


7 unk.

7 eq.

קופסה בעלת משקל W נתמך 6-5
במוט בעל אורך של 0.3m. וגם בקיר חלק
אם משטח המגע בין הקופסה למוט מחוספס ו- $\mu_s=0.3$
וגם משטח המגע בין המוט למשטח A מחוספס ו- $\mu_s=0.4$
מיצאו את המקום שבה התנועה תתחיל?
מה המרחק Δx בין A ל-B כאשר תנועה עומדת לה





FROM WEIGHT $N_B = W$

FOR BAR $\Sigma F_x = 0 \Rightarrow f_A = f_B$ (1)

$\Sigma F_y = 0 \Rightarrow N_A = N_B$ (2) $\Rightarrow$ slipping will occur at either A or B

if A first $\Rightarrow$ $f_A = \mu_A N_A = .4N_A \overset{(2)}{=} .4N_B$ $\quad f_{B_{max}} = .3N_B$

by (1) $f_B = f_A = .4N_B > f_{B_{max}} \Rightarrow$ slip at B first

if B first $\Rightarrow$ $f_B = \mu_B N_B = .3N_B \overset{(2)}{=} .3N_A$ $\quad \cancel{f_{A_{max}} = .4N_A}$

by (1) $f_A = f_B = .3N_A$ $\quad f_{A_{max}} = .4N_A$.

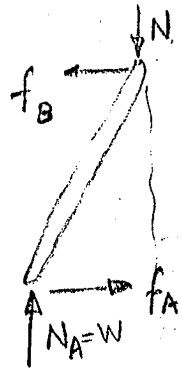
$\therefore f_A < .4N_A$ ✓

$\Sigma M_A = 0 = -N_B x + f_B y = 0 \quad -x + \mu_B y = 0 \quad y = \sqrt{(300)^2 - x^2}$

for impending motion $x = 86.2$ mm.

if A assumed first $x = 111.1$ mm

for $x = 75\,mm$.



① $\Sigma F_x = f_A - f_B = 0 \Rightarrow f_A = f_B$

$\Sigma F_y = N_A - N_B = 0 \Rightarrow N_A = N_B$

but for $x = 75\,mm < 86.7\,mm$.
then no slip at either
surface $\therefore f \neq \mu N$ @ A & B

$$\Sigma M_B = -N_A(75) + f_A(y) = 0 \quad \text{or} \quad f_A = \frac{N_A(75)}{\sqrt{(300)^2 - 75^2}} = .258 N_A$$

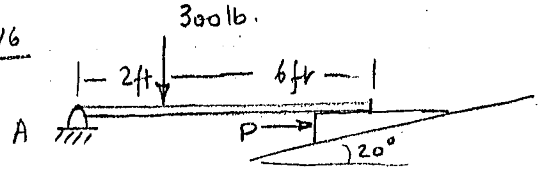
but from ① $\quad f_A = f_B = .258\, N_A$

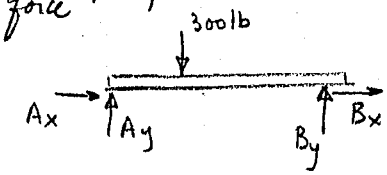
$$N_B \cdot \Delta x = f \Delta y \qquad f = N_B \cdot \frac{\Delta x}{\Delta y} = \frac{75}{\sqrt{300^2 - 75^2}} N_B = .258\, N_B = .258\, W$$

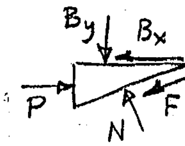
$\mu = .25$

what force is required to move the horiz beam upward

8-16







$\mu = .25$ for both.

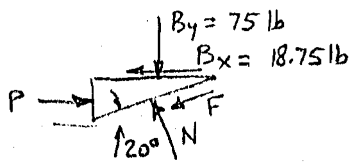
SINCE P MOVES BLOCK FORWARD $B_x = \mu B_y$; $\mu N = F$

FOR BEAM $\Sigma F_x = A_x + B_x = 0$

$\Sigma F_y = A_y + B_y - 300\,lb = 0$

$+\circlearrowleft \Sigma M_A = -2(300) + B_y(8) = 0$ $\quad \underline{B_y = 75\,lb}$ $\quad \underline{B_x = \mu B_y = .25(.75) = 18.75\,lb}$

FOR BLOCK



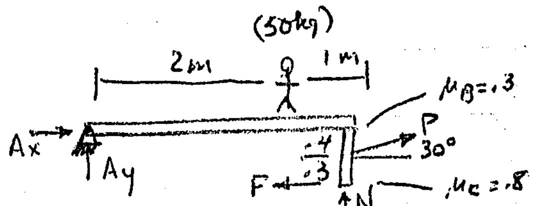
① $\Sigma F_x = P - 18.75 - \overbrace{\mu N}^{F} \cos 20° - N \sin 20° = 0$

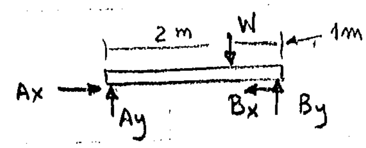
② $\Sigma F_y = -75\,lb + N\cos 20° - \mu N \sin 20° = 0$

From ② $\quad N = \dfrac{75}{\cos 20° - \mu \sin 20°} = 87.8\,lb$ for $\mu = .4$ ③

From ③ & ① $\quad P = 18.75 + N\left[\mu \cos 20° + \sin 20°\right] = 69.41\,lb$

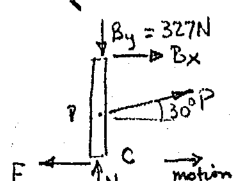
8-31





$+\circlearrowleft \Sigma M_A = 0 \quad - W(2) + B_y(3) = 0 \quad B_y = \frac{2W}{3} = \frac{2}{3}50(9.81) = 327\,N$

POST:



① $\Sigma F_x = B_x + P\cos 30° - F = 0$ $\quad$ assume: SLIP AT C $\quad F = \mu_c N$

② $\Sigma F_y = -B_y + N + P\sin 30° = 0$

③ $+\circlearrowleft \Sigma M_p = -F(.3) + B_x(.4) = 0$ $\quad \underline{B_x = -.75F}$ ④

①+④ $\Rightarrow -1.75F + P\cos 30° = 0$ $\quad$ or $\quad N = \frac{F}{\mu} = \frac{1}{\mu}\frac{P\cos 30°}{1.75}$

$\Sigma F_y = 0 \Rightarrow B_y = \frac{1}{\mu}\frac{P\cos 30°}{1.75} + P\sin 30° = P\left[\frac{1}{.8}\frac{\cos 30°}{1.75} + .5\right] = 1.1186P$

$\boxed{P = B_y/1.1186 = 292.33\,N}$ $\quad \boxed{N = 180.83\,N}$ $\quad \boxed{F = .8N = 144.67\,N}$

$\boxed{B_x = -108.5\,N}$ $\quad$ Now check if $B_x \le \mu_B B_y$

Check @B $\quad F_{max} = B_y(.3) = 98.1\,N \longrightarrow$ since $B_x > F_{max}$ slipping must occur at B first

since slip at B $\quad B_x = \mu_B B_y = 98.1\,N$; $F = -130.8\,N$ FROM ④ ; FROM ① $P = +264.3\,N$

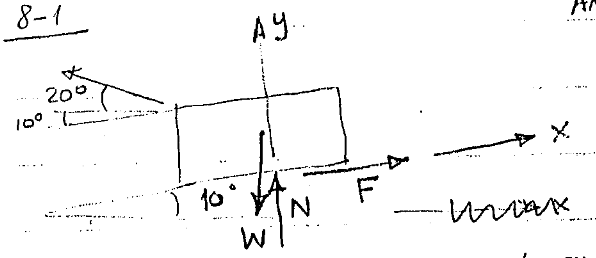
From ② $\quad N = B_y - P\sin 30° = 327 - 264.3(.5) = 194.85\,N$

Check to see if $\quad F < \mu_c N = .8(194.85) = 155.88\,N$

HANDOUT #3 - Statics

TELL STUDENTS TO CHANGE AXES

8-1



$\Sigma F_x = F - W \sin 10° - P \cos 30° = 0$

$\Sigma F_y = N - P \sin 30° - W \cos 10° = 0$

2 eqs ; 2 unknowns (N & P)

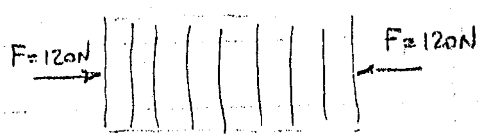
IMPENDING MOTION $\therefore F = \mu_s N = .5N$

$W = 350\,kg \cdot 9.81 \frac{m}{sec^2} = 3433.5N$

$$P = W \left[\frac{\mu_s \cos 10° - \sin 10°}{\mu_s \sin 30° + \cos 30°} \right] = 980.82N$$

רוצים להרים מספר ספרים ביחד

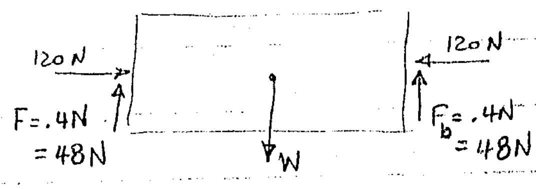
8-10



2 CASES ① slip at hand - book interface $\mu_s = .6$

② slip between book - book $\mu_s = .4$

Suppose ②



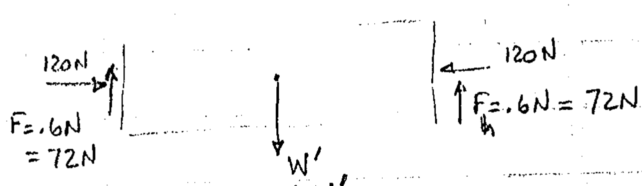
then $\Sigma F_y = 96 - W = 0 \quad W = 96\ N$

HE CAN ONLY CARRY 96N

IF EACH BOOK IS .95 kg MASS . $W = 9.32N$ $\therefore n = \frac{96N}{9.32N} = 10.3$

or 10 BOOKS

Suppose ①.

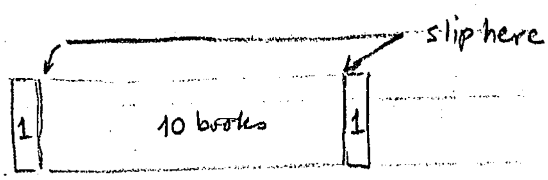


then $\Sigma F_y = 144 - W' = 0 \quad W' = 144N$

$W < W'$

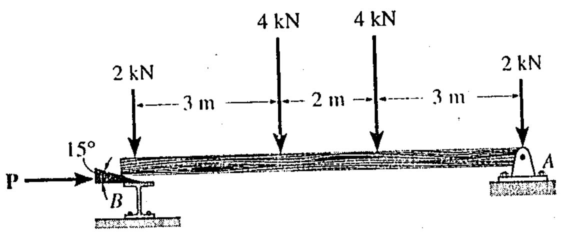
Since: $\cancel{F_b < F_h}$ book - book occurs first

thus

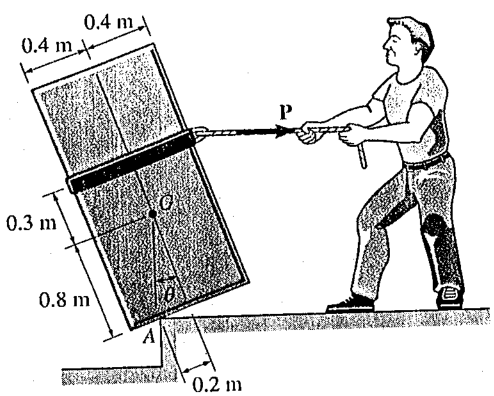


10 + 2 = 12 books

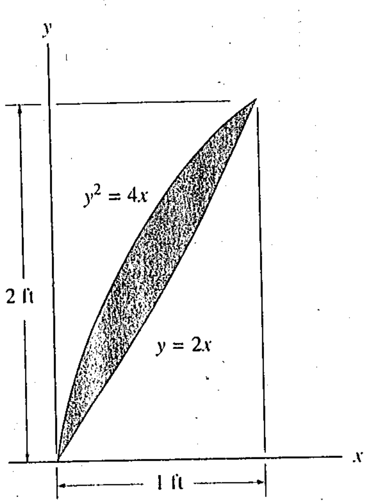
floor loading shown, determine the horizontal force **P** that must be applied to move the wedge forward. The coefficient of static friction between the wedge and the two surfaces of contact is $\mu_s =$ 0.25. Neglect the weight of the wedge.



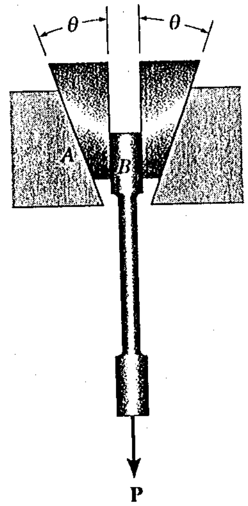
8–33. The man attempts to move the block, which has a weight of 800 lb and a center of gravity at G, over the edge of the step. If the coefficient of static friction at A is $\mu_A = 0.3$, determine the angle θ at which the block begins to slip. Also, with what horizontal force **P** must the man pull for this to occur?



9–21. Determine the location $(\bar{x}, \bar{y})$ of the centroid of the shaded area.

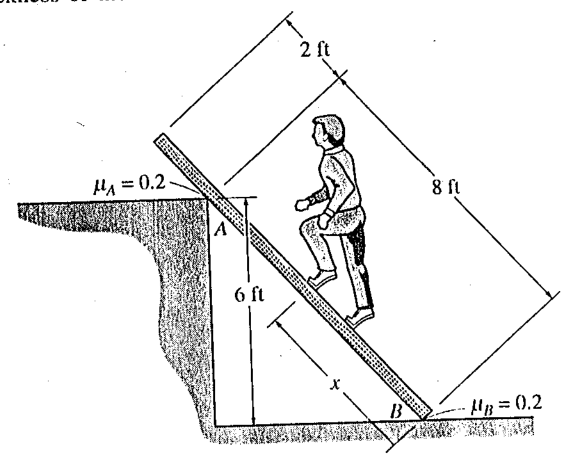


sion-testing machine. Determine the design angle θ of the wedges so that the specimen will not slip regardless of the applied load. The coefficients of static friction are $\mu_A = 0.1$ at A and $\mu_B = 0.6$ at B. Neglect the weight of the blocks.



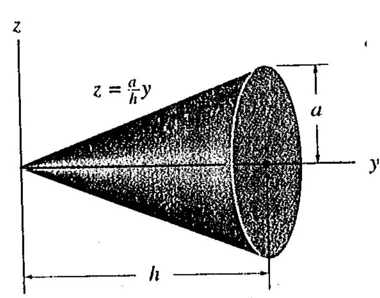
Prob. 8–62

***8–32.** A 50-lb beam rests on two surfaces at points A and B. Determine the maximum distance x to which a boy can walk up the beam before the beam begins to slip. The boy weighs 120 lb and assume he walks up the beam with a constant velocity. Neglect the thickness of the beam.

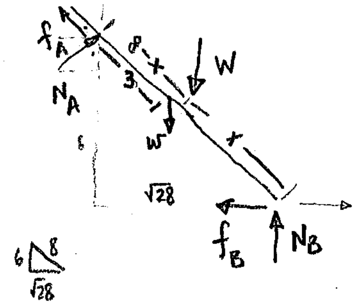


***9–24.** Determine the distance $\bar{y}$ to the centroid of the cone.

9–25. Determine the distance $\bar{y}$ to the center of mass of the cone. The density of the material varies linearly from zero at the origin O to ρ_0 at $x = h$.



Probs. 9–24/9–25



$\cos\theta = \frac{\sqrt{28}}{8} \quad \sin\theta = \frac{6}{8}$

$$\Sigma F_x = -f_A \cdot \frac{\sqrt{28}}{8} + N_A \cdot \frac{6}{8} - f_B = 0 \implies \mu N_B = \left(-\mu\frac{\sqrt{28}}{8} + \frac{6}{8}\right)N_A \implies .6177 N_A - .2 N_B = 0$$

$$\Sigma F_y = f_A \cdot \frac{6}{8} + N_A \frac{\sqrt{28}}{8} - W - w + N_B = 0 \implies (W+w) = N_B + \left(\mu\frac{6}{8} + \frac{\sqrt{28}}{8}\right)N_A = .81144 N_A + N_B$$

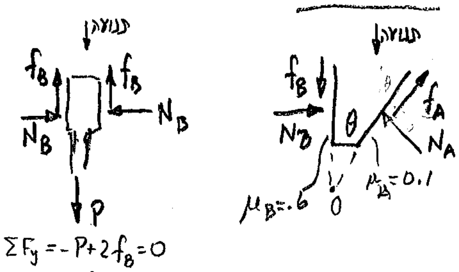
$$N_A = \underline{43.59N} \quad N_B = \underline{134.63N}$$

$$+\circlearrowleft \Sigma M_A = 0 = -w\left(3\cdot\frac{\sqrt{28}}{8}\right) - W(8-x)\frac{\sqrt{28}}{8} - f_B \cdot 6 + N_B\sqrt{28} = 0$$

$$= -50\left(3\frac{\sqrt{28}}{8}\right) - W(8-x)\frac{\sqrt{28}}{8} + \left(-\mu \cdot 6 + \sqrt{28}\right)N_B = 0$$

$$8 - x = 5.69 \text{ m} \qquad \underline{x = 2.31 \text{ m}}$$

8-62



$\Sigma F_y = -P + 2f_B = 0$

$f_B = P/2$

אם המשקולת חולקת כלפי מטה, ותפוס שהטריז חולק כלפי מעלה. $\Sigma F_y = 0$ ו- לפי הסימטריות $\frac{P}{2} = f_B$ של המשקולת. עבור הטריז

$$\Sigma F_y = -f_B + f_A\cos\theta + N_A\sin\theta = 0$$
$$\Sigma F_x = N_B - N_A\cos\theta + f_A\sin\theta = 0$$

כדי שתנועה עומדת להתרחש $f_B = \mu_B N_B$ ו- $f_A = \mu_A N_A$. אם ממירים את אלה במשוואות ומוסיפים את המשוואות נקבל

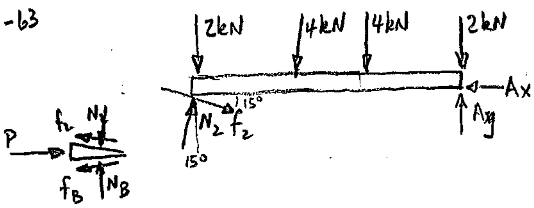
$$-(\mu_A + \mu_B)\cos\theta \cdot N_A + (1 + \mu_B\mu_A)\sin\theta \cdot N_A = 0$$

$$\tan\theta = \frac{-\mu_A + \mu_B}{1 + \mu_B\mu_A} = .4717$$

$$\theta = 25.25^\circ$$

אם נקח $\underline{\theta = 25.3^\circ}$ לא יהיה חליקה

8-63



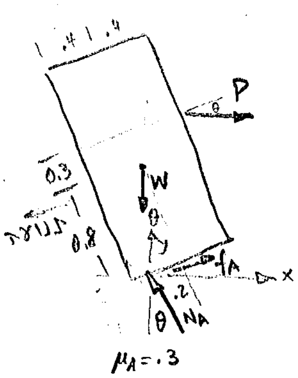
על הטריז

$$\Sigma F_x = P - f_B - f_2\cos 15^\circ - N_2\sin 15^\circ = 0$$
$$\Sigma F_y = N_B + f_2\sin 15^\circ - N_2\cos 15^\circ = 0$$
$$f_B = \mu_s N_B \quad f_2 = \mu_s N_2$$

על הקורה

$$+\circlearrowleft \Sigma M_B = 0 \implies -4\cdot 3 - 4\cdot 5 - 2\cdot 8 + A_y \cdot 8 = 0$$
$$\implies A_y = 6\text{kN}$$
$$\Sigma F_y = 0 \implies A_y - 2 - 4 - 4 - 2 - f_2\sin 15^\circ + N_2\cos 15^\circ = 0$$
$$6 - 12 + (-\mu_s\sin 15^\circ + \cos 15^\circ)N_2 = 0$$
$$N_2 = \frac{6}{-\mu_s\sin 15^\circ + \cos 15^\circ} = \frac{6}{1.0306} = 6.658 \text{ kN}$$
$$f_2 = \mu_s N_2 = 1.665 \text{ kN}$$

מציבים במשוואות של הטריז כדי לקבל $\underline{6\text{ kN} = N_B}$ ו $\underline{1.5\text{ kN} = f_B}$, ו- $\underline{4.831\text{ kN} = P}$



$$\Sigma F_x = +f_A\cos\theta - N_A\sin\theta + P = 0 \qquad P = (-\mu_A\cos\theta + \sin\theta)N_A$$
$$\Sigma F_y = -W + f_A\sin\theta + N_A\cos\theta = 0 \qquad W = (+\mu_A\sin\theta + \cos\theta)N_A$$

$$\frac{P}{W} = \frac{-\mu_A\cos\theta + \sin\theta}{\mu_A\sin\theta + \cos\theta} = \frac{-.2\cos\theta + .8\sin\theta}{1.1\cos\theta + .6\sin\theta}$$

LHS = RHS

$$+\circlearrowleft \Sigma M_A = 0 \implies -W\cos\theta(.2) + W\sin\theta(.8) - P\cos\theta(1.1) - P\sin\theta(.6) = 0$$
$$W[-.2\cos\theta + .8\sin\theta] - P[1.1\cos\theta + .6\sin\theta] = 0$$

θ	LHS		RHS
0	-.3	<	-.1819
10	-.1175	<	-.0489
20	.0576	<	.0692
30	.2364	>	.1811

שינוי אי שיוון

θ	LHS		RHS
20	.0576	<	.0692
21	.0752	<	.0805
22	.0928	>	.0918

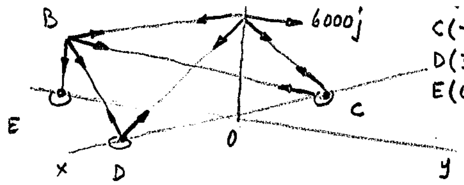
שינוי אי שיוון

θ	LHS		RHS
21.5	.08399	<	.08615
21.6			
21.7	.0875	<	.0884
21.8	.08932	<	.08954
21.9	.0910	>	.09066

שינוי אי שיוון

לכן, $\underline{\theta = 21.9^\circ}$ ו- $N_A = \frac{W}{\mu_A\sin\theta + \cos\theta} = \frac{800}{1.0397} = \underline{769.4N}$

$\underline{P} = (-\mu_A\cos\theta + \sin\theta)N_A = \underline{72.8N}$



C(-3,0,0)
D(3,0,0)
E(0,-5,0)

@A $\begin{cases} \underline{F}_{DA} = F_{DA}\,(3\underline{i}+0\underline{j}-4\underline{k})/5 \\ \underline{F}_{CA} = F_{CA}\,(-3\underline{i}+0\underline{j}-4\underline{k})/5 \\ \underline{F} = 6000\underline{j} \end{cases}$ $\Rightarrow$ $\Sigma F_y = \frac{-5}{\sqrt{29}}F_{BA} + 6000 = 0$ $\quad \underline{F_{BA} = 6462.2\,N}$

$\Sigma F_z = \frac{-2}{\sqrt{29}}F_{BA} - \frac{4}{5}F_{DA} - \frac{4}{5}F_{CA} = 0$ $\quad \underline{F_{CA}} = \underline{F_{DA}} = \underline{-1500\,N}$

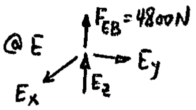
@B $\begin{cases} \underline{F}_{AB} = -F_{BA} = -F_{BA}(0\underline{i}-5\underline{j}-2\underline{k})/\sqrt{29} \\ \underline{F}_{EB} = F_{EB}(0\underline{i}+0\underline{j}-2\underline{k})/2 = -F_{EB}\underline{k} \\ \underline{F}_{CB} = F_{CB}(-3\underline{i}+5\underline{j}-2\underline{k})/\sqrt{38} \\ \underline{F}_{DB} = F_{DB}(3\underline{i}+5\underline{j}-2\underline{k})/\sqrt{38} \end{cases}$

$\Sigma F_x = -\frac{3}{\sqrt{38}}F_{CB} + \frac{3}{\sqrt{38}}F_{DB} = 0$ $\quad F_{CB} = F_{DB}$

$\Sigma F_y = \frac{5}{\sqrt{29}}F_{BA} + \frac{5}{\sqrt{38}}F_{CB} + \frac{5}{\sqrt{38}}F_{DB} = 0$ $\quad \underline{F_{DB} = F_{CB} = -3698.6}$

$\Sigma F_z = \frac{2}{\sqrt{29}}F_{BA} - F_{EB} - \frac{2}{\sqrt{38}}F_{CB} - \frac{2}{\sqrt{38}}F_{DB} = 0$ $\quad \underline{F_{EB} = 4800\,N}$

@E

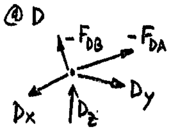


$\Sigma F_x = \underline{0 = E_x}$

$\Sigma F_y = \underline{0 = E_y}$

$\Sigma F_z = 0 = F_{EB} + E_z = 0$ $\quad \underline{E_z = -4800N}$

@D

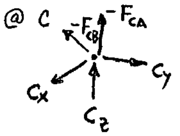


$\Sigma F_x = 0 \quad D_x - \frac{3}{\sqrt{38}}F_{DB} - \frac{3}{5}F_{DA} = 0$ $\quad D_x = -2700N$

$\Sigma F_y = 0 \quad D_y - 0F_{DA} - \frac{5}{\sqrt{38}}F_{DB} = 0$ $\quad D_y = -3000N$

$\Sigma F_z = 0 \quad D_z + \frac{4}{5}F_{DA} + \frac{2}{\sqrt{38}}F_{DB} = 0$ $\quad D_z = 2400N$

@C



$\Sigma F_x = 0 \quad C_x + \frac{3}{\sqrt{38}}F_{CB} + \frac{3}{5}F_{CA} = 0$ $\quad C_x = 2700N$

$\Sigma F_y = 0 \quad C_y - \frac{5}{\sqrt{38}}F_{CB} - 0F_{CA} = 0$ $\quad C_y = -3000N$

$\Sigma F_z = 0 \quad C_z + \frac{2}{\sqrt{38}}F_{CB} + \frac{4}{5}F_{CA} = 0$ $\quad C_z = 2400N$

כל הכוחות החיצוניים שווים אפס:

$$(E_x\underline{i}+E_y\underline{j}+E_z\underline{k}) + (D_x\underline{i}+D_y\underline{j}+D_z\underline{k}) + (C_x\underline{i}+C_y\underline{j}+C_z\underline{k}) + 6000\underline{j} = \underline{0}$$

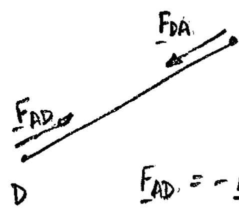
$$0\underline{i}+0\underline{j}-4800\underline{k} + (-2700\underline{i}-3000\underline{j}+2400\underline{k}) + (2700\underline{i}-3000\underline{j}+2400\underline{k}) + 6000\underline{j} = (0-2700+2700)\underline{i} + (0-3000-3000+6000)\underline{j} + (-4800+2400+2400)\underline{k} = \underline{0}$$

כל המומנטים החיצוניים שווים לאפס.

$$\Sigma \underline{M}_0 = \underline{r}_{E0}\times(E_x\underline{i}+E_y\underline{j}+E_z\underline{k}) + \underline{r}_{D/0}\times(D_x\underline{i}+D_y\underline{j}+D_z\underline{k}) + \underline{r}_{C/0}\times(C_x\underline{i}+C_y\underline{j}+C_z\underline{k}) + \underline{r}_{A/0}\times 6000\underline{j} =$$

$$-5\underline{j}\times(-4800\underline{k}) + 3\underline{i}\times(-2700\underline{i}-3000\underline{j}+2400\underline{k}) - 3\underline{i}\times(2700\underline{i}-3000\underline{j}+2400\underline{k}) + 4\underline{k}\times 6000\underline{j} = 24000\underline{i} - 9000\underline{k} - 7200\underline{j} + 9000\underline{k} + 7200\underline{j} - 24000\underline{i} = \underline{0}$$

F_{BA} = 6462 N	N	F_{BC} = 3699 N	δ	$E_x = 0$	C_x = 2700 N ↙x+	D_x = 2700 N ↗x-
F_{CA} = 1500 N	ʃ	F_{BD} = 3699 N	δ	$E_y = 0$	C_y = 3000 N ↖y-	D_y = 3000 N ↖y-
F_{DA} = 1500 N	δ	F_{EB} = 4800 N	N	E_z = 4800 N ↓z-	C_z = 2400 N ↑z+	D_z = 2400 N ↑z+



$\underline{F}_{AD} = -\underline{F}_{DA}$

הערה: צריך להיזהר באברים בני שני כוחות; נניח שיש מוט בין A ו-D. הכוחות הווקטוריים

SESSION # 20 EXAM

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SESSION # 21

ANNOUNCE EXAM FOR SESSION # 24 ON CH. 6 & CH 8 § 1 & 2
(14 Nov)

CHAP 9

- DISCUSS METHODS TO FIND

לאוסף חלקיקים בודדים

CENTER OF GRAVITY
" OF MASS } FOR DISCRETE PARTICLES & ARBITRARY SHAPES

או לצורות כלשהן

- APPLY TO LINES, AREAS & VOLUMES

- CENTER OF GRAVITY OF A SYSTEM OF PARTICLES - PLACE WHERE TOTAL WEIGHT OF ALL PARTICLES IS APPLIED

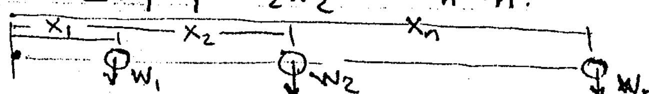
- WEIGHT OF PARTICLES CONSTITUTE A SYSTEM OF PARALLEL FORCES
- IN 2-D FORCES ARE COPLANAR

$$+\downarrow \Sigma F_R = W_1 + W_2 + \dots + W_n = \Sigma W_i = W_T$$

- LOCATION OF THIS WEIGHT $\bar{x}$ IS SUCH THAT

$$\bar{x} \sum_{i=1}^{n} W_i = \Sigma x_1 W_1 + x_2 W_2 + \dots x_n W_n .$$

EQUIV SYSTEM $\Sigma \vec{F_i} = \vec{F_R}$



x_i's are measured from a given reference point

$$\text{ie} \quad +\curvearrowleft \Sigma M_o = \Sigma x_1 W_1 + x_2 W_2 + \dots + x_n W_n .$$

EQUIV TO MOMENT ABOUT Z AXIS.

