ינן קת הגדסית

INDEX

המכללה האקדמית יהודה ושומרון
הנדסה אזרחית

מכניכת הנדסית (40611) תשס"א

פרופ' לוי עזרא

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2. שיווי משקל של חלקיק
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ספרות

1. Beer, F.P., and Johnson, E.R., *Vector Mechanics for Engineers-Statics*, SI Metric Edition, McGraw-Hill
2. Hibbeler, R.C., *Engineering Mechanics-Statics*, McMillan Publishing
3. Meriam and Kraige, *Engineering Mechanics-Statics*

תרגילי בית - להשלימם אבל לא לחזירם למרצה. חשוב לעשותם כדי להתכונן לבוחני תרגילי בית.
אם לא תוכל להיות נוכח בבוחן תרגילי בית, (כמו מילואים, נסיעות למען עבודתך או מחלות)
עליך להתקשר לפרופ לוי מיד ולהודיע לו. הודעות אחרי בוחן כלשהו לא יתקבלו

הציונים תלויים בשלוש דברים:

1. **3** בוחנים מתרגילי בית מחושבים כ-**5%** כל אחד
2. בוחן אמצע סמסטר מחושב כ-**35%**
3. מבחן סופי מחושב כ-**50%**

משרדו של פרופ' לוי נמצא בבניין **2** חדר **8**. שעות קבלה: יום ב בין השעות **12:00-15:00**
מס' הטל: **03-9066211** במשרד, או בזמן חרום **053-771247**

אם יהיו שינויים לסילבוס, קהל הסטודנטים ייודע בו זמני

Suggested Problem Solution Format

Given: *A concise statement of the information supplied.*

Find: *A concise statement of the information sought.*

A sketch of the original system should be included where helpful.

Solution:

- *Make any additional sketches necessary (FBDs) labeling dimensions and coordinate systems needed.*
- *Working from the diagrams, list the basic equations and assumptions to be employed.*
- *Using the equations, work through the analysis, simplifying the algebra as much as possible before substituting the numeric values. (Much insight into the dynamics of a system can be gained from the algebraic form.)*
- *Substitue the proper numerical values to obtain a numerical result.*
- *Inspect the answer to make sure it is reasonable. (Does it at least have the proper magnitude?)*

Note: The neater and more organized the work is kept, the easier it is to learn the material!

gives the student a chance to concentrate on the method and not the unique application. It may help to think, metaphorically, that this book is the step-stool to the ladder of a textbook.

In the opinion of the authors, work done neatly and in an organized ~~manor~~ manner will help a student grasp the material more quickly than if little care is taken in performing the solution. To that end, a suggested problem-solution format has been supplied, which will help the student who is having difficulties in organizing the information contained within a problem. By separating the process into a series of steps, the student can start the problem and progress through it level by level. The first few steps require a thorough examination of the problem, which helps to dissipate the anxiety of "where do I start?" Once mastered on simpler problems, the same technique will help in successfully solving more difficult ones.

The suggested solution format begins by isolating the information supplied in the problem--information in the form of the mass, weight, velocities, accelerations, magnitudes of forces, or specific dimensions. Rather than recopying the problem statement as written, an attempt should be made to read, digest, and decide what information is important. Then, rewriting the information in the student's own words starts a thought process that eventually will lead to the solution.

Reading through the problem a second time, the student isolates the information to be found. This information too, is listed as part of the solution process.

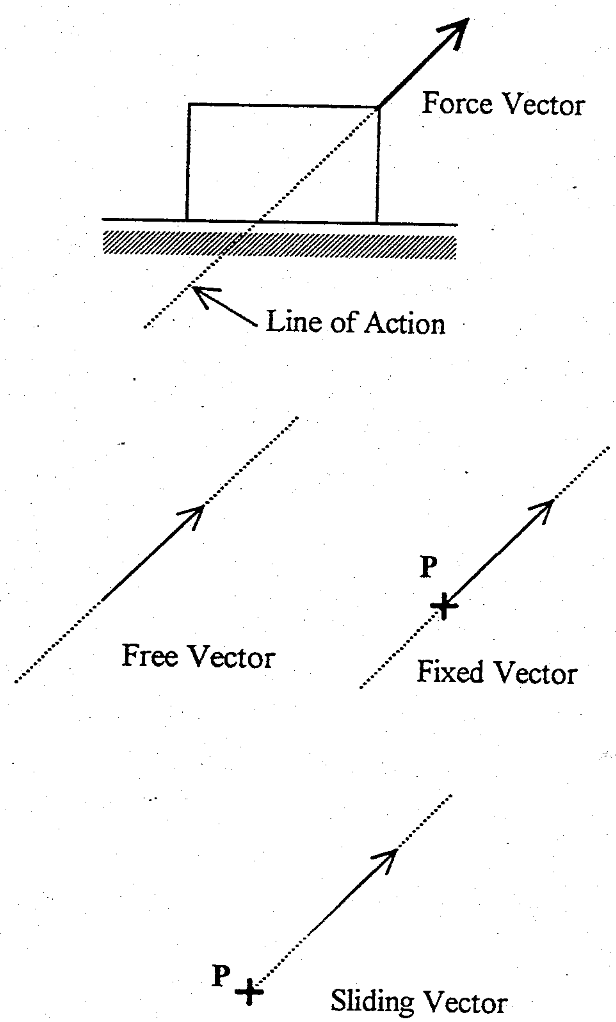
A sketch of the system is usually helpful. This might include a general sketch of the system as supplied in the problem, followed by a free-body diagram, or any other sketches showing important geometric relationships. These sketches should be done as accurately and neatly as possible since they are important tools in the coordination of the supplied information.

Chapter 2

Concurrent Force Systems

Introduction

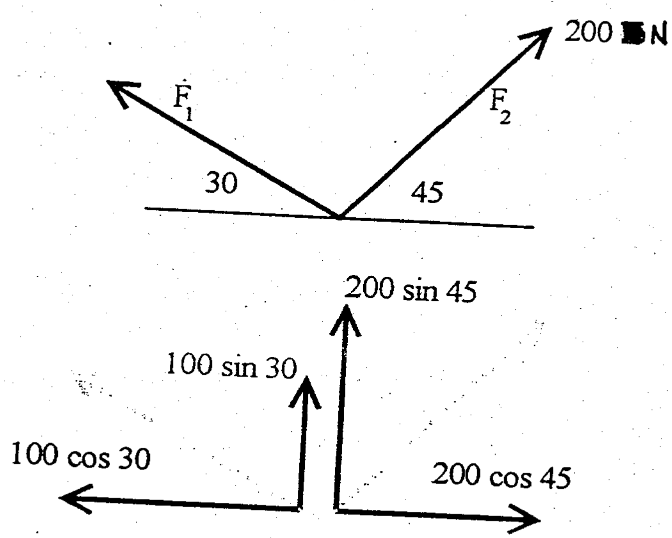
A number of forces acting on a body is referred to as a system of forces. Forces are depicted as vectors. A line that extends this vector is known as the line of action. This is the line of action referred to on page 29 of the text. The three types of vectors are illustrated to the left.



Definitions

Vector	a quantity that has both magnitude and direction
עקרון העברה Principal of Transmissibility	the property of a force that allows it to be moved anywhere along its line of action if only external forces are of concern
כוח מפורס Distributed Force	a force applied over an area
Concentrated or Point Force כוח מרוכז	a force applied to an area that is so small that it may be considered as a point
Concurrent Forces כוחות שותפי נקודה	forces whose line of action pass through a common point

Rectangular Components רכיבים קרטיזיאניים	components which are 90° apart.
Free Vector וקטור חופשי	a vector that can be moved to any location without changing its net effect
Sliding Vector וקטור החלקתי	a vector that is confined to act anywhere along a specific line
Fixed Vector וקטור צמוד	a vector that must act at only one specific point



Resultant of Forces

Example 1

Determine the magnitude and direction of the resultant force for the system of forces shown.

Given:

Forces F_1 and F_2 of given magnitude and direction

Find:

The magnitude and direction of the resultant force

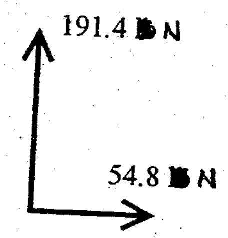
Solution:

Assumptions:

The forces are coplanar

The first step is to break the two forces into components. Rectangular components with the same axes are used. Adding these components yields:

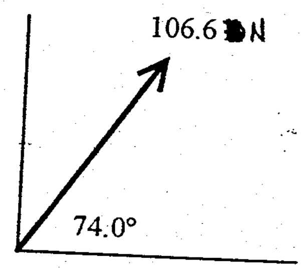
$\Sigma F_x = 200 \cos 45° - 100 \cos 30° = 141.4 - 86.6 = 54.8$ [redacted]N
$\Sigma F_y = 200 \sin 45° + 100 \sin 30° = 141.4 + 50.0 = 191.4$ [redacted]N



We now find the resultant of the vectors as

$$R^2 = 54.8^2 + 191.4^2 = 39{,}697$$
R = 199.1 [redacted] N

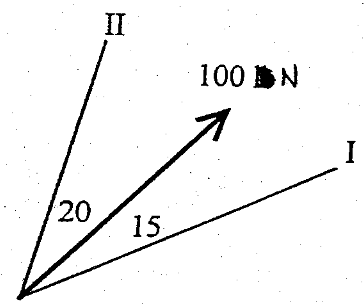
The direction of the force is now found



$$\tan\theta = 191.4/54.8 = 3.49$$
$$\theta = 74.0°$$

Comments : Here the technique of representing forces as components and then adding these components together was applied. The resultant components were then transformed back into a single resultant. This procedure will work for any number of concurrent forces.

Example 2



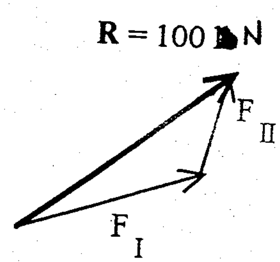
Resolve the 100 [redacted] N force into two components along the axes I and II shown.

Given:

A force of magnitude of 100 [redacted] N acting in the direction shown two non-perpendicular axes along which the components of the force are to be found

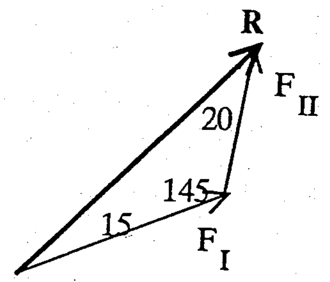
Find:

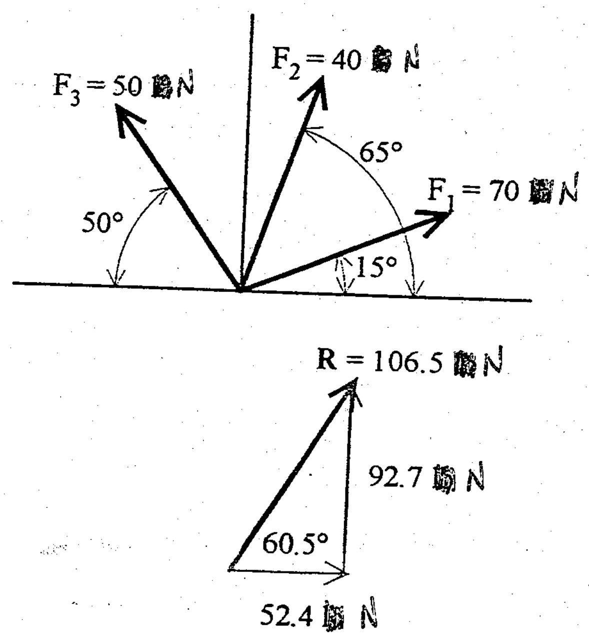
The components along the axes shown



Solution:

Since the two axes are not perpendicular a different approach must be used. Before proceeding, an examination of what is to be done will be helpful. Note that we have by vector addition





$F_i + F_{ii} = R$

This is exactly what was done in the previous problem. Now take the two components and form a parallelogram. The angles are determined using similar triangles. Looking at the lower triangle and using the law of sines :

$100/\sin 145° = F_i/\sin 20° = F_{ii}/\sin 15°$

$F_i = 100 \sin 20°/\sin 145° = 59.6$ [redacted] N
$F_{ii} = 100 \sin 15°/\sin 145° = 45.1$ [redacted] N

Example 3

Three forces act as shown; determine their resultant. כוח שקול

Given:

Three forces as shown

Find:

Their resultant force

Solution:

Assumptions:

The forces are coplanar
The forces act concurrently (through the same point)

First it must be shown that the forces are concurrent. To show this concurrency, it is observed that the three forces all pass through a common point. Now separate the forces into their rectangular components:

$F_1 = 70\cos 15° \Rightarrow + 70\sin 15° \Uparrow = 67.6$ [redacted] N $\Rightarrow + 18.1$ [redacted] N $\Uparrow$
$F_2 = 40\cos 65° \Rightarrow + 40\sin 65° \Uparrow = 16.9$ [redacted] N $\Rightarrow + 36.3$ [redacted] N $\Uparrow$

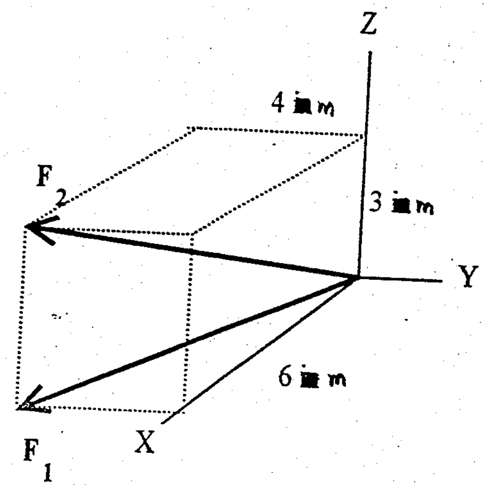
$F_3 = 50\cos 50° \Leftarrow + 50\sin 50° \Uparrow = 32.1$ [redacted] N $\Leftarrow + 38.3$ [redacted] N $\Uparrow$

$R_x = 67.6 + 16.9 - 32.1 = 52.4$ [redacted] N $\Rightarrow = 52.4\ i$ [redacted] N
$R_Y = 18.1 + 36.3 + 38.3 = 92.7$ [redacted] N $\Uparrow = 92.7\ j$ [redacted] N

$R^2 = 52.4^2 + 92.7^2 = 11{,}339$
$R = 106.5$ lb

$\tan\theta = 92.7/52.4 = 1.77$
$\theta = 60.5°$

Note that the resultant must also act through point O.



Example 4

Two forces are oriented in a three-dimensional coordinate system as shown. Determine the angles θ_1 and θ_2 respectively that the two given forces act at.

Given :

Magnitude of the two forces
$F_1 = 20$ [redacted] kN
$F_2 = 10$ [redacted] kN
Direction of the two forces in x,y,z coordinates
(See figure)

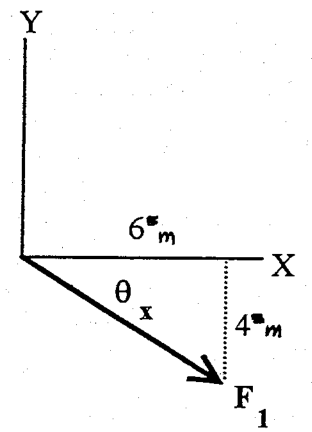
Find:

The angles at which the two forces act

Solution:

Starting with force F_1. In the x-y plane it is observed that

$d^2 = 4^2 + 6^2 = 52$
$d = 7.21$



$$\cos\theta_x = 6 / 7.21 = 0.832 \qquad \theta_x = 33.7°$$

Alternatively the angle could ahve been computed as

$$\tan\theta_x = 4 / 6 = 0.667 \qquad \theta_x = 33.7°$$

However, the cos method was used since this method works in three dimensions. As an illustration examine $\mathbf{F_2}$. $\mathbf{F_2}$ can be written in vector form as

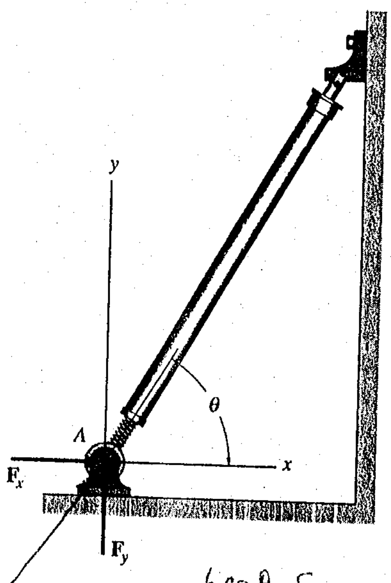
$$\mathbf{F_2} = 10\,[\,6\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}] / [\,6^2 + 4^2 + 3^2\,]^{1/2}$$
$$= 10\,[\,6\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}] / 7.81 \quad \mathbf{kN}$$

The denominator is the resultant of the three displacemet components. Thus

$$\cos\theta_x = 6 / 7.81 = 0.768 \qquad \theta_x = 39.8°$$
$$\cos\theta_y = 4 / 7.81 = 0.512 \qquad \theta_y = 59.2°$$
$$\cos\theta_z = 3 / 7.81 = 0.384 \qquad \theta_z = 67.4°$$

Thus the force $\mathbf{F_2}$ is rotated 39.8° from the x-axis, 59.2° from the y-axis, and 67.4° from the z-axis. Note that the magnitude of force has no implication in the calculation of the directional cosines. The magnitude of the brackets has a value of unity or one.

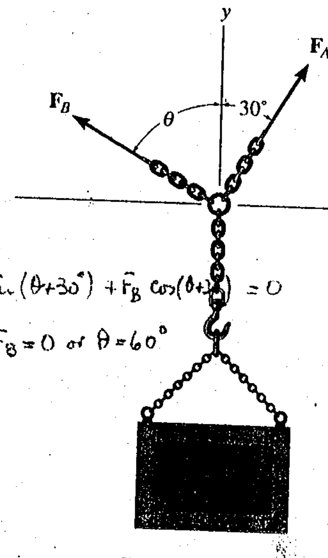
*2–48. The brace is used to support the wall. When used for this purpose, the pin exerts a horizontal force F_x and a vertical force F_y on the brace at A. If the maximum resultant force that can be developed along the brace is 6 kN, and the ratio $F_x/F_y \leq 0.5$, determine the minimum angle θ of placement for the brace.



Prob. 2–48

2–49. The crate is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45°$.

2–50. The crate is to be hoisted using two chains. If the resultant force is to be 600 N, directed along the positive y axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the orientation θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the y axis as shown.



Probs. 2–49/2–50

A (2, −6, 4) $r_{B/A} = r_B - r_A$
B (−1, 6, 0) $r_{B/A} = -3\underline{i} + 12\underline{j} - 4\underline{k}$ $r_{B/A} = 13$

$\underline{u} = -\frac{3}{13}\underline{i} + \frac{12}{13}\underline{j} - \frac{4}{13}\underline{k}$

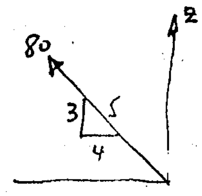
$\underline{F} = F\underline{u} = 24\,kN\left(-\frac{3}{13}\underline{i} + \frac{12}{13}\underline{j} - \frac{4}{13}\underline{k}\right) = -5.54\underline{i} + 22.15\underline{j} - 7.38\underline{k}$ kN

$\alpha = \cos^{-1}\left(-\frac{3}{13}\right) = 103.3°$

$\beta = \cos^{-1}\left(\frac{12}{13}\right) = 22.6°$

$\gamma = \cos^{-1}\left(-\frac{4}{13}\right) = 107.9°$

2-39



$\underline{F}_1 = F_1\underline{u}$

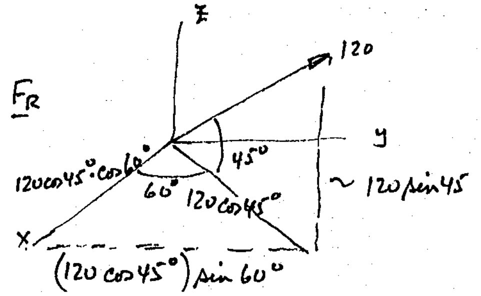
$\underline{u} = \frac{4}{5}\underline{i} + \frac{3}{5}\underline{k}$

$\underline{F}_1 = 80\left(\frac{4}{5}\underline{i} + \frac{3}{5}\underline{k}\right) = 64\underline{i} + 48\underline{k}$ N

$\alpha = \cos^{-1}(4/5) = 36.9°$

$\beta = 90°$

$\gamma = \cos^{-1}(3/5) = 53.1°$



$\underline{F}_R = 120\left[\sin 45°\underline{k} + \cos 45°\cos 60°\underline{i} + \cos 45°\sin 60°\underline{j}\right]$

$\left[\cos 45°\underline{k} + (.707)(.5)\underline{i} + (.707)(.866)\underline{j}\right]$

$84.9\underline{k} + 42.5\underline{i} + 73.5\underline{j}$

$\gamma = 45°$

$\alpha = \cos^{-1}(.354) = 69.3°$

$\beta = \cos^{-1}\left(\frac{\sqrt{6}}{4}\right) = 52.2°$

2-60 A = (0,0,35) B = (25, 43.3, 0) $\therefore \underline{r}_{B/A} = 25\underline{i} + 43.3\underline{j} - 35\underline{k}$

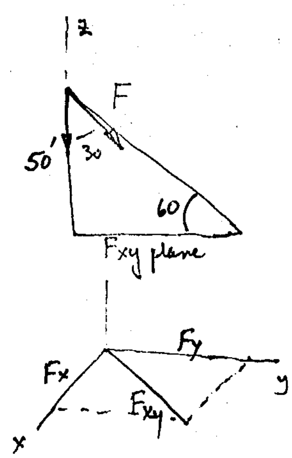
$r_{B/A} = \sqrt{25^2 + (43.3)^2 + (35)^2} = 5\sqrt{149} = 61$ m

$\underline{u} = \frac{\underline{r}}{r} = \frac{5}{\sqrt{149}}\underline{i} + \frac{43.3}{5\sqrt{149}}\underline{j} - \frac{7}{\sqrt{149}}\underline{k} = 0.41\underline{i} + .71\underline{j} - 0.57\underline{k}$

$\therefore \underline{F} = F\underline{u} = 350\left[\frac{5}{\sqrt{149}}\underline{i} + \frac{43.3}{5\sqrt{149}}\underline{j} - \frac{7}{\sqrt{149}}\underline{k}\right]$

$= 143.5\underline{i} + 248.5\underline{j} - 199.5\underline{k}$

2-68



$\frac{F_z}{F} = \cos 30°$ or $\frac{80}{F} = 0.866$ $F = 92.4$ lb

$\frac{F_{xy}}{F} = \sin 30°$ or $F_{xy} = 92.4(0.5) = 46.2$ lb

$F_x = F_{xy}\cos 50° = 29.7$ lb

$F_y = F_{xy}\sin 50° = 35.4$ lb

$\underline{F} = 29.7\underline{i} + 35.4\underline{j} - 80\underline{k}$
$F = 92.4$
$\Big\}$ $\underline{u} = 0.32\underline{i} + 0.38\underline{j} - 0.87\underline{k}$

$\alpha = \cos^{-1}(.32) = 71.3°$

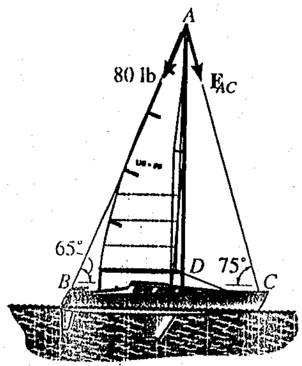
$\beta = \cos^{-1}(.38) = 67.7°$

$\gamma = \cos^{-1}(-.87) = 150°$

תורן סירה

2–13. The mast of the sailboat is subjected to the force of the two cables. If the force in cable AB is 80 lb, determine the required force in AC so that the resultant force caused by both cables is directed vertically downward along the axis AD of the mast. Also, calculate this resultant force.

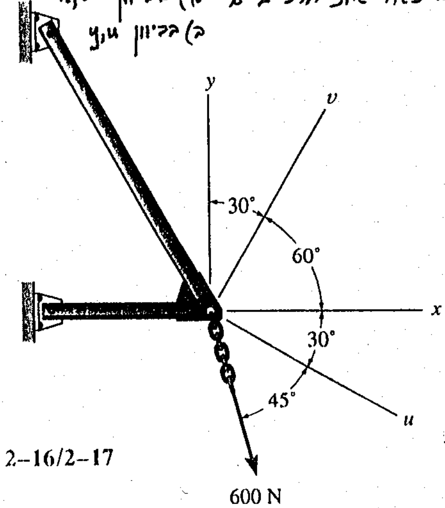
מצאו השקול



Prob. 2–13

2–17. The cable exerts a force of 600 N on the frame. Resolve this force into components acting *(a)* along the x and v axes and *(b)* along the y and u axes. What is the magnitude of each component?

מצאו את הרכיבים א) בכיוון x,v
ב) בכיוון y,u

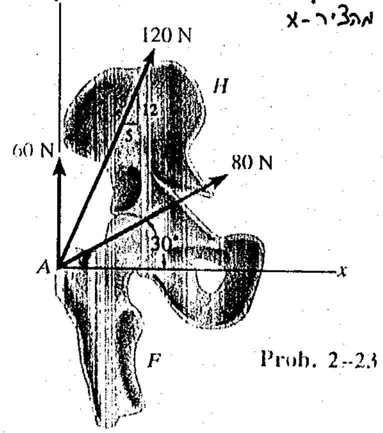


Probs. 2–16/2–17

$F_x = 490\,N$

2–23. The hipbone H is connected to the femur F at A using three different muscles, which exert the forces shown on the femur. Determine the resultant force on the femur and specify its orientation θ measured counterclockwise from the positive x axis.

שלושה שרירים קושרים עצם אחת לשניה. מצאו את השקול
והכיוון θ מהציר-x

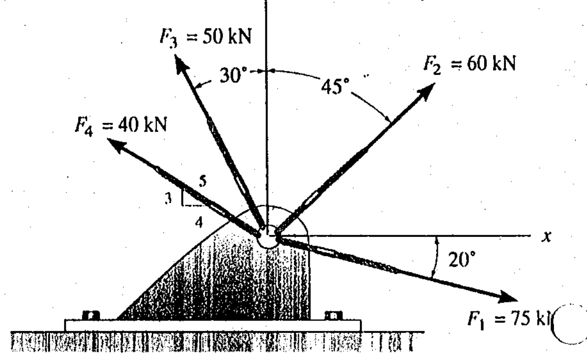


Prob. 2–23

$F_R = 240.31\,N$

***2–40.** Express each force acting on the bracket in Cartesian vector form. What is the magnitude of the resultant force?

מצאו ביטוים לכל כוח בצורה קרטזית.
מהו השקול



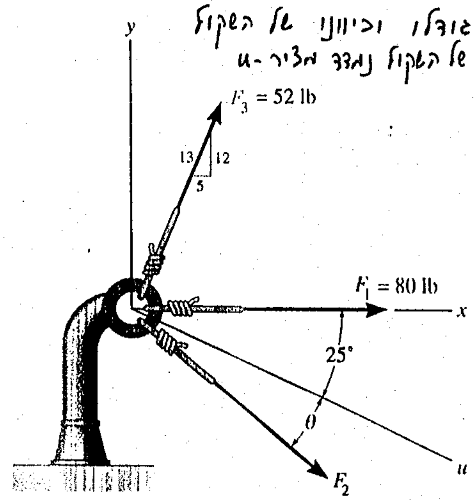
Prob. 2–40

$F_4 = -32\underline{i} + 24\underline{j}\ kN$

$F_x = 55.91\,kN$

orientation, measured clockwise from the positive u axis, of the resultant force of the three forces acting on the bracket.

מצאו גודלו וכיוונו של השקול
כיוונו של השקול נמדד מציר u-

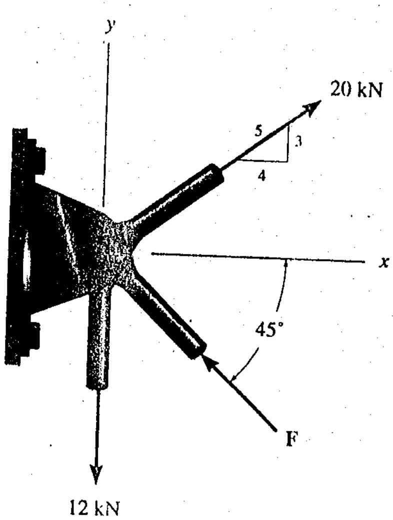


Probs. 2–43/2–44

F_R = 160.73 lb.

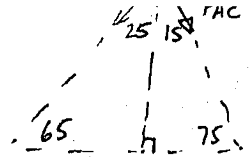
2–51. Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible.

מצאו גודלו של F כדי שהשקול יהיה מינימלי



Prob. 2–51

F_R = 11.31 kN



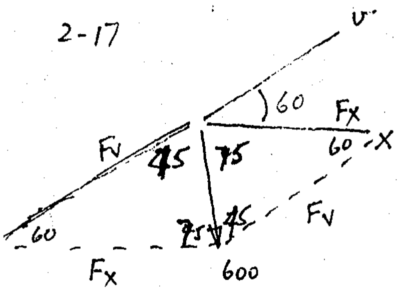
$F_{AC} \sin 15 = 80 \sin 25^\circ$

$F_{AC} = 130.63 \text{ lb}$

Resultant force $= 80 \cos 25 + F_{AC} \cos 15$

$= 198.68 \text{ lb}$

2-17

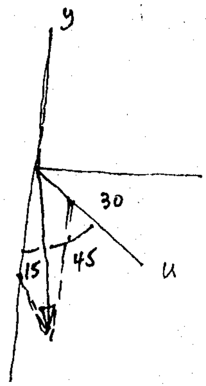


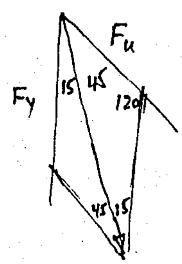
using trig get angles then use law of sines

$$\frac{600}{\sin 60} = \frac{F_v}{\sin 75} = \frac{F_x}{\sin 45}$$

$F_x = 489.9 \text{ N}$

$F_v = 669.2 \text{ N}$



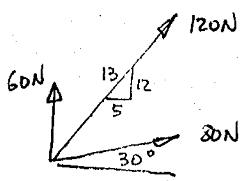


$$\frac{600}{\sin 120} = \frac{F_y}{\sin 45} = \frac{F_u}{\sin 15}$$

$F_y = 489.9 \text{ N}$

$F_u = 179.32 \text{ N}$

2-23

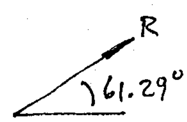


$\Sigma F_x = 120 \cdot \frac{5}{13} + 80 \cos 30^\circ = 115.44 \text{N}$

$\Sigma F_y = 60 + 120 \cdot \frac{12}{13} + 80 \sin 30 = 210.77 \text{N}$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 240.31 \text{ N}$

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = 1.826 \qquad \theta = 61.29^\circ$ from horizontal



2-40

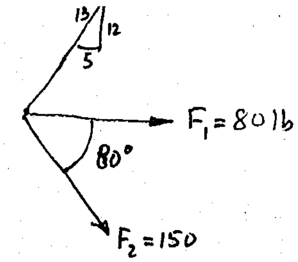
$F_1 = 75(\cos 20\,\underline{i} - \sin 20\,\underline{j}) = 70.48\,\underline{i} - 25.65\,\underline{j}$ kN

$F_2 = 60(\cos 45\,\underline{i} + \sin 45\,\underline{j}) = 42.43\,\underline{i} + 42.43\,\underline{j}$ kN

$F_3 = 50(-\cos 60\,\underline{i} + \sin 60\,\underline{j}) = -25\,\underline{i} + 43.3\,\underline{j}$ kN

$F_4 = 40(-\tfrac{4}{5}\,\underline{i} + \tfrac{3}{5}\,\underline{j}) = -32\,\underline{i} + 24\,\underline{j}$ kN

$\underline{R} = 55.91\,\underline{i} + 84.08\,\underline{j} \qquad R = \sqrt{(55.91)^2 + (84.08)^2} = 100.97 \text{ kN}$



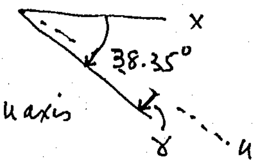
$\Sigma F_x = 52 \cdot \frac{5}{13} + 80 + 150 \cos 80 = 126.05$ lb

$\Sigma F_y = 52 \cdot \frac{12}{13} - 150 \sin 80 = -99.72$ lb

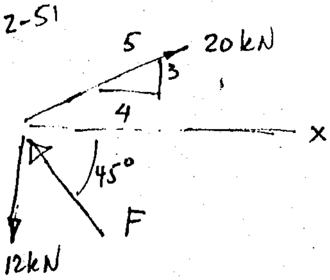
$R = \sqrt{(126.05)^2 + (99.72)^2} = \underline{160.73 \text{ lb}}$

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = -.7911 \qquad \theta = -38.35°$ from horizontal

$\gamma = 38.35 - 25 = \underline{13.35°}$ below u axis



2-51



$\Sigma F_x = 20 \cdot 4/5 - F\cos 45 + 0 = 16 - F\cos 45°$

$\Sigma F_y = 20 \cdot 3/5 - 12 + F\sin 45° = F\sin 45°$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{256 - 32F\cos 45 + F^2\cos^2 45 + F^2\sin^2 45}$

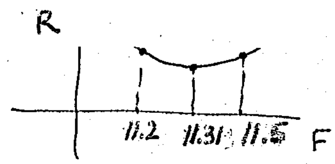
$R = \sqrt{256 - 32F\sin 45 + F^2}$ since $\sin 45 = \cos$

for smallest resultant $\frac{dR}{dF} = 0 = \frac{1}{2} \frac{(-32\sin 45 + 2F)}{\sqrt{256 - 32F\sin 45 + F^2}} \Rightarrow F = 16\sin 45 = \underline{11.31}$

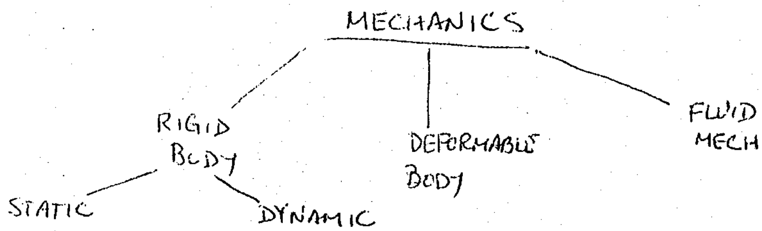
$\therefore \quad \left.\begin{array}{l} \Sigma F_x = 8.00 \\ \Sigma F_y = 8.00 \end{array}\right\} R = 8.00\sqrt{2} = 11.31$ kN

to check: if $F = 11.5$ kN $\quad \left.\begin{array}{l} \Sigma F_x = 7.87 \\ \Sigma F_y = 8.13 \end{array}\right\} R = 11.32$ kN

if $F = 11.2$ kN $\quad \left.\begin{array}{l} \Sigma F_x = 8.08 \\ \Sigma F_y = 7.92 \end{array}\right\} R = 11.314$ kN



Mechanics - Branch of the physical sciences concerned with the state of rest or motion of bodies that are subject to Forces



WILL STUDY Rigid Body Mechanics - forms a basis for design & analysis of many problems & provides background for the other two areas of mech

STATICS - Body is either at rest or moving at a constant velocity
STRUCTURES - bodies that are at rest.

Rigid Body Mechanics - based on laws of SIR ISAAC NEWTON

1. First Law - body at rest or at a constant velocity remains at rest or continues moving at constant velocity until a force acts upon it to change its motion. (axiom)

2. Second Law - If a force $\vec{F}$ acts upon the body it will experience an acceleration $\vec{a}$ in the direction of the force. The relation of $\vec{F}$ to $\vec{a}$
$\vec{F} = m\vec{a}$ m - being the mass of the body.

3. Third law - for every action there is an equal & opposite reaction
Force on particle = force that particle exerts in return
Force is collinear and opposite

Transmissibility - move a force along its line of action without changing motion of the body

4. Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2} = mg \qquad G = 6.673 \times 10^{-11}\ m^3/kg\,sec^2$$

m_1 -

near the surface of the earth $\frac{Gm_2}{R^2} = g = 9.8\ m/sec^2$

$W = F = mg$ m_2 = MASS OF EARTH R = RADIUS OF EARTH

כדור הארץ במצב שטוח

- ASSUME A FLAT EARTH MODEL
  - TYPICAL DISTANCES ARE SMALL IN COMPARISION TO EARTH'S RADIUS
  - CURVATURE WILL NOT AFFECT RESULTS

<u>IDEALIZATIONS</u> - TO SIMPLIFY THE APPLICATION OF THE THEORY

חלקיק PARTICLE - WILL HAVE MASS BUT NO SHAPE

RIGID BODY - A CONGLOMERATION OF PARTICLES, EACH PARTICLE REMAINS AT A FIXED DISTANCE FROM NEIGHBORING PARTICLES BEFORE & AFTER LOAD APPLICATION

כוח מרוכז CONCENTRATED FORCE - FORCE ACTING ON A POINT ON THE BODY. GOOD IF LOADING AREA IS SMALL IN COMPARISON TO THE OVERALL SIZE OF THE BODY.

תיק כוח מפוס

UNITS OF MEASUREMENT - are not <u>arbitrary</u> (שרירותי); connection is $\vec{F} = m\vec{a}$

	LENGTH	METER (m)	FEET (FT)	DISTANCE
זמן	TIME	SECOND (s)	"	
מסה	MASS	KILOGRAM (kg)	SLUGS $\left(\frac{LB\text{-}SEC^2}{FT}\right)$	התנגדות Resistance of matter to a change of velocity
כוח	FORCE	NEWTON (N)	POUNDS (LB)	PUSH OR PULL exerted on a body

$N = kg\text{-}m/sec^2$

מהירות וקטורית	velocity	m/sec	ft/sec
תאוצה	acceleration	m/sec^2	ft/sec^2
תאוצת גרביטציה	acceleration of gravity (g)	$9.81\ m/sec^2$	$32.2\ ft/sec^2$

MASS can be measured at <u>any</u> location; weight is dependent on the gravity field

BE CONSISTENT with your units

CHAPTER 2 נוכל לבטא את הכמויות הפיסיקליות ככמויות סקלריות או ו

Physical quantities can be expressed as either scalars or vectors

סקלאר - a SCALAR is a quantity having only a magnitude. ~~Scalars~~

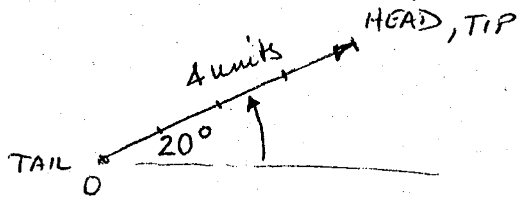
- Scalars can be added, multiplied, divided and subtracted in the usual manner

EX: VOLUME, LENGTH, MASS   דוגמאות: נפח, אורך, מסה

וקטור VECTORS quantity that has magnitude, direction and obeys the parallelogram law of addition   לחבר אותם בחוק המקבילית

Represented by $\vec{A}$: vector is represented by an arrow whose magnitude is length of vector. Direction is direction the arrow points measured from some fixed reference line.

ציר ספציפי כמו
ציר אופקי x
במערכת קרטזית

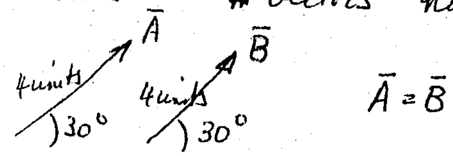


TYPES OF VECTORS

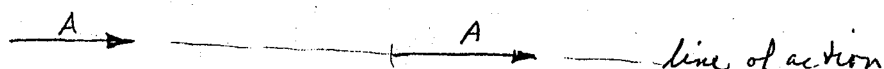
FIXED VECTOR - a vector acting at a fixed point in space

FREE VECTOR - can act anywhere in space only preserves magnitude and direction

Equal vectors - vectors having same magnitude & direction wrt same ref. line

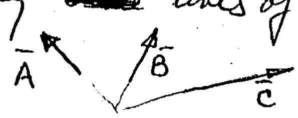


וקטור החלקה Sliding vector - can be moved along its line of action

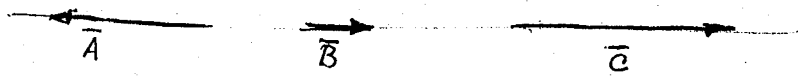


וקטורים מישוריים COPLANAR VECTORS - vectors acting in the same plane

וקטורים משותפי - נקודה CONCURRENT Vectors - having ~~same~~ lines of action pass through same pt in space

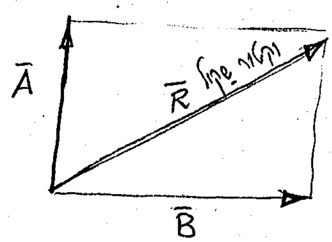


וקטורים מתואמי-קו COLLINEAR VECTORS - having the same line of action (קו פעולה)

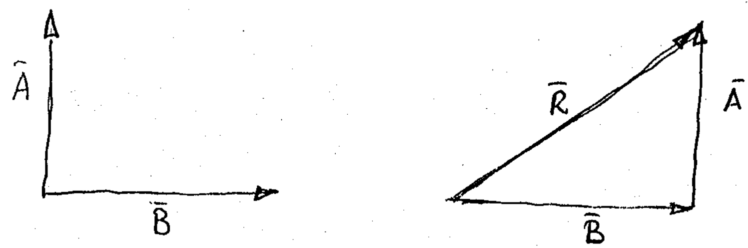


PARALLELOGRAM LAW

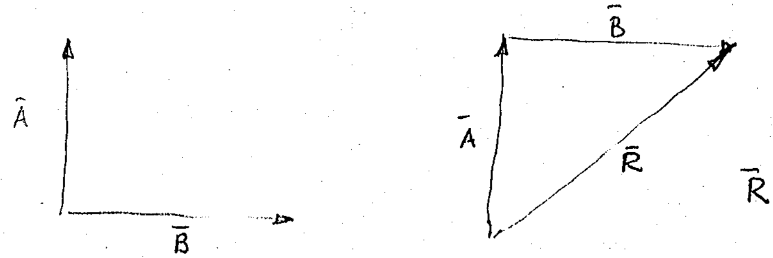
$\bar{A}+\bar{B} = \bar{R}$



slide $\bar{A}$ over so that the tail of $\bar{A}$ & the tip of $\bar{B}$ are the same pt, keeping the magnitude & direction of $\bar{A}$ unchanged



Connect tail of $\bar{B}$ & tip of $\bar{A}$ to get $\bar{R}$

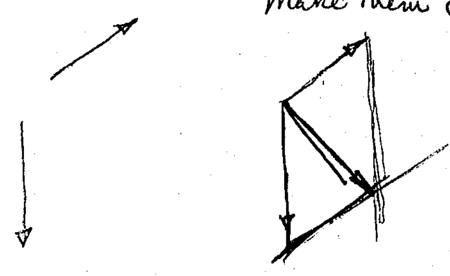


Note addition is commutative

$\bar{R} = \bar{A}+\bar{B} = \bar{B}+\bar{A}$

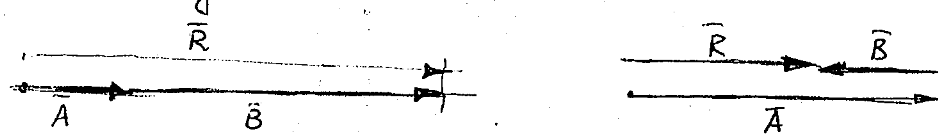
for Non concurrent vectors

Make them concurrent



Form the parallelogram

Addition of Collinear Vectors

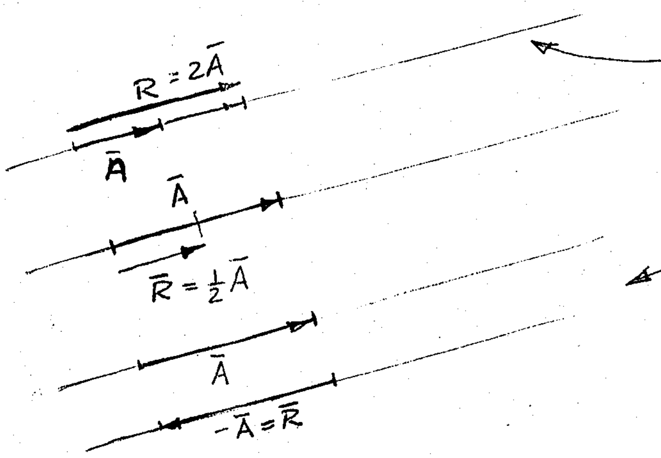


Scalar multiplication or division of a vector. – has same direction but different magnitude; if the scalar is negative direction is opposite

$\bar{R} = n\bar{A}$

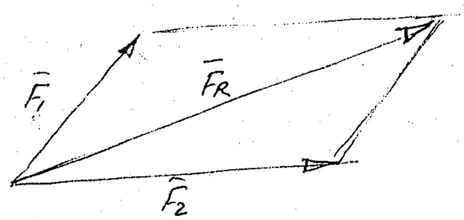
$\bar{R} = 2\bar{A}$ $\qquad \bar{R} = \frac{1}{2}\bar{A} = \bar{A}/2.$

$\bar{R} = -\bar{A}$



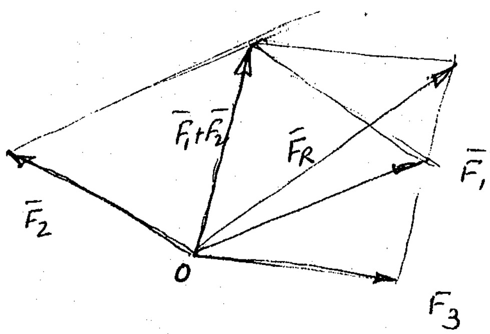
FORCES ARE VECTORS & SATISFY PARALLELOGRAM LAW

- IN STATICS NORMALLY EITHER YOU WANT TO FIND THE RESULTANT FORCE ACTING ON A BODY
  OR KNOWING RESULTANT FORCE (כוח שקול) WANT TO FIND THE COMPONENT (גם רכיב) ALONG GIVEN LINES OF ACTION
  לפי קווי הפעולה הנתונים לנו



IF MORE THAN TWO FORCES ARE ACTING WE CAN USE THE PARALLELOGR LAW MORE THAN ONCE

$\bar{F}_R = \bar{F}_3 + (\bar{F}_1 + \bar{F}_2)$



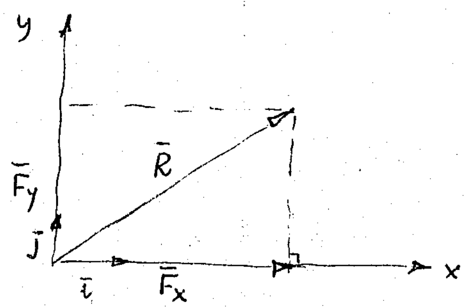
דורש גאומטריה או טריגונומטריה או הרכבה לפי שירטוט

Requires geometric & trigonometric calculation or construction.

