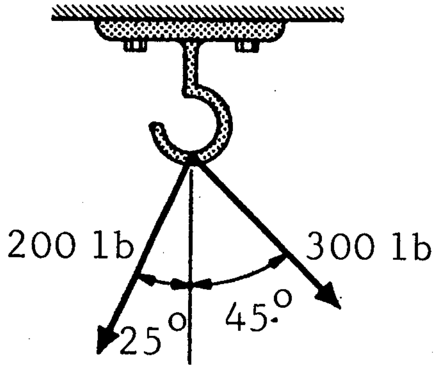
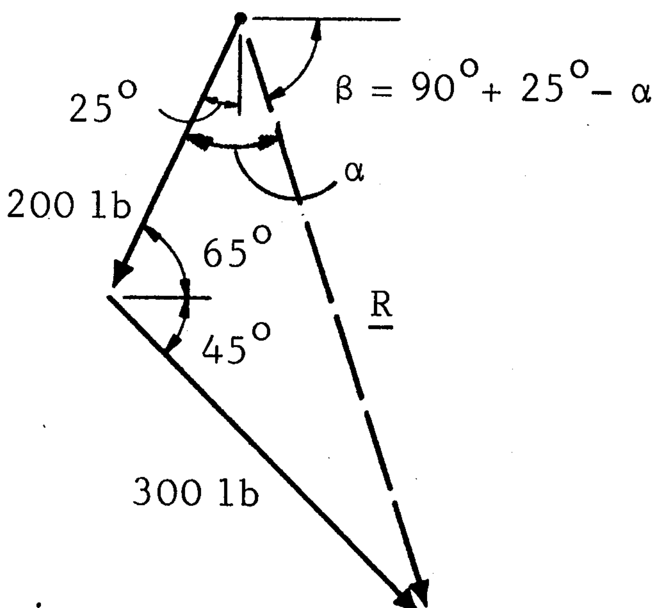


Determine by trigonometry the magnitude and direction of the resultant of the two forces shown.

Triangle Rule. We draw the forces in tip-to-tail fashion.



Law of Cosines

$$R^2 = (200\ \text{lb})^2 + (300\ \text{lb})^2 - 2(200\ \text{lb})(300\ \text{lb})\cos(45^\circ + 65^\circ)$$

$R = 413.6$ lb ◁

Law of Sines

$$\frac{\sin\alpha}{300\ \text{lb}} = \frac{\sin(45^\circ + 65^\circ)}{413.6\ \text{lb}}$$

$\alpha = 42.97^\circ$ ◁

$$\beta = 90^\circ + 25^\circ - 42.97^\circ = 72.03^\circ$$

$\mathbf{R} = 414$ lb ⦪ 72.0° ◀

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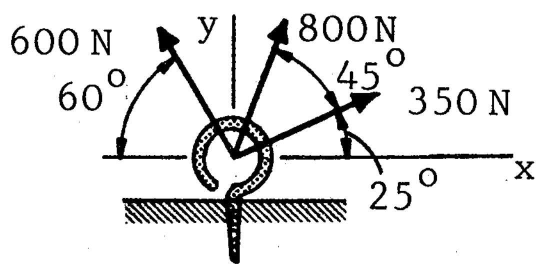


Determine the resultant of the three forces shown.

Rectangular Components

350–N force: $F_x = +(350\,N)\cos 25^\circ = +317\,N$

$F_y = +(350\,N)\sin 25^\circ = +147.9\,N$

800–N force: $F_x = +(800\,N)\cos 70^\circ = +274\,N$

$F_y = +(800\,N)\sin 70^\circ = +752\,N$

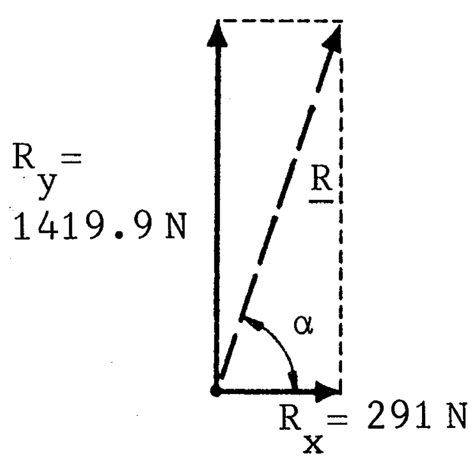
600–N force: $F_x = -(600\,N)\cos 60^\circ = -300\,N$

$F_y = +(600\,N)\sin 60^\circ = +520\,N$

Resultant: $\underline{R} = R_x\underline{i} + R_y\underline{j}$

$R_x = \Sigma F_x = +317\,N + 274\,N - 300\,N$ $\qquad R_x = +291\,N$

$R_y = \Sigma F_y = +147.9\,N + 752\,N + 520\,N$ $\qquad R_y = +1419.9\,N$



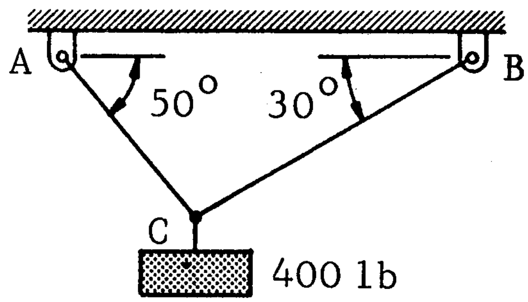
$$\tan\alpha = \frac{R_y}{R_x} = \frac{1419.9\,N}{291\,N}$$

$\alpha = 78.42^\circ$ ◁

$$R = \frac{R_y}{\sin\alpha} = \frac{1419.9\,N}{\sin 78.42^\circ}$$

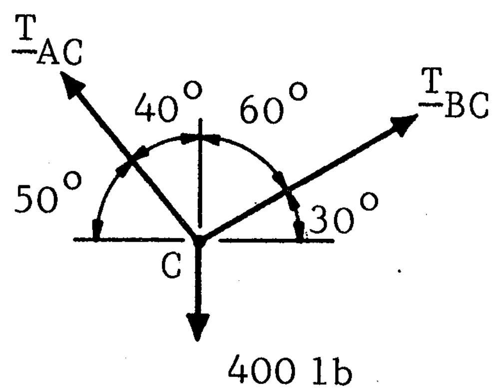
$R = 1449.4\,N$ ◁

$\underline{R} = 1449\,N \measuredangle 78.4^\circ$ ◀

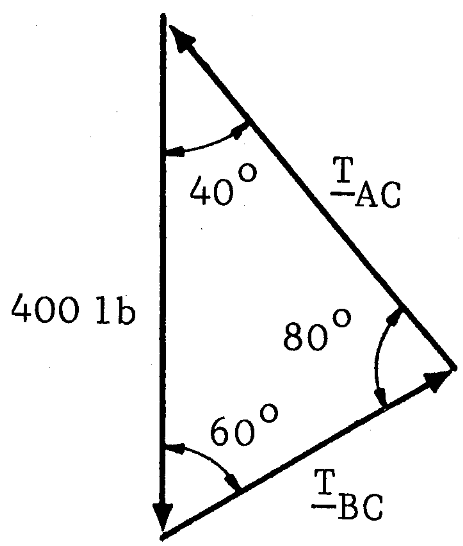


Determine the tension in cable AC and in cable BC.

Free Body. Point C is chosen as a free body.



Free-Body Diagram



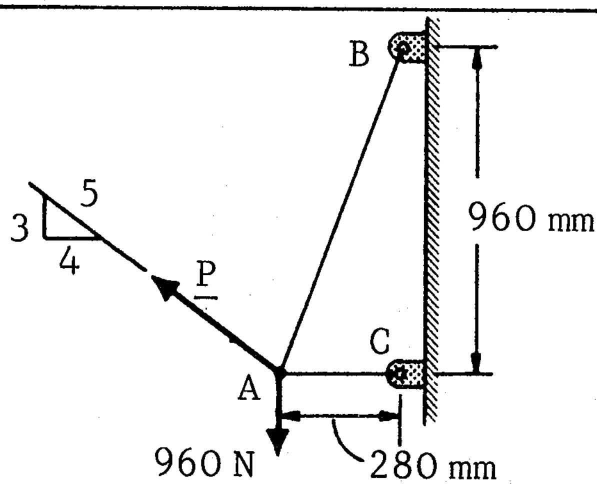
Force Triangle

From the force triangle we write:

Law of Sines
$$\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{400\text{ lb}}{\sin 80^\circ}$$

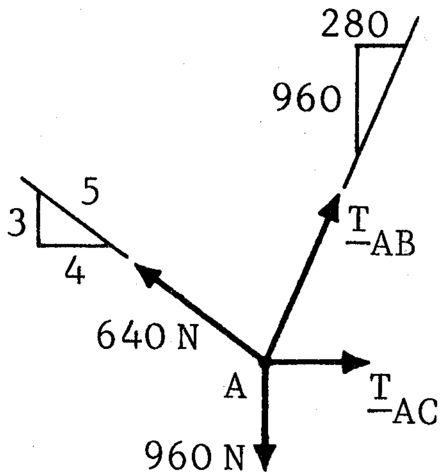
$T_{AC} = 352$ lb ◀

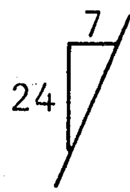
$T_{BC} = 261$ lb ◀



Knowing that $P = 640$ N, determine the tension is cable AB and in cable AC.

Free Body. Point A is chosen as a free body.





$$\sqrt{7^2 + 24^2} = 25$$

Free-Body Diagram

$+\uparrow \Sigma F_y = 0$: $\quad \frac{24}{25} T_{AB} + \frac{3}{5}(640\text{ N}) - 960\text{ N} = 0$

$T_{AB} = +600$ N $\qquad T_{AB} = 600$ N ◀

$\xrightarrow{+} \Sigma F_x = 0$: $\quad \frac{7}{25} T_{AB} - \frac{4}{5}(640\text{ N}) + T_{AC} = 0$

$$\frac{7}{25}(600\text{ N}) - \frac{4}{5}(640\text{ N}) + T_{AC} = 0$$

$T_{AC} = +344$ N $\qquad T_{AC} = 344$ N ◀

A force acts at the origin in a direction defined by the angles $\theta_x = 75^\circ$ and $\theta_z = 130^\circ$. Knowing that the y component of the force is +300 lb, determine (a) the other components and the magnitude of the force, (b) the value of θ_y.

<u>Direction Angles</u>. We have $\theta_x = 75^\circ$ and $\theta_z = 130^\circ$.

Eq.(2.24): $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$$\cos^2 75^\circ + \cos^2\theta_y + \cos^2 130^\circ = 1$$

$$\cos\theta_y = \pm 0.721$$

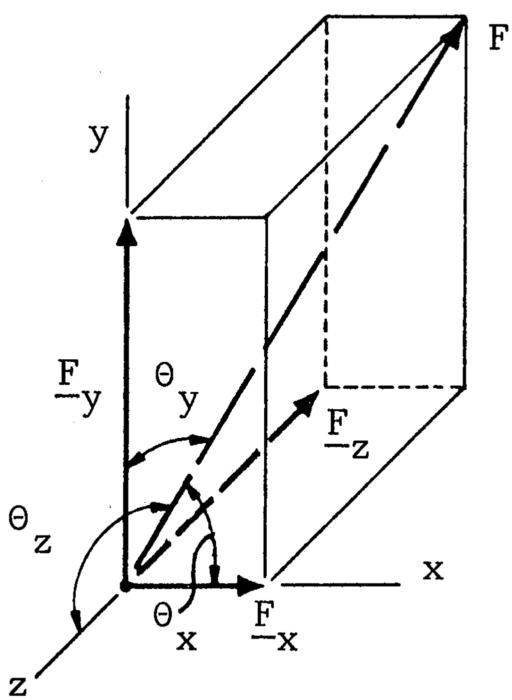
Since $F_y > 0$, we choose:

$$\cos\theta_y = +0.721 \qquad \theta_y = 43.9^\circ \blacktriangleleft$$

<u>Given</u>: $F_y = +300$ lb.

Eq.(2.19): $F_y = F\cos\theta_y$

$$300\ \text{lb} = F\cos 43.9^\circ \qquad F = 416\ \text{lb} \blacktriangleleft$$



For F = 416 lb, we have:

$F_x = F\cos\theta_x$

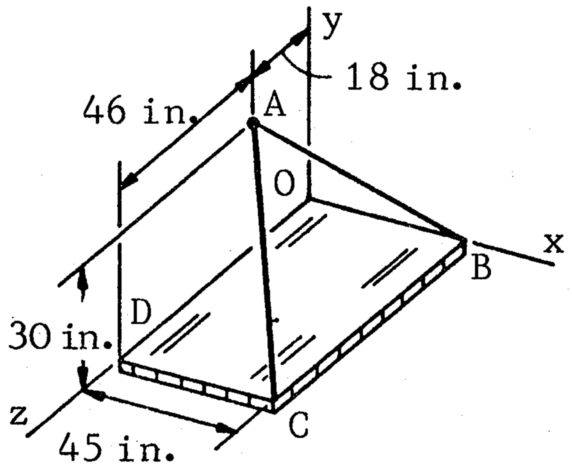
$= (416\ \text{lb})\cos 75^\circ$

$$F_x = +107.7\ \text{lb} \blacktriangleleft$$

$F_z = F\cos\theta_z$

$= (416\ \text{lb})\cos 130^\circ$

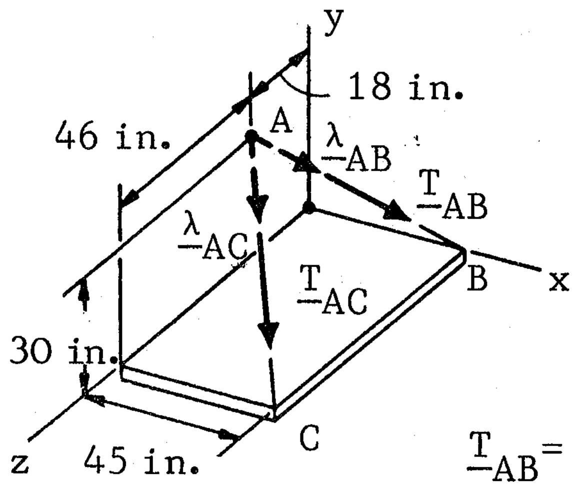
$$F_z = -267\ \text{lb} \blacktriangleleft$$



Given: $T_{AB} = 285\text{ lb}$
$T_{AC} = 426\text{ lb}$

Find: Resultant of forces exerted at A by the two cables.

Forces Exerted at A.



Cable AB.

$$\overrightarrow{AB} = (45\text{ in.})\underline{i} - (30\text{ in.})\underline{j} - (18\text{ in.})\underline{k}$$

$$AB = 57\text{ in.}$$

$$\underline{T}_{AB} = T_{AB}\underline{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= (285\text{ lb})\frac{45\underline{i} - 30\underline{j} - 18\underline{k}}{57}$$

$$\underline{T}_{AB} = (225\text{ lb})\underline{i} - (150\text{ lb})\underline{j} - (90\text{ lb})\underline{k} \quad \triangleleft$$

Cable AC. $\overrightarrow{AC} = (45\text{ in.})\underline{i} - (30\text{ in.})\underline{j} + (46\text{ in.})\underline{k}$

$$AC = 71\text{ in.}$$

$$\underline{T}_{AC} = T_{AC}\underline{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$

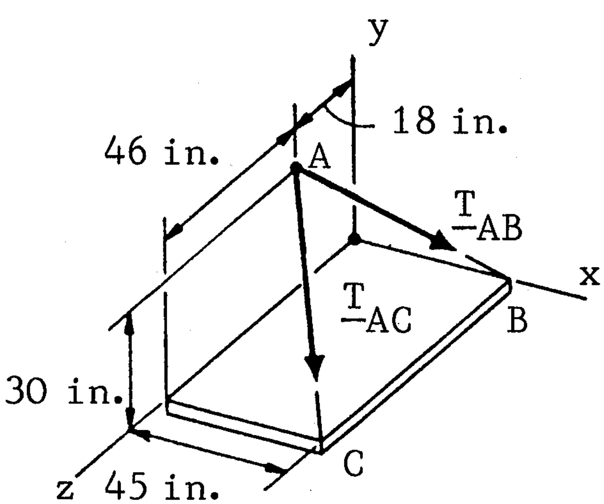
$$= (426\text{ lb})\frac{45\underline{i} - 30\underline{j} + 46\underline{k}}{71}$$

$$\underline{T}_{AC} = (270\text{ lb})\underline{i} - (180\text{ lb})\underline{j} + (276\text{ lb})\underline{k} \quad \triangleleft$$

(continued)

We have: $\underline{T}_{AB} = (225 \text{ lb})\underline{i} - (150 \text{ lb})\underline{j} - (90 \text{ lb})\underline{k}$

$\underline{T}_{AC} = (270 \text{ lb})\underline{i} - (180 \text{ lb})\underline{j} + (276 \text{ lb})\underline{k}$



Resultant

$\underline{R} = R_x\underline{i} + R_y\underline{j} + R_y\underline{k}$

$R_x = \Sigma F_x = 225 \text{ lb} + 270 \text{ lb}$

$R_x = +495 \text{ lb}$ ◁

$R_y = \Sigma F_y = -150 \text{ lb} - 180 \text{ lb}$

$R_y = -330 \text{ lb}$ ◁

$R_z = \Sigma F_z = -90 \text{ lb} + 276 \text{ lb}$

$R_z = +186 \text{ lb}$ ◁

Magnitude of Resultant

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(495)^2 + (-330)^2 + (186)^2}$$

$R = 623.3 \text{ lb}$

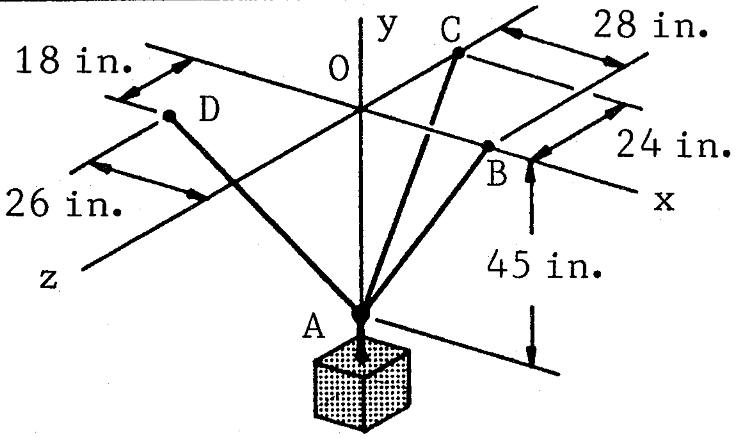
$R = 623 \text{ lb}$ ◀

Direction of Resultant

$$\cos\Theta_x = \frac{R_x}{R} = \frac{495}{623.3} = +0.7942 \qquad \Theta_x = 37.4^\circ$$ ◀

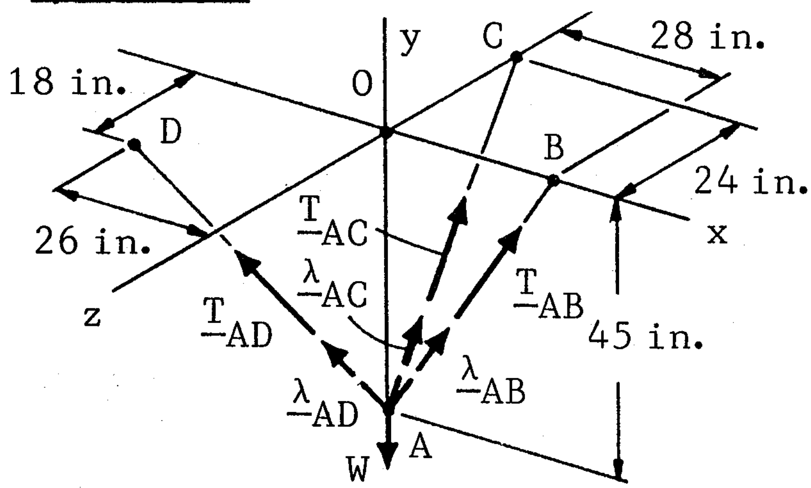
$$\cos\Theta_y = \frac{R_y}{R} = \frac{-330}{623.3} = -0.5294 \qquad \Theta_y = 122.0^\circ$$ ◀

$$\cos\Theta_z = \frac{R_z}{R} = \frac{186}{623.3} = +0.2984 \qquad \Theta_z = 72.6^\circ$$ ◀



Knowing that the tension in cable AD is 924 lb, determine the weight W of the crate.

Free Body. Point A is chosen as a free body.



$\mathbf{W} = -W\mathbf{j}$ ◁

Cable AB. $\overrightarrow{AB} = (28\text{ in.})\mathbf{i} + (45\text{ in.})\mathbf{j}$ $\qquad AB = 53\text{ in.}$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\frac{28\mathbf{i} + 45\mathbf{j}}{53}$$

$$\mathbf{T}_{AB} = \frac{28}{53}T_{AB}\mathbf{i} + \frac{45}{53}T_{AB}\mathbf{j} \quad ◁$$

Cable AC. $\overrightarrow{AC} = (45\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$ $\qquad AC = 51\text{ in.}$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{45\mathbf{j} - 24\mathbf{k}}{51}$$

$$\mathbf{T}_{AC} = \frac{45}{51}T_{AC}\mathbf{j} - \frac{24}{51}T_{AC}\mathbf{k} \quad ◁$$

(continued)

Cable AD. $\overrightarrow{AD} = -(26 \text{ in.})\underline{i} + (45 \text{ in.})\underline{j} + (18 \text{ in.})\underline{k}$

$AD = 55$ in.

$$\underline{T}_{AD} = T_{AD}\underline{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\frac{-26\underline{i} + 45\underline{j} + 18\underline{k}}{55}$$

$$\underline{T}_{AD} = -\frac{26}{55}T_{AD}\underline{i} + \frac{45}{55}T_{AD}\underline{j} + \frac{18}{55}T_{AD}\underline{k} \quad \triangleleft$$

Equilibrium Condition

$\Sigma\underline{F} = 0$: $\underline{T}_{AB} + \underline{T}_{AC} + \underline{T}_{AD} + \underline{W} = 0$

Substitute for $\underline{T}_{AB}$, $\underline{T}_{AC}$, $\underline{T}_{AD}$, and $\underline{W}$ and set the coefficients of $\underline{i}$, $\underline{j}$, $\underline{k}$ equal to zero.

$$\underline{i}: \quad \frac{28}{53}T_{AB} - \frac{26}{55}T_{AD} = 0 \qquad (1)$$

$$\underline{j}: \quad \frac{45}{53}T_{AB} + \frac{45}{51}T_{AC} + \frac{45}{55}T_{AD} - W = 0 \qquad (2)$$

$$\underline{k}: \quad -\frac{24}{51}T_{AC} + \frac{18}{55}T_{AD} = 0 \qquad (3)$$

Given: $T_{AD} = 924$ lb.

Substitute $T_{AD} = 924$ lb,

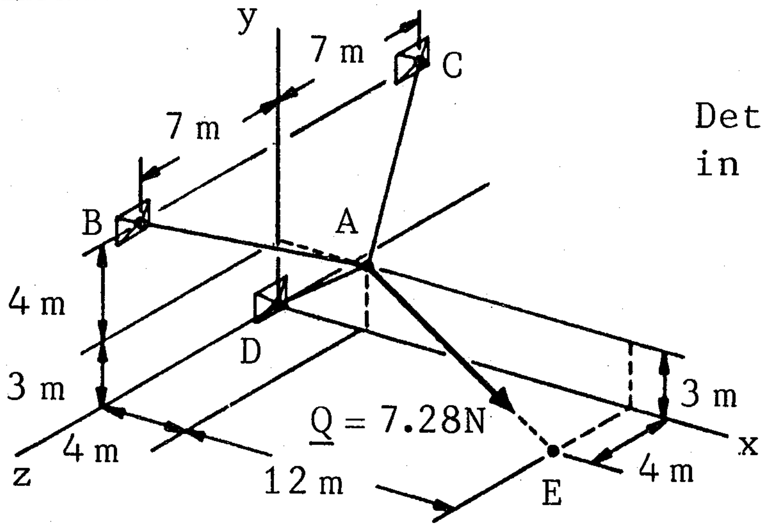
into Eq.(1) and solve for: $T_{AB} = 826.8$ lb

into Eq.(3) and solve for: $T_{AC} = 642.6$ lb

Substitute for T_{AB}, T_{AC}, and T_{AD} into Eq.(2):

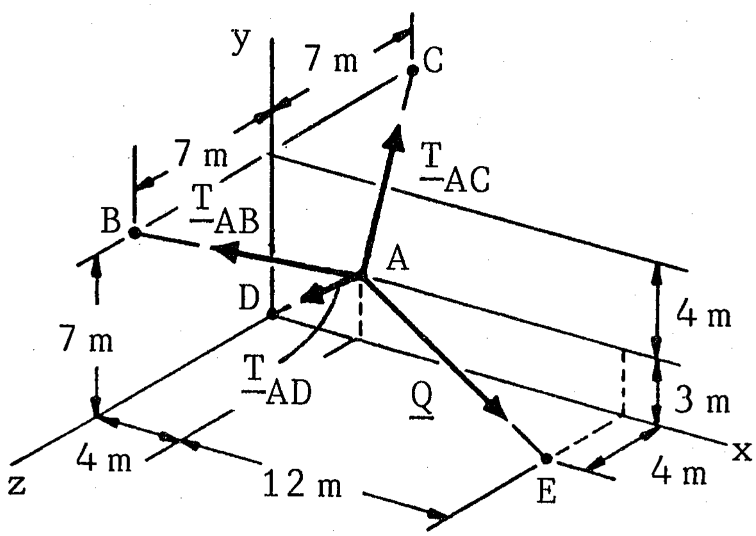
$$\frac{45}{53}(826.8) + \frac{45}{51}(642.6) + \frac{45}{55}(924) - W = 0$$

$$702 + 567 + 756 - W = 0 \qquad W = 2025 \text{ lb} \quad \blacktriangleleft$$



Determine the tension in each cable

Free Body. Point A is chosen as a free body.