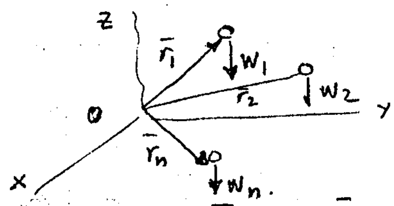
$\downarrow W_T$ $\quad \downarrow W_T$ $\quad M_o = \bar{x} W_T$ $\quad \bar{x} = \frac{M_o}{W_T}$

FIGURE 9-1 pg 325

$$\therefore \quad \bar{x} = \frac{\sum^n x_i W_i}{\sum^n W_i}$$

$\bar{x}$ is the center of gravity in x direction

$\bar{x}$ need not coincide with a particle - it can be a point in space

in 3-D since we have 

then $\sum \vec{F} = -(W_1 + W_2 + W_3 + \dots W_n)\bar{k} = -W_T\bar{k}$; $W_T = \sum^n W_i$

$$\sum \vec{M}_o = \sum \bar{r}_1 \times -W_1\bar{k} + \bar{r}_2 \times (-W_2\bar{k}) + \dots \sum \bar{r}_n \times (-W_n\bar{k})$$

$$= \sum M_{o_x}\bar{i} + \sum M_{o_y}\bar{j} + \sum M_{o_z}\bar{k}$$

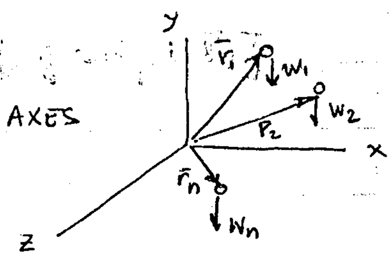
$$= \sum(-r_{1y}W_1 - r_{2y}W_2 + \dots - r_{ny}W_n)\bar{i} + \sum(r_{1x}W_1 + r_{2x}W_2 + \dots + r_{nx}W_n)\bar{j} + 0\bar{k}$$

but $\sum \vec{M}_o = (\bar{x}\vec{i} + \bar{y}\vec{j} + \bar{z}\vec{k}) \times (-W_T\bar{k}) = \bar{x}W_T\bar{j} - \bar{y}W_T\vec{i}$

Then

$$\bar{x}W_T = \sum r_{ix}W_i \quad \text{or} \quad \bar{x} = \frac{\sum_{i=1}^n r_{ix}W_i}{\sum W_i} \Rightarrow \frac{\sum x_i W_i}{\sum W_i}$$

$$\sum -(r_{iy}W_i) = -\bar{y}W_T \quad \text{or} \quad \bar{y} = \frac{\sum_{i=1}^n r_{iy}W_i}{\sum W_i} = \frac{\sum y_i W_i}{\sum W_i}$$

SUPPOSE AXES  $\bar{z} = \dfrac{\sum^n r_{iz}W_i}{\sum W_i}$ $\bar{x} = \dfrac{\sum^n r_{ix}W_i}{\sum^n W_i}$

THEREFORE

center of gravity is $\bar{x} = \dfrac{\sum r_{ix}W_i}{\sum W_i}$ $\bar{y} = \dfrac{\sum r_{iy}W_i}{\sum W_i}$ $\bar{z} = \dfrac{\sum r_{iz}W_i}{\sum W_i}$

in general Center of gravity is $(\bar{x}, \bar{y}, \bar{z})^*$

Center of mass $W_i = m_i g \Rightarrow \sum r_{ix} m_i g = (\sum r_{ix} m_i) g$

$$W_T = \sum W_i = \sum m_i g = (\sum m_i) g$$

thus

$$\bar{x} = \frac{\sum r_{ix} m_i}{\sum m_i} \qquad \bar{y} = \frac{\sum^n r_{iy} m_i}{\sum m_i} \qquad \bar{z} = \frac{\sum^n r_{iz} m_i}{\sum m_i}$$

Thus centroid & center of mass are for all intents & purposes the same

Center of Mass & Centroid for a continuous body

replace $\sum W_i \Rightarrow \int dW$ as $n \to \infty$

$$\sum r_i W_i \Rightarrow \int r_i \, dW$$

~~Centroid~~ Center of ~~mass~~ gravity

Thus
$$\bar{x} = \frac{\int x \, dW}{\int dW} \qquad \bar{y} = \frac{\int y \, dW}{\int dW} \qquad \bar{z} = \frac{\int z \, dW}{\int dW}$$

~~Center of mass replace~~ $\cancel{dW \text{ by } m\,dg + g\,dm = g[d(\rho V)]}$
$\cancel{= g[d\rho \cdot V + \rho \, dV]}$

$$\cancel{\bar{x} = \frac{\int x \, dm}{\int dm} \qquad \bar{y} = \frac{\int y \, dm}{\int dm} \qquad \bar{z} = \frac{\int z \, dm}{\int dm}}$$

צפיפות המשקל

define ρ_w – weight density $= \dfrac{\text{weight}}{\text{unit volume}}$ $\left[\dfrac{\text{force}}{\text{length}^3}\right]$.

then $dW = \rho_w dV$ hence

$$\bar{x} = \frac{\int x \rho_w \, dV}{\int \rho_w \, dV} \qquad \bar{y} = \frac{\int y \rho_w \, dV}{\int \rho_w \, dV} \qquad \bar{z} = \frac{\int z \rho_w \, dV}{\int \rho_w \, dV}$$

צפיפות המסה

if mass density is defined $\rho_m = \dfrac{\text{mass}}{\text{unit volume}}$ the $dm = \rho_m dV$

and
$$\bar{x} = \frac{\int x \, dm}{\int dm} = \frac{\int x \rho_m \, dV}{\int \rho_m \, dV} \quad \text{etc.}$$

This defines the center of mass. NOTE $\underline{\underline{\rho_W = \rho_m g}}$

Centroid is the geometrical center; if the material is homogeneous: $\rho_m = \text{const.}$, $\rho_W = \text{const.}$

$$\bar{x} = \frac{\int x \, dV}{\int dV} \qquad \bar{y} = \frac{\int y \, dV}{\int dV} \qquad \bar{z} = \frac{\int z \, dV}{\int dV}$$

FOR 2-D bodies we can define the centroid by

$$\bar{x} = \frac{\int x\,dA}{\int dA} \qquad \bar{y} = \frac{\int y\,dA}{\int dA} \qquad \bar{z} = \frac{\int z\,dA}{\int dA}$$

For a 1-D body.

$$\bar{x} = \frac{\int x\,dL}{\int dL} \qquad \bar{y} = \frac{\int y\,dL}{\int dL} \qquad \bar{z} = \frac{\int z\,dL}{\int dL}$$

these 3 sets are geometrical properties do not depend on the weight

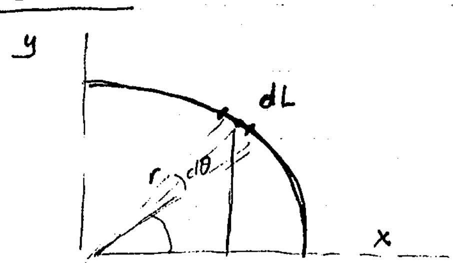
- WHERE USED STRUCTURAL MECHANICS — CENTROID NEEDED TO PROPERLY DESIGN MEMBER
- DISTRIBUTED LOADING – GOOD TO REPLACE BY A FORCE THROUGH ITS CENTROID
- SYMMETRIC BODIES HAVE CENTROID ON LINE OF SYMMETRY.

FIG. 9-6,7 PG 329

x, y, z in these formulas as the moment arms or ⊥ distance to centroid of the elemental area.

DIFFERENTIAL ELEMENTS dL, $dA = \square$, $dV =$  or 

EXAMPLE 9-1



$dL = r\,d\theta$

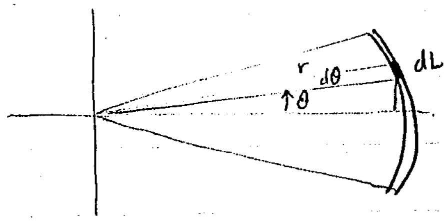
$x = r\cos\theta \qquad y = r\sin\theta$

$$\bar{x} = \frac{\int x\,dL}{\int dL} = \frac{\int_0^{\pi/2} r\cos\theta \cdot r\,d\theta}{\int_0^{\pi/2} r\,d\theta} = \frac{r^2 \sin\theta\big|_0^{\pi/2}}{r\,\theta\big|_0^{\pi/2}} = \frac{r}{\pi/2}$$

$$= \frac{2r}{\pi}$$

$$\bar{y} = \frac{\int y\,dL}{\int dL} = \frac{\int_0^{\pi/2} r\sin\theta\,(r\,d\theta)}{\int_0^{\pi/2} r\,d\theta} = \frac{r^2(-\cos\theta)\Big|_0^{\pi/2}}{r\pi/2} = \frac{r^2}{r\pi/2} = \frac{2r}{\pi}.$$

Problem 9-1



by symmetry $\bar{y} = 0$. $dL = r\,d\theta$

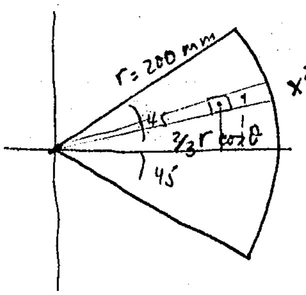
$x = r\cos\theta$

$$\bar{x} = \frac{\int x\,dL}{\int dL} = \frac{\int_{-\alpha}^{\alpha}(r\cos\theta)\,r\,d\theta}{\int_{-\alpha}^{\alpha} r\,d\theta} = \frac{\frac{r^2}{r}\sin\theta\Big|_{-\alpha}^{\alpha}}{\theta\Big|_{-\alpha}^{\alpha}}$$

$$\bar{x} = \frac{r\left[\sin\alpha - \sin(-\alpha)\right]}{\alpha - (-\alpha)} = \frac{2r\sin\alpha}{2\alpha} = \frac{r\sin\alpha}{\alpha}$$

SESSION #22

Problem 9-8 ⟵ **DO**



$x^2+y^2 = 4\times10^4\,mm^2$
$= .04\,m^2$
$r = .2\,m$

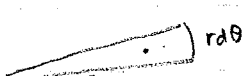
by symmetry $\bar{y} = 0$

$dA = r\,d\theta\,dr$ $\quad x = r\cos\theta$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\iint r\cos\theta \cdot r\,dr\,d\theta}{\iint r\,dr\,d\theta}$$

$$= \frac{\left(\int_0^{.2} r^2 dr\right)\left(\int_{-\pi/4}^{\pi/4}\cos\theta\,d\theta\right)}{\left(\int_0^{.2} r\,dr\right)\int_{-\pi/4}^{\pi/4} d\theta}$$

$$= \frac{r^3/3\Big|_0^{.2}}{r^2/2\Big|_0^{.2}}\;\frac{\sin\theta\Big|_{-\pi/4}^{\pi/4}}{\theta\Big|_{-\pi/4}^{\pi/4}} = \frac{2r}{3}\cdot\frac{\cancel{2}\sin\pi/4}{\cancel{2}\,\pi/4} = \frac{\cancel{2}(.2)}{3}\,\frac{\sqrt{2}}{\cancel{2}}\cdot\frac{4}{\pi} = .12\,m$$



שטח משולש
$\frac{1}{2} r \cdot r\,d\theta$
$\frac{2}{3} r$ מרכז הכבד
$\frac{2}{3} r\cos\theta = x$ אבל בכיוון

$dA = \frac{1}{2} r^2 d\theta \quad x = \frac{2}{3} r\cos\theta$

$$\bar{x} = \frac{\int_{-\pi/4}^{\pi/4}\frac{1}{3} r^3\cos\theta\,d\theta}{\int_{-\pi/4}^{\pi/4}\frac{1}{2} r^2 d\theta} = \frac{2r}{3}\,\frac{\sin\theta}{\theta}\Big|_{-\pi/4}^{\pi/4}.$$

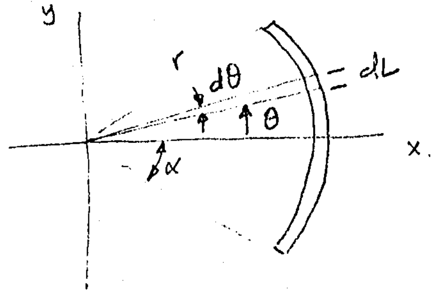
FOR 9-12 WHAT IS $\bar{Y}$, WHAT IS $\bar{X}$ by sym $\bar{y} = \frac{h}{2}$ $\bar{x} = 0$

FROM 9-1 $\bar{z} = \dfrac{a\sin\pi/4}{\pi/4} = \dfrac{4a}{\pi}\cdot\dfrac{\sqrt{2}}{2} = \dfrac{2\sqrt{2}\,a}{\pi} = .9a$

למצוא מרכז הכובד

PROBLEM 9-1

תיל בצורת קשת



by symmetry $\bar{y} = 0$

$dL = r\,d\theta \qquad x = r\cos\theta$

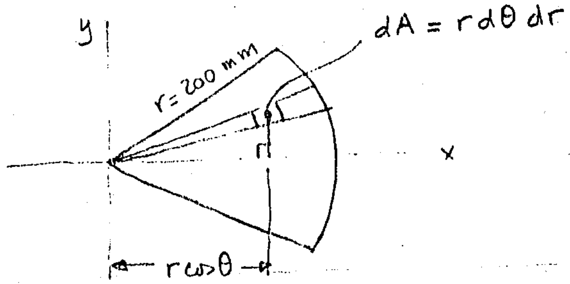
$$\bar{x} = \frac{\int x\,dL}{\int dL} = \frac{\int_{-\alpha}^{\alpha} (r\cos\theta)(r\,d\theta)}{\int_{-\alpha}^{\alpha} r\,d\theta} = \frac{r^2 \sin\theta \Big|_{-\alpha}^{\alpha}}{r\theta \Big|_{-\alpha}^{\alpha}}$$

$$= \frac{r\sin\alpha}{\alpha}$$

למצוא מרכז השטח

PROBLEM 9-8

עיגול



by symmetry $\bar{y} = 0$

$x = r\cos\theta$

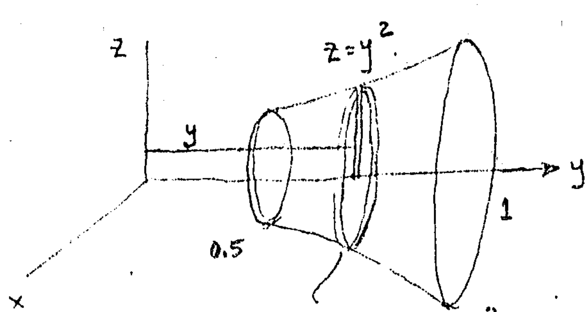
$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int (r\cos\theta)(r\,dr\,d\theta)}{\int r\,dr\,d\theta}$$

$$= \frac{\int_0^{.2m} r^2 dr \int_{-\pi/4}^{\pi/4} \cos\theta\,d\theta}{\int_0^{.2m} r\,dr \int_{-\pi/4}^{\pi/4} d\theta} = \frac{\frac{r^3}{3}\Big|_0^{.2} \sin\theta \Big|_{-\pi/4}^{\pi/4}}{\frac{r^2}{2}\Big|_0^{.2} \theta \Big|_{-\pi/4}^{\pi/4}}$$

$$= .12\text{ m}$$

למצוא מרכז הנפח

PROBLEM 9-19



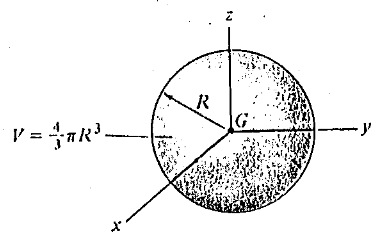
volume disk $= dy \cdot \pi z^2 = \pi y^4 dy$

by symmetry centroid must lie on the y axis

$$\bar{y} = \frac{\int y\,dV}{\int dV} \qquad \bar{x} = \bar{z} = 0$$

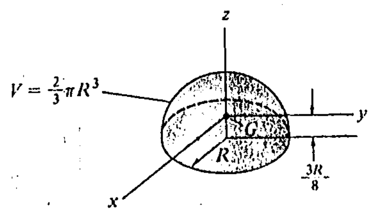
$$= \frac{\int_{.5}^{1} y\,(\pi y^4 dy)}{\int_{.5}^{1} (\pi y^4 dy)} = \frac{y^6/6 \Big|_{.5}^{1}}{y^5/5 \Big|_{.5}^{1}} = .847$$

Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



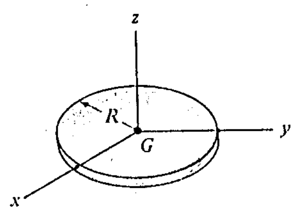
Sphere

$$I_{xx} = I_{yy} = I_{zz} = \tfrac{2}{5}mR^2$$



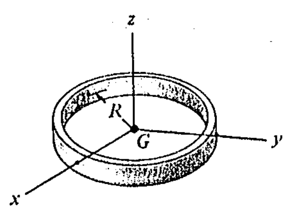
Hemisphere

$$I_{xx} = I_{yy} = 0.259mR^2 \qquad I_{zz} = \tfrac{2}{5}mR^2$$



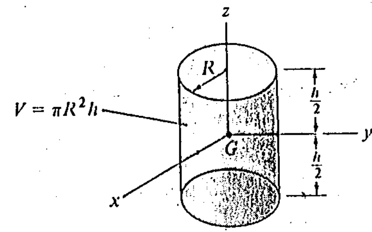
Thin circular disk

$$I_{xx} = I_{yy} = \tfrac{1}{4}mR^2 \qquad I_{zz} = \tfrac{1}{2}mR^2$$



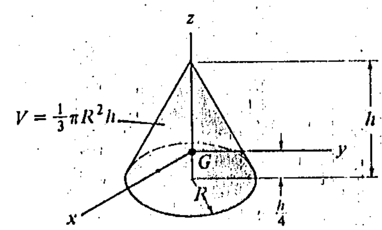
Thin ring

$$I_{xx} = I_{yy} = \tfrac{1}{2}mR^2 \qquad I_{zz} = mR^2$$



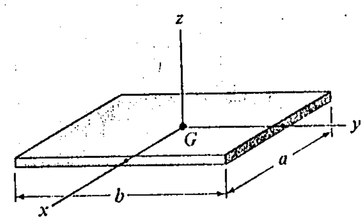
Cylinder

$$I_{xx} = I_{yy} = \tfrac{1}{12}m(3R^2 + h^2) \qquad I_{zz} = \tfrac{1}{2}mR^2$$



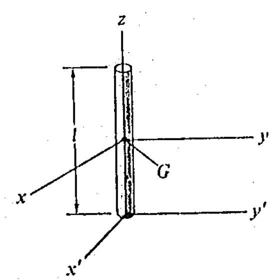
Cone

$$I_{xx} = I_{yy} = \tfrac{3}{80}m(4R^2 + h^2) \qquad I_{zz} = \tfrac{3}{10}mR^2$$



Thin plate

$$I_{xx} = \tfrac{1}{12}mb^2 \qquad I_{yy} = \tfrac{1}{12}ma^2 \qquad I_{zz} = \tfrac{1}{12}m(a^2 + b^2)$$



Slender rod

$$I_{xx} = I_{yy} = \tfrac{1}{12}ml^2 \qquad I_{x'x'} = I_{y'y'} = \tfrac{1}{3}ml^2 \qquad I_{zz} = 0$$

Example 9–7

Locate the centroid of the wire shown in Fig. 9–13*a*.

SOLUTION

Composite Parts. The wire is divided into three segments as shown in Fig. 9–13*b*.

Moment Arms. The location of the centroid for each piece is determined and indicated in the figure. In particular, the centroid of segment 1 is determined either by integration or using the table on the inside back cover.

Summations. The calculations are tabulated as follows:

Segment	L (mm)	$\tilde{x}$ (mm)	$\tilde{y}$ (mm)	$\tilde{z}$ (mm)	$\tilde{x}L$ (mm²)	$\tilde{y}L$ (mm²)	$\tilde{z}L$ (mm²)
1	$\pi(60) = 188.5$	60	−38.2	0	11 310	−7200	0
2	40	0	20	0	0	800	0
3	20	0	40	−10	0	800	−200
	$\Sigma L = 248.5$				$\Sigma\tilde{x}L = 11\ 310$	$\Sigma\tilde{y}L = -5600$	$\Sigma\tilde{z}L = -200$

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{11\ 310}{248.5} = 45.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{-5600}{248.5} = -22.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{-200}{248.5} = -0.805 \text{ mm} \qquad \textit{Ans.}$$

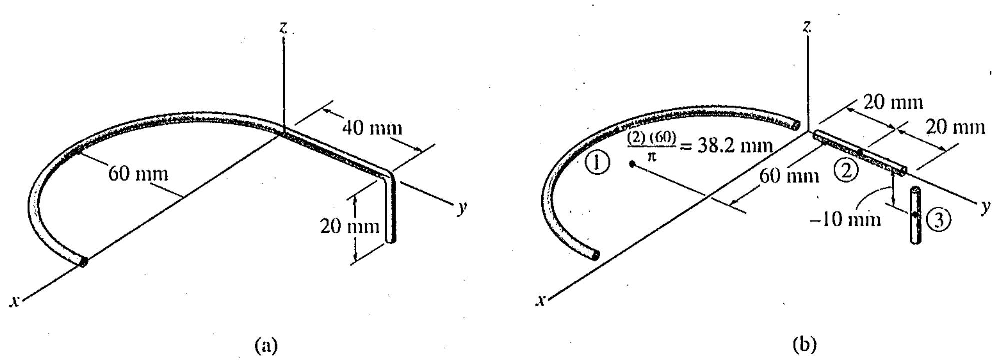
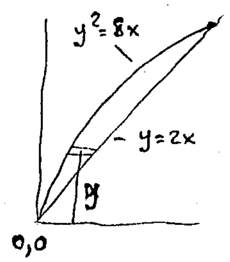


Fig. 9–13

PROB 9-9

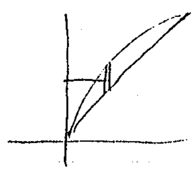


$$dA = (x_2 - x_1)\,dy = \left(\tfrac{y}{2} - \tfrac{y^2}{8}\right) dy$$

$$\bar{y} = \frac{\int_0^4 y\left[\tfrac{y}{2} - \tfrac{y^2}{8}\right] dy}{\int_0^4 \left[\tfrac{y}{2} - \tfrac{y^2}{8}\right] dy} = \frac{\left(\tfrac{y^3}{6} - \tfrac{y^4}{32}\right)\Big|_0^4}{\tfrac{y^2}{4} - \tfrac{y^3}{24}\Big|_0^4}$$

$$y^2 = 8x = 4x^2 \qquad 4x(2-x) = 0 \qquad x=0 \quad x=2 \Rightarrow y=4$$

$$\bar{y} = \frac{\overset{10.67}{\tfrac{64}{6}} - \overset{8}{\tfrac{256}{32}}}{\tfrac{16}{4} - \tfrac{64}{24}} = \frac{2.67}{4 - 2.67} = \frac{2.67}{1.33} = \underline{\underline{2m}}$$



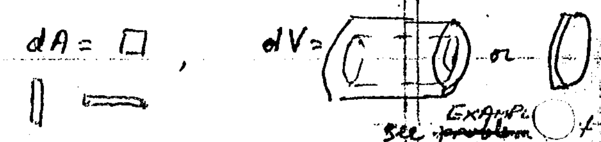
$$dA = (y_2 - y_1)\,dx = \left[\sqrt{8x} - 2x\right] dx$$

$$\bar{x} = \frac{\int x\left(\sqrt{8x} - 2x\right) dx}{\int_0^2 \left(\sqrt{8x} - 2x\right) dx} = \frac{\left(\tfrac{2}{5}\sqrt{8}\, x^{5/2} - \tfrac{2x^3}{3}\right)\Big|_0^2}{1.33\,m^2}$$

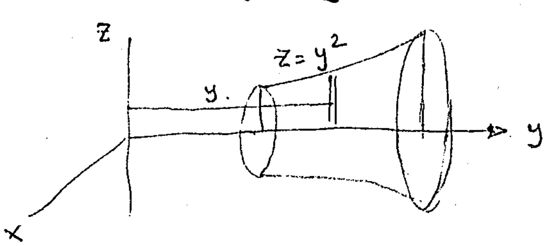
$$= \frac{\tfrac{2}{5}\cdot 16 - \tfrac{16}{3}}{} = \frac{6.4 - 5.33}{1.33} = \frac{1.07}{1.33} = \underline{\underline{.8045\,m}}$$

PROCEDURE

Differential element - DRAW IT AT AN ARBITRARY POINT



Problem 9-19 ⟸ DO

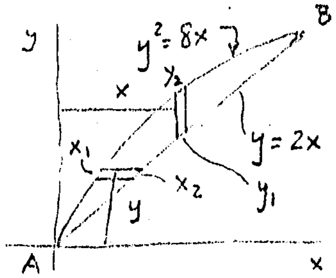


$$\bar{y} = \frac{\int y\,dV}{\int dV} \qquad dV = \pi z^2 dy = \pi y^4 dy$$

$$= \frac{\pi \int_{.5}^{1} y^5 dy}{\pi \int y^4 dy} = \frac{y^6/6\,\Big|_{.5}^{1}}{y^5/5\,\Big|_{.5}^{1}} = \frac{\tfrac{1}{6} - \tfrac{1}{64 \cdot 6}}{\tfrac{1}{5} - \tfrac{1}{32 \cdot 5}} = \frac{5}{6}\cdot\frac{63}{62} = .847$$

by symmetry centroid must lie on y axis $\therefore\ \bar{x} = \bar{z} = 0$

PROBLEM 9-9



TO FIND INTERSECTIONS

A & B $\quad y^2 = 8x$; but $y = 2x$ also

$4x^2 = 8x \qquad \left\{ \begin{matrix} x=0 & x=2 \\ y=0 & y=4 \end{matrix} \right\}$

$dA = (x_2 - x_1)\,dy$

$dA = \left(\frac{y}{2} - \frac{y^2}{8}\right) dy$

$dA = (y_2 - y_1)\,dx$
$= (\sqrt{8x} - 2x)\,dx$

$$\bar{x} = \frac{\int_0^2 x\left(\sqrt{8x} - 2x\right)dx}{\int_0^2 \left(\sqrt{8x} - 2x\right)dx} = \frac{\int x\,dA}{\int dA}$$

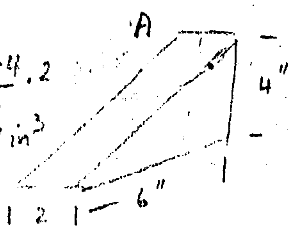
$$= \frac{\left[\frac{\sqrt{8}\,x^{5/2}}{5/2} - \frac{2x^3}{3}\right]_0^2}{\left[\sqrt{8}\,\frac{x^{3/2}}{3/2} - \frac{2x^2}{2}\right]_0^2} = .8045\text{m}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int_0^4 y\left[\frac{y}{2} - \frac{y^2}{8}\right]dy}{\int \left(\frac{y}{2} - \frac{y^2}{8}\right)dy}$$

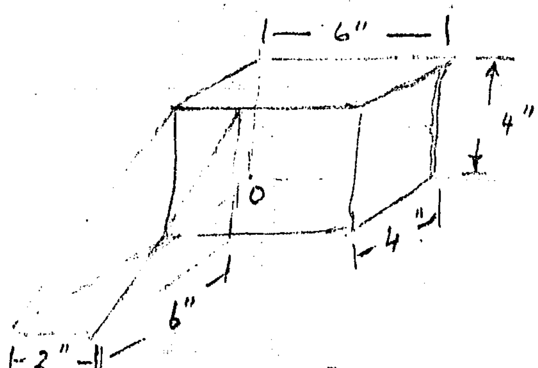
$$= \frac{\left[\frac{y^3}{6} - \frac{y^4}{32}\right]_0^4}{\left[\frac{y^2}{4} - \frac{y^3}{24}\right]_0^4}$$

$\bar{y} = 2\text{ m}$

EXAMPLE



$V_A = \frac{6 \cdot 4}{2} \cdot 2$
$= 24 \text{ in}^3$



centroid of B
$\bar{x} = 2''$
$\bar{y} = 3''$
$\bar{z} = 2''$

$V_B = 6 \cdot 4 \cdot 4 = 96 \text{ in}^3$

centroid is $\bar{y} = 1''$
$\bar{x} = 2''$
$\bar{z} = 2''$

wrt O $\quad \bar{y} = 1''$
$\bar{x} = 6'' = 2'' + 4''$
$\bar{z} = 2''$

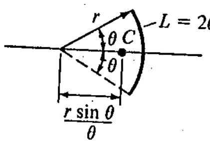
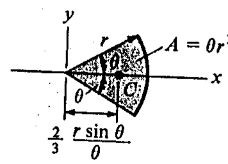
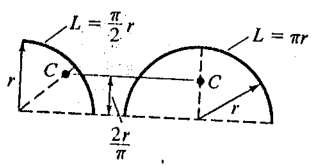
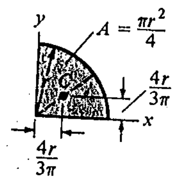
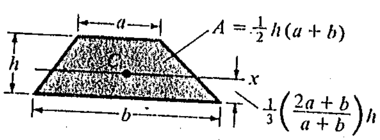
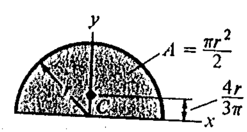
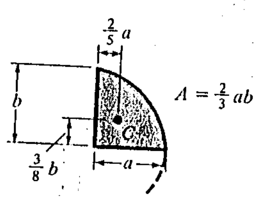
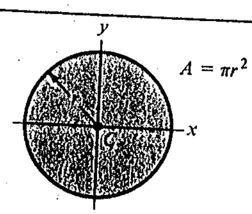
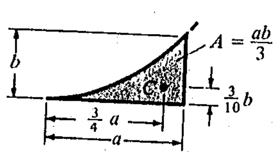
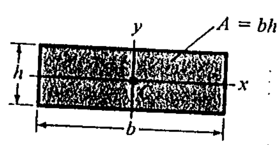
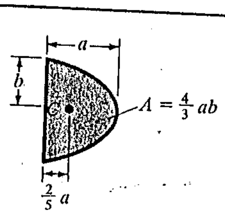
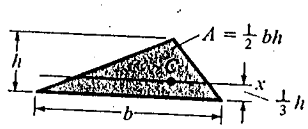
$$\bar{x} = \frac{\sum \bar{x}_i V_i}{\sum V_i} = \frac{6''(24) + 2(96)}{120} = \frac{336}{120} = 2.8\text{ in}$$

$$\bar{y} = \frac{\sum \bar{y}_i V_i}{\sum V_i} = \frac{1''(24) + 3''(96)}{120} = 2.6\text{ in}$$

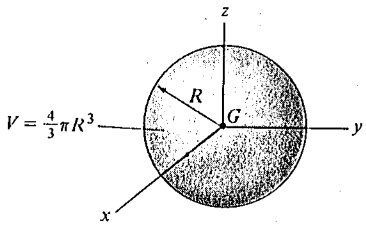
$$\bar{z} = \frac{\sum \bar{z}_i V_i}{\sum V_i} = \frac{2''(24) + 2(96)}{120} = 2\text{ in}$$

must always define centroid of each body wrt origin for the entire system.