——— – ——— – ———

ADDITION OF RECTANGULAR FORCE COMPONENTS

להבריג
לפרק



Any force can be ~~broken up~~ resolved into components along given coordinate axes

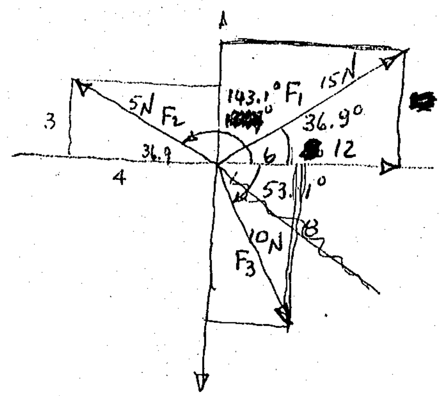
$\bar{R} = \bar{F}_x + \bar{F}_y$

$= F_x \bar{i} + F_y \bar{j}$ Cartesian Vector Form

$\vec{F}_x$ & $\vec{F}_y$ are projection onto x, y axes of $\vec{F}$

$\bar{i}$ & $\bar{j}$ are vectors in the x & y direction whose magnitude is 1 - UNIT VECTORS

are positive when pointing in the +x & +y direction



$\bar{F}_R = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

$(F_{1x}\bar{i} + F_{1y}\bar{j}) + (-F_{2x}\bar{i} + F_{2y}\bar{j})$

$+ (F_{3x}\bar{i} - F_{3y}\bar{j})$

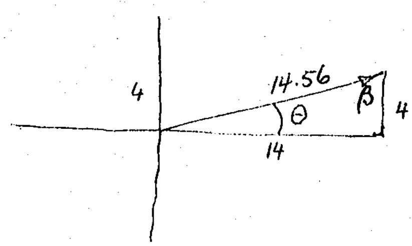
Collinear Collinear

$\bar{F}_R = (F_{1x} - F_{2x} + F_{3x})\bar{i} + (F_{1y} + F_{2y} - F_{3y})$

$= F_{Rx}\bar{i} + F_{Ry}\bar{j}$

$F_{Rx} = \sum_{k=1}^{3} F_{kx}$ $F_{Ry} = \sum_{k=1}^{3} F_{ky}$ $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

$= 12 - 4 + 6 = 14$ $3 + 9 - 8 = 4$ $\frac{F_y}{F_R} = \sin\theta, \cos\beta$, $\frac{F_x}{F_R} = \cos\theta$



$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{212} = 14.56$

$\tan\theta = \frac{4}{14} = \frac{F_{Ry}}{F_{Rx}} \Rightarrow 15.95° = \theta$

$\sin\theta = \frac{F_{Ry}}{F_R} = \frac{4}{14.56} \Rightarrow \theta = 15.95°$

1. DRAW A DIAGRAM SHOWING ALL KNOWN INFORMATION
2. DRAW IN COORDINATE AXES
3. DRAW COMPONENTS OF EACH VECTOR
4. CALCULATE DATA FOR EACH COMPONENT

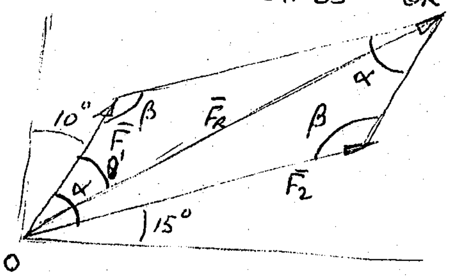
SESSION #2

Use LAW OF SINES OR LAW OF COSINES

DID NOT DO

Example 2-1

A screw eye is subjected to 2 forces. Find mag. & dir of resultant force



$\bar{F}_1 = 150\ N@$ $F_1 = 150N$

$\bar{F}_2 = 100N@$

$\bar{F}_R = ?$

parallelogram all interior angles sum to 360°

$\alpha = 90° - 10° - 15° = 65°$

$360° = 2\alpha + 2\beta = 2 \cdot 65 + 2\beta$

$230° = 2\beta \qquad \beta = 115°$

$F_R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\beta} = 212.6N$ — חוק הקוסינוסים Law of Sines

$\frac{F_R}{\sin 115} = \frac{F_2}{\sin\theta} \qquad \frac{212.6N}{\sin 115} = \frac{100N}{\sin\theta}$ חוק הסינוסים

$\sin\theta = \sin 115° \left(\frac{100N}{212.6N}\right)$

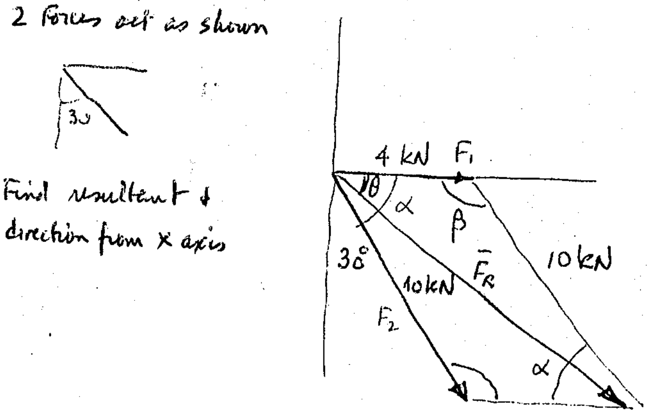
$\theta = 25.3°$

direction of F_R from the vertical is $25.3 + 10 = 35.3°$

PROBLEM 2-2

2 forces act as shown

Find resultant & direction from x axis



$\alpha = 60°$

$360 = 2\alpha + 2\beta$

$360 - 2(60) = 2\beta$

$240 = 2\beta \qquad \beta = 120°$

$F_R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\beta}$

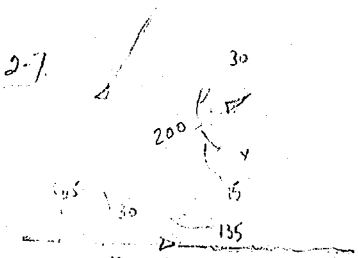
$F_R = 12.49\ kN$

$\frac{F_R}{\sin 120} = \frac{F_2}{\sin\theta}$

$\sin\theta = \sin 120 \cdot \frac{F_2}{F_R}$

$\theta = 43.9°$

PROBLEM 2-7



$\frac{200}{\sin 135} = \frac{y}{\sin 30} = \frac{x}{\sin 15°}$

$\frac{200}{.7071} = \frac{y}{.5} = \frac{x}{.2588}$

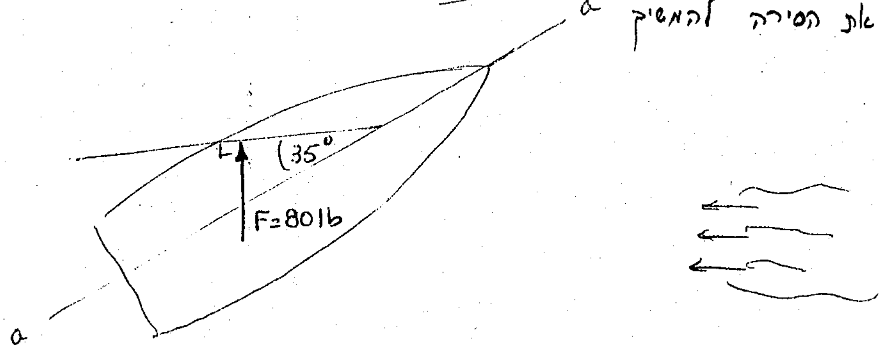
$x = 73.21\ N$

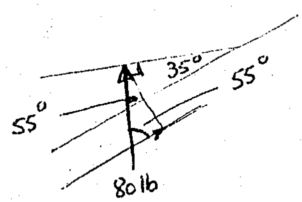
$y = 141.42\ N$

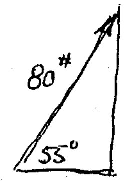
PROBLEM 2-11

DID NOT DO

רוח צדי של 80N פועל על מפרש סירה
מהי כוח רכיב שנותן את הסירה להמשיך
בכיוון שלה



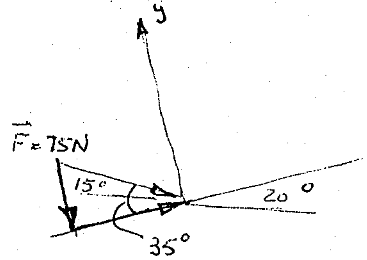




$\sin 55° = F_{\perp}/80 \Rightarrow F_{\perp} = 65.5$

$\cos 55° = F_{\parallel}/80 \Rightarrow F_{\parallel} = 45.9$

PROBLEM 2-10



$\vec{F} = F_x \vec{i} - F_y \vec{j}$

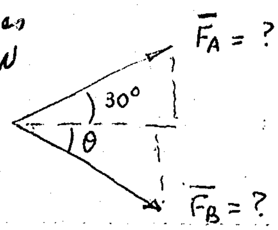
$F = 75N$

$\sin 35° = F_y/F \qquad \cos 35° = F_x/F$

$F_x = 61.4\,N \qquad F_y = 43.0\,N$

PROBLEM - 2-33 (2-23 using components)

truck is towed by 2 ropes. Find the forces F_A & F_B to develop a resultant force of 8 kN



DID NOT DO

$\sum F_{R_{x_0}} = 8kN$

$F_A \cos 30° + F_B \cos\theta = 8kN$

$\sum F_{R_y} = 0$

$F_A \sin 30° - F_B \sin\theta = 0$

$F_A = F_B \dfrac{\sin\theta}{\sin 30°}$

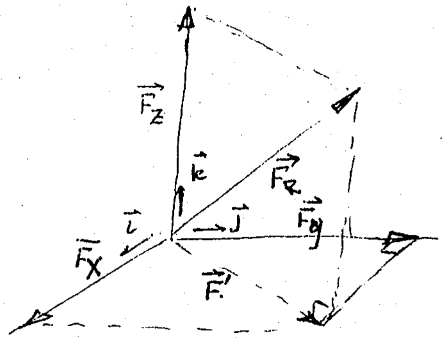
$$F_B \left[\cos 30° \frac{\sin\theta}{\sin 30°} + \cos\theta \right] = 8kN$$

$$F_B \cdot \left[1.532 \right] = 8kN \quad \text{or} \quad F_B = 5.22\,kN$$

$$F_A = F_B \frac{\sin\theta}{\sin 30°} = 3.57\,kN$$

FORCE ~~RESULTANTS~~ COMPONENTS IN 3-DIMENSIONS FOR 1 VECTOR (CAN DO FOR MULTIPLE)

כוח מרחבי



Any force can be resolved into 3 components

$$\vec{F_R} = \vec{F_x} + \vec{F_y} + \vec{F_z}$$
$$= F_x \vec{\imath} + F_y \vec{\jmath} + F_z \vec{k} \qquad \text{CARTESIAN REPR}$$

$\bar{\imath}, \bar{\jmath}, \bar{k}$ are unit vectors

Visualization in 3-D is harder than 2-D. Cartesian Form makes it easier to visualize the vector's components

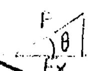
$$F' = \sqrt{F_x^2 + F_y^2} \qquad F_R = \sqrt{F_z^2 + F'^2} \quad \Rightarrow F_R = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

This gives MAGNITUDE of the vector F_R knowing its components. (רכיבים)

Direction of Force is defined by 3 angles DIRECTION COSINES

$$\cos\alpha = \frac{F_x}{F_R} \qquad \cos\beta = \frac{F_y}{F_R} \qquad \cos\gamma = \frac{F_z}{F_R}$$

use 2-D Result as a comparison



$\cos\theta = \frac{F_x}{F}$

$\vec{F_x}, \vec{F_y}, \vec{F_z}$ are the projections (היטלים) of $\vec{F_R}$ onto x, y, z axes

α, β, γ are the ~~project~~ angles between $\bar{F}$ and the projections $\bar{F_x}, \bar{F_y}, \bar{F_z}$

Suppose we take

$$(F_R)^2 = F_x^2 + F_y^2 + F_z^2 \qquad \text{divide by } F_R^2$$
$$1 = \left(\frac{F_x}{F_R}\right)^2 + \left(\frac{F_y}{F_R}\right)^2 + \left(\frac{F_z}{F_R}\right)^2$$
$$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

if 2 angles are known the 3rd is obtained from this

if we take $\vec{F_R}$ and divide it by F_R result is a vector $\vec{u}$

$$\frac{\vec{F_R}}{F_R} = \vec{u} \quad \text{is a vector of magnitude 1 directed along } \vec{F_R}$$

$$\vec{F_R} = F_R \vec{u}$$

$$\frac{F_x}{F_R}\vec{\imath} + \frac{F_y}{F_R}\vec{\jmath} + \frac{F_z}{F_R}\vec{k} = \vec{u}$$

$$\cos\alpha\,\vec{\imath} + \cos\beta\,\vec{\jmath} + \cos\gamma\,\vec{k} = \vec{u}$$

Defines $\vec{u}$ in terms of $\bar{\imath}, \bar{\jmath}, \bar{k}$

$$F_R\vec{u} = F_R\cos\alpha\,\vec{\imath} + F_R\cos\beta\,\vec{\jmath} + F_R\cos\gamma\,\vec{k} = \vec{F_R}$$

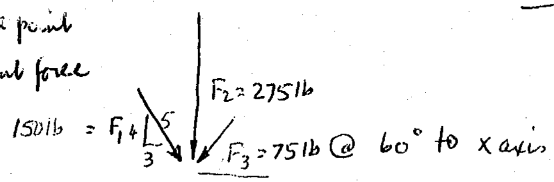
This allows us to find $\vec{F_R}$ if F_R and the angles γ, β, γ are known

Express 3 forces in cartesian form & find resultant, mag & direction

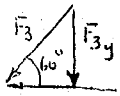
PROBLEM 2-30

DID NOT DO

3 Forces act on a point
find the resultant force

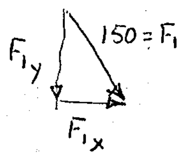


$\overline{F_2} = -275\vec{j}$
$\overline{F_3} = -37.5\,\bar{i}$ $\quad -64.95\bar{j}$
$\vec{F_1} = 90\bar{i}$ $\quad -120\bar{j}$
$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = -222.5\bar{i} \quad -184.95\bar{j}$
$= F_x\bar{i} + F_y\bar{j}$



$\frac{F_{3x}}{F_3} = \cos 60° = .5 \qquad F_{3x} = 37.5 lb$

$\frac{F_{3y}}{F_3} = \sin 60° = .866 \quad F_{3y} = 64.95 lb.$

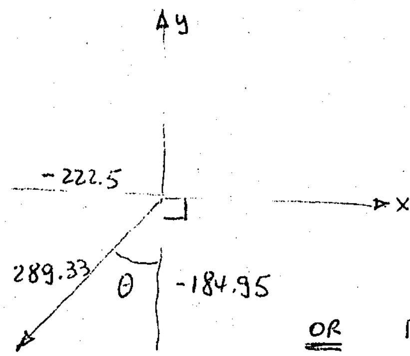


$\frac{F_{1y}}{F_1} = \frac{4}{5} \qquad F_{1y} = F_1 \cdot \frac{4}{5} = 150 \cdot \frac{4}{5} = 120 lb.$

$\frac{F_{1x}}{F_1} = \frac{3}{5} \qquad F_{1x} = F_1 \cdot \frac{3}{5} = 150 \cdot \frac{3}{5} = 90 lb$

TAKE EACH VECTOR & FIND COMPONENTS

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(222.5)^2 + (184.95)^2} = 289.33 lb$$



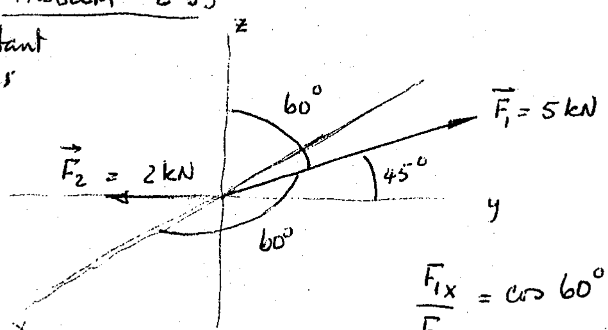
$\cos\theta = \frac{+184.95}{289.33} = .6392$

$\theta = \cos^{-1}(.6392) = 50.27°$

OR FROM THE + x axis 140.27°

PROBLEM 2-39

Find the resultant
Given 2 vectors



$\overline{F_1} \quad \alpha, \beta, \gamma = 60°, 45°, 60°$
$\overline{F_2} \quad \alpha, \beta, \gamma = 90°, 180°, 90°$

$\overline{F_2} = (0\bar{i} - 2\bar{j} + 0\bar{k})\ kN$

$\frac{F_{1x}}{F_1} = \cos 60° \Rightarrow \vec{F_{1x}} = F_1 \cdot \cos 60° = 5 kN \cdot \frac{1}{2} = 2.5 kN$

$\frac{F_{1y}}{F_1} = \cos 45° \Rightarrow F_{1y} = F_1 \cdot \cos 45° = 5 kN (.707) = 3.54 kN$

$\frac{F_{1z}}{F_1} = \cos 60° \Rightarrow F_{1z} = F_1 \cdot \cos 60° = 5 kN (.5) = 2.5 kN$

$\overline{F_1} = (2.5\vec{i} + 3.54\bar{j} + 2.5\vec{k})\ kN$

$$\overline{F}_R = \overline{F}_1 + \overline{F}_2 = (2.5\,\overline{i} + 1.54\,\overline{j} + 2.5\,\overline{k})\,kN = F_{Rx}\,\overline{i} + F_{Ry}\,\overline{j} + F_{Rz}\,\overline{k}$$

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2 + (F_{Rz})^2} = 3.86\ kN$$

$$\frac{\overrightarrow{F_R}}{F_R} = \vec{u} = .648\,\overline{i} + .399\,\overline{j} + .648\,\overline{k}$$

$\cos\alpha = .648 \quad \alpha = 49.6^\circ$

$\cos\beta = .399 \quad \beta = 66.5^\circ$

$\cos\gamma = .648 \quad \gamma = 49.6^\circ$

FOR MANY 3-D VECTORS

$$\overline{F}_R = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \dots$$

$$= (F_{x_1}\,\overline{i} + F_{y_1}\,\overline{j} + F_{z_1}\,\overline{k}) + (F_{x_2}\,\overline{i} + F_{y_2}\,\overline{j} + F_{z_2}\,\overline{k}) + (F_{x_3}\,\overline{i} + F_{y_3}\,\overline{j} + F_{z_3}(\ )) + \dots$$

$$= (F_{x_1} + F_{x_2} + F_{x_3} + \dots)\,\overline{i} + (F_{y_1} + F_{y_2} + F_{y_3} + \dots)\,\overline{j} + (F_{z_1} + F_{z_2} + F_{z_3} + \dots)\,\overline{k}$$

$$F_{Rx}\,\overline{i} + F_{Ry}\,\overline{j} + F_{Rz}\,\overline{k} = \Sigma F_x\,\overline{i} + \Sigma F_y\,\overline{j} + \Sigma F_z\,\overline{k}$$

Thus $\quad F_{Rx} = \Sigma F_x \qquad F_{Ry} = \Sigma F_y \qquad F_{Rz} = \Sigma F_z$

then $\quad \cos\alpha = \dfrac{F_{Rx}}{F_R} \qquad \cos\beta = \dfrac{F_{Ry}}{F_R} \qquad \cos\gamma = \dfrac{F_{Rz}}{F_R}$

where

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

SESSION #3

וקטור מיקום

Position Vector

Normally you may be given a line of action of the force and the magnitude of the force. We want to find the vector representing the force

Previously we stated that

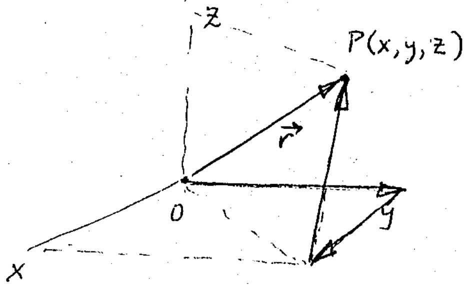
ū - וקטור יחידה

$$\vec{F} = F\vec{u}$$

where $\vec{u}$ was a unit vector in the direction of $\vec{F}$

How do we find $\vec{u}$: by means of a position vector

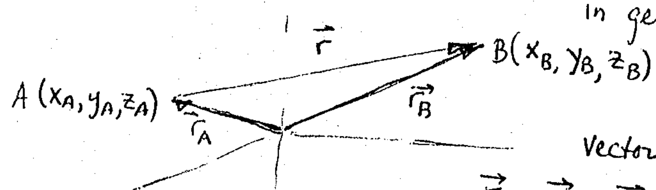
Position vector - a fixed vector which locates a point in space relative to another point.



$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

IF THE TAIL OF THE VECTOR IS AT THE ORIGIN

in general



Vector between pt A and pt B must satisfy

$$\vec{r}_A + \vec{r}_{B/A} = \vec{r}_B \quad \text{or} \quad \vec{r} = \vec{r}_B - \vec{r}_A$$

$$\vec{r}_{B/A} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

$$(x_B\vec{i} + y_B\vec{j} + z_B\vec{k}) - (x_A\vec{i} + y_A\vec{j} + z_A\vec{k})$$

THUS A VECTOR BETWEEN ANY TWO POINTS CAN BE OBTAINED BY SUBTRACTING THE COORDINATE OF THE TAIL FROM THE COORDINATE OF THE TIP FOR EACH COMPON

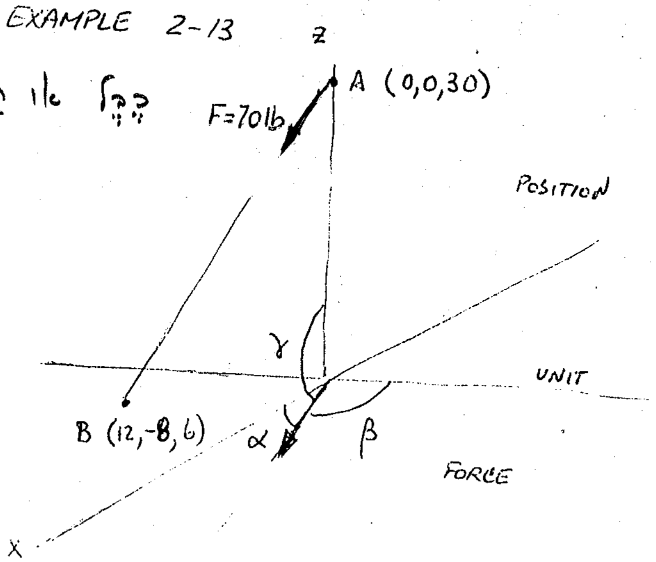
TO FIND A UNIT VECTOR IN THE DIRECTION OF $\vec{r}$: DIVIDE $\vec{r}$ BY ITS MAGNITUDE

$$\therefore \quad r_{B/A} = |\vec{r}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

AND $\vec{u} = \dfrac{\vec{r}}{r}$

EXAMPLE 2-13

(wire)



POSITION

$$\bar{r} = \bar{r}_B - \bar{r}_A = (12\bar{i} - 8\bar{j} + 6\bar{k}) - (0\bar{i} + 0\bar{j} + 30\bar{k})$$

$$\bar{r} = (12\bar{i} - 8\bar{j} - 24\bar{k})\text{ ft}$$

$$r = |\vec{r}| = \sqrt{(12)^2 + (-8)^2 + (-24)^2} = 28\text{ ft}$$

UNIT

$$\bar{u} = \frac{\bar{r}}{r} = \frac{12}{28}\bar{i} - \frac{8}{28}\bar{j} - \frac{24}{28}\bar{k}$$

$$\bar{u} = .429\bar{i} - .286\bar{j} - .857\bar{k}$$

FORCE

$$\bar{F} = F\bar{u} = 70^{\#}[.429\bar{i} - .286\bar{j} - .857\bar{k})$$

$$= (30\bar{i} - 20\bar{j} - 60\bar{k})\text{lb}$$

$$= F_x\bar{i} + F_y\bar{j} + F_z\bar{k}$$

$$\cos\alpha = \frac{F_x}{F} = \frac{30}{70} \qquad \cos\beta = \frac{F_y}{F} = \frac{-20}{70} \qquad \cos\gamma = \frac{-60}{70} = \frac{F_z}{F}$$

$$\alpha = 64.6^\circ \qquad \beta = 106.6^\circ \qquad \gamma = 149.0^\circ$$

n of a force is
. Such a situa-
g the cord AB.
has the *same*
A to point B on
ector $\mathbf{u} = \mathbf{r}/r$.

ote that it has
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oint B, then

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example

Example 2–15

The man shown in Fig. 2–40*a* pulls on the cord with a force of 70 lb. Represent this force, acting on the support A, as a Cartesian vector and determine its direction.

SOLUTION

Force $\mathbf{F}$ is shown in Fig. 2–40*b*. The *direction* of this vector, $\mathbf{u}$, is determined from the position vector $\mathbf{r}$, which extends from A to B, Fig. 2–40*b*. To formulate $\mathbf{F}$ as a Cartesian vector we use the following procedure.

Position Vector. The coordinates of the end points of the cord are $A(0, 0, 30)$ and $B(12, -8, 6)$. Forming the position vector by subtracting the corresponding x, y, and z coordinates of A from those of B, we have

$$\mathbf{r} = (12 - 0)\mathbf{i} + (-8 - 0)\mathbf{j} + (6 - 30)\mathbf{k}$$
$$= \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\} \text{ ft}$$

Show on Fig. 2–40*a* how one can write $\mathbf{r}$ *directly* by going from A $\{12\mathbf{i}\}$ ft, then $\{-8\mathbf{j}\}$ ft, and finally $\{-24\mathbf{k}\}$ ft to get to B.

The magnitude of $\mathbf{r}$, which represents the *length* of cord AB, is

$$r = \sqrt{(12)^2 + (-8)^2 + (-24)^2} = 28 \text{ ft}$$

Unit Vector. Forming the unit vector that defines the direction and sense of both $\mathbf{r}$ and $\mathbf{F}$ yields

$$\mathbf{u} = \frac{\mathbf{r}}{r} = \frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}$$

Force Vector. Since $\mathbf{F}$ has a *magnitude* of 70 lb and a *direction* specified by $\mathbf{u}$, then

$$\mathbf{F} = F\mathbf{u} = 70 \text{ lb}\left(\frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right)$$
$$= \{30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}\} \text{ lb} \qquad \textit{Ans.}$$

As shown in Fig. 2–40*b*, the coordinate direction angles are measured between $\mathbf{r}$ (or $\mathbf{F}$) and the *positive axes* of a localized coordinate system with origin placed at A. From the components of the unit vector:

$$\alpha = \cos^{-1}\left(\frac{12}{28}\right) = 64.6° \qquad \textit{Ans.}$$

$u = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k$

$$\beta = \cos^{-1}\left(\frac{-8}{28}\right) = 107° \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-24}{28}\right) = 149° \qquad \textit{Ans.}$$

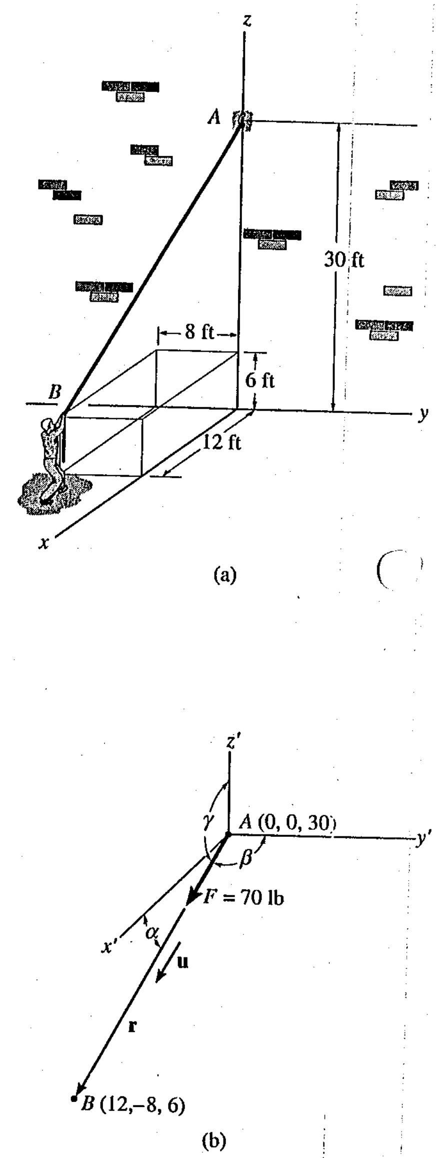


Fig. 2–40

DOT PRODUCT

מכפלה סקלרית

GIVEN 2 VECTORS $\vec{A}$ & $\vec{B}$

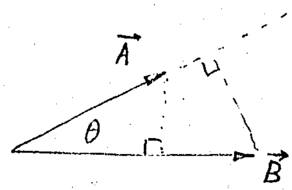
DOT PRODUCT

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

θ ANGLE OF THE INCLUDED ANGLE

RESULT IS A SCALAR

(projection) הטלה



$0^\circ \le \theta \le 180^\circ$

note that $A\cos\theta$ is the projection of $\vec{A}$ on the vector $\vec{B}$ and $B\cos\theta$ is the projection of $\vec{B}$ on $\vec{A}$

Dot products satisfy

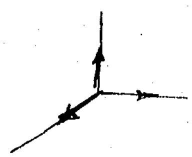
Commutative Law $\quad \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Distributive Law over addition $\quad \vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

Multiplication by a scalar

$$m(\vec{A}\cdot\vec{B}) = (m\vec{A})\cdot\vec{B} = \vec{A}\cdot(m\vec{B}) = (\vec{A}\cdot\vec{B})m$$

Dot Product of Unit vectors



$\bar{i}\cdot\bar{i} = 1\cdot1\cos0^\circ = 1 \qquad \bar{j}\cdot\bar{i} = 0 \qquad \bar{k}\cdot\bar{i} = 0$

$\bar{i}\cdot\bar{j} = 1\cdot1\cos90^\circ = 0 \qquad \bar{j}\cdot\bar{j} = 1 \qquad \bar{k}\cdot\bar{j} = 0$

$\bar{i}\cdot\bar{k} = 1\cdot1\cos90^\circ = 0 \qquad \bar{j}\cdot\bar{k} = 0 \qquad \bar{k}\cdot\bar{k} = 1$

$$\bar{A}\cdot\bar{B} = (A_x\bar{i} + A_y\bar{j} + A_z\bar{k})\cdot(B_x\bar{i} + B_y\bar{j} + B_z\bar{k})$$

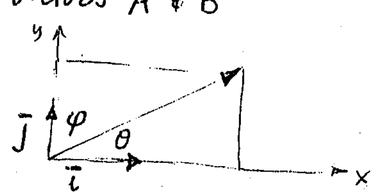
$$= A_xB_x + A_yB_y + A_zB_z$$

TO FIND THE ANGLE BETWEEN 2 VECTORS

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$ — multiplication of the magnitudes

$$= \frac{\vec{A}}{A}\cdot\frac{\vec{B}}{B} = \vec{u}_A\cdot\vec{u}_B$$ DOT product of the unit vectors in the direction of the vectors $\vec{A}$ & $\vec{B}$



Special case: let $\vec{B}$ be a unit vector $\bar{i}$

$\vec{A}\cdot\bar{i} = A\cos\theta$

let $\vec{B}$ be a unit vector $\bar{j}$

$\vec{A}\cdot\bar{j} = A\cos\varphi$ = component of A in the y direction

Now $\vec{A} = A_x\bar{i} + A_y\bar{j} = A\cos\theta\,\bar{i} + A\cos\varphi\,\bar{j}$

$= (\bar{A}\cdot\bar{i})\bar{i} + (\bar{A}\cdot\bar{j})\bar{j}$

נניח כי וקטורי הבסיס הם i, j, k אז קוסינוסי הכוון הם:

$$l = \cos\theta_x \; ; \; m = \cos\theta_y \; ; \; n = \cos\theta_z \; ; \; l^2 + m^2 + n^2 = 1$$

אזי נוכל לרשום את הוקטור $\vec{F}$ כך

$$\vec{F} = F(\vec{i}l + \vec{j}m + \vec{k}n)$$

עכשיו כדי לשמור רכיב הוקטור $\vec{F}$ בעזרת המכפלה הסקלרית

$$F_x = F \cdot \cos\theta_x \equiv \vec{F} \cdot \vec{i}$$

ובכלל:

$$F_n = \vec{F} \cdot \vec{n}$$

ולכן אם נרצה לכתוב את הוקטור הפועל בכוון $\vec{n}$ אזי

$$\vec{F_n} = F_n \cdot n = (\vec{F} \cdot \vec{n})\vec{n} = (\vec{F} \cdot \vec{n})\vec{n}$$

נניח כי ל-n קוסינוסי כוון α, β, γ

$$\vec{n} = \vec{i}\alpha + \vec{j}\beta + \vec{k}\gamma$$

ונניח כי ל-F קוסינוסי כוון l, m, n אזי היטל של $\vec{F}$ על $\vec{n}$ יהיה

$$F_n = \vec{F} \cdot \vec{n} = F(\vec{i}l + \vec{j}m + \vec{k}n) \cdot (i\alpha + j\beta + k\gamma)$$

$$= F(l\alpha + m\beta + n\gamma)$$

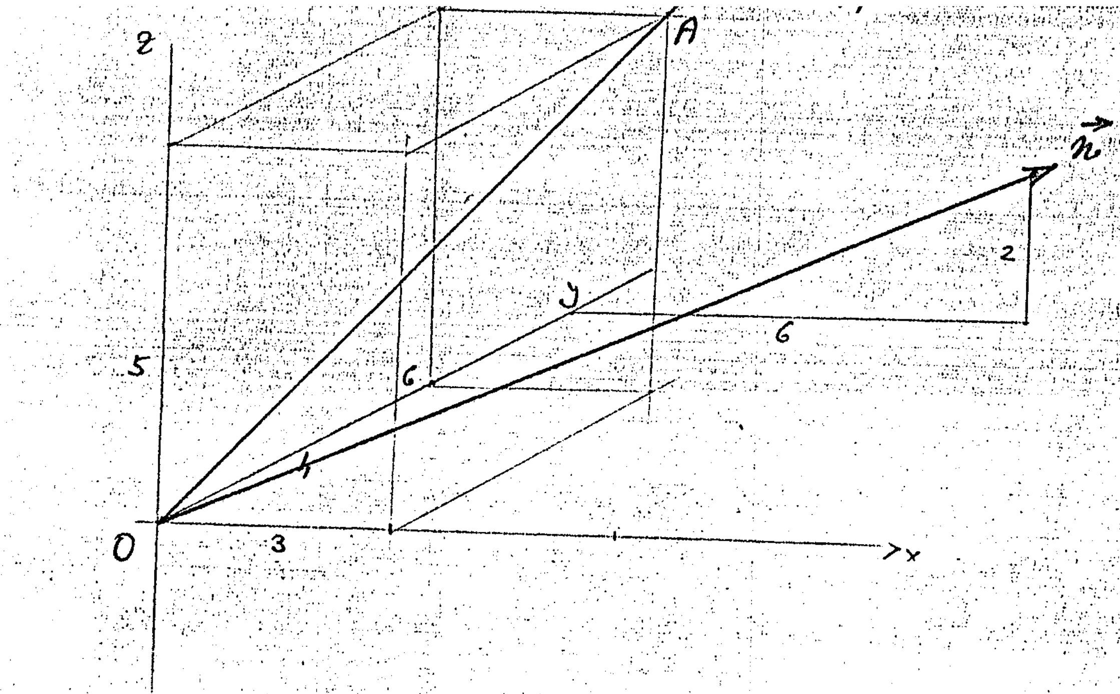
כי יש: $i \cdot i = 1 \quad i \cdot j = 0$

<u>דוגמא</u>

כוח F הוא מהרושת ושעוצמתו $100N$ עובר דרך הנקודה $A(3,4,5)$ א. חשב את רכביו במערכת הנתונה

ב. חשב את היטלו על המישור x, y

ג. חשב את היטלו על הישר אם המשורטט



אורך הוקטור $\overline{OA}$ $\qquad A=\sqrt{3^2+4^2+5^2}=\sqrt{50}=7.071m$

ולכן קוסינוסי הכיוון הם:

$$l=\frac{3}{A}=0.424 \qquad m=\frac{4}{A}=0.566 \qquad n=\frac{5}{A}=0.707$$

א. רכיבי F במערכת יהיו:

$$F_x=F\cos\theta_x=F\cdot l=100\times 0.424=42.4\ N$$

$$F_y=F\cdot m=56.6\ N$$

$$F_z=F\cdot n=70.7\ N$$

ב. קוסינוס הזווית בין $\vec{F}$ למישור xy הוא:

$$\cos\theta_{xy}=\frac{\sqrt{3^2+4^2}}{7.071}=0.707$$

ולכן אורך ההיטל:

$$F_{xy}=F\cos\theta_{xy}=100\cdot 0.707=70.7\ N$$

ג. קוסינוסי הכיוון של $\vec{n}$ הם:

$$\alpha=\beta=\frac{6}{\sqrt{6^2+6^2+2^2}}=0.688 \qquad \gamma=\frac{2}{\sqrt{6^2+6^2+2^2}}=0.229$$

לכן ההיטל אורכו יהיה

$$F_n=\vec{F}\cdot\vec{n}=100\,(l\alpha+m\beta+n\gamma)=100\,(0.424\cdot 0.688+0.566\cdot 0.688+0.707\cdot 0.229)$$

$$F_n=84.4\ N$$

פתרון וקטורי לתרגיל 2.6

וקטור $\underline{OA}$

$$\underline{OA} = 3\underline{i} + 4\underline{j} + 5\underline{k}$$

וקטור יחידה

$$\underline{e}_{OA} = \frac{\underline{OA}}{|\underline{OA}|} = \frac{1}{\sqrt{3^2+4^2+5^2}} \cdot (3\underline{i} + 4\underline{j} + 5\underline{k})$$

$$\underline{e}_{OA} = \frac{\sqrt{2}}{10}(3\underline{i} + 4\underline{j} + 5\underline{k})$$

וקטור הכח

$$\underline{F} = F\underline{e}_{OA} = 100 \cdot \frac{\sqrt{2}}{10}(3\underline{i} + 4\underline{j} + 5\underline{k}) =$$

$$= 10\sqrt{2}\,(3\underline{i} + 4\underline{j} + 5\underline{k}) \text{ [New]}$$

רכיב $\underline{F}$ בכיוון z

$$F_z = \underline{F} \cdot \underline{k} = 50\sqrt{2} \text{ New}$$

$$F_{xy} = \sqrt{F^2 - F_z^2} = \sqrt{100^2 - (50\sqrt{2})^2} = 50\sqrt{2} \text{ New}$$

$$F_{xy} = 70.7 \text{ New}$$

וקטור בכיוון ON

$$\underline{ON} = 6\underline{i} + 6\underline{j} + 2\underline{k}$$

$$\underline{e}_{ON} = \frac{1}{2\sqrt{19}}(6\underline{i} + 6\underline{j} + 2\underline{k}) = \frac{\sqrt{19}}{38}(6\underline{i} + 6\underline{j} + 2\underline{k})$$

היטל של F על ON

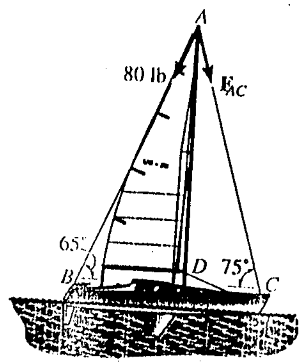
$$F_{On} = \underline{F} \cdot \underline{e}_{On} = 10\sqrt{2} \cdot \frac{\sqrt{19}}{38}(18 + 24 + 10) =$$

$$= \frac{210\sqrt{38}}{19} = 84.4 \text{ New}$$

תרגיל

(2–13) The mast of the sailboat is subjected to the force of the two cables. If the force in cable AB is 80 lb, determine the required force in AC so that the resultant force caused by both cables is directed vertically downward along the axis AD of the mast. Also, calculate this resultant force.

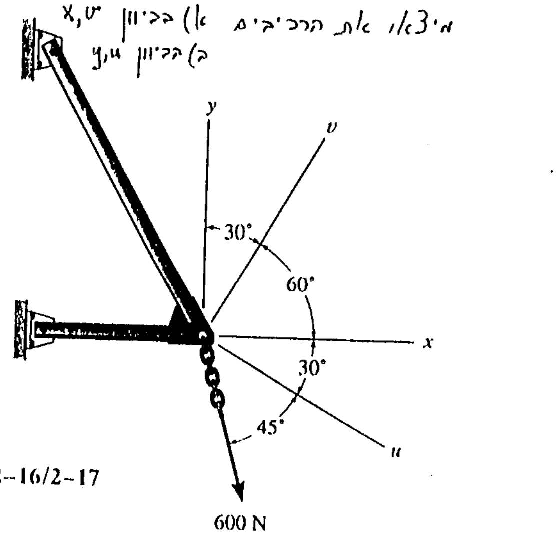
מיצאו השקול



Prob. 2–13

(2–17) The cable exerts a force of 600 N on the frame. Resolve this force into components acting *(a)* along the x and v axes and *(b)* along the y and u axes. What is the magnitude of each component?

מיצאו את הרכיבים א) בכיוון x,v
ב) הכיוון y,u

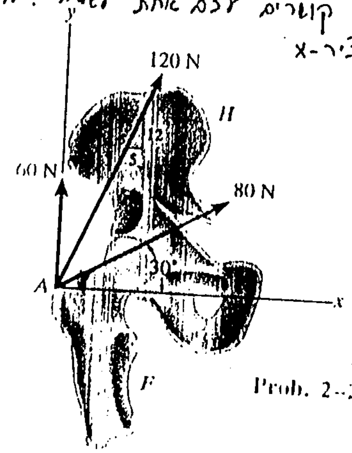


Probs. 2–16/2–17

$F_x = 490 N$

(2–23) The hipbone H is connected to the femur F at A using three different muscles, which exert the forces shown on the femur. Determine the resultant force on the femur and specify its orientation θ measured counterclockwise from the positive x axis.

שלושה שרירים קושרים עצם אחת לשניה. מצאו את השקול
והכיוון θ מהציר-x

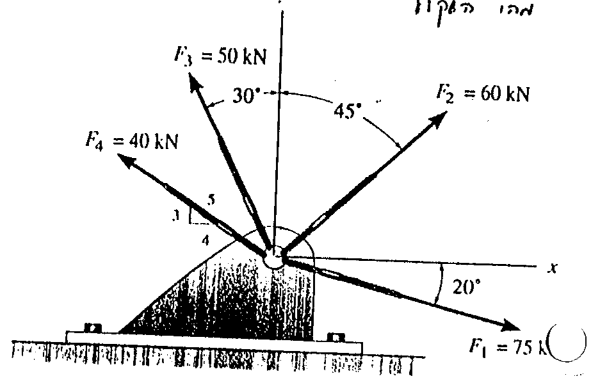


Prob. 2–23

$F_R = 240.31 N$

***2–40.** Express each force acting on the bracket in Cartesian vector form. What is the magnitude of the resultant force?

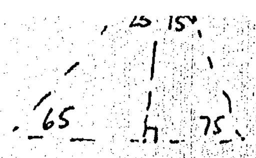
מצאו ביטויים לכל כוח בצורה קרטזית.
מהו השקול



Prob. 2–40

$F_4 = -32\underline{i} + 24\underline{j}$ kN

$F_x = 55.91$ kN



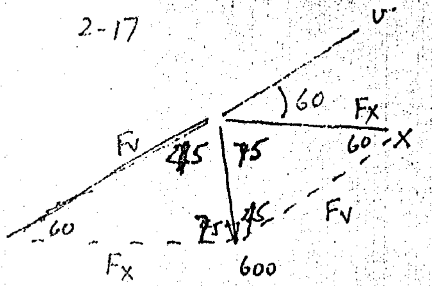
$F_{AC}\sin 15 = 80 \sin 25^\circ$

$F_{AC} = 130.63 \text{ lb}$

Resultant force $= 80 \cos 25 + F_{AC} \cos 15$

$= 198.68 \text{ lb}$

2-17

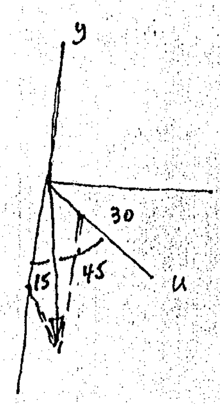


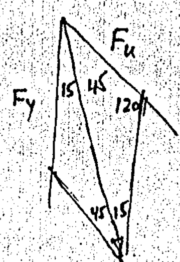
using trig get angles then use law of sines

$$\frac{600}{\sin 60} = \frac{F_v}{\sin 75} = \frac{F_x}{\sin 45}$$

$F_x = 489.9 \text{ N}$

$F_v = 669.2 \text{ N}$



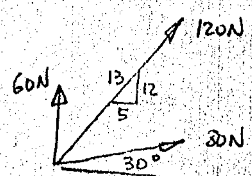


$$\frac{600}{\sin 120} = \frac{F_y}{\sin 45} = \frac{F_u}{\sin 15}$$

$F_y = 489.9 \text{ N}$

$F_u = 179.32 \text{ N}$

2-23

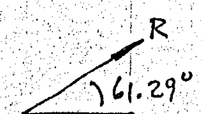


$\Sigma F_x = 120 \cdot \frac{5}{13} + 80 \cos 30^\circ = 115.44 \text{N}$

$\Sigma F_y = 60 + 120 \cdot \frac{12}{13} + 80 \sin 30 = 210.77 \text{N}$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 240.31 \text{N}$

$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = 1.826 \qquad \theta = 61.29^\circ$ from horizontal



2-40

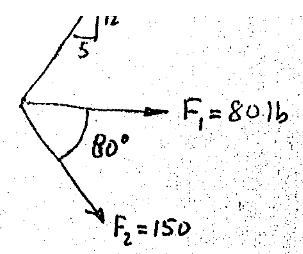
$F_1 = 75(\cos 20\, \underline{i} - \sin 20\, j) = 70.48\, \underline{i} - 25.65\, \underline{j}$ kN

$F_2 = 60\, (\cos 45\, \underline{i} + \sin 45\, j) = 42.43\, \underline{i} + 42.43\, \underline{j}$ kN

$F_3 = 50\, (-\cos 60\, \underline{i} + \sin 60\, j) = -25\, \underline{i} + 43.3\, \underline{j}$ kN

$F_4 = 40\, (-4/5\, \underline{i} + 3/5\, \underline{j}) = -32\, \underline{i} + 24\, \underline{j}$ kN

$\underline{R} = 55.91\, \underline{i} + 84.08\, j \qquad R = \sqrt{(55.91)^2 + (84.08)^2} = 100.97 \text{ kN}$



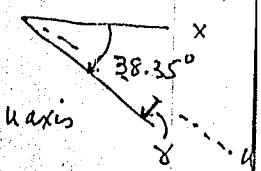
13

$$\Sigma F_y = 52 \cdot \frac{12}{13} - 150 \sin 80 = -99.72 \text{ lb}$$

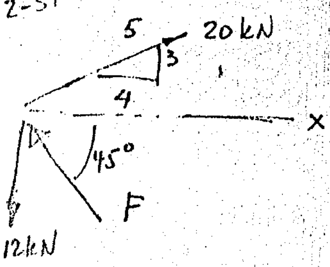
$$R = \sqrt{(126.05)^2 + (99.72)^2} = \underline{160.73 \text{ lb}}$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = -.7911 \qquad \theta = -38.35° \text{ from horizontal}$$

$$\gamma = 38.35 - 25 = \underline{13.35°} \text{ below u axis}$$



2-51



$$\Sigma F_x = 20 \cdot \tfrac{4}{5} - F\cos 45 + 0 = 16 - F\cos 45°$$

$$\Sigma F_y = 20 \cdot \tfrac{3}{5} - 12 + F\sin 45° = F\sin 45°$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{256 - 32F\cos 45 + F^2\cos^2 45 + F^2\sin^2 45}$$

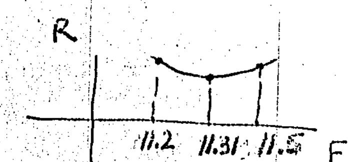
$$R = \sqrt{256 - 32F\sin 45 + F^2} \qquad \text{since } \sin 45 = \cos$$

for smallest resultant $\frac{dR}{dF} = 0 = \frac{1}{2}\frac{(-32\sin 45 + 2F)}{\sqrt{256 - 32F\sin 45 + F^2}} \Rightarrow F = 16\sin 45 = \underline{11.3}$

$\therefore$ $\Sigma F_x = 8.00$, $\Sigma F_y = 8.00$ } $R = 8.00\sqrt{2} = 11.31$ kN

to check: if $F = 11.5$ kN $\Sigma F_x = 7.87$, $\Sigma F_y = 8.13$ } $R = 11.32$ kN

if $F = 11.2$ kN $\Sigma F_x = 8.08$, $\Sigma F_y = 7.92$ } $R = 11.314$ kN



PROBLEM 2-71 (P. 57) DID NOT DO - HANDOUT

$$\bar{r}_1 = \bar{r}_C - \bar{r}_A = (-2\bar{i} - 3\bar{j} + 7.5\bar{k}) - (2\bar{i} + 3\bar{j} + 3\bar{k}) = (-4\bar{i} - 6\bar{j} + 4.5\bar{k})$$

$$\bar{r}_2 = \bar{r}_B - \bar{r}_A = (0\bar{i} - 3\bar{j} + 0\bar{k}) - (2\bar{i} + 3\bar{j} + 3\bar{k}) = (-2\bar{i} - 6\bar{j} - 3\bar{k})\ ft$$

$$\bar{r}_1 \cdot \bar{r}_2 = 8 + 36 - 13.5 = 30.5$$

$$r_1 = \sqrt{(-4)^2 + (-6)^2 + (4.5)^2} = 8.5 \qquad r_2 = \sqrt{(-2)^2 + (-6)^2 + (-3)^2} = 7$$

$$r_1 r_2 = 7 \cdot (8.5) = 59.5$$

$$.5126 = \cos\theta = \frac{\bar{r}_1 \cdot \bar{r}_2}{r_1 r_2} \Rightarrow 59.2^\circ = \theta$$

Also can be solved

$$\bar{F}_1 = F_1 \bar{u}_1 = F_1 \frac{\bar{r}_1}{r_1} \qquad \bar{F}_2 = F_2 \bar{u}_2 = F_2 \frac{\bar{r}_2}{r_1}$$

$$\bar{F}_1 \cdot \bar{F}_2 = \frac{F_1}{r_1}\bar{r}_1 \cdot \frac{F_2}{r_2}\bar{r}_2 = \frac{F_1 F_2}{r_1 r_2}\, \bar{r}_1 \cdot \bar{r}_2$$

$$\cos\theta = \bar{F}_1 \cdot \bar{F}_2 / F_1 F_2 = \frac{\bar{r}_1 \cdot \bar{r}_2}{r_1 r_2} = \frac{\vec{u}_1 \cdot \vec{u}_2}{u_1 u_2} = \vec{u}_1 \cdot \vec{u}_2$$

$$\bar{u}_1 = \frac{\bar{r}_1}{r_1} = \frac{(-4\bar{i} - 6\bar{j} + 4.5\bar{k})\ ft}{8.5\ ft} = -.471\bar{i} - .706\bar{j} + .529\bar{k}$$

$$\bar{F}_1 = F_1 \bar{u}_1 = -28.24\bar{i} - 42.35\bar{j} + 31.76\bar{k} \qquad F_1 = 60\ lb$$

$$\bar{u}_2 = \frac{\bar{r}_2}{r_2} = \frac{(-2\bar{i} - 6\bar{j} - 3\bar{k})\ ft}{7\ ft} = -.286\bar{i} - .857\bar{j} - .429\bar{k}$$

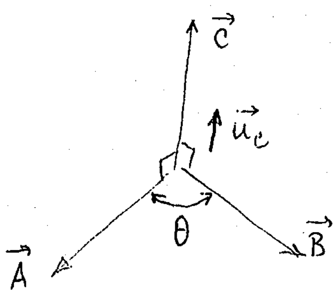
$$\bar{F}_2 = F_2 \bar{u}_2 = -8.57i - 25.7\bar{j} - 12.86\bar{k} \qquad F_2 = 30\ lb$$

$$\bar{F}_1 \cdot \bar{F}_2 = 921.98 \qquad \cos\theta = \frac{\bar{F}_1 \cdot \bar{F}_2}{F_1 F_2} = .5722 \qquad \theta = 59.2^\circ$$

$$\bar{F}_R = \bar{F}_1 + \bar{F}_2 = -36.81\bar{i}\ \ldots$$ [illegible]

CROSS - PRODUCT מכפלה וקטורית

SESSION #4



$\vec{C} = \vec{A} \times \vec{B}$ where $\vec{C}$ is $\perp$ to $\vec{A}$ & $\perp$ to $\vec{B}$

c גודל $c = |\vec{C}| = AB\sin\theta \qquad 0^\circ \le \theta \le 180^\circ$

$\vec{C} = AB\sin\theta\ \vec{u}_c$ — $\vec{u}_c$ is a unit vector in the direction of $\vec{C}$

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} = -\vec{B} \times \vec{A}$$ not commutative

$$m(\vec{A} \times \vec{B}) = m\vec{A} \times \vec{B} = \vec{A} \times m\vec{B} = (\vec{A} \times \vec{B})\,m$$ SCALAR

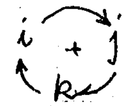
$$\vec{A} \times (\vec{B} + \vec{D}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{D}$$ Distributive

cross product of unit vectors

$\vec{i}\times\vec{j}=\vec{k}$ $\quad\vec{i}\times\vec{i}=0$

$\vec{j}\times\vec{k}=\vec{i}$ $\quad\vec{j}\times\vec{j}=0$

$\vec{k}\times\vec{i}=\vec{j}$ $\quad\vec{k}\times\vec{k}=0$



CLOCKWISE MOTION : +
CCW " : −

$\vec{j}\times\vec{i}=-\vec{k}$ $\quad\vec{i}\times\vec{k}=-\vec{j}$

$\vec{k}\times\vec{j}=-\vec{i}$

$$\vec{A}\times\vec{B}=(A_x\vec{i}+A_y\vec{j}+A_z\vec{k})\times(B_x\vec{i}+B_y\vec{j}+B_z\vec{k})$$

$$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\vec{i}-(A_xB_z-A_zB_x)\vec{j}+(A_xB_y-A_yB_x)\vec{k}$$

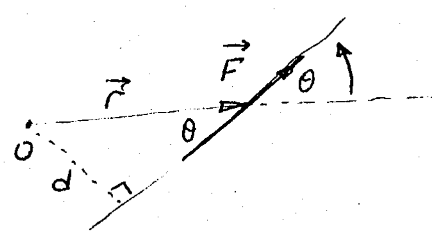
$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}\begin{matrix}\vec{i} & \vec{j}\\ A_x & A_y\\ B_x & B_y\end{matrix}$$ also a representation

SHOW HOW TO OPERATE THE DETERMINANT!