Force Q. $\overrightarrow{AE} = (12\text{ m})\mathbf{i} - (3\text{ m})\mathbf{j} + (4\text{ m})\mathbf{k}$ $\qquad AE = 13\text{ m}$

$$\mathbf{Q} = Q\boldsymbol{\lambda}_{AE} = Q\frac{\overrightarrow{AE}}{AE} = (7.28\text{ N})\frac{12\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}}{13}$$

$$\mathbf{Q} = (6.72\text{N})\mathbf{i} - (1.68\text{N})\mathbf{j} + (2.24\text{N})\mathbf{k} \quad \triangleleft$$

Cable AD. $\overrightarrow{AD} = -(4\text{ m})\mathbf{i} - (3\text{ m})\mathbf{j}$ $\qquad AD = 5\text{ m}$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\frac{-4\mathbf{i} - 3\mathbf{j}}{5}$$

$$\mathbf{T}_{AD} = -\frac{4}{5}T_{AD}\mathbf{i} - \frac{3}{5}T_{AD}\mathbf{j} \quad \triangleleft$$

(continued)

Cable AB. $\overrightarrow{AB} = -(4\text{ m})\underline{i} + (4\text{ m})\underline{j} + (7\text{ m})\underline{k}$ $AB = 9\text{ m}$

$$\underline{T}_{AB} = T_{AB}\underline{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\frac{-4\underline{i} + 4\underline{j} + 7\underline{k}}{9}$$

$$\underline{T}_{AB} = -\frac{4}{9}T_{AB}\underline{i} + \frac{4}{9}T_{AB}\underline{j} + \frac{7}{9}T_{AB}\underline{k} \quad \triangleleft$$

Cable AC. $\overrightarrow{AC} = -(4\text{ m})\underline{i} + (4\text{ m})\underline{j} - (7\text{ m})\underline{k}$ $AC = 9\text{ m}$

$$\underline{T}_{AC} = T_{AC}\underline{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{-4\underline{i} + 4\underline{j} - 7\underline{k}}{9}$$

$$\underline{T}_{AC} = -\frac{4}{9}T_{AC}\underline{i} + \frac{4}{9}T_{AC}\underline{j} - \frac{7}{9}T_{AC}\underline{k} \quad \triangleleft$$

Equilibrium Condition

$\Sigma \underline{F} = 0$: $\underline{T}_{AB} + \underline{T}_{AC} + \underline{T}_{AD} + \underline{Q} = 0$

Substitute for $\underline{T}_{AB}$, $\underline{T}_{AC}$, $\underline{T}_{AD}$, and $\underline{Q}$ and set the coefficients of $\underline{i}$, $\underline{j}$, $\underline{k}$ equal to zero.

$$\underline{i}: \quad -\frac{4}{9}T_{AB} - \frac{4}{9}T_{AC} - \frac{4}{5}T_{AD} + 6.72\text{ N} = 0 \qquad (1)$$

$$\underline{j}: \quad +\frac{4}{9}T_{AB} + \frac{4}{9}T_{AC} - \frac{3}{5}T_{AD} - 1.68\text{ N} = 0 \qquad (2)$$

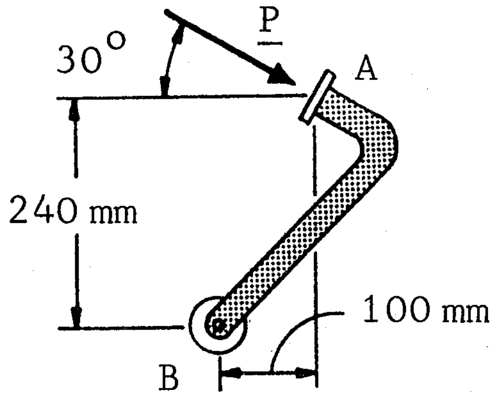
$$\underline{k}: \quad +\frac{7}{9}T_{AB} - \frac{7}{9}T_{AC} + 2.24\text{ N} = 0 \qquad (3)$$

Solving these equations, we obtain:

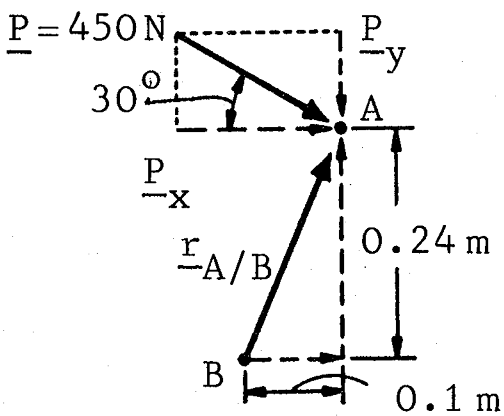
$T_{AB} = 2.88\text{ kN}$ ◀

$T_{AC} = 5.76\text{ kN}$ ◀

$T_{AD} = 3.60\text{ kN}$ ◀



Knowing that P = 450 N, determine the moment of **P** about B.



Solution

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{P}$$

where $\mathbf{r}_{A/B}$ is the position vector drawn from B to A:

$$\mathbf{r}_{A/B} = +(0.1\ \text{m})\,\mathbf{i} + (0.24\ \text{m})\,\mathbf{j}$$

Force **P**.

$$P_x = (450\ \text{N})\cos 30^\circ = 389.7\ \text{N}$$

$$P_y = -(450\ \text{N})\sin 30^\circ = -225\ \text{N}$$

$$\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} = (389.7\ \text{N})\,\mathbf{i} - (225\ \text{N})\,\mathbf{j}$$

Moment about B

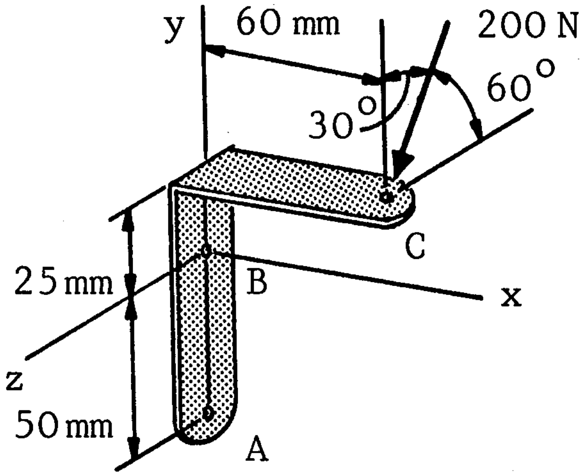
$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{P}:$$

$$= [(0.1\ \text{m})\,\mathbf{i} + (0.24\ \text{m})\,\mathbf{j}] \times [(389.7\ \text{N})\,\mathbf{i} - (225\ \text{N})\,\mathbf{j}]$$

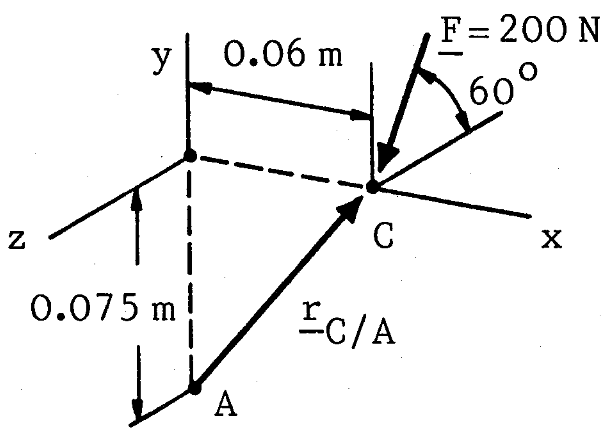
$$= -(22.5\ \text{N}\cdot\text{m})\,\mathbf{k} - (93.5\ \text{N}\cdot\text{m})\,\mathbf{k}$$

$$\mathbf{M}_B = -(116.0\ \text{N}\cdot\text{m})\,\mathbf{k}$$

$$\mathbf{M}_B = 116.0\ \text{N}\cdot\text{m} \circlearrowright$$ ◀



Determine the moment of the 200-N force about point A.



Force F

$F_x = 0$

$F_y = -(200\text{ N})\sin 60^\circ$

$= -172.3\text{ N}$

$F_z = (200)\cos 60^\circ$

$= +100\text{ N}$

$$\underline{F} = -(172.3\text{ N})\underline{j} + (100\text{N})\underline{k}$$ ◁

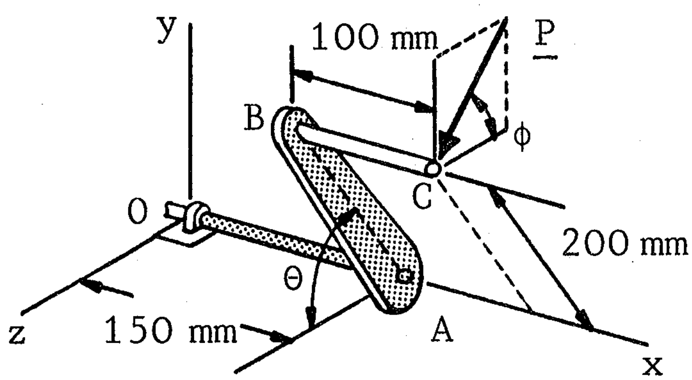
Vector Drawn from A to C

$$\underline{r}_{C/A} = (0.06\text{ m})\underline{i} + (0.075\text{ m})\underline{j}$$ ◁

Moment about A

$$\underline{M}_A = \underline{r}_{C/A} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.06\text{ m} & 0.075\text{ m} & 0 \\ 0 & -172.3\text{N} & 100\text{ N} \end{vmatrix}$$

$$\underline{M}_A = (7.5\text{ N}\cdot\text{m})\underline{i} - (6\text{ N}\cdot\text{m})\underline{j} - (10.39\text{ N}\cdot\text{m})\underline{k}$$ ◀



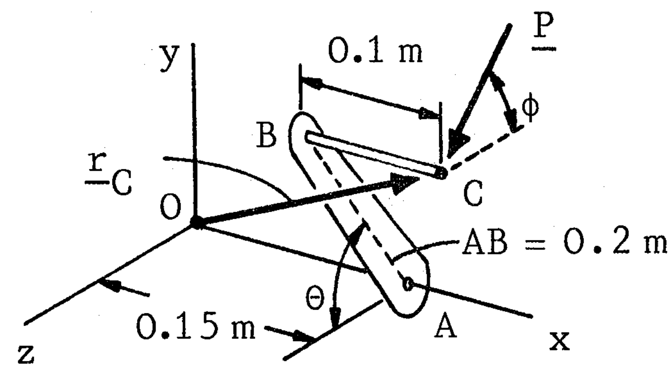
Given:

$M_x = +20\ N\cdot m$

$M_y = -8.75\ N\cdot m$

$M_z = -30\ N\cdot m$

Determine the magnitude of P and the values of φ and Θ.



Solution. Force P

$$\underline{P} = -P\sin\phi\,\underline{j} + P\cos\phi\,\underline{k}$$

Vector from O to C. AB = 0.2 m

$$\underline{r}_C = (0.25\,m)\underline{i} + (0.2\,m)\sin\Theta\,\underline{j} + (0.2\,m)\cos\Theta\,\underline{k}$$

Moment about O

$$\underline{M}_O = \underline{r}_C \times \underline{P} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.25\,m & (0.2\,m)\sin\Theta & (0.2\,m)\cos\Theta \\ 0 & -P\sin\phi & P\cos\phi \end{vmatrix}$$

$$= [(0.2\,m)P(\sin\Theta\cos\phi + \cos\Theta\sin\phi)]\underline{i}$$

$$-[(0.25\,m)P\cos\phi]\underline{j} - [(0.25\,m)P\sin\phi]\underline{k}$$

(continued)

We have $\underline{M}_O = M_x \underline{i} + M_y \underline{j} + M_z \underline{k}$ with:

$$M_x = (0.2\,m)P(\sin\theta\cos\phi + \cos\theta\sin\phi)$$
$$= (0.2\,m)P\sin(\theta+\phi) \qquad (1)$$

$$M_y = -(0.25\,m)P\cos\phi \qquad (2)$$

$$M_z = -(0.25\,m)P\sin\phi \qquad (3)$$

Divide Eq.(3) by Eq.(2): $\tan\phi = \dfrac{M_z}{M_y}$ (4)

Square members of Eqs.(2) and (3) and add:

$$M_y^2 + M_z^2 = (0.25\,m)^2 P^2$$
$$P = 4\ m^{-1}\sqrt{M_y^2 + M_z^2} \qquad (5)$$

Substitute Data: $M_x = +20\,N\cdot m$, $M_y = -8.75\,N\cdot m$, $M_z = -30\,N\cdot m$

Eq.(4): $\tan\phi = \dfrac{-30\,N\cdot m}{-8.75\,N\cdot m} = 3.429$ $\qquad \phi = 73.74^\circ$

$\phi = 73.7^\circ$ ◀

Eq.(5): $P = 4\sqrt{(8.75)^2 + (30)^2}$ $\qquad P = 125.0\,N$ ◀

Eq.(1): $M_x = (0.2\,m)P\sin(\theta+\phi)$

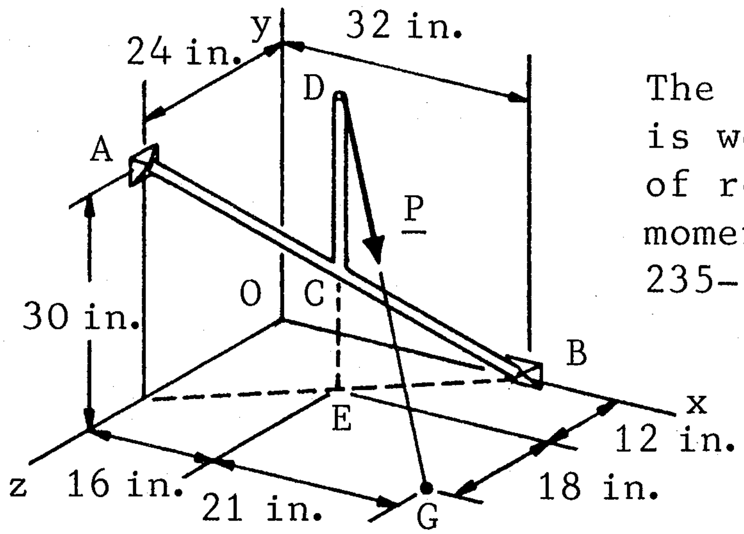
$$+20\,N\cdot m = (0.2\,m)(125\,N)\sin(\theta+\phi)$$

$$\sin(\theta+\phi) = 0.8 \qquad \theta+\phi = 53.13^\circ \text{ and } \theta+\phi = 126.87^\circ$$

Since $\phi = 73.74^\circ$, $\theta + 73.74^\circ = 53.13^\circ$ $\qquad \theta = -20.61^\circ$

or $\theta + 73.74^\circ = 126.87^\circ$ $\qquad \theta = 53.13^\circ$

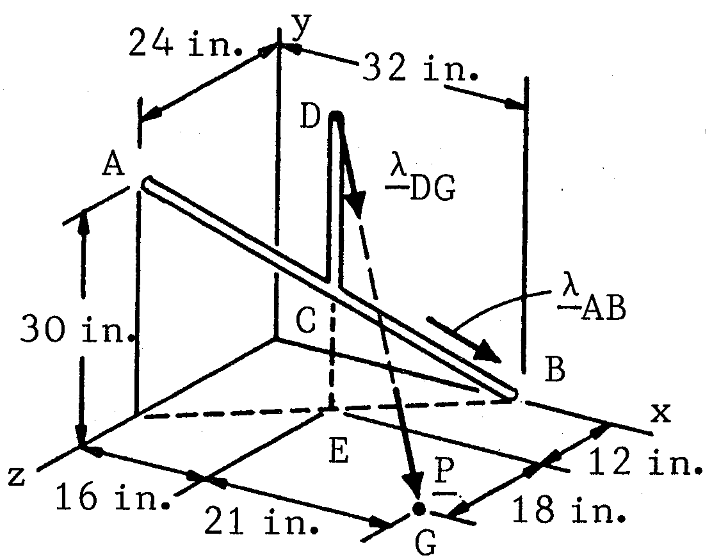
$\theta = 53.1^\circ$ ◀



The 23-in. vertical rod CD is welded to the midpoint C of rod AB. Determine the moment about AB of the 235-lb force $\underline{P}$.

<u>Moment about AB.</u> We shall apply the force $\underline{P}$ at G.

$$M_{AB} = \underline{\lambda}_{AB} \cdot \underline{M}_B = \underline{\lambda}_{AB} \cdot (\underline{r}_{G/B} \times \underline{P}) \qquad (1)$$



$$EC = \frac{1}{2}(30 \text{ in.}) = 15 \text{ in.}$$

$$CD = 23 \text{ in.}$$

$$(DG)_x = +(21 \text{ in.})\underline{i}$$

$$(DG)_y = -(EC + CD)\underline{j}$$

$$= -(15 \text{ in.} + 23 \text{ in.})\underline{j}$$

$$= -(38 \text{ in.})\underline{j}$$

$$(DG)_z = +(18 \text{ in.})\underline{k}$$

$$DG = 47 \text{ in.}$$

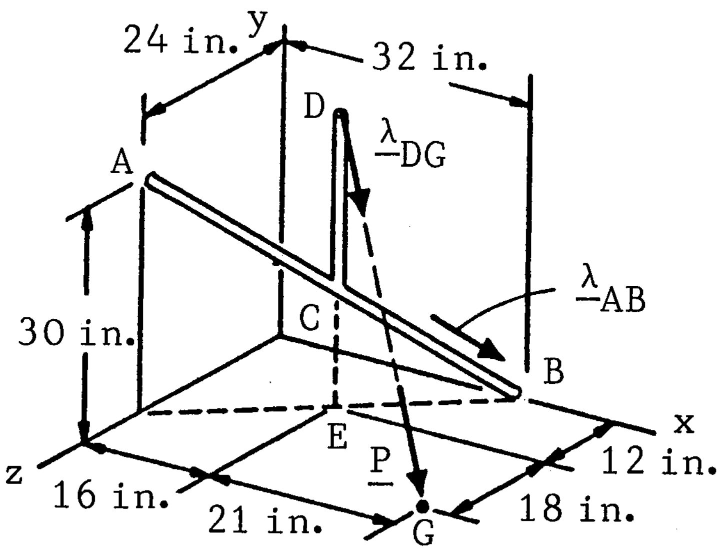
<u>Force</u> $\underline{P}$. $(P = 235 \text{ lb})$

$$\underline{P} = P\underline{\lambda}_{DG} = P\frac{\overrightarrow{DG}}{DG} = (235 \text{ lb})\,\frac{21\underline{i} - 38\underline{j} + 18\underline{k}}{47}$$

$$\underline{P} = (105 \text{ lb})\underline{i} - (190 \text{ lb})\underline{j} + (90 \text{ lb})\underline{k} \quad \triangleleft$$

(continued)

We have: $\underline{P} = (105\text{ lb})\underline{i} - (190\text{ lb})\underline{j} + (90\text{ lb})\underline{k}$ ◁



Vector from B to G

$$\underline{r}_{G/B} = (16\text{ in.} + 21\text{ in.} - 32\text{ in.})\underline{i} + (18\text{ in.} + 12\text{ in.})\underline{k}$$

$$\underline{r}_{G/B} = (5\text{ in.})\underline{i} + (30\text{ in.})\underline{k}$$ ◁

Unit Vector along AB

$$\overrightarrow{AB} = (32\text{ in.})\underline{i} - (30\text{ in.})\underline{j} - (24\text{ in.})\underline{k} \qquad AB = 50\text{ in.}$$

$$\underline{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{32\underline{i} - 30\underline{j} - 24\underline{k}}{50}$$

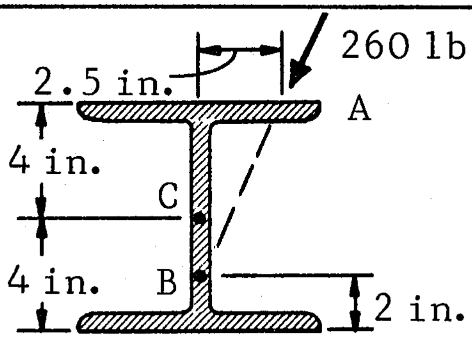
$$\underline{\lambda}_{AB} = 0.64\underline{i} - 0.60\underline{j} - 0.48\underline{k}$$ ◁

Moment about AB

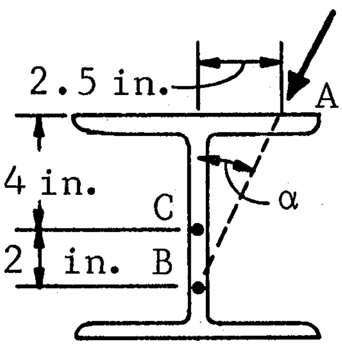
$$M_{AB} = \underline{\lambda}_{AB} \cdot (\underline{r}_{G/B} \times \underline{P}) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ 5\text{ in.} & 0 & 30\text{ in.} \\ 105\text{ lb} & -190\text{ lb} & 90\text{ lb} \end{vmatrix}$$

$$= 0 - 1890 + 456 - 0 + 3648 + 270 = +2484\text{ lb}\cdot\text{in.}$$

$$M_{AB} = +207\text{ lb}\cdot\text{ft}$$ ◀



Replace the 260-lb force shown by an equivalent force-couple system at C.



Solution

$$AB = \sqrt{(2.5)^2 + (6)^2} = 6.5 \text{ in.}$$

$$\sin\alpha = \frac{2.5}{6.5} = \frac{5}{13}\ ; \quad \alpha = 22.6^\circ$$

$$\cos\alpha = \frac{6}{6.5} = \frac{12}{13}$$

Force F. $\underline{F} = -F\sin\alpha\,\underline{i} - F\cos\alpha\,\underline{j}$

$$= -(260 \text{ lb})\frac{5}{13}\underline{i} - (260 \text{ lb})\frac{12}{13}\underline{j}$$

$$\underline{F} = -(100 \text{ lb})\,\underline{i} - (240 \text{ lb})\,\underline{j}$$

Vector from C to A. $\underline{r}_{A/C} = (2.5 \text{ in.})\,\underline{i} + (4 \text{ in.})\,\underline{j}$

Moment of F about C

$$\underline{M}_C = \underline{r}_{A/C} \times \underline{F} = (2.5\underline{i} + 4\underline{j}) \times (-100\underline{i} - 240\underline{j})$$

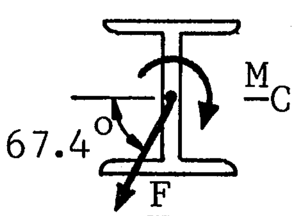
$$= 400\underline{k} - 600\underline{k}$$

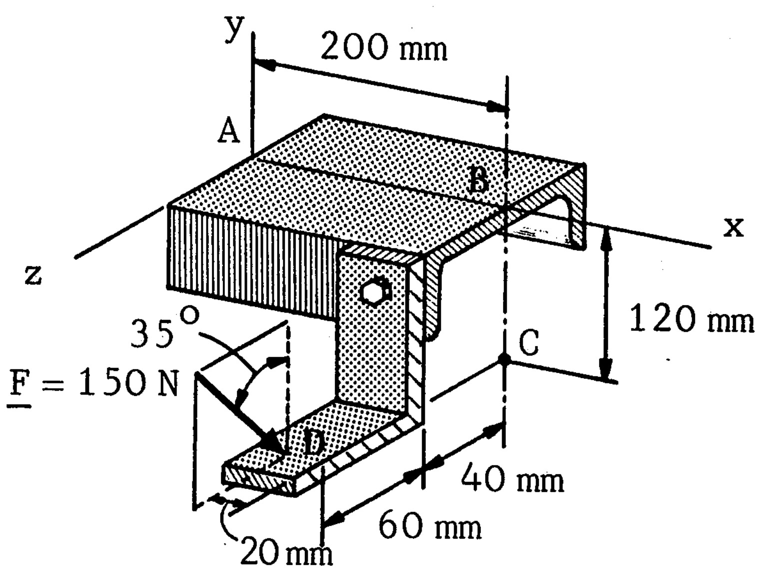
$$\underline{M}_C = -(200 \text{ lb}\cdot\text{in.})\,\underline{k}$$ ◁

Force at C. $\underline{F} = 260^\circ \measuredangle 22.6^\circ = 260 \text{ lb} \measuredangle 67.4^\circ$ ◁

Force-Couple System at C

$\underline{F} = 260 \text{ lb} \measuredangle 67.4^\circ$; $\underline{M}_C = 200 \text{ lb}\cdot\text{in.}$ ↻ ◀





Replace the 150-N force by an equivalent force-couple system at A.

Force at A. We move $\underline{F} = 150\,\text{lb}$ to A

$$\underline{F} = -[(150\,\text{lb})\cos 35^\circ]\underline{j} - [(150\,\text{lb})\sin 35^\circ]\underline{k}$$

$$\underline{F} = -(122.9\,\text{lb})\underline{j} - (86.04\,\text{lb})\underline{k} \quad \triangleleft$$

Vector from A to D

$$\underline{r}_{D/A} = (180\,\text{mm})\underline{i} - (120\,\text{mm})\underline{j} + (100\,\text{mm})\underline{k}$$

Moment about A

$$\underline{M}_A = \underline{r}_{D/A} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.18\,\text{m} & -0.12\,\text{m} & +0.10\,\text{m} \\ 0 & -122.9\,\text{lb} & -86.04\,\text{lb} \end{vmatrix}$$

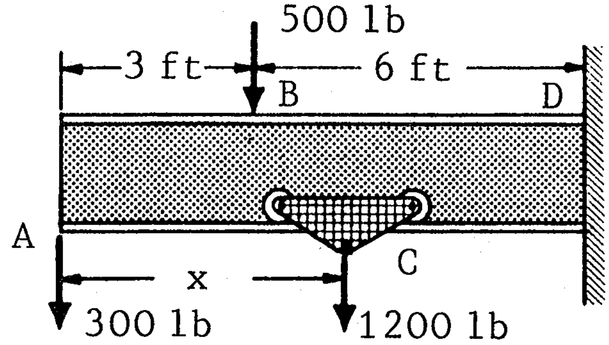
$$= (10.32 + 12.29)\underline{i} + (15.49)\underline{j} - (22.12)\underline{k}$$

$$\underline{M}_A = (22.61\,\text{N}\cdot\text{m})\underline{i} + (15.49\,\text{N}\cdot\text{m})\underline{j} - (22.12\,\text{N}\cdot\text{m})\underline{k} \quad \triangleleft$$

Force-Couple System at A

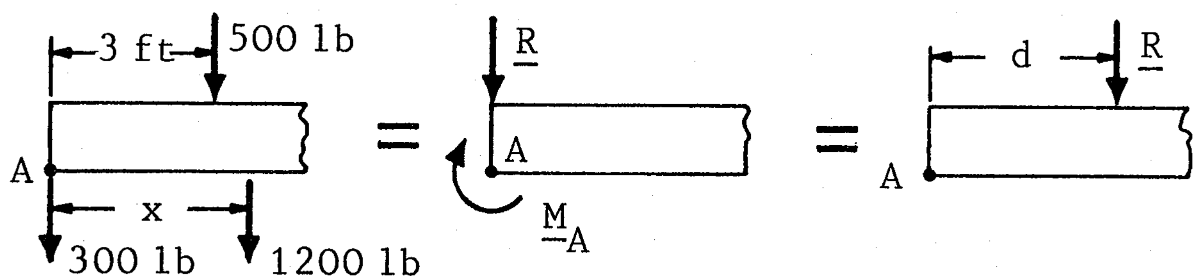
$$\underline{F} = -(122.9\,\text{N})\underline{j} - (86.0\,\text{N})\underline{k} \quad \blacktriangleleft$$

$$\underline{M}_A = (22.6\,\text{N}\cdot\text{m})\underline{i} + (15.49\,\text{N}\cdot\text{m})\underline{j} - (22.1\,\text{N}\cdot\text{m})\underline{k} \quad \blacktriangleleft$$



Determine the distance from point A to the line of action of the resultant of the three loads when:
(a) $x = 1.25$ ft,
(b) $x = 4$ ft,
(c) $x = 8$ ft.