Geometric Properties of Line and Area Elements

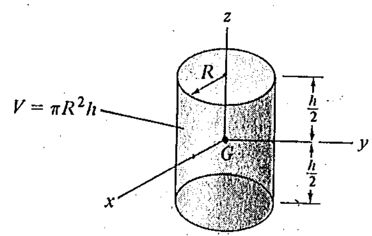
Centroid Location	Centroid Location	Area Moment of Inertia
 Circular arc segment	 Circular sector area	$I_x = \frac{1}{4}r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{1}{4}r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Quarter and semicircular arcs	 Quarter circular area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Trapezoidal area	 Semicircular area	$I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
 Semiparabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
 Exparabolic area	 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Parabolic area	 Triangular area	$I_x = \frac{1}{36}bh^3$

Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



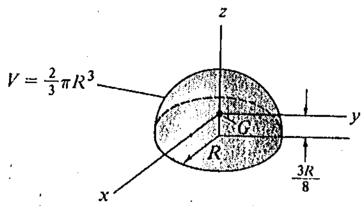
Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mR^2$



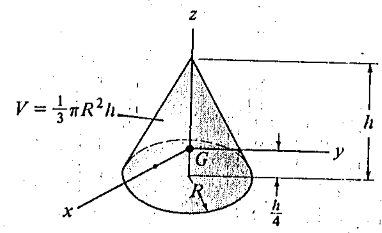
Cylinder

$I_{xx} = I_{yy} = \frac{1}{12}m(3R^2 + h^2)$ $\quad I_{zz} = \frac{1}{2}mR^2$



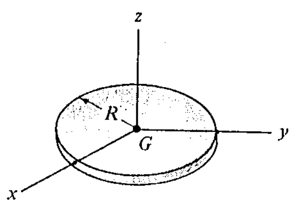
Hemisphere

$I_{xx} = I_{yy} = 0.259mR^2$ $\quad I_{zz} = \frac{2}{5}mR^2$



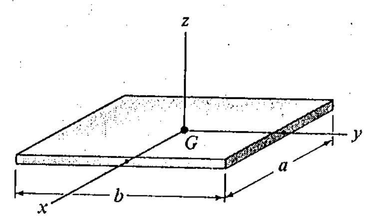
Cone

$I_{xx} = I_{yy} = \frac{3}{80}m(4R^2 + h^2)$ $\quad I_{zz} = \frac{3}{10}mR^2$



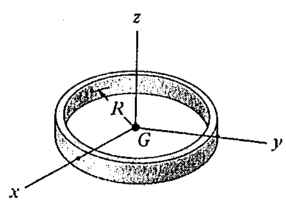
Thin circular disk

$I_{xx} = I_{yy} = \frac{1}{4}mR^2$ $\quad I_{zz} = \frac{1}{2}mR^2$



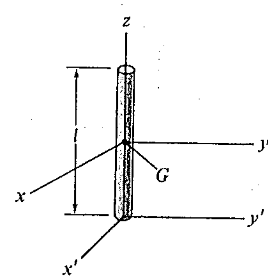
Thin plate

$I_{xx} = \frac{1}{12}mb^2$ $\quad I_{yy} = \frac{1}{12}ma^2$ $\quad I_{zz} = \frac{1}{12}m(a^2 + b^2)$



Thin ring

$I_{xx} = I_{yy} = \frac{1}{2}mR^2$ $\quad I_{zz} = mR^2$

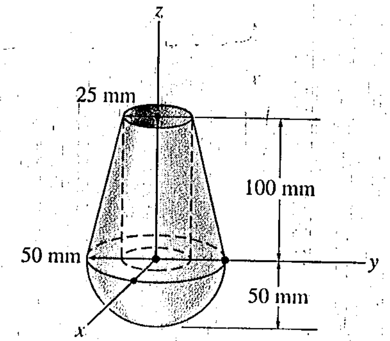


Slender rod

$I_{xx} = I_{yy} = \frac{1}{12}ml^2$ $\quad I_{x'x'} = I_{y'y'} = \frac{1}{3}ml^2$ $\quad I_{zz} = 0$

Example 9–9

Locate the center of mass of the composite assembly shown in Fig. 9–15*a*. The conical frustum has a density of $\rho_c = 8\ \text{Mg/m}^3$, and the hemisphere has a density of $\rho_h = 4\ \text{Mg/m}^3$.



(a)

Fig. 9–15

SOLUTION

Composite Parts. The assembly can be thought of as consisting of four segments as shown in Fig. 9–15*b*. For the calculations, ③ and ④ must be considered as "negative" volumes in order that the four segments, when added together, yield the total composite shape shown in Fig. 9–15*a*.

Moment Arm. Using the table on the inside back cover, the computations for the centroid $\tilde{z}$ of each piece are shown in the figure.

Summations. Because of *symmetry*, note that

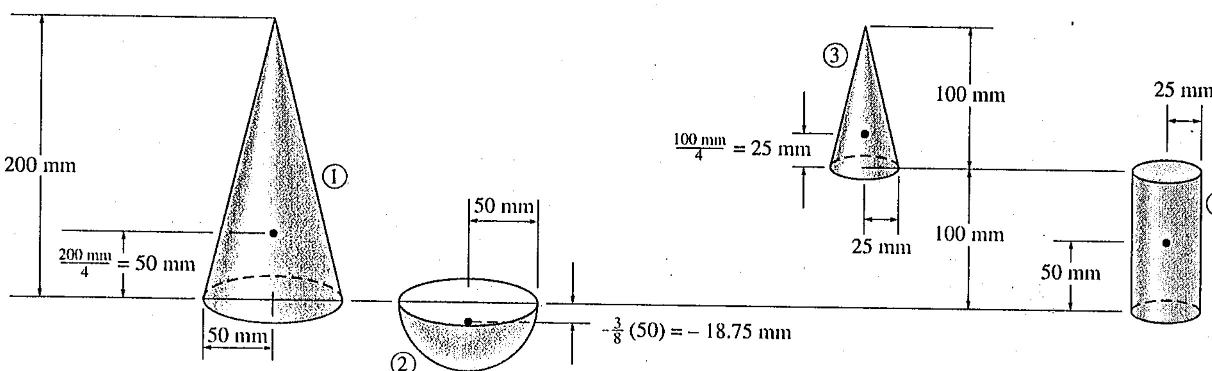
$$\bar{x} = \bar{y} = 0 \qquad \textit{Ans.}$$

Since $W = mg$ and g is constant, the third of Eqs. 9–8 becomes $\bar{z} = \Sigma\tilde{z}m/\Sigma m$. The mass of each piece can be computed from $m = \rho V$ and used for the calculations. Also, $1\ \text{Mg/m}^3 = 10^{-6}\ \text{kg/mm}^3$, so that

Segment	m (kg)	$\tilde{z}$ (mm)	$\tilde{z}m$ (kg · mm)
1	$8(10^{-6})(\frac{1}{3})\pi(50)^2(200) = 4.189$	50	209.440
2	$4(10^{-6})(\frac{2}{3})\pi(50)^3 = 1.047$	−18.75	−19.635
3	$-8(10^{-6})(\frac{1}{3})\pi(25)^2(100) = -0.524$	100 + 25 = 125	−65.450
4	$-8(10^{-6})\pi(25)^2(100) = -1.571$	50	−78.540
	$\Sigma m = 3.141$		$\Sigma\tilde{z}m = 45.815$

Thus,

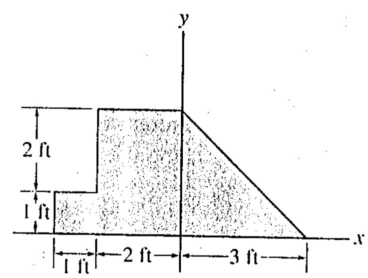
$$\bar{z} = \frac{\Sigma\tilde{z}m}{\Sigma m} = \frac{45.815}{3.141} = 14.6\ \text{mm} \qquad \textit{Ans.}$$



(b)

Example 9–8

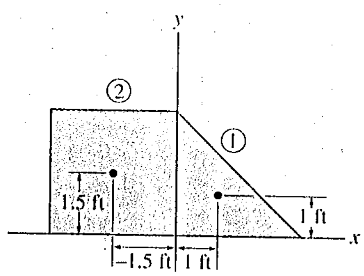
Locate the centroid of the plate area shown in Fig. 9–14*a*.



(a)

SOLUTION

Composite Parts. The plate is divided into three segments as shown in Fig. 9–14*b*. Here the area of the small rectangle 3 is considered "negative" since it must be subtracted from the larger one 2.

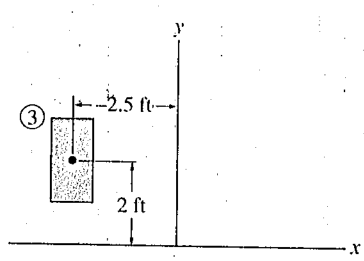


(b)

Moment Arms. The centroid of each segment is located as indicated in the figure. Note that the $\tilde{x}$ coordinates of 2 and 3 are *negative*.

Summations. Taking the data from Fig. 9–14*b*, the calculations are tabulated as follows:

Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	-1.5	1.5	-13.5	13.5
3	$-(2)(1) = -2$	-2.5	2	5	-4
	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$



(c)

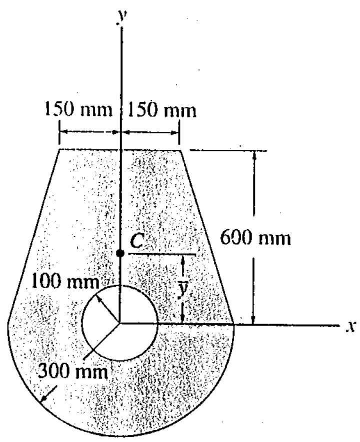
Fig. 9–14

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

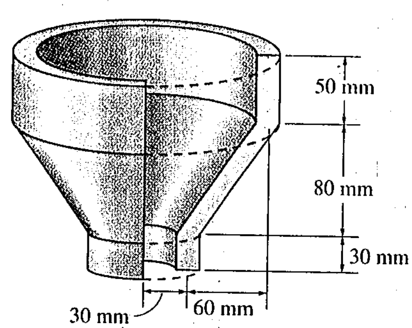
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

9–55. Determine the distance $\bar{y}$ to the centroid of the shaded area.

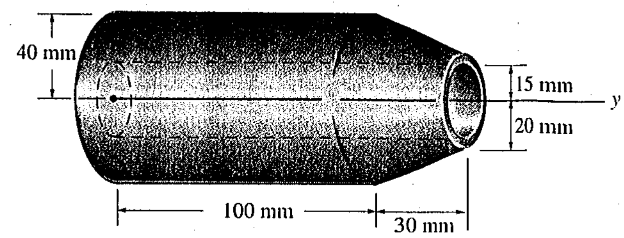


***9–84.** Determine the approximate amount of aluminum necessary to make the funnel. It consists of a full circular part having a thickness of 2 mm. Its cross section is shown in the figure.

9–85. Determine the approximate outer surface area of the funnel. It consists of a full circular part of negligible thickness.



9–67. Determine the distance $\bar{y}$ to the center of mass of the assembly, which has a hole bored through the center. The material has a density of $\rho = 3\ \mathrm{Mg/m^3}$.



Composite bodies

- CAN BE BROKEN UP INTO PARTS FOR WHICH ~~CENTROIDS~~ CENTERS OF GRAVITY ARE KNOWN AS WELL AS THE WEIGHT OF EACH SECTION.

- DON'T NEED TO INTEGRATE – CAN USE FORMULAS WE HAD BEFORE

$$\bar{x} = \frac{\sum \bar{x}_i W_i}{\sum W_i} \qquad \bar{y} = \frac{\sum \bar{y}_i W_i}{\sum W_i} \qquad \bar{z} = \frac{\sum \bar{z}_i W_i}{\sum W_i}$$

- where $\bar{x}_i, \bar{y}_i, \bar{z}_i$ are the centroids of each part measured from the same fixed point (ie the origin of the coord. system).
- W_i is the weight of each part and $\sum W_i$ is the weight of the whole body

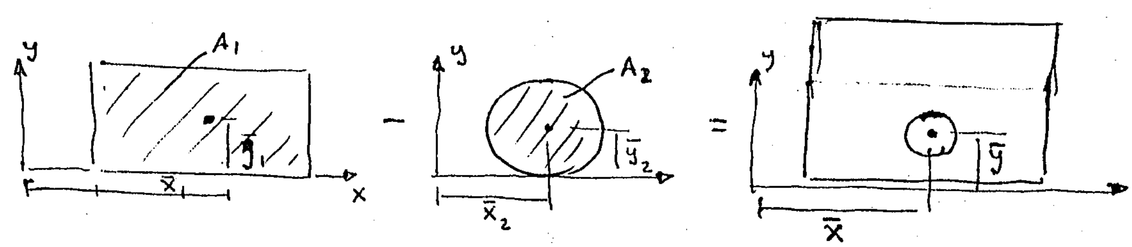
- for CONSTANT density case CENTER OF GRAVITY IS THE CENTROID

FOR COMPOSITE LINES replace W_i by L_i

AREAS " W_i by A_i

VOLUMES " W_i by V_i

- FOR BODIES HAVING HOLES OR MISSING MATERIAL OR AREA OR VOLUME

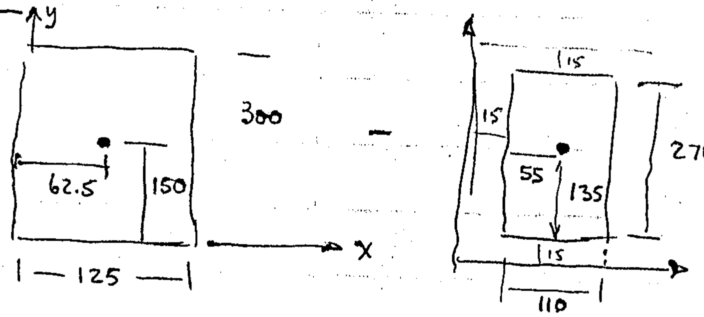


$$\bar{x} = \frac{\sum \bar{x}_1 A_1 - \bar{x}_2 A_2}{\sum (A_1 - A_2)}$$

- IF BODY IS symmetric centroid (center of gravity lies on axis of symmetry.

see EXAMPLES in book pg 342-343

Problem 9-30



$\bar{x} = 62.5$
$\bar{y} = 150$
$A_1 = 125 \times 300 = 37500\ mm^2$

$\bar{x} = 70$
$\bar{y} = 150$
$A_2 = 110 \times 270 = 7425\ mm^2 \times 4 = 29700\ mm^2$

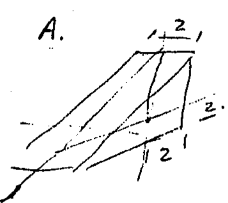
$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = \frac{(\bar{x}_1 A_1 - \bar{x}_2 A_2)}{A_1 - A_2} = \frac{(62.5)(37500) - 70(29700)}{37500 - 29700} = \frac{264750}{7800} = 33.94\ mm$$

by symmetry $\bar{y} = 150\,mm = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{150(37500) - 150(29700)}{37500 - 29700} = 150\frac{\cancel{(37500-29700)}}{\cancel{37500-29700}}$

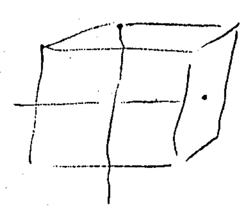
$= 150\ mm.$

Problem 9-45

FOR A.



$\bar{x} = 4''$
$\bar{y} = 1\ in$
$\bar{z}'' = 2\ in$



$\bar{x} = 1''$
$\bar{y} = 3''$
$\bar{z} = 3''$

$V_A = \frac{6 \times 6 \times 2}{2}$
$V_A = 36''$ cube
$= \frac{36}{1728} = .021\ ft^3$

$\bar{x} = \frac{\Sigma \bar{x}_i W_i}{\Sigma W_i}$ $\quad \bar{y} = \frac{\Sigma \bar{y}_i W_i}{\Sigma W_i}$

$W_A = \rho_A V_A$
$= 150(.021)$
$= 3.13\ lb.$

$V = 6 \times 6 \times 2 = 72''$ cubed
$= \frac{72}{1728} = .042\ ft^3$

$W_B = \rho_B V_B$
$= 400(.042) = 16.67\ lb$

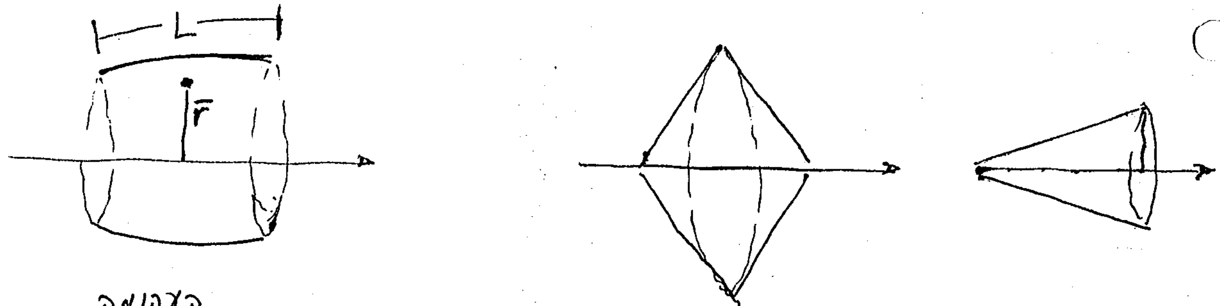
$$\bar{x} = \frac{(4'')(3.13\,lb) + 1''(16.67\,lb)}{19.80\,lb} = 1.47''$$

$\bar{y} = 2.68\ in \quad \bar{z} = 2.84\ in$

SESSION # 23

Theorem of Pappus - Guldinus   תורת פאפוס-גולדינוס

- used to find surface area or volume of ~~revolution~~ any solid of revolution
- Surface of revolution generated by rotating a line (plane curve) about some fixed axis
- Volume of revolution generated by rotating ~~an~~ plane area about some fixed axis



העקומה

NOTE: CURVE OR AREA CANNOT CROSS AXIS ABOUT WHICH THEY ARE ROTATED

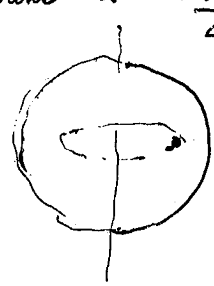
SURFACE AREA = $2\pi\bar{x}L$ or $\theta\bar{x}L$ θ is the angle in radians the curve is rotated

elemental surface area $2\pi r\,dL$ TOTAL AREA is $\int 2\pi r\,dL = 2\pi\int r\,dL = 2\pi\bar{r}L$

VOLUME = $2\pi\bar{r}A$ or $\theta\bar{r}A$

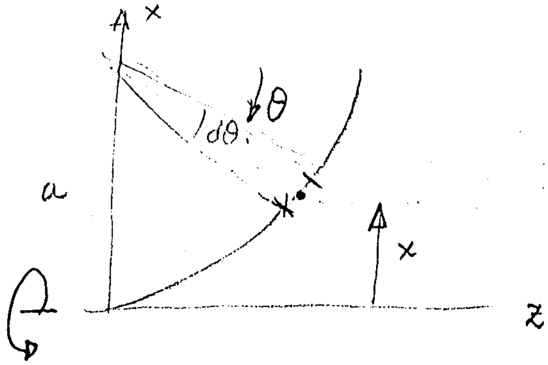
elemental ~~surface area~~ volume = $2\pi r\,dA$ TOTAL volume is $\int 2\pi r\,dA = 2\pi\int r\,dA = 2\pi\bar{r}A$

PROBLEM 9-1   found $\bar{x} = \dfrac{r\sin\alpha}{\alpha}$ let $\alpha = \pi/2$ $\bar{x} = \dfrac{2r}{\pi}$; $\pi r = L$



$S = 2\pi\bar{x}L = 2\pi\cdot\dfrac{2r}{\pi}\cdot\pi r = 4\pi r^2$ = surface area

$\cancel{(x-a)^2 + (y-a)^2 + (z-a)^2}$



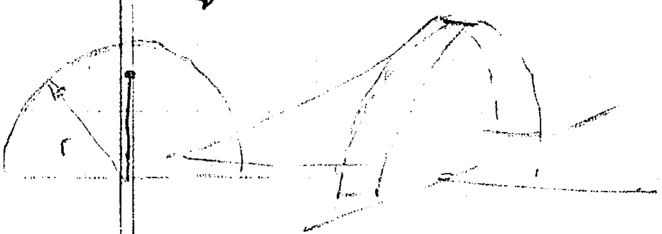
arc length is $a\,d\theta$

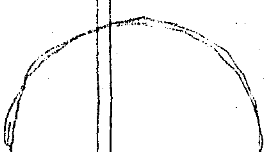
surface area generated is $\pi x\,(a\,d\theta)$

$$x = a - a\sin\theta = a(1-\sin\theta)$$

$$\int dA = \pi a^2 \int_0^{\pi/2} (1-\sin\theta)\,d\theta$$

$$A = \pi a^2 \left[\theta + \cos\theta\right]_0^{\pi/2} = \pi a^2 (\pi/2 - 1)$$





$$X_c = \frac{2r}{\pi}$$

$$\int x_c\,dA = \int_0^{\pi/2} \frac{2a}{\pi}(1-\sin\theta)\,\pi a^2 (1-\sin\theta)\,d\theta$$

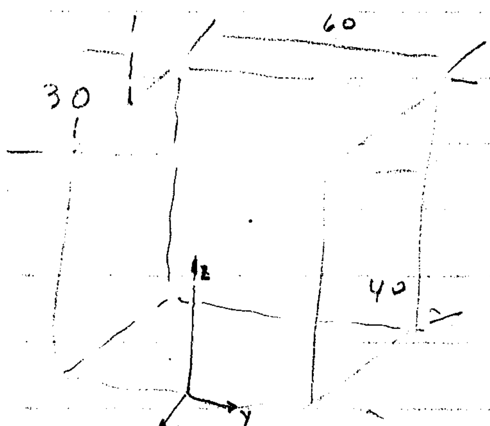
$$= \int 2a^3 \left[1 - 2\sin\theta + \frac{1-\cos 2\theta}{2}\right] d\theta$$

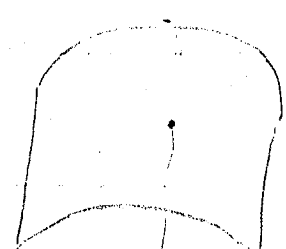
$$= 2a^3 \left[\theta + 2\cos\theta + \theta/2 - \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$

$$= 2a^3 \left[\frac{3\pi}{4} - 2\right] = 4a^3\left[\frac{3\pi}{8} - 1\right]$$

$$\therefore\ \bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{4a^3(3\pi/8 - 1)}{\pi a^2(\pi/2 - 1)} = \frac{a(3\pi - 8)}{\pi(\pi - 2)}.$$

5-51





$V = 30 \times 60 \times 40\ \mathrm{mm}^3$

$\bar{x} = -15 \quad \bar{y} = 0 \quad \bar{z} = 20$

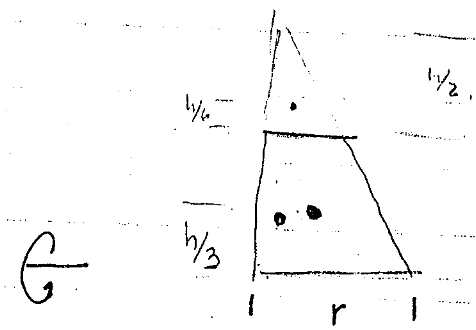
$V = \dfrac{\pi r^2 h}{2} = \dfrac{\pi (20)^2 (30)}{2}$

$\bar{x} = -\dfrac{4r}{3\pi} \quad \bar{z} = 25 \quad \bar{y} = 0$

$$\bar{x} = \Sigma \frac{-15\left[72\times10^3\right] + \frac{4(20)}{3\pi}\cdot\frac{\pi}{2}\left(12\times10^3\right)}{72\times10^3 - \frac{\pi}{2}\left(12\times10^3\right)} = -17.31 \text{ mm}$$

$$\bar{z} = \Sigma \frac{20\left(72\times10^3\right) - 25\left(\pi\times\frac{12}{2}\times10^3\right)}{72\times10^3 - \pi\left(6\times10^3\right)} = 18.23 \text{ mm.}$$

Pappus & Guldinus



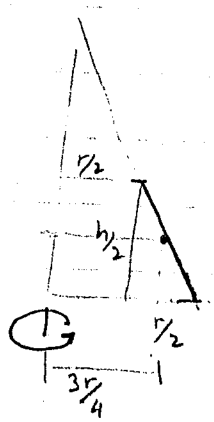
$\bar{y}$	A
$h/3$	$A_{BIG} = \frac{rh}{2}$
$2h/3.$	$A_{small} = \frac{\frac{r}{2}\cdot\frac{h}{2}}{2} = \frac{rh}{8}$

$$\frac{\frac{4rh^2}{24} - \frac{2rh^2}{24}}{\frac{3rh}{8}} = \frac{2rh^2/24}{9rh^3/24}$$

$V = 2\pi\bar{y}A$

$= 2\pi\cdot\frac{2h}{9}\cdot\left[\frac{3rh}{8}\right]$

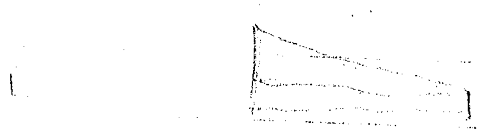
$$\bar{y} = \frac{\frac{rh}{2}\cdot\frac{h}{3} - \frac{rh}{8}\cdot\frac{2h}{3}}{\frac{rh}{2} + \frac{rh}{8}} = \frac{2}{9}h.$$



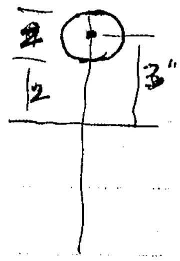
$$\frac{1}{2}\sqrt{r^2+h^2} = L$$

$$S \quad 2\pi\bar{r}L = \cancel{2}\pi\cdot\frac{3r}{4}\cdot\frac{1}{\cancel{2}}\sqrt{r^2+h^2}$$

$$= \frac{3\pi r}{4}\sqrt{r^2+h^2}$$



9-53



$S = 2\pi \bar{r} L$ $\qquad$ $L = \text{circum} = 2\pi R = 2\pi \cdot 1 = 2\pi$ ''

$\bar{r} = 3''$

$S = 2\pi \cdot 3 \cdot 2\pi = 12\pi^2 \text{ in}^2 = 118.4 \text{ in}^2$

$V = 2\pi \bar{r} A$ $\qquad$ $A = \pi R^2 = \pi 1^2 = \pi \text{ in}^2$

$V = 2\pi\, 3'' \cdot \pi \text{ in}^2 = 6\pi^2 \text{ in}^3 = 59.2 \text{ in}^3.$

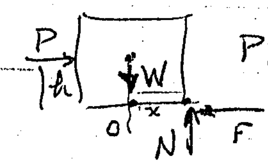
HW # 9-51, 9-47, [illegible], 9-81

Review for exam. Ch 6 & § 8.1, 8.2

TOPICS – Joints / Sections } Methods

Frames.

Friction < slipping / tipping

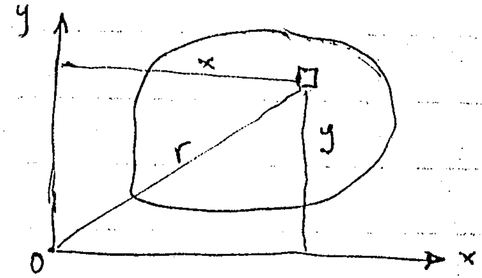


$Ph = Wx = Nx.$

Do handout

SESSION #25

- CENTROID – was first moment of ~~[illegible]~~ area $\bar{x}, \bar{y} \sim \int x\,dA, \int y\,dA$
- CERTAIN CASES need to eval second moment $\int x^2 dA, \int y^2 dA$



$dI_x = y^2 dA$ $\qquad$ $I_x = \int y^2 dA$

$dI_y = x^2 dA$ $\qquad$ $I_y = \int x^2 dA$

$dI_o = r^2 dA$ $\qquad$ $I_o = \int r^2 dA$

$I_o = I_x + I_y$ $\qquad$ $r^2 = x^2 + y^2$

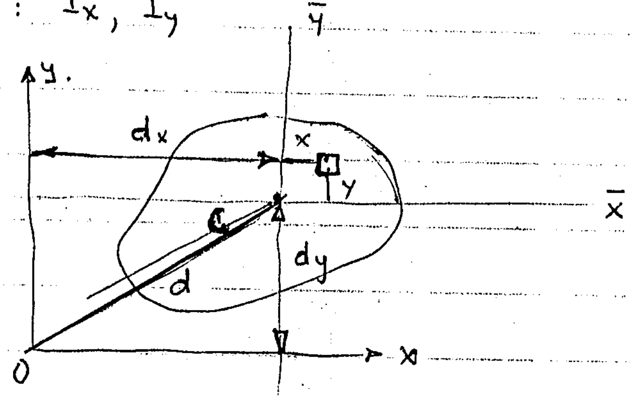
- note I_x, I_y, J_o are always ≥ 0 $\qquad J_o$ polar moment of inertia

- units always $(\text{length})^4$

- PARALLEL AXIS THEOREM

Given: $\bar{I}_{\bar{x}}, \bar{I}_{\bar{y}}$ the moments of inertia about the centroid

Find: I_x, I_y



$$I_x = \int (y+d_y)^2 dA = \int (y^2 + 2y d_y + d_y^2)\, dA = \int y^2 dA + 2 d_y \cancel{\int y\, dA}^{\,0} + d_y^2 \int dA$$
$$= \bar{I}_{\bar{x}} + 0 + A d_y^2$$

$$I_y = \int (x+d_x)^2 dA = \int x^2 dA + 2 d_x \cancel{\int x\, dA}^{\,0} + d_x^2 \int dA$$
$$= \bar{I}_{\bar{y}} + A d_x^2$$

$$J_o = I_x + I_y = \bar{I}_{\bar{x}} + \bar{I}_{\bar{y}} + A(d_x^2 + d_y^2)$$
$$= \bar{J}_c + A d^2 \qquad d^2 = d_x^2 + d_y^2$$

THE MOMENT OF INERTIA OF AN AREA ABOUT AN AXIS IS EQUAL TO THE AREA'S MOMENT OF INERTIA ABOUT A PARALLEL AXIS THROUGH THE AREA'S CENTROID + THE PRODUCT OF THE AREA AND THE SQUARE OF THE $\perp$ DISTANCE BETWEEN THE AXES

$$I = \int x^2 dA = d^2 A \qquad d = \sqrt{\frac{I}{A}}$$

רדיוס האינרציה $\quad I_y = A r_y^2$

RADIUS OF GYRATION : GIVEN I_x, I_y, J_o,

$$k_y = \sqrt{\frac{I_y}{A}} \qquad \cancel{i = x, y, 0} \qquad k_x = \sqrt{\frac{I_x}{A}}, \quad k_o = \sqrt{\frac{J_o}{A}}$$

WHEN AREAS ARE GIVEN BY FUNCTIONS YOU CAN INTEGRATE ~~THEM~~ $\int y^2 dA$
$\int x^2 dA$

PROBLEM 10-2

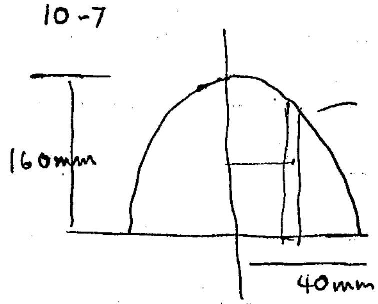
$\bar{J}_C = 23 \text{ in}^4$ given $I_y = 5 \text{ in}^4$ and $I_x' = 40 \text{ in}^4$

$$\bar{J}_C = I_y + I_x = 23 = 5 + I_x \qquad I_x = 18 \text{ in}^4$$

$$I_{x'} = I_x + A d^2 \Rightarrow 40 = 18 + A(3)^2 \qquad A = \frac{22 \text{ in}^4}{9 \text{ in}^2}$$

$$A = 2.44 \text{ in}^2$$

PROBLEM 10-7



find: $k_y \Rightarrow I_y, A$.