THIS IS IMPORTANT TO KNOW FOR THE DEFINITION OF A MOMENT

$\vec{M}_O=\vec{r}\times\vec{F}$ (the moment about point O = position vector $\vec{r}$ from O to any pt on the line of action of the force $\vec{F}$ cross the force vector

מודדים θ מקו פעולת r לקו
פעולת F



θ measured between lines of action of $\vec{r}$ & $\vec{F}$

CONSIDER MOMENTS POSITIVE AS DEFINED THIS WAY.
FROM THE LINE OF ACTION OF $\vec{r}$ TO THE LINE OF ACTION OF $\vec{F}$

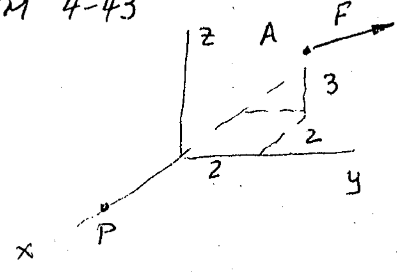
$$M_O\vec{u}_M=\vec{M}_O=\vec{r}\times\vec{F}=(rF\sin\theta)\,\vec{u}_M\Rightarrow\quad M_O=rF\sin\theta=F\,r\sin\theta$$

$r\sin\theta=d$ $\quad\therefore\quad$ M_O = magnitude of the force times the perpendicular distance to the line of action of the force

(P. 121)

PROBLEM 4-43

Find moment of F about P



P (2,0,0) m

A (−2,2,3) m

$\vec{r}=\vec{r}_A-\vec{r}_P=(-4\vec{i}+2\vec{j}+3\vec{k})$ m

$\vec{F}=(-6\vec{i}+4\vec{j}+3\vec{k})$ kN

$$\vec{M}_P = \vec{r}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 2 & 3 \\ -6 & 4 & 3 \end{vmatrix} = \vec{i}\,[2\cdot 3-4\cdot 3] - \vec{j}\,[-4\cdot 3-(-6)3] + \vec{k}\,[(-4)\cdot 4-(-6)$$

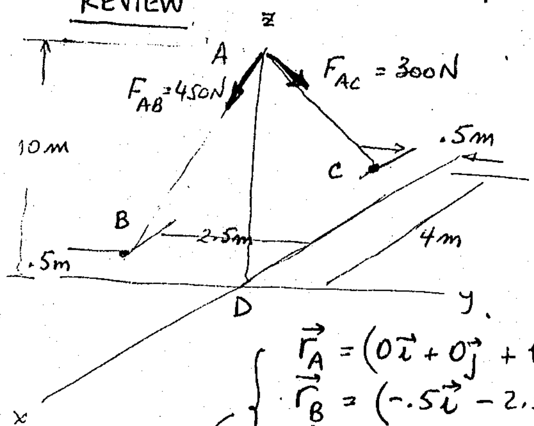
$[m\cdot kN]$

$$= \vec{i}\,(-6) - \vec{j}\,(+6) + \vec{k}\,(-4)$$

$$= (-6\vec{i} - 6\vec{j} - 4\vec{k})\ kN\cdot m$$

REVIEW — IN CLASS + HANDOUT

Find angle between 2 forces



2-83 1) Find $\vec{r}_{AB}$, $\vec{r}_{AC}$; determine r_{AB}, r_{AC}; find $\vec{u}_{AB}$, $\vec{u}_{AC}$

2) $\vec{F}_{AB} = F_{AB}\,\vec{u}_{AB}$ $\quad \vec{F}_{AC} = F_{AC}\,\vec{u}_{AC}$

$\vec{F}_R = \vec{F}_{AB} + \vec{F}_{AC}$

$$\begin{cases} \vec{r}_A = (0\vec{i} + 0\vec{j} + 10\vec{k})\ m \\ \vec{r}_B = (-.5\vec{i} - 2.5\vec{j} + 0\vec{k})\ m \\ \vec{r}_C = (-4\vec{i} - .5\vec{j} + 0\vec{k})\ m \end{cases}$$

$\vec{r}_{AB} = (-.5\vec{i} - 2.5\vec{j} - 10\vec{k})\ m$

$r_{AB} = 10.32m$

$\vec{r}_{AC} = (-4\vec{i} - .5\vec{j} - 10\vec{k})\ m$

$r_{AC} = 10.78\ m$

$\vec{u}_{AB} = -.048\vec{i} - .242\vec{j} - .969\vec{k}$

$\vec{F}_{AB} = F_{AB}\cdot\vec{u}_{AB} = (450N)\,\vec{u}_{AB}$
$= (-21.6\vec{i} - 108.9\vec{j} - 436.05\vec{k})$

$\vec{r}_{AC}/r_{AC} = \vec{u}_{AC} = -.371\vec{i} - .046\vec{j} - .928\vec{k}$

$\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC} = (300N)\,\vec{u}_{AC}$
$= (-111.3\vec{i} - 13.8\vec{j} - 278.4\vec{k})\ N$

$\vec{F}_R = \vec{F}_{AB} + \vec{F}_{AC} = (-132.9\vec{i} - 122.7\vec{j} - 714.45\vec{k})\ N$

2-84 $\vec{F}_{AB}\cdot\vec{u}_{AC}$ = magnitude of F_{AB} along line of action AC = F'

$(-21.6\vec{i} - 108.9\vec{j} - 436.05\vec{k})N \cdot (-.371\vec{i} - .046\vec{j} - .928\vec{k}) = 417.68N$

$\vec{F}' = F'\,\vec{u}_{AC}$

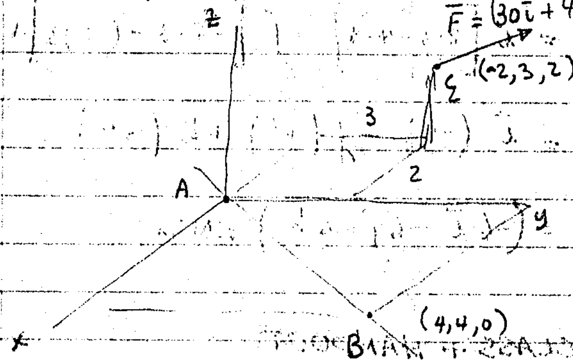
2-85 $$\cos\theta = \frac{\vec{F}_{AB}\cdot\vec{F}_{AC}}{F_{AB}F_{AC}} = \frac{\vec{r}_{AC}\cdot\vec{r}_{AB}}{r_{AB}\,r_{AC}} = \frac{\vec{u}_{AB}\cdot\vec{u}_{AC}}{1\cdot 1} = .928$$

$\theta = \cos^{-1}(.928) = 21.85°$

over

4-53 p. 123

find moment about line AB



$\bar{r}_{C/A} = \bar{r}_C - \bar{r}_A = (-2\bar{i} + 3\bar{j} + 2\bar{k})\,m$

$\bar{r}_{B/A} = \bar{r}_B - \bar{r}_A = (4\bar{i} + 4\bar{j})\,m$

$r_{B/A} = |\bar{r}_{B/A}| = 4(1.414) = 5.656\,m.$

$\bar{u}_{B/A} = \dfrac{\bar{r}_{B/A}}{r_{B/A}} = \dfrac{1}{\sqrt{2}}\bar{i} + \dfrac{1}{\sqrt{2}}\bar{j} + 0\bar{k}$

$$M_P = \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -2 & 3 & 2 \\ 30 & 40 & 20 \end{vmatrix} = \frac{1}{\sqrt{2}}(60) + \frac{1}{\sqrt{2}}(60) + \frac{1}{\sqrt{2}}(40) - \frac{1}{\sqrt{2}}(80) = \frac{80}{\sqrt{2}}\ N\text{-}m$$

$= 56.57\,N\text{-}m.$

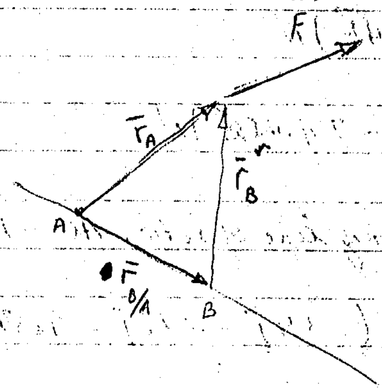
$M_P = \bar{M}_A \cdot \bar{u}_{B/A} = \bar{u}_{B/A} \cdot \bar{M}_A$

$$\vec{M}_A = \begin{vmatrix} i & j & k \\ -2 & 3 & 2 \\ 30 & 40 & 20 \end{vmatrix} = i(60-80) + j(-40+60) + k(-80-90)$$

$= (-20i + 100j - 170k)\ N\text{-}m.$

$u_{B/A} = \dfrac{1}{\sqrt{2}}(i+j)\,(-20i + 100j - 170k) = \dfrac{-20+100}{\sqrt{2}} = \dfrac{80}{\sqrt{2}}$

$\vec{M}_P = M_P\,\vec{u}_{BA} = \dfrac{80}{\sqrt{2}}\left[\dfrac{i}{\sqrt{2}} + \dfrac{j}{\sqrt{2}}\right] = 40(i+j)\ N\text{-}m.$



$\vec{M}_A = \vec{r}_A \times \vec{F}$

$\vec{M}_B = \vec{r}_B \times \vec{F}$

$= (\bar{r}_A + \bar{r}_{B/A}) \times \bar{F}$

$= \bar{r}_A \times \bar{F} + \bar{r}_{B/A} \times \bar{F}$

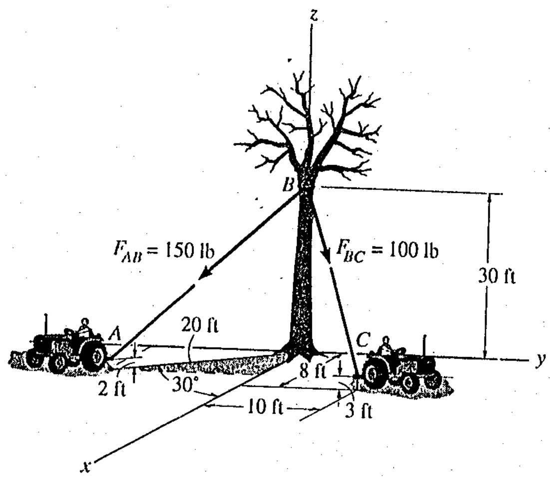
$M_P = \vec{M}_A \cdot \vec{u}_{B/A} = \bar{M}_A \cdot \dfrac{\bar{r}_{B/A}}{r_{B/A}} = \bar{u}_{B/A} \cdot (\bar{r}_A \times \bar{F})$

$\bar{M}_B \cdot \bar{u}_{B/A} = \bar{u}_{B/A} \cdot \bar{M}_B = \bar{u}_{B/A} \cdot (\bar{r}_B \times \bar{F})$

$= \bar{u}_{B/A} \cdot (\bar{r}_A \times \bar{F}) + \underbrace{\bar{u}_{B/A} \cdot (\bar{r} \times \bar{F})}_{=0}$

$= M_P +$

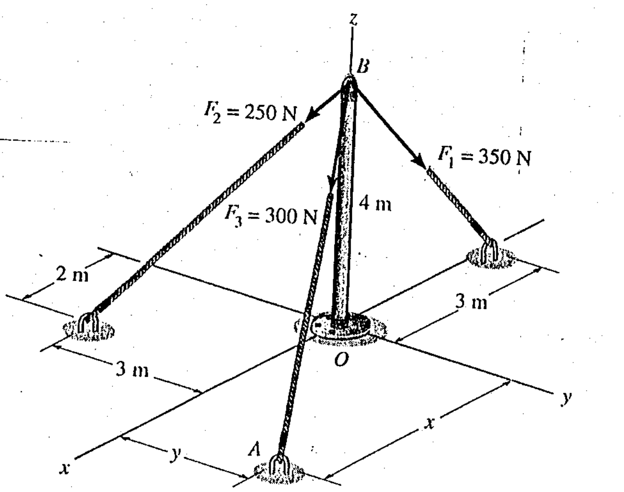
magnitude and coordinate direction angles of the resultant force.



$\underline{F}_1 = 75.5\underline{i} - 43.6\underline{j} - 122\underline{k}$

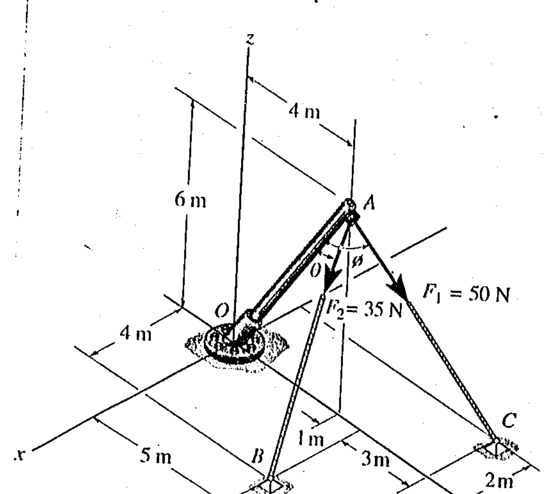
Prob. 2–89

***2–96.** The pole is held in place by three cables. If the force of each cable acting on the pole is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Set $x = 4$ m, $y = 2$ m.



Prob. 2–96

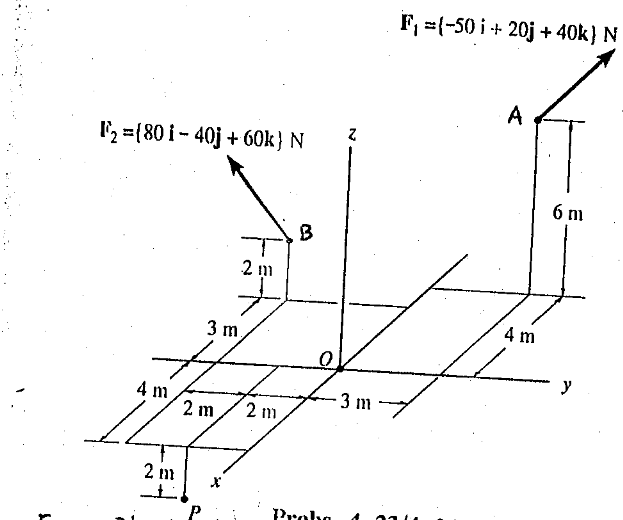
2–113. The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis *OA* of the pole.



$(F_1)_{..} = 18.5N$

point *O*. Express the result as a Cartesian vector.

***4–24.** Determine the resultant moment of the forces about point *P*. Express the result as a Cartesian vector.

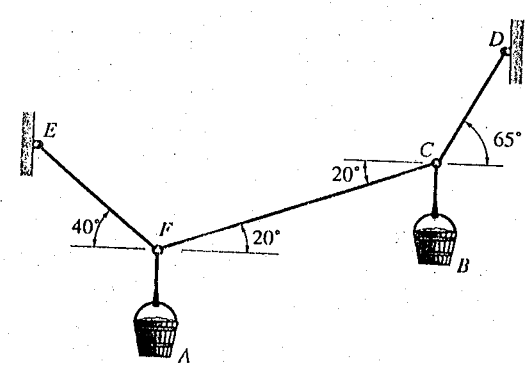


$\underline{r}_{B/o} = -3\underline{i} - 4\underline{j} + 2\underline{k}$

$\underline{M}_o = \underline{r}_{B/o} \times \underline{F}_2$

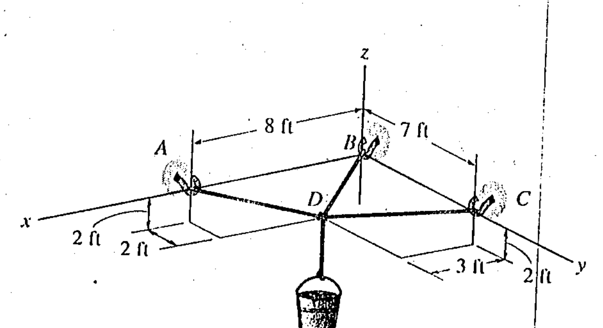
Probs. 4–23/4–24

3–51. The cords suspend the two buckets in the equilibrium position shown. Determine the weight of bucket *B*. Bucket *A* has a weight of 60 lb.



Prob. 3–51

3–53. The bucket has a weight of 20 lb. Determine the tension developed in each cord for equilibrium.



$F_{DA} = 21.5N$

A : (17.32, 10, 2) $r_{A/B} = r_A - r_B = (17.32\underline{i} - 10\underline{j} - 28\underline{k})$ m $r_{A/B} = 34.41$ m $\underline{u}_{A/B} = \frac{\underline{r}_{A/B}}{r_{A/B}} = 0.503\underline{i} - 0.291\underline{j} - 0.814\underline{k}$

C = (8, 10, 3) $\underline{F}_{AB} = F_{AB}\,\underline{u}_{A/B} = 150(.503\underline{i} - .291\underline{j} - .814\underline{k}) = (75.5\underline{i} - 43.7\underline{j} - 122\underline{k})$ N

$\underline{r}_{C/B} = \underline{r}_C - \underline{r}_B = (8\underline{i} + 10\underline{j} - 27\underline{k})$ m $r_{C/B} = 29.88$ m $\underline{u}_{C/B} = \frac{\underline{r}_{C/B}}{r_{C/B}} = 0.268\underline{i} + 0.335\underline{j} - 0.904\underline{k}$

$\underline{F}_{BC} = F_{BC}\,\underline{u}_{C/B} = 100(.268\underline{i} + .335\underline{j} - .904\underline{k}) = (26.8\underline{i} + 33.5\underline{j} - 90.4\underline{k})$ N

$\underline{F}_R = \underline{F}_{AB} + \underline{F}_{BC} = (102.3\underline{i} - 10.2\underline{j} - 212.4\underline{k})$ N $F_R = 235.97$ N $\underline{u}_R = 0.434\underline{i} - .043\underline{j} - .900\underline{k}$

$\alpha = \cos^{-1}(.434) = 64.28°$; $\beta = \cos^{-1}(-.043) = 92.46°$; $\gamma = \cos^{-1}(-.900) = 154.1$

2-96

C (−3, 0, 0)
D = (2, −3, 0)
A = (x, y, 0)
B = (0, 0, 4)

$\underline{r}_{D/B} = (2\underline{i} - 3\underline{j} - 4\underline{k})$ m $r_{D/B} = 5.385$ m $\underline{u}_{D/B} = \underline{r}_{D/B}/r_{D/B} = .371\underline{i} - .557\underline{j} - .743\underline{k}$

$\underline{r}_{C/B} = (-3\underline{i} + 0\underline{j} - 4\underline{k})$ m $r_{C/B} = 5$ m $\underline{u}_{C/B} = \underline{r}_{C/B}/r_{C/B} = -.6\underline{i} + 0\underline{j} - 0.8\underline{k}$

$\underline{r}_{A/B} = (x\underline{i} + y\underline{j} - 4\underline{k})$ m $r_{A/B} = \sqrt{x^2 + y^2 + 16}$ m $= 6$ m $\underline{u}_{A/B} = \underline{r}_{A/B}/r_{A/B} = 0.667\underline{i} + 0.333\underline{j} - 0.667\underline{k}$

$\underline{F}_{DB} = F_{DB}\,\underline{u}_{D/B} = 250(.371\underline{i} - .557\underline{j} - .743\underline{k}) = 92.75\underline{i} - 139.25\underline{j} + 185.75\underline{k}$

$\underline{F}_{CB} = F_{CB}\,\underline{u}_{C/B} = 350(-.6\underline{i} + 0\underline{j} - 0.8\underline{k}) = -210\underline{i} + 0 + 280\underline{k}$

$\underline{F}_{AB} = F_{AB}\,\underline{u}_{A/B} = 300(0.667\underline{i} + 0.333\underline{j} - 0.667\underline{k}) = 200\underline{i} + 100\underline{j} - 200\underline{k}$

$\underline{F}_R = \underline{F}_{DB} + \underline{F}_{CB} + \underline{F}_{AB} = (82.75\underline{i}$ ~~−319.25~~ $-39.25\,\underline{j} - 665.75\underline{k})$ N = ~~[illegible]~~ N **672**

$\alpha = \cos^{-1}\left(\frac{82.75}{672}\right) =$ **82.93**°; $\beta = \cos^{-1}\left(\frac{-39.25}{672}\right) =$ **93.35**°; $\gamma = \cos^{-1}\left(\frac{-665.75}{672}\right) =$ **172.18**°

2-13

A (0, 4, 6)
B (4, 5, 0)
C (−2, 8, 0)

$\underline{r}_{B/A} = (4\underline{i} + \underline{j} - 6\underline{k})$ m $r_{B/A} = 7.28$ m $\underline{u}_{B/A} = .549\underline{i} + .137\underline{j} - .824\underline{k}$

$\underline{r}_{C/A} = (-2\underline{i} + 4\underline{j} - 6\underline{k})$ m $r_{C/A} = 7.48$ m $\underline{u}_{C/A} = -.267\underline{i} + .535\underline{j} - .802\underline{k}$

$\underline{r}_{O/A} = -(0\underline{i} + 4\underline{j} + 6\underline{k})$ m $r_{A/O} = 7.21$ $\underline{u}_{O/A} = -(0\underline{i} + .555\underline{j} + .832\underline{k})$

$\underline{F}_{BA} = F_{BA}\,\underline{u}_{B/A} = 35(.549\underline{i} + .137\underline{j} - .824\underline{k}) = 19.22\underline{i} + 4.80\underline{j} - 28.84\underline{k}$

$\underline{F}_{CA} = F_{CA}\,\underline{u}_{C/A} = 50(-.267\underline{i} + .535\underline{j} - .802\underline{k}) = -13.35\underline{i} + 26.75\underline{j} - 40.10\underline{k}$

$\cos\theta = -.609$, $F_2\cos\theta$
$\cos\phi = -.37$, $F_1\cos\phi$

$(F_1)_{OA} = \underline{F}_{CA} \cdot \underline{u}_{O/A} = (-.555)(26.75) - .832(-40.10) = +18.52$ N

$(F_2)_{OA} = \underline{F}_{BA} \cdot \underline{u}_{O/A} = (4.8)(-.555) - 28.84(-.832) = +21.33$ N

4-13

$\underline{F}_1 = (-50\underline{i} + 20\underline{j} + 40\underline{k})$ N $\underline{r}_{A/O} = (-4\underline{i} + 3\underline{j} + 6\underline{k})$ m

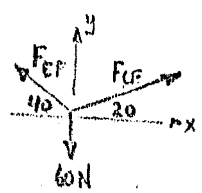
$\underline{F}_2 = (80\underline{i} - 40\underline{j} + 60\underline{k})$ N $\underline{r}_{B/O} = (-3\underline{i} - 4\underline{j} + 2\underline{k})$ m

$\underline{M}_1 = \underline{r}_{A/O} \times \underline{F}_1 = (-80\underline{k} + 160\underline{j} + 150\underline{k} + 120\underline{i} - 300\underline{j} - 120\underline{i})$
$= (0\underline{i} - 140\underline{j} + 70\underline{k})$ N-m

$\underline{M}_2 = \underline{r}_{B/O} \times \underline{F}_2 = (120\underline{k} + 180\underline{j} + 320\underline{k} - 240\underline{i} + 160\underline{j} + 80\underline{i})$
$= (-160\underline{i} + 340\underline{j} + 440\underline{k})$ N-m

$\Sigma\underline{M} = \underline{M}_1 + \underline{M}_2 = -160\underline{i} + 200\underline{j} + 510\underline{k}$

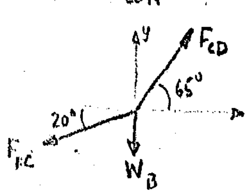
3-51



$\Sigma F_x = F_{CF}\cos 20 - F_{EF}\cos 40° = 0 \Rightarrow F_{CF} = F_{EF}\frac{\cos 40°}{\cos 20°} = .8152 F_{EF}$

$\Sigma F_y = F_{CF}\sin 20 + F_{EF}\sin 40° - 60 = 0$

$= (.8152 F_{EF})(.342) + F_{EF}(.6428) - 60 = 0$ $\underline{F_{EF} = 65.1 \text{ N}}$ $\underline{F_{CF} = .8152 F_{EF} = 53.1 \text{ N}}$



$\Sigma F_x = F_{CD}\cos 65° - F_{FC}\cos 20° = 0 \Rightarrow \underline{F_{CD}} = F_{FC}\frac{\cos 20°}{\cos 65°} = 2.2235 F_{FC} =$ ~~[illegible]~~ **118.1** N

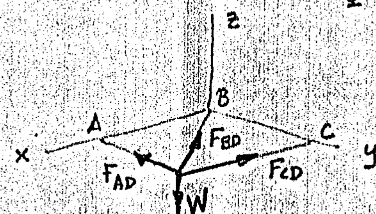
$\Sigma F_y = F_{CD}\sin 65° - F_{FC}\sin 20° - W_B = 0 \Rightarrow \underline{W_B} = F_{CD}\sin 65° - F_{FC}\sin 20° =$ ~~[illegible]~~ **88.9 N**

A (8,0,0) $\quad \underline{r}_{A/D} = \underline{r}_A - \underline{r}_D = ($[illegible]$)\,m \quad r_{A/D} = $ [illegible] $m \quad \underline{u}_{A/D} = .870\underline{i} - .348\underline{j} + .348\underline{k}$

B (0,0,0) $\quad \underline{r}_{B/D} = \underline{r}_B - \underline{r}_D = (-3\underline{i} - 2\underline{j} + 2\underline{k})\,m \quad r_{B/D} = 4.123\,m \quad \underline{u}_{B/D} = -.728\underline{i} - .485\underline{j} + .485\underline{k}$

C (0,7,0) $\quad \underline{r}_{C/D} = \underline{r}_C - \underline{r}_D = (-3\underline{i} + 5\underline{j} + 2\underline{k})\,m \quad r_{C/D} = 6.164\,m \quad \underline{u}_{C/D} = -.487\underline{i} + .811\underline{j} + .324\underline{k}$

$$\underline{F}_{AD} = F_{AD}\,\underline{u}_{A/D} = F_{AD}(.870\underline{i} - .348\underline{j} + .348\underline{k})$$
$$\underline{F}_{BD} = F_{BD}\,\underline{u}_{B/D} = F_{BD}(-.728\underline{i} - .485\underline{j} + .485\underline{k})$$
$$\underline{F}_{CD} = F_{CD}\,\underline{u}_{C/D} = F_{CD}(-.487\underline{i} + .811\underline{j} + .324\underline{k})$$
$$\underline{W} = -20\underline{k}$$



$$\Sigma F_x = 0 \Rightarrow \left.\begin{aligned} .870 F_{AD} - .728 F_{BD} - .487 F_{CD} &= 0 \\ -.348 F_{AD} - .485 F_{BD} + .811 F_{CD} &= 0 \\ +.348 F_{AD} + .485 F_{BD} + .324 F_{CD} - 20 &= 0 \end{aligned}\right\}$$

פותר לפי מחשבון לקבל

$F_{AD} = 21.57\,N$

$F_{BD} = 13.99\,N$

$F_{CD} = 17.62\,N$

דטרמיננט $= -.7665$

SESSION # 5

EQUILIBRIUM OF PARTICLES

- A PARTICLE IS IN EQUILIBRIUM IF ITS ACCELERATION IS ZERO
  - EITHER THE PARTICLE IS AT REST OR IT HAS CONSTANT VELOCITY IF IN MOTION - FIRST LAW OF NEWTON
- WILL USE IDEAS ABOUT RESOLVING FORCES INTO COMPONENTS
- WILL ASSUME THAT THE SIZE OF A PARTICLE DOESN'T MATTER
- FORCES ACTING ON THE PARTICLE ARE CONCURRENT FORCES
- SIMPLEST SYSTEM TO DISCUSS IS 2-D CASE FIRST (ALL FORCES ACT IN SAME PLANE)

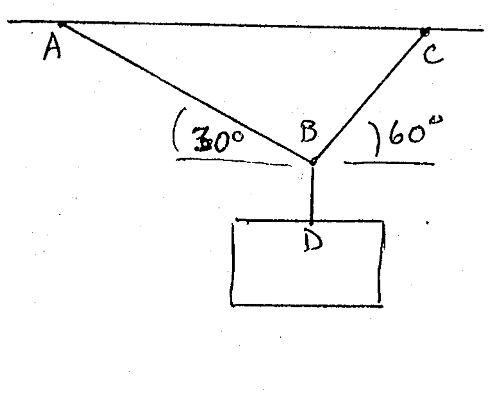
- IF A PARTICLE HAS NO ACCELERATION PARTICLE IS IN STATIC EQUILIBRIUM OR "EQUILIBRIUM"

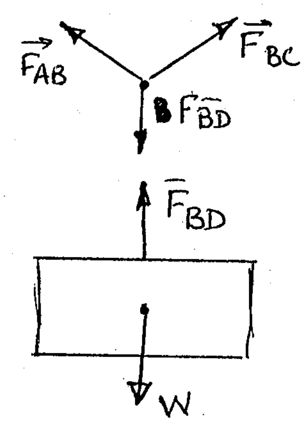
MATHEMATICALLY $\sum \vec{F} = \vec{0}$ VECTOR SUM OF FORCES = 0

- THIS IS A NECESSARY & SUFFICIENT CONDITION FOR EQUILIBRIUM OF PARTICLE

FREE-BODY DIAGRAM

- TO DO THIS DRAW A FREE-BODY DIAGRAM OF POINT ON WHICH FORCES ACT SHOWING ALL EXTERNAL FORCES ACTING ON THE PARTICLE





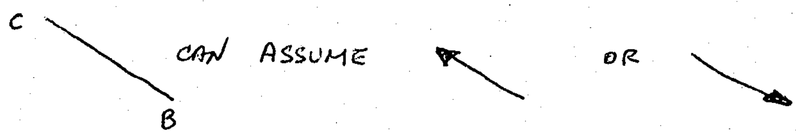
לבודד את החלקיק מסביבתו

1. IMAGINE THE PARTICLE TO BE ISOLATED FROM ITS SURROUNDING

לסמן את כל הכוחות הפועלים עליו

2. INDICATE ALL FORCES ACTING ON THE BODY
   - FORCES THAT CAUSE BODY TO MOVE : WEIGHT, MAGNETIC, ELECTROSTATIC
   - FORCES DUE TO CONSTRAINTS & SUPPORTS THAT PREVENT MOTION

3. LABEL ALL KNOWN FORCES (MAGNITUDE & DIRECTION)
   FOR UNKNOWN FORCES : ASSUME A DIRECTION ALONG THE LINE OF ACTION ~~[illegible]~~



- THE MAGNITUDE OF A FORCE IS CONSIDERED + . IF YOU GET A − IN YOUR ANSWER THAT MEANS THE DIRECTION OF THE FORCE <u>OPPOSITE</u> TO THE DIRECTION YOU ASSUMED.

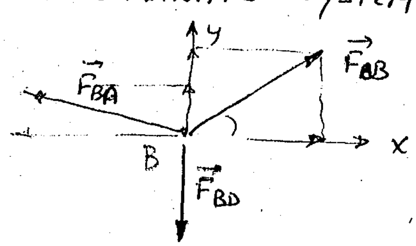
4. ONLY PUT THOSE FORCES THAT ACT ON THE ~~BODY~~ PARTICLE YOU'RE CONSIDERING
   pg 60 FIG 3-1.

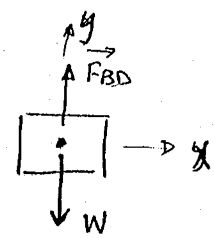
- 5. FOR A RIGID BODY THE SAME HOLDS IF WE ASSUME FORCES ARE CONCURRENT

- FOR THE DIAGRAM LET $\vec{F}_{BA}$ BE AT AN ANGLE OF 30° & $\vec{F}_{BC}$ BE AT 60° TO THE HORIZONTAL. LET W = 100 lb

- בוחר מערכת קואורדינטות
  SET UP A COORDINATE SYSTEM X−Y   IF ONE DOESN'T ALREADY EXIST

COLOR CHALK





<u>FOR THE BLOCK</u> $W = (-100\,\vec{j})\,lb$ $\vec{F}_{BD} = F_{BD}\,\vec{j}$

since BLOCK IN EQUIL. $\sum \vec{F} = \vec{0} = \sum F_x \vec{i} + \sum F_y \vec{j} = \vec{0}$ or $\overset{+}{\rightarrow}\sum F_x = 0,\ \overset{+}{\uparrow}\sum F_y = 0$

NO FORCES IN X DIRECTION $\overset{+}{\rightarrow}\sum F_x = 0$, $+\!\uparrow \sum F_y = F_{BD} - W = 0 \Rightarrow F_{BD} = W = 100\,lb$
MAGNITUDE OF F_{BD}

טבעת

<u>FOR THE RING AT B</u>

$$\sum \vec{F} = \vec{0} = \sum F_x \vec{i} + \sum F_y \vec{j} = \vec{0}$$

in the x DIRECTION $F_{CB_x} - F_{BA_x} = 0 = F_{CB}\cos 60° - F_{AB}\cos 30° = 0$

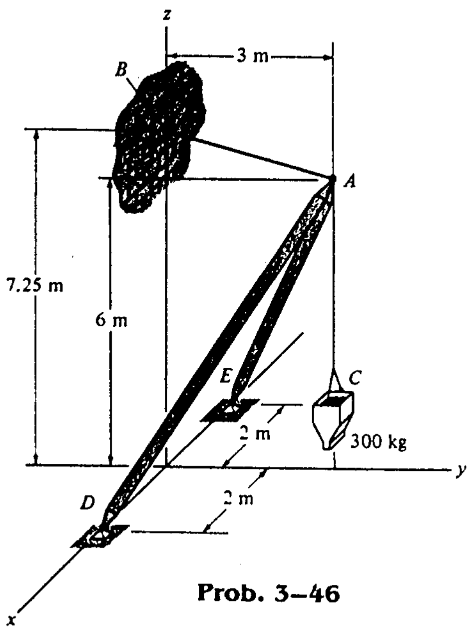
$F_{CB}\cdot(.5) - F_{AB}(.866) = 0$ or $F_{CB} = 2(.866)F_{AB} = \underline{1.732 F_{AB}}$

השם: ____________________

ת.ז. : ____________________

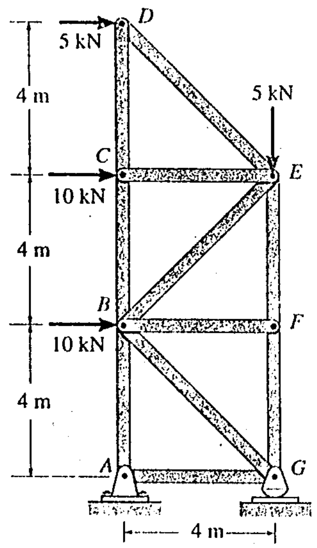
3–46. The boom supports a bucket and its contents, which have a total mass of 300 kg. Determine the forces developed in struts *AD* and *AE* and the tension in cable *AB* for equilibrium. The force in each strut acts along its axis.

מערכת המוטות תומכת בסל ותכניו. מסת הסל והתכנים היא 300kg. מיצאו את הכוחות המפותחים במוטות AD ו- AE וגם מתח בכבל AB כדי שיהיה שיווי משקל. כוח במוט פועל בכיוון ציר המוט.



Prob. 3–46

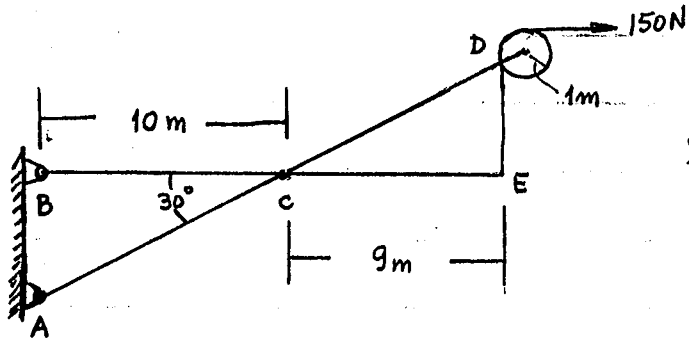
ת.ז. ____________



Prob. 6–24

מצאו את הכוחות באברים BE ו-EF
של המסבך.
מה אפשר לאמר לגבי אבר BF?

ת.ב. ____________



מצאו את הכוחות בפרקים (פינים) A ו־C של המסגרת. הגלגלת ב־D חלקה (אין חיכוך שם).

$+\uparrow \Sigma F_y$

$$F_{CB_y} + F_{AB_y} - F_{BD} = 0$$

$$F_{CB} \sin 60^\circ + F_{AB} \sin 30^\circ - F_{BD} = 0$$

$$.866 F_{CB} + .5 F_{AB} - F_{BD} = 0$$

$F_{CB} = 1.732 F_{AB}$, $F_{BD} = 100 lb$

$$.866 (1.732 F_{AB}) + .5 F_{AB} - 100^{lb} = 0$$

$$F_{AB} (.866(1.732) + .5) - 100^{lb} = 0 \quad +100 lb.$$

$$F_{AB} (2) = 100 lb \qquad \underline{F_{AB} = 50 lb}$$

$$F_{CB} = 1.732 F_{AB} = 1.732 (50 lb) = \underline{86.6 lb}$$

} because they are + we picked the right direction

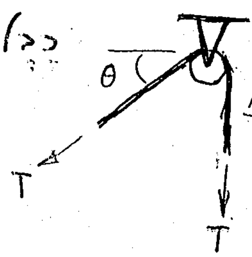
- CAN INTERCHANGE TERMS "PARTICLE" & "RIGID BODY"
  - REPLACE THE RIGID BODY BY A PARTICLE WITH WEIGHT OF RIGID BODY
  - WEIGHT ACTS AS CONCENTRATED FORCE THROUGH ITS GEOMETRIC CENTER
  - REMAINING FORCES PASS THROUGH CENTER : FORCES ARE CONCURRENT & COPLANAR & MUST SATISFY

$$\Sigma \vec{F} = \vec{0}$$

PROBLEMS W/ CABLES & PULLEYS : CABLES ARE WEIGHTLESS & UNSTRETCHABLE

CABLES ONLY SUPPORT TENSION IN THE DIRECTION OF THE CABLE

4.32 f.22

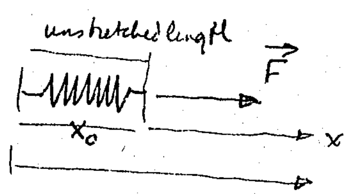


- CONTINUOUS CABLE OVER A PULLEY (FRICTIONLESS) TRANSMITS MAGNITUDE OF THE TENSION FORCE $\vec{T}$ NO MATTER WHAT θ IS.
- REASON IS THE SUPPORT

<u>SPRINGS</u> : FORCE IN THE SPRING $F = k$(elongated length − unstretched length)

if a spring acts in the x direction

$F = kx$ where x is measured + from the unstretched length.

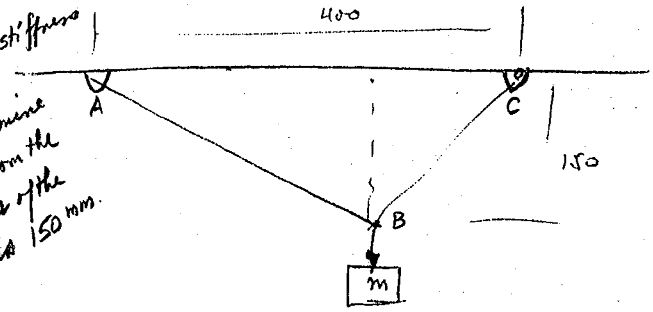


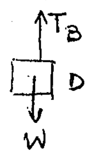
K is the spring constant or the stiffness

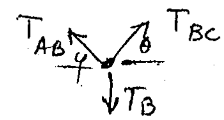
set up

$$F = k(x - x_0)$$

3-11 An elastic cord has a stiffness of 50 N/m & unstretched length of 400 mm. Determine the mass D suspended from the cord if the displacement of the pulley from the ceiling is 150 mm. Neglect size of pulley.





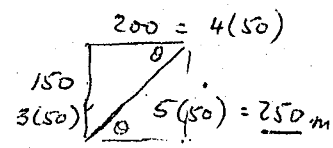


ΣF_x $\quad T_{BC} \cos\theta - T_{AB} \cos\varphi = 0$

ΣF_y $\quad T_{AB} \sin\varphi + T_{BC} \sin\theta - T_B = 0.$

Assume $T_{AB} = T_{BC}$ just like pulley $= \underline{\theta = \varphi}$.

$\Sigma F_y = 0 \Rightarrow \quad 2 T_{AB} \sin\theta - T_B = 0.$



$\sin\theta = \frac{150}{250} = .6$

Now. original length = 200 mm.

new length = 250 mm.

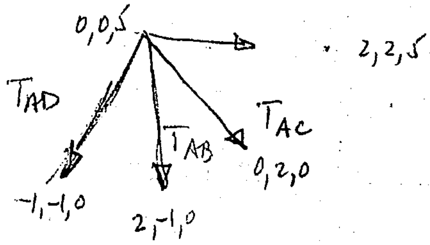
$F = k(x - x_0)$

$= 50 \frac{N}{m} \left(\frac{250 - 200}{1000 \text{ mm/m}} \right)^{mm} = 2.5 N.$

$\Sigma F_y = 0 \Rightarrow 2(2.5)(.6) - T_B = 0. \quad T_B = 3 N.$

$m g = T_B \quad m = \frac{3N}{9.81 \text{ m/sec}^2} \cong .306 kg.$

3-77



$\bar{n} = \frac{2\bar{i}+2\bar{j}}{2\sqrt{2}} = .7071\bar{i} + .7071\bar{j}$

$F = 707.1\bar{i} + 707.1\bar{j}$ N

$\bar{n}_{AD} = \frac{(-\bar{i}-\bar{j}-5\bar{k})}{\sqrt{27}}$ $\bar{n}_{AB} = \frac{2\bar{i}-\bar{j}+5\bar{k}}{\sqrt{30}}$ $\bar{n}_{AC} = \frac{0\bar{i}+2\bar{j}-5\bar{k}}{\sqrt{29}}$

$\Sigma\vec{F} = \vec{0}$ $T_{AD}\bar{n}_{AD} + T_{AB}\bar{n}_{AB} + T_{AC}\bar{n}_{AC} + \bar{F} = \bar{0}$

(-1)× $-\frac{T_{AD}}{\sqrt{27}} + \frac{2}{\sqrt{30}}T_{AB} + 707.1 = 0$

(+1)× (-5)× $-\frac{T_{AD}}{\sqrt{27}} - \frac{T_{AB}}{\sqrt{30}} + 707.1 + T_{AC}\cdot\frac{2}{\sqrt{29}} = 0$

+(1)× $\frac{-5}{\sqrt{27}}T_{AD} - \frac{5}{\sqrt{30}}T_{AB} - \frac{5}{\sqrt{29}}T_{AC} = 0$

$-3535.5 - \frac{15}{\sqrt{29}}T_{AC} = 0$ $\underline{T_{AC}} = -1269\text{ N} = \frac{-3535.5\sqrt{29}}{15}$

$-\frac{3}{\sqrt{30}}T_{AB} + T_{AC}\frac{2}{\sqrt{29}} = 0$ $\underline{T_{AB}} = T_{AC}\cdot\frac{2}{\sqrt{29}}\cdot\frac{\sqrt{30}}{3} = -861\text{ N}$

$T_{AD} = -\left[\frac{\sqrt{27}}{\sqrt{30}}T_{AB} + \frac{\sqrt{27}}{\sqrt{29}}T_{AC}\right] \simeq -[-720 - 1269] = 2041\text{ N}$

AS WE SHOWED $\sum \vec{F} = \vec{0}$ FOR EQUILIBRIUM

IF WE WANT THEM IN TERMS OF THE CARTESIAN COMPONENTS $\vec{i}, \vec{j}$

$$\sum \vec{F} = \vec{0} = \sum F_x \vec{i} + \sum F_y \vec{j} \quad \text{or} \quad \sum F_x = 0 \text{ or } \sum F_y = 0$$

suppose we want to express them along any 2 lines

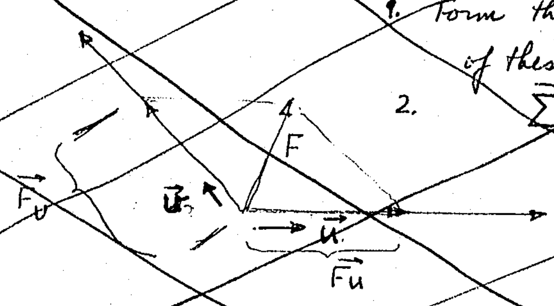
1. Form the unit vectors in the directions of these lines $\vec{u}_1$ & $\vec{u}_2$

2. $\sum \vec{F} = 0 = \sum F_{u_1} \vec{u}_1 + \sum F_{u_2} \vec{u}_2$

$\sum F_{u_1} = 0$

$\sum F_{u_2} = 0$

NOT TRUE!



where F_u, F_v is the component of any of the forces in the direction of $\vec{u}, \vec{v}$

TO OBTAIN: $F_u = \vec{F} \cdot \vec{u}$ $\quad F_v = \vec{F} \cdot \vec{v}$

IN 3-D

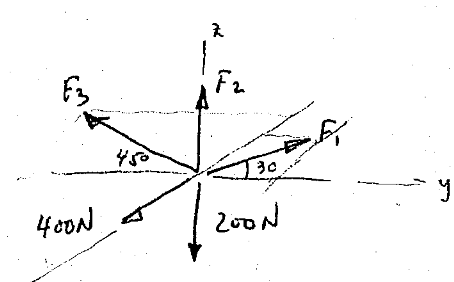
JUST AS IN 2-D WE MUST SATISFY $\sum \vec{F} = \vec{0}$

$$\sum \vec{F} = \vec{0} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k}$$ in terms of the CARTESIAN

$$\left.\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array}\right\} \text{ for 3D equilibrium}$$

1. Draw Free-Body Diagram, label all known & unknown forces
2. Establish coordinates x, y, z at the particle & ~~apply the equations~~
3. Express each force in its cartesian components
4. Apply the equations

Problem 3-22



$\sum \vec{F} = \vec{0}$

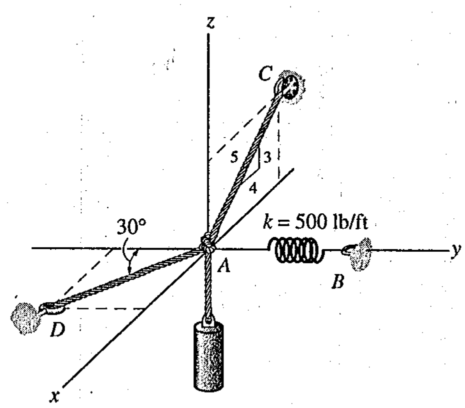
$\sum F_x = F_1 \cos 30° - F_3 \cos 45° = 0$

$\sum F_y = 400 - F_1 \sin 30° - F_3 \sin 45°$

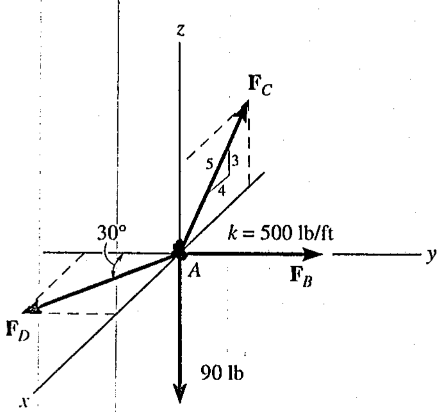
$\sum F_z = F_2 - 200$

Example 3–6

The 90-lb cylinder shown in Fig. 3–9a is supported by two cables and a spring having a stiffness $k = 500$ lb/ft. Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.



(a)



(b)

Fig. 3–9

SOLUTION

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the analysis since the cable forces are concurrent at this point. Fig. 3–9b.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be directly applied. Considering components directed along the positive axes as "positive," we have

$$\Sigma F_x = 0; \quad F_D \sin 30° - \tfrac{4}{5}F_C = 0 \quad (1)$$

$$\Sigma F_y = 0; \quad -F_D \cos 30° + F_B = 0 \quad (2)$$

$$\Sigma F_z = 0; \quad \tfrac{3}{5}F_C - 90 \text{ lb} = 0 \quad (3)$$

Solving Eq. 3 for F_C, then Eq. 1 for F_D, and finally Eq. 2 for F_B, we get

$$F_C = 150 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 240 \text{ lb} \qquad \textit{Ans.}$$

$$F_B = 208 \text{ lb} \qquad \textit{Ans.}$$

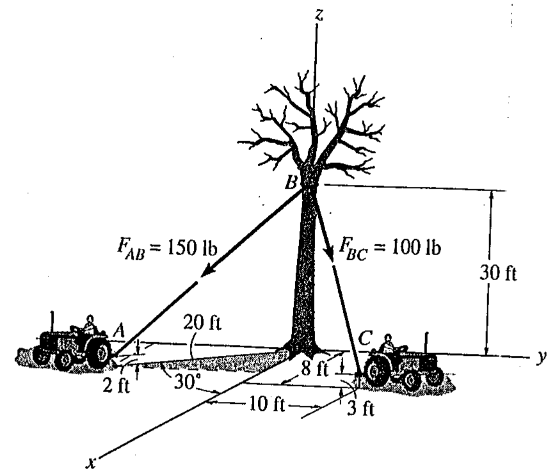
The stretch of the spring is therefore

$$F_B = ks_{AB}$$

$$208 \text{ lb} = 500 \text{ lb/ft} \, (s_{AB})$$

$$s_{AB} = 0.416 \text{ ft} \qquad \textit{Ans.}$$

... direction angles of the resultant force.