Equivalent Systems of Forces



Given System = Force and Couple at A = Resultant

$+\downarrow R = 300\text{ lb} + 500\text{ lb} + 1200\text{ lb}$ $\qquad \mathbf{R} = 2000\text{ lb}\downarrow$

$+\circlearrowright M_A = (500\text{ lb})(3\text{ ft}) + (1200\text{ lb})x$

$+\circlearrowright M_A = 1500\text{ lb}\cdot\text{ft} + (1200\text{ lb})x$

Distance from A to Resultant

$M_A = Rd$: $\quad 1500\text{ lb}\cdot\text{ft} + (1200\text{ lb})x = (2000\text{ lb})d$

$d = 0.75 + 0.6x$

(a) For $x = 1.25$ ft:

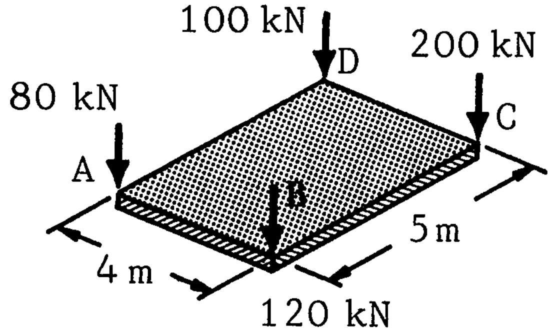
$d = 0.75 + 0.6(1.25)$ $\qquad d = 1.50$ ft ◀

(b) For $x = 4$ ft:

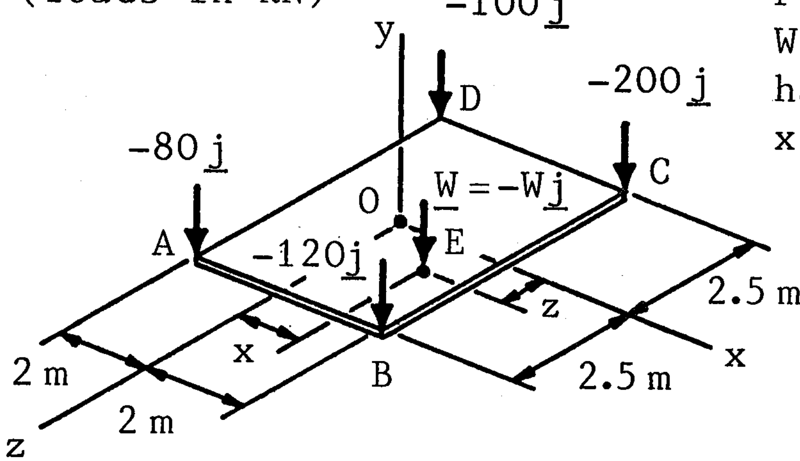
$d = 0.75 + 0.6(4)$ $\qquad d = 3.15$ ft ◀

(c) For $x = 8$ ft:

$d = 0.75 + 0.6(8)$ $\qquad d = 5.55$ ft ◀



A rectangular foundation mat supports four column loads as shown. Determine the magnitude and point of application of the smallest additional load which must be applied to the mat if the resultant of the five loads is to pass through the center of the mat.



Place additional load $\mathbf{W} = -W\mathbf{j}$ at point E having coordinates x and z.

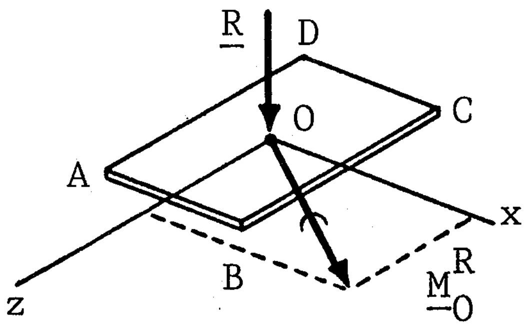
Force Couple at O

$$\mathbf{R} = \Sigma \mathbf{F}$$

$$\mathbf{M}_O^R = \Sigma(\mathbf{r} \times \mathbf{F})$$

	$\mathbf{r}$, m	$\mathbf{F}$, kN	$\mathbf{r} \times \mathbf{F}$, kN·m
A	$-2\mathbf{i} + 2.5\mathbf{k}$	$-80\mathbf{j}$	$200\mathbf{i} + 160\mathbf{k}$
B	$2\mathbf{i} + 2.5\mathbf{k}$	$-120\mathbf{j}$	$300\mathbf{i} - 240\mathbf{k}$
C	$2\mathbf{i} - 2.5\mathbf{k}$	$-200\mathbf{j}$	$-500\mathbf{i} - 400\mathbf{k}$
D	$-2\mathbf{i} - 2.5\mathbf{k}$	$-100\mathbf{j}$	$-250\mathbf{i} + 200\mathbf{k}$
E	$x\mathbf{i} + z\mathbf{k}$	$-W\mathbf{j}$	$Wz\mathbf{j} - Wx\mathbf{k}$
			$\mathbf{M}_O^R = (-250 + Wz)\mathbf{i} - (280 + Wx)\mathbf{k}$

(continued)



We have:

$$\mathbf{M}_O^R = (-250 + Wz)\,\mathbf{i} - (280 + Wx)\,\mathbf{k}$$

For Resultant at O:

We set coefficients of $\mathbf{i}$ and $\mathbf{k}$ equal to zero.

$$Wz = 250 \qquad Wx = -280 \qquad (1)$$

Since $|x| \leq 2$ m, we must have: $W \geq 280/2 = 140$ kN ◁

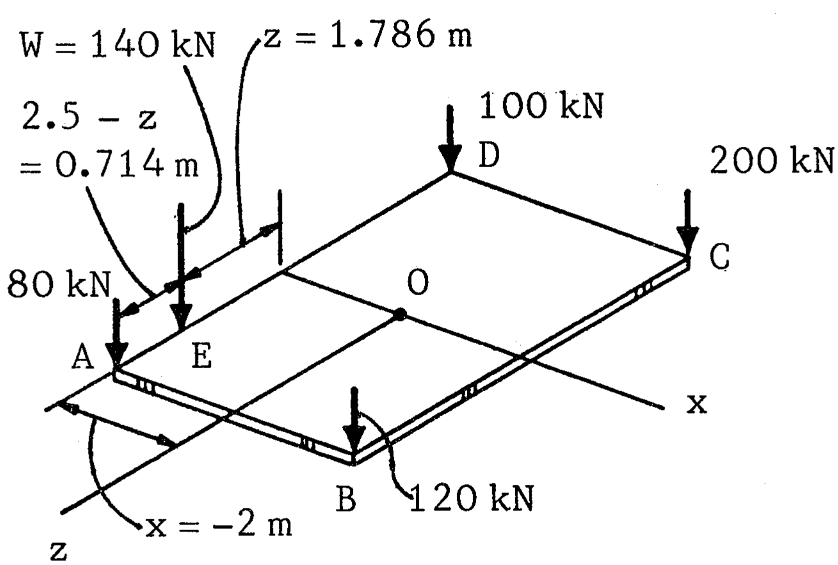
Since $|z| \leq 2.5$ m, we must have: $W \geq 250/2.5 = 100$ kN ◁

Thus, smallest additional load is: $W = 140$ kN ◀

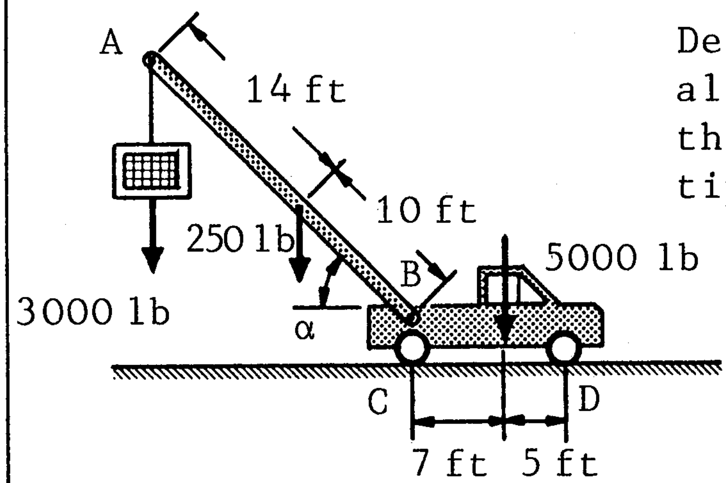
Substitute $W = 140$ kN into Eqs.(1):

$$(140\text{ kN})\,z = 250\text{ kN·m} \qquad z = 1.786\text{ m}$$

$$(140\text{ kN})\,x = -280\text{ kN·m} \qquad x = -2\text{ m}$$

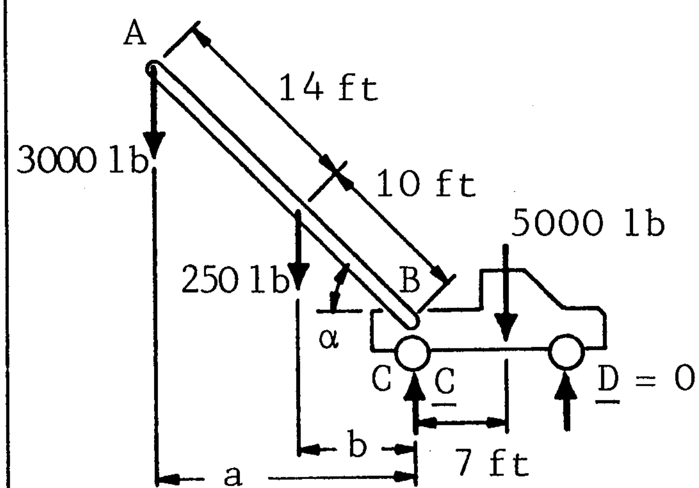


$\mathbf{W}$ is applied at E, 0.714 m from A on edge AD ◀



Determine the smallest allowable value of α if the truck is not to tip over.

Free Body. The truck and crane are chosen as a free body. When the truck is about to tip, we have $\underline{D} = 0$.



$$a = (24\text{ ft})\cos\alpha$$

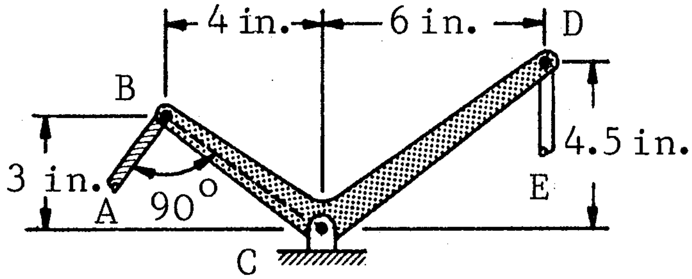
$$b = (10\text{ ft})\cos\alpha$$

$+\circlearrowright \Sigma M_C = 0:$

$$(3000\text{ lb})(24\text{ ft})\cos\alpha + (250\text{ lb})(10\text{ ft})\cos\alpha - (5000\text{ lb})(7\text{ ft}) = 0$$

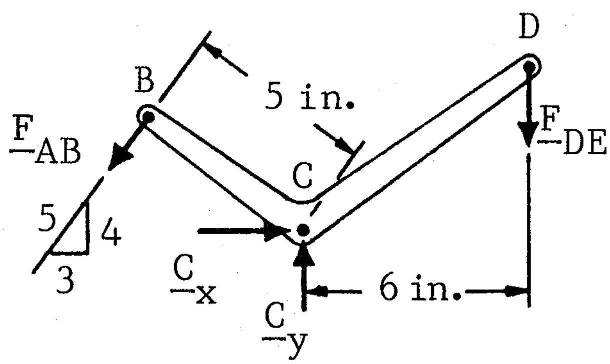
$$74{,}500\cos\alpha - 35{,}000 = 0$$

$$\cos\alpha = 0.4698 \qquad \alpha = 62.0^\circ$$



Determine the maximum force which may be exerted by link AB on the crank if the maximum allowable value of the reaction at C is 400 lb.

Free Body. The crank is chosen as a free body.



$+\circlearrowright \Sigma M_C = 0$:

$F_{AB}(5\text{ in.}) - F_{DE}(6\text{ in.}) = 0$

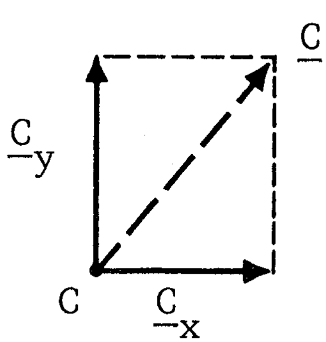
$$F_{DE} = \frac{5}{6}F_{AB} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = 0$: $\quad -\frac{3}{5}F_{AB} + C_x = 0 \qquad C_x = \frac{3}{5}F_{AB}$

$+\uparrow \Sigma F_y = 0$: $\quad -\frac{4}{5}F_{AB} + C_y - F_{DE} = 0$

$$-\frac{4}{5}F_{AB} + C_y - \frac{5}{6}F_{AB} = 0 \qquad C_y = \frac{49}{30}F_{AB}$$

Reaction at C



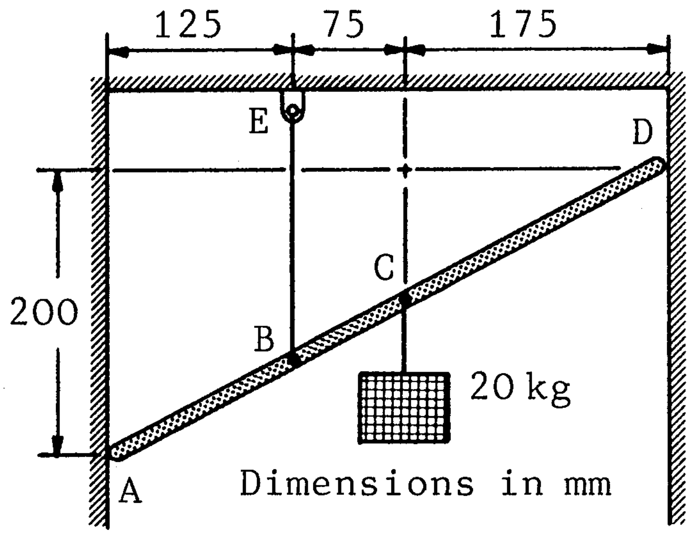
$$C^2 = C_x^2 + C_y^2$$

$$C^2 = \left(\frac{3}{5}F_{AB}\right)^2 + \left(\frac{49}{30}F_{AB}\right)^2$$

$$C = 1.740\,F_{AB}$$

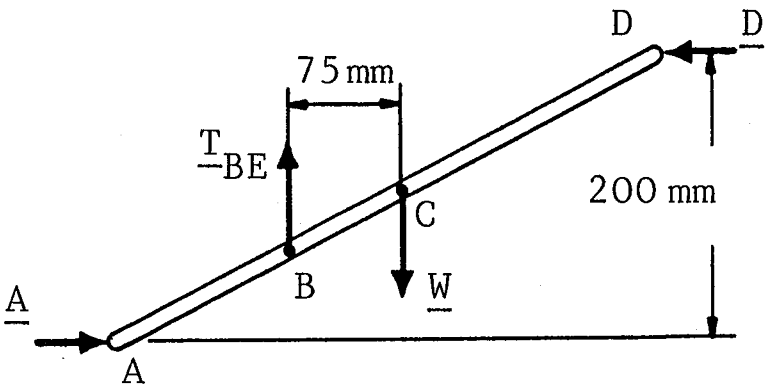
For $C = 400$ lb, $\quad 400\text{ lb} = 1.740\,F_{AB}$

$$F_{AB} = 230\text{ lb} \blacktriangleleft$$



Determine the tension in cable BE and the reactions at A and D.

Free Body. The rod ABCD is chosen as a free body.



$W = mg$
$= (20\,\text{kg})(9.81\,\text{m/s}^2)$
$W = 196.2\,\text{N}$

$\Sigma F_x = 0$: $A = D$

$\Sigma F_y = 0$: $T_{BE} = W$ $T_{BE} = 196.2\,\text{N}$ ◀

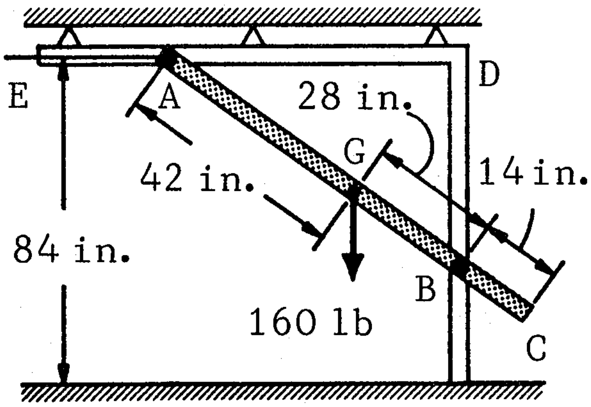
The forces acting on the free body form two couples.

$+\circlearrowright \Sigma M = 0$: $A(200\,\text{mm}) - (196.2\,\text{N})(75\,\text{mm}) = 0$

$A = 73.6\,\text{N}$

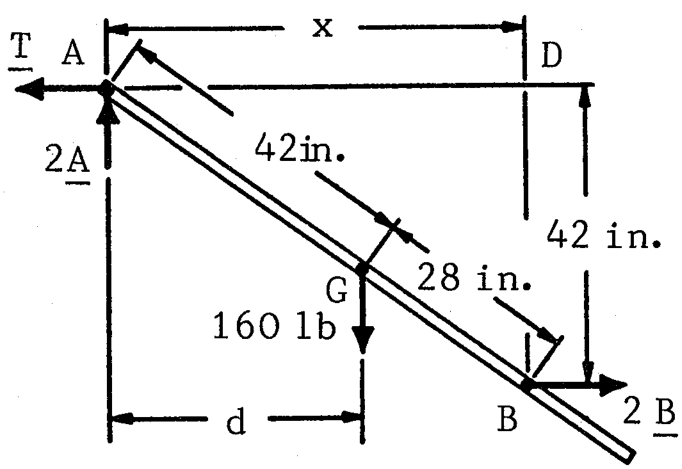
$\underline{A} = 73.6\,\text{N} \longrightarrow$ ◀

$\underline{D} = 73.6\,\text{N} \longleftarrow$ ◀



The 160-lb overhead garage door is supported by the cable AE attached at the middle of its upper edge and by two sets of rollers at A and B. For the position when BD = 42 in., determine (a) the tension in cable AE, (b) the reaction at each of the four rollers.

Free Body. The door is chosen as a free body. We denote by A and by B the reaction at one roller.



$$x^2 + (42\text{ in.})^2 = (70\text{ in})^2$$

$$x = 56\text{ in.}$$

$$d = \left(\frac{42\text{ in.}}{70\text{ in.}}\right) x = 0.6x$$

$$d = 0.6(56\text{ in.}) = 33.6\text{ in.}$$

$+\circlearrowright \Sigma M_A = 0$: $(2B)(42\text{ in.}) - (160\text{ lb})d = 0$

$(2B)(42\text{ in.}) - (160\text{ lb})(33.6\text{ in.}) = 0$

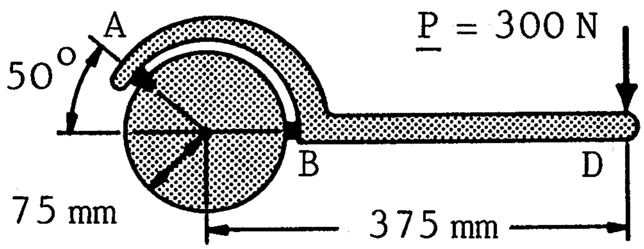
$B = 64.0\text{ lb}$ $\quad\quad$ $\underline{B} = 64.0\text{ lb} \rightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $-T + 2B = 0$

$-T + 2(64.0\text{ lb}) = 0$ $\quad\quad$ $T = 128.0\text{ lb}$ ◀

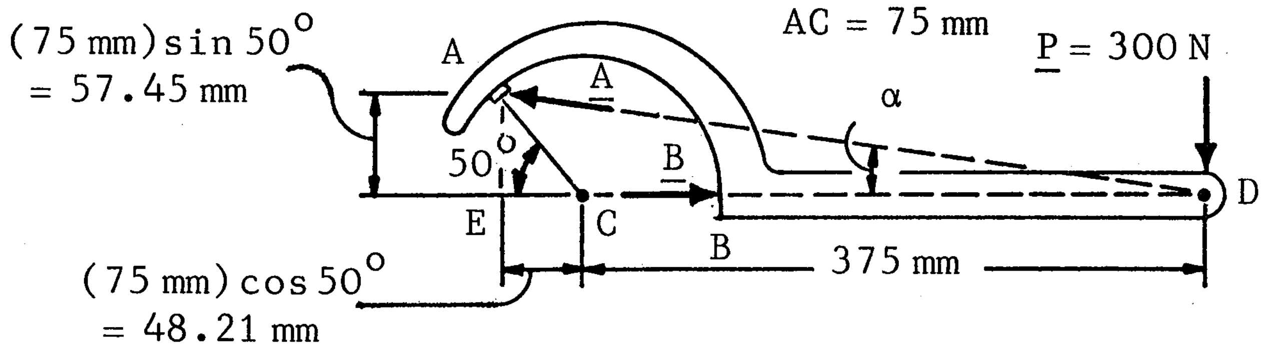
$+\uparrow \Sigma F_y = 0$: $2A - 160\text{ lb} = 0$

$A = 80\text{ lb}$ $\quad\quad$ $\underline{A} = 80\text{ lb} \uparrow$ ◀



In the spanner shown a pin fits in a hole at A, while a flat, frictionless surface rests against the shaft at B. For the loading shown, determine the reactions at A and B.

Free Body. The spanner is chosen as a free body.



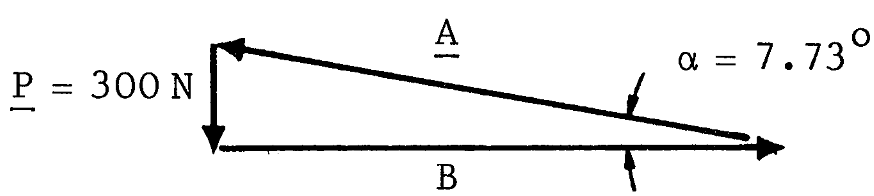
Three-Force Body. Forces concurrent at D.

In triangle ADE:

$$\tan\alpha = \frac{57.45}{48.21 + 375} = 0.13575$$

$$\alpha = 7.73^\circ$$

Force Triangle

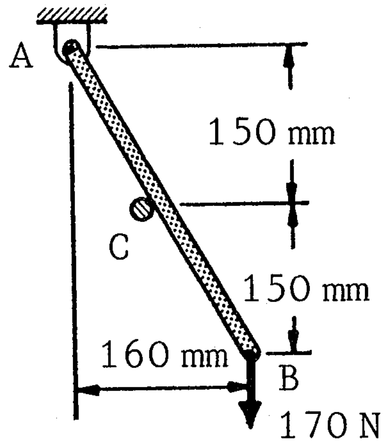


$A(\sin 7.73^\circ) = 300\ \mathrm{N}$

$A = 2230\ \mathrm{N}$ $\qquad$ $\underline{A} = 2230\ \mathrm{N} \searrow 7.73^\circ$ ◀

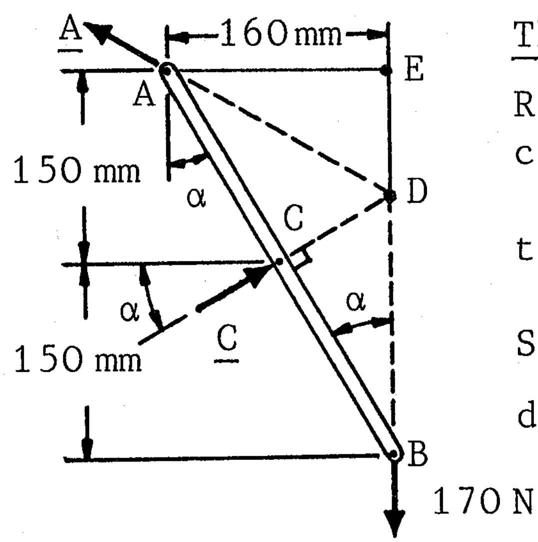
$B(\tan 7.73^\circ) = 300\ \mathrm{N}$

$B = 2210\ \mathrm{N}$ $\qquad$ $\underline{B} = 2210\ \mathrm{N} \rightarrow$ ◀



Rod AB is supported by a pin and bracket at A and rests against a frictionless peg at C. Determine the reactions at A and C.

Free Body. The rod AB is chosed as a free body.



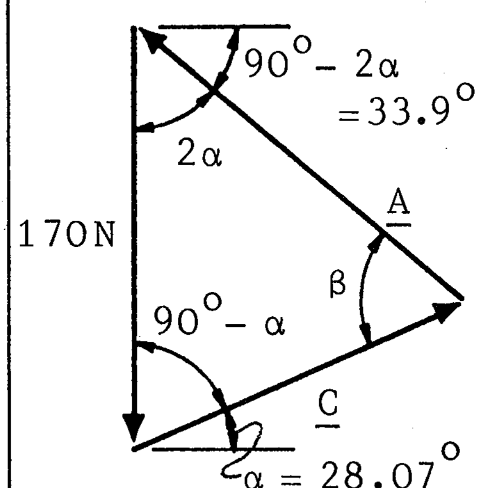
Three-Force Body

Reaction at C ⊥ to AB. Forces concurrent at D.

$\tan\alpha = \dfrac{160\text{ mm}}{300\text{ mm}}$ $\qquad \alpha = 28.07^\circ$

Since AC = CB, ∢CAD = α, and direction of $\underline{A}$ is ◣ 2α.

Force Triangle



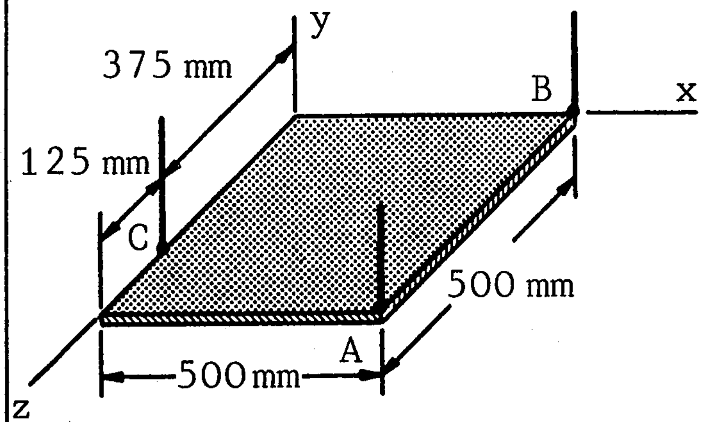
$(90^\circ - \alpha) + 2\alpha + \beta = 180^\circ$
$\beta = 90^\circ - \alpha$

This is the same as angle between $\underline{C}$ and 170-N force; triangle is isosceles and we have:

$A = 170\text{ N}$
$C = 2(170\text{ N})\sin\alpha = 160.0\text{ N}$

$\underline{A} = 170\text{ N}$ ◣ 33.9° ◀

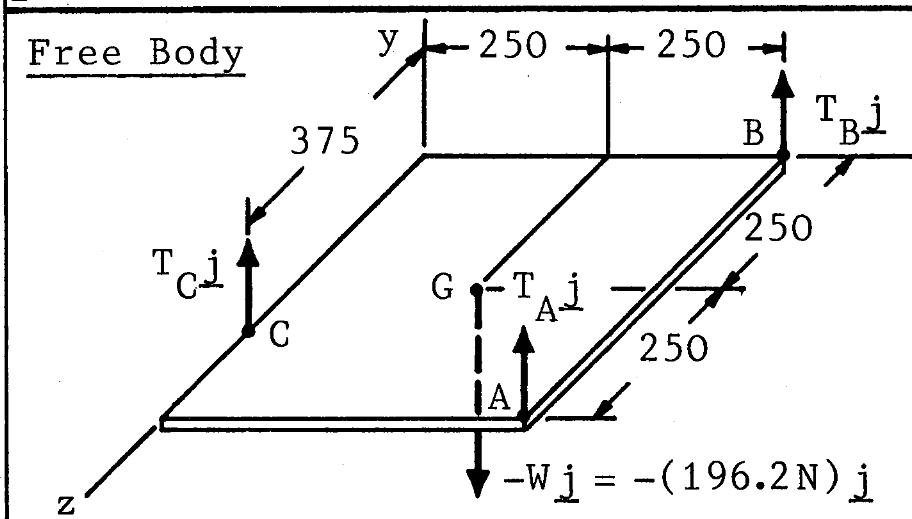
$\underline{C} = 160\text{ N}$ ◢ 28.1° ◀



The 20-kg plate is supported by three wires. Determine the tension in each wire.