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$I_y = \int_{-40}^{40} x^2 dA = \int x^2 (y\,dx)$$
$$= \int x^2 (.1)(1600 - x^2) dx$$
$$= .1 \int (1600x^2 - x^4)$$
$$= .1 \left[1600\frac{x^3}{3} - \frac{x^5}{5} \right]_{-40}^{40}$$
$$= 2730666.7 \text{ mm}^4$$

$$dA = y\,dx$$

$$\int dA = \int y\,dx = \int .1\,(1600 - x^2) dx$$
$$= .1 \int (1600 - x^2) dx$$
$$= .1 \left[1600x - \frac{x^3}{3} \right]_{-40}^{40} = .1 \left[85333.33 \right] = 8533.33$$

$$k_y = \sqrt{\frac{I_y}{A}} = 17.89 \text{ mm}$$

MOMENTS OF INERTIA OF COMPOSITE AREAS

- FOR EACH FIND THE MOMENT OF INERTIA ABOUT A COMMON AXIS
- TAKE THE ALGEBRAIC SUM
- IF ONE AREA IS A "HOLE" SUBTRACT

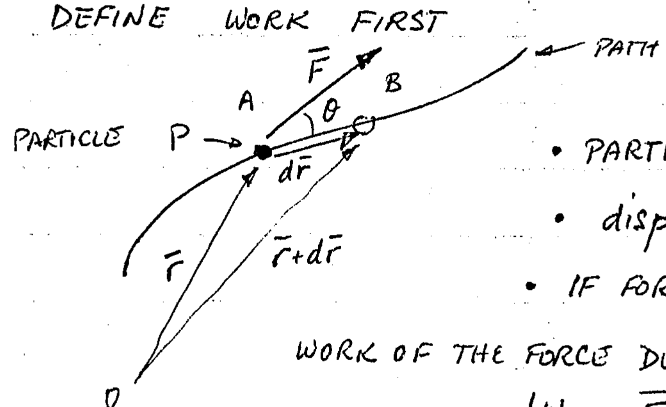
IF NOT CENTROIDAL AXIS USE $I = \bar{I} + Ad^2$

TALK ABOUT PROBLEMS ON Pg 379-380

SESSION #26

- IN CHAPTER 5, 6 & 8: SOLVED PROBLEMS BY MEANS OF EQUIL. EQS.
- FOR FRAMES & MACHINES – WE HAD TO DISMEMBER STRUCTURE TO SOLVE FOR UNKNOWN EXAMPLE PROB 8-17 Pg 299
- HOWEVER WE CAN SOLVE PROBLEMS BY
  - METHOD OF POTENTIAL ENERGY
  - PRINCIPLE OF VIRTUAL WORK
- THIS GIVES RESULTS DIRECTLY W/O DISMEMBERING SYSTEM.
- POTENTIAL ENERGY WILL ALLOW US TO ~~SAY~~ EXPLAIN THE IDEA OF EQUILIBRIUM

- DEFINE WORK FIRST

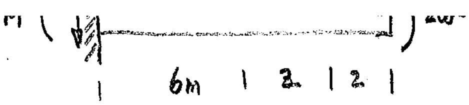


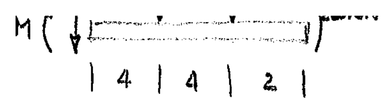
- PARTICLE MOVES FROM PT A TO B
- displacement = $d\bar{r}$
- IF FORCE $\bar{F}$ ACTS ON PARTICLE

WORK OF THE FORCE DUE TO THIS DISPLACEMENT

$$dU = \bar{F} \cdot d\bar{r} \quad \text{scalar}$$
$$= F ds \cos\theta$$

IF $\theta < \pi/2$ $dU > 0$ $\theta = \pi/2$ $dU = 0$ $\theta > \pi/2$ $dU < 0$

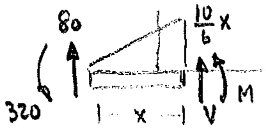




$\Sigma F_y = -V - 80\text{ kN} \qquad V = -80\text{ kN}$

$+\curvearrowleft \Sigma M = -M - 30\cdot 4 - 50\cdot 8 + 200 = 0 \qquad M = -320\text{ kN-m}$

$F = \int \frac{10}{6} x\,dx = \frac{10x^2}{12}$

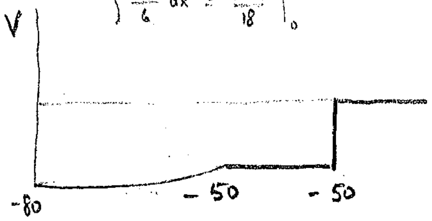


$\Sigma F = V + 80 - \frac{10}{6}\frac{x^2}{2} = \quad V = -80 + \frac{5}{6}x^2$

$\curvearrowleft \Sigma M_0 = 320 + M - \frac{10x}{6}\cdot\frac{x}{2}\cdot\frac{2x}{3} + Vx = 0; \quad M = -320 + \frac{10}{18}x^3 - Vx = -320 + 80x - \frac{5}{18}x^3$

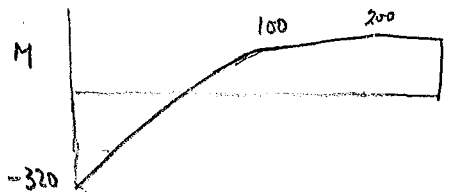
$M = \int \frac{10x^2}{6}dx = \frac{10x^3}{18}\Big|_0^x$

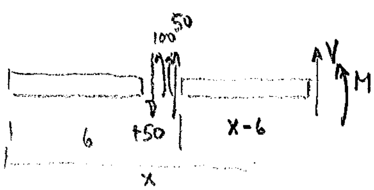
$-320 + 480 - \frac{5}{18}(6^2\cdot 6)$



$V = -\frac{dM}{dx} \qquad M = +80x - \frac{5}{18}x^3 + C \qquad M\big|_{x=0} = -320$

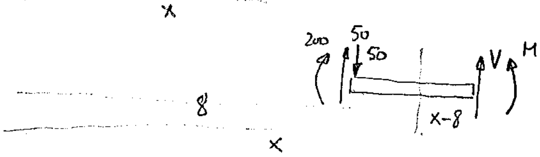
$M = 80x - \frac{5}{18}x^3 - 320$





$\Sigma F_y = V + 50 = 0 \qquad V = -50$

$\curvearrowleft \Sigma M \quad M - 100 - 50(x-6) = 0 \qquad M = 100 + 50(x-6) =$



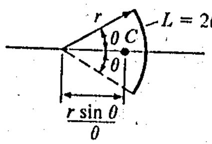
$\Sigma F_y = 50 - 50 + V \quad V = 0$

$\Sigma M = -200 + \cancel{V}(x-8) + M = 0$

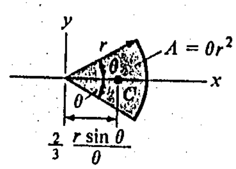
$M = 200$

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia

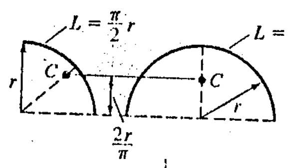


Circular arc segment

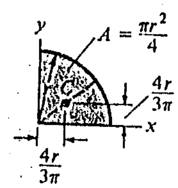


Circular sector area

$$I_x = \tfrac{1}{4} r^4(\theta - \tfrac{1}{2}\sin 2\theta)$$
$$I_y = \tfrac{1}{4} r^4(\theta + \tfrac{1}{2}\sin 2\theta)$$

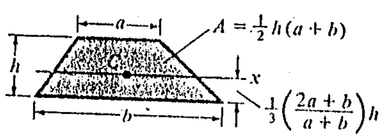


Quarter and semicircular arcs

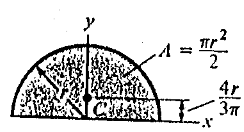


Quarter circular area

$$I_x = \tfrac{1}{16}\pi r^4$$
$$I_y = \tfrac{1}{16}\pi r^4$$

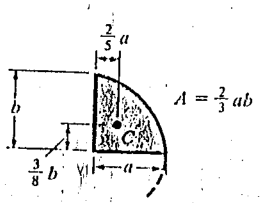


Trapezoidal area

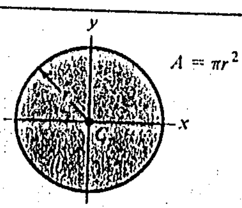


Semicircular area

$$I_x = \tfrac{1}{8}\pi r^4$$
$$I_y = \tfrac{1}{8}\pi r^4$$

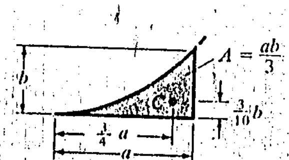


Semiparabolic area

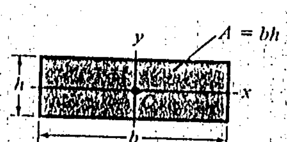


Circular area

$$I_x = \tfrac{1}{4}\pi r^4$$
$$I_y = \tfrac{1}{4}\pi r^4$$

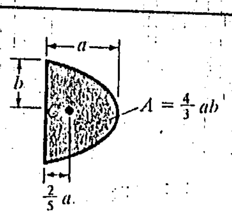


Exparabolic area

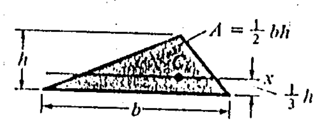


Rectangular area

$$I_x = \tfrac{1}{12} bh^3$$
$$I_y = \tfrac{1}{12} hb^3$$

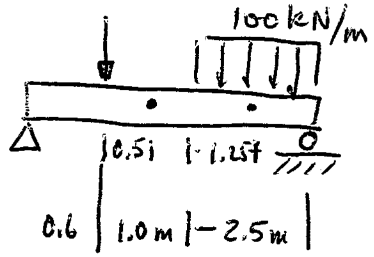


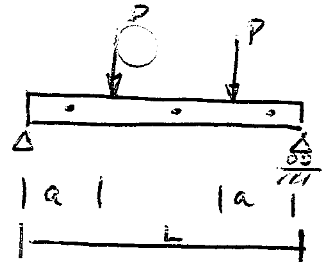
Parabolic area

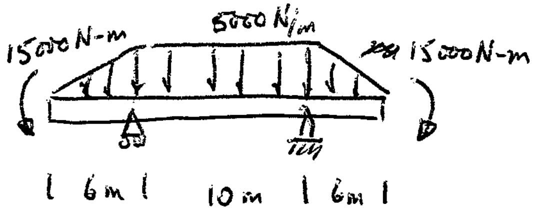


Triangular area

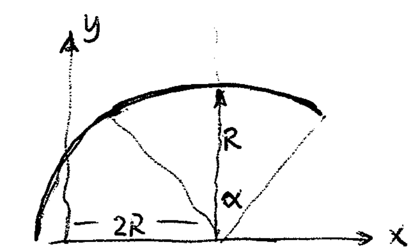
$$I_x = \tfrac{1}{36} bh^3$$



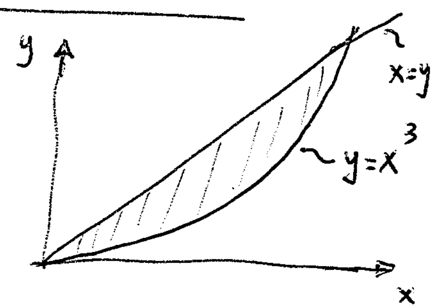




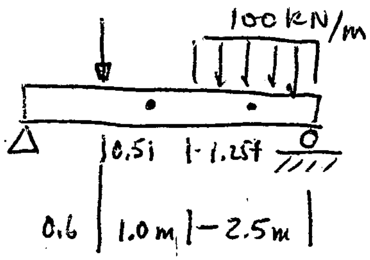
שרטטו את מהלך מומנט וכוח גזירה.
בכל נקודה (•) מצאו את N,V,M

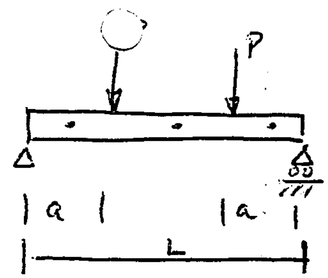


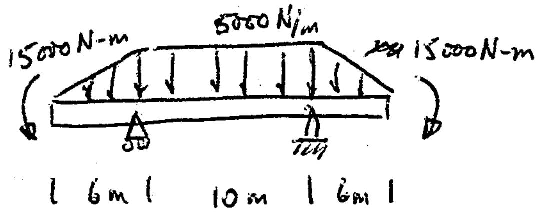
מצאו את משטח כא.
מסובבים את הקשת
ציר x



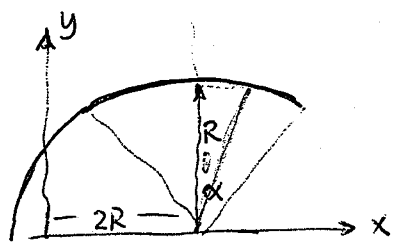
מצאו מרכז הכובד
והנפח של השטח המקווקו
כאשר מסובבים אותו סביב
ציר x וציר y







שרטטו את מהלך מומנט וכוח גזירה.
בכל נקודה (•) מצאו את N,V,M

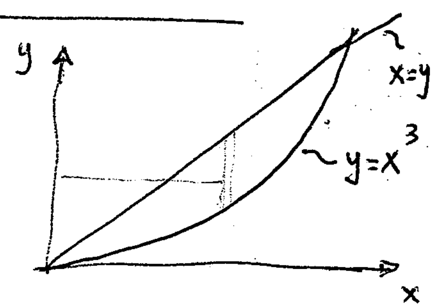


מצאו את משטח כאן,
מסובבים את הקשת ציר x

$$\bar{y} = \frac{\int y\,dL}{\int dL} = \frac{\int_{-\alpha}^{\pi/2} R\cancel{\cos}\theta \cdot R\,d\theta}{\int_{-\alpha}^{\pi/2} R\,d\theta}$$

$$+R^2 \cancel{\sin}\theta \Big|_{-\alpha}^{\pi/2} = \frac{R^2 (1+\sin\alpha)}{R(\pi/2+\alpha)} = \frac{R(1+\sin\alpha)}{\pi/2+\alpha}$$

$$S = 2\pi \bar{y} L = 2\pi \frac{R(1+\sin\alpha)}{\cancel{\pi/2+\alpha}} \cdot R(\cancel{\pi/2+\alpha}) = 2\pi R^2 (1+\sin\alpha)$$



מצאו מרכז הכובד
והנפח של השטח המקווקו
כאשר מסובבים אותו מסביב
ציר x וציר y

$x=0\ \ y=0$
$x=1\ \ y=1$

$$\frac{\int x\,dA}{\int dA}$$

Cable Subjected to a Distributed Load. Consider the weightles cable shown in Fig. 7–22*a*, which is subjected to a loading function $w = w(x$ *as measured in the x direction*. The free-body diagram of a small segment o the cable having a length Δs is shown in Fig. 7–22*b*. Since the tensile force i the cable changes continuously in both magnitude and direction along th cable's length, this change is denoted on the free-body diagram by $\Delta \mathbf{T}$. Th distributed load is represented by its resultant force $w(x)(\Delta x)$, which acts at fractional distance $k(\Delta x)$ from point O, where $0 < k < 1$. Applying the equa tions of equilibrium yields

$$\xrightarrow{+} \Sigma F_x = 0; \quad -T \cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad -T \sin\theta - w(x)(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \quad w(x)(\Delta x)k(\Delta x) - T\cos\theta\,\Delta y + T\sin\theta\,\Delta x = 0$$

Dividing each of these equations by Δx and taking the limit as $\Delta x \to 0$, and hence $\Delta y \to 0$, $\Delta\theta \to 0$, and $\Delta T \to 0$, we obtain

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad (7\text{–}7)$$

$$\frac{d(T\sin\theta)}{dx} - w(x) = 0 \qquad (7\text{–}8)$$

$$\frac{dy}{dx} = \tan\theta \qquad (7\text{–}9)$$

Integrating Eq. 7–7, we have

$$T\cos\theta = \text{constant} = F_H \qquad (7\text{–}10)$$

Here F_H represents the horizontal component of tensile force at *any point* along the cable.

Integrating Eq. 7–8 gives

$$T\sin\theta = \int w(x)\,dx \qquad (7\text{–}11)$$

Dividing Eq. 7–11 by Eq. 7–10 eliminates T. Then, using Eq. 7–9, we can obtain the slope

$$\tan\theta = \frac{dy}{dx} = \frac{1}{F_H}\int w(x)\,dx$$

ɒ **a Distributed Load.** Consider the weightless '2a, which is subjected to a loading function $w = w(x)$ *rection*. The free-body diagram of a small segment of h Δs is shown in Fig. 7–22*b*. Since the tensile force in inuously in both magnitude and direction along the nge is denoted on the free-body diagram by $\Delta \mathbf{T}$. The sented by its resultant force $w(x)(\Delta x)$, which acts at a :) from point O, where $0 < k < 1$. Applying the equa- elds

$\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$

$\theta - w(x)(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$

$x)k(\Delta x) - T\cos\theta\,\Delta y + T\sin\theta\,\Delta x = 0$

equations by Δx and taking the limit as $\Delta x \rightarrow 0$, and), and $\Delta T \rightarrow 0$, we obtain

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad (7\text{–}7)$$

$$\frac{d(T\sin\theta)}{dx} - w(x) = 0 \qquad (7\text{–}8)$$

$$\frac{dy}{dx} = \tan\theta \qquad (7\text{–}9)$$

'e have

$$T\cos\theta = \text{constant} = F_H \qquad (7\text{–}10)$$

: horizontal component of tensile force at *any point*

} gives

$$T\sin\theta = \int w(x)\,dx \qquad (7\text{–}11)$$

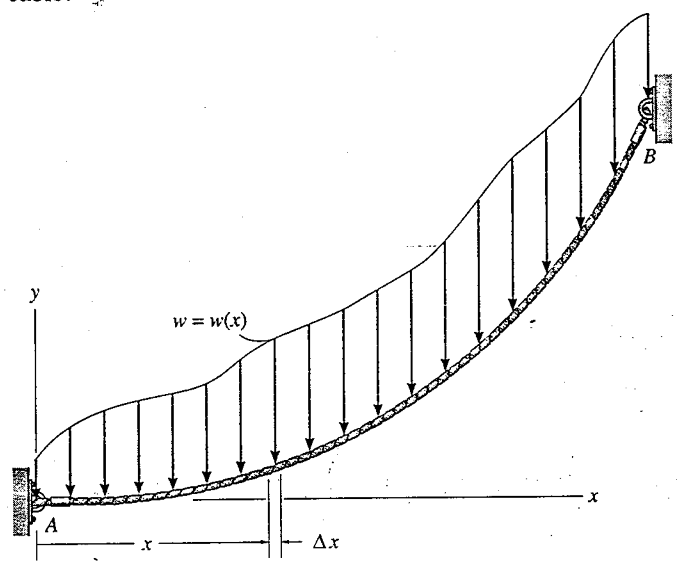
Eq. 7–10 eliminates T. Then, using Eq. 7–9, we can

$$\tan\theta = \frac{dy}{dx} = \frac{1}{F_H}\int w(x)\,dx$$

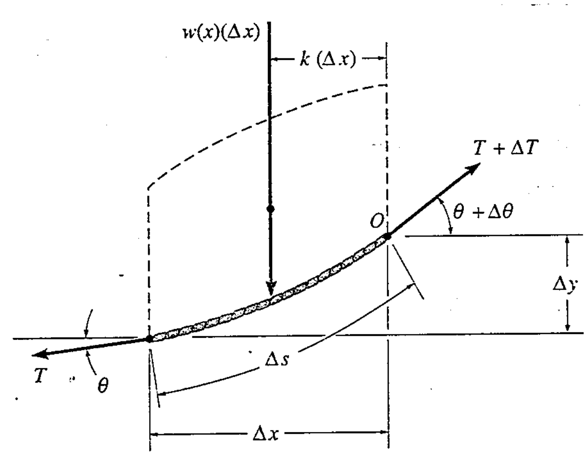
Performing a second integration yields

$$\boxed{y = \frac{1}{F_H}\int\left(\int w(x)\,dx\right)dx} \qquad (7\text{–}12)$$

This equation is used to determine the curve for the cable, $y = f(x)$. The horizontal force component F_H and the two constants, say C_1 and C_2, resulting from the integration are determined by applying the boundary conditions for the cable.



(a)

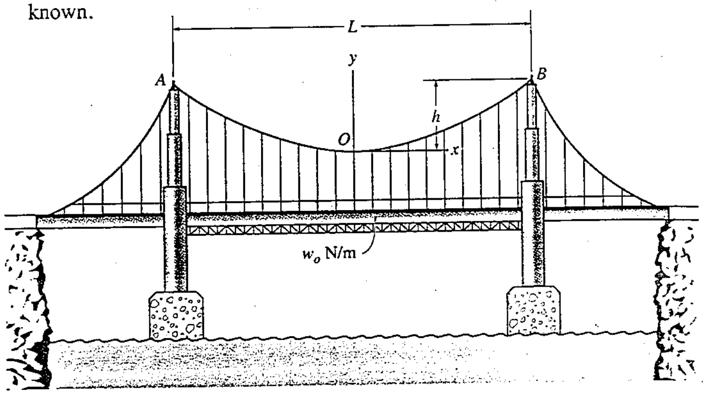


(b)

Fig. 7–22

7-14

The cable of a suspension bridge supports half of the uniform road surface between the two columns at A and B, as shown in Fig. 7–23a. If this loading is w_o N/m, determine the maximum force developed in the cable and the cable's required length. The span length L and sag h are known.



(a)

Fig. 7–23

SOLUTION

We can determine the unknowns in the problem by first finding the curve that defines the shape of the cable by using Eq. 7–12. For reasons of symmetry, the origin of coordinates has been placed at the cable's center. Noting that $w(x) = w_o$, we have

$$y = \frac{1}{F_H}\int\left(\int w_o\,dx\right)dx$$

Integrating this equation twice gives

$$y = \frac{1}{F_H}\left(\frac{w_o x^2}{2} + C_1 x + C_2\right) \tag{1}$$

The constants of integration may be determined by using the boundary conditions $y = 0$ at $x = 0$ and $dy/dx = 0$ at $x = 0$. Substituting into Eq. 1 yields $C_1 = C_2 = 0$. The curve then becomes

$$y = \frac{w_o}{2F_H}x^2 \tag{2}$$

This is the equation of a *parabola*. The constant F_H may be obtained by using the boundary condition $y = h$ at $x = L/2$. Thus,

$$F_H = \frac{w_o L^2}{8h} \tag{3}$$

Therefore, Eq. 2 becomes

$$y = \frac{4h}{L^2}x^2 \tag{4}$$

Since F_H is known, the tension in the cable may be determined using Eq. 7–10, written as $T = F_H/\cos\theta$. For $0 \le \theta < \pi/2$, the maximum tension will occur when θ is *maximum*, i.e., at point B, Fig. 7–23a. From Eq. 2, the slope at this point is

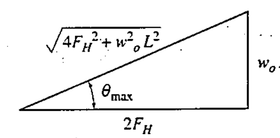
$$\left.\frac{dy}{dx}\right|_{x=L/2} = \tan\theta_{\max} = \left.\frac{w_o}{F_H}x\right|_{x=L/2}$$

or

$$\theta_{\max} = \tan^{-1}\left(\frac{w_o L}{2F_H}\right) \tag{5}$$

Therefore,

$$T_{\max} = \frac{F_H}{\cos(\theta_{\max})} \tag{6}$$



(b)

Using the triangular relationship shown in Fig. 7–23b, which is based on Eq. 5, Eq. 6 may be written as

$$T_{\max} = \frac{\sqrt{4F_H^2 + w_o^2 L^2}}{2}$$

Substituting Eq. 3 into the above equation yields

$$T_{\max} = \frac{w_o L}{2}\sqrt{1 + \left(\frac{L}{4h}\right)^2} \qquad \textit{Ans.}$$

For a differential segment of cable length ds, we can write

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Hence, the total length of the cable, $\mathcal{L}$, can be determined by integration. Using Eq. 4, we have

$$\mathcal{L} = \int ds = 2\int_0^{L/2}\sqrt{1 + \left(\frac{8h}{L^2}x\right)^2}\,dx \tag{7}$$

Integrating and substituting the limits yields

$$\mathcal{L} = \frac{L}{2}\left[\sqrt{1 + \left(\frac{4h}{L}\right)^2} + \frac{L}{4h}\sinh^{-1}\left(\frac{4h}{L}\right)\right] \qquad \textit{Ans.}$$

Cable Subjected to Its Own Weight. When the weight of the cable becomes important in the force analysis, the loading function along the cable becomes a function of the arc length s rather than the projected length x. A generalized loading function $w = w(s)$ acting along the cable is shown in Fig. 7–24*a*. The free-body diagram for a segment of the cable is shown in Fig. 7–24*b*. Applying the equilibrium equations to the force system on this diagram, one obtains relationships identical to those given by Eqs. 7–7 through 7–9, but with ds replacing dx. Therefore, it may be shown that

$$T\cos\theta = F_H \qquad (7\text{–}13)$$

$$T\sin\theta = \int w(s)\,ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}\int w(s)\,ds \qquad (7\text{–}14)$$

To perform a direct integration of Eq. 7–14, it is necessary to replace dy/dx by ds/dx. Since

$$ds = \sqrt{dx^2 + dy^2}$$

then

$$\frac{dy}{dx} = \sqrt{\left(\frac{ds}{dx}\right)^2 - 1}$$

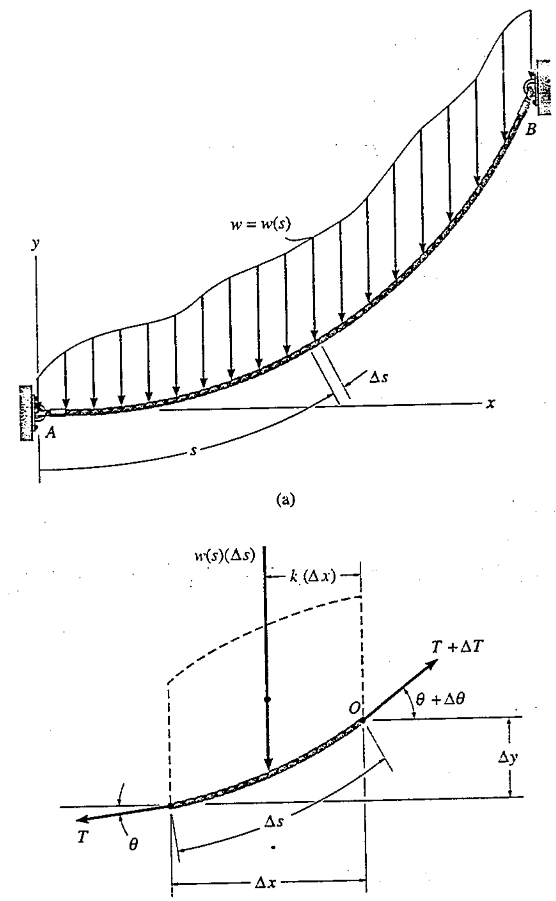
Therefore,

$$\frac{ds}{dx} = \left\{1 + \frac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right\}^{1/2}$$

Separating the variables and integrating yields

$$\boxed{x = \int \frac{ds}{\left\{1 + \dfrac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right\}^{1/2}}} \qquad (7\text{–}15)$$

The two constants of integration, say C_1 and C_2, are found using the boundary conditions for the cable.



(a)

(b)

Fig. 7–24

Example 7–15

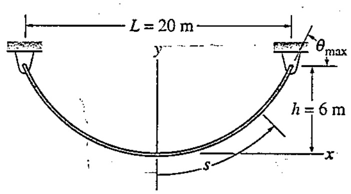


Fig. 7–25

Determine the deflection curve, the length, and the maximum tension in the uniform cable shown in Fig. 7–25. The cable weighs $w_o = 5$ N/m.