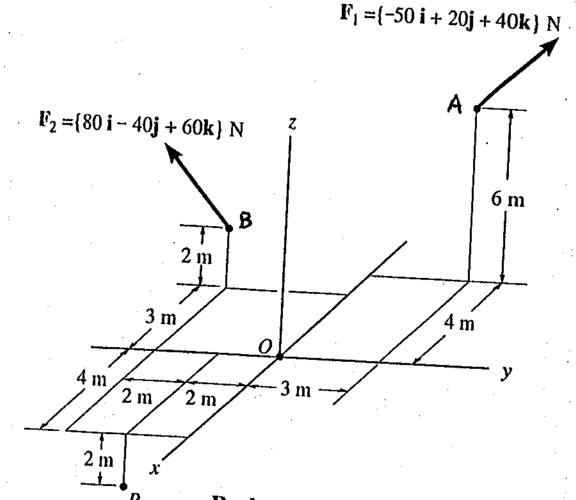


$\underline{F_1} = 75.5\underline{i} - 43.6\underline{j} - 122\underline{k}$

Prob. 2–89

... Cartesian vector.

*4–24. Determine the resultant moment of the forces about point P. Express the result as a Cartesian vector.

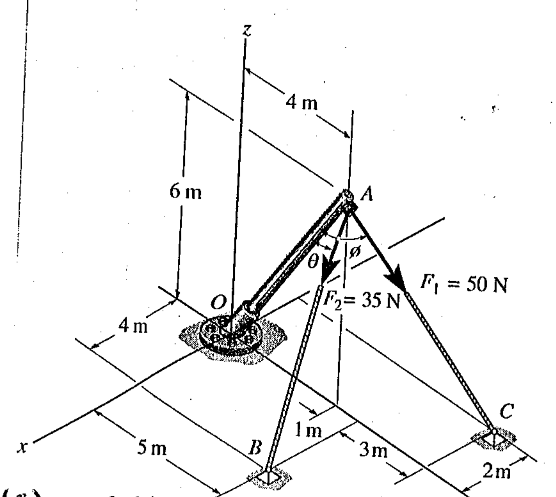


$\underline{r}_{B/O} = -3\underline{i} - 4\underline{j} + 2\underline{k}$

$\underline{M}_O = \underline{r}_{B/O} \times \underline{F}_2$

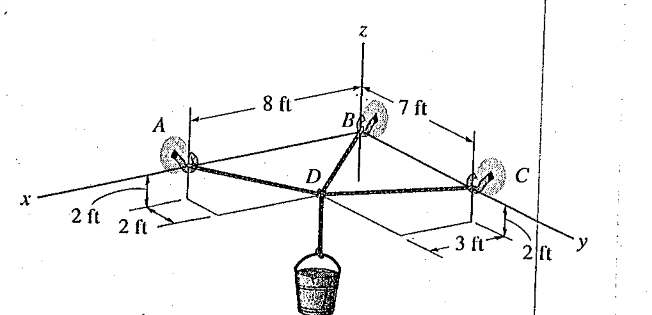
Probs. 4–23/4–24

2–113. The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis OA of the pole.



$(F_1)_{AO} = 18.5N$

3–53. The bucket has a weight of 20 lb. Determine the tension developed in each cord for equilibrium.



$F_{DA} = 21.5N$

Prob. 3–53

A : (17.32, 10, 2)
C = (8, 10, 3)

$\underline{r}_{A/B} = \underline{r}_A - \underline{r}_B = (11.32\underline{i} - 10\underline{j} - 28\underline{k})\ m \qquad r_{A/B} = 34.41\ m \qquad \underline{u}_{A/B} = \frac{\underline{r}_{A/B}}{r_{A/B}} = 0.503\underline{i} - 0.291\underline{j} - 0.814\underline{k}$

$\underline{F}_{AB} = F_{AB}\,\underline{u}_{A/B} = 150(.503\underline{i} - .291\underline{j} - .814\underline{k}) = (75.5\underline{i} - 43.7\underline{j} - 122\underline{k})\ N$

$\underline{r}_{C/B} = \underline{r}_C - \underline{r}_B = (8\underline{i} + 10\underline{j} - 27\underline{k})\ m \qquad r_{C/B} = 29.88\ m \qquad \underline{u}_{C/B} = \frac{\underline{r}_{C/B}}{r_{C/B}} = 0.268\underline{i} + 0.335\underline{j} - 0.904\underline{k}$

$\underline{F}_{BC} = F_{BC}\,\underline{u}_{C/B} = 100(.268\underline{i} + .335\underline{j} - .904\underline{k}) = (26.8\underline{i} + 33.5\underline{j} - 90.4\underline{k})\ N$

$\underline{F}_R = \underline{F}_{AB} + \underline{F}_{BC} = (102.3\underline{i} - 10.2\underline{j} - 212.4\underline{k})\ N \qquad F_R = 235.97\ N \qquad \underline{u}_R = 0.434\underline{i} - .043\underline{j} - .900\underline{k}$

$\alpha = \cos^{-1}(.434) = 64.28^\circ;\quad \beta = \cos^{-1}(-.043) = 92.46^\circ;\quad \gamma = \cos^{-1}(-.900) = 15$

2-96

C (−3, 0, 0)
D = (2, −3, 0)
A = (x, y, 0)
B = (0, 0, 4)

$\underline{r}_{D/B} = (2\underline{i} - 3\underline{j} - 4\underline{k})\ m \qquad r_{D/B} = 5.385\ m \qquad \underline{u}_{D/B} = \underline{r}_{D/B}/r_{D/B} = .371\underline{i} - .557\underline{j} - .743\underline{k}$

$\underline{r}_{C/B} = (-3\underline{i} + 0\underline{j} - 4\underline{k})\ m \qquad r_{C/B} = 5\ m \qquad \underline{u}_{C/B} = \underline{r}_{C/B}/r_{C/B} = -.6\underline{i} + 0\underline{j} - 0.8\underline{k}$

$\underline{r}_{A/B} = (x\underline{i} + y\underline{j} - 4\underline{k})\ m \qquad r_{A/B} = \sqrt{x^2 + y^2 + 16}\ m = 6\ m \qquad \underline{u}_{A/B} = \underline{r}_{A/B}/r_{A/B} = 0.667\underline{i} + 0.333\underline{j} - 0.667\underline{k}$

$\underline{F}_{DB} = F_{DB}\,\underline{u}_{D/B} = 250(.371\underline{i} - .557\underline{j} - .743\underline{k}) = 92.75\underline{i} - 139.25\underline{j} + 185.75\underline{k}$

$\underline{F}_{CB} = F_{CB}\,\underline{u}_{C/B} = 350(-.6\underline{i} + 0\underline{j} - 0.8\underline{k}) = -210\underline{i} + 0 + 280\underline{k}$

$\underline{F}_{AB} = F_{AB}\,\underline{u}_{A/B} = 300(0.667\underline{i} + 0.333\underline{j} - 0.667\underline{k}) = 200\underline{i} + 100\underline{j} - 200\underline{k}$

$\underline{F}_R = \underline{F}_{DB} + \underline{F}_{CB} + \underline{F}_{AB} = (82.75\underline{i} - 39.25\underline{j} - 665.75\underline{k})\ N \Rightarrow 672.03\ N = F_R$

$\alpha = \cos^{-1}\left(\frac{82.75}{672.03}\right) = 82.9^\circ;\quad \beta = \cos^{-1}\left(\frac{-39.25}{672.03}\right) = 93.3^\circ;\quad \gamma = \cos^{-1}\left(\frac{-665.75}{672.03}\right) = 172.2^\circ$

2-13

A (0, 4, 6)
B (4, 5, 0)
C (−2, 8, 0)

$\underline{r}_{B/A} = (4\underline{i} + \underline{j} - 6\underline{k})\ m \qquad r_{B/A} = 7.28\ m \qquad \underline{u}_{B/A} = .549\underline{i} + .137\underline{i} - .824\underline{k}$

$\underline{r}_{C/A} = (-2\underline{i} + 4\underline{j} - 6\underline{k})\ m \qquad r_{C/A} = 7.48\ m \qquad \underline{u}_{C/A} = -.267\underline{i} + .535\underline{i} - .802\underline{k}$

$\underline{r}_{O/A} = -(0\underline{i} + 4\underline{j} + 6\underline{k})\ m \qquad r_{A/O} = 7.21 \qquad \underline{u}_{O/A} = -(0\underline{i} + .555\underline{j} + .832\underline{k})$

$\underline{F}_{BA} = F_{BA}\,\underline{u}_{B/A} = 35(.549\underline{i} + .137\underline{j} - .824\underline{k}) = 19.22\underline{i} + 4.80\underline{j} - 28.84\underline{k}$

$\underline{F}_{CA} = F_{CA}\,\underline{u}_{C/A} = 50(-.267\underline{i} + .535\underline{j} - .802\underline{k}) = -13.35\underline{i} + 26.75\underline{j} - 40.10\underline{k}$

$(F_1)_{OA} = \underline{F}_{CA} \cdot \underline{u}_{O/A} = (-.555)(26.75) - .832(-40.10) = +18.52\ N$

$(F_2)_{OA} = \underline{F}_{BA} \cdot \underline{u}_{O/A} = (4.8)(-.555) - 28.84(-.832) = +21.33\ N$

4-13

$\underline{F}_1 = (-50\underline{i} + 20\underline{j} + 40\underline{k})\ N \qquad \underline{r}_{A/O} = (-4\underline{i} + 3\underline{j} + 6\underline{k})\ m$

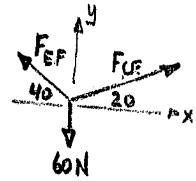
$\underline{F}_2 = (80\underline{i} - 40\underline{j} + 60\underline{k})\ N \qquad \underline{r}_{B/O} = (-3\underline{i} - 4\underline{j} + 2\underline{k})\ m$

$\underline{M}_1 = \underline{r}_{A/O} \times \underline{F}_1 = (-80\underline{k} + 160\underline{j} + 150\underline{k} + 120\underline{i} - 300\underline{j} - 120\underline{i}) = (0\underline{i} - 140\underline{j} + 70\underline{k})\ N\text{-}m$

$\underline{M}_2 = \underline{r}_{B/O} \times \underline{F}_2 = (120\underline{k} + 180\underline{j} + 320\underline{k} - 240\underline{i} + 160\underline{j} + 80\underline{i}) = (-160\underline{i} + 340\underline{j} + 440\underline{k})\ N\text{-}m$

$\Sigma\underline{M} = \underline{M}_1 + \underline{M}_2 = -160\underline{i} + 200\underline{j} + 510\underline{k}$

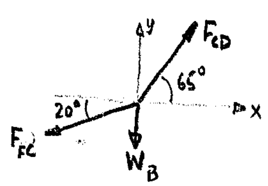
3-51



$\Sigma F_x = F_{CF}\cos 20 - F_{EF}\cos 40^\circ = 0 \Rightarrow F_{CF} = F_{EF}\frac{\cos 40^\circ}{\cos 20^\circ} = .8152 F_{EF}$

$\Sigma F_y = F_{CF}\sin 20 + F_{EF}\sin 40^\circ - 60 = 0$

$= (.8152 F_{EF})(.342) + F_{EF}(.6428) - 60 = 0 \qquad \underline{F_{EF} = 65.1\ N} \qquad \underline{F_{CF} = .8152 F_{EF} = 53.1\ N}$

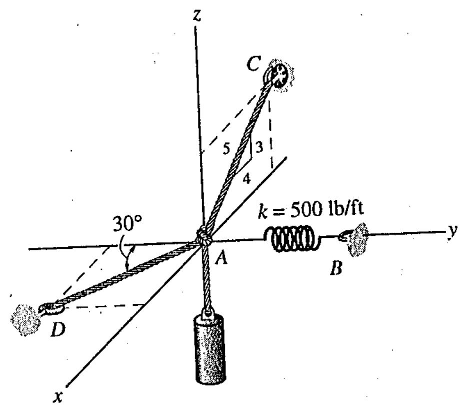


$\Sigma F_x = F_{CD}\cos 65^\circ - F_{FC}\cos 20^\circ = 0 \Rightarrow \underline{F_{CD}} = F_{FC}\frac{\cos 20^\circ}{\cos 65^\circ} = 2.2235 F_{FC} = \underline{118.1\ N}$

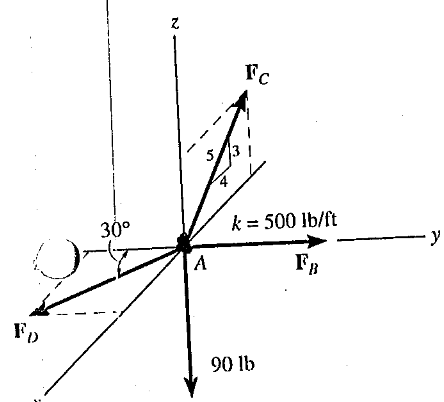
$\Sigma F_y = F_{CD}\sin 65^\circ - F_{FC}\sin 20^\circ - W_B = 0 \Rightarrow \underline{W_B} = F_{CD}\sin 65^\circ - F_{FC}\sin 20^\circ = \underline{88.9\ N}$

Example 3–6

The 90-lb cylinder shown in Fig. 3–9*a* is supported by two cables and a spring having a stiffness $k = 500$ lb/ft. Determine the force in the cables and the stretch of the spring for equilibrium. Cable *AD* lies in the x–y plane and cable *AC* lies in the x–z plane.



(a)



(b)

Fig. 3–9

SOLUTION

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at *A* is chosen for the analysis since the cable forces are concurrent at this point. Fig. 3–9*b*.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be directly applied. Considering components directed along the positive axes as "positive," we have

$$\Sigma F_x = 0; \qquad F_D \sin 30° - \tfrac{4}{5}F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30° + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \tfrac{3}{5}F_C - 90 \text{ lb} = 0 \qquad (3)$$

Solving Eq. 3 for F_C, then Eq. 1 for F_D, and finally Eq. 2 for F_B, we get

$$F_C = 150 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 240 \text{ lb} \qquad \textit{Ans.}$$

$$F_B = 208 \text{ lb} \qquad \textit{Ans.}$$

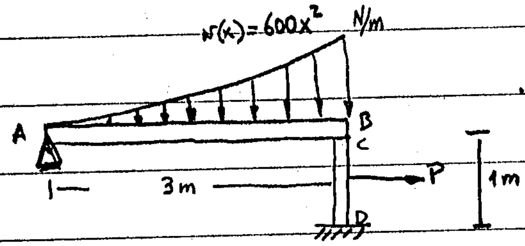
The stretch of the spring is therefore

$$F_B = k s_{AB}$$

$$208 \text{ lb} = 500 \text{ lb/ft } (s_{AB})$$

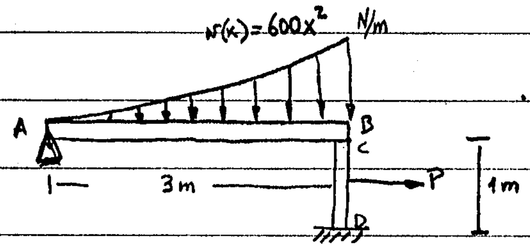
$$s_{AB} = 0.416 \text{ ft} \qquad \textit{Ans.}$$

ת.ז.: ______



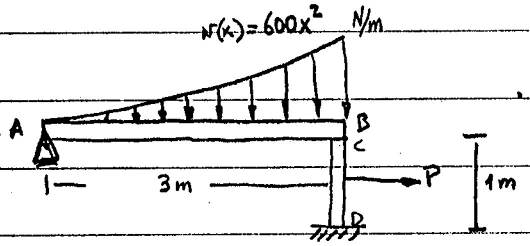
קורה AB נתמכת בסמך נייח ב-A ובקורה CD ב-B. קורה AB סובלת בכוח מפורס $w(x)=600x^2 \frac{N}{m}$.
אם ב-C מקדם חיכוך הוא $\mu_C=0.4$ וב-D מקדם חיכוך הוא $\mu_D=0.3$, מהו הכוח האופקי המינימלי, P, הנדרש כדי להזיז את קורה CD
הוכיחו את תשובתכם! P פועל באמצע CD

ת.ז.: ________



קורה AB נתמכת בסמך נ"ת ב-A ובקורה CD ב-B. קורה AB סובלת בכוח מפורס $w(x)=600x^2 \frac{N}{m}$. אם ב-C מקדם חיכוך הוא $\mu_C=0.4$ וב-D מקדם חיכוך הוא $\mu_D=0.3$, מהו הכוח האופקי המינימלי, P, הנדרש כדי להזיז את קורה CD הוכיחו את תשובתכם! P פועל באמצע CD

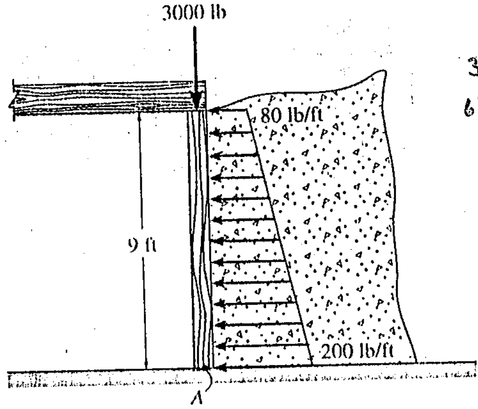
ת.ז.: ______



קורה AB נתמכת בסמך נייח ב-A ובקוה CD ב-B. קורה AB סובלת בכוח מפורס $w(x)=600x^2 \frac{N}{m}$. אם ב-C מקדם חיכוך הוא $\mu_C=0.4$ וב-D מקדם חיכוך הוא $\mu_D=0.3$, מהו הכוח האופקי המינימלי, P, הנזקק כדי להזיז את קורה CD

הוכיחו את תשובתכם! P פועל באמצע CD

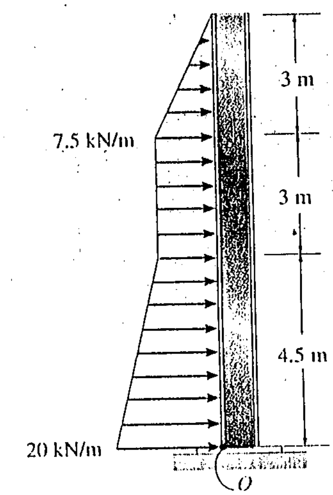
4–130. The column is used to support the floor which exerts a force of 3000 lb on the top of the column. The effect of soil pressure along the side of the column is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column measured from its base A.



3.25 kip
67.2°
3.86 ft

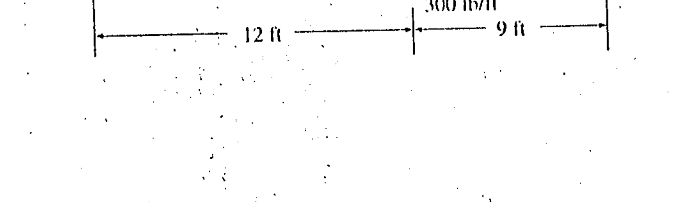
Prob. 4–130

***4–132.** Replace the loading by an equivalent resultant force and couple moment acting at point O.



Prob. 4–132

4–131. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location measured from point O.

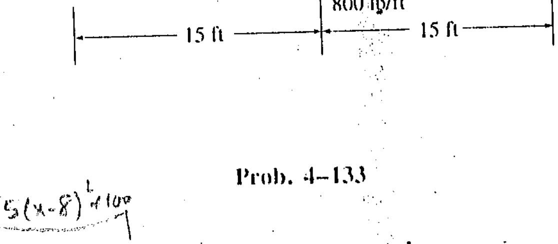


Prob. 4–131

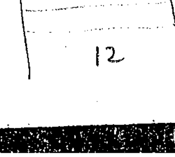
4-141 1.87 kip 3.66 ft

4–133. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam measured from the pin at C.

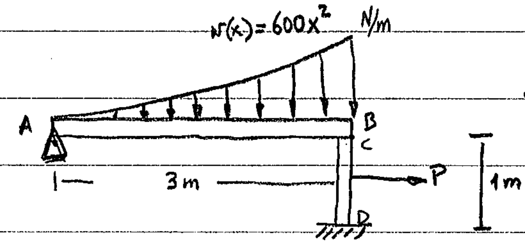
18 kip ↓ 11.7 ft



Prob. 4–133



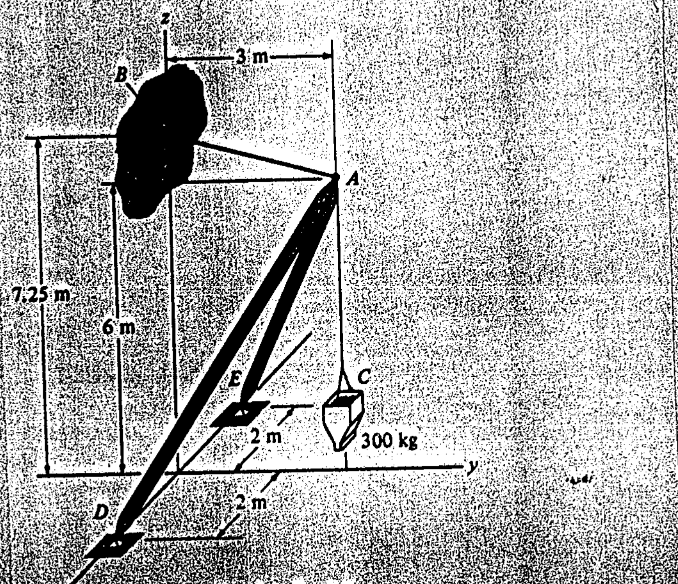
נ.ב.:



קורה AB נתמכת בסמך נ"ת ב-A ובקיר CD ב-B. קורה AB סובלת בכוח מפורס $w(x)=600x^2\frac{N}{m}$. אם ב-C מקדם חיכוך הוא $\mu_C=0.4$ וב-D מקדם חיכוך הוא $\mu_D=0.3$, מהו הכוח האופקי המינימלי, P, הנדרש כדי להזיז את קורה CD. הוכיחו את תשובתכם! P פועל באמצע CD

ת.ז. : ______

3–46. The boom supports a bucket and its contents, which have a total mass of 300 kg. Determine the forces developed in struts *AD* and *AE* and the tension in cable *AB* for equilibrium. The force in each strut acts along its axis.



Prob. 3–46

מערכת המוטות תומכת בסל ותכניו. מסת הסל והתכנים היא 300kg. מיצאו את הכוחות המפותחים במוטות AD ו- AE וגם מתח בכבל AB כדי שיהיה שיווי משקל. כוח במוט פועל בכיוון ציר המוט.

שם: ____________

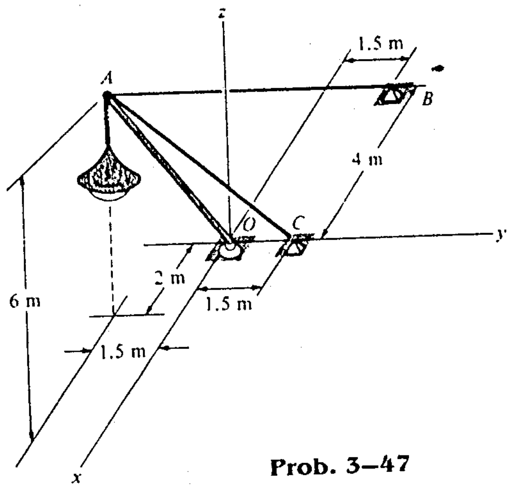
ת.ז. ____________

האור שנמצא בתרשים נתמך בנקודה A בשני כבלים AB ו-AC ובמוט OA. מסת האור שווה ל-15kg.

א) כתבו ביטויים בשביל הכוחות הקרטזיים $\underline{F}_{AB}$ ו-$\underline{F}_{AC}$

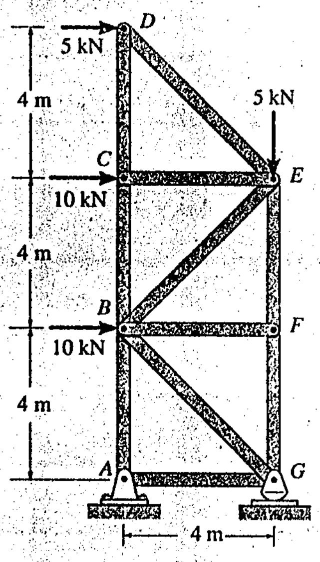
ב) מצאו את הזווית בין שני הכוחות האלה.

ג) מצאו את המומנט M_O (מומנט מסביב O) שבא מהכוח $\underline{F}_{AC}$



Prob. 3–47

ת.ז. ____________

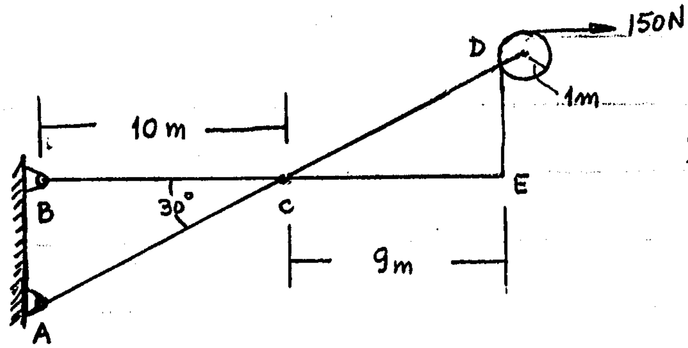


Prob. 6–24

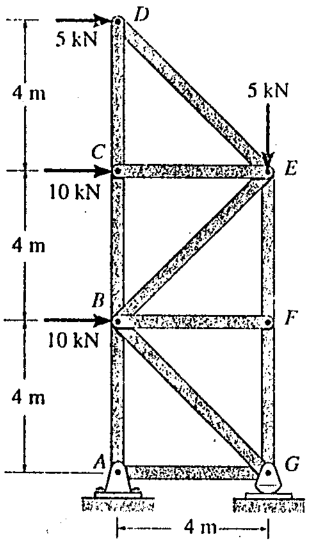
מצאו את הכוחות באברים BE ו-EF של המסבך.

מה אפשר לאמר לגבי אבר BF?

מצאו את הכוחות בפרקים (פינים) A ו-C של המסגרת. הגלגלת ב-D חלקה (אין חיכוך שם).



ת.ז. ____________



Prob. 6–24

מצאו את הכוחות באיברים BE ו-EF של המסבך.

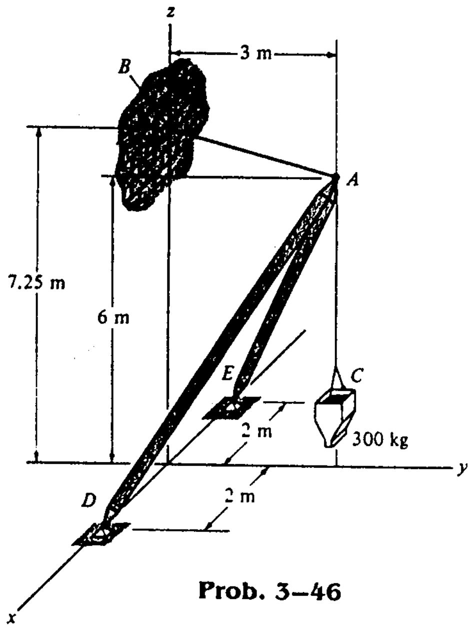
מה אפשר לאמר לגבי איבר BF?

השם: ____________

ת.ז. : ____________

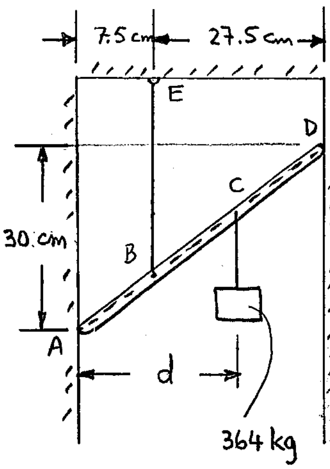
3–46. The boom supports a bucket and its contents, which have a total mass of 300 kg. Determine the forces developed in struts AD and AE and the tension in cable AB for equilibrium. The force in each strut acts along its axis.

מערכת המוטות תומכת בסל ותכניו. מסת הסל והתכנים היא 300kg. מיצאו את הכוחות המפותחים במוטות AD ו- AE וגם מתח בכבל AB כדי שיהיה שיווי משקל. כוח במוט פועל בכיוון ציר המוט.



Prob. 3–46

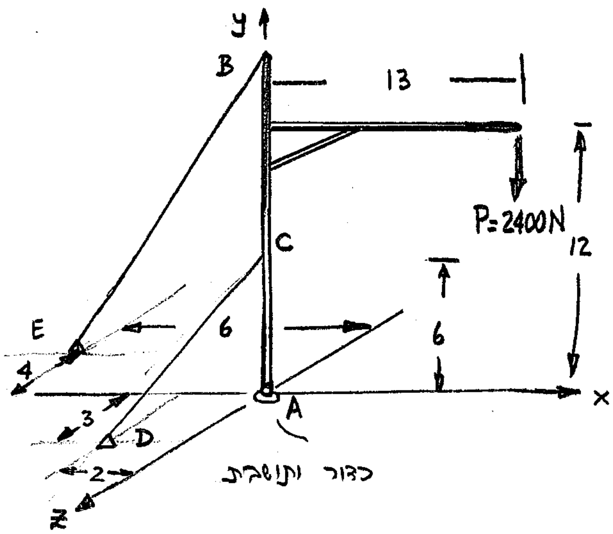
ת.ז. ________



מוט קל AD נתלה מכבל BE ותומך במסה של 364kg ב-C.
הקצוות A ו-D של המוט נוגעים בקירות מאונכות וחלקות.

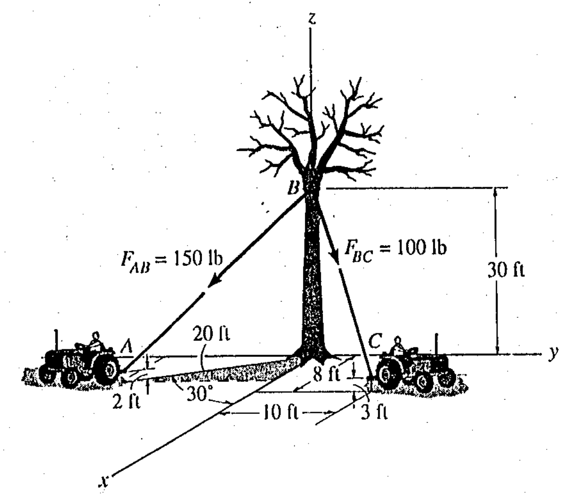
א) מצאו המתח בכבל BE והתגובות ב-A ו-D, אם $d=20cm$.

ב) מצאו את המרחק המקסימלי, d, שתוכלו להשתמש בו, אם ערך המקסימלי המותר לתגובה (ריאקציה) ב-A הוא 2225N



מצאו את הכוחות ב BE ו-CD והראקציות ב-A

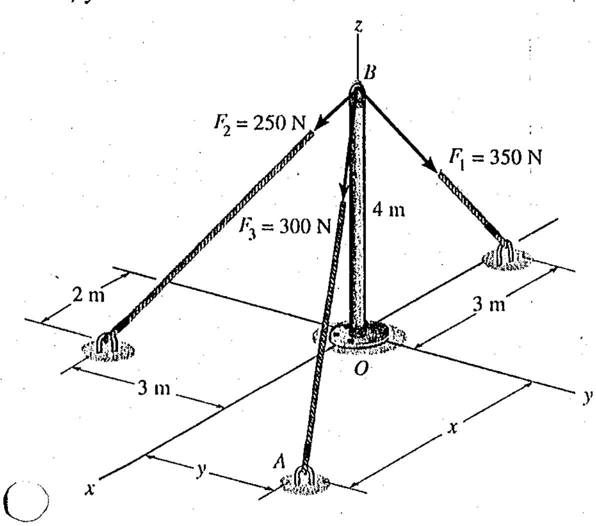
magnitude and coordinate direction angles of the resultant force.



$F_1 = 75.5i - 43.6j - 122k$

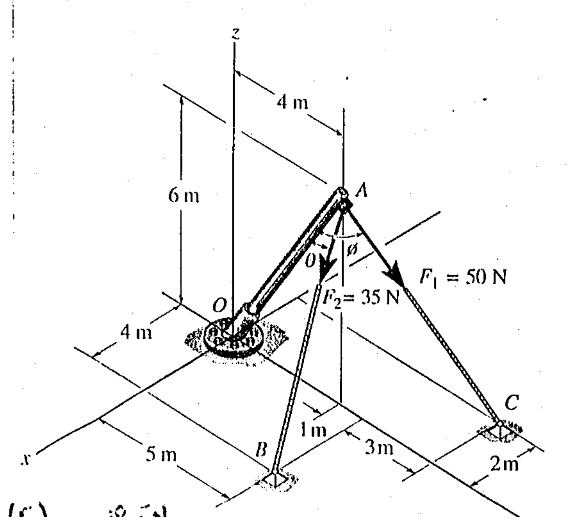
Prob. 2–89

*2[illegible]. The pole is held in place by three cables. If the force of each cable acting on the pole is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Set $x = 4$ m, $y = 2$ m.

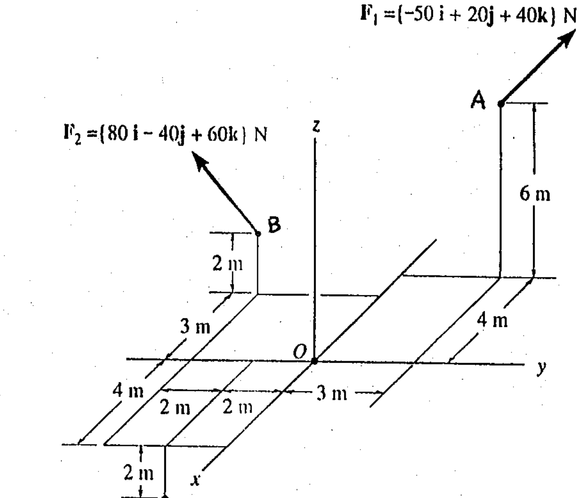


Prob. 2–96

2–113. The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis OA of the pole.



*4–24. Determine the resultant moment of the forces about point P. Express the result as a Cartesian vector.

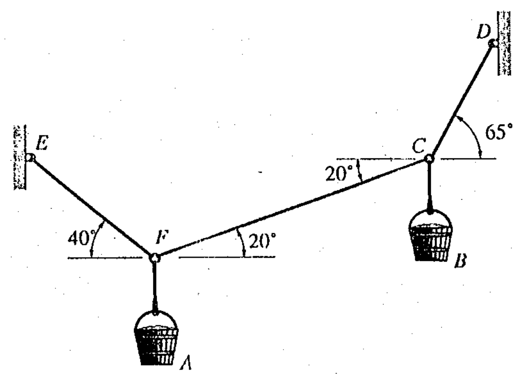


$r_{B/O} = -3i - 4j + 2k$

$M_O = r_{B/O} \times F_2$

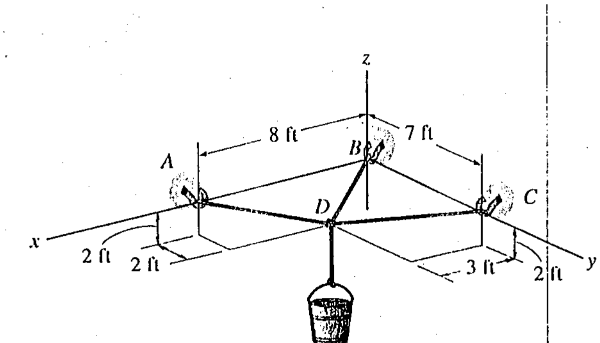
Probs. 4–23/4–24

3–51. The cords suspend the two buckets in the equilibrium position shown. Determine the weight of bucket B. Bucket A has a weight of 60 lb.



Prob. 3–51

3–53. The bucket has a weight of 20 lb. Determine the tension developed in each cord for equilibrium.



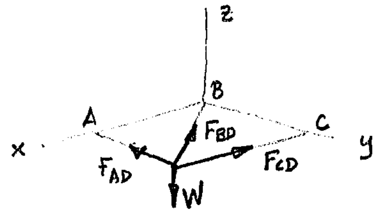
$F_{DA} = 21.5N$

A (8,0,0) $\quad \underline{r}_{A/D} = \underline{r}_A - \underline{r}_D =$ ([illegible]) m $\quad r_{A/D} =$ [illegible] m $\quad \underline{u}_{A/D} = .870\underline{i} - .348\underline{j} + .348\underline{k}$

B (0,0,0) $\quad \underline{r}_{B/D} = \underline{r}_B - \underline{r}_D = (-3\underline{i} - 2\underline{j} + 2\underline{k})\,m \quad r_{B/D} = 4.123\,m \quad \underline{u}_{B/D} = -.728\underline{i} - .485\underline{j} + .485\underline{k}$

C (0,7,0) $\quad \underline{r}_{C/D} = \underline{r}_C - \underline{r}_D = (-3\underline{i} + 5\underline{j} + 2\underline{k})\,m \quad r_{C/D} = 6.164\,m \quad \underline{u}_{C/D} = -.487\underline{i} + .811\underline{j} + .324\underline{k}$

$$\underline{F}_{AD} = F_{AD}\,\underline{u}_{A/D} = F_{AD}(.870\underline{i} - .348\underline{j} + .348\underline{k})$$
$$\underline{F}_{BD} = F_{BD}\,\underline{u}_{B/D} = F_{BD}(-.728\underline{i} - .485\underline{j} + .485\underline{k})$$
$$\underline{F}_{CD} = F_{CD}\,\underline{u}_{C/D} = F_{CD}(-.487\underline{i} + .811\underline{j} + .324\underline{k})$$
$$\underline{W} = -20\underline{k}$$



$$\Sigma F_x = 0 \Rightarrow \left.\begin{aligned} &.870\,F_{AD} - .728\,F_{BD} - .487\,F_{CD} = 0 \\ &-.348\,F_{AD} - .485\,F_{BD} + .811\,F_{CD} = 0 \\ &+.348\,F_{AD} + .485\,F_{BD} + .324\,F_{CD} - 20 = 0 \end{aligned}\right\}$$

פותר לפי מחשבון לקבל

$F_{AD} = 21.57\,N$

$F_{BD} = 13.99\,N$

$F_{CD} = 17.62\,N$

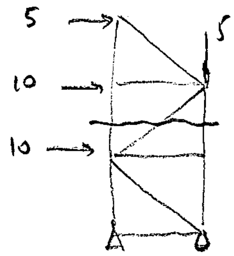
דטרמיננט = $-.7665$

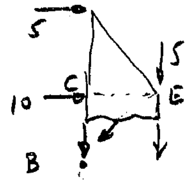
028798

73.33 $\bar{x}$ || 80.8

19.73 σ || 10.79

(1)





$\Sigma M_E = -5 \cdot 4 + F_{CB} \cdot 4 = 0$ $\quad$ $F_{CB} = 5\ kN$ 2

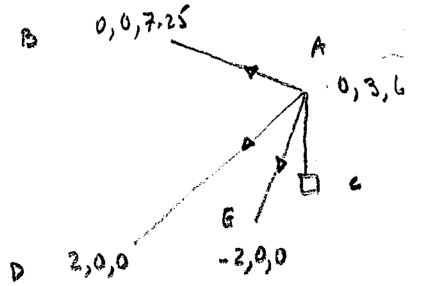
$\Sigma M_B = 0 = -5 \cdot 8 + 10 \cdot 4 - 5 \cdot 4 - F_{EF} \cdot 4 = 0$ 8 $\quad$ $F_{EF} = -25\ kN$ 2

$\Sigma F_y = 0 = +5 + 10 - F_{BE} \cdot \frac{1}{\sqrt{2}} = 0$ 4 $\quad$ $F_{BE} = 15\sqrt{2}\ kN$ 2

$BF = 0$ 2

(24) $\cdot \frac{4}{3} = 32$

(2)



$r_{D/A} = 2i - 3j - 6k$ $\quad$ $r_{D/A} = 7$; $u_{D/A} = \frac{2}{7}i - \frac{3}{7}j - \frac{6}{7}k$ 4

$r_{E/A} = -2i - 3j - 6k$ $\quad$ $r_{E/A} = 7$; $u_{E/A} = -\frac{2}{7}i - \frac{3}{7}j - \frac{6}{7}k$ 4

$r_{B/A} = 0i - 3j + 1.25k$ $\quad$ $r_{B/A} = 3.25$; $u_{B/A} = 0i - \frac{3}{3.25}j + \frac{1.25}{3.25}k$ 4

$300 \times 9.81 = 2943\ N$

$\Sigma F = F_{D/A} + F_{E/A} + F_{B/A} - 2943k = 0$

$\Sigma F_x = \frac{2}{7}F_{DA} - \frac{2}{7}F_{EA} + 0 = 0$ 4 $\quad$ $F_{DA} = F_{EA}$ 2

$\Sigma F_y = 0 \quad -\frac{3}{7}F_{DA} - \frac{3}{7}F_{EA} - \frac{3}{3.25}F_{BA} = 0 \Rightarrow -\frac{6}{7}F_{DA} - \frac{3}{3.25}F_{BA} = 0$ 4 $\quad$ $F_{BA} = -\frac{3.25}{3} \cdot \frac{6}{7} F_{DA} = -.92857 F_{DA}$

5

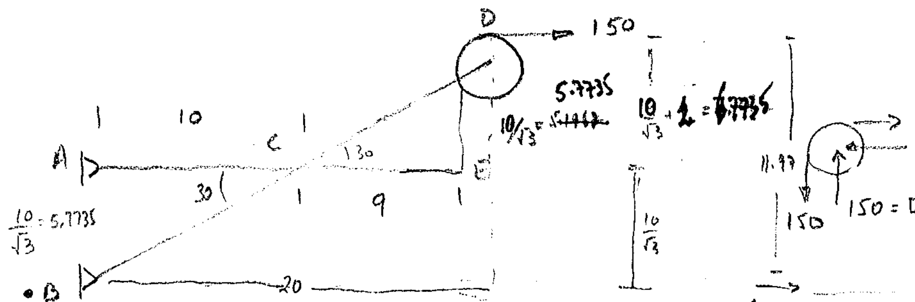
$\Sigma F_z = 0 \quad -\frac{6}{7}F_{DA} - \frac{6}{7}F_{EA} + \frac{1.25}{3.25}F_{BA} - 2943 = 0 \Rightarrow -\frac{12}{7}F_{DA} + \frac{1.25}{3.25}\left(-\frac{3.25}{3} \cdot \frac{6}{7}F_{DA}\right) - 2943 = 0$

$-2.07 F_{DA} - 2943 = 0$

$F_{DA} = -1420.8\ N = F_{EA}$ 2

$F_{BA} = +1319.3\ N$ 2

(31)



4 FBD ◯ 2 Dx Dy

5 FBD ACE

6 FBD BCD

ΣM_A = 6 Bx

~~ΣM_B 6 Ax~~

ΣM_C 8 By 2

$\Sigma F_x \Rightarrow C_x$ 3 + 2

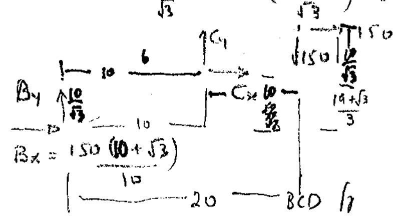
ΣF_y 3 Cy 2

~~Ay = Cy - 150 3~~ Ay 2

$A_x = C_x \quad A_y = C_y - 150$ 5

$\Sigma M_A = 0 \quad B_x \cdot \frac{10}{\sqrt{3}} - 150\left(\frac{10+\sqrt{3}}{\sqrt{3}}\right) = 0$ 4 $\quad$ $B_x = \frac{150(10+\sqrt{3})}{10} = 176$ 2

~~$\Sigma M_B = 0 \quad A_x \frac{10}{\sqrt{3}} - 150\left(\frac{20+\sqrt{3}}{\sqrt{3}}\right) = 0$ 4 $\quad A_x = -\frac{150(19+\sqrt{3})}{10} = -326$~~ 2



$\Sigma M_C = 0 \Rightarrow -B_y(10) + B_x\left(\frac{10}{\sqrt{3}}\right) - 150\left(\frac{10}{\sqrt{3}}\right) - 150\left(\frac{10}{\sqrt{3}}\right) = 0$ 8 $\quad$ $B_y = -135$ 2

$B_x + C_x + 150 = 0$ $\quad$ $C_x = -326$ 3+2 2

~~$A_x = C_x = -150 - 150(9+\sqrt{3})$~~ $= -326 = A_x$ $\quad$ $C_x = -311$

$B_y + C_y - 150 = 0$ $\quad$ $C_y = 285$ 3+2

~~$\Sigma M_B = 0 \Rightarrow C_y = \left(+150(29) + C_x \frac{10}{\sqrt{3}} + 150\left(\frac{20}{\sqrt{3}}\right)\right) / 10$~~ = 150 = Cy

~~$-B_y + C_y - 150 = 0$~~ 4 $\quad$ $B_y = -150$ 2

שם: ______________

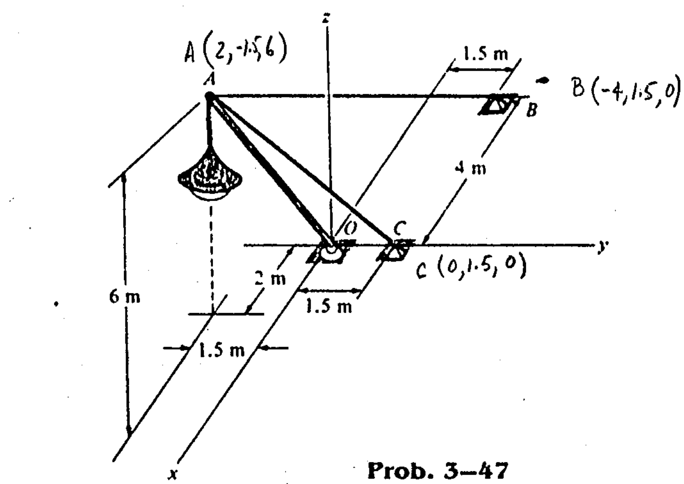
ת.ז. ______________

האור שנמצא בתרשים נתמך בנקודה A בשני כבלים AB ו-AC ובמוט OA. מסת האור שווה ל-15kg.

א) כתבו ביטוים בשביל הכוחות הקרטזיים F_{AB} ו-F_{AC}

ב) מצאו את הזווית בין שני הכוחות האלה.

ג) מצאו את המומנט M_o (מומנט מסביב 0) שבאו מהכוח F_{AC}



Prob. 3–47

$\underline{r}_{B/A} = -6i + 3j - 6k \qquad r_{B/A} = 9 \qquad \underline{u}_{B/A} = -.67\underline{i} + .33\underline{j} - .67\underline{k}$ (5)

$\underline{r}_{C/A} = -2i + 3j - 6k \qquad r_{C/A} = 7 \qquad u_{C/A} = -.29\underline{i} + .43\underline{j} - .86\underline{k}$ (5)

$\underline{F}_{AB} = F_{AB}(-.67\underline{i} + .33\underline{j} - .67\underline{k}) = F_{AB}\underline{u}_{B/A} = \left(-\frac{2}{3}\underline{i} + \frac{1}{3}\underline{j} - \frac{2}{3}\underline{k}\right)F_{AB}$

$\underline{F}_{AC} = F_{AC}(-.29\underline{i} + .43\underline{j} - .86\underline{k}) = F_{AC}\underline{u}_{C/A} = \left(-\frac{2}{7}\underline{i} + \frac{3}{7}\underline{j} - \frac{6}{7}\underline{k}\right)F_{AC}$

$\cos\theta = \underline{u}_{B/A} \cdot \underline{u}_{C/A} = \left(-\frac{2}{3}\left(-\frac{2}{7}\right) + \frac{1}{3}\left(\frac{3}{7}\right) - \frac{2}{3}\left(-\frac{6}{7}\right)\right)$

$= \frac{19}{21}$ (5) $\quad .9048 \qquad 25.2° = \theta$

$\underline{r}_{C/O} = (0i + 1.5\underline{j} + 0\underline{k})$

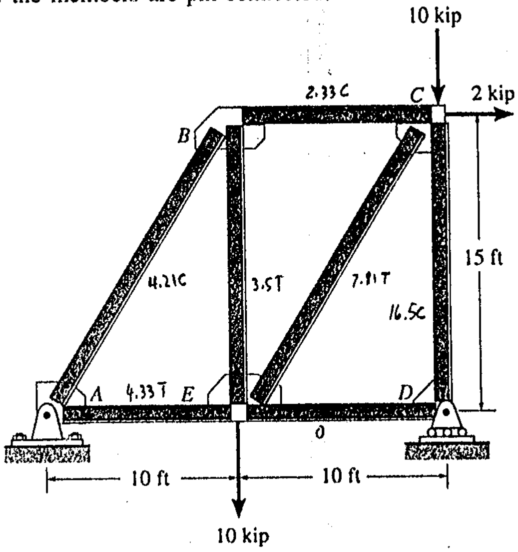
$$\underline{r}_{C/O} \times \underline{F} = \underline{M}_o = \begin{vmatrix} i & j & k \\ 0 & 1.5 & 0 \\ -.27F_{AC} & .43F_{AC} & -.86F_{AC} \end{vmatrix} = -.86F_{AC}(1.5)\underline{i} + .27F_{AC}(1.5)\underline{k}$$

$= \left(-\frac{6}{7} \cdot \frac{3}{2}\,i + \frac{2}{7} \cdot \frac{3}{2}\,\underline{k}\right)F_{AC}$ (5)

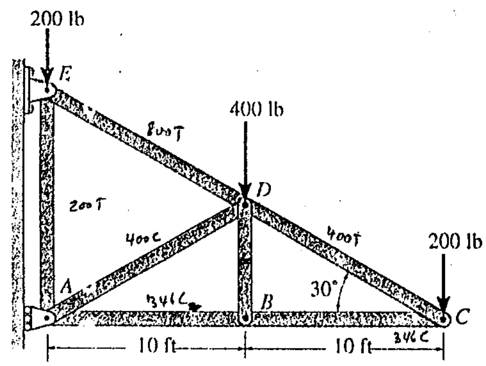
$\underline{M}_o = \frac{-18i + 6k}{14} F_{AC}$

$= (-1.29\underline{i} + .43\underline{k})F_{AC}$

cate whether the members are in tension or compression. Assume that all the members are pin-connected.