Free Body

Dimensions in mm



$W = mg$
$= (20\,\text{kg})(9.81\,\text{m/s}^2)$

$W = 196.2\ \text{N}$

$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A\mathbf{j} + \mathbf{r}_{C/B} \times T_C\mathbf{j} + \mathbf{r}_{G/B} \times (-196.2\ \text{N})\,\mathbf{j} = 0$

$500\mathbf{k} \times T_A\mathbf{j} + (-500\,\mathbf{i} + 375\mathbf{k}) \times T_C\mathbf{j} + (-250\mathbf{i} + 250\text{k}) \times (-196.2\mathbf{j}) = 0$

$-500T_A\,\mathbf{i} - 500T_C\mathbf{k} - 375T_C\mathbf{i} + 49.05 \times 10^3\mathbf{k} + 49.05 \times 10^3\mathbf{i} = 0$

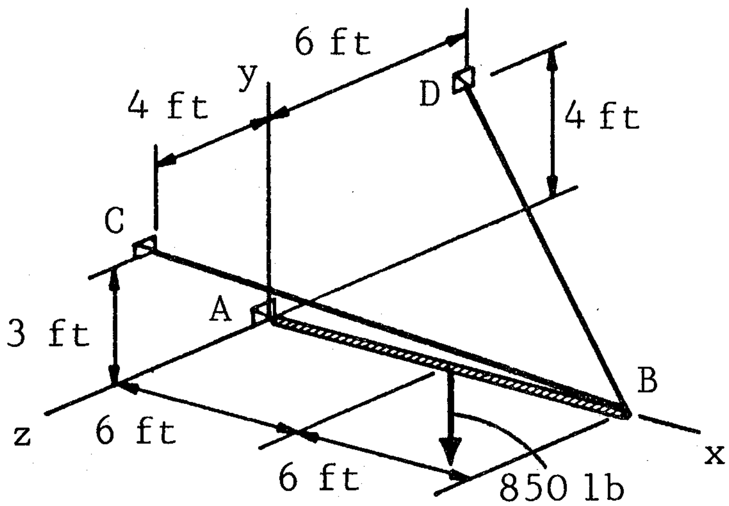
Equate to zero the coefficients of unit vectors.

$\mathbf{k}: \quad -500T_C + 49.05 \times 10^3 = 0$ $\qquad T_C = 98.1\ \text{N}$ ◀

$\mathbf{i}: \quad -500T_A - 375(98.1) + 49.05 \times 10^3 = 0$ $\qquad T_A = 24.5\ \text{N}$ ◀

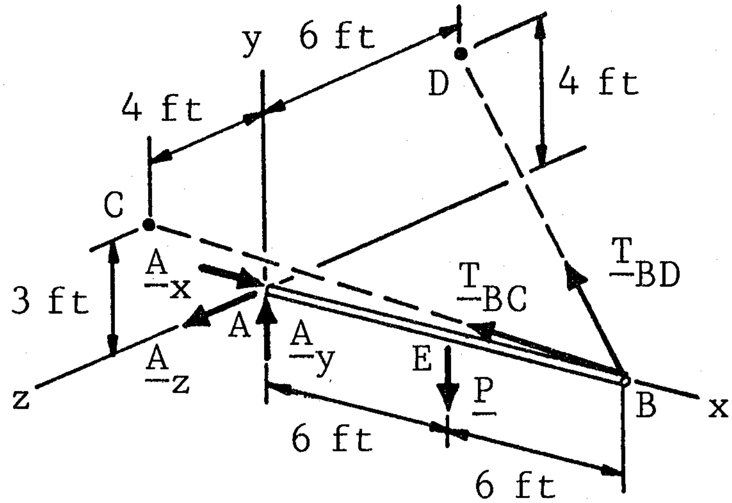
$\Sigma F_y = 0: \quad T_A + T_B + T_C - 196.2\ \text{N} = 0$

$24.5 + T_B + 98.1 - 196.2 = 0$ $\qquad T_B = 73.6\ \text{N}$ ◀



Boom AB is supported by two cables and by a ball and socket at A. Determine:
(a) the tension in each cable,
(b) the reaction at A.

Free Body. Boom AB is chosen as a free body.



$\underline{A}_x = A_x \underline{i}$

$\underline{A}_y = A_y \underline{j}$

$\underline{A}_z = A_z \underline{k}$

$\underline{P} = -(850\ \text{lb})\underline{j}$

Cable BC

$\overrightarrow{BC} = -(12\ \text{ft})\underline{i} + (3\ \text{ft})\underline{j} + (4\ \text{ft})\underline{k}$ $\quad BC = 13\ \text{ft}$

$\underline{T}_{BC} = T_{BC}\dfrac{\overrightarrow{BC}}{BC}$ $\quad \underline{T}_{BC} = -\dfrac{12}{13}T_{BC}\underline{i} + \dfrac{3}{13}T_{BC}\underline{j} + \dfrac{4}{13}T_{BC}\underline{k}$ ◁

Cable BD

$\overrightarrow{BD} = -(12\ \text{ft})\underline{i} + (4\ \text{ft})\underline{j} - (6\ \text{ft})\underline{k}$ $\quad BD = 14\ \text{ft}$

$\underline{T}_{BD} = -\dfrac{12}{14}T_{BD}\underline{i} + \dfrac{4}{14}T_{BD}\underline{j} - \dfrac{6}{14}T_{BD}\underline{k}$ ◁

(continued)

Equilibrium Equations

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BC} + \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_E \times \mathbf{P} = 0$$

$$12\mathbf{i} \times \left(-\frac{12}{13}T_{BC}\mathbf{i} + \frac{3}{13}T_{BC}\mathbf{j} + \frac{4}{13}T_{BC}\mathbf{k}\right)$$

$$+ 12\mathbf{i} \times \left(-\frac{12}{14}T_{BD}\mathbf{i} + \frac{4}{14}T_{BD}\mathbf{j} - \frac{6}{14}T_{BD}\mathbf{k}\right) + 6\mathbf{i} \times (-850\mathbf{j}) = 0$$

$$\frac{36}{13}T_{BC}\mathbf{k} - \frac{48}{13}T_{BC}\mathbf{j} + \frac{48}{14}T_{BD}\mathbf{k} + \frac{72}{14}T_{BD}\mathbf{j} - 5100\mathbf{k} = 0$$

Equate coefficients of $\mathbf{j}$ and $\mathbf{k}$ to zero.

$$\mathbf{j}: \quad -\frac{48}{13}T_{BC} + \frac{72}{14}T_{BD} = 0$$

$$\mathbf{k}: \quad \frac{36}{13}T_{BC} + \frac{48}{14}T_{BD} - 5100 = 0$$

Solving simultaneously: $T_{BC} = 975$ lb; $T_{BD} = 700$ lb ◀

$$\Sigma \mathbf{F} = 0: \quad A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} + \mathbf{T}_{BC} + \mathbf{T}_{BD} + \mathbf{P} = 0$$

$$\left(A_x - \frac{12}{13}T_{BC} - \frac{12}{14}T_{BD}\right)\mathbf{i} + \left(A_y + \frac{3}{13}T_{BC} + \frac{4}{14}T_{BD} - 850\right)\mathbf{j}$$

$$+ \left(A_z + \frac{4}{13}T_{BC} - \frac{6}{14}T_{BD}\right)\mathbf{k} = 0$$

$\mathbf{i}$: $A_x - \frac{12}{13}(975) - \frac{12}{14}(700) = 0$ $\qquad A_x = 1500$ lb ◁

$\mathbf{j}$: $A_y + \frac{3}{13}(975) + \frac{4}{14}(700) - 850 = 0$ $\qquad A_y = 425$ lb ◁

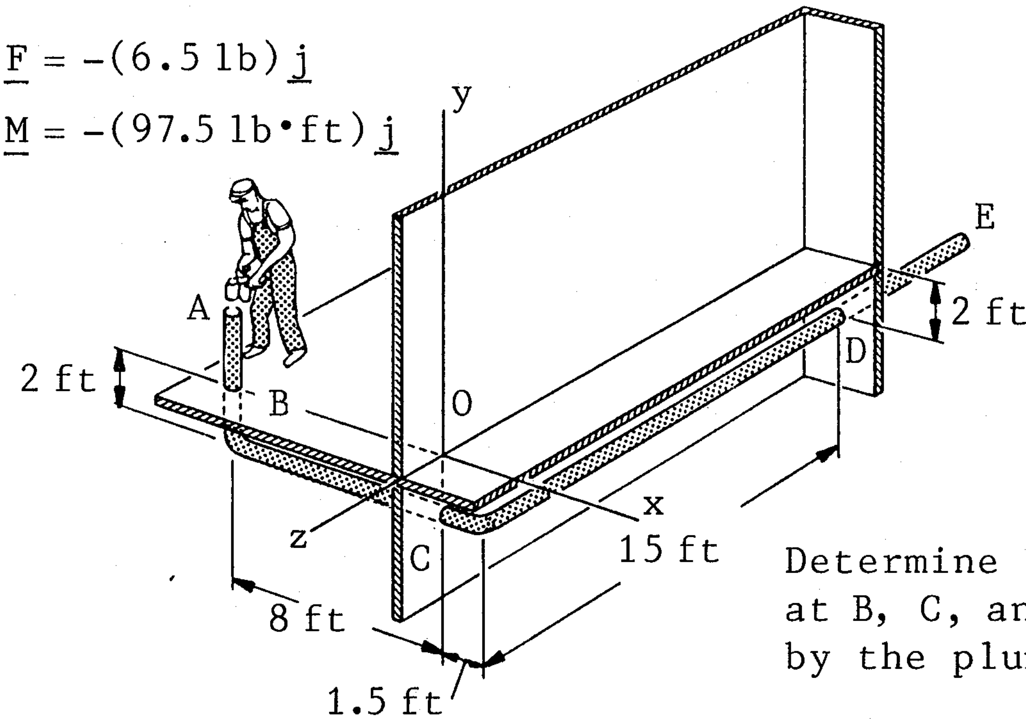
$\mathbf{k}$: $A_z + \frac{4}{13}(975) - \frac{6}{14}(700) = 0$ $\qquad A_z = 0$ ◁

$$\mathbf{A} = (1500 \text{ lb})\mathbf{i} + (425 \text{ lb})\mathbf{j}$$ ◀

Plumber exerts at A:

$\underline{F} = -(6.5\text{ lb})\underline{j}$

$\underline{M} = -(97.5\text{ lb}\cdot\text{ft})\underline{j}$

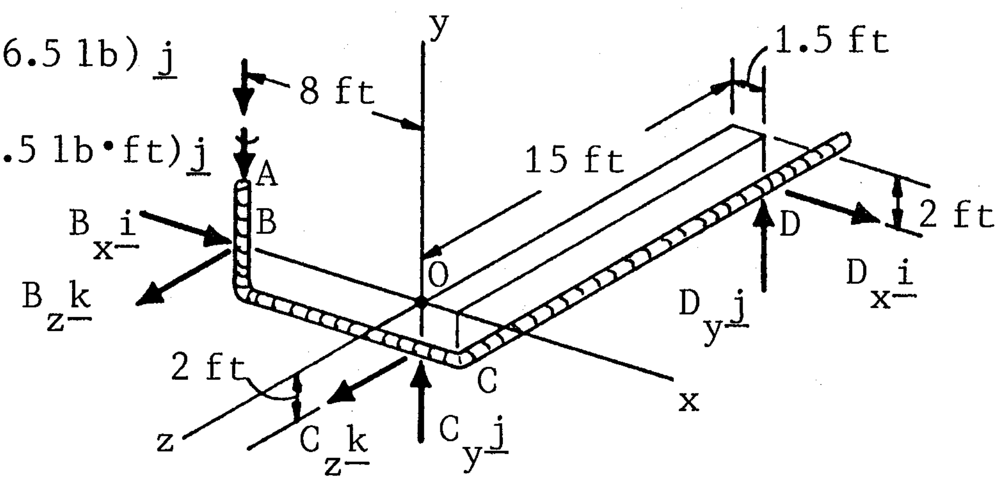


Determine reactions at B, C, and D caused by the plumber.

Free Body. The entire pipe is chosen as a free body.

$\underline{F} = -(6.5\text{ lb})\underline{j}$

$\underline{M} = -(97.5\text{ lb}\cdot\text{ft})\underline{j}$



Equilibrium Equations:

$\Sigma\underline{M}_O = 0: \quad \underline{r}_B \times (\underline{B} + \underline{F}) + \underline{r}_C \times \underline{C} + \underline{r}_D \times \underline{D} + \underline{M} = 0$

$\underline{r}_B = -(8\text{ ft})\underline{i}, \quad \underline{r}_C = -(2\text{ ft})\underline{j}, \quad \underline{r}_D = (1.5\text{ ft})\underline{i} - (2\text{ ft})\underline{j} - (15\text{ ft})\underline{k}$

(continued)

Equilibrium Equations

$\Sigma \underline{M}_O = 0: \quad \underline{r}_B \times (\underline{B} + \underline{F}) + \underline{r}_C \times \underline{C} + \underline{r}_D \times \underline{D} + \underline{M} = 0$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -8 & 0 & 0 \\ B_x & -6.5 & B_y \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -2 & 0 \\ 0 & C_y & C_z \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.5 & -2 & -15 \\ D_x & D_y & 0 \end{vmatrix} - 97.5\underline{j} = 0$$

Equate coefficients of $\underline{i}$, $\underline{j}$, $\underline{k}$ to zero.

$\underline{i}: \quad 0 - 2C_z + 15D_y = 0 \longrightarrow C_z = 7.5D_y$ (1)

$\underline{j}: \quad 8B_z + 0 - 15D_x - 97.5 = 0$ (2)

$\underline{k}: \quad 52 + 0 + 1.5D_y + 2D_x = 0$ (3)

$\Sigma F_x = 0: \quad B_x + D_x = 0$ (4)

$\Sigma F_y = 0: \quad C_y + D_y - 6.5 = 0$ (5)

$\Sigma F_z = 0: \quad B_z + C_z = 0 \longrightarrow B_z = -C_z = -7.5D_y$ (6)

Substitute from (6) into (2): $8(-7.5D_y) - 15D_x - 97.5 = 0$

or $15D_x + 60D_y = -97.5$ (7)

Solve (3) and (7) simultaneoulsy: $D_x = -30.5$, $D_y = +6$

$\underline{D} = -(30.5\text{ lb})\underline{i} + (6\text{ lb})\underline{j}$ ◀

From (5): $C_y + 6 - 6.5 = 0 \qquad C_y = +0.5$

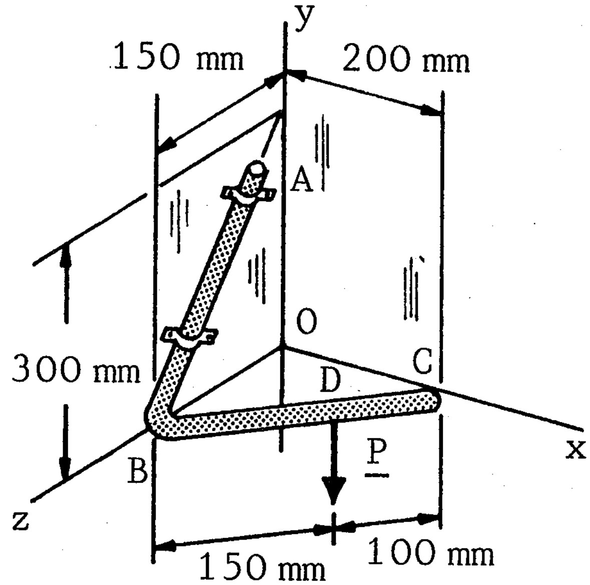
From (1): $C_z = 7.5(6) \qquad C_z = +45.0$

$\underline{C} = (0.5\text{ lb})\underline{j} + (45.0\text{ lb})\underline{k}$ ◀

From (4): $B_x - 30.5 = 0 \qquad B_x = +30.5$

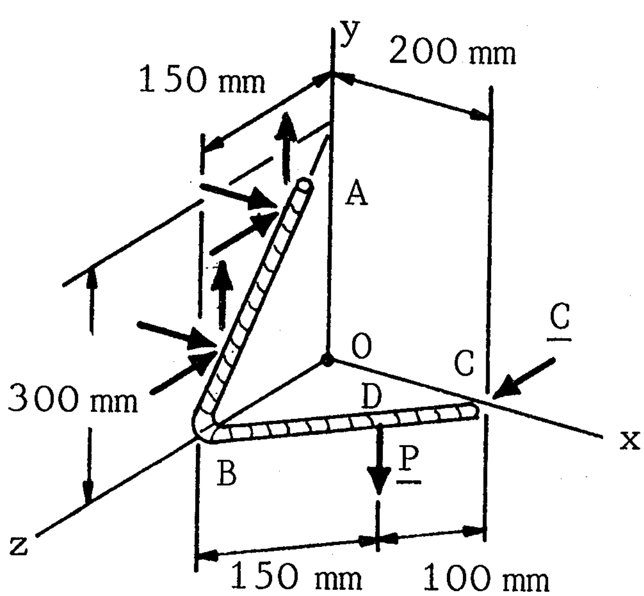
From (6): $B_z = -7.5(6) \qquad B_z = -45.0$

$\underline{B} = (30.5\text{ lb})\underline{i} - (45.0\text{ lb})\underline{k}$ ◀



Rod ABC is hinged to a vertical wall and bears at C against another vertical wall. For the load $P = 150\,N$, find the reaction at C. (Neglect friction.)

Free Body. Rod ABC is chosen as a free body.



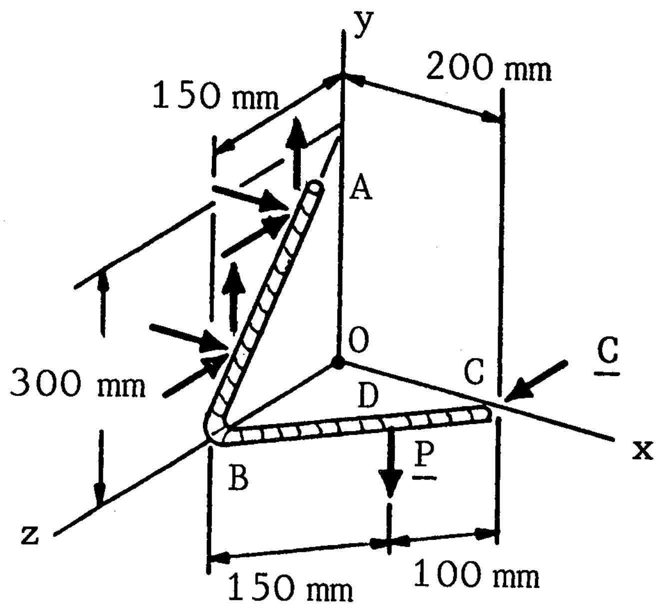
$\underline{P} = -(150\,N)\,\underline{j}$

Since $\underline{C}$ is ⊥ to xy plane,

$\underline{C} = C\,\underline{k}$

Equilibrium Equation. To eliminate reactions at the brackets we shall write: $\Sigma M_{BA} = \Sigma\left(\underline{\lambda}_{BA} \cdot \underline{M}_B\right) = 0$

(continued)



$\underline{P} = -(150\,\text{N})\,\underline{j}$

$\underline{C} = C\underline{k}$

Equilibrium Equation. $\Sigma M_{BA} = \Sigma\left(\underline{\lambda}_{BA} \cdot \underline{M}_B\right) = 0$

$$\underline{\lambda}_{BA} \cdot [\underline{r}_{D/B} \times (-150\underline{j})] + \underline{\lambda}_{BA} \cdot (\underline{r}_{C/B} \times C\underline{k}) = 0 \qquad (1)$$

where $\underline{r}_{C/B} = (200\,\text{mm})\,\underline{i} - (150\,\text{mm})\,\underline{k}$

$$\underline{r}_{D/B} = \frac{150}{250}\underline{r}_{C/B} = (120\,\text{mm})\,\underline{i} - (90\,\text{mm})\,\underline{k}$$

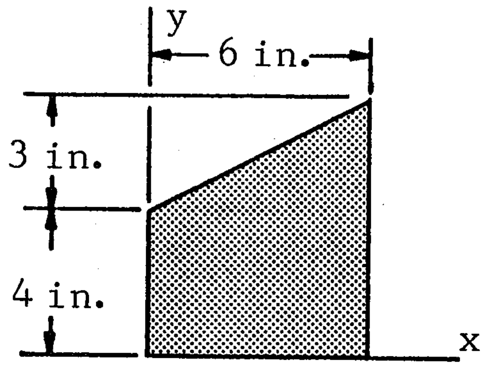
$\overrightarrow{BA} = (300\,\text{mm})\,\underline{j} - (150\,\text{mm})\,\underline{k}$ $\qquad BA = 150\sqrt{5}\ \text{mm}$

$$\underline{\lambda}_{BA} = \frac{300\underline{j} - 150\underline{k}}{150\sqrt{5}} \qquad \underline{\lambda}_{BA} = \frac{1}{\sqrt{5}}(2\underline{j} - \underline{k})$$

$$\text{Eq.(1):}\ \begin{vmatrix} 0 & 2 & -1 \\ 120 & 0 & -90 \\ 0 & -150 & 0 \end{vmatrix} \frac{1}{\sqrt{5}} + \begin{vmatrix} 0 & 2 & -1 \\ 200 & 0 & -150 \\ 0 & 0 & C \end{vmatrix} \frac{1}{\sqrt{5}} = 0$$

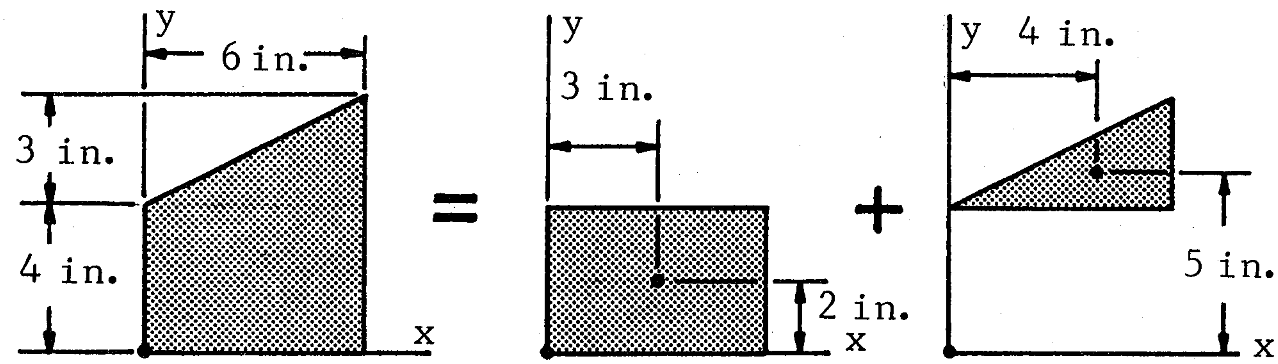
$$(18\,000 - 400C)/\sqrt{5} = 0$$

$C = 45\,\text{N}$ $\qquad \underline{C} = (45\,\text{N})\,\underline{k}$ ◀

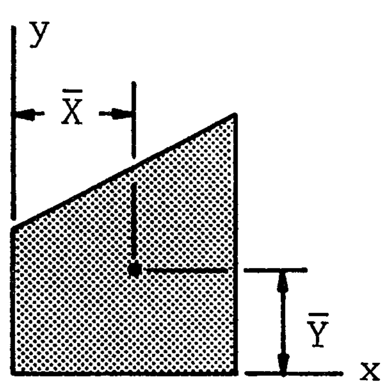


Locate the centroid of the plane area shown.

Solution. The area is obtained by adding a rectangle and a triangle.



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
Rectangle	$(4)(6) = 24$	3	2	72	48
Triangle	$\frac{1}{2}(3)(6) = 9$	4	5	36	45
Σ	33			108	93



$\bar{X}\,\Sigma A = \Sigma\,\bar{x}A$

$\bar{X}(33 \text{ in}^2) = 108 \text{ in}^3$

$\bar{X} = 3.27$ in. ◀

$\bar{Y}\,\Sigma A = \Sigma\,\bar{y}A$

$\bar{Y}(33 \text{ in}^2) = 93 \text{ in}^3$

$\bar{Y} = 2.82$ in. ◀

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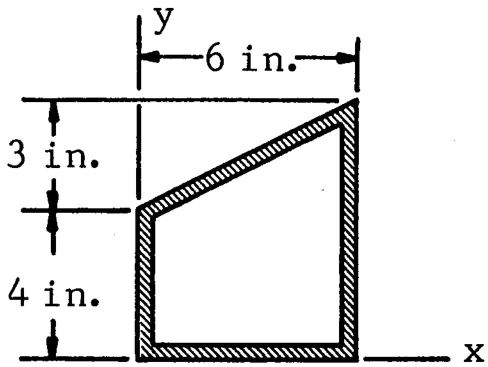
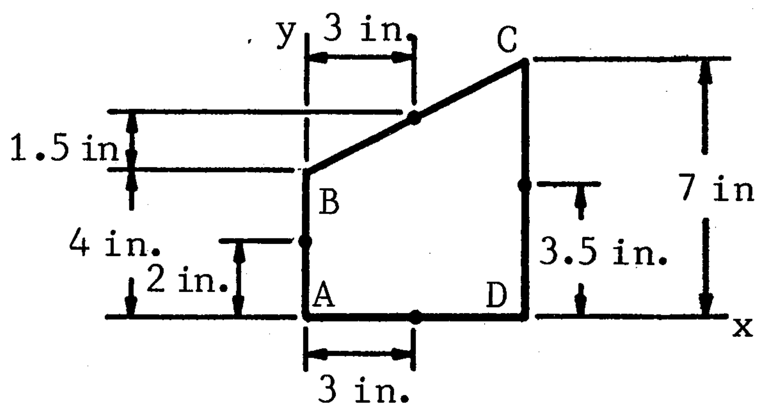


A thin homogeneous wire is bent to form the figure shown. Locate the center of gravity of the wire figure.

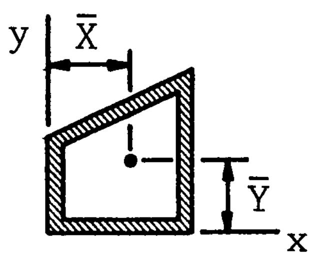
Solution. Center of gravity of homogeneous wire is located at the centroid of the corresponding line.



$$BC^2 = (3 \text{ in.})^2 + (6 \text{ in.})^2$$

$$BC = 6.708 \text{ in.}$$

	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
AB	4	0	2	0	8
BC	6.708	3	5.5	20.124	36.894
CD	7	6	3.5	42	24.5
DA	6	3	0	18	0
Σ	23.708			80.124	69.394



$$\bar{X}\,\Sigma L = \Sigma \bar{x} L$$

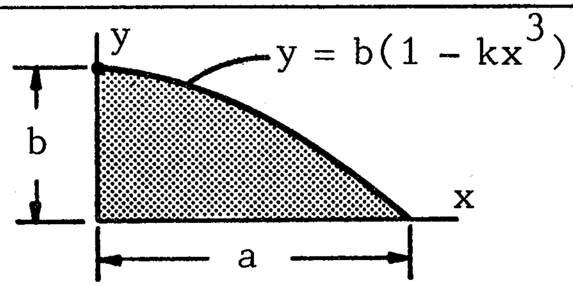
$$\bar{X}(23.708 \text{ in.}) = 80.124 \text{ in}^2$$

$$\bar{X} = 3.38 \text{ in.}$$ ◀

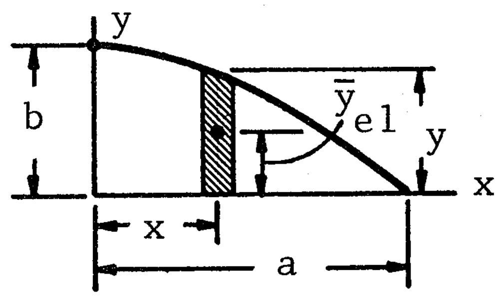
$$\bar{Y}\,\Sigma L = \Sigma \bar{y} L$$

$$\bar{Y}(23.708 \text{ in.}) = 69.394 \text{ in}^2$$

$$\bar{Y} = 2.93 \text{ in.}$$ ◀



Determine the centroid of the area shown.