SOLUTION

By reasons of symmetry, the origin of coordinates is located at the center of the cable. The deflection curve is expressed as $y = f(x)$. We can determine it by first applying Eq. 7–15, where $w(s) = w_o$.

$$x = \int \frac{ds}{[1 + (1/F_H^2)(\int w_o\, ds)^2]^{1/2}}$$

Integrating the term under the integral sign in the denominator, we have

$$x = \int \frac{ds}{[1 + (1/F_H^2)(w_o s + C_1)^2]^{1/2}}$$

Substituting $u = (1/F_H)(w_o s + C_1)$ so that $du = (w_o/F_H)\, ds$, a second integration yields

$$x = \frac{F_H}{w_o}(\sinh^{-1} u + C_2)$$

or

$$x = \frac{F_H}{w_o}\left\{\sinh^{-1}\left[\frac{1}{F_H}(w_o s + C_1)\right] + C_2\right\} \tag{1}$$

To evaluate the constants note that, from Eq. 7–14,

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_o\, ds \qquad \text{or} \qquad \frac{dy}{dx} = \frac{1}{F_H}(w_o s + C_1)$$

Since $dy/dx = 0$ at $s = 0$, then $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_o s}{F_H} \tag{2}$$

The constant C_2 may be evaluated by using the condition $s = 0$ at $x = 0$ in Eq. 1, in which case $C_2 = 0$. To obtain the deflection curve, solve for s in Eq. 1, which yields

$$s = \frac{F_H}{w_o}\sinh\left(\frac{w_o}{F_H}x\right) \tag{3}$$

Now substitute into Eq. 2, in which case

$$\frac{dy}{dx} = \sinh\left(\frac{w_o}{F_H}x\right)$$

Hence

$$y = \frac{F_H}{w_o}\cosh\left(\frac{w_o}{F_H}x\right) + C_3 \tag{4}$$

If the boundary condition $y = 0$ at $x = 0$ is applied, the constant $C_3 = -F_H/w_o$, and therefore the deflection curve becomes

$$y = \frac{F_H}{w_o}\left[\cosh\left(\frac{w_o}{F_H}x\right) - 1\right]$$

This equation defines the shape of a *catenary curve*. The constant F_H is obtained by using the boundary condition that $y = h$ at $x = L/2$, in which case

$$h = \frac{F_H}{w_o}\left[\cosh\left(\frac{w_o L}{2F_H}\right) - 1\right] \tag{5}$$

Since $w_o = 5$ N/m, $h = 6$ m, and $L = 20$ m, Eqs. 4 and 5 become

$$y = \frac{F_H}{5\text{ N/m}}\left[\cosh\left(\frac{5\text{ N/m}}{F_H}x\right) - 1\right] \tag{6}$$

$$6\text{ m} = \frac{F_H}{5\text{ N/m}}\left[\cosh\left(\frac{50\text{ N}}{F_H}\right) - 1\right] \tag{7}$$

Equation 7 can be solved for F_H by using a trial-and-error procedure. The result is

$$F_H = 45.8\text{ N}$$

and therefore the deflection curve, Eq. 6, becomes

$$y = 9.16[\cosh(0.109x) - 1]\text{ m} \qquad \textit{Ans.}$$

Using Eq. 3, with $x = 10$ m, the half-length of the cable is

$$\frac{\mathscr{L}}{2} = \frac{45.8\text{ N}}{5\text{ N/m}}\sinh\left[\frac{5\text{ N/m}}{45.8\text{ N}}(10\text{ m})\right] = 12.1\text{ m}$$

Hence,

$$\mathscr{L} = 24.2\text{ m} \qquad \textit{Ans.}$$

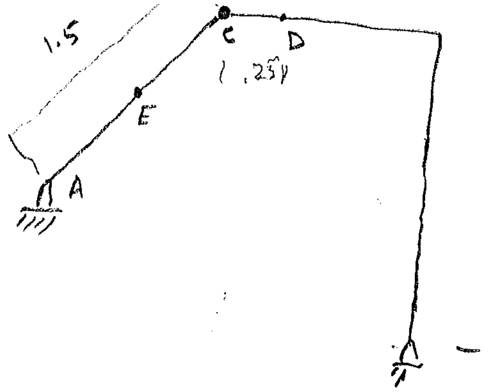
Since $T = F_H/\cos\theta$, Eq. 7–13, the maximum tension occurs when θ is maximum, i.e., at $s = \mathscr{L}/2 = 12.1$ m. Using Eq. 2 yields

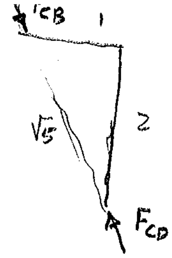
$$\left.\frac{dy}{dx}\right|_{s=12.1\text{ m}} = \tan\theta_{max} = \frac{5\text{ N/m}(12.1\text{ m})}{45.8\text{ N}} = 1.32$$

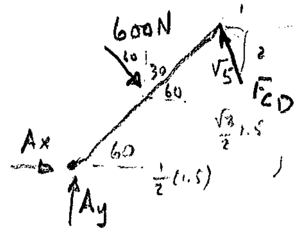
$$\theta_{max} = 52.9^\circ$$

Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{45.8\text{ N}}{\cos 52.9^\circ} = 75.9\text{ N} \qquad \textit{Ans.}$$







$$\Sigma F_x = -F_{CD}\cdot\frac{1}{\sqrt5} + 600\sin 60 + A_x = 0$$

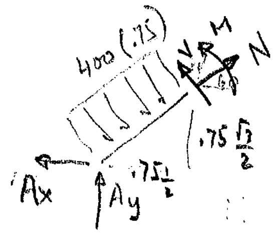
$$\Sigma F_y = F_{CD}\cdot\frac{2}{\sqrt5} - 600\cos 60 + A_y = 0$$

$$+\circlearrowleft \Sigma M_A = -600(.75) + F_{CD}\cdot\frac{2}{\sqrt5}\cdot\frac{1.5}{2} + F_{CD}\cdot\frac{1}{\sqrt5}\cdot\frac{\sqrt3}{2}1.5 = 0$$

$F_{CD} = 359.5\,N$ 310.6

$A_x = -359\,N$ -300

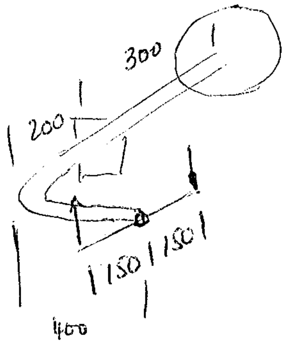
$A_y = 139.25\,N$ 80.4

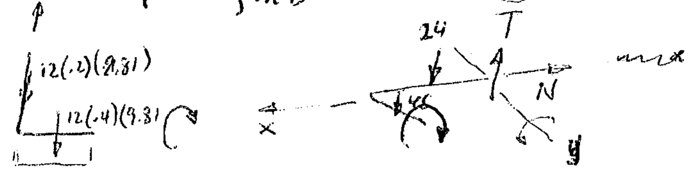


$$\Sigma M = M - A_x\left(.75\frac{\sqrt3}{2}\right) - A_y\left(.75/2\right) + 400(.75)(.375)$$

$$M = 285.7\ N\text{-}m$$

Pipe has mass of 12 kg/m, Fixed into wall at A. Find internal loads acting on B





$\Sigma F_x = 0 \quad N = 0$

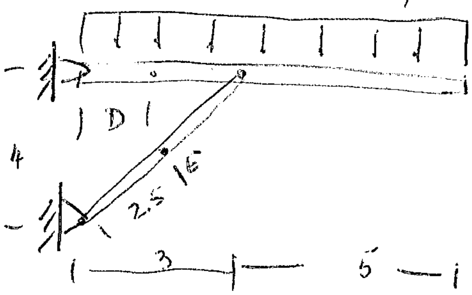
$\Sigma F_z = 0 \quad -96 + T = 0 \quad T = 96$

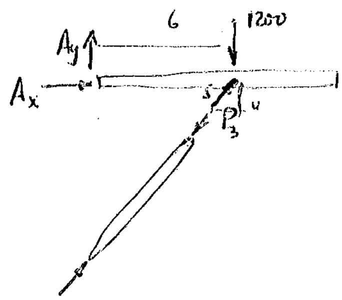
$\Sigma F_y = 0 \quad Z = 0$

$\Sigma M_z = 24(.1) + 48(.2) - 18 = 2.4 + 9.6 - 18 = -6$ N-m

$\Sigma M_x = 48(.2) = 9.6$ N-m

7-6 Find internal at D & E





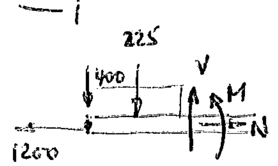
$A_y - 1200 + P \cdot \frac{4}{5} = 0$

$A_x + P \cdot \frac{3}{5} = 0$

$\cancel{1200} \, 400 \cdot \cancel{4} - P \cdot \frac{4}{5} \cdot \cancel{3} = 0$

$P = 2000 \quad A_y = -400$

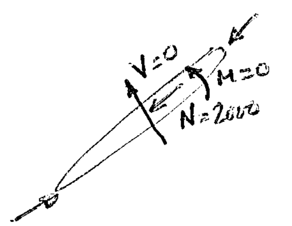
$A_x = -1200$



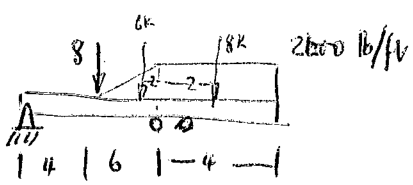
$N = 1200$

$V = 625$

$M = 225(.75) - 625(1.5) = -769$ lb-ft.



Pg 334 Ex 7-16 Draw Shear & Moment Diag.

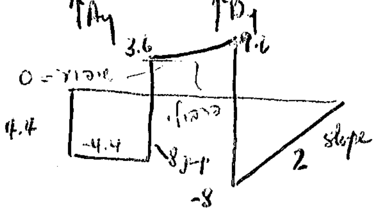


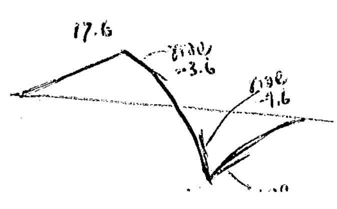
$A_y + B_y = \frac{2000 \cdot 6}{2} + 2000 \cdot 4 = \cancel{20000}$ +8000

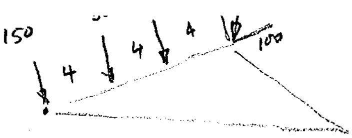
$B_y \cdot 10 - 6 \cdot 8 - 8 \cdot 12 = 0$ −8.4

$B_y = \cancel{17.6}\,k$

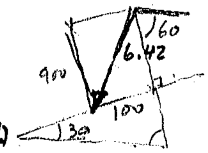
$A_y = 4.4k$



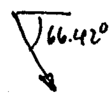




מצוא את השקול וממקם את השקול בנקודה שבקו AB



$\alpha = \tan \frac{900}{100} \cong 6.42°$ (2)



(1) $\Sigma F_x = 898.2$
(1) $\Sigma F_y = 100$ } $F_R = \sqrt{898.2^2 + 100^2} = 903.75$ lb (1)

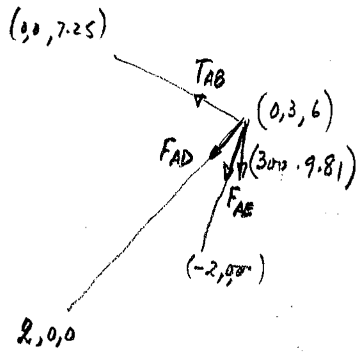
$\circlearrowright \Sigma M_A = \cancel{150 \cdot 0} + 300 \cdot 4 + 275 \cdot 8 + (173.2)12 = 898.3\, d.$ (4)

$4(300 + 550 + 520) = 898.2\, d$

$\frac{4(1370)}{898.2} \cong 6.10$ ft (1)

18

3.46



מערכת המוטות תומכת בסל ותכנים שלו. המסה של הסל ותכניו היא 300kg. מיצאו את הכוחות המפותחים בתמוכות (מוטות) ומתח הכבל לשיווי משקל. כוח במוט פועל בכיוון ציר המוט.

$\Sigma \underline{F} = \underline{F}_{AD} + \underline{F}_{AE} + \underline{T}_{AB} + \underline{W} = 0$

$\Sigma F_x = \frac{2}{7} F_{AD} - \frac{2}{7} F_{AE} + 0 = 0$ $\quad F_{AD} = F_{AE}$ 4

$\Sigma F_y = -\frac{3}{7} F_{AD} - \frac{3}{7} F_{AE} - \frac{3}{3.25} T_{AB} = 0 \Rightarrow -\frac{6}{7} F_{AD} - \frac{3}{3.25} T_{AB} = 0$ 4

$T_{AB} = -\frac{19.5}{21} F_{AD}$

$\underline{u}_{D/A} = \frac{\underline{r}_{D/A}}{r_{D/A}} = \frac{2i - 3j - 6k}{7}$ (5)

$\Sigma F_z = -\frac{6}{7} F_{AD} - \frac{6}{7} F_{AE} + \frac{1.25}{3.25} T_{AB} - 300 \cdot 9.81 = 0$ 5

$\underline{u}_{E/A} = \frac{\underline{r}_{E/A}}{r_{E/A}} = \frac{-2i - 3j - 6k}{7}$ (5)

$-\frac{12}{7} F_{AD} + \frac{1.25}{3.25}\left(-\frac{19.5}{21} F_{AD}\right) - 300 \cdot 9.81 = 0$

$-\frac{12}{7} F_{AD} - \frac{2.5}{21} F_{AD} = 300(9.81)$

$\underline{u}_{B/A} = \frac{\underline{r}_{B/A}}{r_{B/A}} = \frac{0i - 3j + 1.25k}{3.25}$ (5)

$-\frac{12}{7} F_{AD} - \frac{2.5}{7} F_{AD} = 300(9.81)$

$-\frac{14.5}{7} F_{AD} = 300(9.81)$

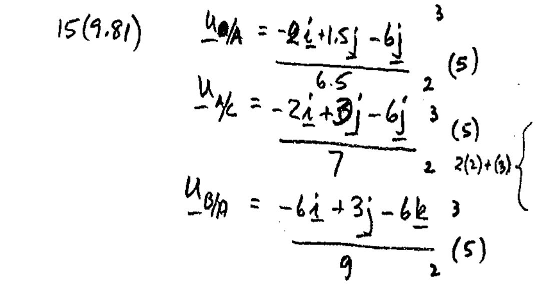
$F_{AD} = -\frac{2100(9.81)}{14.5} \cong -1421$ N 2

$F_{AE} = -1421$ N 2

$T_{AB} = 1319$ N 2

34

3-47



לאור יש מסה של 15kg ונתמך בשני כבלים AB, AC ובמוט AO. אם קו הפעולה של כוח המוט הוא לאורך ציר המוט, מצאו את הכוחות F_{AO}, F_{AB}, F_{AC} לשיווי משקל.

15(9.81)

$\underline{u}_{O/A} = \frac{-2i + 1.5j - 6k}{6.5}$ (5)

$\underline{u}_{A/C} = \frac{-2i + 3j - 6k}{7}$ (5)

$\underline{u}_{B/A} = \frac{-6i + 3j - 6k}{9}$ (5)

$-\frac{2}{6.5} F_{OA} - \frac{2}{7} F_{AC} - \frac{6}{9} F_{AB} = 0$ 4

$\frac{1.5}{6.5} F_{OA} + \frac{3}{7} F_{AC} + \frac{3}{9} F_{BA} = 0$ 4

$-\frac{6}{6.5} F_{OA} - \frac{6}{7} F_{AC} - \frac{6}{9} F_{BA} - 15(9.81) = 0$ 5

(1) + (2)·2 $\quad \left(-\frac{2}{6.5} + \frac{3}{6.5}\right) F_{OA} + \frac{4}{7} F_{AC} = 0$

$-\frac{7}{4}\left(\frac{1}{6.5}\right) F_{OA} = F_{AC} = +85.8$ N 2

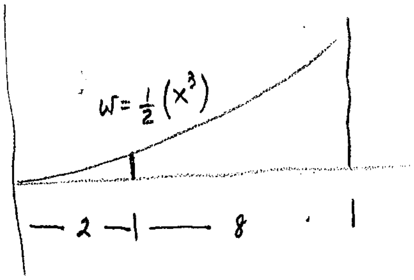
$F_{AB} = 110.4$ N 2

$\frac{9}{6}\left(-\frac{2}{6.5} F_{OA} - \frac{2}{7} F_{AC}\right) = F_{AB}$

34

2(2)+(3) = $-\frac{3}{6.5} F_{OA} - 15(9.81) = 0$

$F_{OA} = -\frac{6.5}{3}(15)(9.81) \cong -318.8$ N 2



רוח נשב חול על לוח ~~על כבי~~ כביש עוצמת העומס
נחשב בפונקציה $w=\frac{1}{2}x^3\frac{N}{m}$. החליפו את הכוח
המפורס בכוח שקול ומצאו את גודל השקול
ומקם את השקול מנקודה A על הקו AB. החול נמצא בין $x=2$ ל- $x=10m$.

$$F = \int_0^{10} w(x)\,dx = \frac{1}{2}\frac{x^4}{4}\Big|_2^{10} = 1250N - 2 = 1248 \quad 4$$

$$M = \int_0^{10} x\,w(x)\,dx = \frac{1}{2}\frac{x^5}{5}\Big|_2^{10} = \frac{100000}{10} - \frac{32}{10} = 10{,}000 - 3.2 \quad 4 = 9996.8$$

$$\bar{x} = \frac{M}{F} = 8$$

$$\frac{9996.8}{1248} = 8.01$$

$$\frac{2}{10}$$

$n=8$ $\quad \bar{x}=6.625 \quad \sigma=2.395$

$\bar{x}=26.25 \quad =8.60$

32.875 $\quad$ 9.71

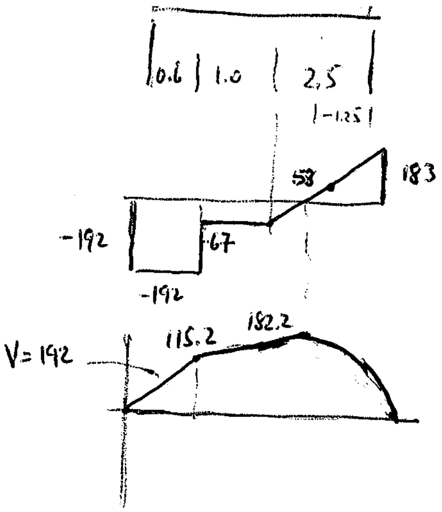
74.72

$\bar{x}=9.67 \quad \sigma=.67$

$\bar{x}=28 \quad \sigma=3.-$

36.56 $\quad$ 5.27

83.09

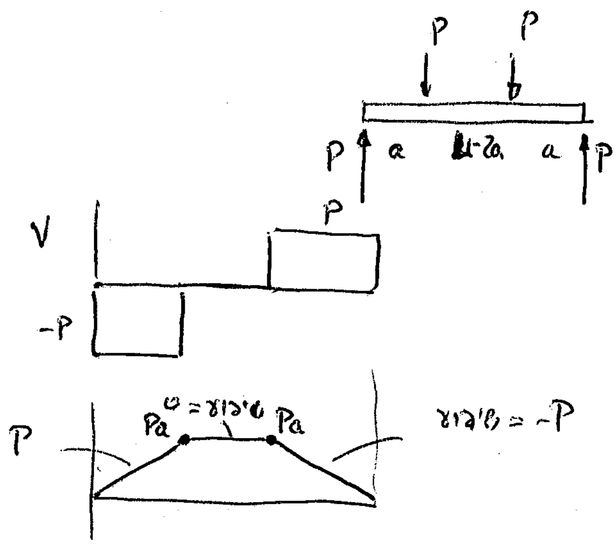


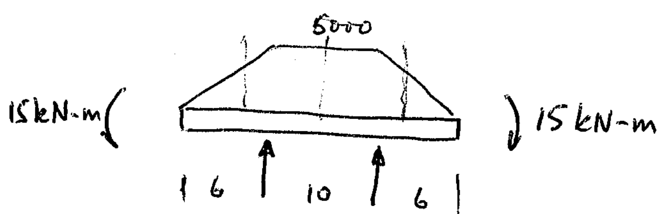
$125(.6) + 250(\cancel{2.845}) + R_B(4.1) = 0$

$R_B = 192.1 \qquad R_A = 183$

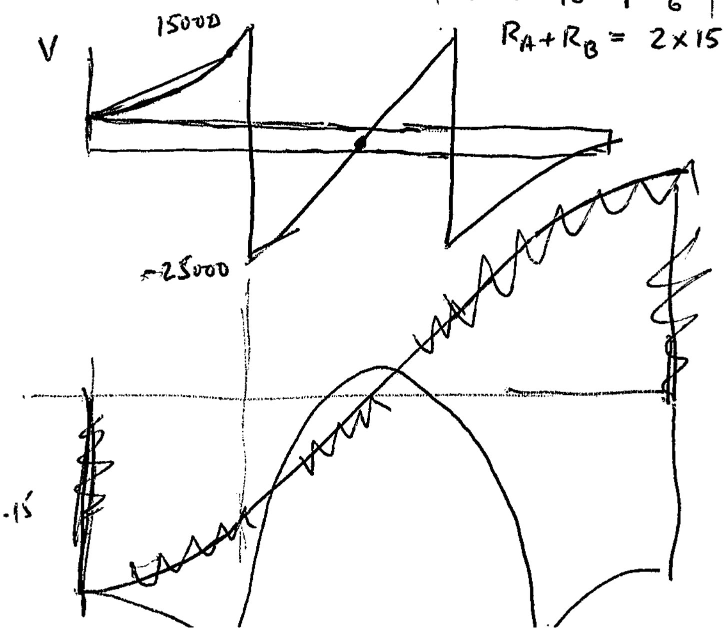
$M = -\int V dx = 192(.6) = 115.2$

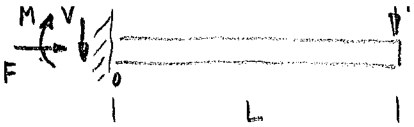
$M = -\int V dx = 67(1) + 115.2 = 182.2$





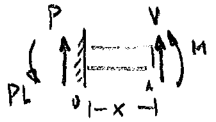
$R_A + R_B = 2 \times 15 + 50 = 0 \qquad R_A = 40\ \mathrm{kN}$





$+\!\uparrow \sum \hat{F}_y = -V - P = 0 \qquad V = -P$

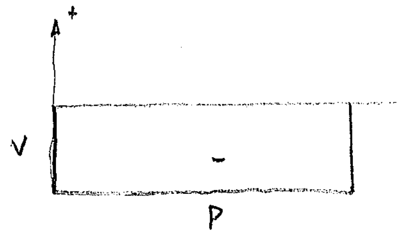
$+\!\circlearrowleft \sum M_0 = -PL - M = 0 \qquad M = -PL$

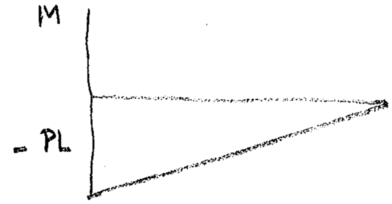


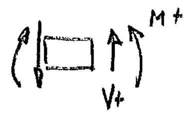
$\sum F_x = 0$

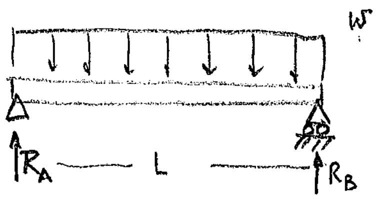
$\sum F_y = V + P = 0 \qquad V = -P$

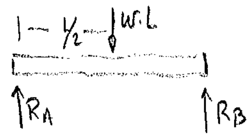
$+\!\circlearrowleft \sum M_0 = M + Vx + PL = 0 \quad M = -P(L-x) = P(x-L)$







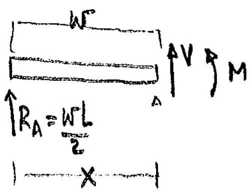




$\sum F_x = 0$

$\sum F_y = R_A + R_B - wL = 0$

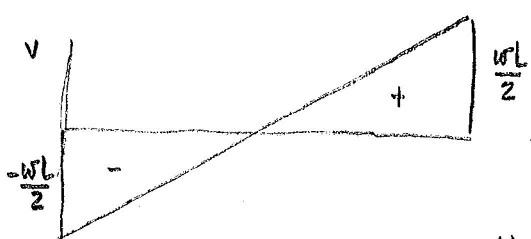
$+\!\circlearrowleft \sum M_A = R_B \cdot L - \dfrac{w \cdot L^2}{2} = 0 \quad R_B = \dfrac{wL}{2} \quad R_A = \dfrac{wL}{2}$

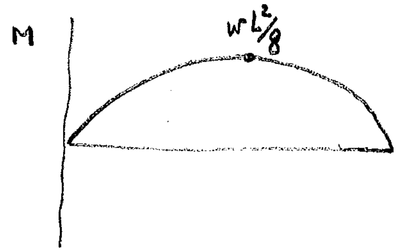


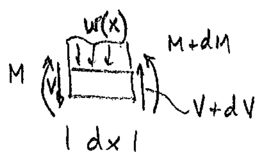
$\sum F_x = 0$

$\sum F_y = V - w \cdot x + \dfrac{wL}{2} = 0 \qquad V = w(x - L/2)$

$\sum M_A = M + w \cdot x \cdot \dfrac{x}{2} - \dfrac{wL}{2} \cdot x = 0 \quad M = \dfrac{w}{2}(Lx - x^2)$







$\sum F_y = V + dV - w(x)dx - V = 0 \qquad \boxed{\dfrac{dV}{dx} = w(x)}$

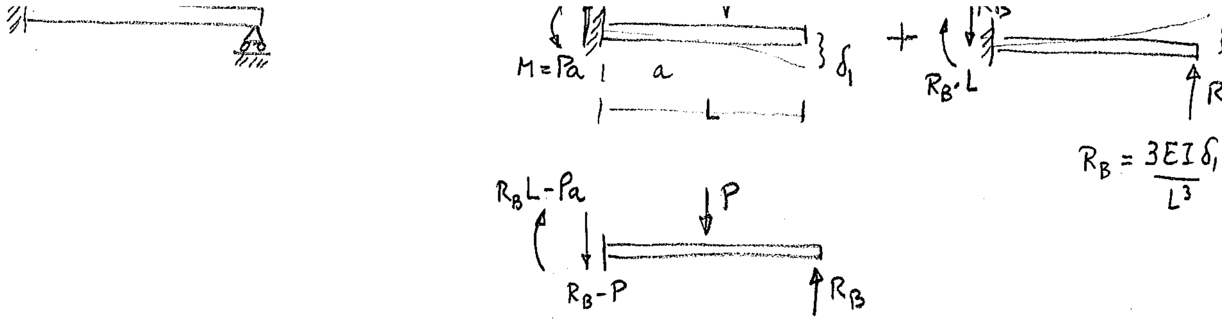
$\sum M = M + dM + (V + dV)dx - M - w(x)dx \cdot \dfrac{dx}{2} = 0$

$dM + V dx = 0 \qquad \boxed{V = -\dfrac{dM}{dx}}$

① $V = -P = -\dfrac{dM}{dx} \qquad M = Px + C \qquad M\big|_{x=0} = -PL \qquad C = -PL \qquad M = Px - PL = P(x-L)$

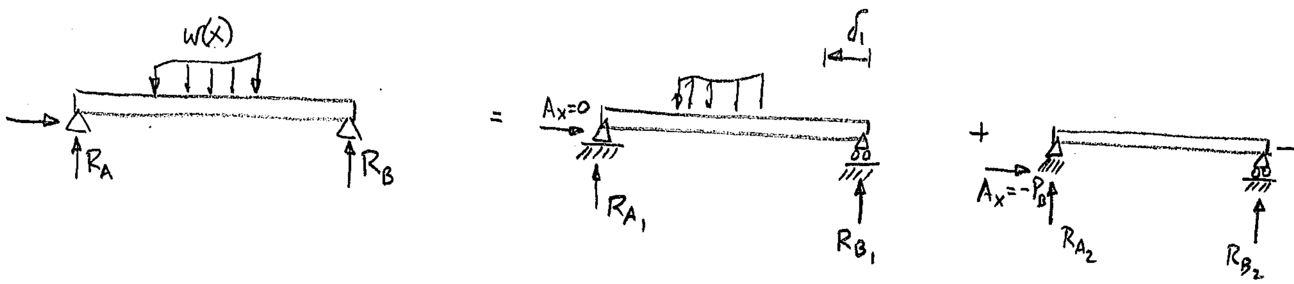
② $\dfrac{dV}{dx} = w \qquad V = wx + C \qquad V\big|_{x=0} = -\dfrac{wL}{2} \qquad C = -\dfrac{wL}{2} \qquad V = wx - \dfrac{wL}{2} = w(x - L/2)$

$V = -\dfrac{dM}{dx} \qquad M = -w\left(\dfrac{x^2}{2} - \dfrac{Lx}{2}\right) + C \qquad M\big|_{x=0} = 0 \qquad C = 0 \qquad M = \dfrac{w}{2}(Lx - x^2)$



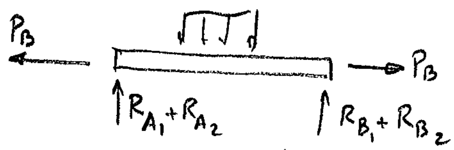
$$\delta = \frac{R_B L^3}{3IE}$$

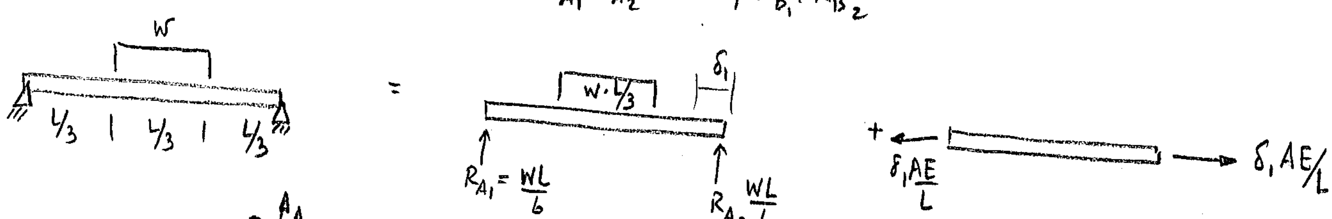
$$R_B = \frac{3EI\delta_1}{L^3}$$



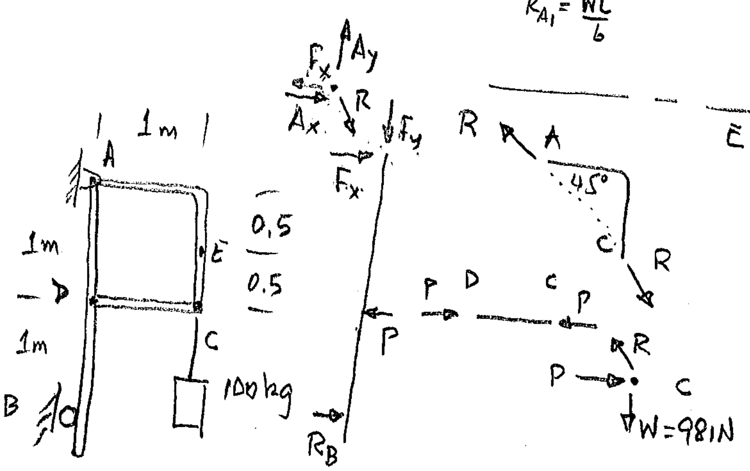
$$\delta = \frac{PL}{AE}$$

$$P_B = \frac{\delta_1 AE}{L}$$





מצאו את כוחות ומומנטים ב - E



@ C

$\Sigma F_y = +R\sin 45° - 981N = 0 \quad \underline{R = 1388N}$

$\Sigma F_x = P - R\sin 45° = 0 \quad \underline{981N = P}$

$-V + 1388\sin 45° = 0$

$\underline{V = 981N}$

$-N + 1388\cos 45° = 0$

$\underline{N = 981N}$

$+\circlearrowleft \; -M + V \cdot \frac{1}{2} = 0$

$\underline{M = 491 N\text{-}m}$

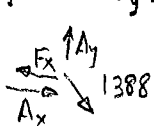
$\Sigma M_A \circlearrowleft + = R_B \cdot 2 - 981 \cdot 1 = 0 \quad R_B = 491N$

$\Sigma M_A \circlearrowleft + = R_B \cdot 2 - P \cdot 1 = 0 \quad P = 981N$

$\Sigma F_x = F_x + R_B - P = 0 \quad F_x = 491N$

$\Sigma F_y = 0 \quad F_y = 0$

בנקודה A



$A_x - F_x + R\cos 45° = 0$

$\underline{A_x = -491N}$

$A_y - R\sin 45° = 0$

$\underline{A_y = 981N}$

and bending-moment diagrams for the beam shown in

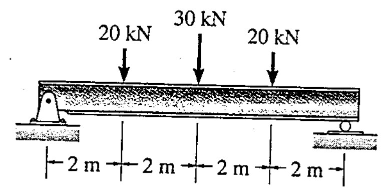
The reactions at the fixed support have been calcu-
ı on the free-body diagram of the beam, Fig. 7–17*b*.