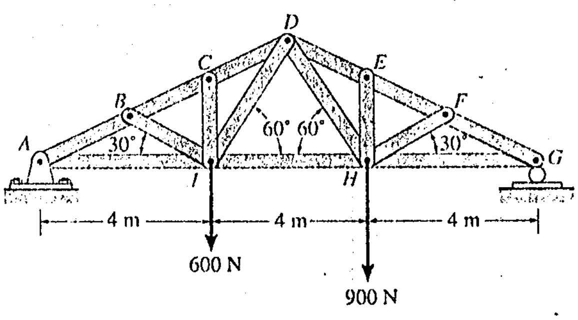


6–11. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



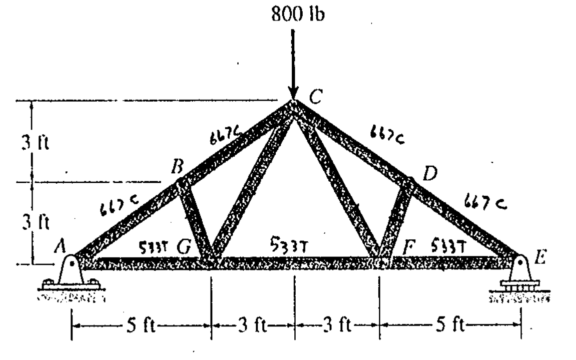
Prob. 6–11

***6–12.** Determine the force in each member of the truss and indicate whether the members are in tension or compression.



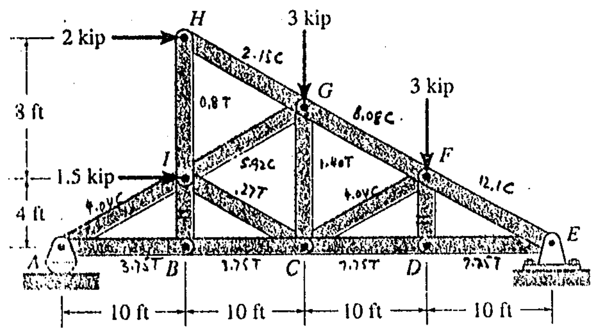
Prob. 6–12

6–13. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



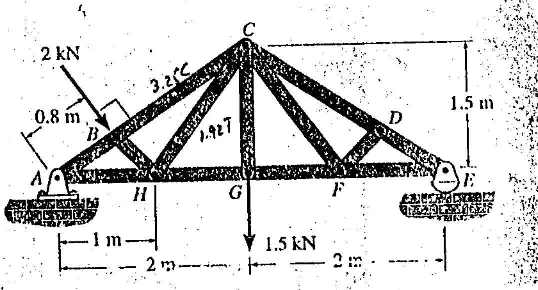
Prob. 6–13

6–18. Determine the force in each member of the truss and indicate whether the members are in tension or compression.

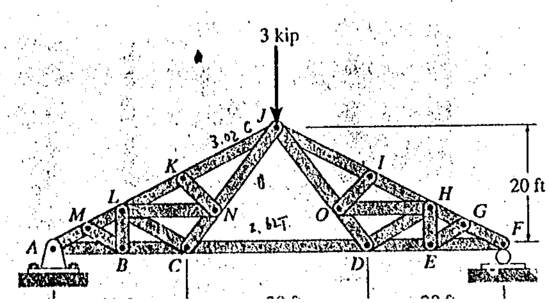


Prob. 6–18

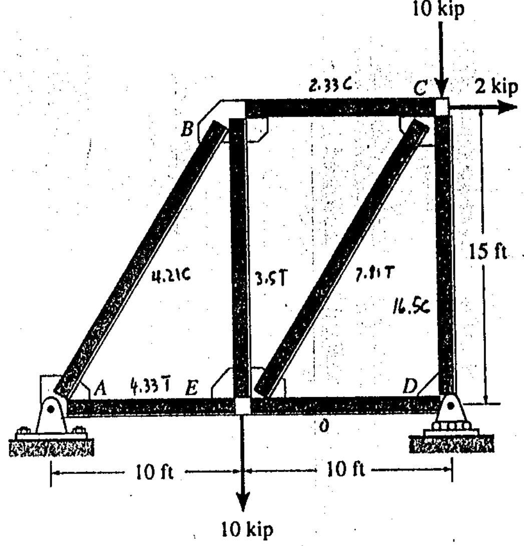
6–29. Determine the force developed in members *BC* and *CH* of the roof truss and indicate whether the members are in tension or compression.



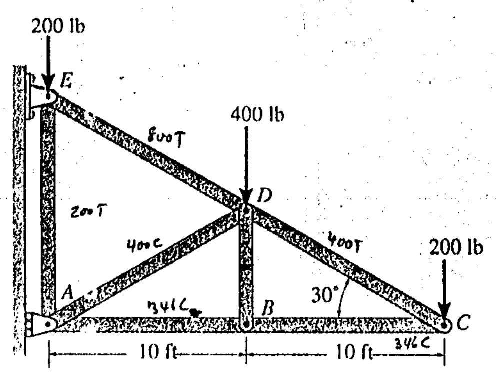
6–37. Determine the force in members *KJ*, *JN*, and *CD*, and indicate whether the members are in tension or compression.



cate whether the members are in tension or compression. Assume that all the members are pin-connected.

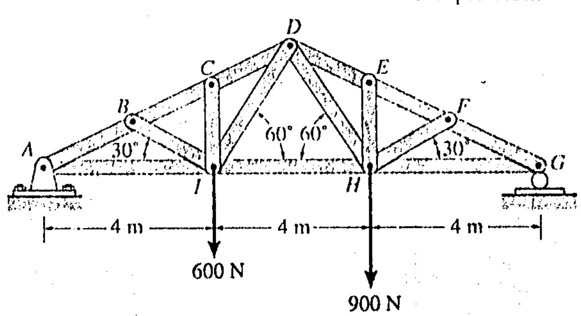


6–11. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



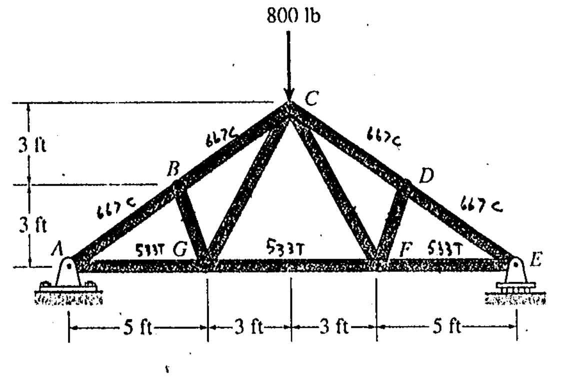
Prob. 6–11

***6–12.** Determine the force in each member of the truss and indicate whether the members are in tension or compression.



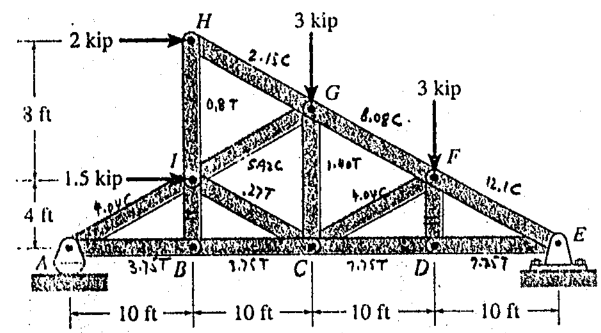
Prob. 6–12

6–13. Determine the force in each member of the truss and indicate whether the members are in tension or compression.



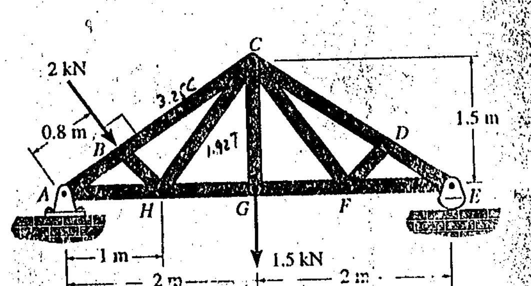
Prob. 6–13

6–18. Determine the force in each member of the truss and indicate whether the members are in tension or compression.

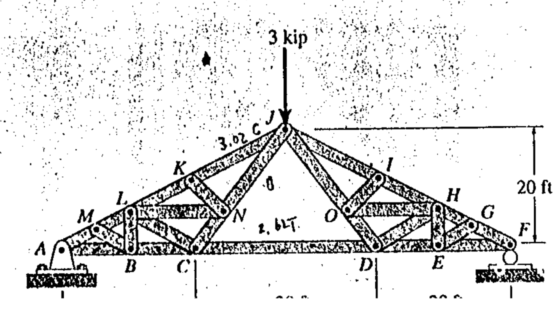


Prob. 6–18

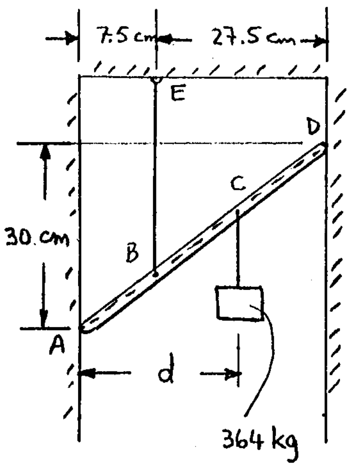
6–29. Determine the force developed in members *BC* and *CH* of the roof truss and indicate whether the members are in tension or compression.



6–37. Determine the force in members *KJ*, *JN*, and *CD*, and indicate whether the members are in tension or compression.

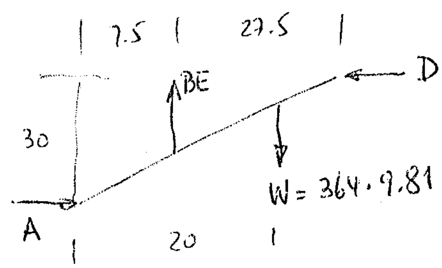


ת.ז. ____________



מוט קל AD נתלה מכבל BE ותומך במסה של 364kg ב-C.
הקצוות A ו-D של המוט נוגעים בקירות מאונכות וחלקות.

א) מצאו המתח בכבל BE והתגובות ב A ו-D, אם d=20cm.
ב) מצאו את המרחק המקסימלי, d, שתוכלו להשתמש בו, אם ערך המקסימלי המותר לתגובה (ריאקציה) ב-A הוא 2225N



$D = A$

$BE = 364(9.81) = 3571$

$D \cdot 30 = W \cdot (12.5)$

$A = D = W \cdot \frac{12.5}{30} = 364(9.81)\left(\frac{12.5}{30}\right) = 1488\ N$

$d' = \frac{D \cdot 30}{W} = \frac{2225 \cdot 30}{364(9.81)} = 18.69\,cm$

$d = d' + 7.5\,cm = 26.19\,cm$

$F_2 = 200\,N$

$$F_1 = F_3 \frac{\cos 45^\circ}{\cos 30^\circ} = F_3 \frac{(.707)}{.866} = F_3\,(.8164)$$

$$400 = F_3\,(.8164)\sin 30 + F_3 \sin 45^\circ$$
$$= F_3\,.4082 + .7071\,F_3 = 1.1153\,F_3$$
$$F_3 = 358.65\ lb$$

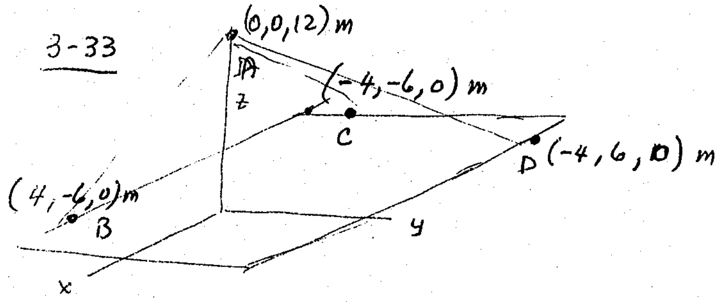
$$F_1 = F_3\,(.8164) = 292.8\ lb$$

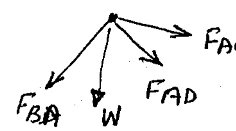
rods support a weight whose mass is 150 kg
3 ~~cables in tension~~ ~~due to an upward load W of 1472N~~
find forces in ~~cables~~ the rods.

SESSION #6

REVIEW

3-33





$\bar{r}_{AC} = \bar{r}_C - \bar{r}_A = (-4\bar{i} - 6\bar{j} + 12\bar{k})\ m$

$r_{AC} = \sqrt{4^2 + 6^2 + (12)^2} = 14\ m$

$\bar{r}_{AD} = \bar{r}_D - \bar{r}_A = (-4\bar{i} + 6\bar{j} + 12\bar{k})\ m$

$r_{AD} = 14\ m$

$\bar{r}_{AB} = \bar{r}_B - \bar{r}_A = (4\bar{i} - 6\bar{j} + 12\bar{k})\ m$

$r_{AB} = 14\,m$

$\bar{u}_{AC} = -.286\bar{i} - .429\bar{j} + .857\bar{k}\ ;\ \bar{F}_{AC} = F_{AC}\bar{u}_{AC}$

$\bar{u}_{AD} = -.286\bar{i} + .429\bar{j} + .857\bar{k}\ ;\ \bar{F}_{AD} = F_{AD}\bar{u}_{AD}$

$\bar{u}_{AB} = +.286\bar{i} - .429\bar{j} - .857\bar{k}\ ;\ \bar{F}_{AB} = F_{AB}\bar{u}_{AB}$

$\Sigma\vec{F} = \vec{0} \Rightarrow \bar{F}_{AB} + \bar{F}_{AD} + \bar{F}_{AC} + \bar{W} = \bar{0}$

$$\Sigma F_x = -.286\,F_{AC} - .286\,F_{AD} + .286\,F_{AB} = 0$$
$$\Sigma F_y = -.429\,F_{AC} + .429\,F_{AD} - .429\,F_{AB} = 0$$
$$\Sigma F_z = -.857\,F_{AC} - .857\,F_{AD} - .857\,F_{AB} - W = 0$$

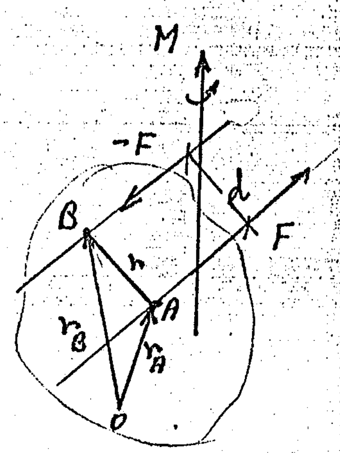
$-F_{AC} - F_{AD} + F_{AB} = 0$ or $F_{AB} = F_{AC} + F_{AD}$

$-F_{AC} + F_{AD} - F_{AB} = 0$ or $F_{AD} = F_{AC} + F_{AB} = F_{AC} + F_{AC} + F_{AD} = 2F_{AC} + F_{AD}$.

THUS $\boxed{F_{AC} = 0} \Rightarrow \underline{F_{AB} = F_{AD}}$

$-F_{AC} - F_{AD} - F_{AB} - \frac{W}{.857} = 0$ or $-2F_{AD} - \frac{W}{.857} = 0$ or $\underline{F_{AD} = -\frac{W}{2\cdot(.857)} = F_{AB}}$

Thus $F_{AD} = F_{AB} = -\frac{W}{2} = -\frac{150\,kg \cdot 9.81}{2(.857)} = \frac{-735.75\,N}{.857} = -858.52\,N$



צמדים

הגדרת הצמד במרחב דומה לזו שבמישור:

$$\vec{M} = \vec{r}_A \times F + \vec{r}_B \times (-F) = (r_A - r_B) \times F$$

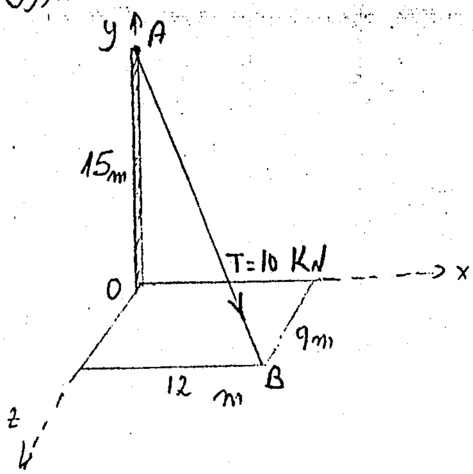
$$= \vec{r} \times \vec{F}$$

גודלו של הצמד יהיה $M = Fd$

* עלינו לזכור שהמומנט שבצמד הינו וקטור חופשי. מומנט רגיל הינו וקטור מחליק.
* גם במקרה התלת מימדי ניתן להמיר כח בכח שווה ומקביל + צמד.

דוגמא

התורן המתואר בשרטוט נתון לכח מתח $T = 10KN$. חשב את המומנט שיוצר הכח T סביב הציר z



יש דרכים שונות לפתרון הבעיה:

דרך א

נחשב את המומנט שיוצר T סביב הנקודה O ולאחר מכן נטילו על z

1. נחשב את רכיבי T ונרשום אותו כוקטור

$$l = \frac{12}{\sqrt{9^2+12^2+15^2}} = 0.566 \qquad m = \frac{-15}{AB} = -0.707 \qquad n = \frac{9}{AB} = 0.424$$

$$T = 10(0.566\,i - 0.707\,j + 0.424\,k)\ KN$$

נבחר כ $\vec{r}$ את $\vec{OA}$

$$\vec{r} = 15\,j\ m$$

פתרון וקטורי ל[illegible]

$\underline{r}_{B/A} = 12\underline{i} - 15\underline{j} + 9\underline{k}$

$$\underline{u}_{AB} = \frac{1}{\sqrt{144+225+81}}(12\underline{i} - 15\underline{j} + 9\underline{k}) = \frac{\sqrt{2}}{30}(12\underline{i} - 15\underline{j} + 9\underline{k})$$

$$= \frac{\sqrt{2}}{10}(4\underline{i} - 5\underline{j} + 3\underline{k})$$

$$\underline{T} = T\underline{u}_{AB} = \sqrt{2}(4\underline{i} - 5\underline{j} + 3\underline{k})[KN]$$

$r_{A/O} = 15\underline{j}$

נחשב מומנט סביב O -

$$\underline{M}_O = \underline{r}_{A/O} \times \underline{T} = \sqrt{2}\begin{bmatrix} i & j & k \\ 0 & 15 & 0 \\ 4 & -5 & 3 \end{bmatrix} = \sqrt{2}(45\underline{i} - 60\underline{k}) \quad KNm$$

$$= 15\sqrt{2}(3\underline{i} - 4\underline{k}) \quad KNm$$

$$M_z = \underline{M}_A \cdot \underline{k} = -60\sqrt{2} \quad KNm \cong -84.9 \quad KN\,m$$

דרך ב'

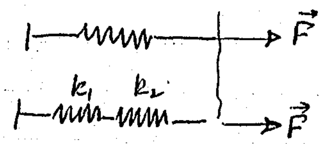
$$\underline{M}_z = \underline{u}_z \cdot \underline{r}_{A/O} \times \underline{F} = \sqrt{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 15 & 0 \\ 4 & -5 & 3 \end{vmatrix} = \sqrt{2}(-60) = 84.9 \text{ kN-m}$$

$$\underline{M}_B = \underline{r}_{B/O} \times \underline{T} = (12\underline{i} + 9\underline{k}) \times \sqrt{2}(4\underline{i} - 5\underline{j} + 3\underline{k}) = \sqrt{2}\begin{vmatrix} i & j & k \\ 12 & 0 & 9 \\ 4 & -5 & 3 \end{vmatrix}\begin{matrix} i & j \\ 12 & 0 \\ 4 & -5 \end{matrix}$$

$$= \sqrt{2}(36\underline{j} - 60\underline{k} - 36\underline{j} + 45\underline{i}) = \sqrt{2}(45\underline{i} - 60\underline{k})\ !$$

$$M_z = \underline{u}_z \cdot \underline{r}_{B/O} \times \underline{T} = \underline{k} \cdot (\sqrt{2})(45\underline{i} - 60\underline{k}) = -60\sqrt{2} \text{ kN-m} = -84.9 \text{ kN-m}$$

PROBLEM 3-15



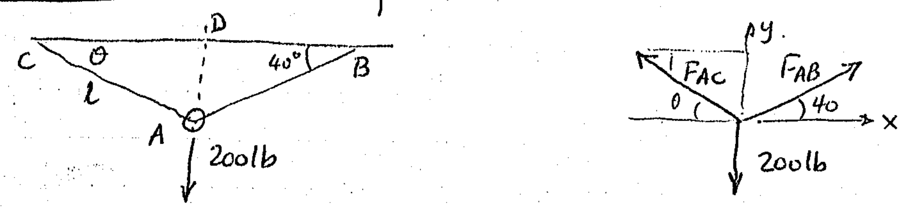
$\vec{F} = k_T(x_1 + x_2)$

same $\vec{F}$ acts on k_1 & also on k_2

$\vec{F} = k_1 x_1$ & $\vec{F} = k_2 x_2$ $\quad \frac{\vec{F}}{k_1} = x_1 \quad \frac{\vec{F}}{k_2} = x_2 \quad \vec{F}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = x_1 + x_2 = \frac{\vec{F}}{k}$

THUS $\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2}$

PROBLEM 3-13 Find length l



$F_{AC} = 160 lb.$

$$\Sigma F_x = F_{AB}\cos 40° - F_{AC}\cos\theta = 0$$
$$\Sigma F_y = F_{AB}\sin 40° + F_{AC}\sin\theta - 200\,lb = 0$$
$$F_{AB} = F_{AC}\cos\theta / \cos 40° \Rightarrow F_{AC}\left[\cos\theta \frac{\sin 40°}{\cos 40°} + \sin\theta\right] = 200\,lb$$
$$F_{AC}\left[\sin(\theta + 40°)\right] = 200\,lb \cdot \cos 40°$$
$$\sin(\theta + 40°) = 200\,lb \cdot \cos 40° / 160\,lb = .9576 \qquad \theta + 40° = 133.25°$$
$$\theta = +33.25°$$

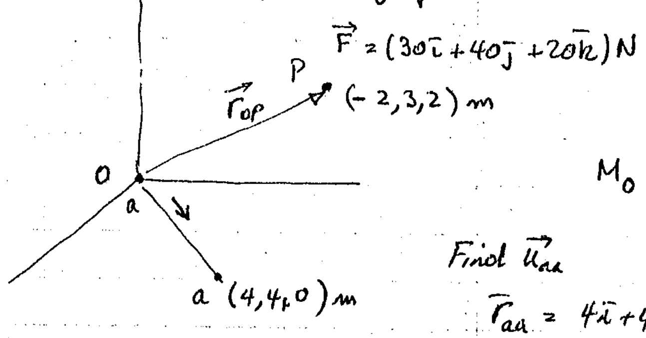
Now since $AB = 2\,ft$ $\quad \perp = AD = AB \sin 40° = 2\,ft \sin 40° = 1.286\,ft$

$AD = AC \sin 33.25° = 2.35\,ft$

or

$$\frac{AC}{\sin 40°} = \frac{AB}{\sin 33.25°} \quad \text{by law of sines}$$

4-53 find proj of moment due to $\vec{F}$ about a,a axis



Find $M_o \Rightarrow \vec{r}_{op}, \vec{F}$

$\vec{r}_{op} = \vec{r}_p - \vec{r}_o = (-2\hat{i} + 3\hat{j} + 2\hat{k})\,m$

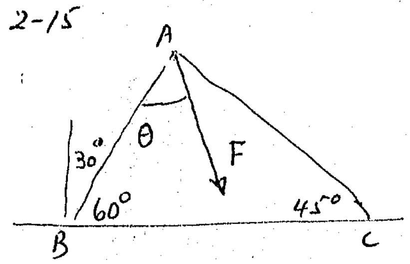
$$M_o = \vec{r}_{op} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 2 \\ 30 & 40 & 20 \end{vmatrix} = -20\hat{i} + \hat{j}(100) + \hat{k}$$ N-m

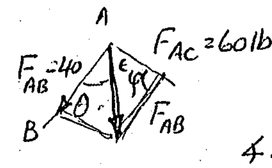
Find $\vec{u}_{aa}$

$\vec{r}_{aa} = 4\hat{i} + 4\hat{j} + 0\hat{k} \qquad \vec{u}_{aa} = .707\hat{i} + .707\hat{j} + 0\hat{k}$

also $\bar{u} \cdot M_0 = \bar{u} \cdot (\bar{r} \times \bar{F}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

2-15





$\measuredangle ABC = 60°$
$\measuredangle ACB = 45°$
$\measuredangle BAC = 75°$
$\varphi = [360° - 2(75°)]/2 = 105°$

$F = \sqrt{(F_{AB})^2 + (F_{AC})^2 - 2F_{AB}F_{AC}\cos\varphi} = $ ~~20 lb.~~ 80.26 lb

$\frac{F}{\sin 105°} = \frac{F_{AB}}{\sin \epsilon}$ or $\sin \epsilon = \frac{F_{AB}}{F} \sin 105° = .4814$ $\epsilon = 28.78°$

$\theta = \measuredangle BAC - 28.78° = \underline{46.22°}$

SESSION # 7

- $\Sigma \vec{F} = \vec{0}$ IS A NECESSARY CONDITION BUT NOT SUFFICIENT FOR EQUILIBRIUM

  (תנאי דרוש / תנאי שלא יספיק)

- FURTHER RESTRICTION ON ~~[illegible]~~ NON CONCURRENT ~~[illegible]~~ FORCES MUST ALSO BE MADE

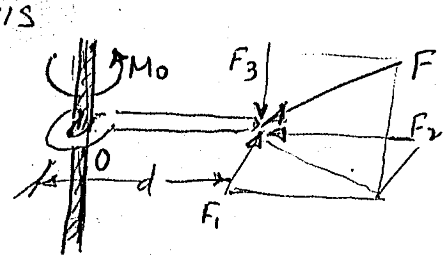
  צריך הגבלות אחרות כאשר הכוחות לא שותפי נקודה

- NON CONCURRENCY GIVES RISE TO CONCEPT OF A MOMENT

  - WILL DEFINE CONCEPT
  - HOW TO OBTAIN IT
  - EQUIVALENT SYSTEMS TO ONE COMPRISED OF FORCES & MOMENTS

- LOOK AT THE SIMPLEST CASE FIRST - MOMENT DUE TO COPLANAR FORCES

A FORCE CAN CAUSE TURNING ABOUT AN AXIS IF ITS LINE OF ACTION IS NOT THROUGH THAT AXIS OR DOES NOT ACT PARALLEL TO THE AXIS



F_2 has line of action through 0
F_3 ∥ to axis
F_1 only contributes.

USE BOOK (ON TABLE) TO SHOW THIS

• Rotational effect of $\vec{F_1}$ is called the moment $\vec{M_O}$. Note that the axis of rotation is ⊥ to F_1 & to d. Also axis intersects the plane that contains F_1 & d at O.

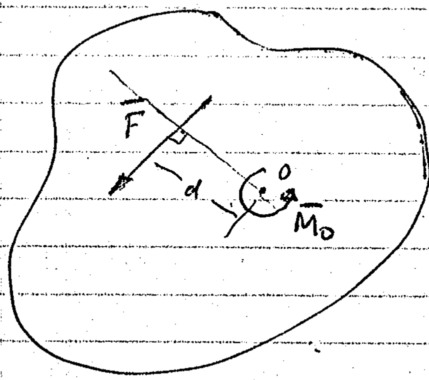
סיבוב שנגרם על ידי F נקרא מומנט גם פה. ציר הסיבוב ⊥ ל F_1 והציר עובר את המישור שבו F ו-d בנקודה O.

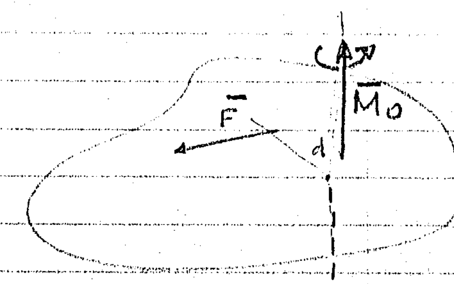
• $\vec{M_O}$ is a vector quantity

• magnitude of M_O is Fd, F is magnitude of Force and d is ⊥ distance from O to the line of action of F

• d is the moment arm Units are force·distance (N-m) (lb-ft)

• Direction is using RH rule (לפי יד ימינה) ⊥ to d & F SHOW THE ○.
PUT FINGERS IN DIRECTION OF moment arm, sweep toward +ive direction of force THUMB gives direction.





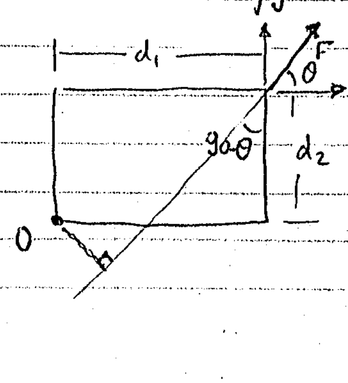
• MOMENT CAN BE CONSIDERED A SLIDING VECTOR ALONG THE MOMENT AXIS BUT IT IS BOUND (נועד) TO THE AXIS PASSING THRU O.

מומנט הוא וקטור החלקה לאורך ציר המומנט אבל נועד לציר שעובר את הנקודה O.

VARIGNON'S THEOREM – MOMENT OF A FORCE ABOUT A POINT IS EQUAL TO THE SUM OF THE MOMENTS OF THE FORCE'S COMPONENTS ABOUT THE PT

מומנט הכוח שווה לסך הכול של מומנטים הבאים מרכיבי הכוח מסביב אותה נקודה

$$\vec{M}_O = (F_x\bar{i} + F_y\bar{j} + F_z\bar{k})(d \text{ to each component})$$



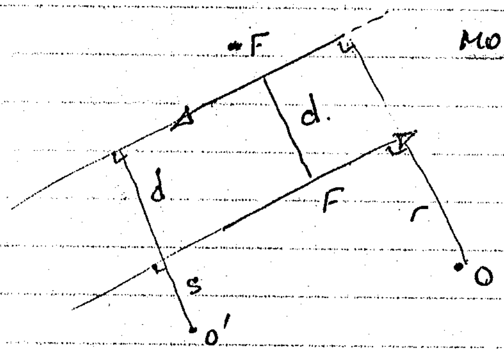
$$M_O = F_y d_1 \circlearrowright + F_x d_2 \circlearrowright$$
$$M_O = F_y d_1 \circlearrowright - F_x d_2 \circlearrowright = (F_y d_1 - F_x d_2) \circlearrowright$$

COUPLES זוג כוחות

- A COUPLE – 2 FORCES, //, SAME MAG. OPPOSITE DIRECTIONS SEPARATED BY A ⊥ DISTANCE d.

- $\sum \vec{F}$ OF COUPLE = $\vec{F} - \vec{F} = \vec{O}$

  ONLY A MOMENT IS GENERATED (נוצר) magnitude is Fd



MOMENT OF A COUPLE IS A FREE VECTOR NOT TIED TO ANY POINT.

$$(+\vec{M}_O = (r+d)(+F)\circlearrowleft + rF\circlearrowright = -(r+d)F\circlearrowright + rF\circlearrowright = -Fd\circlearrowright = Fd\circlearrowleft$$

$$(+M_{O'} = (d+s)F\circlearrowleft + F\cdot s\circlearrowright = -Fd\circlearrowright = Fd\circlearrowleft$$

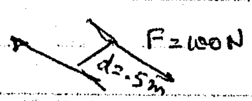
thus the same moment is obtained about any point, thus

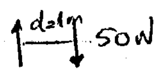
$M = Fd$ depends only on ⊥ distance & not on location

- EQUIVALENT COUPLES – COUPLES THAT PRODUCE THE SAME MOMENT

  MOMENT OF COUPLE A = MOMENT OF COUPLE B

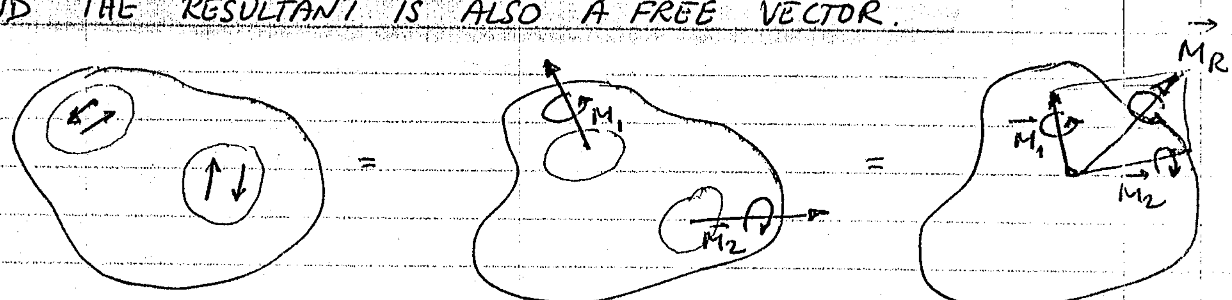
  FORCES THAT PRODUCE THE COUPLE A MUST LIE IN SAME PLANE OR PARALLEL PLANE TO THOSE FORCE THAT PRODUCE COUPLE B





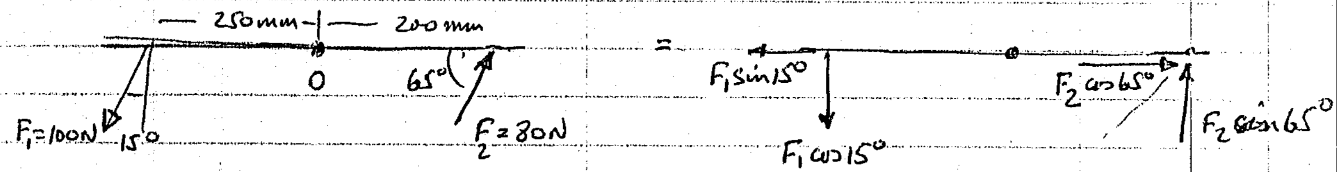
- RESULTANT COUPLES - שקול זוגות כוחות

  CAN BE ADDED VECTORIALLY : SINCE EACH IS A FREE VECTOR AND THE RESULTANT IS ALSO A FREE VECTOR.



SINCE FREE VECTORS CAN BE APPLIED ANYWHERE - APPLY IT TO A PT P ON THE BODY WHICH IS CONVENIENT

Problem
4-3



$F_1 \sin 15°$ & $F_2 \cos 65°$ give no moment.

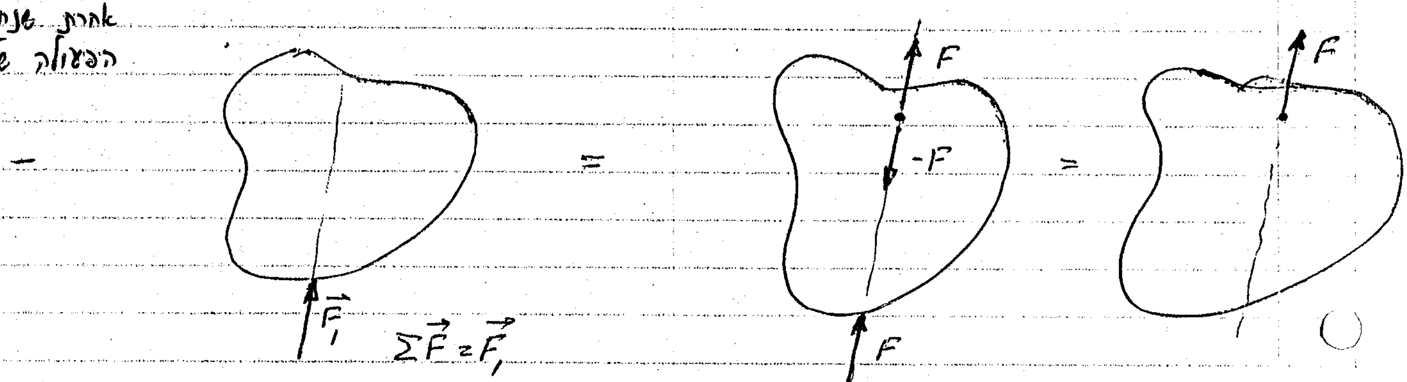
$\curvearrowright (F_1 \cos 15°)\, 250\,mm + \curvearrowright ((F_2 \sin 65°)\, 200\,mm = (96.59)(250) + (~~33.81~~\ 72.50)(200)$

$= 24148.15\ Nmm + ~~6761.89~~\ 14500.93\ Nmm = ~~30910.04~~\ 38649.08\ Nmm = ~~30.91~~\ 38.65\ Nm$

PRINCIPLE OF TRANSMISSIBILITY   עקרון העברת כוח

EXTERNAL EFFECTS ON A RIGID BODY REMAIN UNCHANGED WHEN A FORCE ACTING AT A GIVEN POINT ON THE BODY, IS APPLIED TO ANOTHER POINT LYING ON THE LINE OF ACTION OF THE FORCE.
(THE FORCE IS A SLIDING VECTOR)

השפעות חיצוניות על גוף קשיח לא משתנות כאשר כוח שפועל בנקודה מסויימת על הגוף מיושם בנקודה אחרת שנחה על קו הפעולה של הכוח.

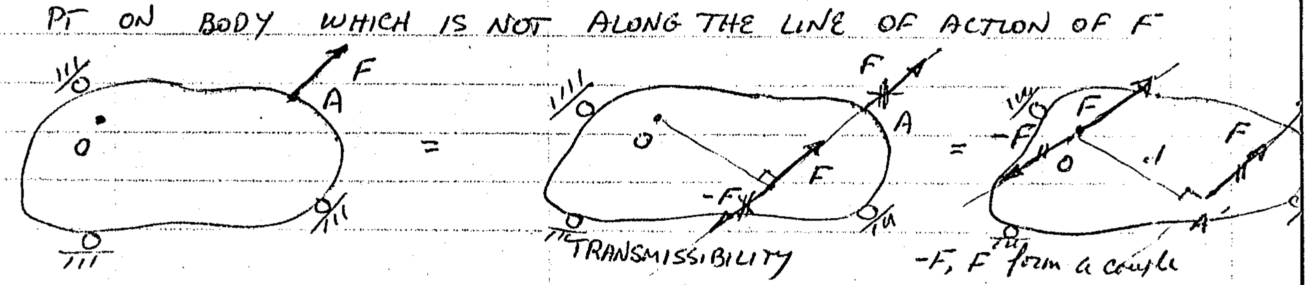


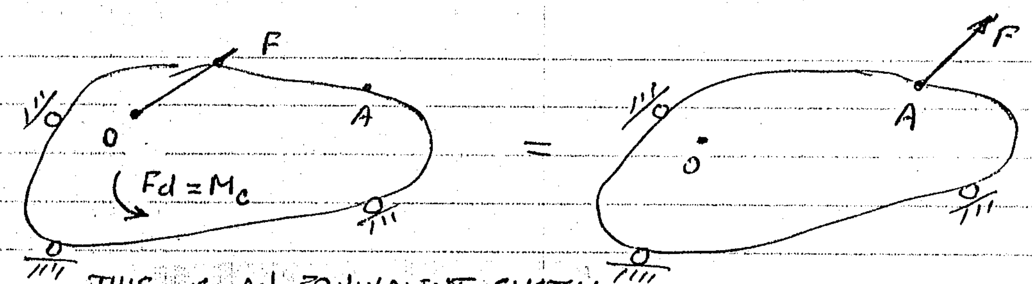
NO MOMENTS ARE GENERATED

WE SAY THE FORCE HAS BEEN TRANSMITTED ALONG ITS LINE OF ACTION
אומרים שהכוח הועבר דרך הקו הפעולה שלו.

RESOLUTION OF A FORCE INTO A FORCE & COUPLE

- GIVEN A BODY WITH A FORCE ACTING ON IT AT A PT "A" WANT TO FIND WHAT HAPPENS IF WE MOVE FORCE TO ANOTHER PT ON BODY WHICH IS NOT ALONG THE LINE OF ACTION OF F





מערכת אקוויוולנטית THIS IS AN EQUIVALENT SYSTEM TO THE ORIGINAL SYSTEM WILL PRODUCE SAME REACTIONS AT SUPPORTS AS ORIGINAL SYS

M_C CAN BE APPLIED ANY WHERE USUALLY A

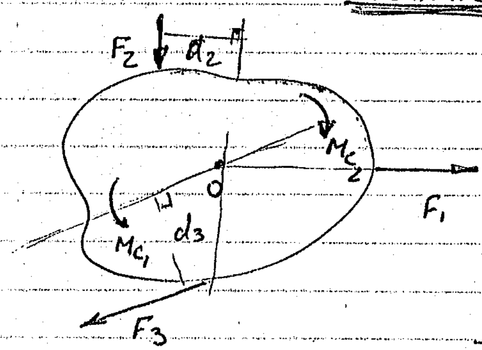
NOTE DIRECTION & MAGNITUDE OF F IS UNCHANGED
DIRECTION & MAGNITUDE OF M_C CAN BE OBTAINED FROM ORIGINAL SY ABOUT O

סיכום הדבר

- IF A FORCE IS MOVED TO A PT "O" ALONG ITS LINE OF ACTION SIMPLY MOVE FORCE (PRINCIPLE OF TRANSMISSIBILITY)
- IF PT "O" IS NOT ON LINE OF ACTION FIND EQUIVALENT SYSTEM. MAGNITUDE & DIRECTION OF MOMENT IS MOMENT ABOUT "O".

כל כוחות שוקפי מישור אבל לא שוקפי נקודה

FOR A SET OF COPLANAR (NON COLLINEAR) FORCES & COUPLES



1. MOVE THE FORCES SO THAT THEY ARE COLLINEAR & ACCOUNT FOR THE COUPLE MOMENTS THEY PRODUCE ie F_3d_3, F_2d_2
2. ADD UP THE FORCES $\sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_R$
3. ADD UP THE MOMENTS

$$\vec{M}_{R_o} = \sum\vec{M}_O = \cancel{\vec{M}_1} + \vec{M}_2 + \vec{M}_3 + \vec{M}_{c_1} + \vec{M}_{c_2}$$

$\vec{M}_{c_1}$ & $\vec{M}_{c_2}$ are free vectors & can be moved to O
$\cancel{\vec{M}_1}$, $\vec{M}_2$, $\vec{M}_3$ depend on the ⊥ distance between O & the forces $\cancel{F_1}$, F_2, F_3

$|\vec{F}_R|$ IS INDEPENDENT OF "O" & so is direction of $\vec{F}_R$

$\vec{M}_{R_o}$ will be ⊥ $\vec{F}_R$ in a coplanar case

ובסכום לקביעת גודלו וקו פעולתו של השקול נדרשות שלוש משוואות

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

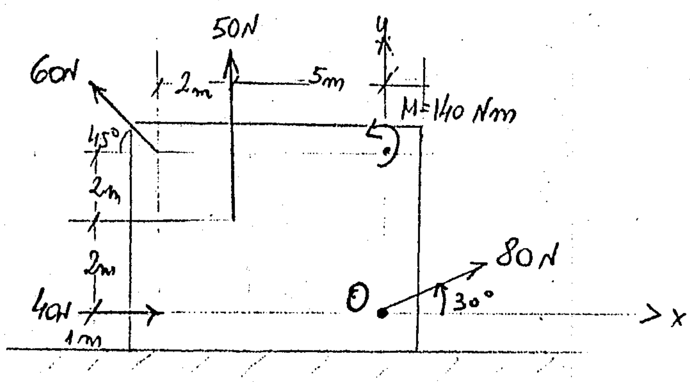
$$R \cdot d = \sum M_o$$

* במקרה הפרטי בו קווי הפעולה של כל הכוחות נחתכים בנקודה אחת מתמלאת המשוואה השלישית באופן טריוויאלי.

* עובדת היות שקול הכוחות $=0$ אינה בהכרח גוררת שהמומנט יהיה $=0$ כמו בדוגמה למטה. גם המשפט ההפוך נכון למשל בכוחות הנחתכים בנקודה אחת.

<u>דוגמא</u>

חשב את השקול וכוונו



$$R_x = \sum F_x = 40 + 80\cos 30° - 60\cos 45° = 66.9\ N$$

$$R_y = 50 + 80\sin 30° + 60\sin 45° = 132.4\ N$$

$$R = \sqrt{R_x^2 + R_y^2} = 148.3\ N$$

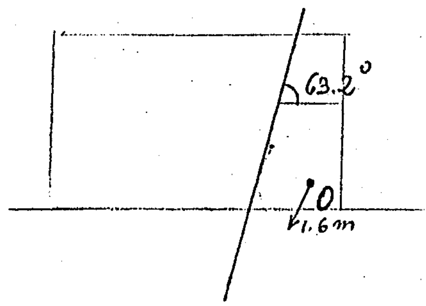
$$\theta = \tan^{-1}\frac{R_y}{R_x} = 63.2°$$

כעת נחפש למצוא את קו הפעולה של הכח R. לשם כך נבדוק מומנטים סביב O.

$$148.3d = 140 - 50\times 5 + 60\cos 45°\times 4 - 60\sin 45°\times 7 = -237.28\ N\text{-}m$$

$$d = -1.6\ m$$

הסימן השלילי פירושו ש-R צריך לייצר מומנט עם כוון השעון כלומר
עליו להמצא שמאלה ל-O במרחק 1.6m וכוון קו הפעולה θ



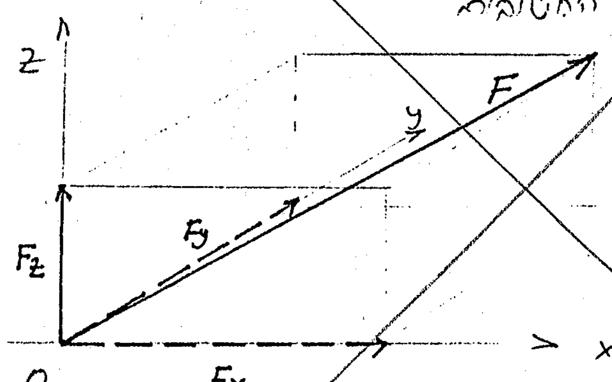
<u>הערות:</u>

1. הרכיב M אינו תורם דבר לשקול R אך משפיע על מקומו
ב. בחירת O כראשית הצירים וכמרכז לחשוב המומנטים נעשתה כדי שעקרונות כוחות (מ80 , מ40) יעברו דרכה ולא יתרמו למומנט.

<u>חלק ב' מערכת תלת מימדית</u>

2.6 <u>רכיבים קרטזיים</u>

במעבר לתצורת תלת מימדית נשתמש בסימון וקטורי אשר יקל עלינו בהרבה את החשובים



כח F הפועל ב-O ניתן לפרק במערכת xyz כדלקמן:

$$F_x = F\cos\theta_x$$

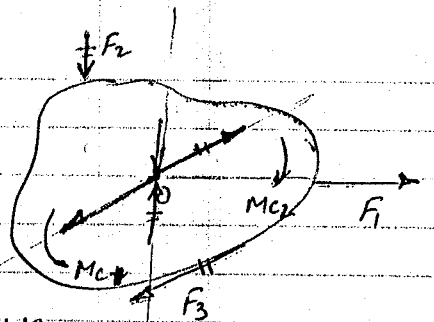
$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

ובכתיב וקטורי:

SESSION #8



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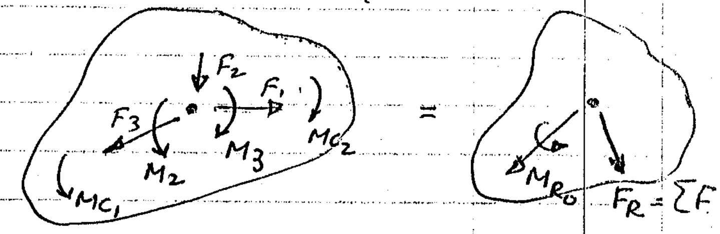


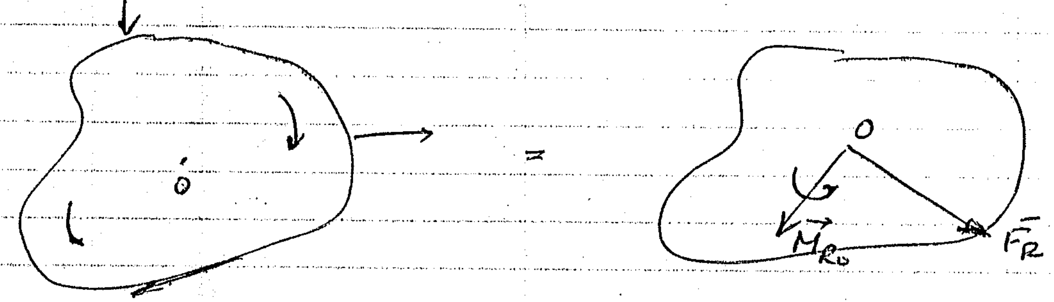
THUS

$$\vec{F}_R = F_{R_x}\bar{\imath} + F_{R_y}\bar{\jmath} = \Sigma\left(F_{1x}\bar{\imath} + F_{1y}\bar{\jmath}\right) + \left(F_{2x}\bar{\imath} + F_{2y}\bar{\jmath}\right) + \ldots$$

$$\left.\begin{aligned} F_{R_x} &= \Sigma F_x \\ F_{R_y} &= \Sigma F_y \end{aligned}\right\} \text{ Just as before.}$$

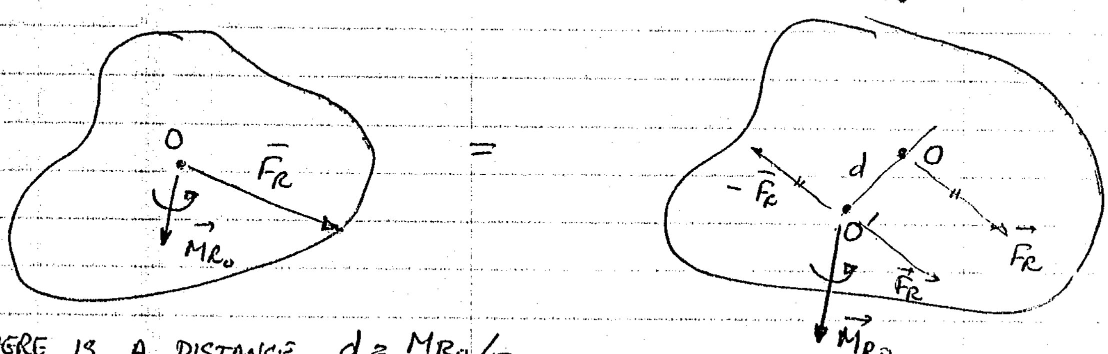
$$\& \quad \vec{M}_{R_0} = \Sigma \vec{M}_0 = \Sigma M_{C_i}$$

WE HAVE REDURED THE SYSTEM OF FORCES & COUPLES TO AN EQUIVALENT SYSTEM OF ONE FORCE & 1 COUPLE



WE CAN REDUCE THIS EVEN FURTHER

$\bar{M}_{R_0}$ is $\perp$ $\bar{F}_R$



THERE IS A DISTANCE $d = M_{R_0}/F_R$

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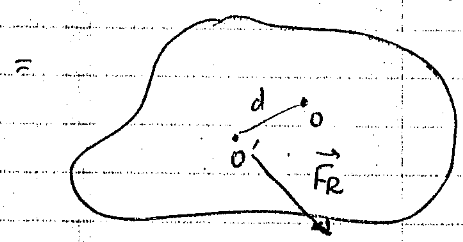


where $F_R \cdot d = M_{R_0}$ but has opposite direction to M_{R_0}

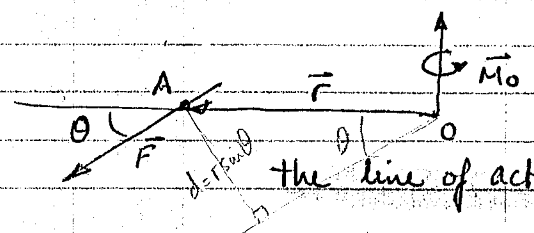
thus THE MOMENT M_{R_0} cancels with $F_R \cdot d$.