Value of k. For $x = a$, $y = 0$

$0 = b(1 - ka^3) \qquad k = 1/a^3$

$$y = b\left(1 - \frac{x^3}{a^3}\right)$$

Vertical Element. $\bar{x}_{el} = x$, $\bar{y}_{el} = y/2$, $dA = y\,dx$

$$A = \int dA = \int_0^a y\,dx = \int_0^a b\left(1 - \frac{x^3}{a^3}\right)dx = b\left[x - \frac{x^4}{4a^3}\right]_0^a = \frac{3}{4}ab$$

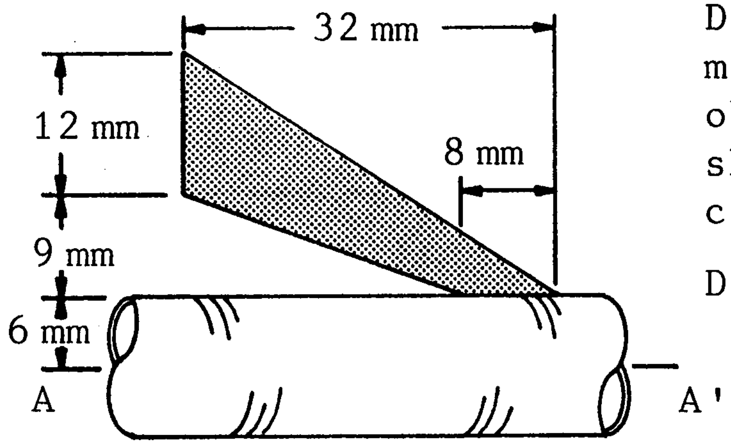
$$Q_y = \int \bar{x}_{el}\,dA = \int_0^a xy\,dA = \int_0^a b\left(x - \frac{x^4}{a^3}\right)dx = b\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]_0^a = \frac{3}{10}a^2b$$

$$Q_x = \int \bar{y}_{el}\,dA = \int_0^a \left(\frac{y}{2}\right)y\,dx = \frac{1}{2}\int_0^a b^2\left(1 - \frac{x^3}{a^3}\right)^2 dx$$

$$= \frac{b^2}{2}\int_0^a \left(1 - \frac{2x^3}{a^3} + \frac{x^6}{a^6}\right)dx = \frac{b^2}{2}\left[x - \frac{x^4}{2a^3} + \frac{x^7}{7a^6}\right]_0^a = \frac{9}{28}ab^2$$

$\bar{x}A = Q_y: \quad \bar{x}\left(\frac{3}{4}ab\right) = \frac{3}{10}a^2b \qquad \bar{x} = \frac{2}{5}a$ ◀

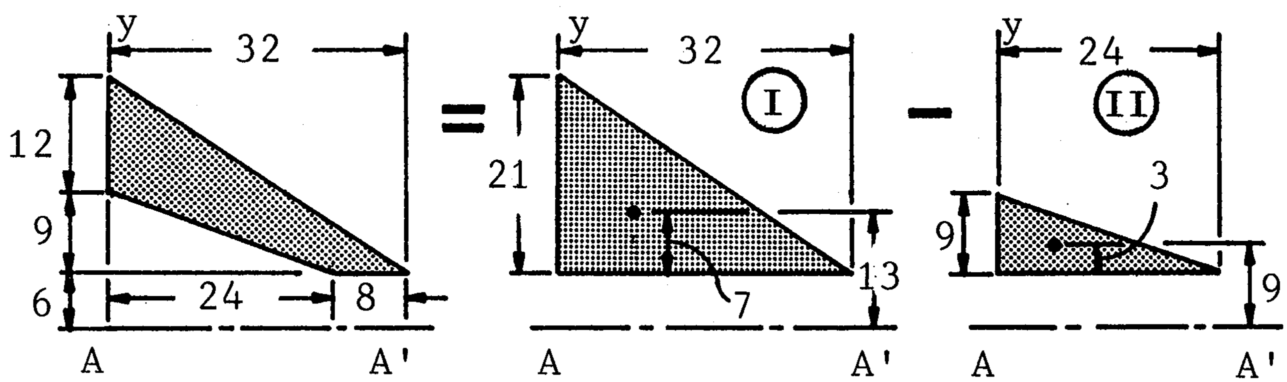
$\bar{y}A = Q_x: \quad \bar{y}\left(\frac{3}{4}ab\right) = \frac{9}{28}ab^2 \qquad \bar{y} = \frac{3}{7}b$ ◀



Determine the volume and mass of the pipe collar obtained by rotating the shaded area about the centerline AA' of the pipe.

Density: $\rho = 7200\ \text{kg/m}^3$

Solution. Shaded area is difference of two triangles. (Dimmensions in mm)



Triangle I: $A_I = \frac{1}{2}(21)(32) = 336\ \text{mm}^2$ $\bar{y}_I = 13\ \text{mm}$

Triangle II: $A_{II} = \frac{1}{2}(9)(24) = 108\ \text{mm}^2$ $\bar{y}_{II} = 9\ \text{mm}$

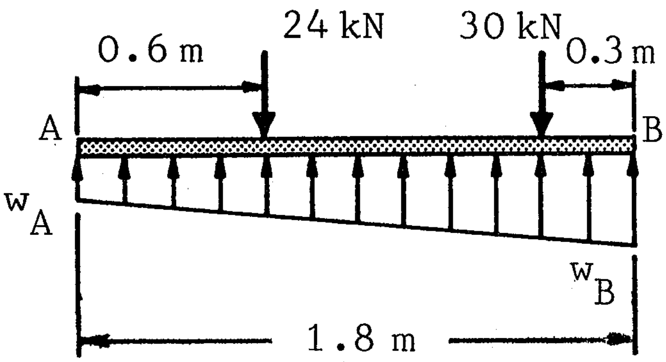
Volume. $V = 2\pi A_I \bar{y}_I - 2\pi A_{II}\bar{y}_{II}$

$= 2\pi[(336\ \text{mm}^2)(13\ \text{mm}) - (108\ \text{mm}^2)(9\ \text{mm})]$

$= 21.34 \times 10^3\ \text{mm}^3$ $V = 21.3 \times 10^3\ \text{mm}^3$ ◀

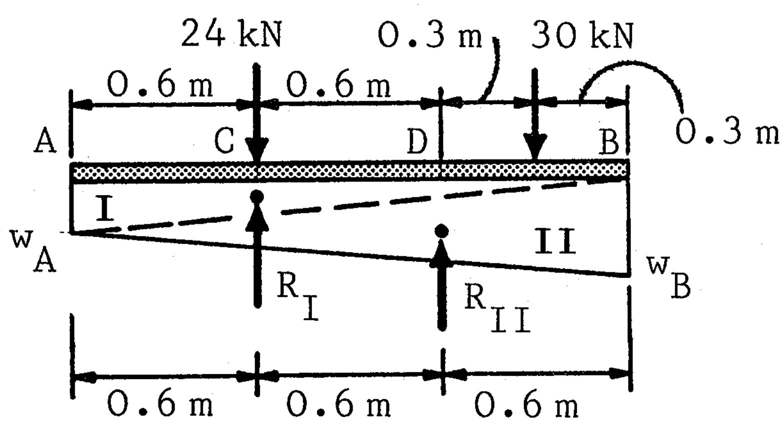
Mass. $m = \rho V = (7200\ \text{kg/m}^3)(21.34 \times 10^3)(10^{-9}\ \text{m}^3)$

$= 153.6 \times 10^{-3}\ \text{kg}$ $m = 153.6\ \text{g}$ ◀



Beam AB rests on soil which exerts a linearly distributed upward load as shown. For the given loading, determine values of w_A and w_B corresponding to equilibrium.

Solution. Divide area of load curve into two triangles.



$$R_I = \frac{1}{2} w_A (1.8\text{ m}) = 0.9 w_A$$

$$R_{II} = \frac{1}{2} w_B (1.8\text{ m}) = 0.9 w_B$$

Equilibrium Equations

$+\circlearrowleft \Sigma M_D = 0$: $(24\text{ kN})(0.6\text{ m}) - (30\text{ kN})(0.3\text{ m}) - R_I(0.6\text{ m}) = 0$

$14.4 - 9 - (0.9 w_A)(0.6) = 0$

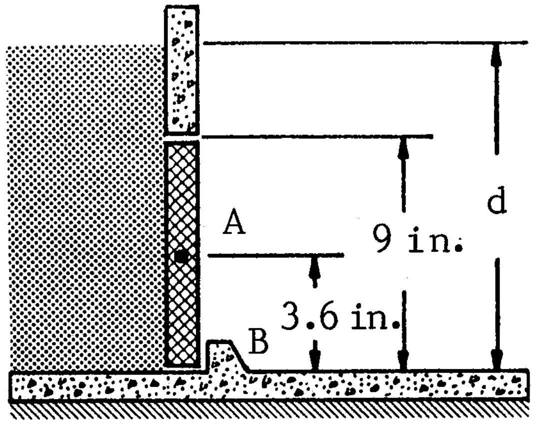
$5.4 - 0.54 w_A = 0$ $\qquad w_A = 10\text{ kN/m}$ ◀

$+\uparrow \Sigma F_y = 0$: $-24\text{ kN} - 30\text{ kN} + R_I + R_{II} = 0$

$-54 + (0.9) w_A + (0.9) w_B = 0$

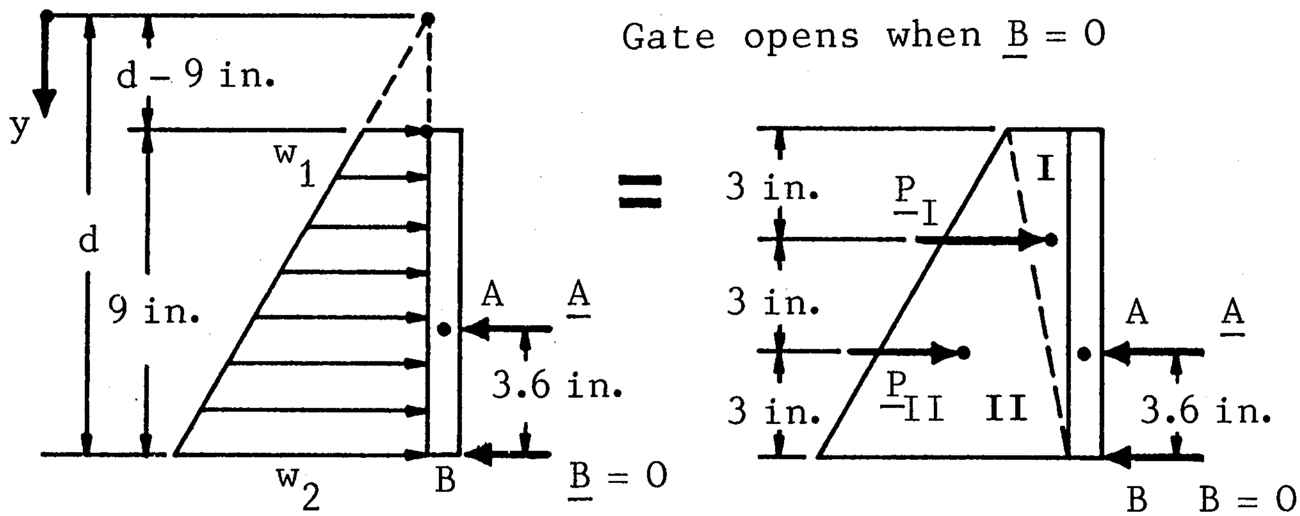
$-54 + (0.9)(10) + 0.9 w_B = 0$

$-45 + 0.9 w_B = 0$ $\qquad w_B = 50\text{ kN/m}$ ◀



An automatic valve consists of a 9 × 9 in. square plate which is pivoted about a horizontal axis at A. Determine the depth of water d for which the gate will open.

Free Body. The gate is chosen as a free body.



Gate opens when B = 0

Since gate is 9 in. wide, $w = 9p = 9\gamma y$

$$w_1 = 9\gamma(d-9) \qquad P_I = \frac{1}{2}9w_1 = \frac{1}{2}(9)(9\gamma)(d-9)$$

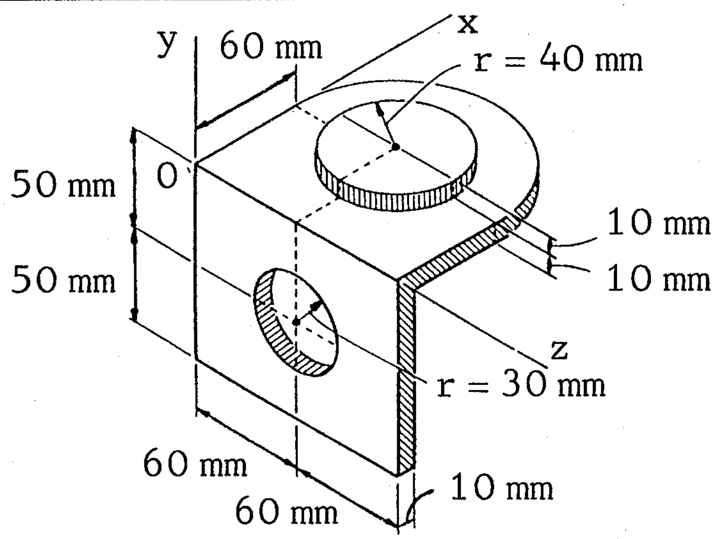
$$w_2 = 9\gamma d \qquad P_{II} = \frac{1}{2}9w_2 = \frac{1}{2}(9)(9\gamma)d$$

$$+\circlearrowright \Sigma M_A = 0: \quad P_I(6\text{ in.} - 3.6\text{ in.}) - P_{II}(3.6\text{ in.} - 3\text{ in.}) = 0$$

$$\left[\frac{1}{2}(9)(9\gamma)(d-9)\right](2.4) - \left[\frac{1}{2}(9)(9\gamma)d\right](0.6) = 0$$

$$(d-9)(2.4) - d(0.6) = 0$$

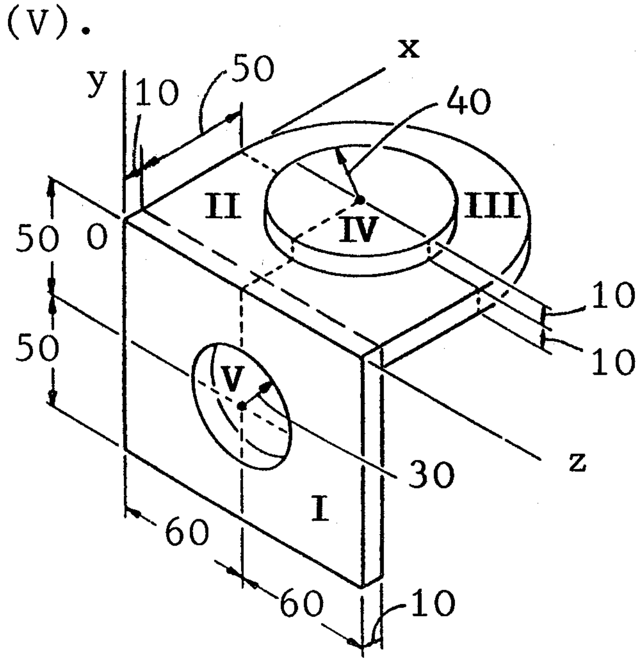
$$1.8d - 21.6 = 0 \qquad d = 12\text{ in.}$$

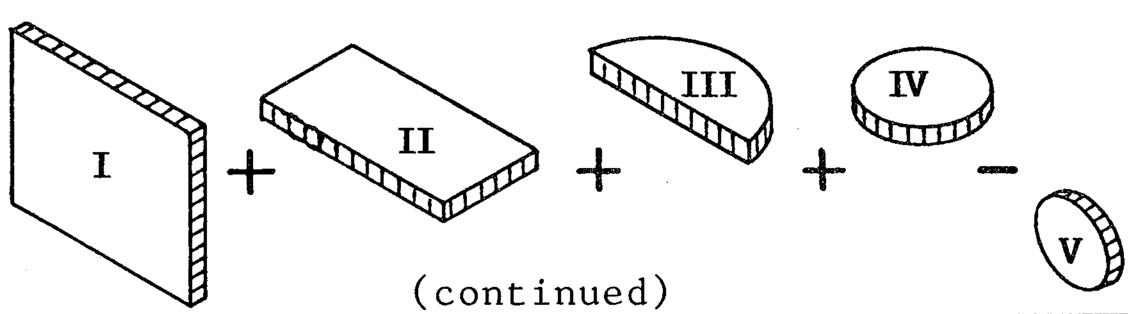


For the machine element shown, locate the y coordinate of the center of gravity.

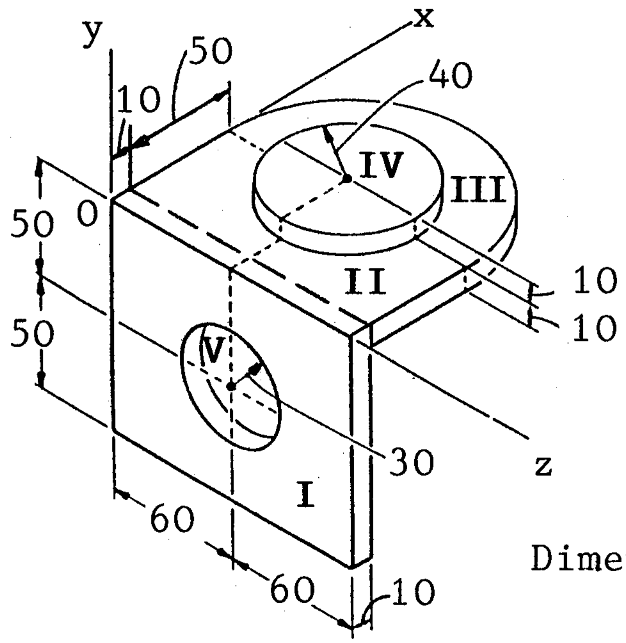
Solution. The machine element consists of two rectangular plates (I and II), plus a half cylinder (III), plus a 40-mm-radius cylinder (IV), minus a 30-mm-radius cylinder (V).

Dimensions in mm





(continued)

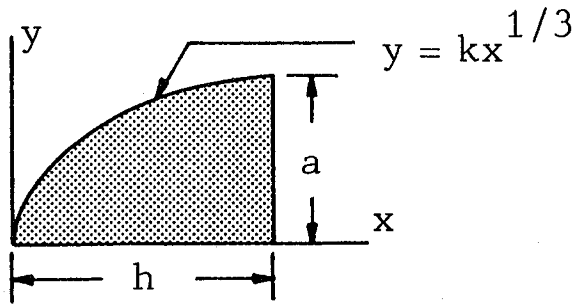


Dimemsions in mm

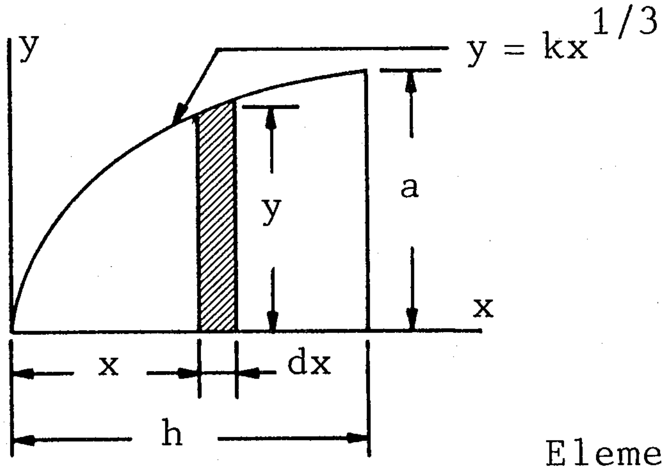
		V, mm^3	$\bar{y}$, mm	$\bar{y}V$, mm^4
I	Rectangular Plate	$(120)(100)(10) = 120.00\times10^3$	-50	-6.00×10^6
II	Rectangular Plate	$(120)(50)(10) = 60.00\times10^3$	-5	-0.300×10^6
III	Half Cylinder	$\frac{\pi}{2}(60)^2(10) = 56.55\times10^3$	-5	-0.283×10^6
IV	Cylinder	$\pi(40)^2(10) = 50.27\times10^3$	$+5$	$+0.251\times10^6$
V	−(Cylinder)	$-\pi(30)^2(10) = -28.27\times10^3$	-50	$+1.414\times10^6$
	Σ	$+258.6\times10^3$		-4.918×10^6

$\bar{Y}\Sigma V = \Sigma\bar{y}V$: $\quad \bar{Y}(258.6\times10^3\ mm^3) = -4.918\times10^6\ mm^4$

$\bar{Y} = -19.02$ mm ◀



Locate the centroid of the volume obtained by rotating the shaded area about the x axis.



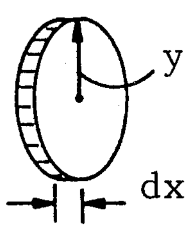
<u>Value of k</u>

For $x = h$, $y = a$

$$a = kh^{1/3} \qquad k = \frac{a}{h^{1/3}}$$

$$y = \frac{a}{h^{1/3}} x^{1/3}$$

<u>Element of Volume</u>. Disk of radius y and thickness dx.



$$dV = \pi y^2 dx = \frac{\pi a^2}{h^{2/3}} x^{2/3} dx$$

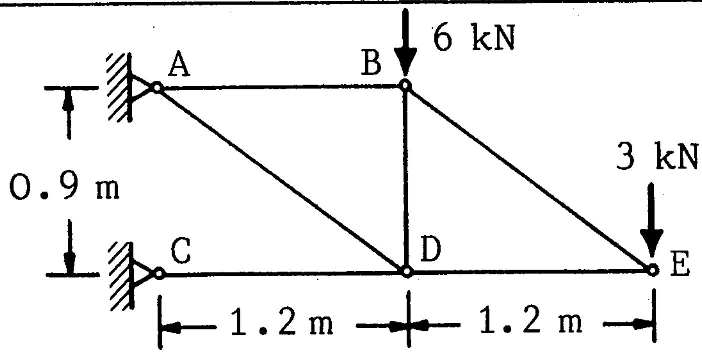
$$\bar{x}_{el} = x$$

$$V = \int dV = \frac{\pi a^2}{h^{2/3}} \int_0^h x^{2/3} dx = \frac{\pi a^2}{h^{2/3}} \left[\frac{3}{5} x^{5/3} \right]_0^h = \frac{3}{5} \pi a^2 h$$

$$\int \bar{x}_{el} dV = \frac{\pi a^2}{h^{2/3}} \int_0^h x^{5/3} dx = \frac{\pi a^2}{h^{2/3}} \left[\frac{3}{8} x^{8/3} \right]_0^h = \frac{3}{8} \pi a^2 h^2$$

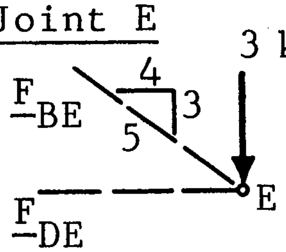
$$\bar{x} V = \int \bar{x}_{el} dV$$

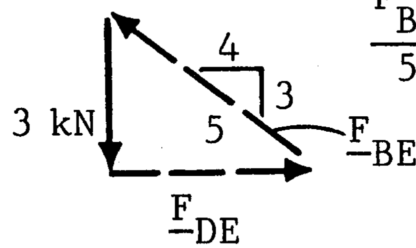
$$\bar{x}\left(\frac{3}{5}\pi a^2 h\right) = \frac{3}{8}\pi a^2 h^2 \qquad \bar{x} = \frac{5}{8} h \blacktriangleleft$$



Determine the force in each member of the truss shown.

Joint E



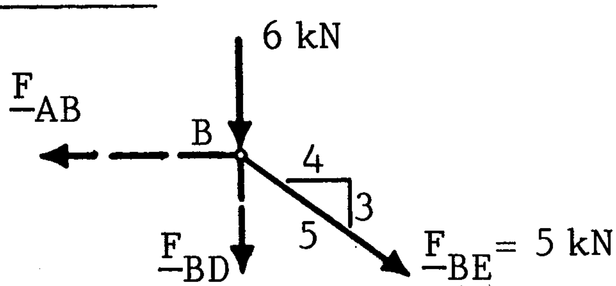


$$\frac{F_{BE}}{5} = \frac{F_{DE}}{4} = \frac{3\text{ kN}}{3}$$

$F_{BE} = 5\text{ kN T}$ ◀

$F_{DE} = 4\text{ kN C}$ ◀

Joint B



$\xrightarrow{+} \Sigma F_x = 0$:

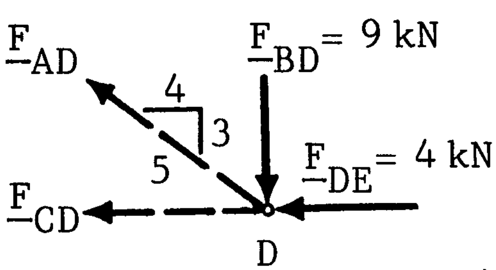
$$\frac{4}{5}(5\text{ kN}) - F_{AB} = 0$$

$F_{AB} = +4\text{ kN}$ $F_{AB} = 4\text{ kN T}$ ◀

$+\uparrow \Sigma F_y = 0$: $-6\text{ kN} - \frac{3}{5}(5\text{ kN}) - F_{BD} = 0$

$F_{BD} = -9\text{ kN}$ $F_{BD} = 9\text{ kN C}$ ◀

Joint D



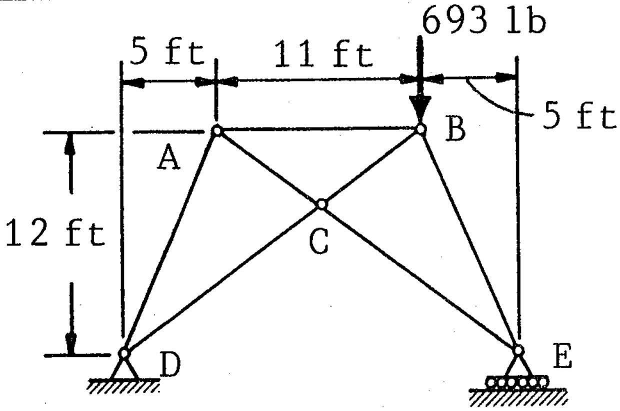
$+\uparrow \Sigma F_y = 0$:

$$-9\text{ kN} + \frac{3}{5}F_{AD} = 0$$

$F_{AD} = +15\text{ kN}$ $F_{AD} = 15\text{ kN T}$ ◀

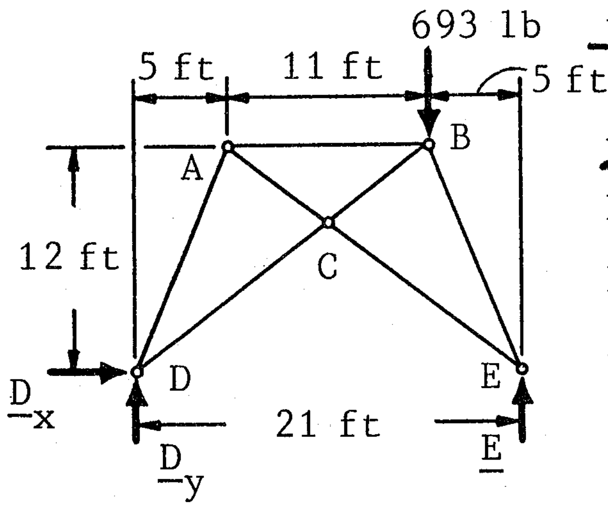
$\xrightarrow{+} \Sigma F_x = 0$: $-4\text{ kN} - \frac{4}{5}(15\text{ kN}) - F_{CD} = 0$

$F_{CD} = -16\text{ kN}$ $F_{CD} = 16\text{ kN C}$ ◀



Determine the force in each member of the truss shown.