Jsing the established sign convention, Fig. 7–11, the
f the beam is plotted first; i.e., $x = 0$, $V = +1080$;
Fig. 7–17*c*.
n distributed load is downward and *constant*, the slope
ı is $dV/dx = -w = -40$ for $0 \leq x \leq 12$ as indicated.
of shear at $x = 12$ is $V = +600$. This can be deter-
g the area under the load diagram between $x = 0$ and
nts the change in shear. That is, $\Delta V = -\int w(x)\,dx =$
Thus $V|_{x=12} = V|_{x=0} + (-480) = 1080 - 480 =$
obtain this value by using the method of sections,
or equilibrium $V = +600$.
ween $12 < x \leq 20$ is $w = 0$, the slope $dV/dx = 0$ as
s the shear to the required value of $V = +600$ at

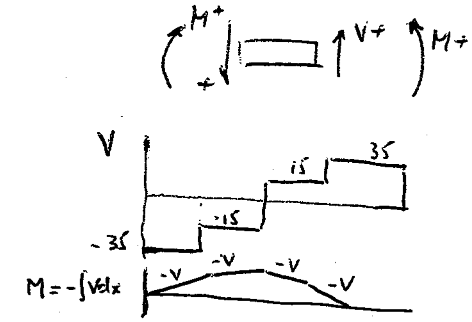
Again, using the established sign convention,
nts at the ends of the beam are plotted first; i.e.,
; $x = 20$, $M = -1000$, Fig. 7–17*d*.
ear gives the slope of the moment diagram since
ated, at $x = 0$, $dM/dx = +1080$; and at $x = 12$,
$< x < 12$, specific values of the shear diagram are
ecreasing. Hence, the moment diagram is parabolic
ng, positive slope.
noment at $x = 12$ ft is -5800. This can be found by
pezoidal area under the shear diagram, which repre-
nent, $\Delta M = \int V dx = 600(12) + \frac{1}{2}(1080 - 600)(12)$
$I|_{x=12} = M|_{x=0} + 10{,}080 = -15{,}880 + 10{,}080 =$
sic'' method of sections can also be used, where
equires $M = -5800$, Fig. 7–17*e*.
n has a constant slope for $12 < x \leq 20$ since, from
$/dx = V = +600$. This brings the value of $M =$
quired.

Example 7–11

Draw the shear and moment diagrams for the beam in Fig. 7–18*a*.



(a)



SOLUTION

Support Reactions. The reactions at the supports are shown on the free-body diagram in Fig. 7–18*b*.

Shear Diagram. The end points $x = 0$, $V = +35$ and $x = 8$, $V = -35$ are plotted first, as shown in Fig. 7–18*c*.

Since there is no distributed load on the beam, the slope of the shear diagram throughout the beam's length is zero; i.e., $dV/dx = -w = 0$. There is a discontinuity or ''jump'' of the shear diagram, however, at each concentrated force. From Eq. 7–5, $\Delta V = -F$, the change in shear is negative when the force acts downward and positive when the force acts upward. Stated another way, the ''jump'' follows the force, i.e., a downward force causes a downward jump, and vice versa. Thus, the 20-kN force at $x = 2$ m changes the shear from 35 kN to 15 kN; the 30-kN force at $x = 4$ m changes the shear from 15 kN to -15 kN, etc. We can *also* obtain numerical values for the shear at a specified point in the beam by using the method of sections, as for example, $x = 2^+$ m, $V = 15$ kN in Fig. 7–18*e*.

Moment Diagram. The end points $x = 0$, $M = 0$ and $x = 8$, $M = 0$ are plotted first, as shown in Fig. 7–18*d*.

Since the shear is constant in each region of the beam, the moment diagram has a corresponding constant positive or negative slope as indicated on the diagram. Numerical values for the change in moment at any point can be computed from the *area* under the shear diagram. For example, at $x = 2$ m, $\Delta M = \int V\,dx = 35(2) = 70$. Thus, $M|_{x=2} = M|_{x=0} + 70 = 0 + 70 = 70$. Also, by the method of sections, we can determine the moment at a specified point, as for example, $x = 2^+$ m, $M = 70$ kN · m, Fig. 7–18*e*.

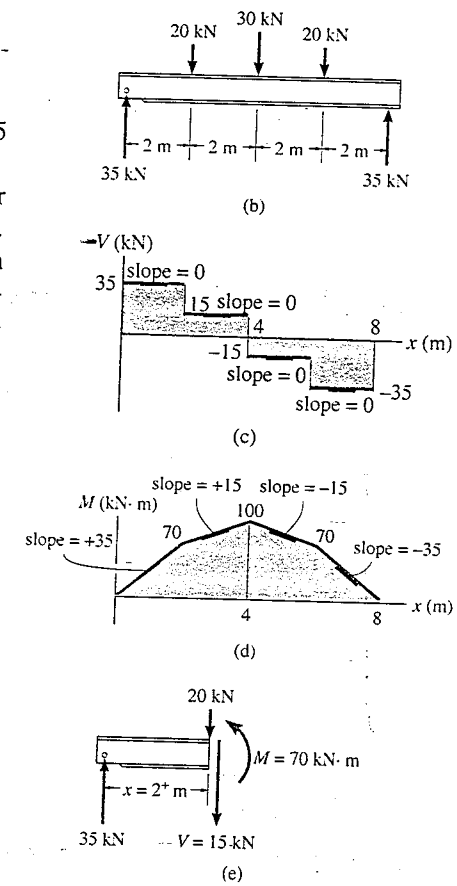
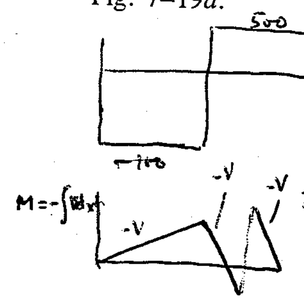
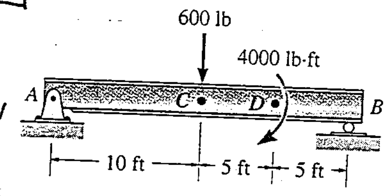


Fig. 7–18

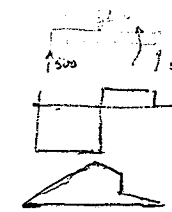
Example 7–12

Sketch the shear and bending-moment diagrams for the beam shown in Fig. 7–19*a*.





(a)



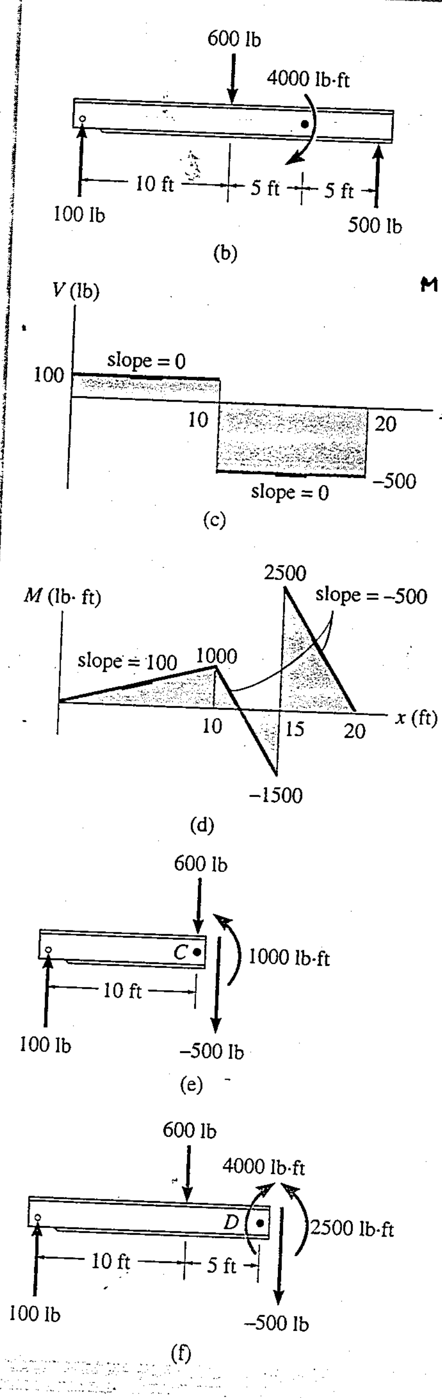


Fig. 7–19

SOLUTION

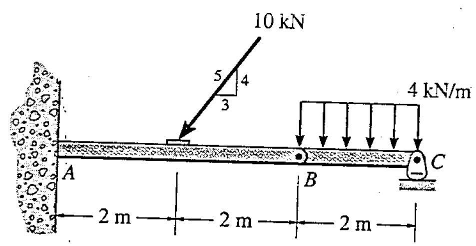
Support Reactions. The reactions are calculated and indicated on the free-body diagram, Fig. 7–19*b*.

Shear Diagram. As in Example 7–11, the shear diagram can be constructed by "following the load" on the free-body diagram. In this regard, beginning at A, $V_A = +100$ lb. No load acts between A and C, so the shear remains constant; i.e., $dV/dx = -w(x) = 0$. At C the 600-lb force acts downward, so the shear jumps down 600 lb, from 100 lb to -500 lb. Again the shear is constant (no load) and ends at -500 lb, point B. Notice that no jump or discontinuity in shear occurs at D, the point where the 4000-lb · ft couple moment is applied, Fig. 7–19*a*. This is because, for force equilibrium, $\Delta V = 0$ in Fig. 7–15*b*.

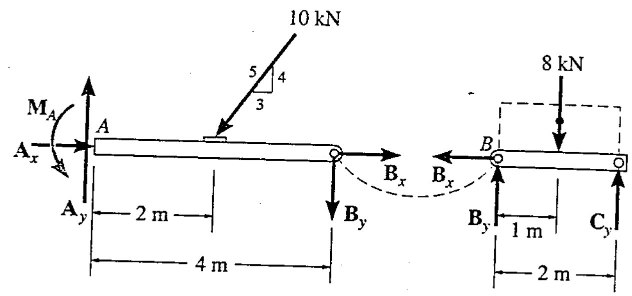
Moment Diagram. The moment at each end of the beam is zero. These two points are plotted first, Fig. 7–19*d*. The slope of the moment diagram from A to C is constant since $dM/dx = V = +100$. The value of the moment at C can be determined by the method of sections or by first computing the rectangular area under the shear diagram between A and C. This gives the change in moment $\Delta M_{AC} = (100 \text{ lb})(10 \text{ ft}) = 1000$ lb · ft. Since $M_A = 0$, then $M_C = 0 + 1000$ lb · ft $= 1000$ lb · ft. From C to D the slope of the moment diagram is $dM/dx = V = -500$, Fig. 7–19*c*. The area under the shear diagram between points C and D is $\Delta M_{CD} = M_D - M_C = (-500 \text{ lb})(5 \text{ ft}) = -2500$ lb · ft, so that $M_D = 1000 - 2500 = -1500$ lb · ft. A jump in the moment diagram occurs at point D, which is caused by the concentrated couple moment of 4000 lb · ft. From Eq. 7–6, the jump is *positive* since the couple moment is *clockwise*. Thus, at $x = 15^+$ ft, the moment is $M_D = -1500 + 4000 = 2500$ lb · ft. This value can *also* be determined by the method of sections. From point D the slope of $dM/dx = -500$ is maintained until the diagram closes to zero at B, Fig. 7–19*d*

Example 6–15

The beam shown in Fig. 6–28*a* is pin-connected at *B*. Determine the reactions at its supports. Neglect its weight and thickness.



(a)



(b)

Fig. 6–28

SOLUTION

Free-Body Diagrams. By inspection, if we consider a free-body diagram of the entire beam *ABC*, there will be three unknown reactions at *A* and one at *C*. These four unknowns cannot all be obtained from the three equations of equilibrium, and so it will become necessary to dismember the beam into its two segments as shown in Fig. 6–28*b*.

Equations of Equilibrium. The six unknowns are determined as follows:

Segment BC

$\xrightarrow{+}\Sigma F_x = 0;$ $\quad B_x = 0$

$\circlearrowleft+\Sigma M_B = 0;$ $\quad -8\text{ kN}(1\text{ m}) + C_y(2\text{ m}) = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad B_y - 8\text{ kN} + C_y = 0$

Segment AB

$\xrightarrow{+}\Sigma F_x = 0;$ $\quad A_x - 10\text{ kN}(\tfrac{3}{5}) + B_x = 0$

$\circlearrowleft+\Sigma M_A = 0;$ $\quad M_A - 10\text{ kN}(\tfrac{4}{5})(2\text{ m}) - B_y(4\text{ m}) = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad A_y - 10\text{ kN}(\tfrac{4}{5}) - B_y = 0$

Solving each of these equations successively, using previously calculated results, we obtain

$A_x = 6\text{ kN} \quad A_y = 12\text{ kN} \quad M_A = 32\text{ kN}\cdot\text{m}$ *Ans.*

$B_x = 0 \quad B_y = 4\text{ kN}$

$C_y = 4\text{ kN}$ *Ans.*

Example 6–16

Determine the horizontal and vertical componer
pin at *C* exerts on member *ABCD* of the frame sl

SOLUTION

Free-Body Diagrams. By inspection, the three co
that the supports exert on *ABCD* can be determined
gram of the entire frame, Fig. 6–29*b*. Also, the free-
frame member is shown in Fig. 6–29*c*. Notice that
force member. As shown by the colored dashed line
and *E* have equal magnitudes but opposite directions
body diagrams.

Equations of Equilibrium. The six unknowns A_x,
will be determined from the equations of equilibriun
frame and then to member *CEF*. We have

Entire Frame

$\circlearrowleft+\Sigma M_A = 0;$ $\quad -981\text{ N}(2\text{ m}) + D_x(2.8\text{ m}) = 0$

$\xrightarrow{+}\Sigma F_x = 0;$ $\quad A_x - 700.7\text{ N} = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad A_y - 981\text{ N} = 0$

Member CEF

$\circlearrowleft+\Sigma M_C = 0;$ $\quad -981\text{ N}(2\text{ m}) - (F_B\sin 45°)(1.6\text{ m}$

$F_B = -1734.2\text{ N}$

$\xrightarrow{+}\Sigma F_x = 0;$ $\quad -C_x - (-1734.2\cos 45°\text{ N}) =$

$C_x = 1230\text{ N}$

$+\uparrow\Sigma F_y = 0;$ $\quad C_y - (-1734.2\sin 45°\text{ N}) - 981\text{ N}$

$C_y = -245\text{ N}$

Since the magnitudes of forces $\mathbf{F}_B$ and $\mathbf{C}_y$ were
quantities, they were assumed to be acting in the
free-body diagrams, Fig. 6–29*c*. The correct sense
have been determined "by inspection" *before* appl
equilibrium to member *CEF*. As shown in Fig. 6–
rium about point *E* on member *CEF* indicates that
downward to counteract the moment created by th
point *E*. Similarly, summing moments about point
vertical component of force $\mathbf{F}_B$ must actually act *up*
act upward to the right.

The above calculations can be checked by apply
rium equations to member *ABCD*, Fig. 6–29*c*.

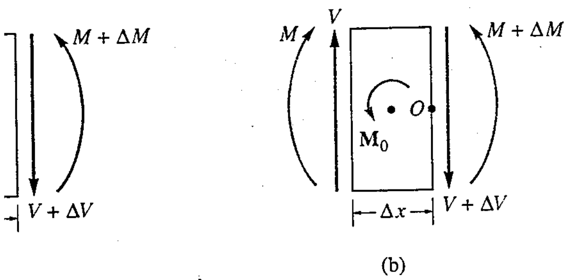


Fig. 7–17

ncentrated Force and Moment. From the deriva- 7.2, it should be apparent that Eqs. 7–1 and 7–3 cannot be ere an external force **F** acts, since these equations do not dden change in shear that occurs at these points. Similarly, 7–4 cannot be used at points where an external couple pplied because of a discontinuity of moment. In order to two cases, we must consider the free-body diagrams of nts of the beam that are located at a concentrated force and Fig. 7–16*a*. These diagrams are shown in Fig. 7–17*a* and ectively.

17*a* it is seen that force equilibrium requires the change in

$$V - F - (V + \Delta V) = 0$$

$$\Delta V = -F \tag{7–5}$$

s *downward* on the beam, as in Fig. 7–16*a*, ΔV is *negative* nps'' *downward,* Fig. 7–16*b*. Likewise, if **F** acts *upward, upward.*

-17*b*, moment equilibrium requires the change in moment

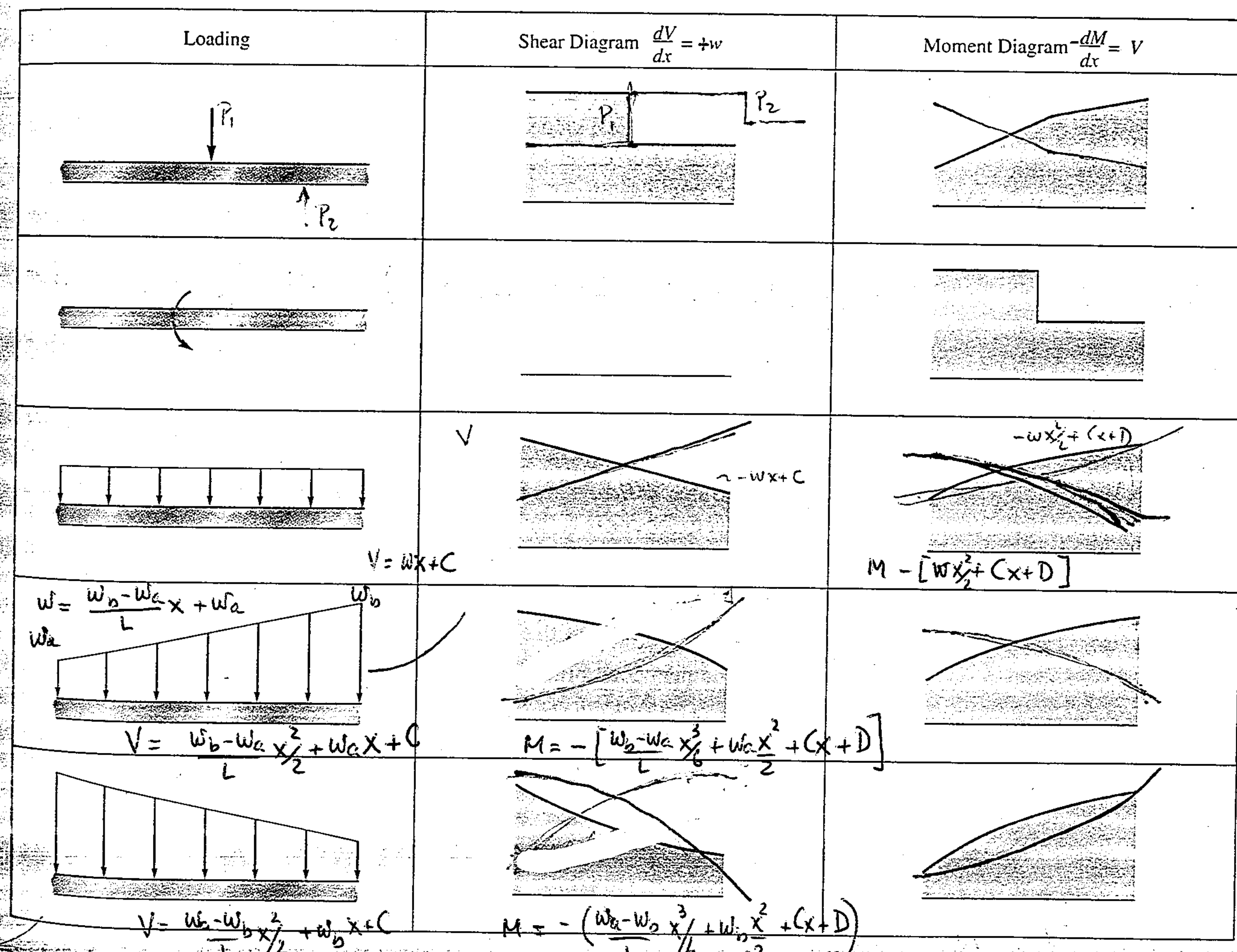
$$M + \Delta M + M_0 - V\,\Delta x - M = 0$$

we get

$$\Delta M = -M_0 \tag{7–6}$$

M_0 is applied *counterclockwise,* as in Fig. 7–16*a*, ΔM is moment diagram jumps *downward,* Fig. 7–16*c*. Likewise,

Table 7–1 illustrates application of Eqs. 7–1, 7–2, 7–5, and 7–6 to some common loading cases. None of these results should be memorized; rather, each should be *carefully studied* so that you become fully aware of how the shear and moment diagrams can be constructed on the basis of knowing the *variation of the slope* from the load and shear diagrams, respectively. It would be well worth the time and effort to self-test your understanding of these concepts by covering over the shear and moment diagram columns in the table and then trying to reconstruct these diagrams on the basis of knowing the loading.



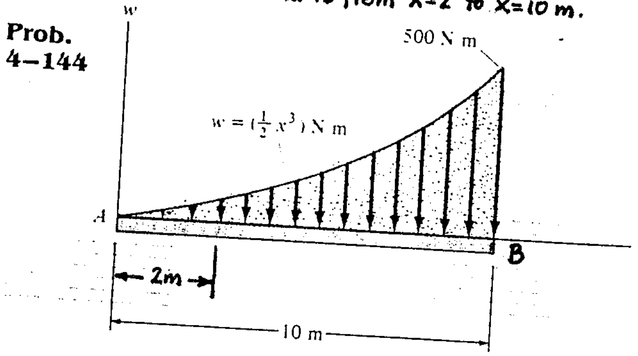
שם : ______________________

ת.ז. : ______________________

בוחן א' מכניקת הנדסית

***4–144.** Wind has blown sand over a platform such that the intensity of load can be approximated by the function $w = (\frac{1}{2}x^3)$ N/m. Simplify this distributed loading to a resultant force and specify the magnitude and location of the force measured from A. **The sand is from x=2 to x=10 m.**

Prob. 4–144

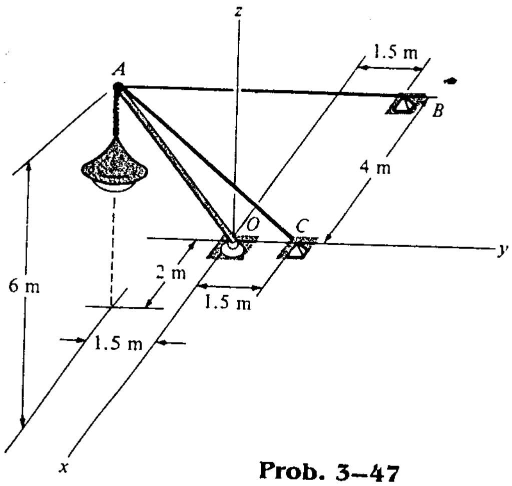


חול נשנב על הלוח AB כדי שעוצמת העומס
נחשב כפונקציה $w = \frac{x^3}{2}\,\frac{N}{m}$. החליפו הכוח
המפורס בכוח שקול ומיצאו גודל השקול.
מקמו את השקול על הקו AB מנקודה A.
הערות: החול נמצא על הלוח בין x=2m
ו- x=10m

השם: ____________________

ת.ז.: ____________________

■3-47. The lamp has a mass of 15 kg and is supported by a pole AO and cables AB and AC. If the force in the pole acts along its axis, determine the forces in AO, AB, and AC for equilibrium.



Prob. 3–47

האור שנתלה מנקודה A נתמך בשני כבלים BA ו- AC והמוט OA. המסה של האור שווה ל- 15 kg אם כוח במוט פועל רק בכיוון ציר המוט, מיצאו את הכוחות F_{OA}, F_{AC} ו- F_{AB} כדי שיהיה שיווי משקל.

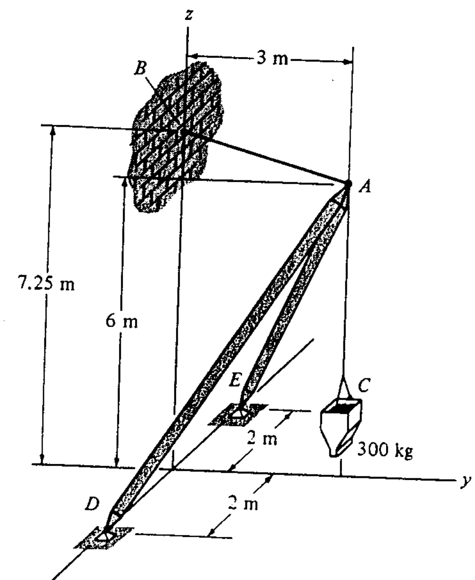
רמז: השתמשו במשוואה של ΣF_y כדי לבודד אחד מהנעלמים במשוואה של ΣF_z.

השם: ____________________

ת.ז. : ____________________

3–46. The boom supports a bucket and its contents, which have a total mass of 300 kg. Determine the forces developed in struts *AD* and *AE* and the tension in cable *AB* for equilibrium. The force in each strut acts along its axis.

מערכת המוטות תומכת בסל ותכניו. מסת הסל והתכנים היא 300kg. מצאו את הכוחות המפותחים במוטות AD ו- AE וגם מתח בכבל AB כדי שיהיה שיווי משקל. כוח במוט פועל בכיוון ציר המוט.



Prob. 3–46

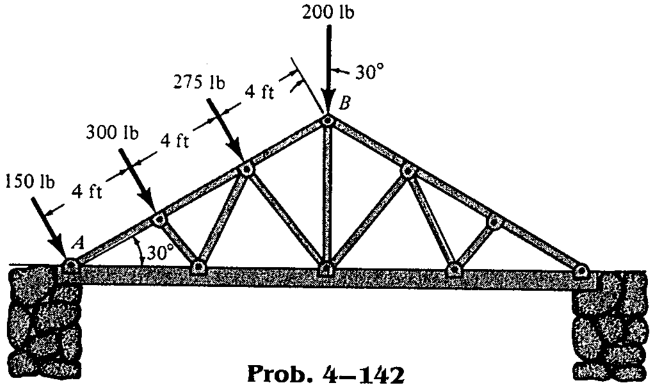
השם : ____________________

ת.ז. ____________________

בוחן ב' מכניקת הנדסית

מערכת של ארבע כוחות פועלת על מסבך הגג כמו מסורטט בציור. מיצאו את השקול ומקמו. את השקול בנקודה מסויימת בקו AB

4–142. The system of four of the system forces acts on the roof truss. Determine the resultant force and specify its location along *AB*, measured from point *A*.



Prob. 4–142

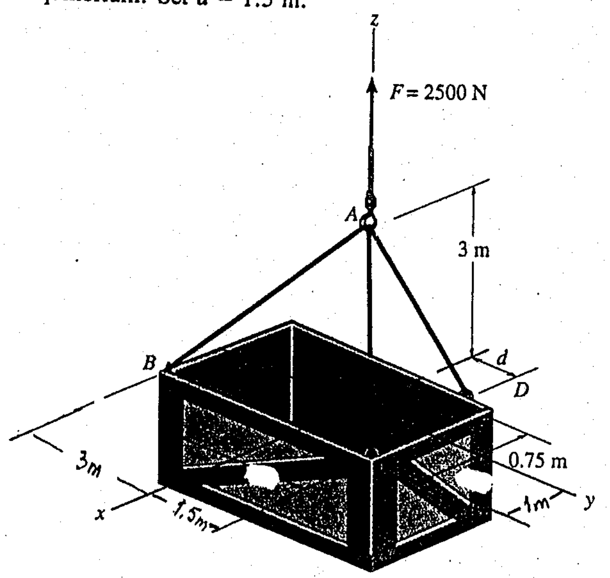
מכניקה הנדסית - בוחן א' גירסה ג'

השם ____________________

ת.ז. ____________________

3–46. The crate is hoisted from the hold of a ship using the cable arrangement shown. If a vertical force of 2500 N is applied to the hook at A, determine the tension in each of the three cables for equilibrium. Set $d = 1.5$ m.

תיבה הורמה מאמצע אונייה בדרך מערכת כבלים כמשורטט בציור. אם כוח מאונך של 2500N מיושם בנקודה A, מיצאו את הכוחות שפועלים בשלושת הכבלים כדי שיהיה שיווי משקל. קחו d=1.5m



Prob. 3–46

B (1, −3, 0)
C (1, 1.5, 0)
D (−0.75, 1.5, 0)
A (0, 0, 3)

$\underline{r}_{B/A} = \underline{i} - 3\underline{j} - 3\underline{k}$ $\sqrt{19} = r_{BA} = 4.36$ $\underline{F}_{AB} = \frac{F_{AB}}{\sqrt{19}}(i - 3j - 3k)$ 5

$\underline{r}_{C/A} = \underline{i} + 1.5\underline{j} - 3\underline{k}$ $\sqrt{12.25} = r_{CA} = 3.50$ $\underline{F}_{CA} = \frac{F_{CA}}{\sqrt{12.25}}(i + 1.5j - 3k)$ 5

$\underline{r}_{D/A} = -.75i + 1.5j - 3k$ $\sqrt{11.81} = r_{D/A} = 3.44$ $\underline{F}_{DA} = \frac{F_{DA}}{\sqrt{11.81}}(-.75i + 1.5j - 3k)$ 5

$\underline{F} = 2500\,\underline{k}$

$$\overset{.2294}{\frac{F_{AB}}{\sqrt{19}}}(1) + \overset{.2857}{\frac{F_{CA}}{\sqrt{12.25}}}(1) - \overset{.2182}{\frac{.75}{\sqrt{11.81}}}F_{DA} = 0$$ 4

2(2)+(3)

$$-F_{AB}\overset{.6882}{\frac{3}{\sqrt{19}}} + \overset{.4286}{\frac{1.5F_{CA}}{\sqrt{12.25}}} + \overset{.4365}{\frac{1.5F_{DA}}{\sqrt{11.81}}} = 0$$ 4

$$-F_{AB}\overset{.6882}{\frac{3}{\sqrt{19}}} - 3\overset{.8571}{\frac{F_{CA}}{\sqrt{12.25}}} - 3\overset{.8730}{\frac{F_{DA}}{\sqrt{11.81}}} + 2500 = 0$$ 5

$$-\frac{9}{\sqrt{19}}F_{AB} + 2500 = 0 \qquad F_{AB} = 2500\frac{\sqrt{19}}{9} = 1210.9$$ 2

3(1) + (3)

$$\cancel{\frac{6}{\sqrt{19}}F_{AB}} - \frac{5.25}{\sqrt{11.81}}F_{DA} + 2500 = 0$$

$$\cancel{\frac{2\cancel{6}}{\sqrt{19}}\frac{2500\sqrt{19}}{\cancel{9}\,3}} - \frac{5.25}{\sqrt{11.81}}F_{DA} + 2500 = 0 \Rightarrow F_{DA} = \frac{2500\sqrt{11.81}}{5.25} = 1636.6$$ 2

(1)

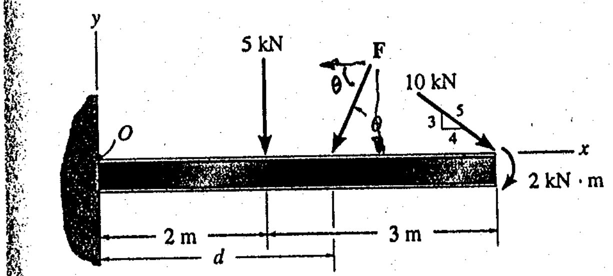
$$\frac{2500}{\sqrt{19}}\frac{\sqrt{19}}{9} + \frac{F_{CA}}{\sqrt{12.25}} - \frac{.75}{\sqrt{11.81}}\frac{2500\sqrt{11.81}}{5.25} = 0$$

$F_{CA} = \cancel{\sqrt{12.25}\left(\frac{.75}{5.25}\left(\frac{7500}{7}\right) - \frac{2500}{9}\right) = \sqrt{12.25}(1071.43 - 833.33) = \sqrt{12.25}(238.10\ N)}$ 2

$$\sqrt{12.25}\left(-\frac{2500}{9} + \frac{2500}{7}\right) = \sqrt{12.25}\left(\overset{357.14}{\cancel{1071.43}} - 277.77\right) = \sqrt{12.25}(79.37) = 277.6$$

34

4–104. Determine the magnitude and orientation θ of force **F** and its placement d on the beam so that the loading system is equivalent to a resultant force of 20 kN acting vertically downward at point O, and a clockwise couple moment of 80 kN · m.