CAN BE DONE IN COPLANAR SYSTEMS SINCE $\bar{F}_R$ is $\perp$ $\bar{M}_{R_O}$

[redacted]

- EXAM SEPT 24 1 PAGE BRING CALCULATOR
- MR LAMBERTI WILL PROCTOR
- COVER CH 1 – 3 SECT 4.7, 4.8, 4.9
- 3 OR 4 PROBLEMS 1 HR & 15 MIN
- SEE ME NOW IF YOU CAN'T MAKE IT
- I WILL BE IN MY OFFICE monday until 8PM · TUES TIL' 1PM.

MOMENT OF A VECTOR ABOUT A POINT O



$\vec{M}_O = \vec{r} \times \vec{F}$

$\vec{r}$ IS vector from O to any pt on the line of action of $\bar{F}$

$\vec{r} = \vec{r}_A - \vec{r}_O$

$M_o = |\vec{M}_o| = rF\sin\theta$ $\quad r\sin\theta = d$ $\perp$ distance between O & Line of Action

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \bar{i}\left(r_y F_z - r_z F_y\right) - \bar{j}\left(r_x F_z - r_z F_x\right) + \bar{k}\left(r_x F_y - r_y F_x\right)$$

direction of $\vec{M}_o$ is by the rh rule of the cross product

$\vec{M}_O$ is $\perp$ to $\vec{r}$ & $\vec{F}$

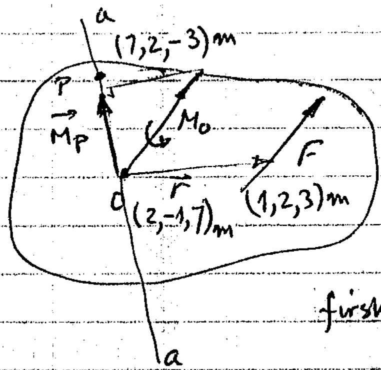
PRINCIPLE OF MOMENTS (VARIGNION'S THEOREM FOR NON CONCURRENT FORCES)

IF $\bar{F} = \bar{F}_1 + \bar{F}_2$ $\quad \vec{M} = \vec{r} \times \vec{F} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 = \vec{r} \times (\bar{F}_1 + \bar{F}_2)$

MOMENT OF A FORCE ABOUT A SPECIFIED AXIS

LIKE FINDING THE PROJECTION OF A MOMENT ONTO A SPECIFIC AXIS

SUPPOSE WE WANT THE MOMENT ABOUT A POINT "O"



$\vec{M}_o = \vec{r} \times \vec{F}$

suppose we want to find the projection along the line aa that passes through "O"

first find unit vector along aa $\vec{u}_a$

now $M_P = |\vec{M}_P| = \vec{M}_o \cdot \vec{u}_a$ magnitude of projection

now $\vec{M}_P = M_P \vec{u}_a$ PROJECTION OF $\vec{M}_o$ along aa

$$\vec{M}_P = M_P \vec{u}_a = (\vec{M}_o \cdot \vec{u}_a)\vec{u}_a = (\vec{u}_a \cdot \vec{M}_o)\vec{u}_a$$

$$M_P = \vec{u}_a \cdot \vec{M}_o = \vec{u}_a \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}.$$

$$= u_{ax}(r_y F_z - r_z F_y) - u_{ay}(r_x F_z - r_z F_x) + u_{az}(r_x F_y - r_y F_z)$$

THE PT "O" IS AN ARBITRARY PT ON THE AXIS aa

Problem suppose $\vec{F} = -(5\bar{i} + 3\bar{j} + 2\bar{k})$ kN

$$\vec{r} = \bar{r}_A - \bar{r}_o = (1\bar{i} + 2\bar{j} + 3\bar{k})\,m - (2\bar{i} - \bar{j} + 7\bar{k})\,m$$
$$= (-1\bar{i} + 3\bar{j} - 4\bar{k})\,m$$

along aa $\bar{r}_{aa} = \bar{r}_p - \bar{r}_o = (7\bar{i} + 2\bar{j} - 3\bar{k})\,m - (2\bar{i} - \bar{j} + 7\bar{k})\,m$
$= (5\bar{i} + 3\bar{j} - 10\bar{k})\,m.$ $r_{aa} = \sqrt{5^2 + 3^2 + (\ )^2} = 11.$

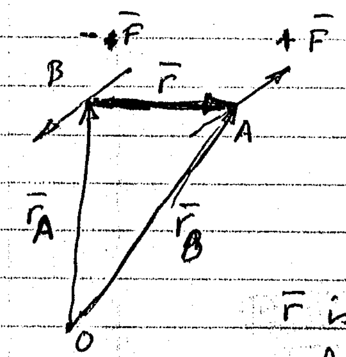
$$\bar{u}_a = \frac{\bar{r}_{aa}}{r_{aa}} = .4319\bar{i} + .2592\bar{j} - .8503\bar{k}$$

$$\underset{kN\text{-}m}{M_P} = \begin{vmatrix} .4319 & +.2592 & -.8503 \\ -1 & 3 & -4 \\ -5 & -3 & -2 \end{vmatrix} = -(.4319)[3 \cdot 2 - 3(-4)] + .2592[-1(2) - 5(-4)] + .8503[-1(3) - 5(3)]$$

$$= -.4319(18) + .2592(18) + .8503(-18) =$$
$$-7.7742 + 4.6656 - 15.3054 = -18.414 \text{ kN-m}$$

$$\vec{M}_P = M_P \vec{u}_a = -18.414\,\text{kNm}(.4319\bar{i} + .2592\bar{j} - .8503\bar{k}) = (-7.95\bar{i} - 4.77\bar{j} + 15.66\bar{k})\ \text{kN-m}$$

SECT 4.10 MOMENT OF A COUPLE



$\bar{M}_O = \bar{r}_A \times \bar{F} + \bar{r}_B \times (+\bar{F})$

$= (\bar{r}_B - \bar{r}_A) \times \bar{F} = \bar{r} \times \bar{F}$

$\bar{r}_B = \bar{r} + \bar{r}_A \quad \therefore \quad \bar{r}_B - \bar{r}_A = \bar{r}$

$\bar{r}$ is the ~~distance~~ vector from a pt on the line of action of one force to the line of action of the other force

$\bar{M}_O$ is dependent on $\bar{r}$ & not $\bar{r}_A$ or $\bar{r}_B$ thus $\bar{M}_O$ is a free vector

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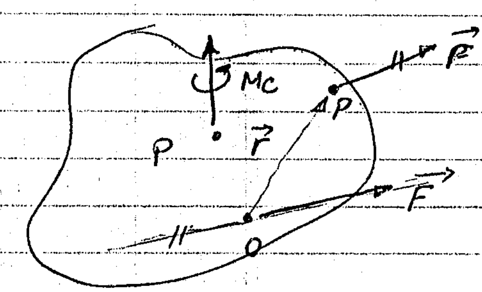
SESSION # 9

EXAM

SESSION #10

MOVING A FORCE IN SPACE TO ANOTHER POINT

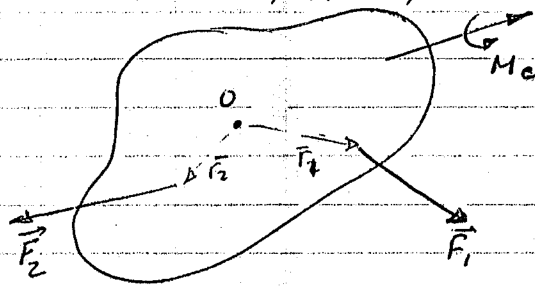


Couple is $M_C = \vec{r} \times \vec{F}$ acts at any Pt

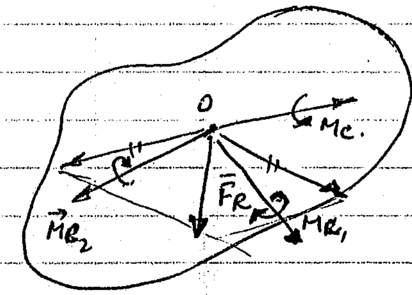
$\vec{r} = \vec{r}_P - \vec{r}_O$

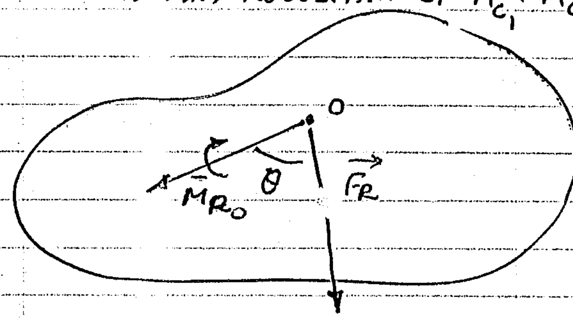
FOR A COUPLE OF FORCES

ORIGINALLY THE BODY



1. FIND POSITION VECTOR FROM O TO EACH LINE OF ACTION
2. MOVE VECTORS TO O & ADD TO BODY moment of couple $M_{c_1} = \bar{r}_1 \times \bar{F}_1$, $M_{c_2} = \bar{r}_2 \times \bar{F}_2$
3. FIND RESULTANT OF $\bar{F}_1$ & $\bar{F}_2$
4. MOVE $\bar{M}_c$ to O (free vector)
5. FIND RESULTANT OF $\bar{M}_{c_1} + \bar{M}_{c_2} + \bar{M}_{c_3} = M_{R_0}$





$\bar{F}_R$ & $\bar{M}_{R_0}$ WILL IN GENERAL NOT BE ⊥ TO EACH OTHER ie $\theta \neq 90°$

HOWEVER $\bar{M}_{R_0}$ CAN BE RESOLVED INTO A COMPONENT $\bar{M}_{\parallel}$ & $\bar{M}_{\perp}$
$\bar{M}_{\parallel}$ is parallel to $\bar{F}_R$ & $\bar{M}_{\perp}$ is ⊥ to $\bar{F}_R$

OPEN BOOK TO <u>PAGE 130</u> - EXPLAIN HOW TO ELIMINATE $\bar{M}_{\perp}$

① $\bar{M}_{\parallel}$ is obtained as follows : $\bar{u}_R = \frac{\bar{F}_R}{F_R}$ now $M_{\parallel} = |M_{\parallel}| = \bar{M}_{R_0} \cdot \bar{u}_R$

and $\bar{M}_{\parallel} = M_{\parallel}\, \bar{u}_R = (\bar{M}_{R_0} \cdot \bar{u}_R)\, \bar{u}_R$

② $\bar{M}_{\perp} = \bar{M}_{R_0} - \bar{M}_{\parallel}$ is ⊥ to $\bar{F}_R$ (why?) parallelogram law.

Now to eliminate $\bar{M}_{\perp}$ find $M_{\perp} = |\bar{M}_{\perp}|$ $|\bar{F}_R| = F_R$
and $d = M_{\perp}/F_R$.

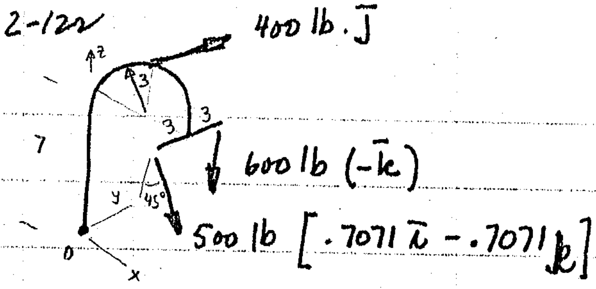
now move $\bar{F}_R$ to a pt d units away from O so that it causes a moment of equal magnitude to $M_{\perp}$ but of <u>opposite direction</u>

③ This is an equivalent force system where the Force $\bar{F}_R$ and $\bar{M}_{\parallel}$ are collinear (since $\bar{M}_{\parallel}$ is a free vector) ≡ WRENCH.

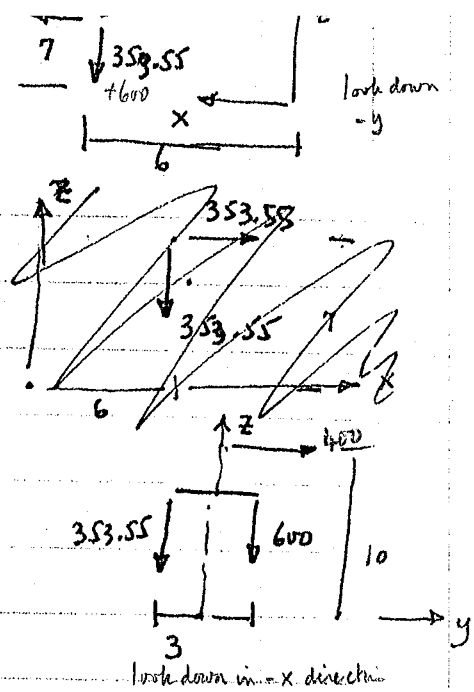
מפתח ברגים

④ Any system of forces & moment can be reduced to a wrench.

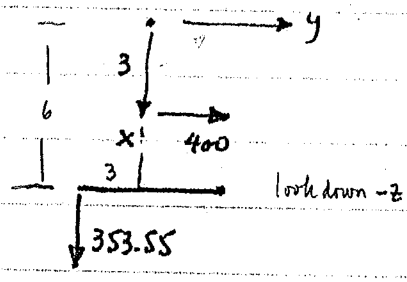
Proble 2-122

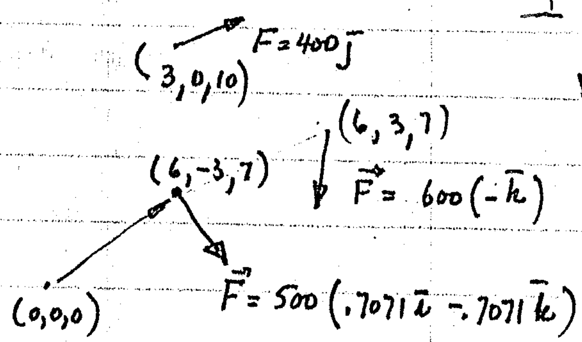


הפעולה המאוחדת שבאה
משלושה הכוחות האלה
גורמת לכוח שקול ומומנט שקול



$$M_y = 6(353.55) + 7(353.55) + 600(6)$$
$$M_x = 353.55(3) - 600(3) - 400(10)$$
$$M_z = 353.55(3) + 400(3) = 753.55 \times 3 = 2260.75$$





$$\Sigma \vec{M}_O = \begin{vmatrix} i & j & k \\ 6 & -3 & 7 \\ 353.55 & 0 & -353.55 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 6 & +3 & 7 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3 & 0 & 10 \\ 0 & 400 & 0 \end{vmatrix}$$

$$M_R = \Sigma M = i[+3(353.55) - 3(600) - 400(10)] + j[+6(353.55) + 7(353.55) + 6(600)] + k[3(353.55) + 3(400)]$$
$$= i[-4739.35] + j[8196.15] + k[2260.65]$$

$M_R = 9733.9$ lb-ft

Also $\Sigma F = 353.55 i - 353.55 k - 600 k + 400 j = 353.55 i + 400 j - 953.55 k$

$F_R = 1092.82$ lb-ft

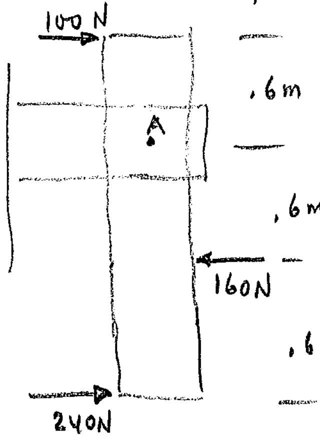
$$u_R = \frac{F_R}{F_R} = .324 i + .366 j - .873 k$$

$$M_{\parallel} = (M_R \cdot u_R) u_R = [(.324)(-4739.35) + (.366)(8196.15) + (-.873)(2260.65)] u_R = -509.31(.324 i + .366 j - .873 k)$$
$$= (-165.02 i - 186.41 j + 444.62 k) \text{ lb-ft}$$

$$M_{\perp} = M_R - M_{\parallel} = (-4574.33 i + 8382.56 j - 2074.24 k) \text{ lb-ft} \qquad M_{\perp} = 9772.12 \text{ lb-ft}$$

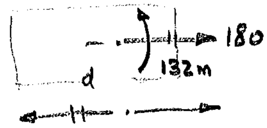
d = M⊥/F = 8.94 ft

בכוח שקול יחידי מצוא את המרחק d מתחת נקודה A לקו הפעולה שלו



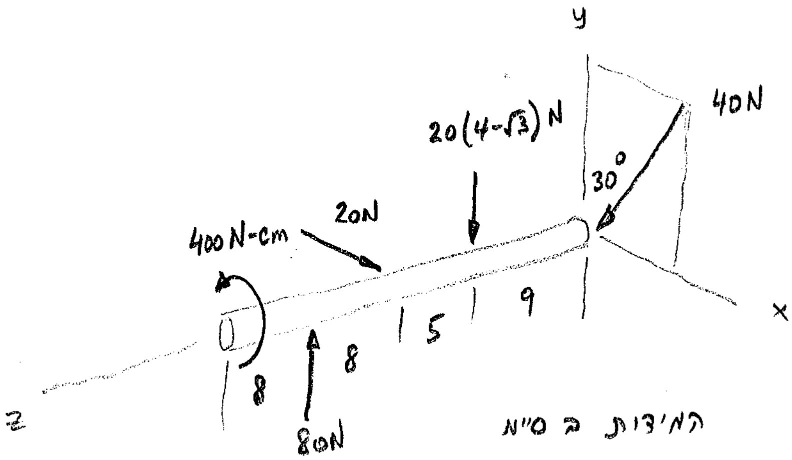
$$\Sigma F = 100 - 160 + 240 = 180\ N$$

$$+\circlearrowleft \Sigma M_A = 100(.6) + 160(.6) - 240(1.2) = -132\ N\text{-}m$$



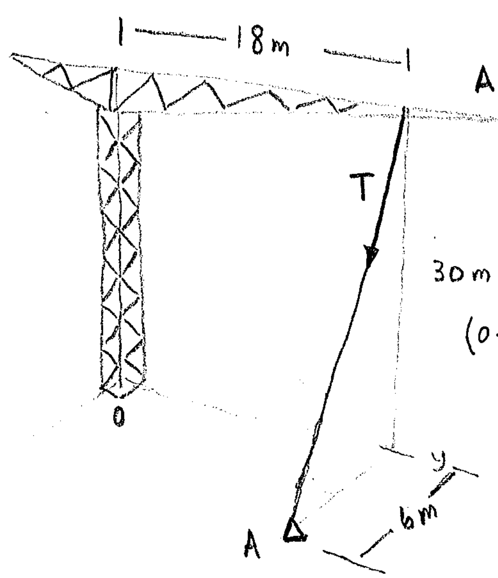
$$d = \frac{132}{180} = 0.733\ m$$

תרגילי בית



המידות בס"מ

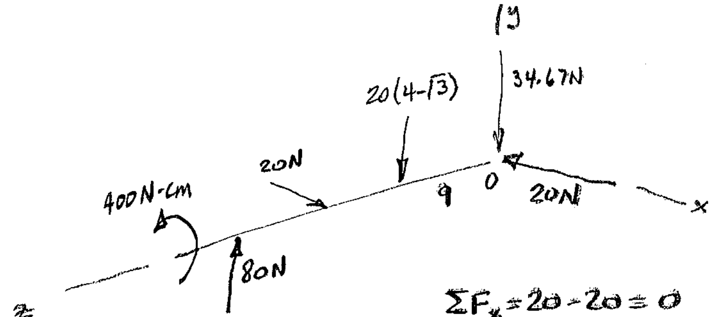
- מצוא כוח שקול ומומנט שקול
- מצוא מפתח לברגים



<u>עגורן</u> להרים משקל A

מפתח T=21kN בכבל.

- מצוא המומנט שמיוצר בכוח T מסביב בסיס העגורן (ב-O)



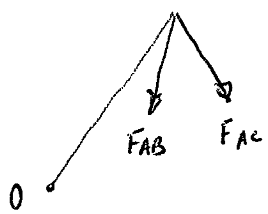
$\Sigma F_x = 20 - 20 = 0$

$\Sigma F_y = -34.64 - 20(4-\sqrt{3}) + 80 = -34.64 + 34.64 = 0$

$\Sigma \underline{M}_0 = 20(4-\sqrt{3})9\underline{i} + 20(14)\underline{j} - 80(22)\underline{i} + 400\underline{k}$ N-m

---

2-113



מצאנו הכוחות

$\underline{F}_{AB} = 19.22\underline{i} + 4.80\underline{j} - 28.84\underline{k}$ N

$\underline{F}_{AC} \; -13.35\underline{i} + 26.75\underline{j} - 40.10\underline{k}$ N

$\underline{F}_R = \underline{F}_{AB} + \underline{F}_{AC} = -5.87\underline{i} + 31.55\underline{j} - 68.94\underline{k}$

מומנט מסביב O

$\underline{r}_{A/0} = (+4\underline{j} + 6\underline{k})$ m

לכן

$$(\underline{r}_{A/0} \times \underline{F}_R) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 6 \\ -5.87 & 31.55 & -68.94 \end{vmatrix} \begin{matrix} \underline{i} & \underline{j} \\ 0 & 4 \\ -5.87 & 31.55 \end{matrix}$$

$= \underline{i}\left(4(-68.94) - 6(31.55)\right) - \underline{j}\left(0(-68.94) - 6(-5.87)\right) + \underline{k}\left(0\cdot 31.55 - 4(-5.87)\right)$

$\underline{M}_0 = -465.06\underline{i} - 35.22\underline{j} + \underline{k}\,23.48$ N-m

$\underline{M}_0 \perp \underline{F}_R$

לפרק את $\underline{M}_\parallel$ ו- $\underline{M}_\perp$: $\underline{M}_\parallel$ קודם

$F_R = \sqrt{(5.87)^2 + (31.55)^2 + (68.94)^2} = 76.04$ N

$\underline{u}_R = \frac{\underline{F}_R}{F_R} = -0.077\underline{i} + .415\underline{j} - .9066\underline{k}$

לכן

$M_\parallel = \underline{M}_0 \cdot \underline{u}_R = 14.48$ N-m

ו

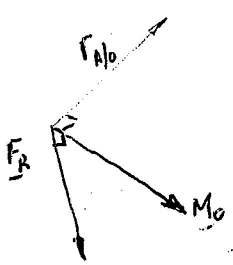
$\underline{M}_\parallel = M_\parallel \underline{u}_R = 14.48(-0.077\underline{i} + .415\underline{j} - .907\underline{k})$

$= (-1.12\underline{i} + 6.01\underline{j} - 13.13\underline{k})$ N-m

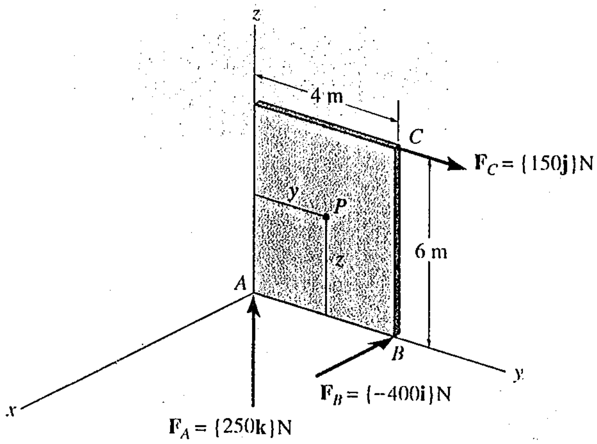
$\underline{M}_\perp = \underline{M}_0 - \underline{M}_\parallel = (-654.36\underline{i} - 35.22\underline{j} + 23.48\underline{k}) - (-1.12\underline{i} + 6.01\underline{j} - 13.13\underline{k})$

$= (-653.24\underline{i} - 41.23\underline{j} + 36.61\underline{k})$ N-m

כדי להסיר את $\underline{M}_\perp$

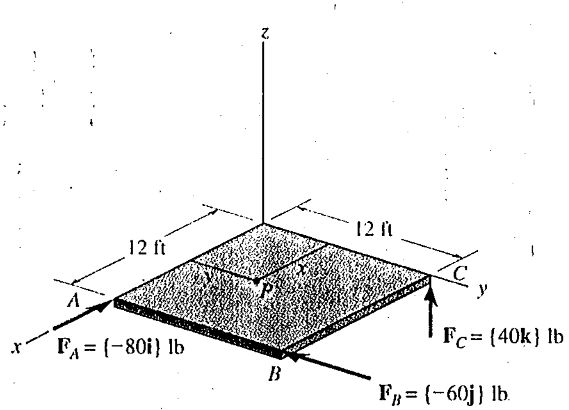


4–121. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.



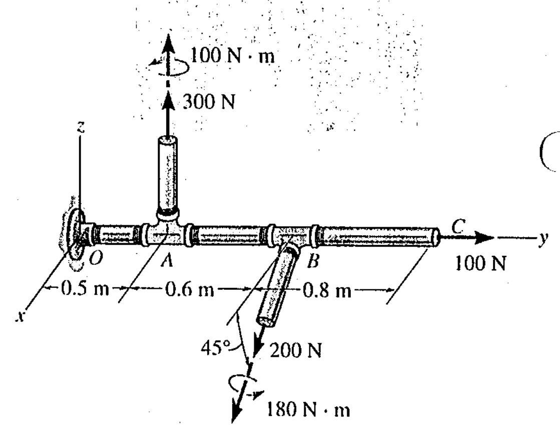
Prob. 4–121

4–122. Replace the three forces acting on the plate by a wrench. Specify the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



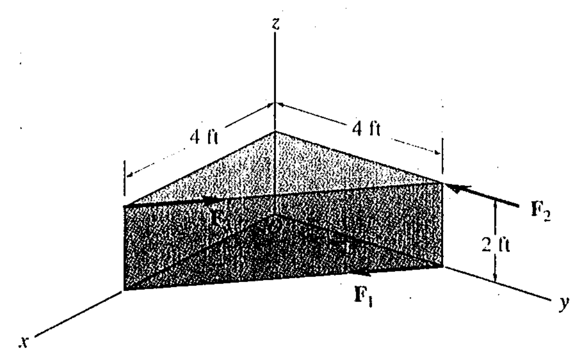
Prob. 4–122

4–123. Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



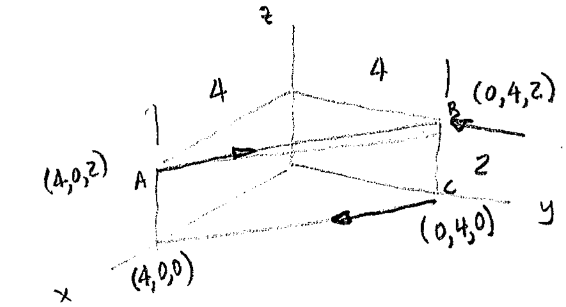
Prob. 4–123

*__4–124.__ The three forces acting on the block each have a magnitude of 10 lb. Replace this system by a wrench and specify the point where the wrench intersects the z axis measured from point O.



Prob. 4–124

4-124



$$\underline{u}_3 = \frac{-4\underline{i}+4\underline{j}+0\underline{k}}{4\sqrt{2}} = -\frac{\underline{i}}{\sqrt{2}}+\frac{\underline{j}}{\sqrt{2}}$$

$$\underline{u}_1 = \frac{4\underline{i}-4\underline{j}+0\underline{k}}{4\sqrt{2}} = \frac{\underline{i}}{\sqrt{2}}-\frac{\underline{j}}{\sqrt{2}}$$

$$\underline{u}_2 = -\underline{j}$$

$$\underline{r}_{A/o} = 4\underline{i}+0\underline{j}+2\underline{k}$$

$$\underline{r}_{B/o} = 0\underline{i}+4\underline{j}+2\underline{k}$$

$$\underline{r}_{C/o} = 0\underline{i}+4\underline{j}+0\underline{k}$$

$$\underline{F}_R = \underline{F}_1+\underline{F}_2+\underline{F}_3 = 10\left(\frac{\underline{i}-\underline{j}}{\sqrt{2}}\right)+(-10\underline{j})+10\left(\frac{-\underline{i}+\underline{j}}{\sqrt{2}}\right) = -10\underline{j}$$

$F_R = 10 \quad \underline{u}_R = -\underline{j}$

$$\sum M_O = \underline{r}_{A/o}\times\underline{F}_3 + \underline{r}_{C/o}\times\underline{F}_1 + \underline{r}_{B/o}\times\underline{F}_2$$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & 0 & 2 \\ -\frac{10}{\sqrt{2}} & \frac{10}{\sqrt{2}} & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 0 \\ \frac{10}{\sqrt{2}} & -\frac{10}{\sqrt{2}} & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 2 \\ 0 & -10 & 0 \end{vmatrix}$$

$$\underline{M}_O = -\frac{20}{\sqrt{2}}\underline{j}+\frac{40}{\sqrt{2}}\underline{k}-\frac{20}{\sqrt{2}}\underline{i}-\frac{40}{\sqrt{2}}\underline{k}+20\underline{i} = +20\underline{i}-\frac{20}{\sqrt{2}}\underline{i}-\frac{20}{\sqrt{2}}\underline{j}$$

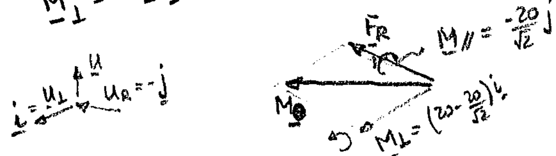
$$M_{\parallel} = \underline{M}_O\cdot\underline{u}_R = \left[\left(20-\frac{20}{\sqrt{2}}\right)\underline{i}-\frac{20}{\sqrt{2}}\underline{j}\right]\cdot(-\underline{j}) = \frac{20}{\sqrt{2}}$$

$\underline{u}_R = \underline{u}_{\parallel}$

$$\underline{M}_{\parallel} = M_{\parallel}\underline{u}_R = -\frac{20}{\sqrt{2}}\underline{j}$$

$$\underline{M}_{\perp} = \underline{M}_O-\underline{M}_{\parallel} = \left(20-\frac{20}{\sqrt{2}}\right)\underline{i} \Rightarrow \underline{u}_{\perp} = \underline{i}$$

$$\underline{M}_{\perp} = M_{\perp}\underline{u}_{\perp} = \left(20-\frac{20}{\sqrt{2}}\right)\underline{i}$$

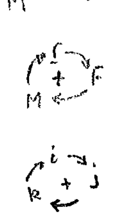


$d = F \times M$

$$\underline{u}_3 = \underline{u}_{\parallel}\times\underline{u}_{\perp} = (-\underline{j})\times\underline{i} = -(-\underline{k}) = \underline{k}$$

$$d = \frac{M_{\perp}}{F_R} = \frac{20-20/\sqrt{2}}{10} = 2-\sqrt{2} = 2-1.414 = .586$$

$M = r \times F$



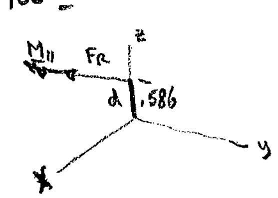
$$\underline{M}_{\perp} = d\,\underline{u}_3\times\underline{F}_R$$

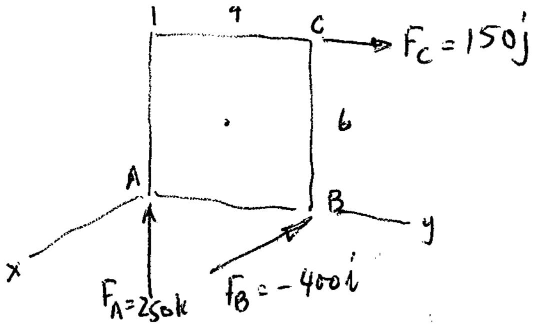
$$\left(20-\frac{20}{\sqrt{2}}\right)\underline{i} = d\,\underline{u}_3\times(-10\underline{j})$$

$$= d\,\underline{k}\times(-10\underline{j})$$

$$\left(20-\frac{20}{\sqrt{2}}\right)\underline{i} = 10d\,\underline{i}$$

$d = .586$ m





החלף את הכוחות האלו שפועלים על הטבלה במפתח לברגים. תפרט את השקול והמומנט של המפתח ומיקום הפעולה P(y,z) שבו קו הפעולה חותך את הטבלה

$\underline{F_R} = -400\underline{i} + 150\underline{j} + 250\underline{k}$ N $\quad F_R = 494.975$N

$\underline{M_A} = (6k \times 150j) + 4j \times (-400\underline{i})$

$= (-900\underline{i} + 1600\underline{k})$ Nm

$\underline{F_R} \cdot \underline{M_A} = 36\times10^4 + 40\times10^4 \neq 0 \Rightarrow \underline{F_R} \not\parallel \underline{M_A}$

$\underline{u_R} = \frac{\underline{F_R}}{F_R} = -.808\underline{i} + .303\underline{j} + .505\underline{k}$

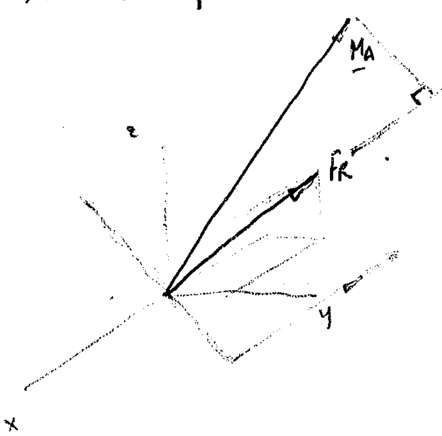
$M_{\parallel} = \underline{M_A} \cdot \underline{u_R} = -900(-.808) + 1600(.505) = 1535.2$ N-m

$\underline{M_{\parallel}} = M_{\parallel}\underline{u_R} = (1535.2)(-.808\underline{i} + .303\underline{j} + .505\underline{k})$

$= -1240.4\underline{i} + 465.2\underline{j} + 775.3\underline{k}$ N-m $\quad M_{\parallel} = 1535$Nm

$\underline{M_\perp} = \underline{M_A} - \underline{M_{\parallel}} = (-900\underline{i} + 1600\underline{k}) - (-1240.4\underline{i} + 465.2\underline{j} + 775.3\underline{k})$

$= 340.4\underline{i} - 465.2\underline{j} + 824.7\underline{k}$ N-m



$M_\perp = 1006.2$ N-m

$\underline{u_\perp} = \underline{M_\perp}/M_\perp = .338\underline{i} - .462\underline{j} + .820\underline{k}$

$d = \frac{M_\perp}{F_R} = \frac{1006.2}{494.975} = 2.03$m

$\underline{u_R} \times \underline{u_\perp} = \underline{u_b}$

$\begin{bmatrix} i & j & k \\ -.808 & .303 & .505 \\ .338 & -.462 & .820 \end{bmatrix}$

$P(y,z) =$

~~$\underline{d} = d\,\underline{u_\perp} = -.686\underline{i} + .938\underline{j} + 1.665\underline{k}$~~

$\underline{d} = d\,\underline{u_b}$

u_R (0, y, z)

$-.686, +.938, -1.665$

$\underline{r_{P/d}} = +.686\underline{i} + (y+.938)\underline{j} + (z+1.665)\underline{k}$

$r_{P/D} = \sqrt{(.686)^2 + (y+.938)^2 + (z+1.665)^2}$

$+.686/r_{PD} = -.808 \quad (y+.938)/r_{PD} = .303$

~~$r_{PD} = \frac{.686}{-.808} = -.849$~~

~~$y = (.303)(-.849) + .938 = +.68$~~

~~$z = .505(-.849) + 1.665 = 2.09$~~

~~$r_{B/D} = (-6\underline{i} + 4\underline{j})$~~

~~$\underline{M_D} = r_{B/D} \times \underline{F_B} = \begin{vmatrix} i & j & k \\ -6 & 4 & 0 \\ -400 & 0 & 0 \end{vmatrix}$~~

~~$= +1600\underline{k}$~~

~~$\underline{F_R} = -400\underline{i} + 150\underline{j} + 250\underline{k}$~~

~~$u_R = -.808\underline{i} + .303\underline{j} + .505\underline{k}$~~

~~$M_{\parallel} = \underline{M_D} \cdot \underline{u_R} = 808$ Nm~~

~~$\underline{M_{\parallel}} = M_{\parallel} u_R = 808(-.808\underline{i} + .303\underline{j} + .505\underline{k})$~~

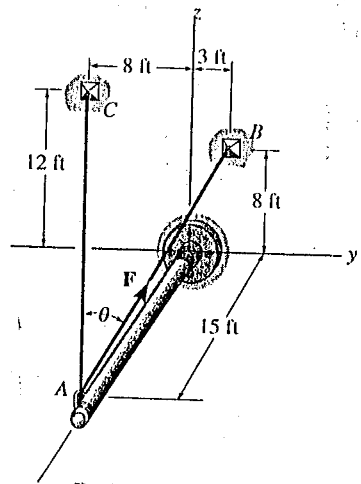
~~$= -652.9\underline{i} + 244.8\underline{j} + 408.0\underline{k}$ N-m~~

~~$\underline{M_\perp} = \underline{M_D} - \underline{M_{\parallel}} = 652.9\underline{i} - 244.8\underline{j} + 1192\underline{k} = 1381$ N-m~~

[illegible]

2–110. Determine the angle θ between cables AB and AC.

2–111. If **F** has a magnitude of 55 lb, determine the magnitude of its projected component acting along the x axis and along cable AC.

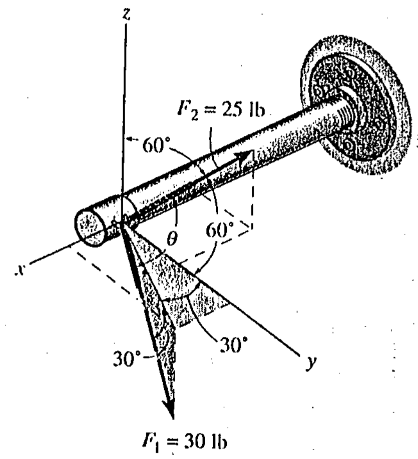


Probs. 2–110/2–111

***2–112.** Determine the angles θ and ϕ between the axis OA of the pole and each cable, AB and AC.

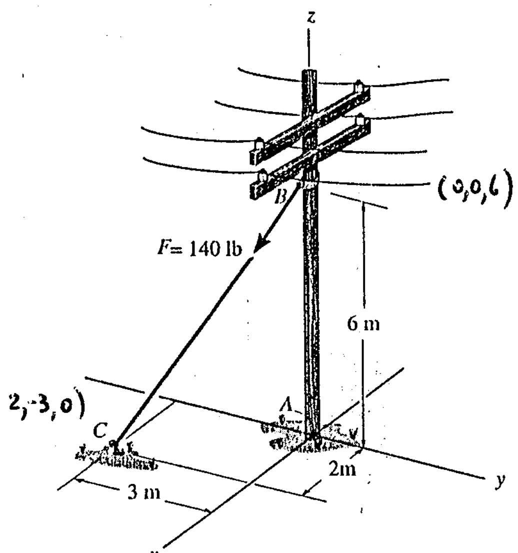
2–114. Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.

2–115. Determine the angle θ between the two cables attached to the pipe.



Probs. 2–114/2–115

4–25. The cable exerts a 140-N force on the telephone pole as shown. Determine the moment of this force at the base A of the pole. Solve the problem two ways, i.e., by using a position vector from A to C, then A to B.



Prob. 4–25

$M_A = 360i + 240j$

$r_{C/B} = 2i - 3j - 6k$

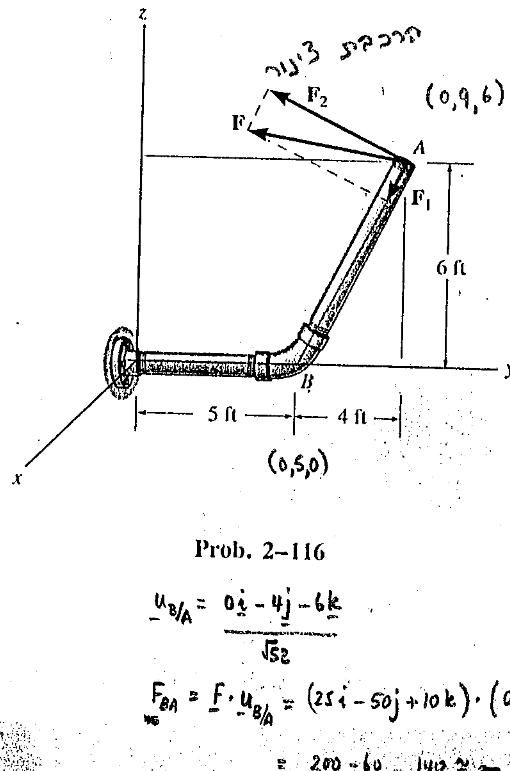
$r_{C/B} = 7$

$u_{CB} = 2/7 i - 3/7 j - 6/7 k \Rightarrow F = 40i - 60j - 120k$

$r_{B/A} = 0i + 0j + 6k$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 6 \\ 40 & -60 & -120 \end{vmatrix} = -6(-60i - 40j) = 360i + 240j$$

***2–116.** The force $\mathbf{F} = \{25\mathbf{i} - 50\mathbf{j} + 10\mathbf{k}\}$ N acts at the end A of the pipe assembly. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which act along the axis of AB and perpendicular to it.



Prob. 2–116

$u_{B/A} = \frac{0i - 4j - 6k}{\sqrt{52}}$

$F_{BA} = F \cdot u_{B/A} = (25i - 50j + 10k) \cdot \frac{(0i - 4j - 6k)}{\sqrt{52}}$

$= \frac{200 - 60}{\sqrt{52}} = \frac{140}{\sqrt{52}} = 19.41\ N$

$F_1 = F_{BA} = \frac{140}{\sqrt{52}} u_{B/A} = \frac{140}{52}(0i - 4j - 6k)$

$= \frac{-560j - 840k}{52} = -10.77j - 16.15k$

$F_2 = F - F_1 = (25i - 50j + 10k) + \frac{560j + 840k}{52}$

$= 25i - (50 - \frac{560}{52})j - (10 - \frac{840}{52})k$

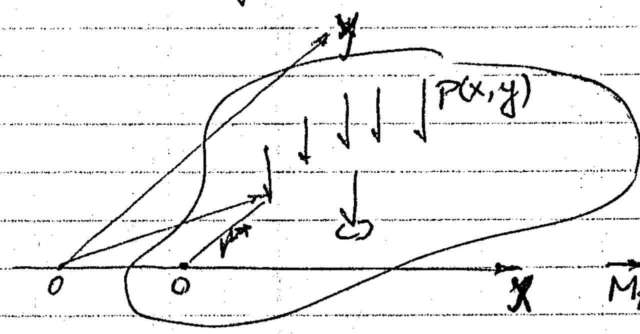
$= 25i - (39.23)j + 6.15k$

- WRENCH causes translation (due to $\bar{F}_R$) and rotation (due to $\bar{M}_{||}$).
- Axis of the wrench & pt through which it acts are unique.

DISTRIBUTED LOADINGS **SESSION #13**

- CAUSED BY WIND, FLUIDS, MATERIAL WEIGHT

INTENSITY OF A LOAD IS A FORCE/UNIT AREA - intensity are infinite number of parallel forces acting on a differential dxdy



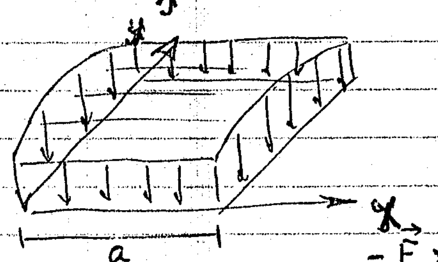
$$p(x,y) = \lim_{\Delta A} \frac{\Delta \vec{F}}{\Delta A} = \frac{d\vec{F}}{dA}$$

$$\vec{F}_R = \sum d\vec{F} = \iint -p(x,y)\,dx\,dy\,\vec{k}$$

$$\vec{M}_{R_0} = \sum \vec{r} \times d\vec{F} = -\iint p(x,y)\vec{k} \times (r_x \vec{i} + r_y \vec{j})\,dx\,dy$$
$$= +\iint [r_x p(x,y)\vec{j} - r_y p(x,y)\vec{i}]\,dx\,dy$$

- REPLACE THIS FORCE SYSTEM BY AN EQUIVALENT FORCE F_R SO THAT ABOUT SOME PT THE SAME OVERALL MOMENT IS PRODUCED

SUPPOSE (1) body is symmetric and the loading $p(x,y)$ is a constant in the x direction ie $\int dx\, p(x,y) = W(y) = p(y)a$ [lb/ft] are units



$$\vec{F}_R = \sum d\vec{F} = -\int W(y)\,dy\,\vec{k}$$ EQUAL TO AREA UNDER

$$\vec{M}_{R_0} = +\int r_x W(y)\,dy\,\vec{j} - \int r_y W(y)\,dy\,\vec{i}$$

$$-\vec{F}_R \times \vec{r} = (\bar{x}\vec{i} + \bar{y}\vec{j}) \times \left[-\int W(y)\,dy\,\vec{k}\right]$$
$$= \bar{x}\int W(y)\,dy\,\vec{j} - \bar{y}\int W(y)\,dy\,\vec{i} = \vec{M}_{R_0}$$

thus $\bar{x}\int W(y)\,dy = +\int r_x W(y)\,dy$ or $\bar{x} = \dfrac{+\int r_x W(y)\,dy}{\int W(y)\,dy}$ $\bar{y} = r_y$

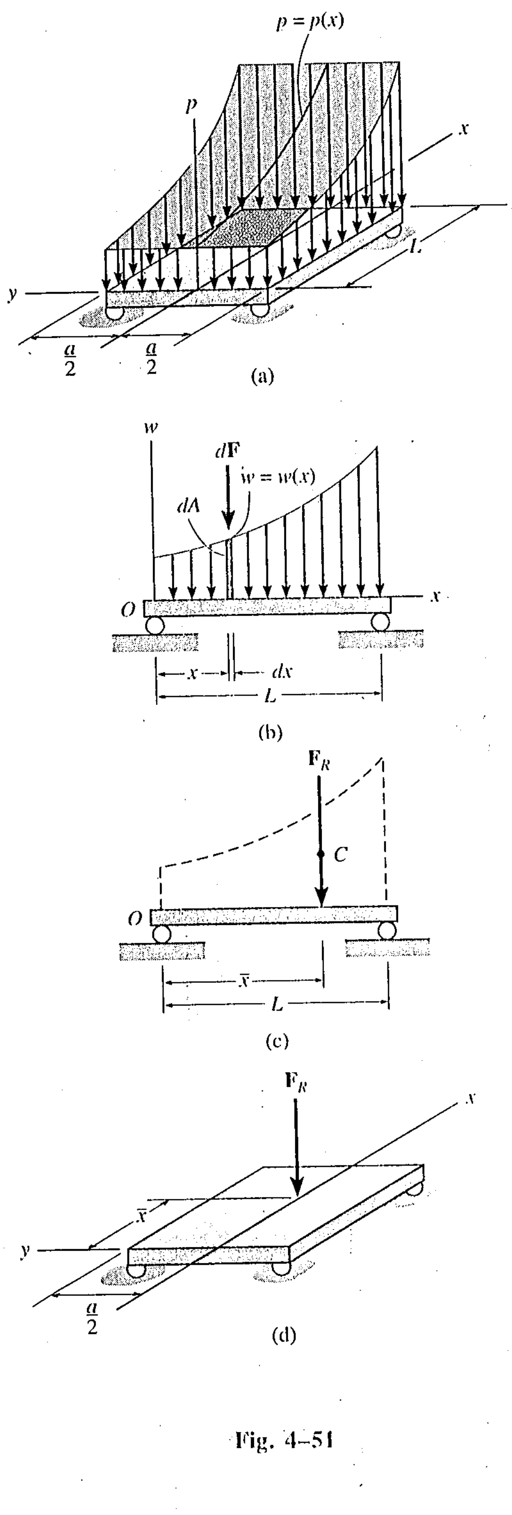
area under loading curve

$$\bar{x} = \frac{\iint x\,p(x,y)\,dx\,dy}{\iint p(x,y)\,dx\,dy}$$ for $p(x,y) = P(y)$ only

$$= \frac{\int x\,dx \int P(y)\,dy}{\int dx \int P(y)\,dy} = \frac{x^2}{2}\Big|_0^a \Big/ x\Big|_0^a = \frac{a}{2}$$

$W(y) = \int p(x,y)\,dx = \int P(y)\,dx = P(y)a$

LOADING, unit [lb/ft]



Magnitude of Resultant Force. From Eq. 4–19 ($F_R = \Sigma F$), the magnitude of $\mathbf{F}_R$ is equivalent to the sum of all the forces in the system. In this case integration must be used, since there is an infinite number of parallel forces $d\mathbf{F}$ acting along the plate, Fig. 4–51*b*. Since $d\mathbf{F}$ is acting on an element of length dx, and $w(x)$ is a force per unit length, then at the location x, $dF = w(x)\ dx = dA$. In other words, the magnitude of $d\mathbf{F}$ is determined from the colored differential *area* dA under the loading curve. For the entire plate length,

$$+\downarrow F_R = \Sigma F; \qquad F_R = \int_L w(x)\ dx = \int_A dA = A \tag{4–21}$$

Hence, the magnitude of the resultant force is equal to the total area under the loading diagram $w = w(x)$.