Reactions. The entire truss is chosen as a free body.



$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad D_x = 0$

$+\circlearrowleft \Sigma M_E = 0$:

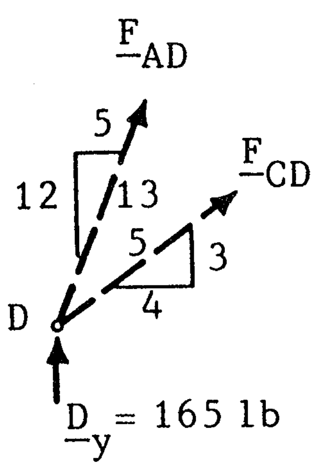
$D_y(21\text{ ft}) - (693\text{ lb})(5\text{ ft}) = 0$

$D_y = +165\text{ lb} \qquad D_y = 165\text{ lb}\uparrow$

$+\uparrow\Sigma F_y = 0$: $\quad 165\text{ lb} - 693\text{ lb} + E = 0$

$E = +528\text{ lb} \qquad E = 528\text{ lb}\uparrow$

Joint D



$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad \frac{5}{13}F_{AD} + \frac{4}{5}F_{CD} = 0 \qquad (1)$

$+\uparrow\Sigma F_y = 0$:

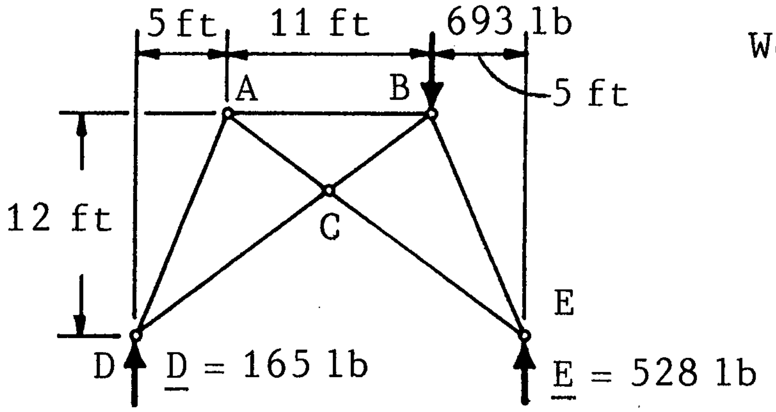
$$\frac{12}{13}F_{AD} + \frac{3}{5}F_{CD} + 165\text{ lb} = 0 \qquad (2)$$

Solve (1) and (2) simultaneously:

$F_{AD} = -260\text{ lb} \qquad F_{AD} = 260\text{ lb C}$ ◀

$F_{CD} = +125\text{ lb} \qquad F_{CD} = 125\text{ lb T}$ ◀

(continued)

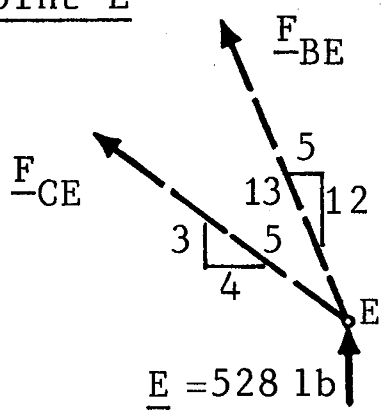


We have:

$F_{AD} = 260$ lb C ◀

$F_{CD} = 125$ lb T ◀

Joint E



$\xleftarrow{+} \Sigma F_x = 0: \quad \frac{5}{13} F_{BE} + \frac{4}{5} F_{CE} = 0 \qquad (3)$

$+\uparrow \Sigma F_y = 0:$

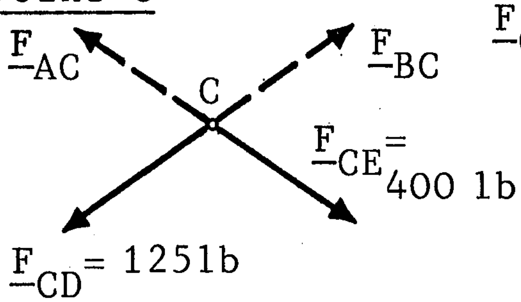
$$\frac{12}{13} F_{BE} + \frac{3}{5} F_{CE} + 528 \text{ lb} = 0 \qquad (4)$$

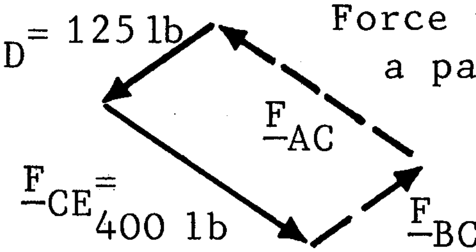
Solve (3) and (4) simultaneously:

$F_{BE} = -832$ lb $\qquad F_{BE} = 832$ lb C ◀

$F_{CE} = +400$ lb $\qquad F_{CE} = 400$ lb T ◀

Joint C





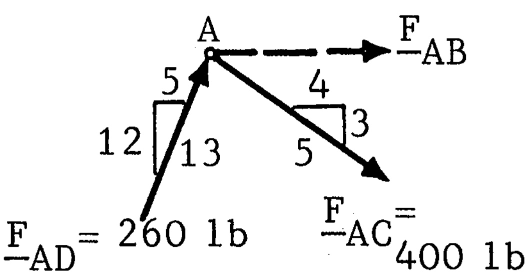
Force polygon is a parallelogram

$F_{AC} = F_{CE}$

$F_{BC} = F_{CD}$

$F_{AC} = 400$ lb T; $\quad F_{BC} = 125$ lb T ◀

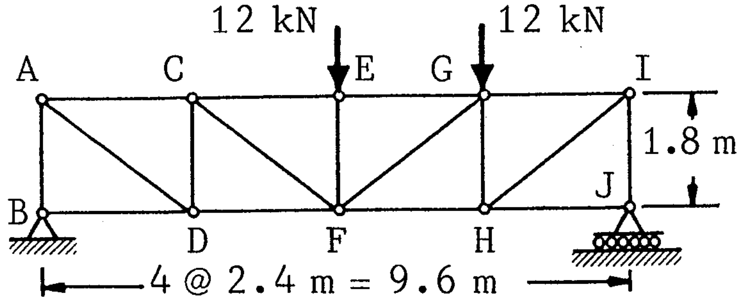
Joint A



$\xrightarrow{+} \Sigma F_x = 0:$

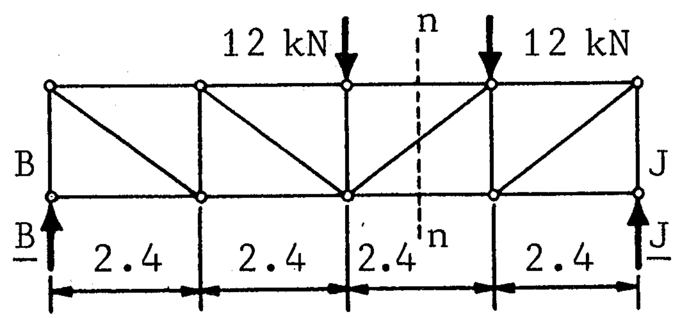
$$\frac{5}{13}(260 \text{ lb}) + \frac{4}{5}(400 \text{ lb}) + F_{AB} = 0$$

$F_{AB} = -420$ lb $\qquad F_{AB} = 420$ lb C ◀



Determine the force in members FG and FH.

Reactions. The entire truss is chosen as a free body.



Dimensions in m

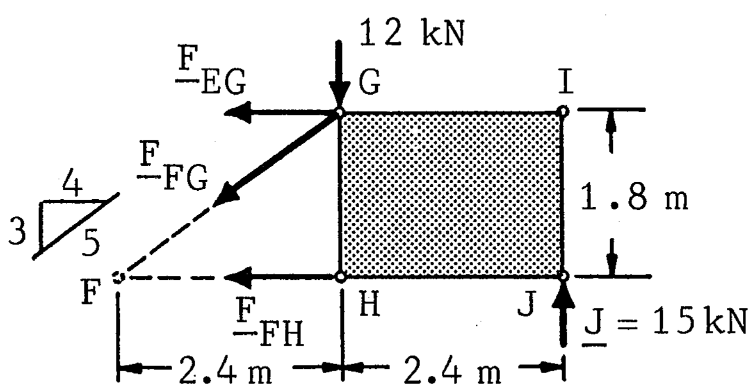
$+\circlearrowleft \Sigma M_J = 0$: $(12\,\text{kN})(4.8\,\text{m}) + (12\,\text{kN})(2.4\,\text{m}) - B(9.6\,\text{m}) = 0$

$B = +9\,\text{kN}$ $\qquad$ $\underline{B} = 9\,\text{kN}\uparrow$

$+\uparrow \Sigma F_y = 0$: $9\,\text{kN} - 12\,\text{kN} - 12\,\text{kN} + J = 0$

$J = +15\,\text{kN}$ $\qquad$ $\underline{J} = 15\,\text{kN}\uparrow$

Force in Members. Consider portion of truss to right of section n-n as a free body.



Member FG

$+\uparrow \Sigma F_y = 0$:

$-\frac{3}{5}F_{FG} - 12\,\text{kN} + 15\,\text{kN} = 0$

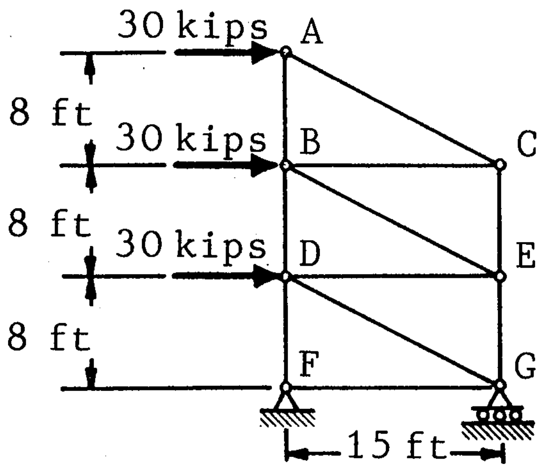
$F_{FG} = +5\,\text{kN}$

$F_{FG} = 5\,\text{kN T}$ ◀

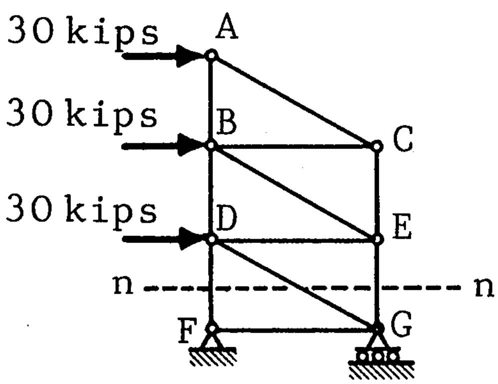
Member FH

$+\circlearrowleft \Sigma M_G = 0$: $(15\,\text{kN})(2.4\,\text{m}) - F_{FH}(1.8\,\text{m}) = 0$

$F_{FH} = +20\,\text{kN}$ $\qquad$ $F_{FH} = 20\,\text{kN T}$ ◀

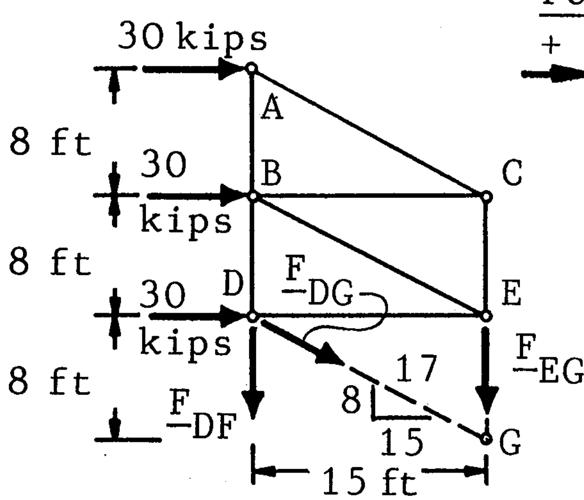


Determine the force in members DG and EG.



Solution

Section n–n is passed through the truss. The portion of the truss above the section will be chosen as a free body.



Force in Member DG

$\xrightarrow{+} \Sigma F_x = 0$:

$$3(30\text{ kips}) + \frac{15}{17} F_{DG} = 0$$

$$F_{DG} = -102\text{ kips}$$

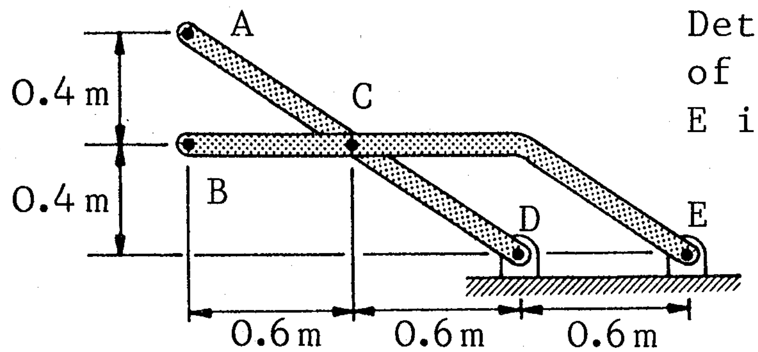
$$F_{DG} = 102\text{ kips C}$$ ◀

Force in Member EG

$+\circlearrowright \Sigma M_D = 0$: $(30\text{ kips})(16\text{ ft}) + (30\text{ kips})(8\text{ ft}) + F_{EG}(15\text{ ft}) = 0$

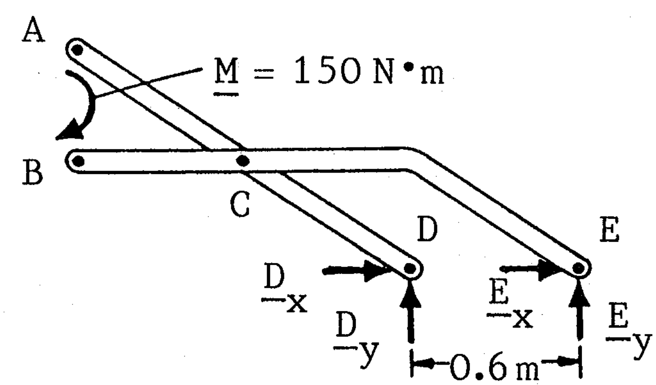
$$F_{EG} = -48\text{ kips}$$

$$F_{EG} = 48\text{ kips C}$$ ◀



Determine the components of the reactions at D and E if a couple $\underline{M} = 150\ \text{N}\cdot\text{m}$ is applied (a) at A, (b) at B.

Entire Frame. The entire frame is chosen as a free body. The point of application of the couple on the free body is immaterial.



$+\circlearrowright\ \Sigma M_D = 0: \quad E_y(0.6\ \text{m}) - 150\ \text{N}\cdot\text{m} = 0$

$E_y = +250\ \text{N} \qquad \underline{E}_y = 250\ \text{N}\uparrow$ ◄

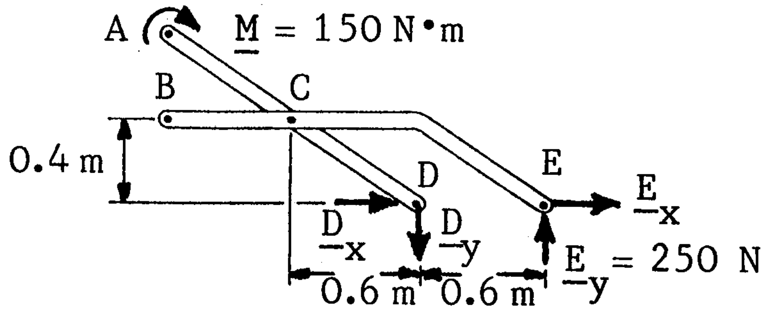
$+\uparrow \Sigma F_y = 0: \quad D_y + 250\ \text{N} = 0$

$D_y = -250\ \text{N} \qquad \underline{D}_y = 250\ \text{N}\downarrow$ ◄

$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + E_x = 0 \qquad (1)$

(continued)

(a) Couple Applied at A.

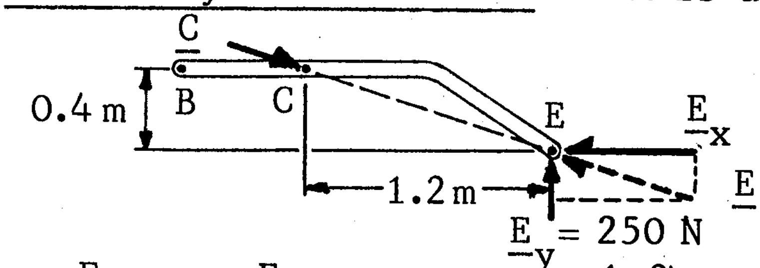


We have: $D_x + E_x = 0$ (1)

$D_y = 250\ \text{N}\downarrow$ ◀

$E_y = 250\ \text{N}\uparrow$ ◀

Free Body: Member BCE. It is a two-force member.



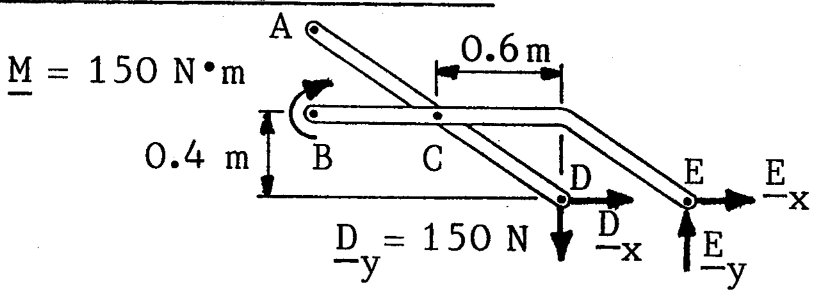
$$\frac{E_x}{1.2\ \text{m}} = \frac{E_y}{0.4\ \text{m}} \qquad E_x = \frac{1.2}{0.4}(250\ \text{N}) = 750\ \text{N}$$

$E_x = 750\ \text{N} \leftarrow$ ◀

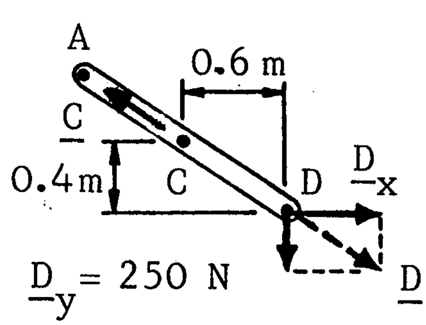
Eq.(1): $D_x - 750\ \text{N} = 0$

$D_x = 750\ \text{N} \rightarrow$ ◀

(b) Couple Applied at B.



Free Body: Member ACD. It is a two-force member.

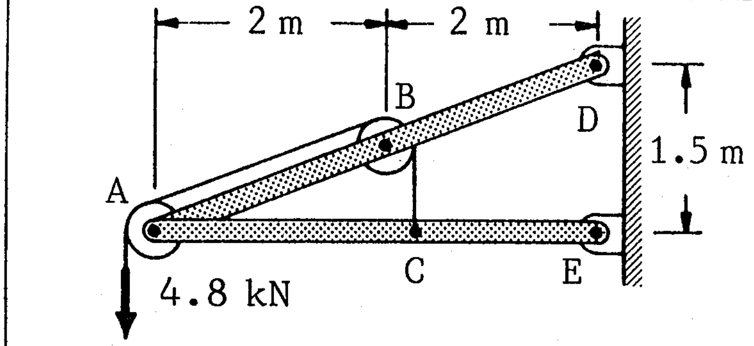


$$\frac{D_x}{0.6\ \text{m}} = \frac{D_y}{0.4\ \text{m}}$$

$$D_x = \frac{0.6}{0.4}(250\ \text{N}) = 375\ \text{N}$$

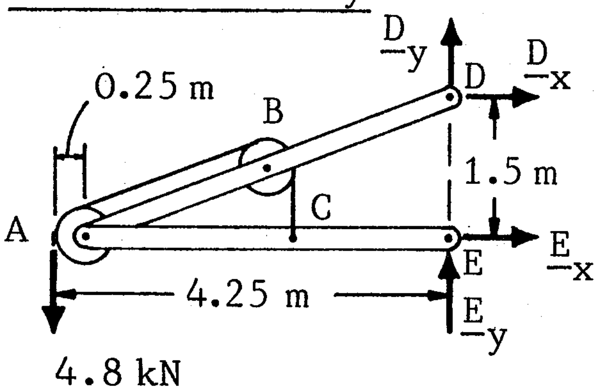
$D_x = 375\ \text{N} \rightarrow$ ◀

Eq.(1): $E_x = 375\ \text{N} \leftarrow$ ◀



Each pulley has a radius of 250 mm. Determine the components of the reactions at D and E.

Entire Assembly



$+\circlearrowleft \Sigma M_E = 0$:

$(4.8\text{ kN})(4.25\text{ m}) - D_x(1.5\text{ m}) = 0$

$D_x = +13.60\text{ kN} \qquad \mathbf{D}_x = 13.60\text{ kN} \rightarrow$ ◀

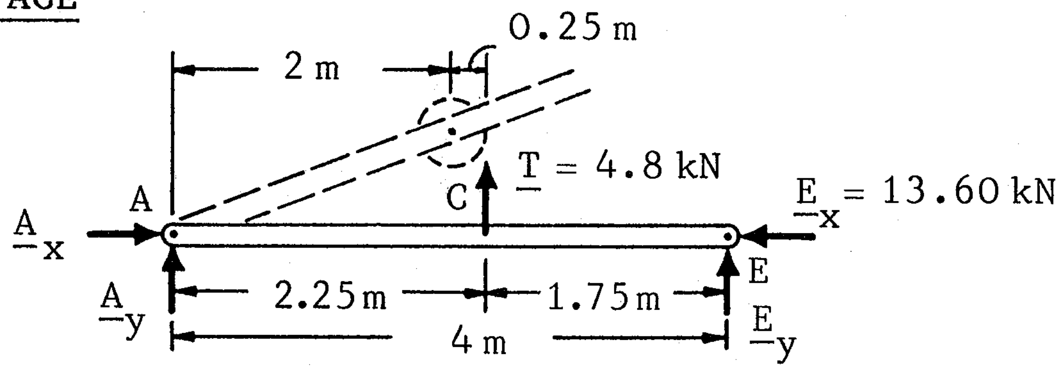
$\xrightarrow{+} \Sigma F_x = 0$:

$E_x + 13.60\text{ kN} = 0$

$E_x = -13.60\text{ kN} \qquad \mathbf{E}_x = 13.60\text{ kN} \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $\qquad D_y + E_y - 4.8\text{ kN} = 0$ (1)

Member ACE

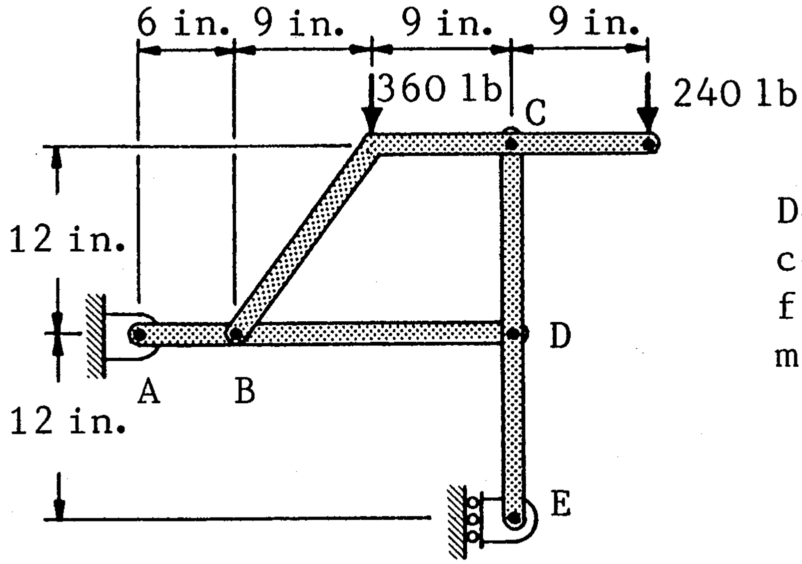


$+\circlearrowleft \Sigma M_A = 0$: $\qquad (4.8\text{ kN})(2.25\text{ m}) + E_y(4\text{ m}) = 0$

$E_y = -2.70\text{ kN} \qquad \mathbf{E}_y = 2.70\text{ kN} \downarrow$ ◀

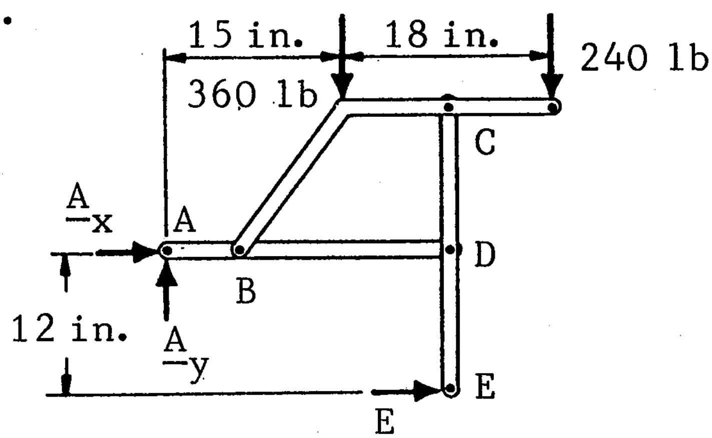
From Eq.(1): $\qquad D_y - 2.70\text{ kN} - 4.80\text{ kN} = 0$

$D_y = +7.50\text{ kN} \qquad \mathbf{D}_y = 7.50\text{ kN} \uparrow$ ◀



Determine the components of all forces acting on member ABD.

Entire Frame. The entire frame is chosen as a free body.