Probs. 4–103/4–104

מיצאו את גודל הכוח F, והזוית θ, ומיקום הכוח d,
על הקורה כדי שמערכת הכוחות והמומנט תהיה
אקוויוולנטית לשקול של 20kN שפועל בכיוון y-
בנקודה O, ולמומנט של 80kN-m בכיוון השעון

4 $\Sigma F_y = \overset{-20}{\cancel{20}}\ kN = -(5kN + F\sin\theta + 6kN)$

2 $\Sigma F_x = 0 = -F\cos\theta + 8kN = 0$ $\quad F\cos\theta = 8\,kN$

$F\sin\theta = 9\,kN$

2 $F = \sqrt{9^2+8^2} = \sqrt{145} \cong 12.$

2 $\tan\theta = 9/8 \quad \theta = 48.4^\circ$

5 $\circlearrowright + \ \Sigma M_o = 5kN \cdot 2 + F\sin\theta\, d + 6kN \cdot 5 \overset{+2}{=} 80$

$= 10 + 9d + 30 + 2 = 80$

$9d = 38 \qquad d = \frac{38}{9} = 4.22\,m$

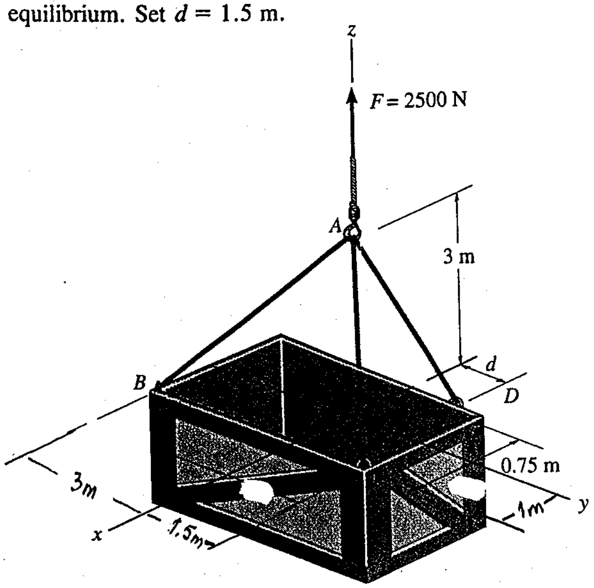
$\frac{1}{17}$

מכניקת הנדסית - בוחן א' גירסה ג'

השם ______________________

ת.ז. ______________________

3–46. The crate is hoisted from the hold of a ship using the cable arrangement shown. If a vertical force of 2500 N is applied to the hook at A, determine the tension in each of the three cables for equilibrium. Set $d = 1.5$ m.



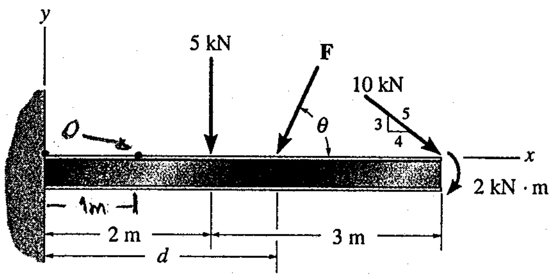
Prob. 3–46

התיבה הורמה מאמצע אונייה בדרך
מערכת כבלים כמשורטט בציור. אם כוח
מאונך של 2500N מיושם בנקודה A,
מיצאו את הכוחות שפועלים בשלושת
הכבלים כדי שיהיה שיווי משקל. קחו d=1.5m

שם

מיצאו את גודל הכוח $\underline{F}$, וכיוונו θ, ומיקום הכוח d,
על הקורה כדי שמערכת הכוחות והמומנט תהיה
אקוויוולנטית לשקול של 20kN שפועל בכיוון y-
בנקודה O, ולמומנט של 80kN-m בכיוון השעון

•4–104. Determine the magnitude and orientation θ of force **F** and its placement d on the beam so that the loading system is equivalent to a resultant force of 20 kN acting vertically downward at point O, and a clockwise couple moment of 80 kN · m.

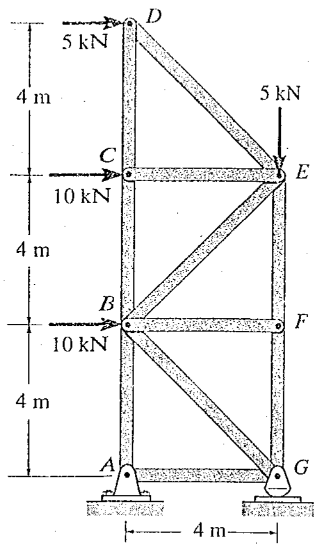


Probs. 4–103/4–104

ת.ז. ______________

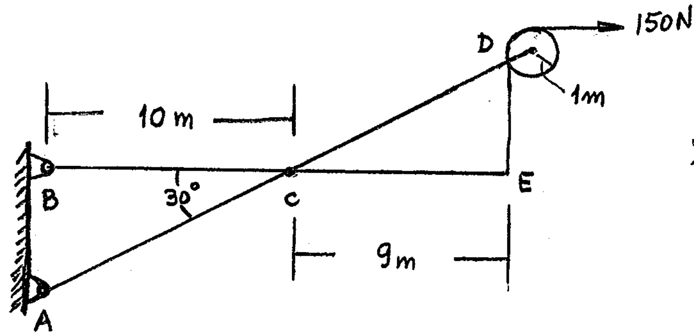
מצאו את הכוחות באברים BE ו-EF של המסבך.

מה אפשר לאמר לגבי אבר BF ?



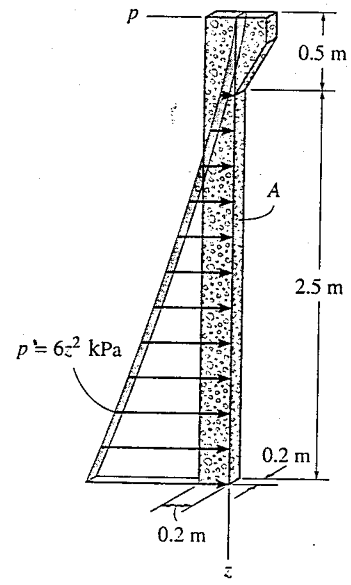
Prob. 6–24

ת.ז. ____________



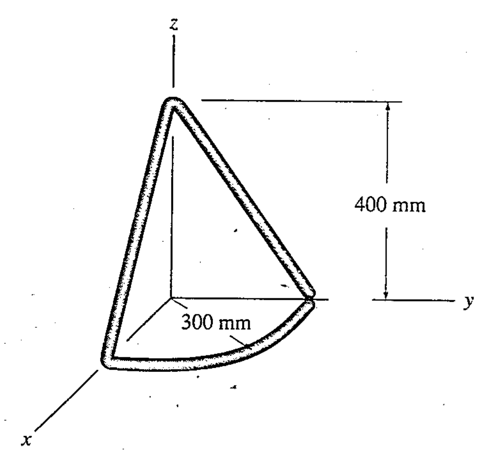
מצאו את הכוחות בפרקים (פינים) A ו-C של המסגרת. הגלגלת ב-D חלקה (אין חיכוך שם).

the resultant force that wet concrete exerts along the plate A, $0.5\text{ m} \le z \le 3\text{ m}$, if the pressure due to the concrete varies as shown. Specif[illegible] the location of the resultant force, measured from the top of the [illegible]umn.

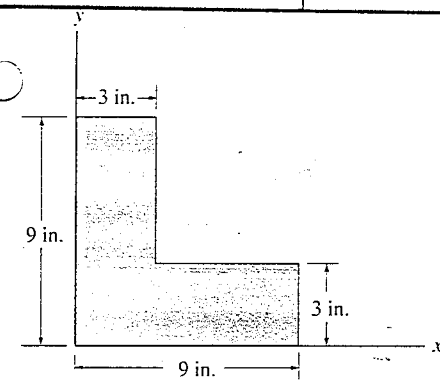


Prob. 9–120

9–121. Determine the location ($\bar{x}$, $\bar{y}$, $\bar{z}$) of the center of gravity of the homogeneous wire.

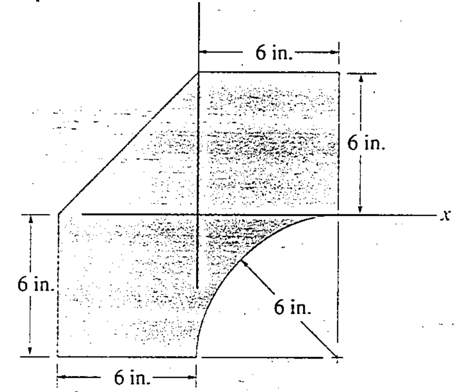


Prob. 9–121



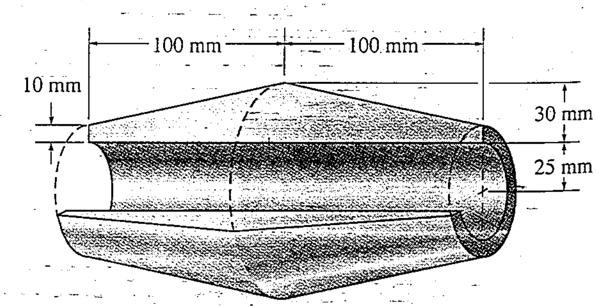
Prob. 9–122

9–123. Determine the location ($\bar{x}$, $\bar{y}$) of the centroid of the homogeneous plate.



Prob. 9–123

*9–124. Determine the volume of steel needed to produce the tapered part. The cross section is shown, although the part is 360° around. Also, compute the outside surface area of the part, excluding the ends.



Prob. 9–124

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מכניכת הנדסית-מועד א
פרופ עזרא לוי

מבחן סופי

17 ביולי 2000

אורך המבחן הזה 180 דקות. במבחן יש ארבע שאלות. עליכם לענות לכל השאלות.
רק להשתמש בקלסר שלכם, תרגילי בית שלכם, ומחשבון. ודברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:
לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

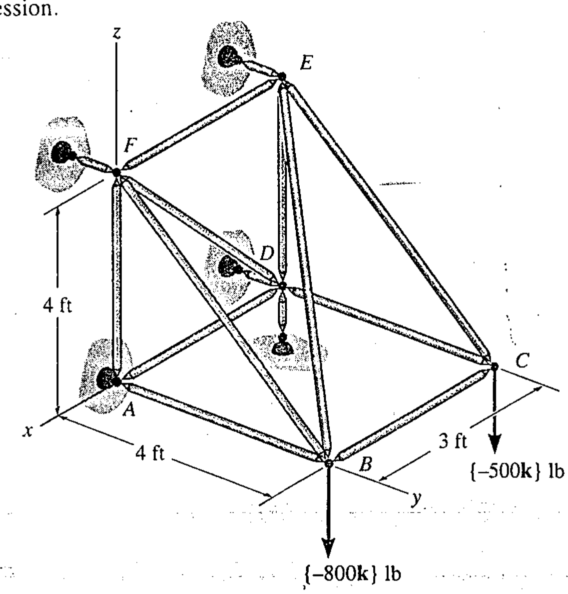
חתימת הסטודנט____________________

תרגיל מס. 1

נתון מסבך תלת־ממדי הניתמך בכדור ותושבת בנקודה A ובאברים קטנים ב־ D, E, ו־F כמשורטט בציור. כוחות הפועלים על המסבך כצוין בציור.

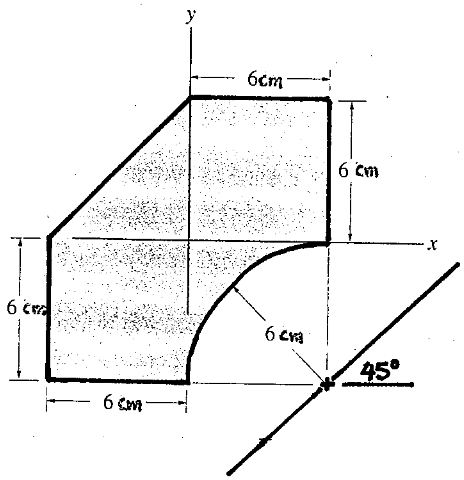
א) מיצאו את הכוחות במוטות CD, EC, EB, AB ו-FB וציינו אם הם לחוצים או מתוחים.

The space truss is supported by a ball-and-socket joint at A and short links at D, E, and F. Determine the forces in CD, EC, EB, AB, FB indicate whether the members are in tension or compression.



תרגיל מס. 2

נתון גוף עם מדות שלו כמשורטט בציור בצד שמאל.



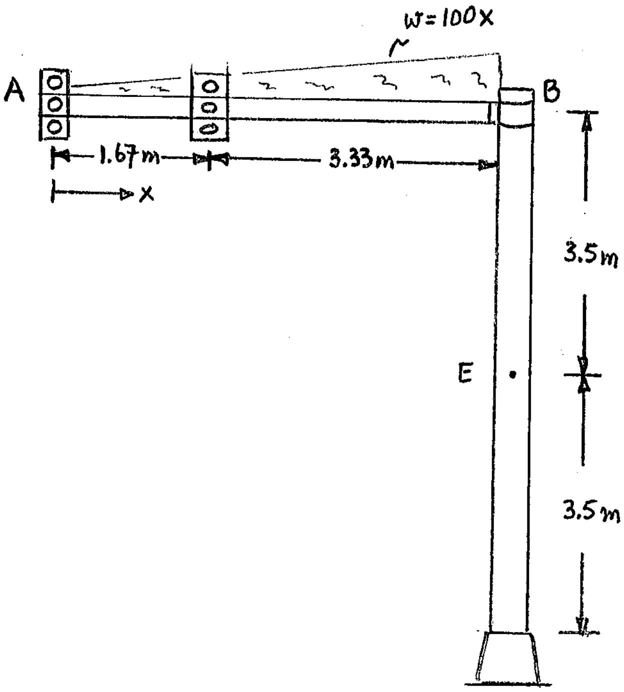
א) מוצאו את מרכז הכובד ($\bar{x}, \bar{y}$) של הגוף הזה.

ב) מיצאו את הנפח של הגוף המיוצר כאשר מסובבים את הגוף 360 מעלות מסביב הקו המשופע 45 מעלות לציר x.

ג) מיצאו את מומנט האינרציה I_{xx} מסביב הציר x.

ד) (5 נקודות נוספות) מיצאו את מומנט האינרציה $I_{x'x'}$ מסביב הציר x' שעובר דרך מרכז הכובד.

תרגיל מס. 3



נתון הציור בצד שמאל והנתונים האלה:
כל רמזור ורמזור שוקל 90 N. הזרוע AB התומך ברמזורים שוקל 67 N/m. בחורף, גם שלג פועל על הזרוע כפונקציה ליניארית $w(x)=100x$ N/m ו־x נמדד מנקודה A ימינה.

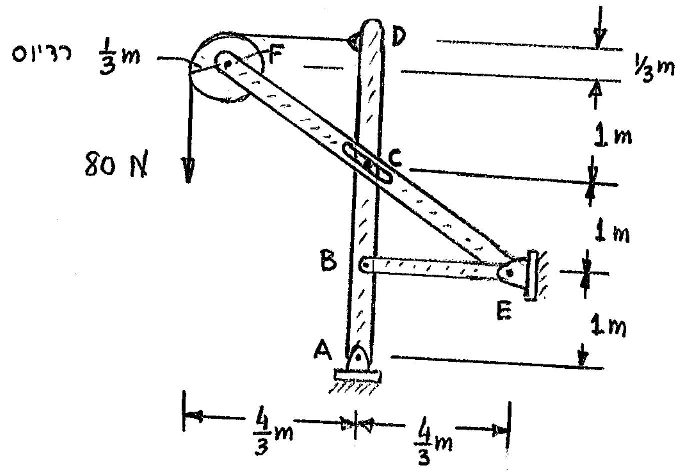
א) מיצאו את מהלכי מומנט וכוח הגזירה של הזרוע AB
והע,

ב) מיצאו את הכוחות ומומנטים פנימיים בנקודה E.

הניחו שהקשר בין הזרוע והעמוד פועל כסמך רתום לזרוע.

תרגיל מס. 4

למסגרת המשורטטת בציור בצד שמאל:



א) מיצאו את הכוחות האופקיים והכוחות האנכיים שהסיכות מפעילות על איבר ABCD.

הערה: הסיכה ב-C מחוברת לאיבר ABCD ועוברת דרך פתח חלק הנמצא באיבר ECF.

מכניכת הנדסית-מועד ב　　　　　　מבחן סופי　　　　　　20 בספטמבר 2000

פרופ עזרא לוי

אורך המבחן הזה 180 דקות. במבחן יש ארבע שאלות. עליכם לענות לכל השאלות.
רק להשתמש בקלסר שלכם, תרגילי בית שלכם, ומחשבון, ודברים הנתונים לכם עם המבחן בלבד.

לחתום על סעיף הבא בבקשה:
לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט________________

תרגיל מס. 1

נתון מסבך תלת-ממדי הניתמך בכדור ותושבת בנקודה A ובאברים קטנים ב- D, E, ו-F כמשורטט בציור. כוחות הפועלים על המסבך כצוין בציור.

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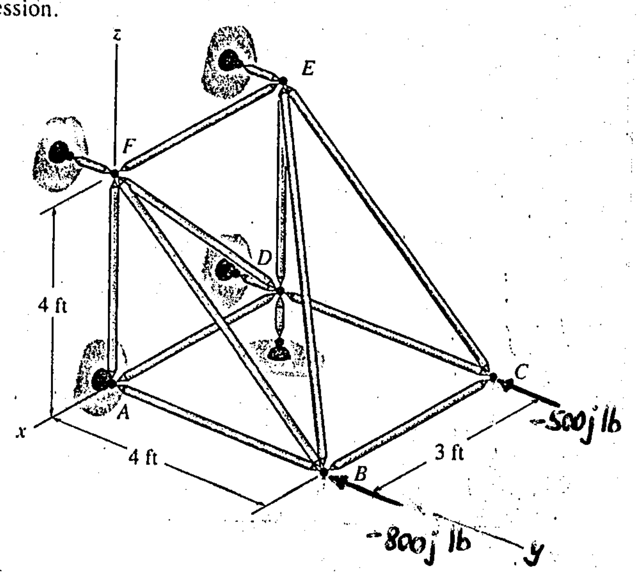
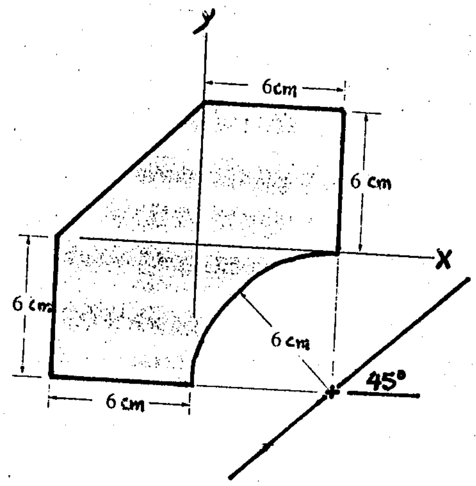
The space truss is supported by a ball-and-socket joint at *A* and short links at *D*, *E*, and *F*. Determine the forces in CD, EC, EB, AB, FB indicate whether the members are in tension or compression.



תרגיל מס. 2

נתון גוף עם מדות שלו כמשורטט בציור בצד שמאל.



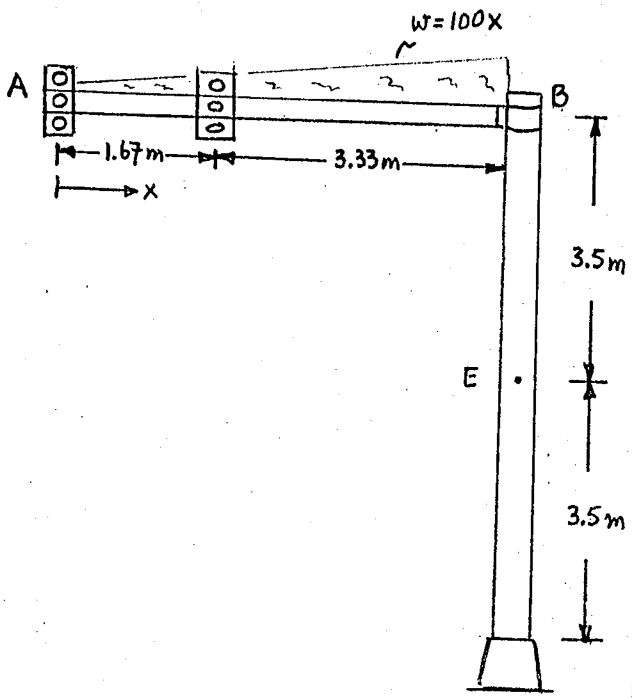
א) מוצאו את מרכז הכובד ($\bar{x}$, $\bar{y}$) של הגוף הזה.

ב) מיצאו את הנפח של הגוף המיוצר כאשר מסובבים את הגוף 360 מעלות מסביב הקו המשופע 45 מעלות לציר x.

ג) מיצאו את מומנט האינרציה I_{xx} מסביב הציר x.

ד) (10 נקודות נוספות) מיצאו את מומנט האינרציה $I_{x'x'}$ מסביב הציר x' שעובר דרך מרכז הכובד.

תרגיל מס. 3



נתון הציור בצד שמאל והנתונים האלה:
כל רמזור ורמזור שוקל 90 N. הזרוע AB התומך ברמזורים שוקל 67 N/m. בחורף, גם שלג פועל על הזרוע כפונקציה ליניארית $w(x)=100x$ N/m ו־x נמדד מנקודה A ימינה.

א) מיצאו את מהלכי מומנט וכוח הגזירה של הזרוע AB

ב) מיצאו את הכוחות ומומנטים פנימיים בנקודה E.

הניחו שהקשר בין הזרוע והעמוד פועל כסמך רתום לזרוע.

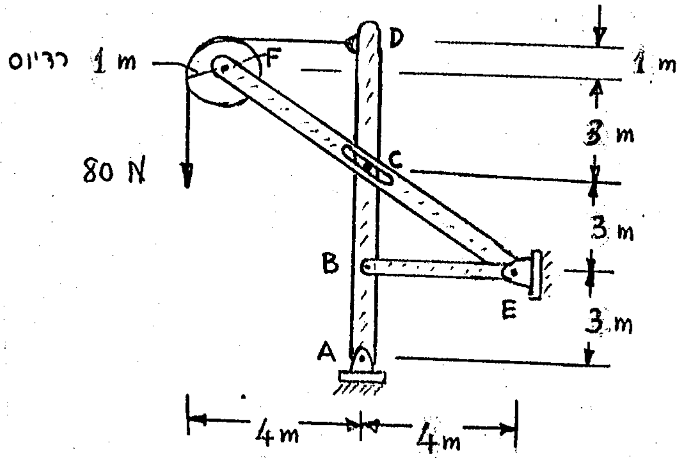
הערה: עוצמת השלג היא $w=100x \frac{N}{m}$

תרגיל מס. 4

למסגרת המשורטטת בציור בצד שמאל:

א) מיצאו את הכוחות האופקיים והכוחות האנכיים שהסיכות מפעילות על איבר ABCD.

<u>הערה:</u> הסיכה ב־C מחוברת לאיבר ABCD ועוברת דרך פתח חלק הנמצא באיבר ECF.



20 בספטמבר 2000

מבחן סופי

מכניקת הנדסית-מועד ב

פרופ עזרא לוי

אורך המבחן הזה 180 דקות. במבחן יש ארבע שאלות. עליכם לענות לכל השאלות.
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לחתום על סעיף הבא בבקשה:
לאורך המבחן אני לא רשאי(ת) לקבל עזרה מאף אחד או לתת עזרה לאף אחד. אם אני אעבור על אלה, אני אכשול את המבחן.

חתימת הסטודנט..........................

תרגיל מס. 1

נתון מסבך תלת-ממדי הניתמך בכדור ותושבת בנקודה A ובאבזים קטנים ב- D, E, ו-F כמשורטט בציור. כוחות הפועלים על המסבך כצוין בציור.

א) מיצאו את הכוחות במוטות CD, EC, EB, AB ו-FB וציינו אם הם לחוצים או מתוחים.

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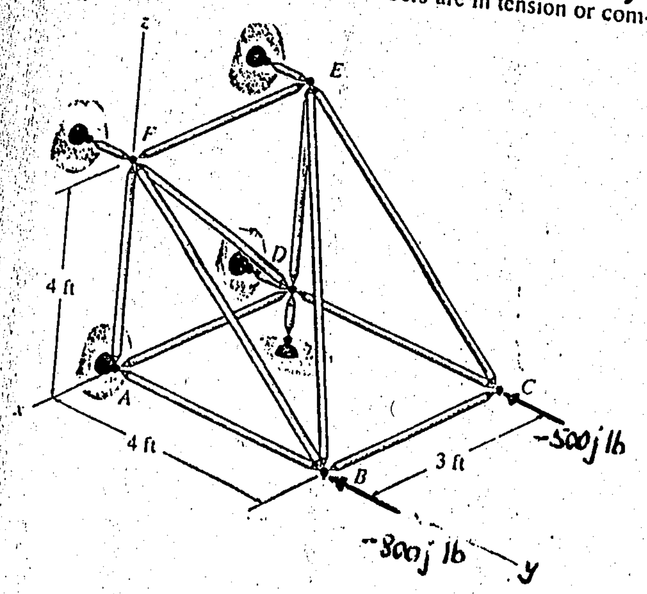


תרגיל מס 2

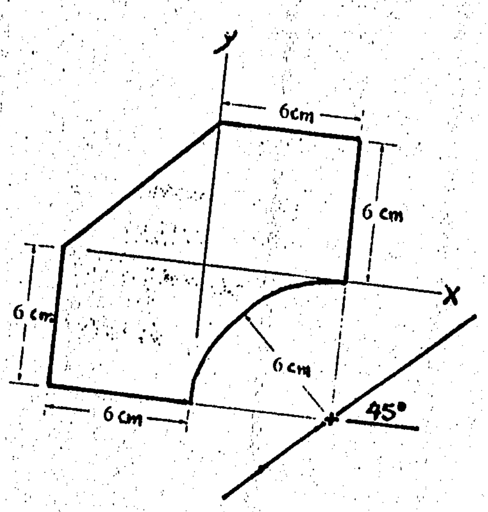
נתון גוף עם מדות שלו כמשורטט בציור בצד שמאל.

א) מוצאו את מרכז הכובד ($\bar{x}$, $\bar{y}$) של הגוף הזה.

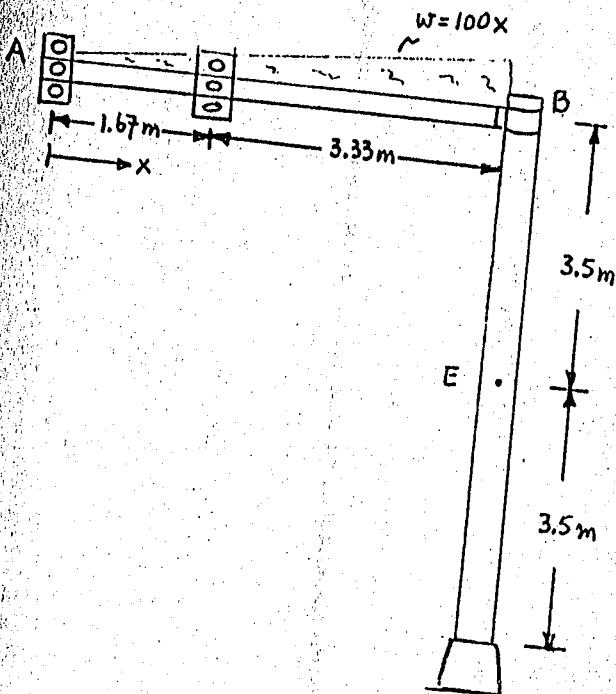
ב) מיצאו את הנפח של הגוף המיוצר כאשר מסובבים את הגוף 360 מעלות מסביב הקו המשופע 45 מעלות לציר x.

ג) מיצאו את מומנט האינרציה I_{xx} מסביב הציר x.

ד) (10 נקודות נוספות) מיצאו את מומנט האינרציה $I_{x'x'}$ מסביב הציר 'x שעובר דרך מרכז הכובד.



תרגיל מס. 3



נתון הציור בצד שמאל והנתונים האלה:
כל רמזור ורמזור שוקל 90 N. הזרוע AB התומך ברמזורים שוקל 67 N/m. בחורף גם שלג פועל על הזרוע כפונקציה ליניארית $w(x)=100x$ N/m ו־x נמדד מנקודה A ימינה.