Location of Resultant Force. Applying Eq. 4–20 ($M_{R_O} = \Sigma M_O$), the location $\bar{x}$ of the line of action of $\mathbf{F}_R$ can be determined by equating the moments of the force resultant and the force distribution about point O (the y axis). Since $d\mathbf{F}$ produces a moment of $x\ dF = x\ w(x)\ dx$ about O, Fig. 4–51*b*, then for the entire plate, Fig. 4–51*c*,

$$\curvearrowleft + M_{R_O} = \Sigma M_O; \qquad \bar{x} F_R = \int_L x\ w(x)\ dx$$

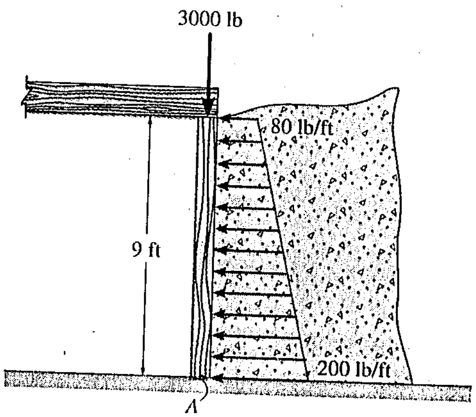
Solving for $\bar{x}$, using Eq. 4–21, we can write

$$\bar{x} = \frac{\displaystyle\int_L x\ w(x)\ dx}{\displaystyle\int_L w(x)\ dx} = \frac{\displaystyle\int_A x\ dA}{\displaystyle\int_A dA} \tag{4–22}$$

This equation represents the x coordinate for the geometric center or *centroid* of the *area* under the distributed-loading diagram $w(x)$. *Therefore, the resultant force has a line of action which passes through the centroid C (geometric center) of the area defined by the distributed-loading diagram* $w(x)$, Fig. 4–51*c*.

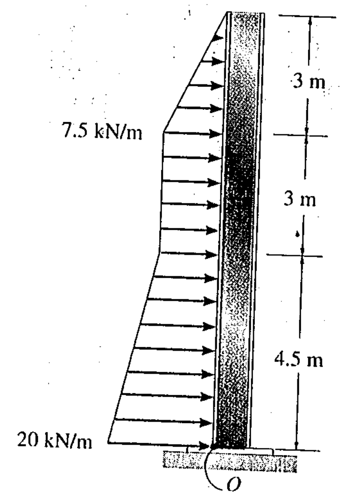
Once $\bar{x}$ is determined, $\mathbf{F}_R$ by symmetry passes through point ($\bar{x}$, 0) on the surface of the plate, Fig. 4–51*d*. If we now consider the three-dimensional pressure loading $p(x)$, Fig. 4–51*a*, we can therefore conclude that *the resultant force has a magnitude equal to the volume under the distributed-loading curve* $p = p(x)$ *and a line of action which passes through the centroid (geometric center) of this volume*. Detailed treatment of the integration techniques for computing the centroids of volumes or areas is given in Chapter 9. In many cases, however, the distributed-loading diagram is in the shape of a rectangle, triangle, or other simple geometric form. The centroids for such common shapes do not have to be determined from Eq. 4–22; rather, they can be obtained directly from the tabulation given on the inside back cover.

4–130. The column is used to support the floor which exerts a force of 3000 lb on the top of the column. The effect of soil pressure along the side of the column is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column measured from its base A.



Prob. 4–130

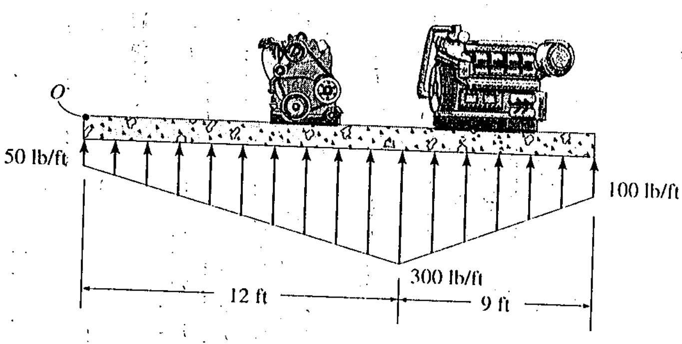
***4–132.** Replace the loading by an equivalent resultant force and couple moment acting at point O.



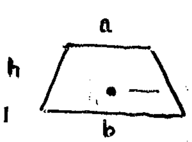
Prob. 4–132

הפלגת עומס האדמה מתחת לבלוק בניין

4–131. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location measured from point O.



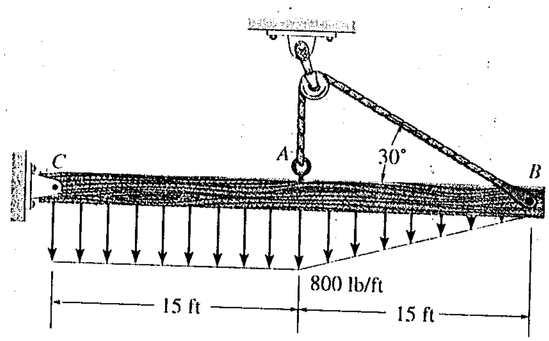
Prob. 4–131



$A = \frac{1}{2}h(a+b)$

$y_c = \frac{1}{3}\left(\frac{2a+b}{a+b}\right)h$

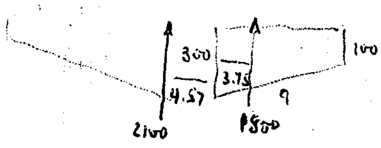
4–133. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam measured from the pin at C.



Prob. 4–133

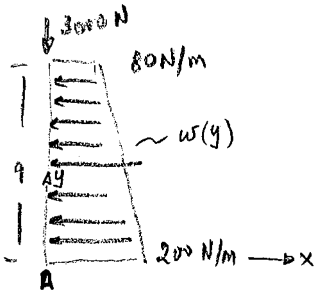
$A = \frac{1}{2}12(350) = 2100$ 50

$y_c = \frac{1}{3}\cdot 12\left(\frac{400}{350}\right) = 4.57$



$y_c = \left(\frac{500}{400}\right)3$

$A = \frac{1}{2}\cdot 9(400) = 1800$



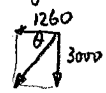
$w(y) = \frac{80-200}{9}y + 200$ $\qquad \frac{80-200}{9} =$ שיפוע של עומס המפורס

$F_x = \int_0^9 w(y)dy = \frac{80-200}{9}\frac{y^2}{2}\Big|_0^9 + 200y\Big|_0^9 = -160(9) + 200(9) = 1260\ N$

$M_A = \int_0^9 y\,w(y)dy = \frac{80-200}{9}\frac{y^3}{3}\Big|_0^9 + 200\frac{y^2}{2}\Big|_0^9 = -120(27) + 200(40.5) = 4860\ N$ +↻

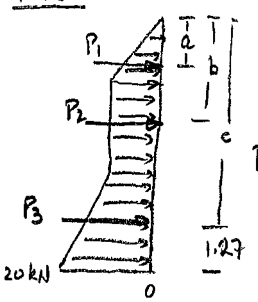
$d = \frac{M_A}{F_x} = \frac{4860}{1260} = 3.857\ m$

$F_R = (F_x^2 + F_y^2)^{1/2} = ((1260)^2 + (3000)^2)^{1/2} = 3254\ N$



$\tan\theta = \frac{-3000}{-1260} = +2.381 \qquad \theta = 67.22^\circ$

4-132



$P_1 = 7.5 \times \frac{3}{2} = 11.25\,kN \qquad a = 2\,m$

$P_2 = 7.5 \times 3 = 22.5\,kN \qquad b = 4.5\,m$

$P_3 = \frac{(20+7.5)}{2} \cdot 4.5 = \sout{41.25}$ 61.88 kN $\qquad c = 9 - \frac{3}{3}\left(\frac{2\cdot 7.5+20}{27.5}\right) = 9 - \frac{35}{27.5} = 7.73$

$F_R = P_1 + P_2 + P_3 = \sum_{i=1}^{3} P_i = \sout{75}$ 95.63 kN

↻ $M_0 = P_1(9-a) + P_2(9-b) + P_3(9-c) = 11.25(7) + 22.5(4.5) + \sout{41.25}$ 61.88 $(1.27) = \sout{232.5}$ 258.6 kN-m ↻

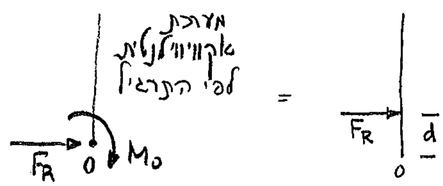
לזכרכם

$(A+B)\frac{H}{2} =$ שטח

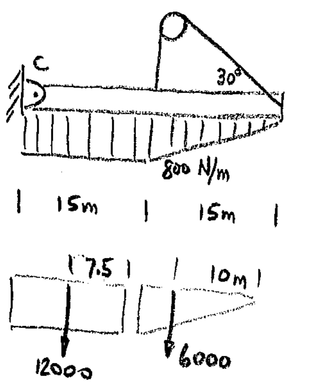
$\frac{2A+B}{A+B}\frac{H}{3} = \bar{x}$

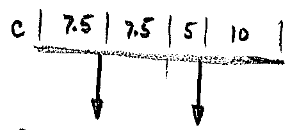
אם רציתי מערכת אקוויוולנטית בלי מומנט.

$d = \frac{M_0}{F_R} = \frac{232.5}{75} = 3.1\ m$



4-133





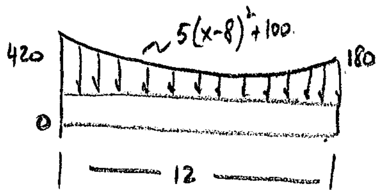
$800 \times 15 = 12000\ N \qquad \frac{800 \times 15}{2} = 6000\,N$

$F_R = 12000 + 6000 = 18000\,N$ ↓

$M_c = 12000(7.5) + 6000(20) = 210000\ N\text{-}m$

$d = \frac{M_c}{F_R} = \frac{210 \times 10^3}{18 \times 10^3} = 11.67\,m$: מסביב C

תרגיל אחרון



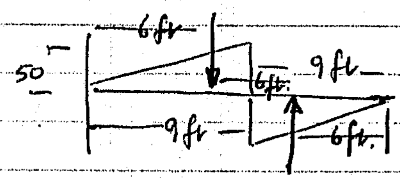
$F_R = \int_0^{12} w(x)dx = \int_0^{12}[5(x-8)^2 + 100]dx = 5\frac{(x-8)^3}{3} + 100x\Big|_0^{12} = 1306.67 + 853.33 = 2160\ N$ ↓

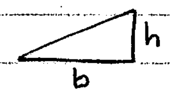
$M_0 = \int_0^{12} x\,w(x)dx = \int_0^{12}(5x^3 - 80x^2 + 420x)dx = \frac{5x^4}{4} - \frac{80}{3}x^3 + \frac{420x^2}{2}\Big|_0^{12} = 10080\ N\text{-}m$

$d = \frac{M_0}{F_R} = 4.67\ m$

Do Problem 4-89

SESSION #11





$\int dA = \frac{1}{2} bh.$

$\int x\,dA = \int x \cdot y\,dx = \int x \cdot \frac{x}{b} h\,dx$

$= \frac{h}{b} \int x^2 dx = \frac{x^3}{3b} h \Big|_0^b = \frac{b^2 h}{3}$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{1}{3} b^2 h}{\frac{b h}{2}} = \frac{2b}{3}$$

$\bar{x} =$ 2/3 base of the triangle

$F_R = \int dA = \frac{bh}{2} = 9ft \cdot \frac{50}{2} lb/ft = 225 lb.$

$\Sigma F = \downarrow 225 lb + \uparrow 225 lb = 0$

$\Sigma M =$ couple $= 225 lb \cdot 6ft = 1350$ lbft ↺

EQUILIBRIUM OF A RIGID BODY

$\Sigma \vec{F} = 0$ and $\Sigma \vec{M_o} = 0$ to prevent translation & rotation of a body

- First study equilibrium in 2-D
  - FREE BODY DIAGRAM – Show body & all forces acting on it
  - Supports also produce forces. What are they – forces that result from resistance to movement or rotation

העומס בשל מההתנגדות
לתנועה או לסביבה

Pg 152 #4, #8, #9

השאלה שהעומסים
השלה מציגים את
שיקול כוח מפוזר שקיים
במקום המגע בין הגוף
לבין הסמך.
כאשר שטח המגע קטן,
יכולים להשתמש בכוח
מרוכב

- Reality – these forces represent the resultant of the distributed load that exists where the contact occurs. If the contact area is small ~~the~~ with respect ~~the~~ to the ~~length~~ area of the support, then the concentrated load is a good representation

- FBD
  - Isolate the body — בודדים את הגוף
  - Identify all the forces, moments, support reactions that are external — מכירים את כל הכוחות, מומנטים, תגובות החיצוניים.
  - Indicate the dimensions — מציינים את המידות

ΣM_O is sum of all moments due to force components about an axis ⊥ to x-y plane passing through pt. O.

- STATIC EQUILIBRIUM means $\vec{F}_R = \vec{0}$ and $\vec{M}_{R_O} = \vec{0}$

$$\vec{F}_R = \Sigma F_x \vec{i} + \Sigma F_y \vec{j} = \vec{0} = 0\vec{i} + 0\vec{j} \quad \text{thus} \quad \Sigma F_x = 0 \quad \Sigma F_y = 0$$

אפשר גם לעבוד בסט אחד אך ורק שהנקודות A ו-B לא נחות על קו שהוא אנכי לציר-y

ALTERNATIVE SETS ARE $\Sigma F_y = 0$, $\Sigma M_A = 0$ $\Sigma M_B = 0$

- A, B do not lie on line ⊥ to y axis
-

$$\Sigma M_A, \Sigma M_B, \Sigma M_C = 0$$

- A, B, C do not lie on the same line.

או אם A, B, C לא נחות על אותו קו

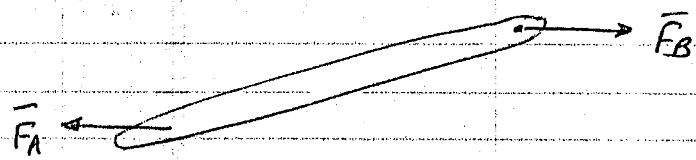
SIMPLIFICATIONS

מצבים מיוחדים

- FORCES APPLIED AT TWO POINTS ONLY — TWO FORCE MEMBERS (איברים שני כוחות)
- FORCE EQUILIB. REQUIRES (כדי שיהיה שיווי משקל)

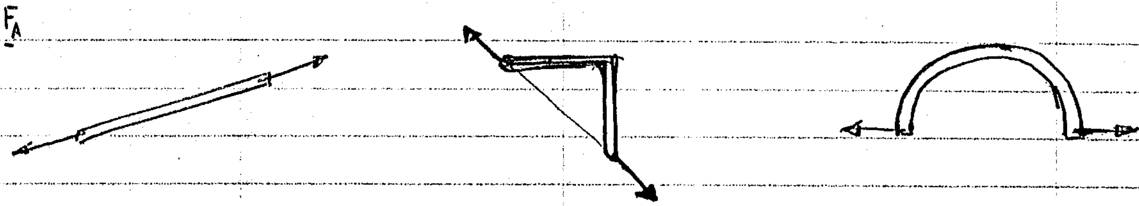
הכוח F_A ו-F_B שווים בגודל אבל בכיוון הפוך

$\bar{F}_A = \bar{F}_B$ same magnitude opposite direction



- MOMENT EQUIL REQUIRES LINE OF ACTION OF $\bar{F}_A$ & $\bar{F}_B$ BE SAME.

שיווי משקל של מומנטים צריך שקו הפעולה של F_A ו-F_B יהיה אותו קו



- IF A BODY IS SUBJECTED TO 3 FORCES THAT ARE COPLANAR (אם לגוף יש שלוש כוחות)
  - FORCES MUST BE CONCURRENT $(\Sigma M_O) = 0$ SATISFIED
- OR • " " " PARALLEL

must only satisfy $\Sigma \vec{F} = \vec{0}$

הכוחות צריכים להיות שותפי מישור וגם שותפי נקודה או שותפי קו פעולה אחד

Gautam did 5.22, 5.27, 5.17, 5.12 (Set up), 5.1, 5.2

Table 5–1 . Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

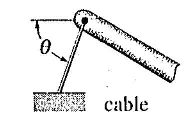
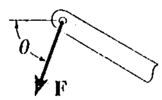
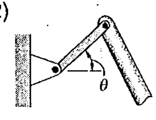
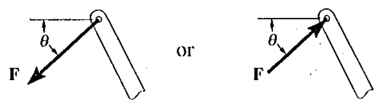
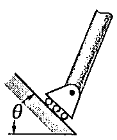
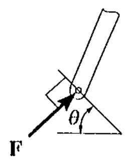
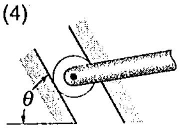
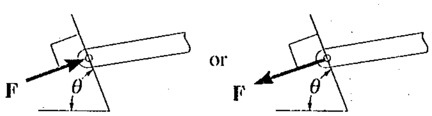
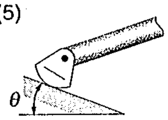
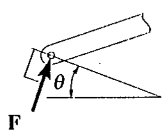
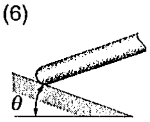
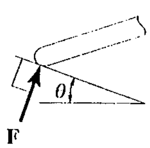
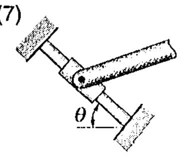
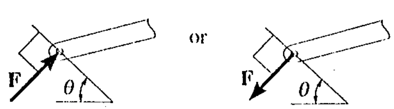
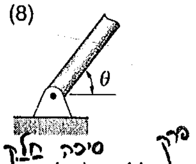
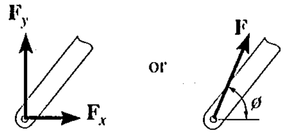
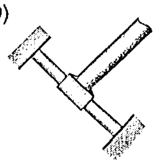
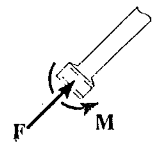
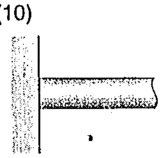
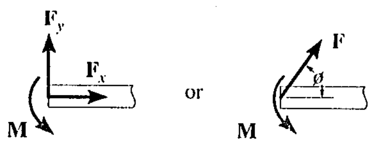
Types of Connection	*Reaction*	*Number of Unknowns*
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link		One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

Table 5–1 (Contd.)

Types of Connection	*Reaction*	*Number of Unknowns*
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Support Reactions. Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of support between bodies subjected to coplanar force systems.

The principles involved for determining these reactions can be illustrated by considering three ways in which a horizontal member, such as a beam, is commonly supported at its end. The first method of support consists of a *roller* or cylinder, Fig. 5–2*a*. Since this type of support only prevents the beam from translating in the vertical direction, it is necessary that the roller exerts a force on the beam in this direction, Fig. 5–2*b*.

The beam can be supported in a more restrictive manner by using a *pin* as shown in Fig. 5–3*a*. The pin passes through holes in the beam and two leaves which are fixed to the ground. Here the pin will prevent translation of the beam in *any direction* ϕ, Fig. 5–3*b*, and so it must exert a force $\mathbf{F}$ on the beam in this direction. For purposes of analysis, it is generally easier to represent this effect by its two components $\mathbf{F}_x$ and $\mathbf{F}_y$, Fig. 5–3*c*.

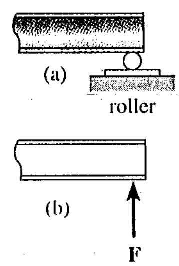


Fig. 5–2

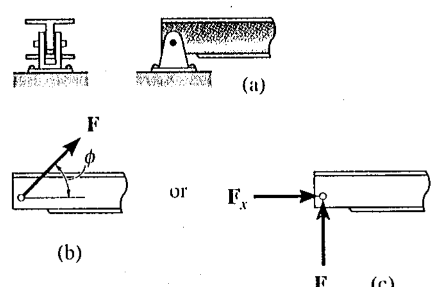


Fig. 5–3

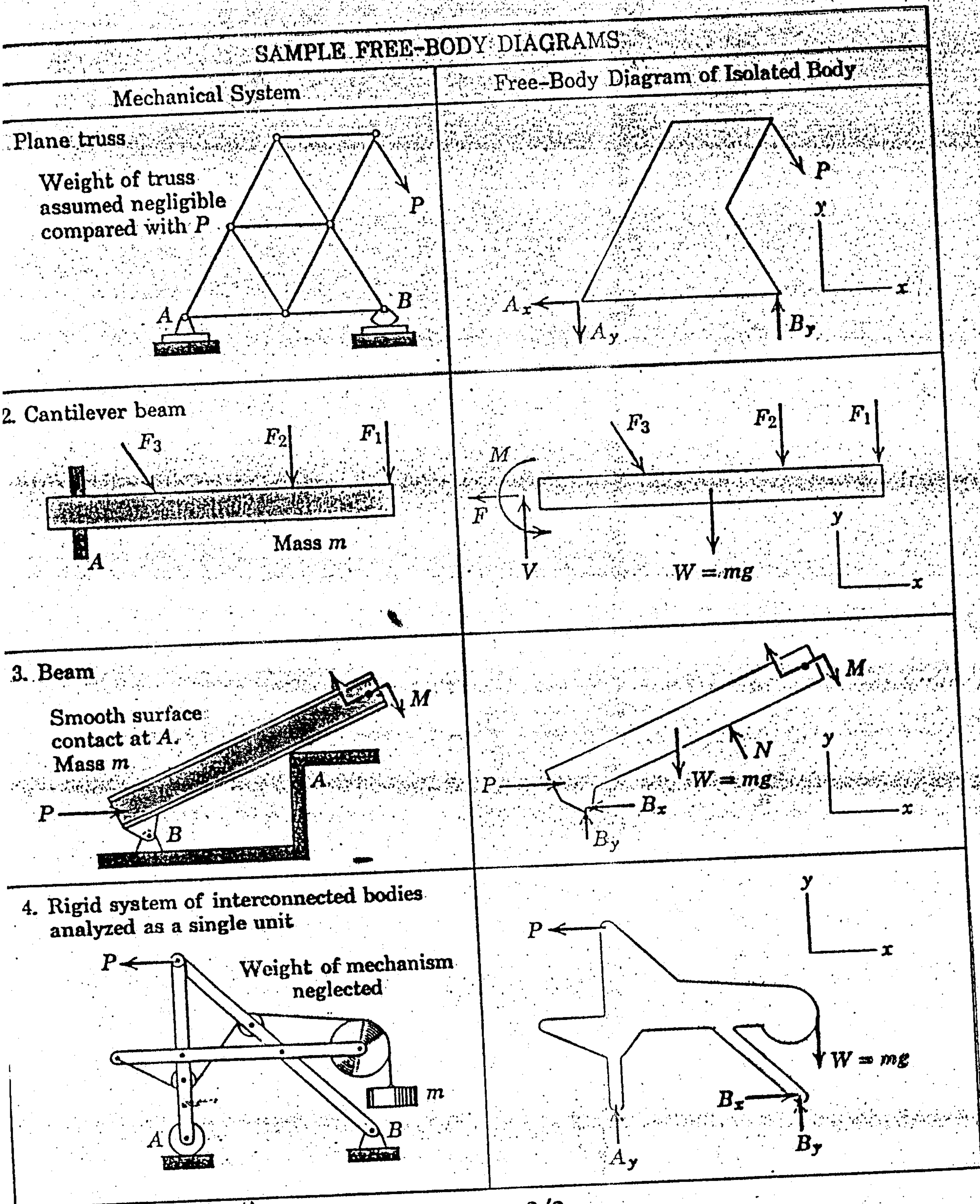
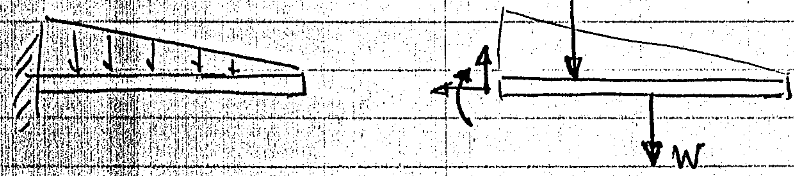
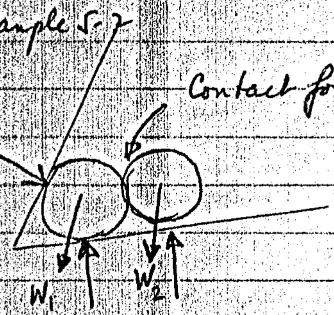

SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
Plane truss
Weight of truss assumed negligible compared with P
P
A
B
P
y
x
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
Mass m
A
F_3
F_2
F_1
M
F
V
W = mg
y
x
3. Beam
Smooth surface contact at A.
Mass m
M
A
P
B
M
N
W = mg
y
P
B_x
B_y
x
4. Rigid system of interconnected bodies analyzed as a single unit
P
Weight of mechanism neglected
m
A
B
P
y
x
W = mg
B_x
A_y
B_y

Figure 3/2

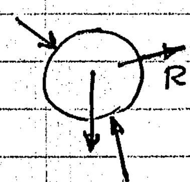
Example 5-1



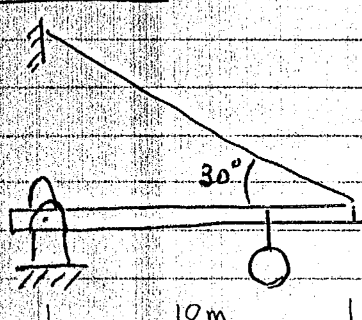
Example 5-2

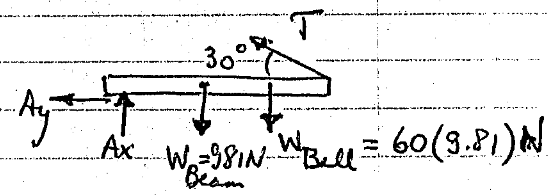


Contact forces here are neglected as they are internal wrt the FBD of both balls

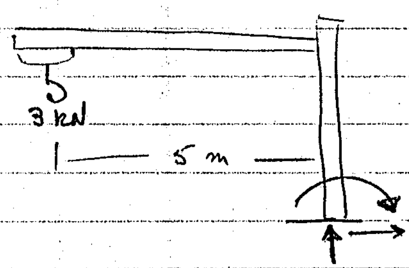


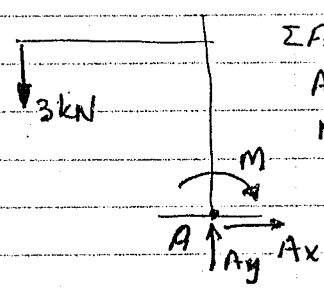
Problem 5-1





Problem 5-4





$\Sigma F_x = 0$ $\quad \Sigma F_y = 0$

$A_x = 0$ $\quad A_y = $ [illegible] kN

$M_A = 15$ kN-m

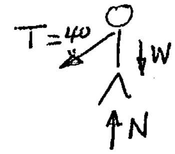
- Equilibrium is $\Sigma F_x, \Sigma F_y, \Sigma M_o = 0$ for a body whose forces are coplanar,

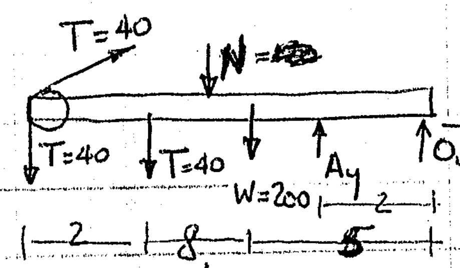
3 eqs give at most 3 unknowns.

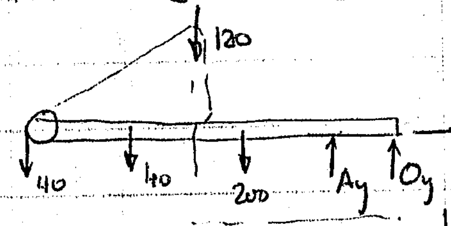
ראינו שמערכת כלשהי של כוחות ומומנטים מערכת כזאת ניתן להחליף בכוח השקול ושקול המומנט בנקודה מסויימת

- in Chapter 4 have shown that any system of forces can be reduced to a resultant force $\vec{F_R}$ and a resultant moment about some point $\vec{M_{R_O}}$.

3-21







$280 = A_y + O_y$

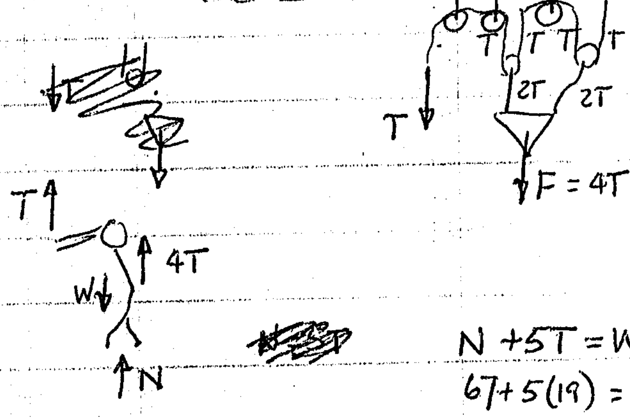
$O_x = 0$

$\Sigma M_o = A_y(2) - 200(5) - 120(6) + 40(8) - 40(10) = 0$

$A_y = \frac{2440}{2} = 1220 \text{ lb}$

$O_y = -940$

3/18



$N + 5T = W$

$67 + 5(19) = 162 \text{ lb.}$

14/April 28/April

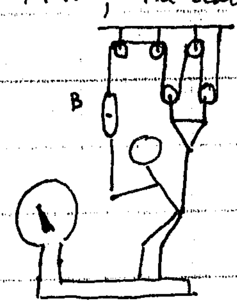
$ ~~496~~ 616 : 40x12

52x10 + 24x4

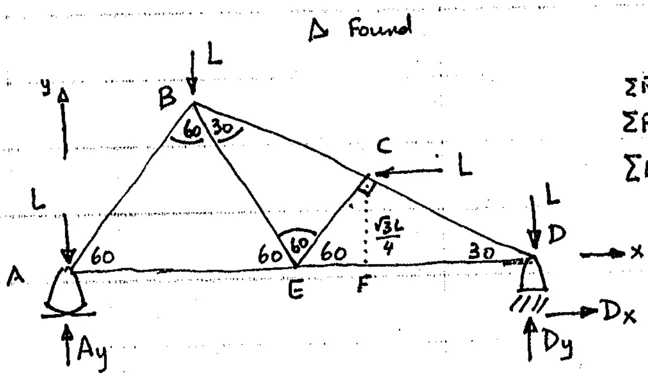
Sixt

14 313 p/NT

A former student of mechanics wants to weigh himself but has access to a scale A with a max. capacity of 100 lb & a small 20 lb spring dynamometer B.
With the rig shown, he finds that when he pulls with a force of that registers 19 lb, the scale reads 67 lb. What is his weight



Δ Found



$\Sigma F_y = 0 \quad A_y + D_y = 3L$

$\Sigma F_x = 0 \quad D_x = L$

$\Sigma M_D = 0 \quad A_y \cdot 2l - L \cdot 2l - L \cdot \frac{3l}{2} - L\frac{\sqrt{3}l}{2} = 0$

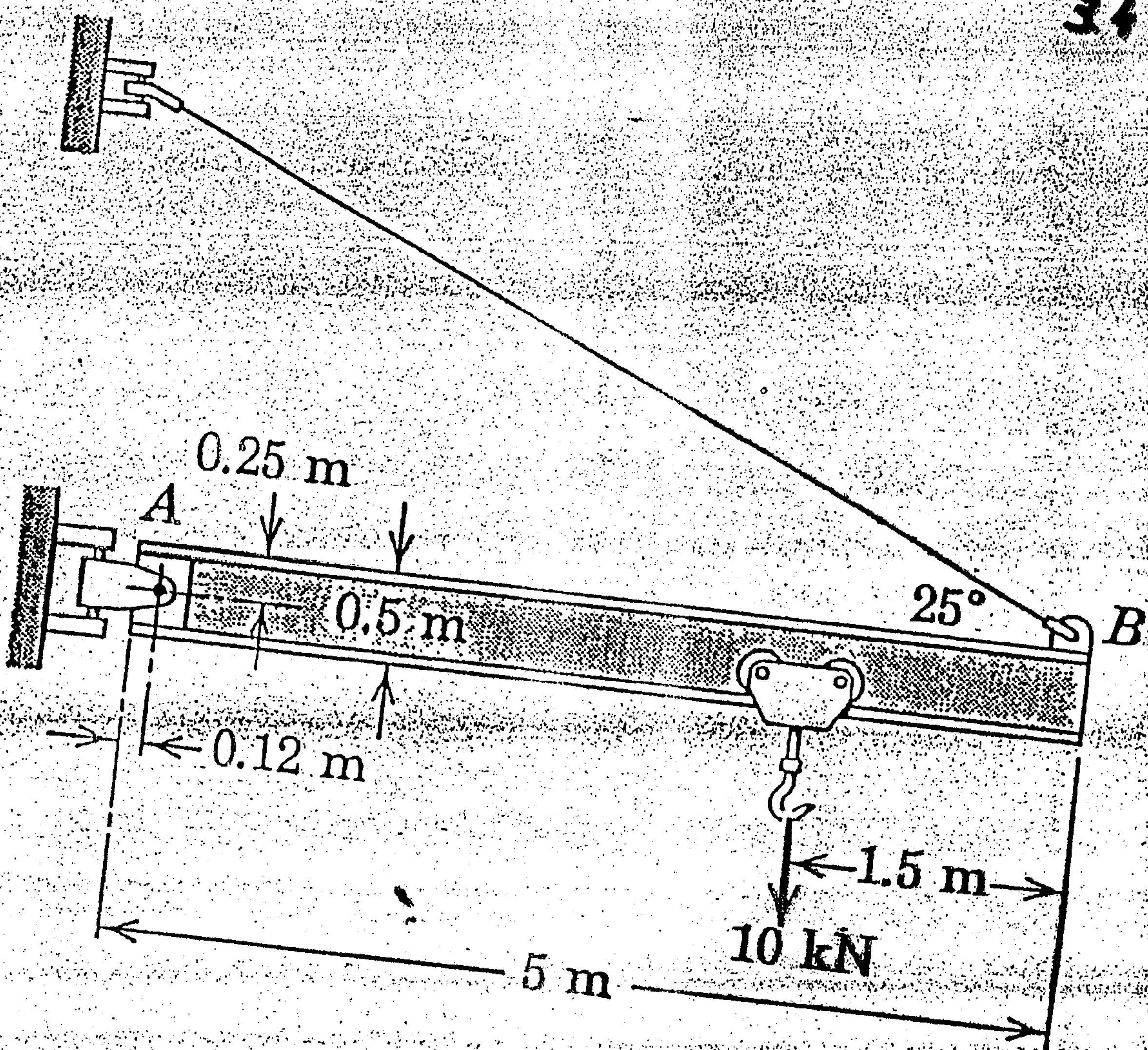
$A_y \cdot 2 - \frac{7L}{2} - \frac{\sqrt{3}}{4}L = 0$

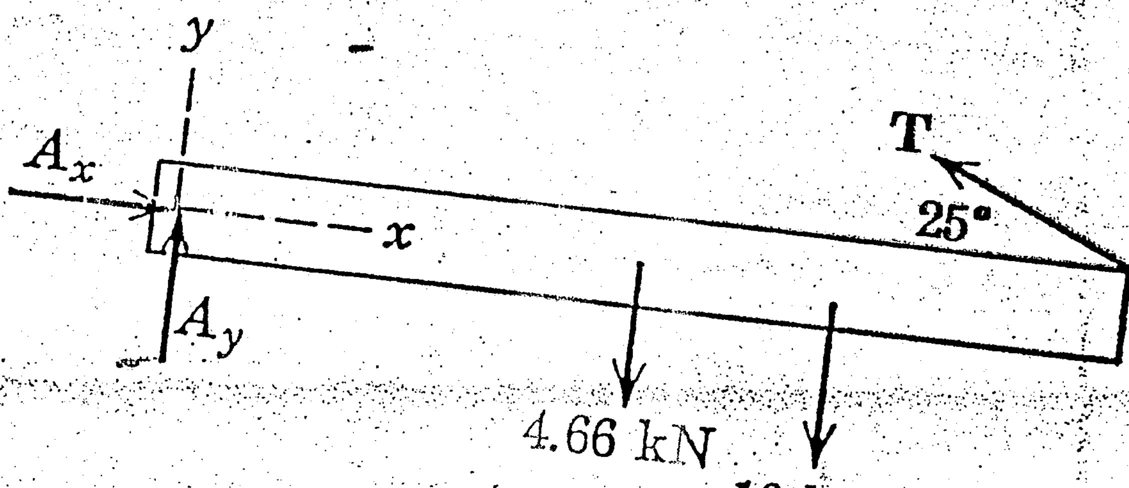
$A_y = \frac{(3.5 + .433)L}{2} = \frac{3.933L}{2} = 1.967L$

$D_y = 3L - A_y = 1.033L$

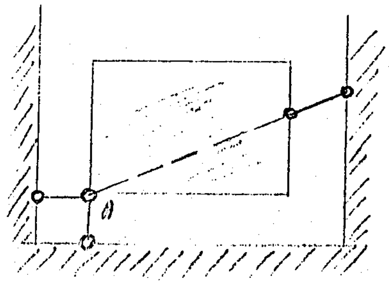
✓

ED-8



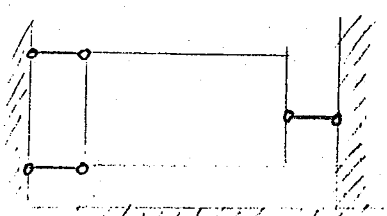


Free–Body Diagram



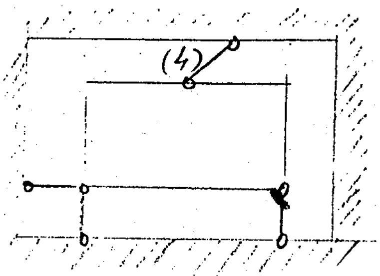
מצב ב'

במקרה זה הכח שיפעיל המוט הימני יעבור דרך הנקודה A. לכן לא תהיה כל מגבלה תנועה התחלתית בנגד לבן המרוסן. זהו מצב של הגבלה בלתי מושלמת.
הריתקות נותנות $M_O=0$ אך האיל לכן אין התנגדות לסבוב סביב A.



מצב ג'

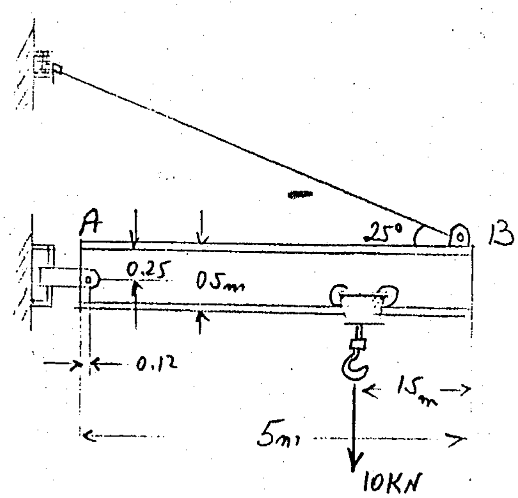
אין התנגדות לכח בכיוון y!
(מתקיימות רק 2 מ.ש.מ.)



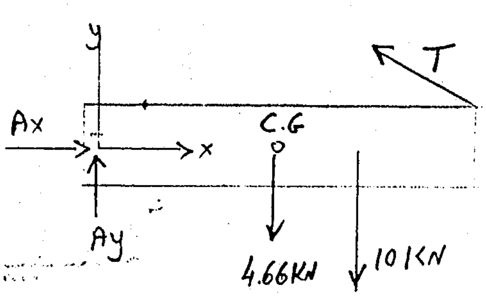
מצב ד'

מצב של ריתום יתר במוט (4)
מבנה בלתי מסוים סטטית פעמיים.

דוגמא (3/2)



חשב את כח המתיחה בכבל והכוחות הפועלים על הפין A במנוף המתואר. מסת הקורה 95kg/m (שקף 3.4)



1. דיאגרמת גוף חפשי
ישנם שלושה נעלמים: T, Ax, Ay

2. משקל הקורה הנכנס במרכז הכובד

$mg = 95 \times 5 \times 9.81 = 4.66$ kN

ב. פתרון ע"י שווי המשקל

1) נתחיל במשוואת מומנטים סביב A שבה יהיה נעלם אחד T

$$\Sigma M_A = 0 \qquad (T\cos 25^\circ)\,0.25 + (T\sin 25^\circ)(5-0.12) - 10(5-1.5-0.12) +$$

$$- 4.66 \times (2.5-0.12) = 0$$

$$T = 19.61\ KN$$

2) שווי משקל בכיוונים x ו- y

$$\Sigma F_x = 0 \qquad A_x - 19.61\cos 25^\circ = 0 \qquad A_x = 17.77\ KN$$

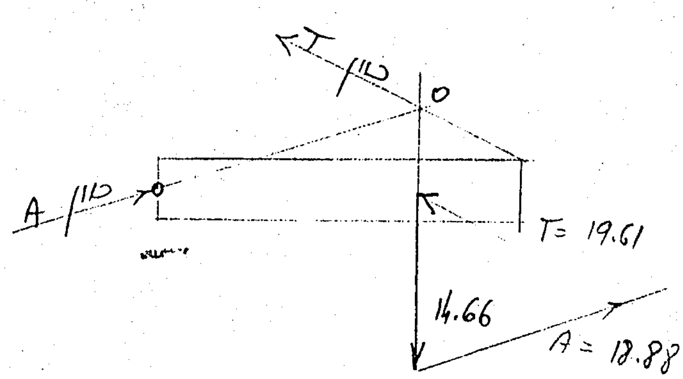
$$\Sigma F_y = 0 \qquad A_y + 19.61\sin 25^\circ - 4.66 - 10 = 0 \qquad A_y = 6.37$$

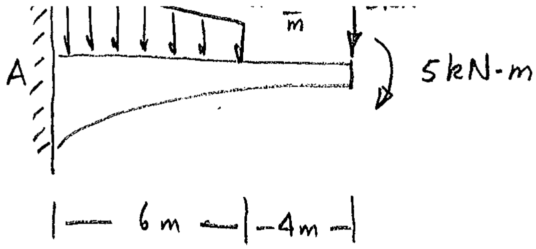
$$A = \sqrt{A_x^2 + A_y^2} \qquad A = 18.88\ KN$$

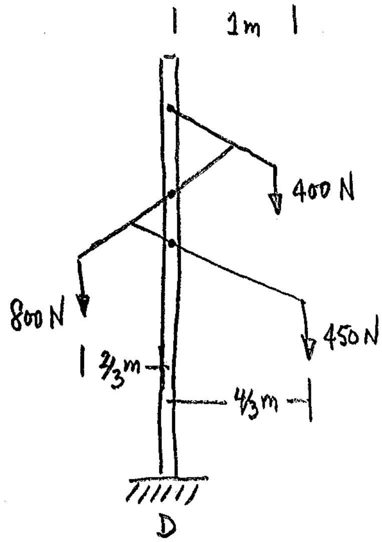
ג. פתרון גרפי:

1. נמצא גרפית את שקול שני הכוחות 4.66, 10 (R)

2. שלושת הכוחות T ו- R ו- A חייבים לעבור דרך נקודה אחת ולסגור פוליגון. לכן כיוון הכח A הוא מ- A דרך O

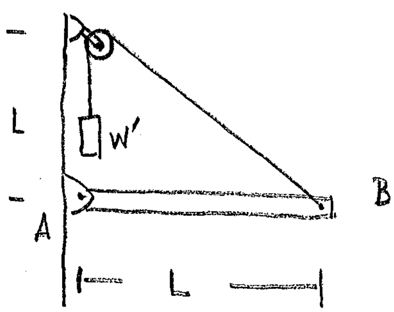




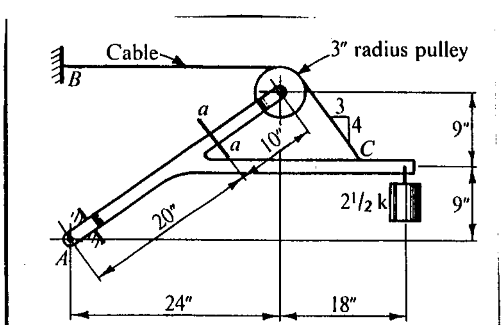


מצאו את התגובות ב-D

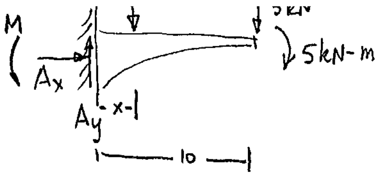
$M_D \cong 0.47$ kN-m



למוט יש משקל W. מצאו את המשקל W' כדי שיהיה שיווי משקל



PROB. 2-13

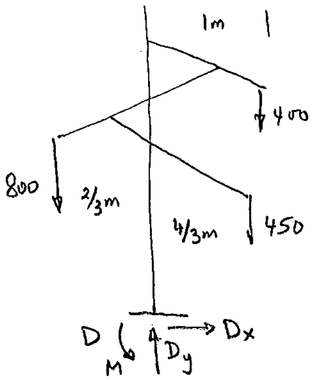


$$x = \left(\frac{2(0.5)+0.8}{0.5+0.8}\right)\frac{6}{3} = \frac{3.6}{1.3} = 2.77\ m$$

$\Sigma F_x = A_x = 0 \qquad \underline{A_x = 0}$

$\Sigma F_y = A_y - Q - 3 = 0 \qquad \underline{A_y = Q + 3 = 6.9\ kN}$

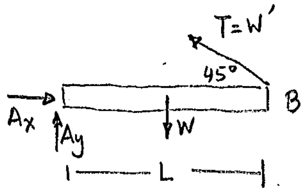
$+\circlearrowright \Sigma M_A = -M + Q\cdot x + 3\cdot 10 + 5 = 0 \qquad M = (3.9)(2.77) + 30 + 5 = 45.8\ kN\text{-}m$



$\Sigma F_x = \underline{D_x = 0}$

$\Sigma F_y = -400 - 800 - 450 + D_y = 0 \qquad \underline{D_y = 1650N}$

$+\circlearrowright \Sigma M = -800(2/3) + 400(1) + 450(4/3) - M = 0 \qquad M = 467\ N\text{-}m$



$\Sigma F_x = A_x - W'\cos 45 = 0$

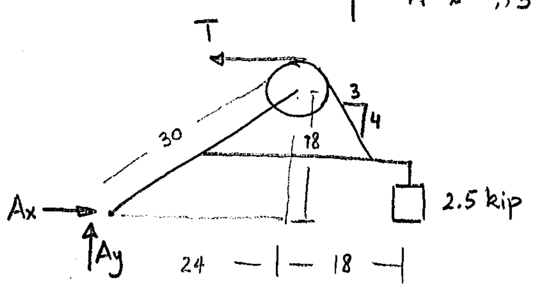
$\Sigma F_y = A_y - W + W'\sin 45°$

$+\circlearrowright \Sigma M_A = W\cdot L/2 - W'\sin 45\cdot L = 0 \qquad W' = \frac{W}{2\sin 45} = \frac{W}{\sqrt{2}}$

$\underline{A_x} = W'\cos 45 = \frac{W}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \underline{\frac{W}{2}}$

$\underline{A_y} = W - W'\sin 45 = \underline{W/2}$

לפי הנתונים אי אפשר לפתור התרגיל הרביעי. אבל בסיכה ב-A כן אפשר

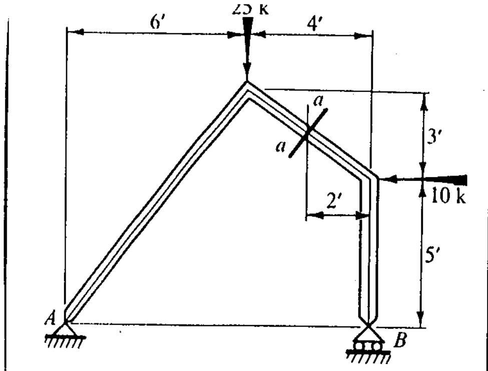


$\Sigma F_x = -T + A_x = 0$

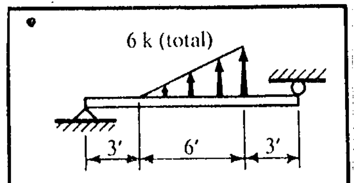
$\Sigma F_y = A_y - 2500 = 0 \qquad A_y = 2500$

$+\circlearrowright \Sigma M_A = -T(21) + 2500(42) = 0 \qquad T = 5000$

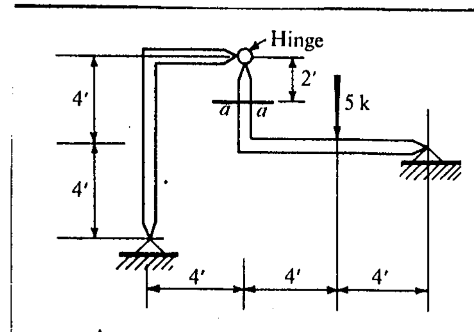
$A_x = T = 5000$



PROB. 2-10

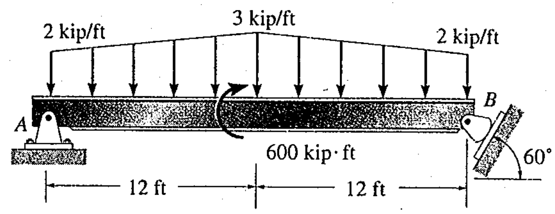


PROB. 2-5

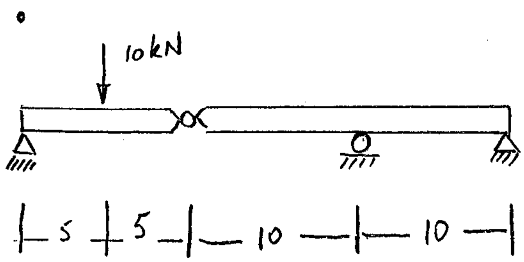


PROB. 2-8

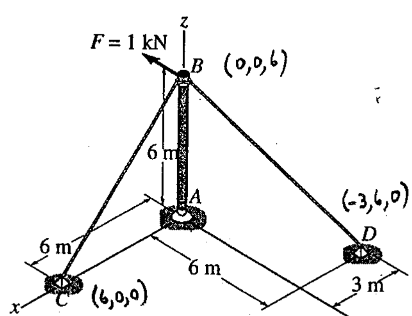
5–29. Determine the support reactions on the beam.