$+\circlearrowright \Sigma M_A = 0$: $E(12 \text{ in.}) - (360 \text{ lb})(15 \text{ in.}) - (240 \text{ lb})(33 \text{ in.}) = 0$

$E = +1110 \text{ lb}$ $\quad \mathbf{E} = 1110 \text{ lb} \rightarrow$

$\xrightarrow{+} \Sigma F_x = 0$: $A_x + 1110 \text{ lb} = 0$

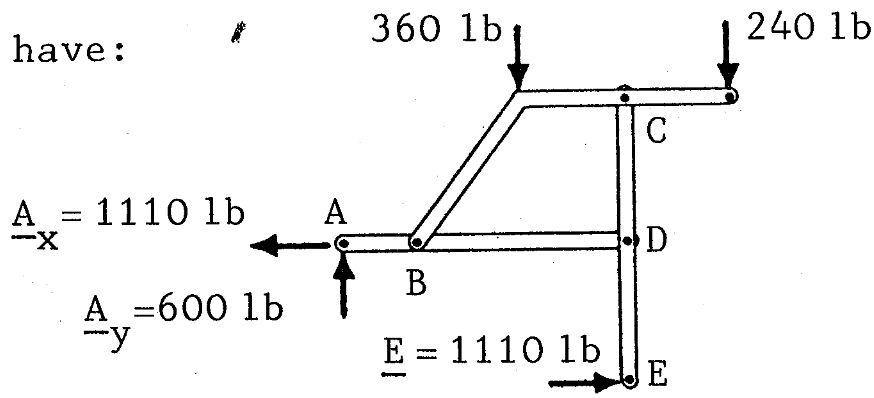
$A_x = -1110 \text{ lb}$ $\quad \mathbf{A}_x = 1110 \text{ lb} \leftarrow$ ◀

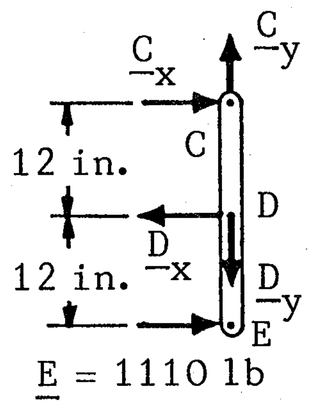
$+\uparrow \Sigma F_y = 0$: $A_y - 360 \text{ lb} - 240 \text{ lb} = 0$

$A_y = +600 \text{ lb}$ $\quad \mathbf{A}_y = 600 \text{ lb} \uparrow$ ◀

(continued)

We have:





__Member CDE__

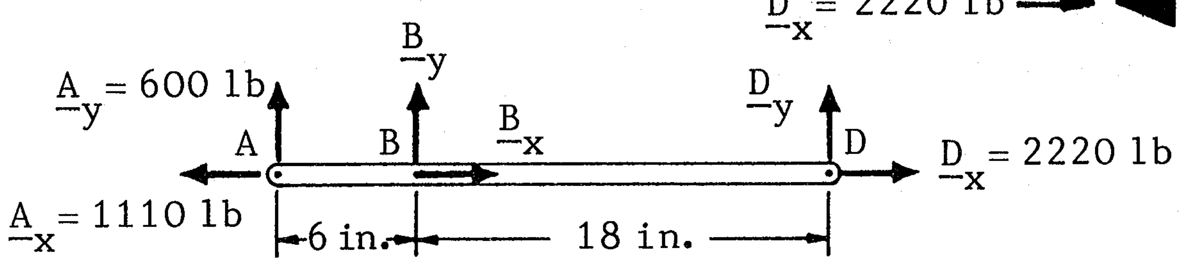
$+\circlearrowright \Sigma M_C = 0$:

$(1110 \text{ lb})(24 \text{ in.}) - D_x(12 \text{ in.}) = 0$

$D_x = +2220 \text{ lb}$ $\quad \mathbf{D}_x = 2220 \text{ lb} \leftarrow$ (on CDE)

__Member ABD__. From above, we note

$\mathbf{D}_x = 2220 \text{ lb} \rightarrow$ ◀



$+\circlearrowright \Sigma M_B = 0$: $\quad D_y(18 \text{ in.}) - (600 \text{ lb})(6 \text{ in.}) = 0$

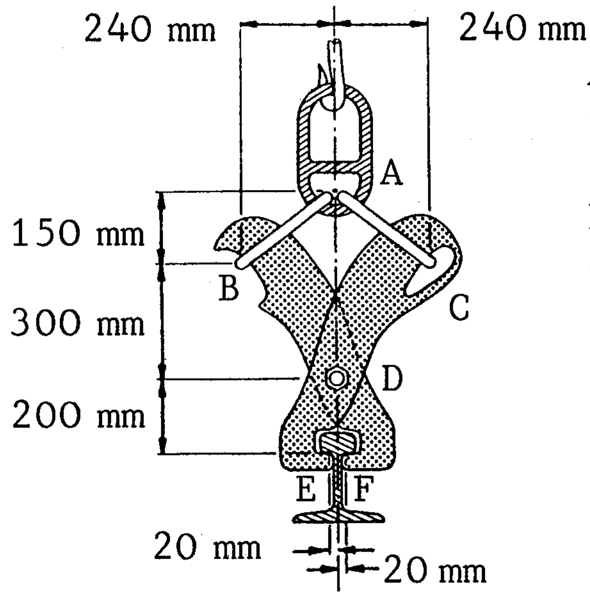
$D_y = +200 \text{ lb}$ $\quad \mathbf{D}_y = 200 \text{ lb} \uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $\quad B_x + 2220 \text{ lb} - 1110 \text{ lb} = 0$

$B_x = -1110 \text{ lb}$ $\quad \mathbf{B}_x = 1110 \text{ lb} \leftarrow$ ◀

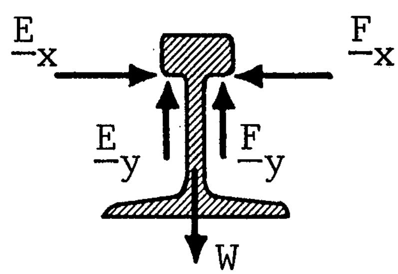
$+\uparrow \Sigma F_y = 0$: $\quad B_y + 200 \text{ lb} + 600 \text{ lb} = 0$

$B_y = -800 \text{ lb}$ $\quad \mathbf{B}_y = 800 \text{ lb} \downarrow$ ◀



A 9-m length of railroad track of mass 40 kg/m is lifted by the tongs shown. Determine the forces exerted at D and F on tong BDF.

Free Body: Rail. Weight of 9-m length of rail:



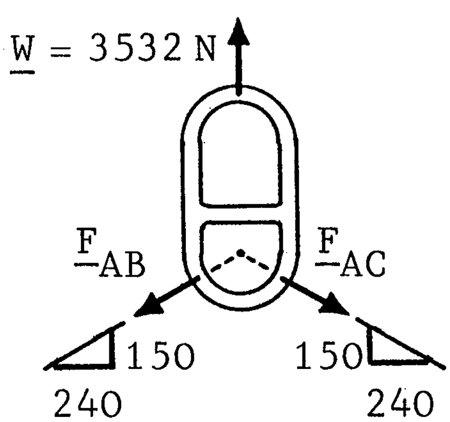
$$W = (9\text{ m})(40\text{ kg/m})(9.81\text{ m/s}^2)$$

$$W = 3532\text{ N}$$

By symmetry:

$$E_y = F_y = \frac{1}{2}W = \frac{1}{2}(3532\text{ N}) = 1766\text{ N}$$

Free Body: Upper Link. By symmetry: $F_{AC} = F_{AB}$



$$(F_{AB})_y = \frac{1}{2}(3532\text{ N}) = 1766\text{ N}$$

Since AB is a two-force member,

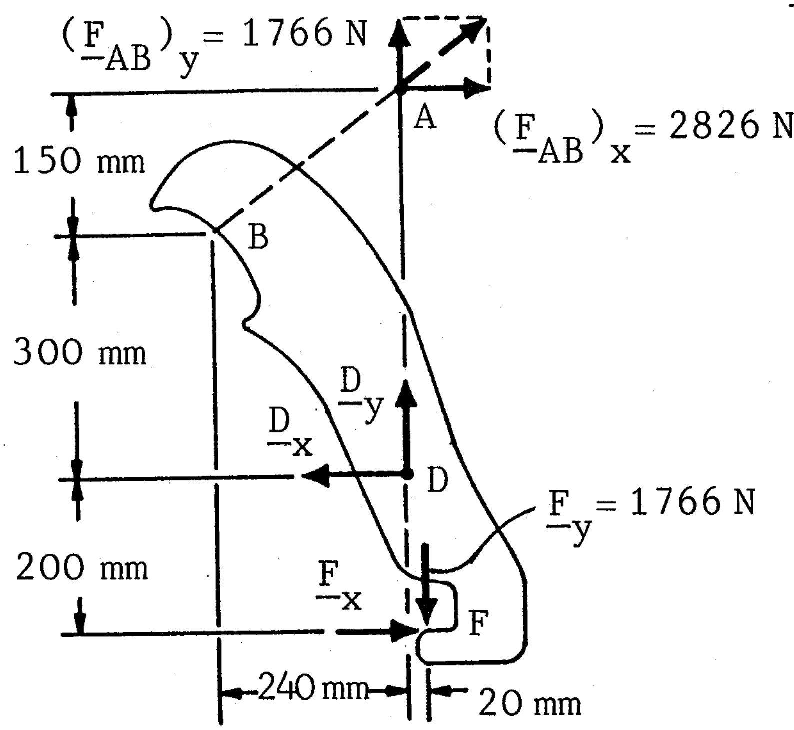
$$\frac{(F_{AB})_x}{240} = \frac{(F_{AB})_y}{150}$$

$$(F_{AB})_x = (1766\text{ N})\left(\frac{240}{150}\right) = 2826\text{ N}$$

(continued)

We have: $F_y = 1766\text{ N}$, $(F_{AB})_x = 2826\text{ N}$, $(F_{AB})_y = 1766\text{ N}$

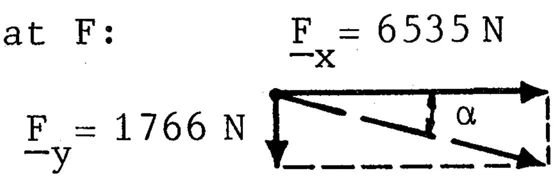
Free Body: Tong BDF. Attach $\underline{F}_{AB}$ at A.



$+\circlearrowright \Sigma M_D = 0$: $F_x(200\text{ mm}) - (2826\text{ N})(450\text{ mm}) - (1766\text{ N})(20\text{ mm}) = 0$

$F_x = 6535\text{ N}$

Force at F:



$\underline{F} = 6770\text{ N} \searrow 15.1^\circ$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $-D_x + (F_{AB})_x + F_x = 0$

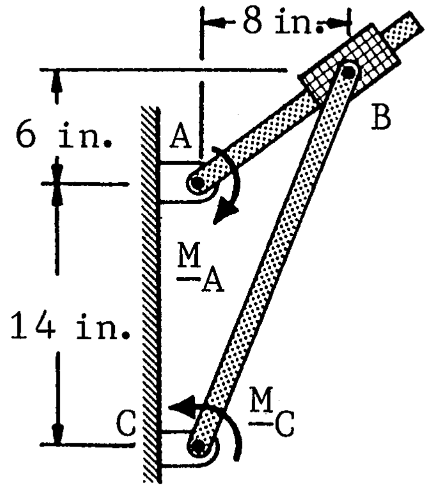
$-D_x + 2826\text{ N} + 6535\text{ N} = 0$ $D_x = 9360\text{ N}$

$+\uparrow \Sigma F_y = 0$: $D_y + (F_{AB})_y - F_y = 0$

$D_y + 1766\text{ N} - 1766\text{ N} = 0$ $D_y = 0$

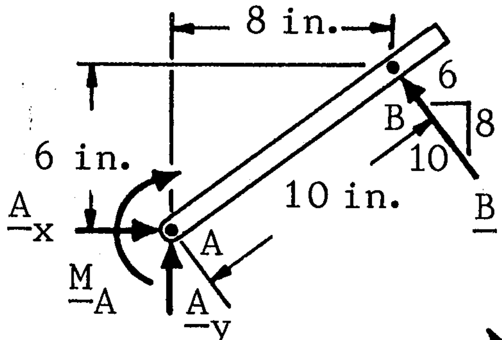
Force at D: $\underline{D} = 9360\text{ N} \leftarrow$ ◀

BEER & JOHNSTON VECTOR MECHANICS FOR ENGINEERS 4/e	STATICS	Problem 6.110 Page 244



Two rods are connected by a frictionless collar at B. Knowing that the magnitude of the couple $\underline{M}_A$ is 500 lb•in., determine (a) the couple $\underline{M}_C$ required for equilibrium, (b) the components of the reaction at C.

<u>Rod AB</u>. Force $\underline{B}$ exerted by collar on AB is ⊥ to AB.



$\underline{M}_A = 500$ lb•in. ↻

$+↻ \Sigma M_A = 0$: $B(10 \text{ in.}) - M_A = 0$

$B(10 \text{ in.}) - 500 \text{ lb•in.} = 0$

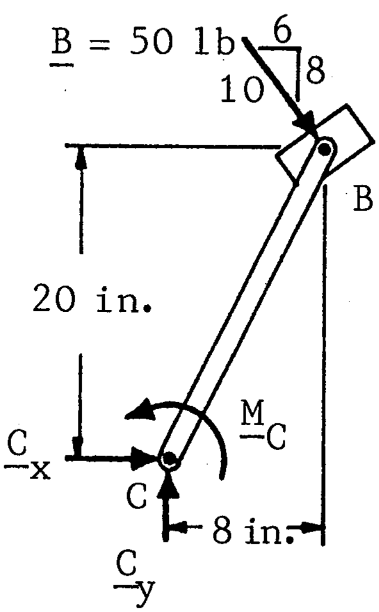
$B = 50$ lb

<u>Rod BC and Collar</u>. $+↻ \Sigma M_C = 0$:

$M_C - (0.6\,B)(20 \text{ in.}) - (0.8\,B)(8 \text{ in.}) = 0$

$M_C - (30 \text{ lb})(20 \text{ in.}) - (40 \text{ lb})(8 \text{ in.}) = 0$

$M_C = 920$ lb•in. $\quad \underline{M}_C = 920$ lb•in. ↻ ◀



$\xrightarrow{+} \Sigma F_x = 0$: $C_x + 0.6\,B = 0$

$C_x + 0.6(50 \text{ lb}) = 0$

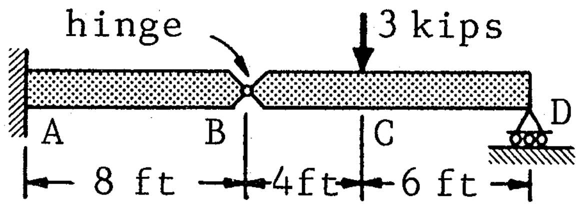
$C_x = -30$ lb $\quad \underline{C}_x = 30$ lb ← ◀

$+\uparrow \Sigma F_y = 0$: $C_y - 0.8\,B = 0$

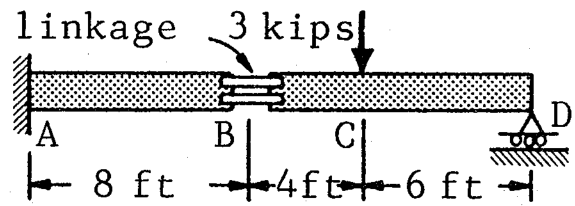
$C_y - 0.8(50 \text{ lb}) = 0$

$C_y = +40$ lb $\quad \underline{C}_y = 40$ lb ↑ ◀

Determine the reactions at the supports.

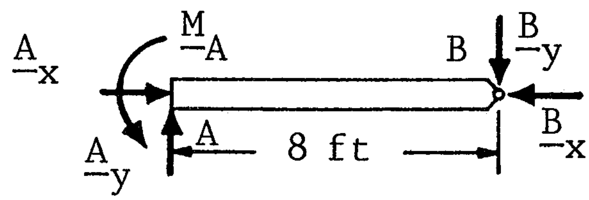


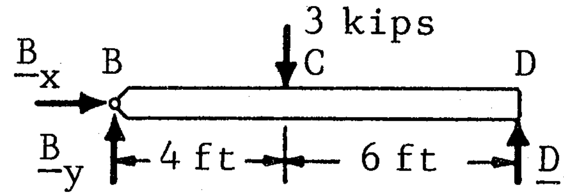
(a)



(b)

(a) <u>Free Bodies.</u> We choose AB and BD as free bodies.





<u>Free Body: Member BD</u>

$+\circlearrowright \Sigma M_B = 0$: $D(10\text{ ft}) - (3\text{ kips})(4\text{ ft}) = 0$

$D = +1.2\text{ kips}$ $\qquad \underline{D} = 1.2\text{ kips}\uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $B_x = 0$ $\qquad B_x = 0$

$+\uparrow \Sigma F_y = 0$: $B_y - 3\text{ kips} + 1.2\text{ kips} = 0$ $\qquad B_y = 1.8\text{ kips}$

<u>Free Body: Member AB</u>

$+\circlearrowright \Sigma M_A = 0$: $M_A - (1.8\text{ kips})(8\text{ ft}) = 0$

$M_A = +14.4\text{ kip}\cdot\text{ft}$ $\qquad \underline{M}_A = 14.4\text{ kip}\cdot\text{ft}\circlearrowright$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - B_x = 0$ $\qquad \underline{A}_x = 0$

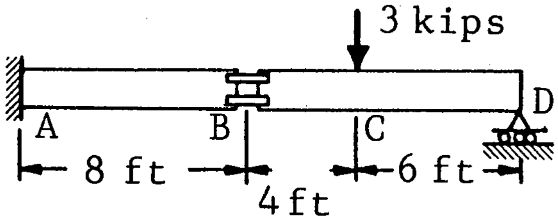
$+\uparrow \Sigma F_y = 0$: $A_y - B_y = 0 \qquad A_y - 1.8\text{ kips} = 0$

$A_y = +1.8\text{ kips}$ $\qquad \underline{A}_y = 1.8\text{ kips}\uparrow$

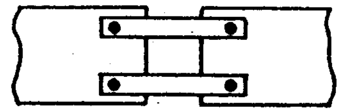
$\underline{A} = 1.8\text{ kips}\uparrow$ ◀

(continued)

(b)

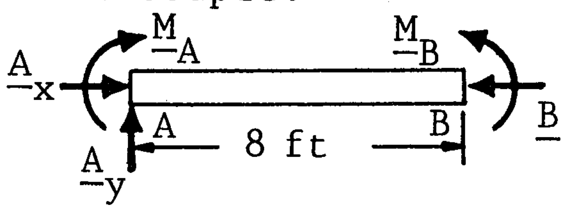


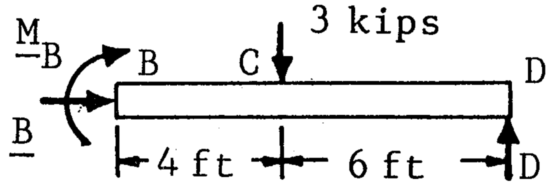
Detail of linkage



Free Bodies. We choose AB and BD as free bodies.

Linkage exerts on each member two horizontal forces which are equivalent to a horizontal force and a couple.





Free Body: Member BD $\xrightarrow{+} \Sigma F_x = 0$: $B = 0$

$+\uparrow \Sigma F_y = 0$: $D - 3\text{ kips} = 0$ $\qquad \underline{D} = 3\text{ kips} \uparrow$ ◀

Note: 3-kip load and $D = 3$ kips form a couple.

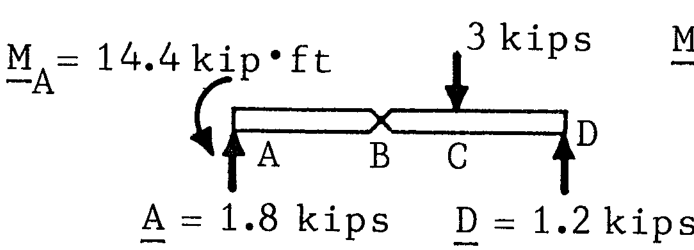
$+\circlearrowright \Sigma M_B = 0$: $(3\text{ kips})(6\text{ ft}) - M_B = 0$ $\qquad M_B = 18\text{ kip}\cdot\text{ft}$

Free Body: Member AB

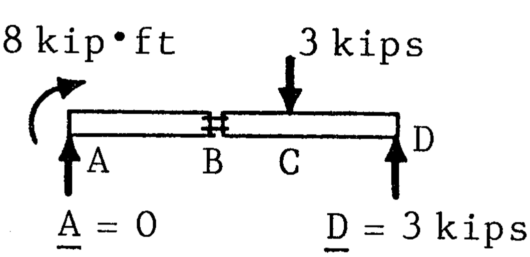
$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$ $\qquad +\uparrow \Sigma F_y = 0$: $A_y = 0$ $\qquad \underline{A} = 0$ ◀

$+\circlearrowright \Sigma M_A = 0$: $18\text{ kip}\cdot\text{ft} - M_A = 0$ $\qquad \underline{M}_A = 18\text{ kip}\cdot\text{ft} \circlearrowright$ ◀

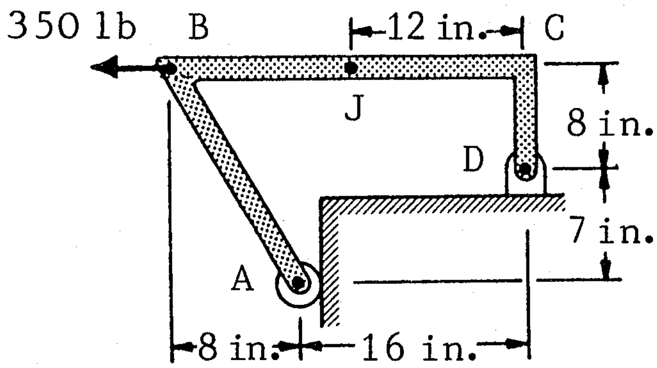
Summary



(a)

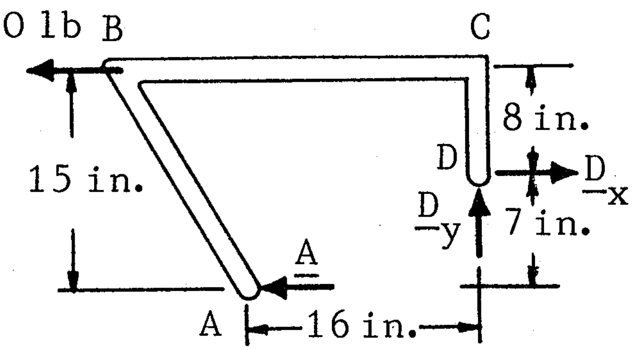


(b)



For the bracket shown determine the internal forces at point J.

Free Body: Entire Bracket.



$+\uparrow \Sigma F_y = 0$: $\quad \mathbf{D}_y = 0$ ◁

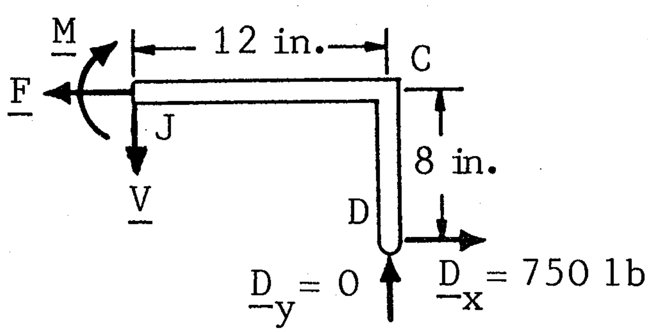
$+\circlearrowright \Sigma M_A = 0$:

$(350\text{ lb})(15\text{ in.}) - D_x(7\text{ in.}) = 0$

$D_x = +750\text{ lb}$

$\mathbf{D}_x = 750\text{ lb} \rightarrow$ ◁

Free Body: Portion JCD.



$\xrightarrow{+} \Sigma F_x = 0$: $\quad 750\text{ lb} - F = 0$

$F = +750\text{ lb}$

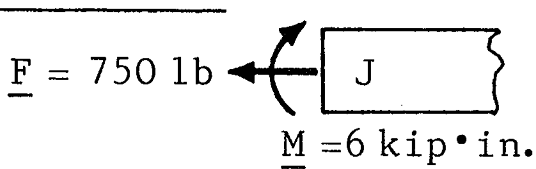
$\mathbf{F} = 750\text{ lb} \leftarrow$ ◀

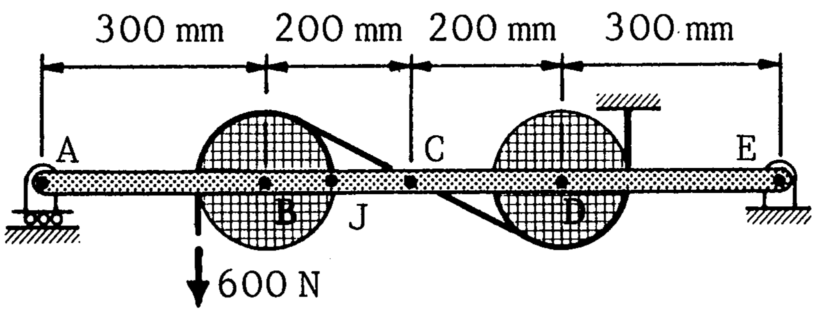
$+\uparrow \Sigma F_y = 0$: $\quad \mathbf{V} = 0$ ◀

$+\circlearrowright \Sigma M_J = 0$: $\quad -M + (750\text{ lb})(8\text{ in.}) = 0$

$M = +6000\text{ lb}\cdot\text{in.}$ $\qquad \mathbf{M} = 6\text{ kip}\cdot\text{in.} \circlearrowright$ ◀

Internal forces on JCD

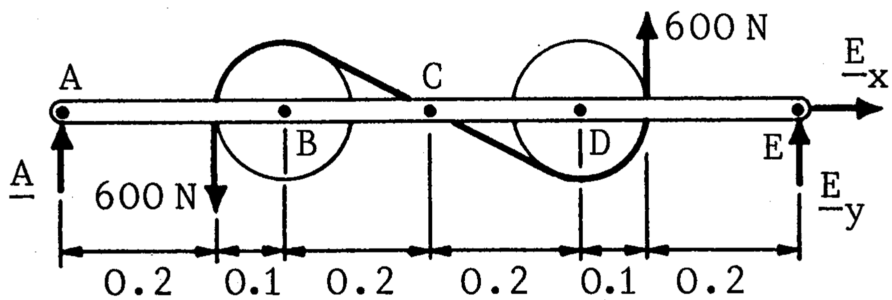




Radius of each pulley: $r = 100$ mm. Determine internal forces (a) at C, (b) at J.

Free Body: Entire Assembly

Dimensions in meters.



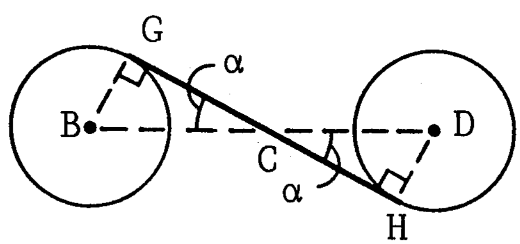
$\Sigma F_x = 0$: $E_x = 0$

$+\circlearrowright \Sigma M_E = 0$: $(600\text{ N})(0.2\text{ m}) - (600\text{ N})(0.8\text{ m}) + A(1\text{ m}) = 0$

$A = +360$ N $\qquad$ $A = 360$ N $\uparrow$

$+\uparrow \Sigma F_y = 0$: $360\text{ N} + E_y = 0$ $\qquad$ $E_y = -360$ N $\qquad$ $E_y = 360$ N $\downarrow$

Slope of Cable



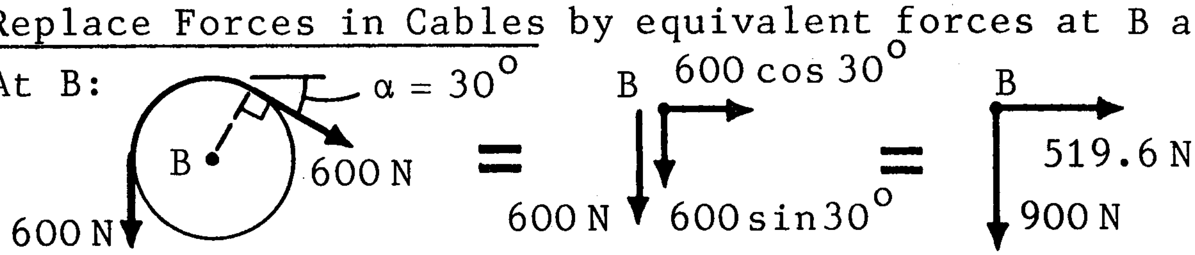
$BG = DH = r = 100$ mm

$BC = CD = 200$ mm

$\sin\alpha = \dfrac{100\text{ mm}}{200\text{ mm}}$ $\qquad$ $\alpha = 30^\circ$

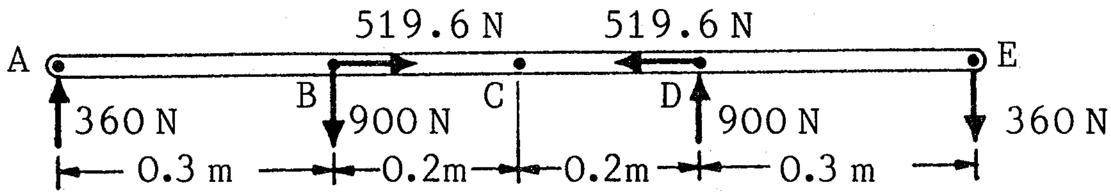
Replace Forces in Cables by equivalent forces at B and D.