א) מיצאו את מהלכי מומנט וכוח הגזירה של הזרוע AB

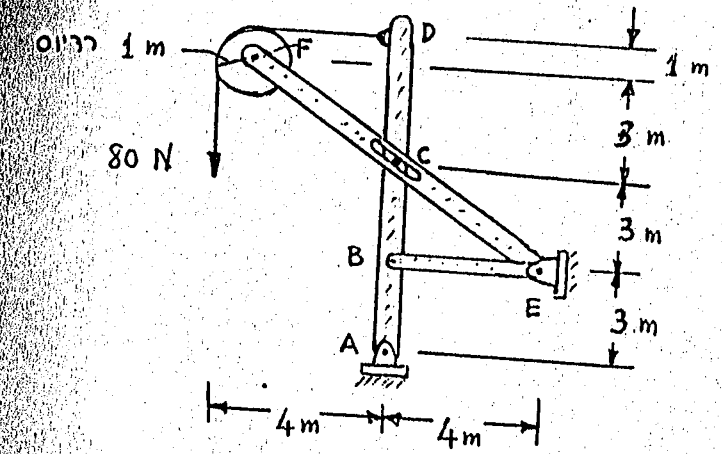
ב) מיצאו את הכוחות ומומנטים פנימיים בנקודה E.

הניחו שהקשר בין הזרוע והעמוד פועל כסמך רתום לזרוע.

הערה: עוצמת השלג היא $w=100x\ \frac{N}{m}$

תרגיל מס. 4

למסגרת המשורטטת בציור בצד שמאל.



א) מיצאו את הכוחות האופקיים והכוחות האנכיים שהסיכות מפעילות על איבר ABCD.

<u>הערה:</u> הסיכה ב־C מחוברת לאיבר ABCD ועוברת דרך פתח חלק הנמצא באיבר ECF.

18 ביוני 2001

מבחן סופי
מועד א

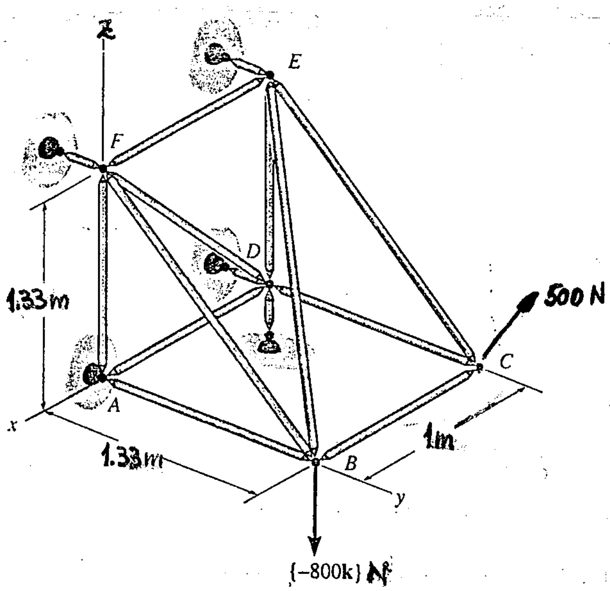
מכניכת הנדסית-מועד א
פרופ עזרא לוי

אורך המבחן הזה 180 דקות. במבחן יש ארבע שאלות. עליכם לענות לכל השאלות.
רק להשתמש בקלסר שלכם, תרגילי בית שלכם, ומחשבון. ודברי'ם הנתונים עם המבחן בלבד.

תרגיל מס. 1

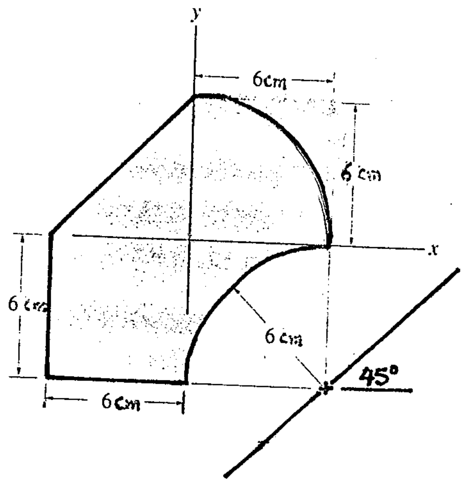
נתון מסבך תלת-ממדי הניתמך בכדור ותושבת בנקודה A ובאברים קטנים ב- D, E, ו-F כמשורטט בציור. כוחות הפועלים על המסבך הם הכוח P=500 N בזוויות כיווניות של (α,β,γ =90,90,180) וכוח בנקודה B כצוין בציור.

א) מיצאו את הכוחות במוטות CD, EC, EB, AB ו-FB וציינו אם הם לחוצים או מתוחים.



תרגיל מס. 2

נתון גוף עם מדות שלו כמשורטט בציור בצד שמאל.



א) מיצאו את מרכז הכובד $(\bar{x}, \bar{y})$ של הגוף הזה.

ב) מיצאו את הנפח של הגוף המיוצר כאשר מסובבים את הגוף 360 מעלות מסביב הקו המשופע 45 מעלות לציר x.

ג) מיצאו את מומנט האינרציה I_{xx} מסביב הציר x.

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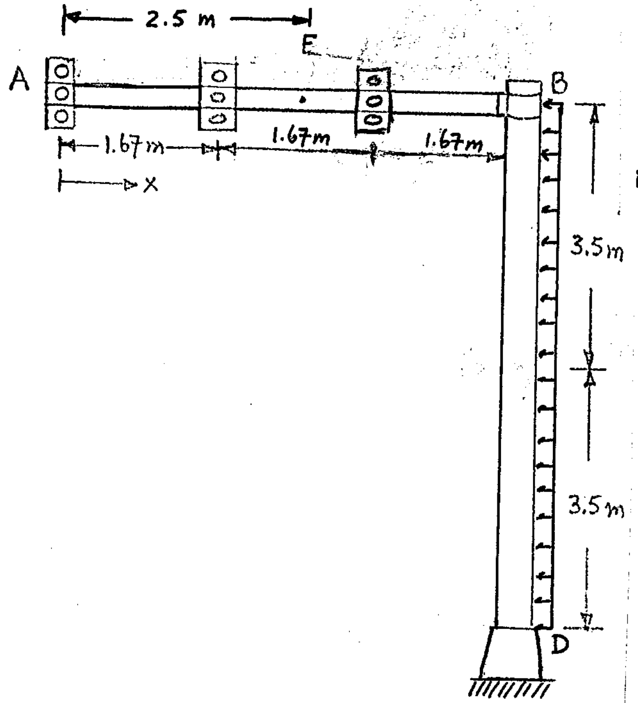
תרגיל מס. 3



נתון, הציור בצד שמואל והנתונים האלה:
כל רמזור ורמזור שוקל 90 N. הזרוע AB התומך ברמזורים שוקל 67 N/m. בחורף, גם הרוח נושב על העמוד שמחזיק את הזרוע בכוח מפורס בגודל $w(y)=100$ N/m מימין שמואלה, ו־y נמדד מנקודה D כלפי מעלה.

א) מיצאו את מהלכי מומנט וכוח הגזירה של העמוד CD

ב) מיצאו את הכוחות ומומנטים פנימיים בנקודה E.

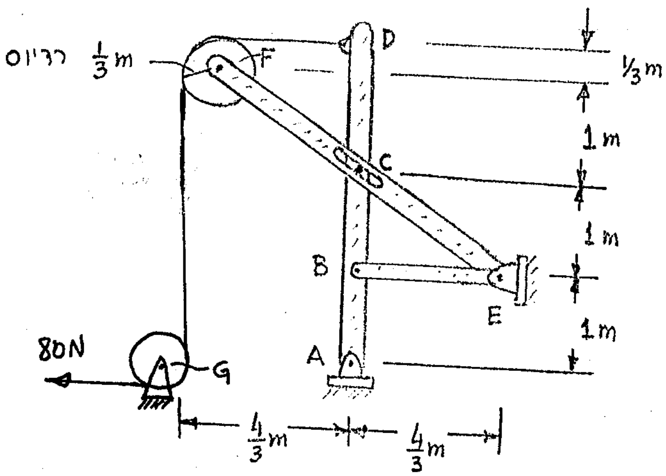
הניחו שהקשר בין הזרוע והעמוד פועל כסמך רתום לזרוע.

תרגיל מס. 4

למסגרת המשורטטת בציור בצד שמאל:

א) מיצאו את הכוחות האופקיים והכוחות האנכיים שהסיכות מפעילות על איבר ABCD.

הערה: הסיכה ב־C מחוברת לאיבר ABCD ועוברת דרך פתח חלק הנמצא באיבר ECF.



מכללת יהודה ושומרון
חטיבה עצמאית-אקדמית

מכלול 4.0
ראשים - תכנון וביצוע מערכות מידע בע"מ

תאריך עברי: כ"ד שבט תש"ס
ת.לועזי...: 17:2531/01/2000
דף........: 13

שעור....: 4061170-01 מכניקה הנדסית
נ.זיכוי: 4.50
ש"ש....: 3.00

תשס הרצאה
כיתה:

סמסטר	יום	משעה	עד שעה	חדר	מרצה	תא
ב	ב	14:00	20:00	יובר(?) 3.6	ד"ר לוי עזרא	

רשימת התלמידים בשעור למילוי בוחנות

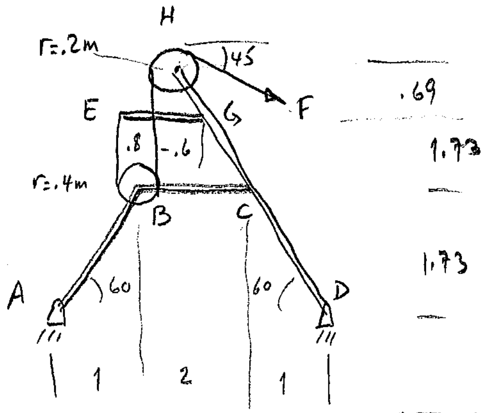
50% 15% 35%

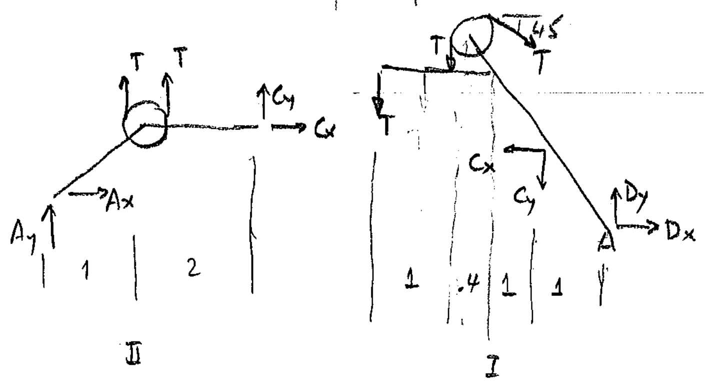
#	שם		מס.זיהוי	שלי	40	60	10	40	30	40	30	30	40	20	QUIZ 1 44	HW	Fin	
1	אבדייאב	מרק	1142899-2	2	39+5	59	5	32	28	40	24	26	39	19	42	89.08	78	85.77
2	אדלר	הדסה	3175445-0	2														
3	אופק	שגיא	2574879-9	2														
4	אסתרי	עמית	565407-4	2	36+30	53	7	37	27	25	23	23	29	16	33	87.42	60	69.36
5	באור	שמואל	3309318-8	2	39	59	-	37	25	37	30	20	38	20	44	82.58	64	79.39
6	ברגר	שלומי	2896000-3	2	40	56	10	40	29	48	28	24	38	20	41	97.83	82	88.29
7	דיגמי	רוית	3353715-0	2	40	53	-	-	28	37	26	-	38	19	32/51	65.08	50	56.72
8	חגי	עדיאל	3413719-0	2	-	-	-	36	25	28	28	20	38	20	19	59.83	54	51.09
9	טלמור	אלי	2717793-0	2	39	59	-	40	25	39½	28	25	37.5	19	41	84.21	63	76.75
10	מזור	רונן	3319091-9	2	37+5	57	-	40	25	28	29	23	38	20	35	82.17	54	67.17
11	מנדל	סטיבן	1156336-8	2	36	57	10	38	27	38	30	27	38	19	33	94.5	66	73.43
12	נידם	לירון	3613898-0	2														
13	נתן	אורן	3157265-4	2	34+5	-	-	-	26	27	26	18	35	19	31/51	58.08	54	56.99
14	סבטבי	יורם	3350740-1	2	38	59	-	40	24	24	27	25	35	19	29	78.92	45	57.41
15	עמרם	גלעד	3879465-7	2	38+10	59	-	38	25	36	27	25	39.5	20	41	85.88	82	86.5
16	פאר	אורי	3866598-0	2	34+15	33	10	35	30	40	26	21	37	19	44/51	90.92	60	73.83
17	פלדבאום	טל	3389102-9	2	35+5	58	-	39	23	36	27	מילו'	-	19	16	71.76	56	51.49
18	פלץ	שי	3182651-4	2	39	57	10	40	26	37	24	28.5	39.5		38	84.54	60	72.91
19	פקטור	אביגדור	2743241-8	2	33+12													
20	פרדיאן	שחר	3310448-0	2	-	59	-	-	28	-	20	-	38	19	-	44.83	0	6.72
21	פרידמן	גדליה	30612624-4	2	36	45	-	38	25	32	29	27	38.5	20	37	80.62	57	70.02
22	רוחמה	רועי	3859090-7	2	39+5	59	-	37	25	40	26	23	37	18	36	83	70	76.08
23	רומי	אבי	3138703-8	2	35+15	60	10	40	27	39	28	23	38	20	40	97.75	79	85.98
24	שגיא	אור	3361318-3	2	35+15	-	-	-	28	36	24	-	38	"	34/51	48.33	23	42.08
	אלמני	יצחק	2457242-2		39	42	-	~~37~~	-	-	15	33.5	30	19	0+25/51	59.17	25	38.53
	אבו שאח	מוסטפא	04066867-5		16	50	10	-	-	-	20	22	36.5	19	38	54.96	40	58.47
	שילוני	[illegible]	57287914		-	-	-	-	30	40	26	29.5	39	20	39	58.25	76	77.76

לפרק לשני חלקים

למצוא את הכוחות בכל פין

מהלכי מומנטים





I $\quad \Sigma M_C = 0$

$$D_y + 1.73D_x + T\frac{\sqrt{2}}{2}\cdot 1.4 - T\frac{\sqrt{2}}{2}(0.69+1.73+.2) + T(1.6) + T(2.4) = 0$$

II $\quad \Sigma M_C = 0$

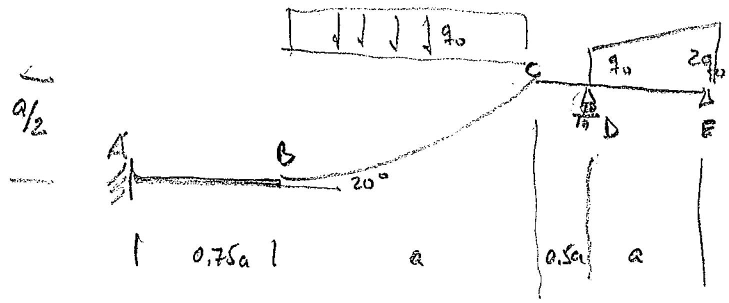
$$-3A_y + 1.73A_x - T(2.4+1.6) = 0$$

כל המבנה

$$A_x + D_x + T\frac{\sqrt{2}}{2} = 0$$

$$A_y + D_y = T\frac{\sqrt{2}}{2}$$

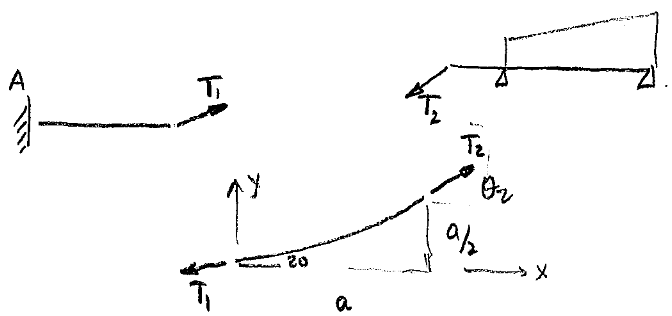
$$\left.\begin{aligned} A_x &= 1.716T \\ A_y &= -0.344T \\ D_x &= -0.42T \\ D_y &= 1.051T \end{aligned}\right\} \longrightarrow \quad \begin{aligned} C_x &= -A_x = -1.716T \\ C_y &= -2T - A_y = -1.656T \end{aligned}$$



$T\cos\theta = H$

$T\sin\theta = \int w(x)\,dx$

$\frac{dy}{dx} = \tan\theta$



$$y = \frac{1}{F_H}\int\left(\int q_0\,dx\right)dx$$

$$y = \frac{1}{F_H}\left[q_0\frac{x^2}{2} + C_1x + C_2\right]$$

$$\frac{dy}{dx} = \frac{1}{F_H}\left[q_0x + C_1\right]$$

$C_2 = 0 \Leftarrow y=0 \quad x=0$

$$= q_0\frac{x}{F_H} + \tan 20^\circ = \tan\theta_2$$

$\frac{dy}{dx} = \frac{C_1}{F_H} = \tan 20^\circ \quad x=0$

$C_1 = F_H\tan 20^\circ$

$$y = \frac{1}{F_H}\left[q_0\frac{a^2}{2} + C_1a\right] = a/2 \quad \Leftarrow \quad y = a/2 \quad x = a$$

$$\left[\frac{q_0}{F_H}\frac{a^2}{2} + \tan 20^\circ a\right] = a/2 \longrightarrow F_H$$

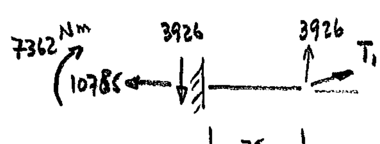
$$\frac{4000(2.5)^2}{2F_H} + .364(2.5) = 1.25$$

$F_H = 10785\ N$

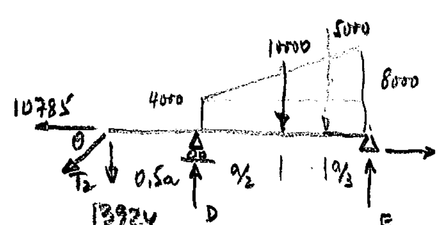
$F_H = T_1\cos 20^\circ \quad T_1 = 11477\ N$

$$\left.\frac{dy}{dx}\right|_{x=a} = \tan\theta_2 = q_0\frac{a}{F_H} + \tan 20^\circ = 1.29 \Rightarrow \theta_2 = 52.84$$

$F_H = T_2\cos 52.84^\circ \Rightarrow T_2 = 17612.\ N$



$10785 = T_1\cos 20^\circ$

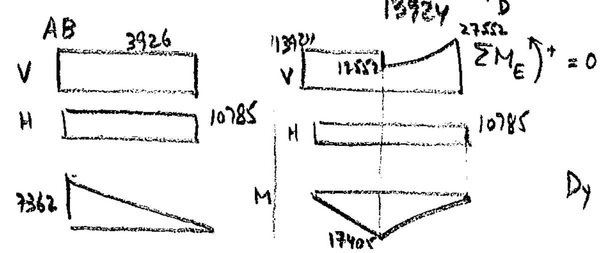


$E_x = 10785$

כוח מקסימלי בכבל 17612

מהלכי מומנט ל CDE, AB

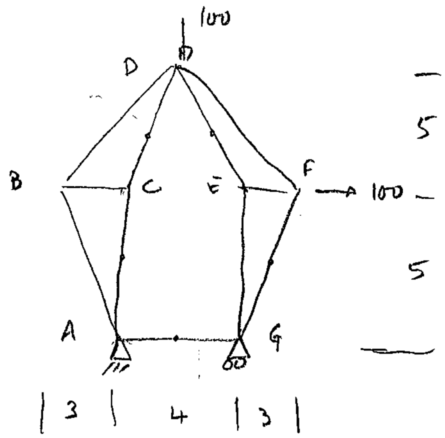
מהלכ' כוח גזירה וכוח צירי



$\Sigma M_E \curvearrowleft^{+} = 0 \quad 10000\cdot 1.25 + 5000(.833) - D_y(2.5) + 13924(3.75) = 0$

$D_y = \underline{27552}\ N$

$D_y + E_y = 28924 \Rightarrow E_y = \underline{1372\ N}$



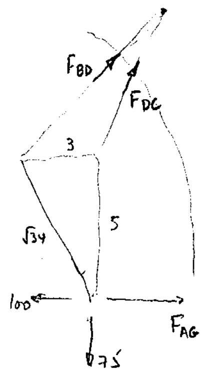
$\checkmark AG,\ \checkmark GF,\ \checkmark CA,\ \checkmark DE,\ \checkmark DC$

$\Sigma M_A \circlearrowleft = 0 \qquad 4G_y - 100\cdot 5 - 100\cdot 2 = 0$

$G_y = 175\,kN$

$\Sigma F_y = 0 \qquad A_y + G_y = 100 \qquad A_y = -75\,kN$

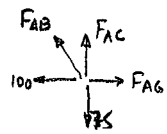
$\Sigma F_x = 0 \qquad A_x + 100 = 0 \qquad A_x = -100\,kN$



$\Sigma M_D \circlearrowleft \qquad -100\cdot 10 + F_{AG}\cdot 10 + 75\cdot 2 = 0$

$F_{AG} = \dfrac{1000-150}{10} = 85\ kN$

A @

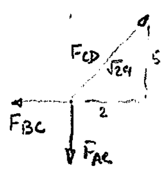


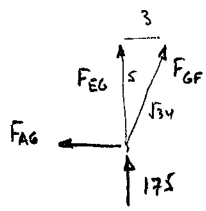
$F_{AB}\cdot\dfrac{5}{\sqrt{34}} + F_{AC} - 75 = 0$

$-F_{AB}\cdot\dfrac{3}{\sqrt{34}} - 100 + F_{AG} = 0 \qquad F_{AB} = -\dfrac{15\sqrt{34}}{3} = -5\sqrt{34}$

$\underline{F_{AC}} = 75 - F_{AB}\cdot\dfrac{5}{\sqrt{34}} = \underline{100\ kN}$

G @

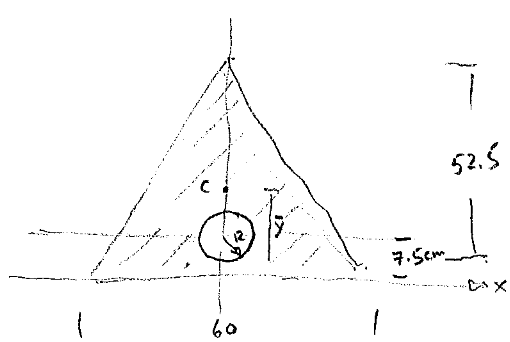




$\Sigma F_x = 0 \qquad F_{GF}\cdot\dfrac{3}{\sqrt{34}} - F_{AG} = 0 \qquad F_{GF} = \dfrac{\sqrt{34}}{3}\cdot 85\ kN$

$F_{AC} = F_{CD}\cdot\dfrac{5}{\sqrt{29}} \qquad \underline{F_{CD}} = \dfrac{\sqrt{29}}{5}F_{AC} = \underline{20\sqrt{29}\,kN}$

$F_{CD} = F_{DE}$ סימטריות

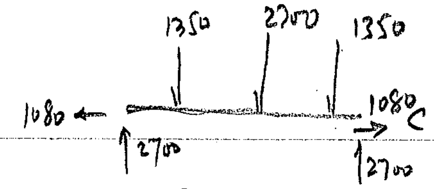


	A	$\bar{y}$	
משולש	$52.5(60)/2$	$\dfrac{52.5}{3}$	$(52.5)^2(60)/6$
עיגול	πR^2	7.5	$7.5\pi R^2$

$\bar{y} = \dfrac{(52.5)^2(10) - 7.5\pi R^2}{52.5(30) - \pi R^2}$

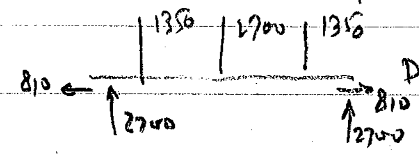
אם AB ע/ אם ב-C

$f_A > 810 = .3(2700)$ לכן ב-A כבר תנועה



אם AB ע/ אם ב-D

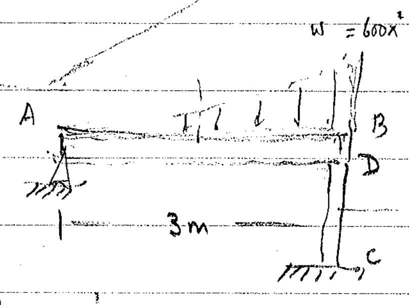
$f_A = 810 = .3(2700)$



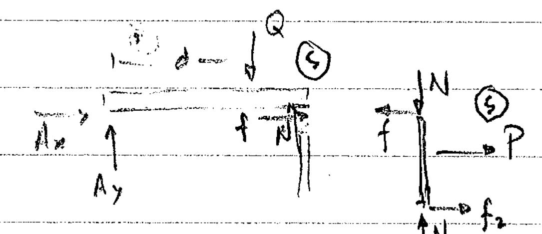
$Q = \int_0^3 w(x)dx = \frac{600x^3}{3} = 200(27) = 5400\,N$ (2)

$\int_0^3 w(x)\cdot x\,dx \quad \frac{600x^4}{4} = 150(81) = 12150\,N\cdot m$ (2)

$d = \frac{12150}{5400} = 2.25$ (2)



קורה AB נתמכת
בסמך נייח ב-A
וקורה CD ב-B, 1m
סובלת בכוח מפורס -
אם במשטח מגע D מקדם חיכוך הוא $\mu_D = 0.4$, $w(x) = 600x^2$ N/m
ובמשטח מגע C המקדם חיכוך הוא $\mu_C = 0.3$, מה הכוח האופקי
המינימלי, P, הנדרש כדי להזיז את קורה CD.
הוכיחו את תשובתכם

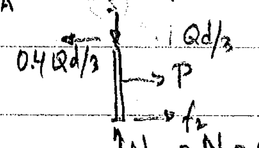


Goes @ D first

$f = -A_x$

$A_y + N = Q.$

$\curvearrowleft \Sigma M_A = -Qd + N\cdot 3 = 0 \quad N = Qd/3 \qquad f = 0.4Qd/3$



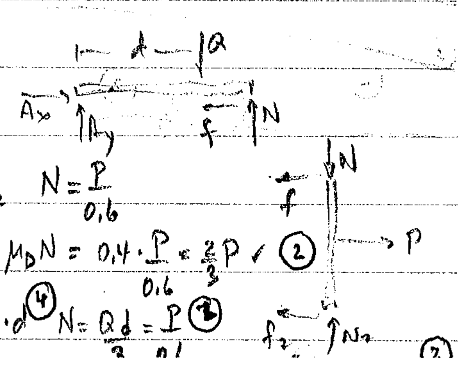
$\Sigma F_y \quad f_2 + P = 0.4Qd/3$

$N_2 = N = Qd/3 \quad \curvearrowleft \Sigma M_C = -P\cdot 1/2 + 0.4Qd/3 \cdot 1 = 0$

$P = 0.8Qd/3$

$f_2 = -0.4Qd/3$

$f_2 > \mu_C N_2 = 0.3Qd/3$



Goes at C first

$A_x = f = P/2 \quad N = \frac{P}{0.6}$

$P/2 = f < \mu_D N = 0.4 \cdot \frac{P}{0.6} = \frac{2}{3}P$ ✓ (2)

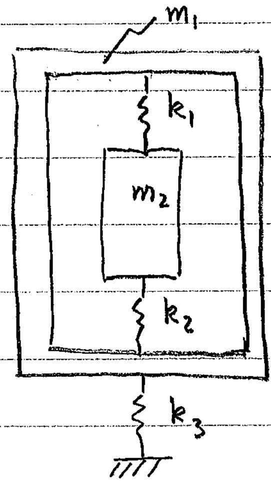
$\Sigma M_A \quad N\cdot 3 = Q\cdot d$ (4) $N = \frac{Qd}{3} = \frac{P}{0.6}$ (2)

$N_2 = N$ (3)

$P = f + f_2$ (3)

$-P/2 + f\cdot 1 = 0$ (4) $\quad f = P/2$ (2) $\Rightarrow f_2 = P/2$ (2) $= 0.3N_2$ (2)

$N_2 = \frac{P}{0.6} = N$



15 נקודות נוספות

כתבו את המשוואות הדיפרנציאליות
של המערכת בעלת שתי דרגות
חופשיות, אם למסות m_1 ו- m_2 יש
קשיחות אין סוף. רק להתייחס
לתנועה אנכית.

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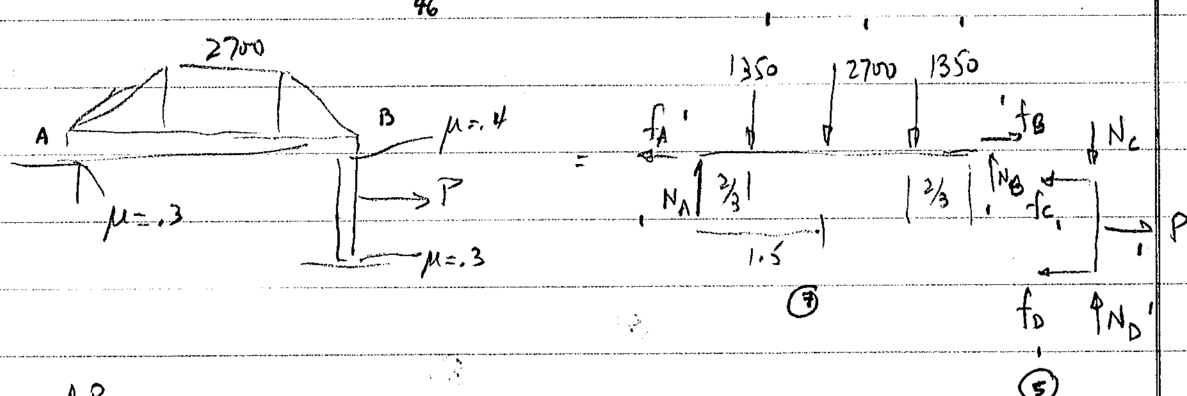

AB

$\Sigma F_x = 0 \quad f_A = f_B$ ②

$\Sigma F_y = 0 \quad N_A + N_B = 5400$ ③

⑧

$\Sigma M \quad \Sigma M_A = N_B \cdot 3 - 1350\left(3 - \tfrac{2}{3}\right) - 2700(1.5) - 1350\left(\tfrac{2}{3}\right) = 0$

$N_B \cdot 3 - 2700(3) = 0 \qquad N_B = 2700$ ② $\quad N_A = 2700$ ②

$N_B = N_C = 2700$

$\Sigma F_y = 0 \quad CD \qquad N_C = N_D = 2700$ ③

$\Sigma F_x = 0 \qquad P = f_C + f_D$ ③

$\Sigma M \qquad \Sigma M_C = P/2 - f_D \cdot 1 = 0$ ④ $\quad f_D = P/2$ ② $\Rightarrow \quad f_C = P/2$ ②

$f_D > 810 = .3 N_D \quad \text{ו-} \quad f_D = 1080$ // $P = 2f_c = 2160 \leftarrow \quad f_c = .4N_c = 1080$ אם ב- C

D הולך קודם