Prob. 5–29



Determine the tension in cables BC and BD and the reactions at the ball-and-socket joint A for the mast shown in Fig. 4–28a.



(a)

SOLUTION (*VECTOR ANALYSIS*)

Free-Body Diagram. There are five unknown force magnitudes shown on the free-body diagram, Fig. 4–28b.

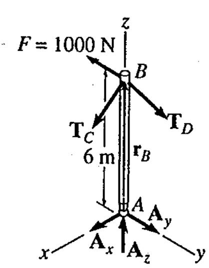
Equations of Equilibrium. Expressing each force in Cartesian vector form, we have

$$\mathbf{F} = \{-1000\mathbf{j}\}\ \text{N}$$

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{T}_C = 0.707T_C\mathbf{i} - 0.707T_C\mathbf{k} \qquad \underline{r}_{c/B} = 6i - 6k \qquad \underline{u}_{c/B} = \frac{1}{\sqrt{2}}\underline{i} - \frac{1}{\sqrt{2}}\underline{k}$$

$$\mathbf{T}_D = T_D\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = -0.333T_D\mathbf{i} + 0.667T_D\mathbf{j} - 0.667T_D\mathbf{k}$$



(b)

Applying the force equation of equilibrium gives

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F} + \mathbf{F}_A + \mathbf{T}_C + \mathbf{T}_D = \mathbf{0}$$

$$(A_x + 0.707T_C - 0.333T_D)\mathbf{i} + (-1000 + A_y + 0.667T_D)\mathbf{j} + (A_z - 0.707T_C - 0.667T_D)\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad A_x + 0.707T_C - 0.333T_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad A_y + 0.667T_D - 1000 = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad A_z - 0.707T_C - 0.667T_D = 0 \qquad (3)$$

Summing moments about point A, we have

$$\Sigma\mathbf{M}_A = \mathbf{0}; \qquad \mathbf{r}_B \times (\mathbf{F} + \mathbf{T}_C + \mathbf{T}_D) = \mathbf{0}$$

$$6\mathbf{k} \times (-1000\mathbf{j} + 0.707T_C\mathbf{i} - 0.707T_C\mathbf{k} - 0.333T_D\mathbf{i} + 0.667T_D\mathbf{j} - 0.667T_D\mathbf{k}) = \mathbf{0}$$

Evaluating the cross product and combining terms yields

$$(-4T_D + 6000)\mathbf{i} + (4.24T_C - 2T_D)\mathbf{j} = \mathbf{0}$$

$$\Sigma M_x = 0; \qquad -4T_D + 6000 = 0 \qquad (4)$$

$$\Sigma M_y = 0; \qquad 4.24T_C - 2T_D = 0 \qquad (5)$$

The moment equation about the z axis, $\Sigma M_z = 0$, is automatically satisfied.

$T_C = 707$ N $\quad T_D = 1500$ N

$A_x = 0$ N $\quad A_y = 0$ N $\quad A_z = 1500$ N

מפני שהמוט המאונך AB הוא אבר של
שני כוחות (כי כל הכוחות פועלים או
ב-A או ב-B) רק מבדיקת מצב
הכוחות אפשר לאמר ש- $A_x = A_y = 0$!

SESSION # 12

EQUILIBRIUM IN 3-D – Draw FBD as usual

- Support Reactions
  developed when translation or rotation is restricted

pg 180 take about (6), (8), (5)

<u>Equilibrium equations</u>

For <u>non concurrent</u> forces $\sum \vec{F} = \vec{0}$
$\sum \vec{M}_0 = \vec{0}$

Results in 6 equations

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum F_z = 0$$

$$\sum M_x = 0, \ \sum M_y = 0, \ \sum M_z = 0$$

sum of the moment components ~~along~~ ∥ to the x, y, z axes passing through O.

O can be on or off the body.

at most 6 unknowns can be found.

1. Draw FBD for the body
2. Indicate all forces acting on the body
3. Indicate all dimensions
4. If it is difficult to determine moments arms or forces use vector analysis $\sum \vec{F} = \vec{0}$, $\sum \vec{M}_0 = 0$
5. <u>Not necessary for axes for force sum be the same as that for moment summation.</u>
6. <u>Choose direction of moment axes so that as many lines of actions of unknowns ~~can be eliminated~~ intersect it. Moment due to these unknowns will be zero.</u>
7. Can choose any set of 3 non orthogonal axes for force or moment summation as long as they are <u>not parallel to each other</u>

allowed to rotate freely about *any* axis, no couple moment is resisted by a ball-and-socket joint.

It should be noted that the *single* bearing supports (5) and (7), the *single* pin (8), and the *single* hinge (9) are shown to support both force and couple-moment components. If, however, these supports are used in conjunction with *other* bearings, pins, or hinges to hold the body in equilibrium, and provided the physical body maintains its *rigidity* when loaded and the supports are *properly aligned* when connected to the body, then the *force reactions* at these supports may *alone* be adequate for supporting the body. In other words, the couple moments become redundant and may be neglected on the free-body diagram. The reason for this will be clear after studying the examples which follow, but essentially the couple moments will not be developed at these supports since the rotation of the body is prevented by the reactions developed at the other supports and not by the supporting couple moments.

Table 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

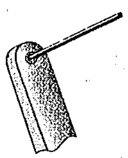
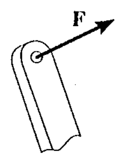
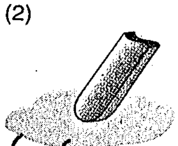
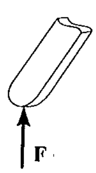
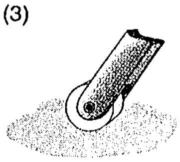
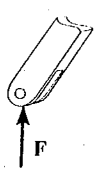
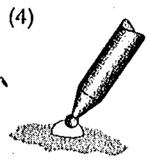
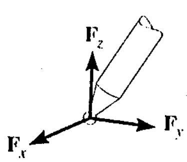
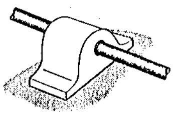
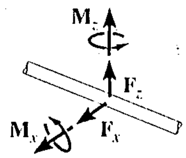
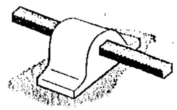
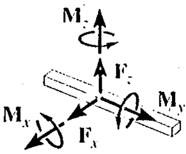
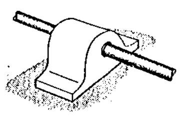
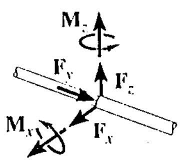
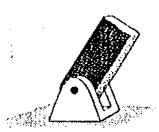
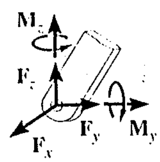
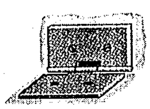
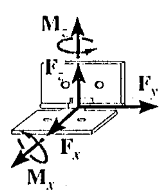
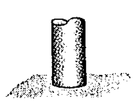
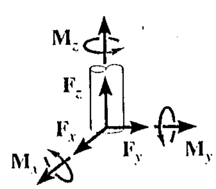
Types of Connection	*Reaction*	*Number of Unknowns*
(1)  cable		One unknown. The reaction is a force which acts away from the member in the direction of the cable.
(2)  סמך שטח חלק smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. Rough surface שטח מחוספס גורם לעזרכות - כוח החיכוך מקביל לשטח.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.

Table 5–2 (Contd.)

Types of Connection	*Reaction*	*Number of Unknowns*
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft.
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

ECHANICAL ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS

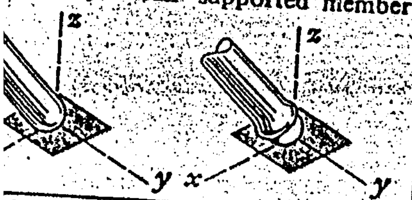
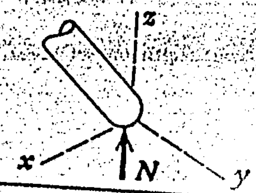
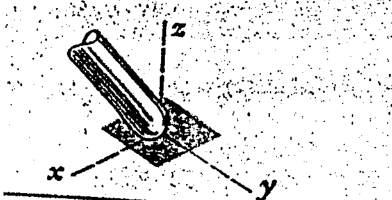
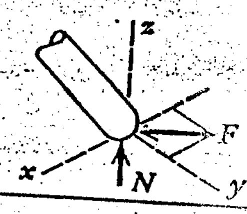
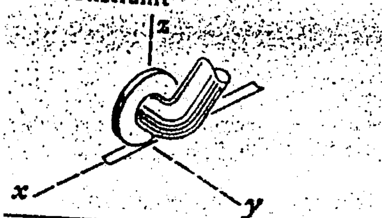
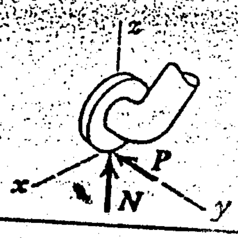
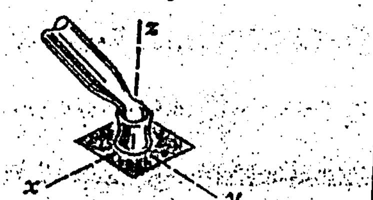
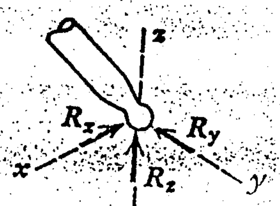
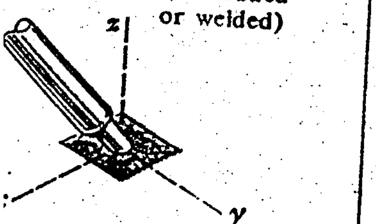
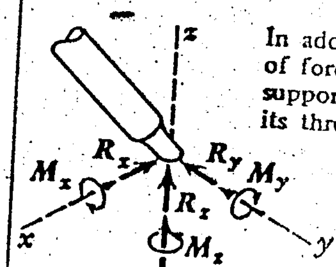
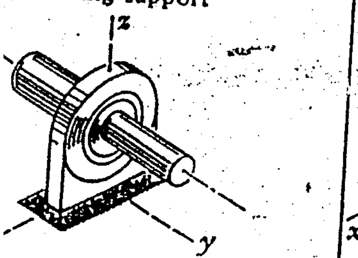
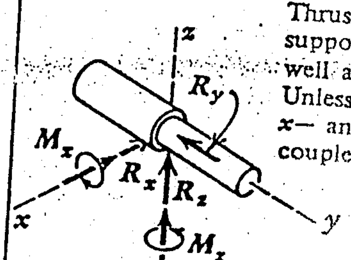
e of Contact and Force Origin	Action on Body to be Isolated
Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.
Member in contact with rough urface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
oller or wheel support with teral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
l–and–socket joint 	 A ball–and–socket joint free to pivot about the center of the ball can support a force R with all three components.
d connection (embedded or welded) 	 In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
t–bearing support 	 Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Unless bearing is pivoted about x– and z–axes, it can support couples M_x and M_z.

Figure 3/8

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS

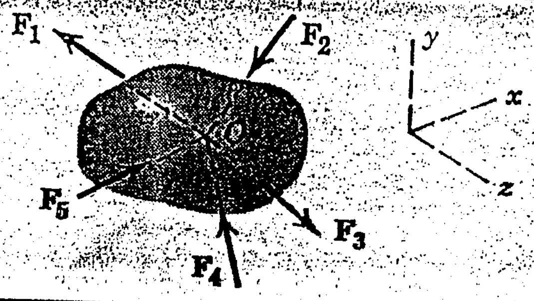
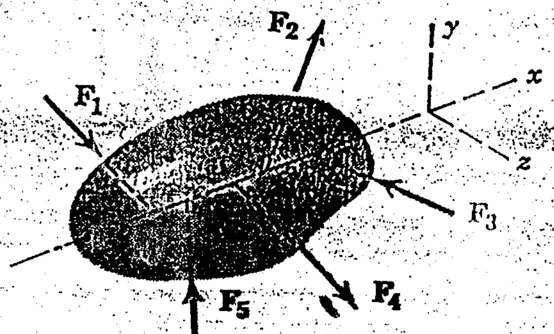
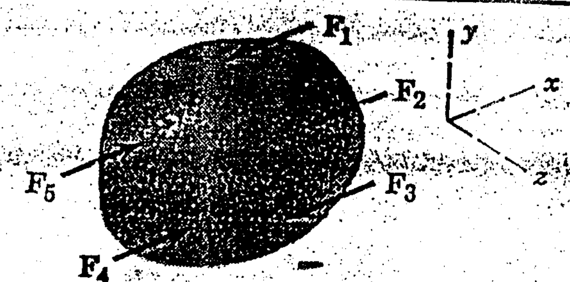
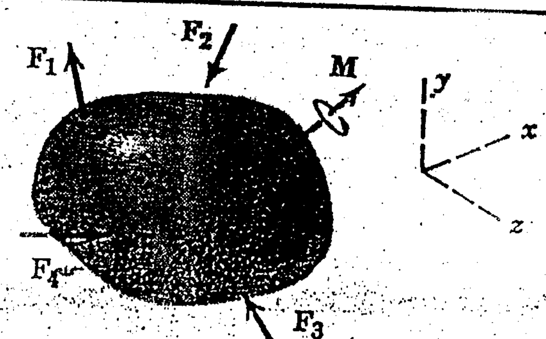
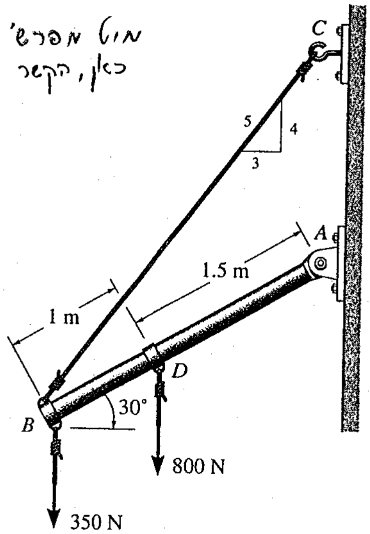
System	Free-Body Diagram	Independent Equations
urrent point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
urrent a line Concurrent w/ a line		All forces pass through one line ∥ to x axis $\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
el Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
l General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Figure 3/9

size of the collars at D and B and the thickness of the boom, determine the horizontal and vertical components of force at pin A and the force in cable CB.

boom – [illegible]
collar – [illegible]

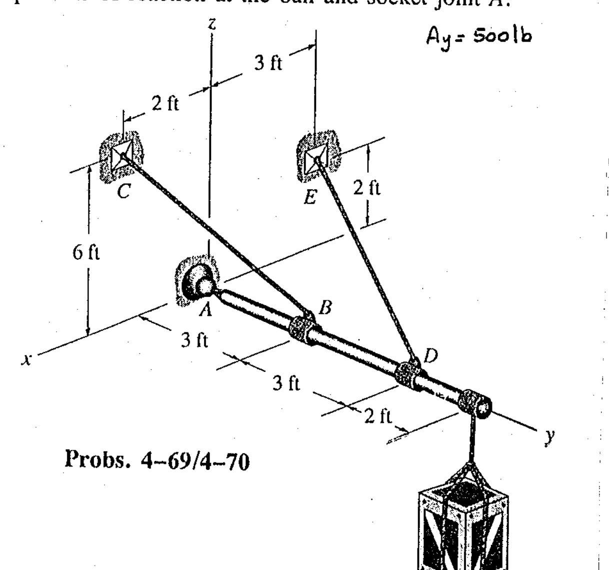


4–69. The boom supports a load having a weight of $W =$ 850 lb. Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in cables BC and DE.

$T_{DE} = 721$ lb $A_z = 1.21$ kip

4–70. Cable BC or DE can support a maximum tension of 700 lb before it breaks. Determine the greatest weight W that can be suspended from the end of the boom. Also, determine the x, y, z components of reaction at the ball-and-socket joint A.

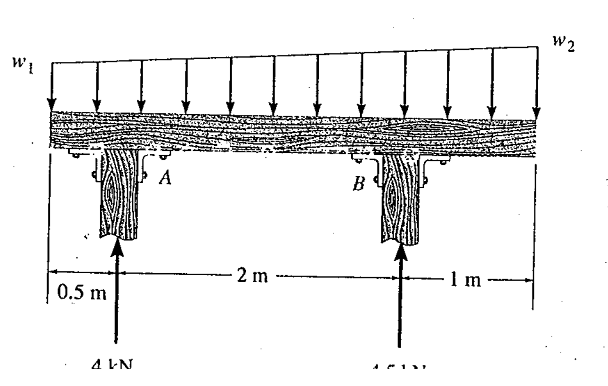
$A_y = 500$ lb



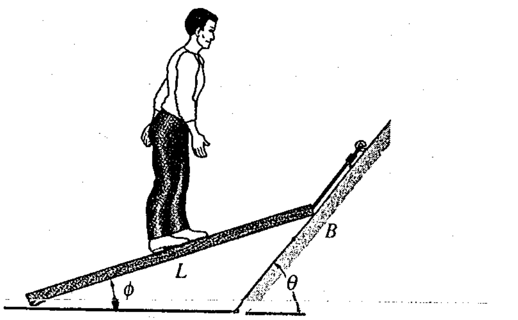
Probs. 4–69/4–70

5–50. Determine the intensity w_1 and w_2 of the trapezoidal loading if the supports at A and B exert forces of 4 kN and 4.5 kN, respectively, on the beam.

$w_2 = 1.63$ kN/m

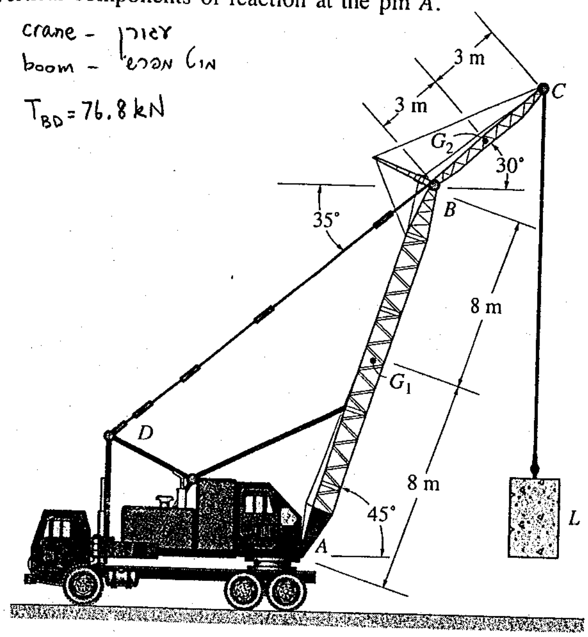


5–48. The man has a weight W and stands at the center of the plank. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.



4–33. The crane lifts a 400-kg load L. If the primary boom AB has a mass of 1.20 Mg and a center of mass at G_1, whereas the secondary boom BC has a mass of 0.6 Mg and a center of mass at G_2, determine the tension in the cable BD and the horizontal and vertical components of reaction at the pin A.

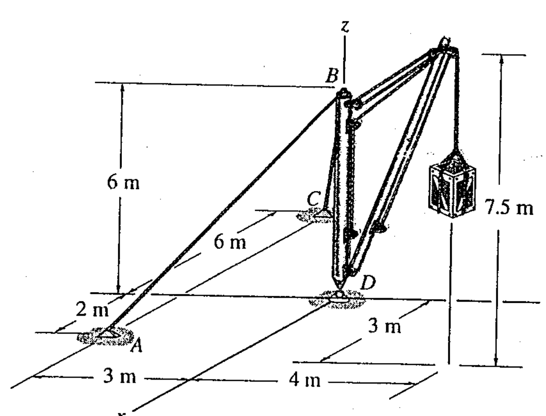
crane – עגורן
boom – [illegible]
$T_{BD} = 76.8$ kN



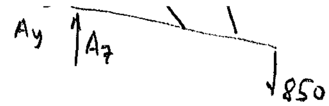
Prob. 4–33

***5–96.** The stiff-leg derrick used on ships is supported by a ball-and-socket joint at D and two cables BA and BC. The cables are attached to a smooth collar ring at B, which allows rotation of the derrick about the z axis. If the derrick supports a crate having a mass of 100 kg, determine the tension in the supporting cables and the x, y, z components of reaction at D.

stiff leg derrick – [illegible]



$\bar{F}_{DE} = F_{DE} \frac{\{\ldots\}}{\sqrt{(-3)^2+(-6)^2+2^2}} = F_{DE}\{-.429\underline{i} - .857\underline{j} + .286\underline{k}\}$



$\bar{F}_{BC} = F_{BC} \frac{\{2i-3j+6k\}}{\sqrt{2^2+3^2+6^2}} = F_{BC}\{.286i - .429\underline{j} + .857\underline{k}\}$

$\Sigma M_x = 0 \quad .286 F_{DE} 6 + .8571 F_{BC} \cdot 3 - 850 \cdot 8 = 0$

$\Sigma M_z = 0 \quad .429 F_{DE} \cdot 6 - .286 F_{BC} \cdot 3 = 0$

$F_{DE} = 721 \text{ lb} \qquad F_{BC} = 2164 \text{ lb}$

$\Sigma F_x = 0 \quad A_x + .286(2164) - .429(721) = 0$

$A_x = -309 \text{ lb}$

$\Sigma F_y = 0 \quad A_y - .857(721) - .429(2164) = 0$

$A_y = 1545 \text{ lb}$

$\Sigma F_z = 0 \quad A_z - 850 + .286(721) + .857(2164) = 0$

$A_z = -1211 \text{ lb}$

לפי מה שמצאנו למעלה

4-70

$\underline{F_{DE}} = F_{DE}\{-.429\underline{i} - .857\underline{j} + .286\underline{k}\} = (-\frac{3}{7}\underline{i} - \frac{6}{7}\underline{j} + \frac{2}{7}\underline{k})F_{DE}$

$\underline{F_{BC}} = F_{BC}\{.286\underline{i} - .429\underline{j} + .857\underline{k}\} = (\frac{2}{7}\underline{i} - \frac{3}{7}\underline{j} + \frac{6}{7}\underline{k})F_{BC}$

$\underline{W} = -W\underline{k}$

$\Sigma M_{x_A} = .286 F_{DE} \cdot 6 + .857 F_{BC} \cdot 3 - W \cdot 8 = 0$

$\Sigma M_{z_A} = .429 F_{DE} \cdot 6 - .286 F_{BC} \cdot 3 = 0 \implies F_{BC} = 3F_{DE} \implies$

לכן F_{BC} צריך להיות
לא ה 700 lb

$233.3 \text{ lb} = \frac{1}{3}F_{BC} = F_{DE}$

$275 \text{ lb} = W$

$\Sigma F_x = 0 \quad A_x + .286 F_{BC} - .429 F_{DE} = 0$

$A_x = -100 \text{ lb}$

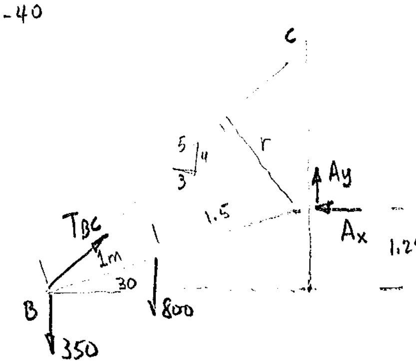
$\Sigma F_y = 0 \quad A_y - .857 F_{DE} - .429 F_{BC} = 0$

$A_y = 500 \text{ lb}$

$\Sigma F_z = 0 \quad A_z - W + .286 F_{DE} + .857 F_{BC} = 0$

$A_z = -392 \text{ lb}$

4-40



$\Sigma M_A = T_{BC} \cdot r - 350(2.5\cos 30) - 800(1.5\cos 30) = 0$

זווית $\measuredangle CBA = \tan^{-1}(4/3) - 30° = 23.13°$

$r = 2.5 \sin 23.13° = .9821$

$\Sigma M_A = 0 \implies T_{BC} = \frac{[350(2.5) + 800(1.5)]\cos 30°}{r} = 1830 \text{ N}$

$\Sigma F_x = 0 = -A_x + T_{BC}\cos 53.13° = -A_x + 1830 \cdot \frac{3}{5} = 0$

$A_x = 1098 \text{ N}$

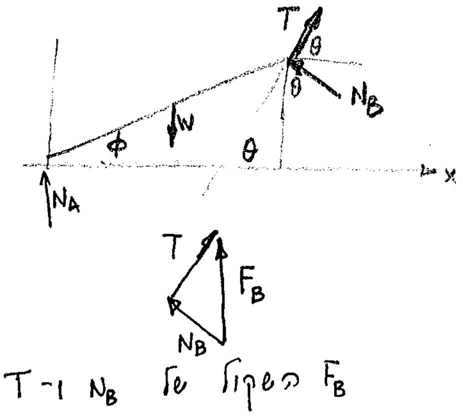
$\Sigma F_y = 0 = -350 - 800 + T_{BC}\sin 53.13° + A_y = 0$

$A_y = -314 \text{ N}$

$$\ldots N_A j - W j + F_B j = 0$$

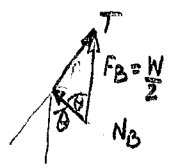
$$F_B = -N_A + W$$

F_B הוא השקול של הרכיבים N_B ו-T



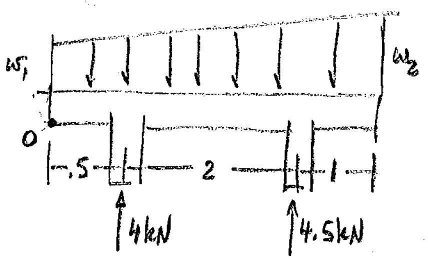
F_B השקול של N_B ו-T

$$\Sigma M_A \circlearrowleft + = 0 \qquad W \cdot \frac{L}{2}\cos\phi - F_B \cdot L\cos\phi = 0 \qquad \underline{F_B = \frac{W}{2}}$$



$$\underline{T = F_B \sin\theta = \frac{W}{2}\sin\theta}$$

5-50



$$w(x) = \frac{w_2 - w_1}{3.5}x + w_1$$

$$\Sigma F_y = 0 = 4 + 4.5 - \int_0^{3.5} w(x)dx = 8.5 - \left[\frac{w_2 - w_1}{3.5}\frac{x^2}{2} + w_1 x\right]_0^{3.5} = 0$$

$$8.5 - \left\{1.75(w_2 - w_1) + 3.5w_1\right\} = 0$$

$$8.5 - 1.75(w_2 + w_1) = 0 \Rightarrow w_2 + w_1 = \frac{8.5}{1.75} = 4.857$$

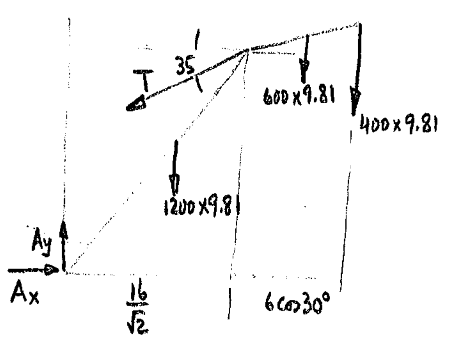
$$\Sigma M_o \circlearrowleft + = 4(0.5) + 4.5(2.5) - \int_0^{3.5} w(x)\cdot x\,dx = 0$$

$$= 13.25 - \left[\frac{w_2 - w_1}{3.5}\frac{x^3}{3} + w_1\frac{x^2}{2}\right]_0^{3.5} = 0$$

$$= 13.25 - \left[(w_2 - w_1)4.08\bar{3} + w_1(6.125)\right] = 13.25 - (4.083w_2 + 2.042w_1) = 0$$

$$w_1 = 3.224\frac{kN}{m} \qquad w_2 = 1.633\frac{kN}{m}$$ לפי המחשבון :

4-33



$$\Sigma M_A \circlearrowleft + = 0 \quad 1200\times9.81\times\frac{8}{\sqrt{2}} + 600\times9.81\left(\frac{16}{\sqrt{2}} + 3\cos30°\right) + 400\times9.81\left(\frac{16}{\sqrt{2}} + 6\cos30°\right)$$

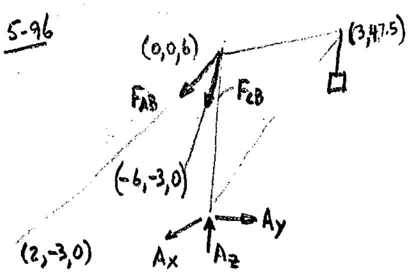
$$+ T\sin35°\cdot\frac{16}{\sqrt{2}} - T\cos35°\cdot\frac{16}{\sqrt{2}} = 0$$

$$\underline{T = 76{,}758\ N}$$

$$\Sigma F_y = 0 \qquad A_y - T\sin35° - 2200\times9.81 \qquad A_y = T\sin35° + 2200\times9.81 = 65609\ N$$

$$\Sigma F_x = 0 \qquad A_x - T\cos35° = 0 \qquad A_x = T\cos35° = \underline{62877\ N}$$

5-96



$$\Sigma \underline{F} = \underline{0} = \underline{F}_{AB} + \underline{F}_{CB} - W\underline{k} + (A_x\underline{i} + A_y\underline{j} + A_z\underline{k}) = \underline{0}$$

$$\underline{F}_{AB} = F_{AB}(2\underline{i} - 3\underline{j} - 6\underline{k})/7 \qquad \underline{F}_{BB} = F_{CB}(-6\underline{i} - 3\underline{j} - 6\underline{k})/9$$

$$\Sigma F_x = 0 : \frac{2}{7}F_{AB} - \frac{6}{9}F_{CB} + A_x = 0$$

$$\Sigma F_y = 0 : -\frac{3}{7}F_{AB} - \frac{3}{9}F_{CB} + A_y = 0$$

$$\Sigma F_z = 0 : -6/7 F_{AB} - 6/9 F_{CB} + A_z - W = 0$$

$$\Sigma \underline{M}_A = \underline{0} : 6\underline{k}\times F_{AB}(2\underline{i} - 3\underline{j} - 6\underline{k})/7 + 6\underline{k}\times F_{CB}(-6\underline{i} - 3\underline{j} - 6\underline{k})/9 + (3\underline{i} + 4\underline{j} + 7.5\underline{k})\times(-981\underline{k}) = 0$$

$$\frac{12}{7}F_{AB}\underline{j} + \frac{18}{7}F_{AB}\underline{i} - \frac{36}{9}F_{BC}\underline{j} + \frac{18}{9}F_{BC}\underline{i} + 2943\underline{j} - 3924\underline{i} = 0$$

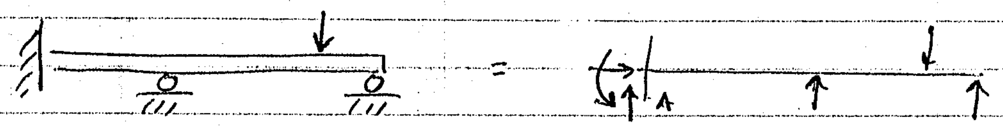
$$\left.\begin{array}{l}\frac{12}{7}F_{AB} - 4F_{BC} + 2943 = 0 \\ \frac{18}{7}F_{AB} + 2F_{BC} - 3924 = 0\end{array}\right\} \longrightarrow \begin{array}{l}F_{AB} = 715.3\ N \\ F_{BC} = 1042.3\ N\end{array}$$

$A_x = 490.5\ N$ מעולה, נקבל

$A_y = +654\ N$

- If unknowns are determined from eqs of equilib - statically determinate
- if more unknowns than equations indeterminate
  - must use other means

Degree of indeterminacy = # of unknowns − # of available equations



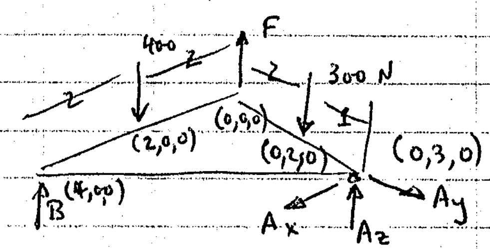
5 unknowns 3 eqs $\Sigma F_x, \Sigma F_y, \Sigma M_A$

indeterminate of degree 2

Read sections 5.6 on indeterminate

DO 5-53, 5-45

triangle is supported by 2 balls @ B + C and a ball/socket at A



$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y = 0$

$\Sigma F_z = B - 400 + F - 300 + A_z = 0$

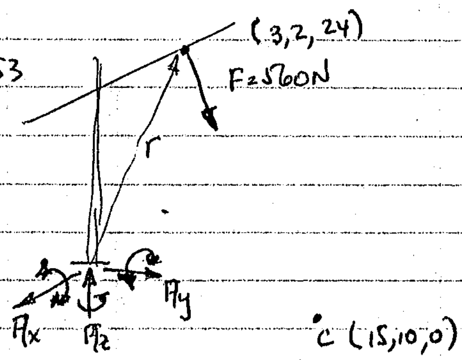
$\Sigma \vec{M}_A = \vec{0} = \Sigma M_{Ax}\bar{i} + \Sigma M_{Ay}\bar{j} + \Sigma M_{Az}\bar{j}$

$(300 \cdot 1\,\bar{i}) - 3F\bar{i} + 400(3)\bar{i} + (400 \cdot 2\,\bar{j}) - B(3)\bar{i} - B(4)\bar{j}$

$\Sigma M_{Ax} = 300 - 3F + 1200 - 3B = 0$ $F = 300\ N$

$\Sigma M_{Ay} = 800 - 4B = 0$ $B = 200\ N$ $A_z = 200\ N.$

5-53



$\vec{r} = (15-3,\ 10-2,\ 0-24)$

$= (12, 8, -24) = 4(3, 2, -6)$

$r = 4 \cdot 7 = 28m.$

$\bar{u} = .429\bar{i} + .286\bar{j} - .857\bar{k}$

$\bar{F} = F\bar{u} = (240\bar{i} + 160\bar{j} - 480\bar{k})N$

$\Sigma \vec{F} = 0$ $\Sigma F_x = A_x + 240 = 0$ $A_x = -240\ N$

$\Sigma F_y = A_y + 160 = 0$ $A_y = -160N$

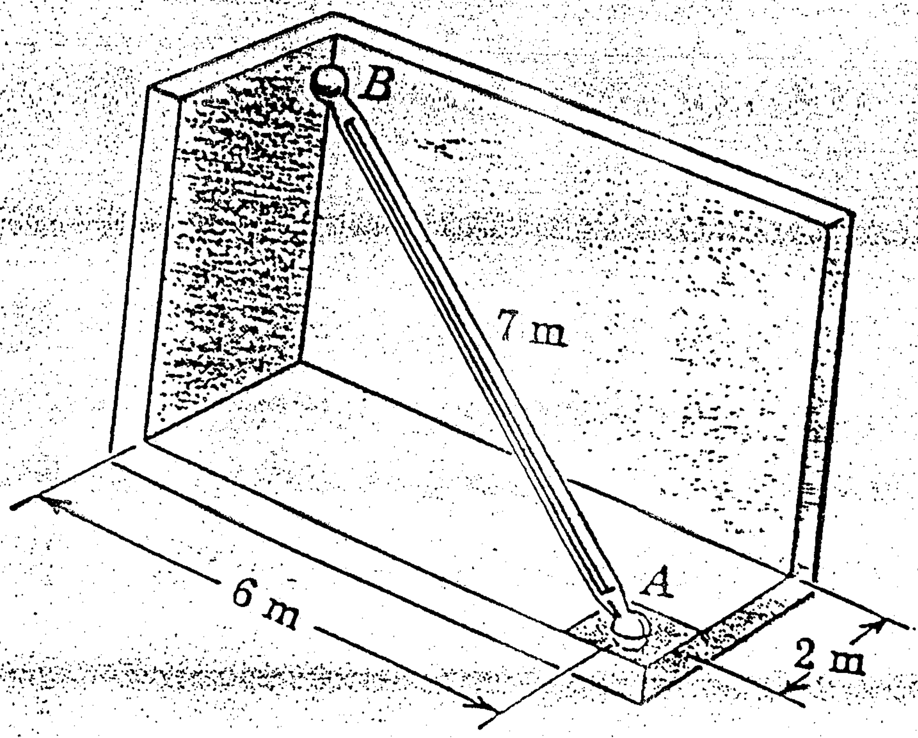
$\Sigma F_z = A_z - 480 = 0$ $A_z = 480\ N$

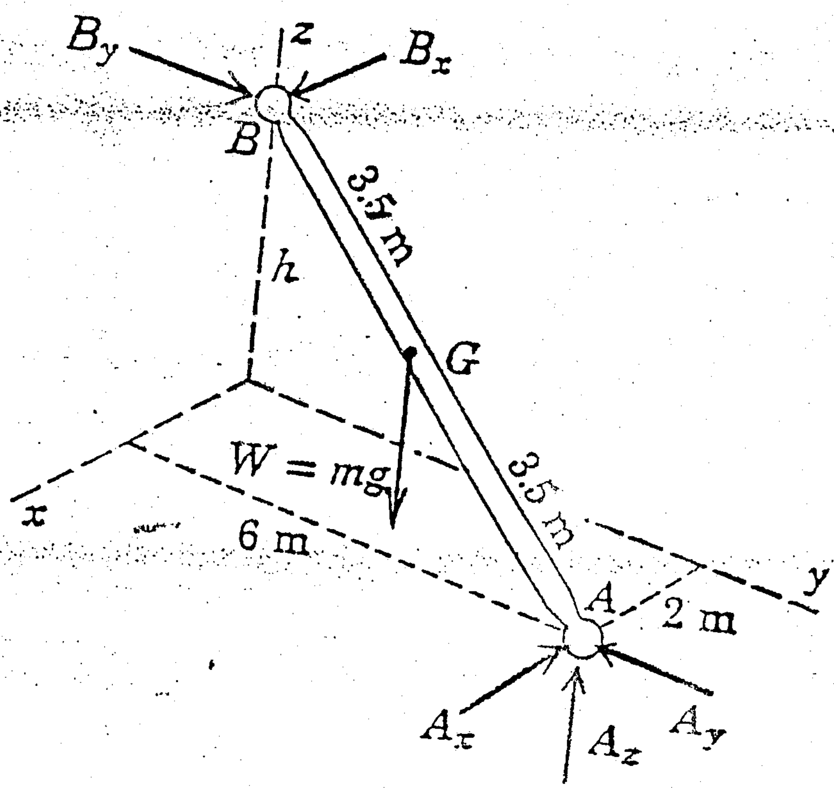
$$\bar{M}_A = \vec{r} \times \bar{F} = \begin{vmatrix} i & j & k \\ 3 & 2 & 24 \\ 240 & 160 & 480 \end{vmatrix} = \bar{i}\,(2 \cdot 480 - 160 \cdot 24) - \bar{j}\,(3 \cdot 480 - 240 \cdot 24) + k\,(3 \cdot 160 - 2 \cdot 240)$$

$= i(-18 \cdot 160) - j(-9 \cdot 480) + k(0)$

$M_{Ax}\underline{i} + M_{Ay}\underline{j} + M_{Az}\underline{k} + \underline{M}_A = \underline{0} \Rightarrow (M_{Ax} - 18 \cdot 160)\underline{i} + (M_{Ay} + 9 \cdot 480)\underline{j} + (M_{Az} + 0)\underline{k} = \underline{0}$

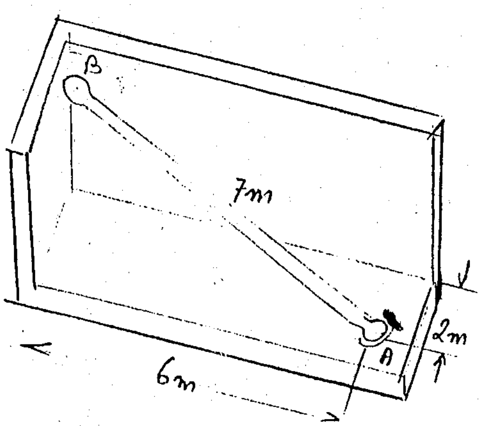
3.7 f,e





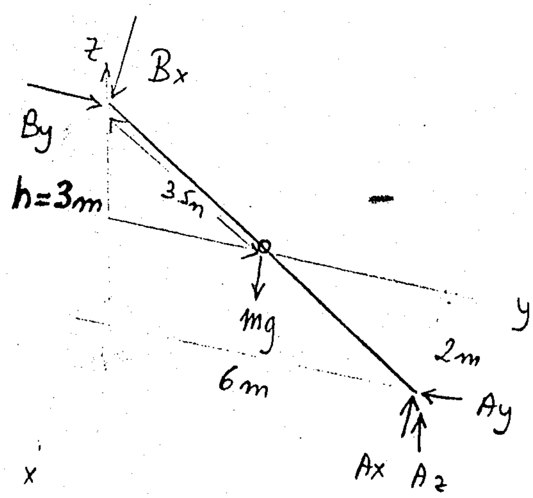
3.4.3 מסוימות סטטית

יש מסוימות של המשקל על הנותן הכרחיות ומספיקות לקבוע את ש. המשקל ואין הן בהכרח מספיקות להגדרתם של כל הכוחות הנעלמים. קיומם של כוחות מרובים גורם לאי מסוימות של המבנה ונשנה מצבים בהם הסמכים אינם מתאימים במקרה הקו - מומנט, ובכן נוסף פרק. (עמ' 106-107)



דוגמא 3/3

נתון המוט שבשרטוט הנשען על קיר חלק ועותק בגומה חלקה בתחתיתו. חשב את תגובות הקיר על המוט. אורך המוט 7m ומסתו 200 ק"ג (ראה שקף 3.7)



א. פתרון

נחשב הכל בשיטה וקטורית. נשתמש ב- A כמרכז מומנטים. לצורך זה יש לחשב רדיוסי וקטורים

$$r_{G/A} = -1i - 3j + 1.5k \ [m]$$

$$r_{B/A} = -2i - 6j + 3k \ [m]$$

$$\sum \vec{M}_A = r_{B/A} \times (iB_x + jB_y) + r_{G/A} \times (-mg\vec{k}) = 0$$

$$mg = 200 \times 9.81 = 1962 \ N$$

$$(-i - 3j + 1.5k) \times (iB_x + jB_y) + (-2i - 6j + 3k) \times (-1962k) = 0$$

$$(-3B_y + 5886)\, i + (3B_x - 1962)\, j - (2B_y + 6B_x)\, k = 0$$

לכן כל רכיב צ"ל - 0

$B_y = 1962\ N$ $\quad B_x = 654\ N$ $\quad B = 2068$

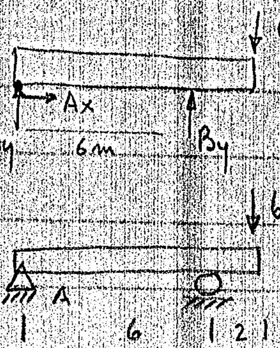
נחשב את הכוחות ב-A משווי משקל בשלושת הכיוונים:

$$\sum \vec{F} = 0 \qquad (654 - A_x)i + (1962 - A_y)j + (-1962 + A_z)k = 0$$

$A_x = 654\ N$ $\quad A_y = 1962\ N$ $\quad A_z = 1962\ N$

$A = 2851\ N$

5-7



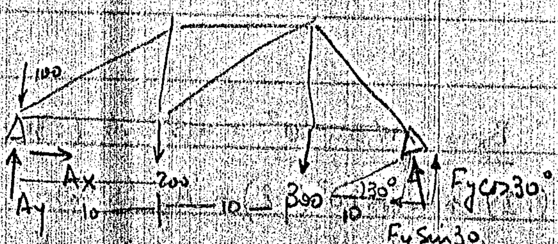
$\Sigma F_x = 0 = A_x = 0$

$\Sigma F_y = A_y + B_y - 6\text{ kN} = 0$

$\Sigma M_A = B_y \cdot 6 - 6 \cdot 8 = 0 \quad B_y = 8\text{ kN}$

$A_y = -2\text{ kN}$

5-22



$\Sigma F_x = A_x - F_y \sin 30^\circ = 0$

$\Sigma F_y = A_y - 100 - 200 - 300 + F_y \cos 30^\circ = 0$

$\Sigma M_A = 0$

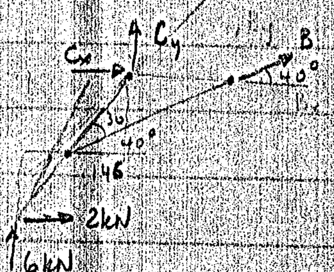
$-200 \cdot 10 - 300 \cdot 20 + F_y \cos 30^\circ \cdot 30 = 0$

$F_y = \dfrac{8000}{30 \cos 30^\circ} = 30{,}792\text{ lb}$

$A_x = 15\,396\text{ lb} = F_y \sin 30^\circ$

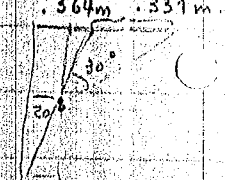
$A_y = 600 - F_y \cos 30^\circ = 333.33\text{ lb}$

5-26



$\Sigma M_B = C_y(.331\text{ m})$

$- 2\text{kN}(1\text{m}) + 6\text{kN}(.695\text{m}) = 0$



$C_y = -6.56\text{ kN}$

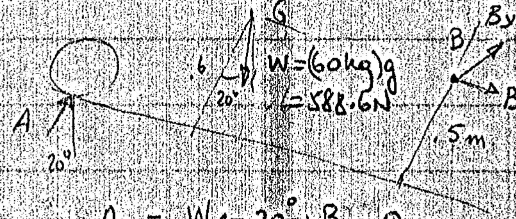
$\Sigma M_C = B \sin 40^\circ(.331) + 2\text{kN}(1\text{m}) - 6\text{kN}(.364\text{m}) = 0$

$B = .865\text{ kN}$

$\Sigma M_A = C_y(.146) - C_x(.4) - 6\text{kN}(.218) + 2(.6) = 0$

$C_x = -2.664\text{ kN}$

5-28



$A_y - W\cos 20° + B_y = 0$

$W\sin 20° + B_x = 0$ $\quad B_x = -W\sin 20° = -588.6(.342) = -201.31\,N$

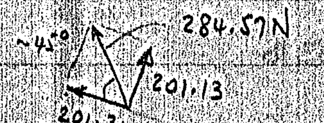
$\Sigma M_B = -W\sin 20°(.1) + W\cos 20°(.8) - A_y(1.2m) = 0$

$A_y = \dfrac{W\sin 20°(.1) - W\cos 20°(.8)}{1.2m}\,Nm = \dfrac{20.13N - (553)(.8)}{1.2m}$ (442.48)

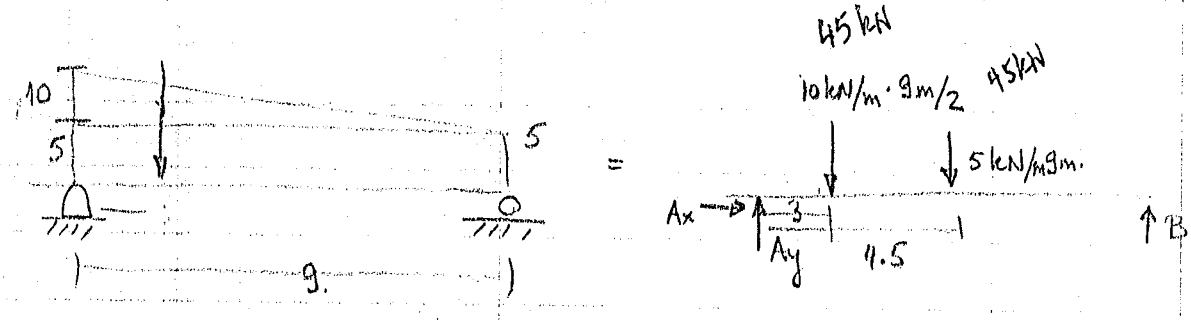
$B_y = W\cos 20° - A_y$ $\quad = \dfrac{422.35}{1.2m}\,Nm = 351.96N$

$B_y = (553.1 - 351.96) = 201.13\,N.$

thus



4-88



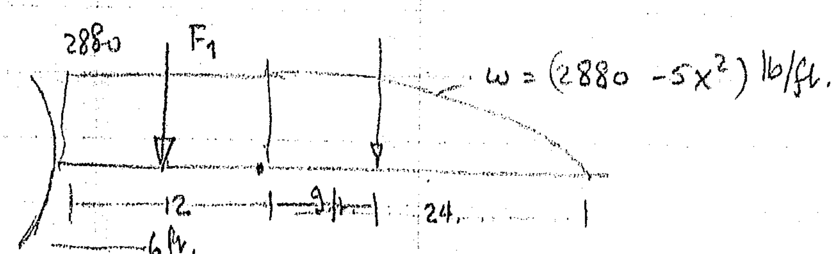
$\Sigma F_x = A_x = 0$

$\Sigma F_y = A_y + B - 45\text{ kN} - 45(\ ) = 0$ kN

$+\circlearrowleft \Sigma M_0 = -45 \cdot 3 - 45(4.5) + B(9) = 0$ $\qquad B = \frac{5}{9}\cdot(7.5) = 37.5$ kN

$A_y = 90 - B = 90 - 37.5 = 52.5$ kN.

4-92



$F_1 = 2880\text{ lb/ft} \cdot 12\text{ ft} = 34560$ lb. acting 6 ft from A or B.

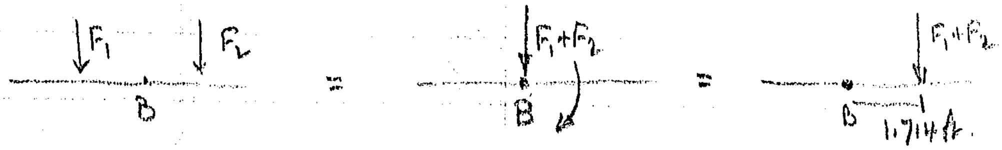
$$F_2 = \int_0^{24} w\,dx = \int_0^{24} (2880 - 5x^2)\,dx = \left[2880x - \frac{5x^3}{3}\right]_0^{24} = 46080 \text{ lb.}$$

$$F_2\,\bar{x} = \int w\,x\,dx = \int (2880x - 5x^3)\,dx = \left[1440x^2 - \frac{5}{4}x^4\right]_0^{24} = 414720 \text{ lb}$$

$\bar{x} = 9$ ft.

$\Sigma F_y = F_1 + F_2 = 34560 + 46080 = 80640$ lb.

$\Sigma M_B = F_1 \cdot 6\text{ ft} + F_2 \cdot 9\text{ ft} = 138240$ lb ft.



$d = \frac{\Sigma M}{\Sigma F_y} =$