At B:



Follow same procedure at D. (continued)

Equivalent Loading of Beam ABCDE

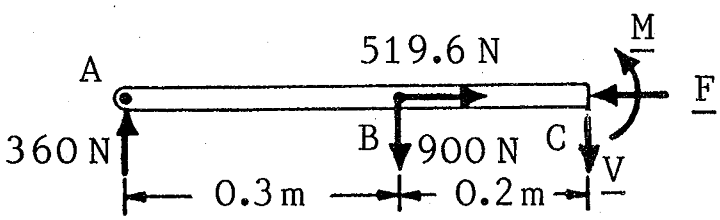


(a) Free Body: Portion AC.

$\xrightarrow{+} \Sigma F_x = 0$: $519.6\text{ N} - F = 0$

$F = 519.6\text{ N}$

$\underline{F} = 520\text{ N} \longleftarrow$ ◀



$+\uparrow \Sigma F_y = 0$: $360\text{ N} - 900\text{ N} - V = 0$

$V = -540\text{ N}$ $\qquad \underline{V} = 540\text{ N} \uparrow$ ◀

$+\circlearrowright \Sigma M_C = 0$: $-(360\text{ N})(0.5\text{ m}) + (900\text{ N})(0.2\text{ m}) + M = 0$

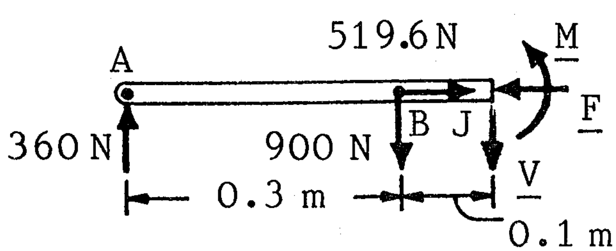
$\underline{M} = 0$ ◀

(b) Free Body: Portion AJ.

$\xrightarrow{+} \Sigma F_x = 0$: $519.6\text{ N} - F = 0$

$F = 519.6\text{ N}$

$\underline{F} = 520\text{ N} \longleftarrow$ ◀

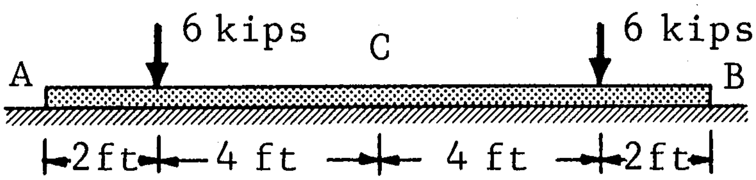


$+\uparrow \Sigma F_y = 0$: $360\text{ N} - 900\text{ N} - V = 0$

$V = -540\text{ N}$ $\qquad \underline{V} = 540\text{ N} \uparrow$ ◀

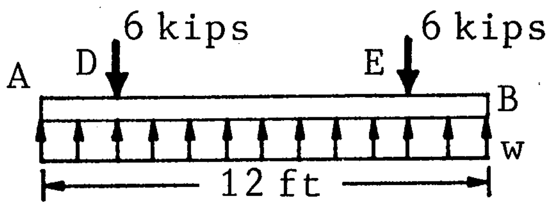
$+\circlearrowright \Sigma M_J = 0$: $-(360\text{ N})(0.4\text{ m}) + (900\text{ N})(0.1\text{ m}) + M = 0$

$M = +54\text{ N}\cdot\text{m}$ $\qquad \underline{M} = 54\text{ N}\cdot\text{m} \circlearrowright$ ◀



Draw the shear and bending-moment diagrams for the beam AB.

Reaction. Uniformly distributed upward reaction.

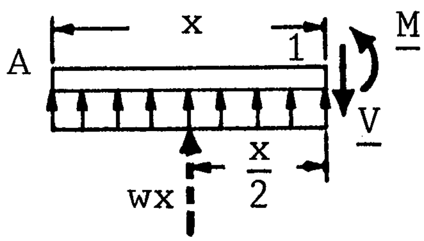


$+\uparrow \Sigma F_y = 0:$

$-2(6\text{ kips}) + w(12\text{ ft}) = 0$

$w = 1\text{ kip/ft}$

From A to D



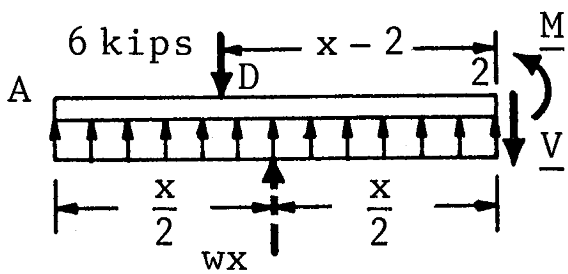
$+\uparrow \Sigma F_y = 0: \quad wx - V = 0$

$V = wx = (1)x \qquad V = x$

$+\circlearrowleft \Sigma M_1 = 0: \quad M - \frac{1}{2}wx^2 = 0$

$M = \frac{1}{2}wx^2 = \frac{1}{2}(1)x^2 \qquad M = \frac{1}{2}x^2$

From D to C

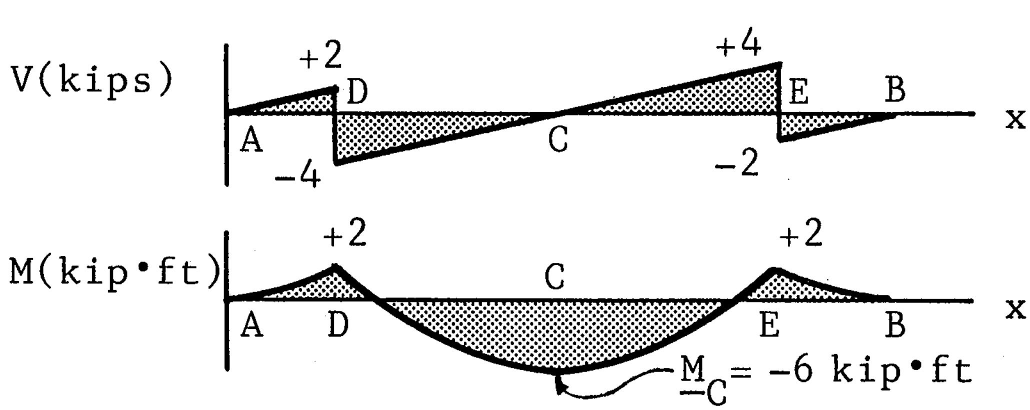


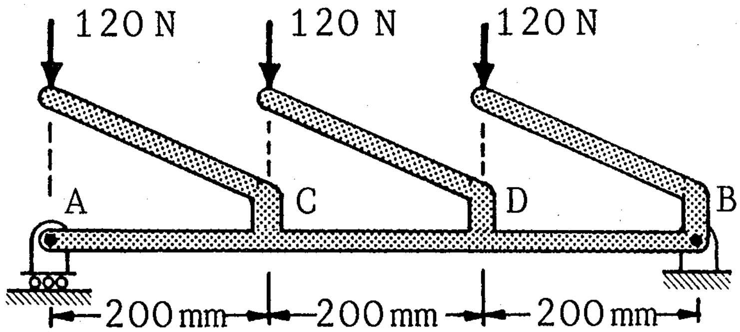
$+\uparrow \Sigma F_y = 0: \quad wx - 6\text{ kips} - V = 0$

$(1)x - 6 - V = 0 \qquad V = -6 + x$

$+\circlearrowleft \Sigma M_2 = 0: \quad (6)(x-2) - wx\frac{x}{2} + M = 0$

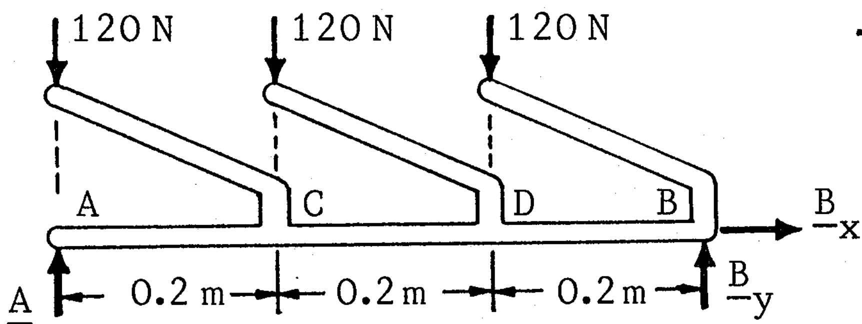
$M = 12 - 6x + (1)x\frac{x}{2} \qquad M = 12 - 6x + \frac{1}{2}x^2$





Draw the shear and bending-moment diagrams for the beam AB.

Reactions



$\xrightarrow{+} \Sigma F_x = 0$:

$\underline{B}_x = 0$

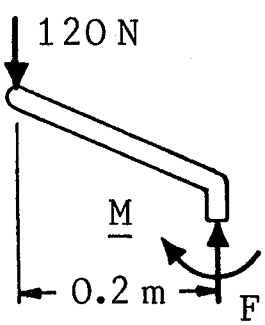
$+\circlearrowright \Sigma M_A = 0$: $\quad B_y(0.6\,\text{m}) - (120\,\text{N})(0.2\,\text{m}) - (120\,\text{N})(0.4\,\text{m}) = 0$

$B_y = +120\,\text{N}$ $\qquad \underline{B}_y = 120\,\text{N} \uparrow$

$+\uparrow \Sigma F_y = 0$: $\quad A - 3(120\,\text{N}) + 120\,\text{N} = 0$

$A = +240\,\text{N}$ $\qquad \underline{A} = 240\,\text{N} \uparrow$

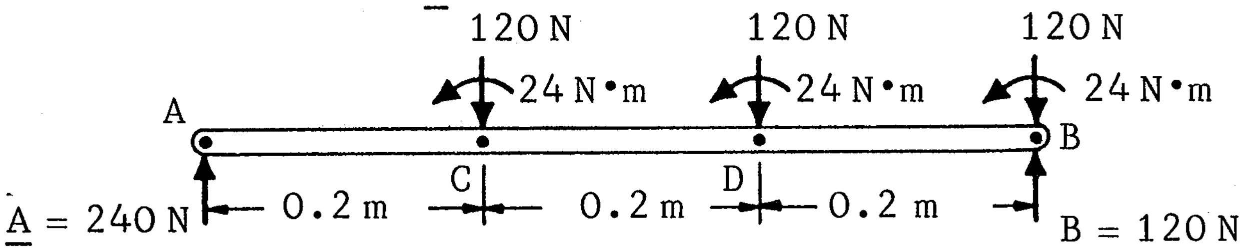
Equivalent Loading of Beam AB. For each arm we have



$\Sigma F = 0$: $\quad F = 120\,\text{N}$

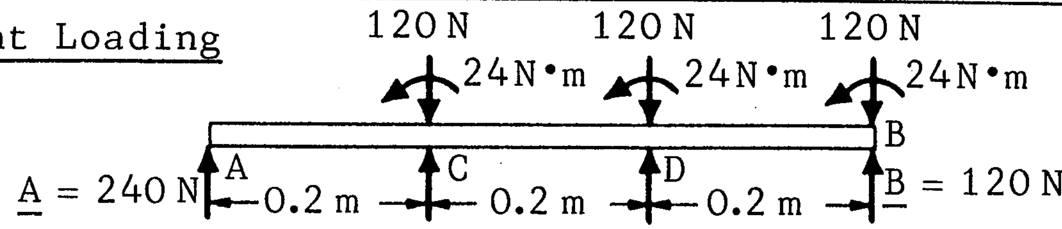
$\Sigma M = 0$: $\quad M = (120\,\text{N})(0.2\,\text{m})$

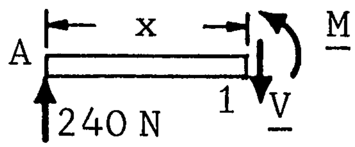
$= 24\,\text{N}\cdot\text{m}$



(continued)

Equivalent Loading

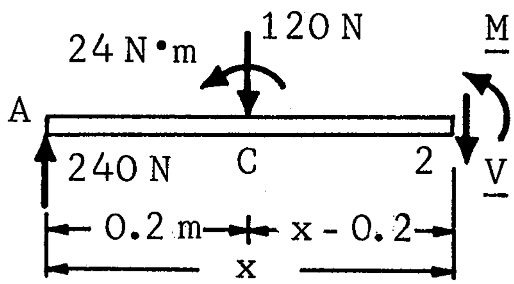




From A to C

$\Sigma F_y = 0: \quad V = 240\text{ N}$

$\Sigma M_1 = 0: \quad M = 240x$



From C to D

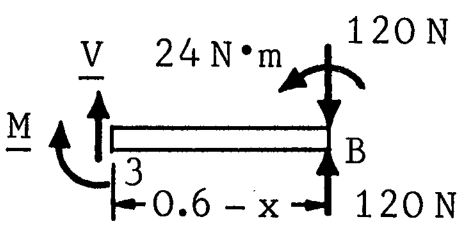
$+\uparrow \Sigma F_y = 0: \quad 240 - 120 - V = 0$

$V = 120\text{ N}$

$+\circlearrowright \Sigma M_2 = 0:$

$-240x + 24 + 120(x - 0.2) + M = 0$

$M = 120x$



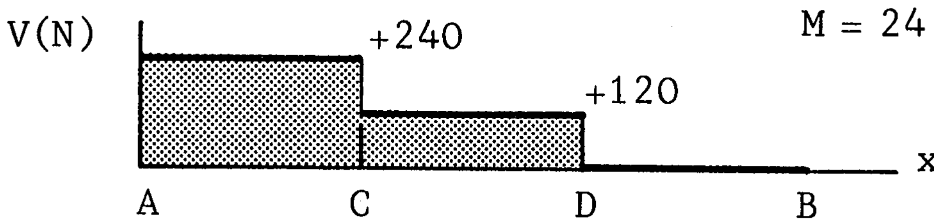
From D to B

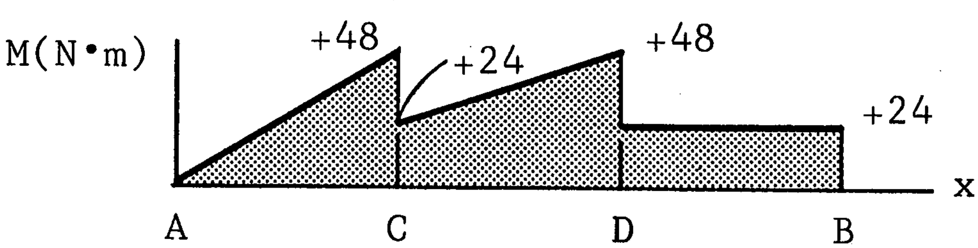
$\Sigma F_y = 0: \quad 120 - 120 - V = 0$

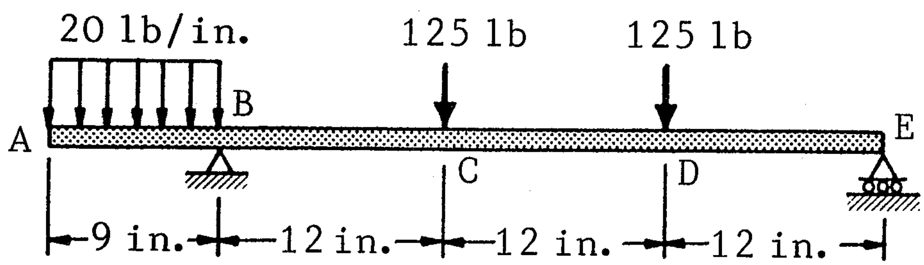
$V = 0$

$+\circlearrowright \Sigma M_B = 0: \quad M - 24 = 0$

$M = 24\text{ N•m}$

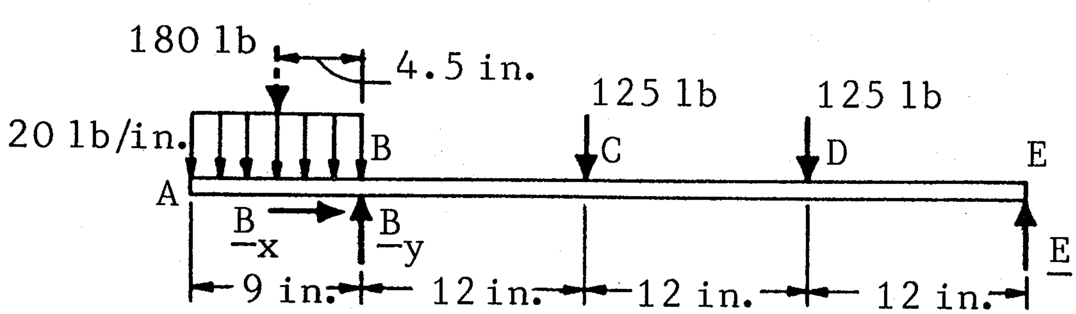






Draw the shear and bending-moment diagrams for the beam and loading shown.

Reactions. Consider entire beam as a free body.



$+\circlearrowright \Sigma M_B = 0$:

$$(180\text{ lb})(4.5\text{ in.}) - (125\text{ lb})(12\text{ in.}) - (125\text{ lb})(24\text{ in.}) + E(36\text{ in.}) = 0$$

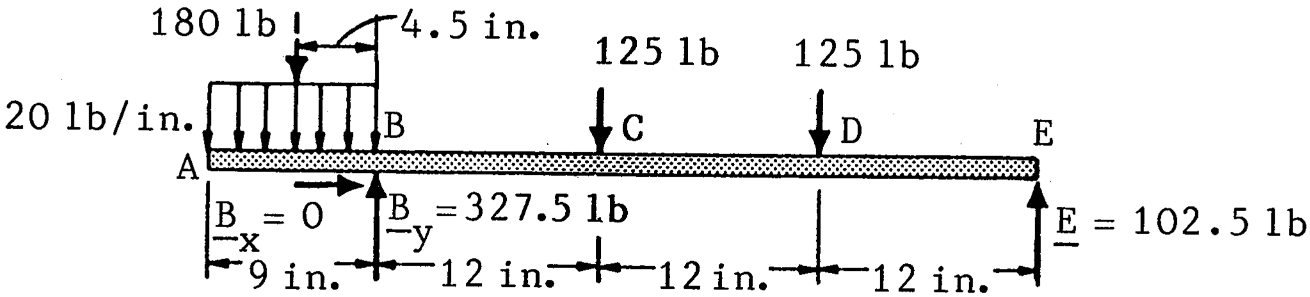
$E = +102.5$ lb $\qquad \mathbf{E} = 102.5$ lb $\uparrow$ ◁

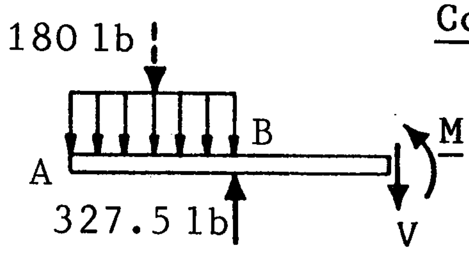
$+\uparrow \Sigma F_y = 0$: $B_y - 180\text{ lb} - 125\text{ lb} - 125\text{ lb} + 102.5\text{ lb} = 0$

$B_y = +327.5$ lb $\qquad \mathbf{B}_y = 327.5$ lb $\uparrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0$: $B_x = 0$ $\qquad \mathbf{B}_x = 0$ ◁

(continued)



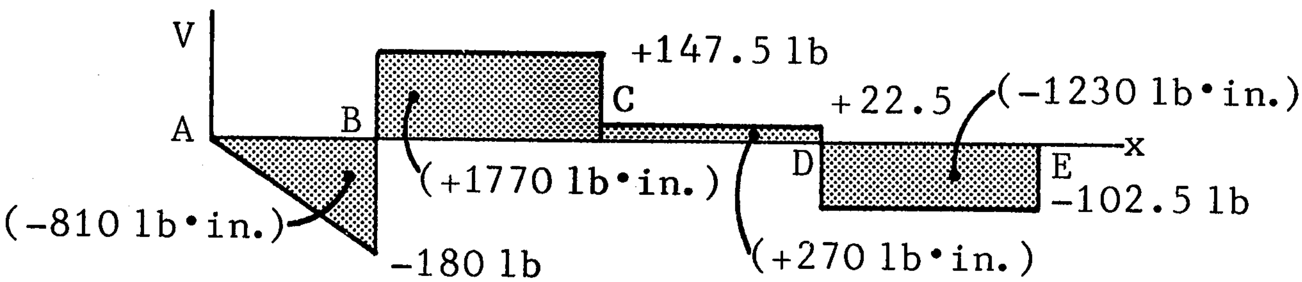


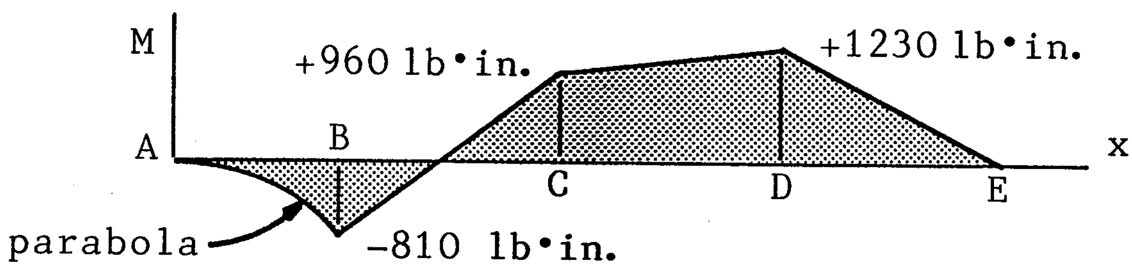
<u>Computation of Shear</u> (sample)

$+\uparrow \Sigma F_y = 0$:

$-180 + 327.5 - V = 0$

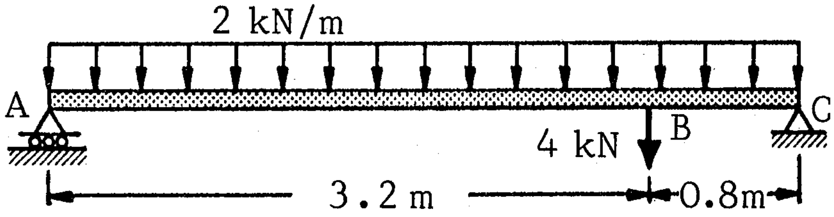
$V = +147.5$ lb



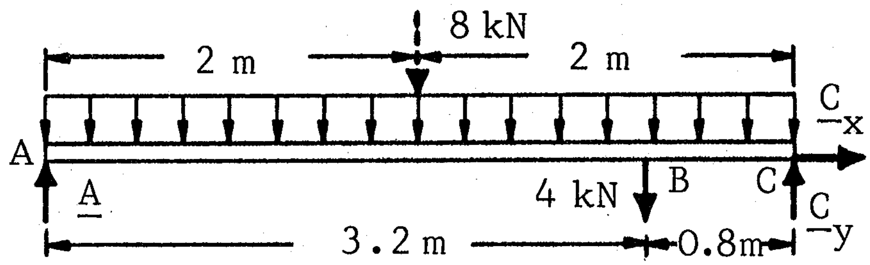


<u>Computation of Bending Moments</u>:

	$M_A = 0$
$M_B - M_A = -\ 810$	$M_B = -\ 810$ lb•in.
$M_C - M_B = +1770$	$M_C = +\ 960$ lb•in.
$M_D - M_C = +\ 270$	$M_D = +1230$ lb•in.
$M_E - M_D = -1230$	$M_E = 0$ (checks)



Draw shear and bending-moment diagrams.

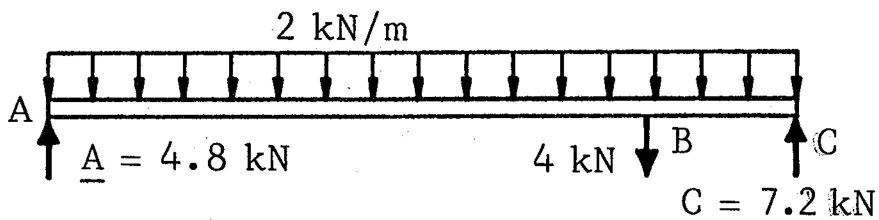


Reactions:

$\Sigma M_C = 0$: $A = 4.8\,\text{kN}\uparrow$

$\Sigma F_x = 0$: $C_x = 0$

$\Sigma F_y = 0$: $C_y = 7.2\,\text{kN}\uparrow$



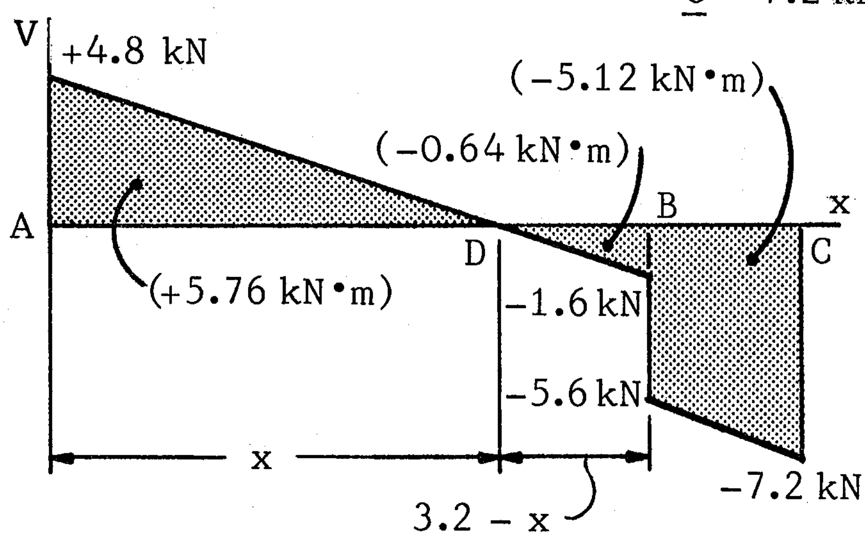
Just to right of A

$V_A = +4.8$ kN

Just to left of B,

$V_B = 4.8 - (2)(3.2)$

$= -1.6$ kN

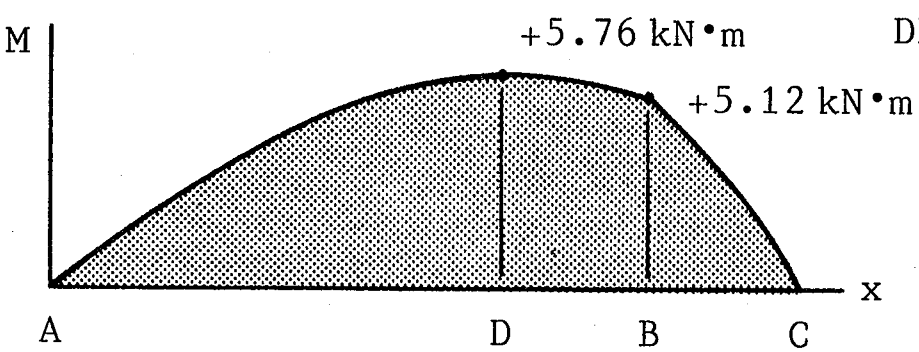


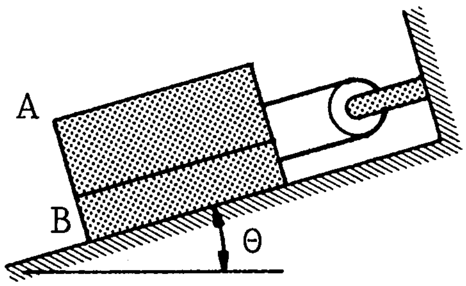
M_{max} at D, where V = 0

$$\frac{x}{4.8} = \frac{3.2 - x}{1.6} = \frac{3.2}{6.4}$$

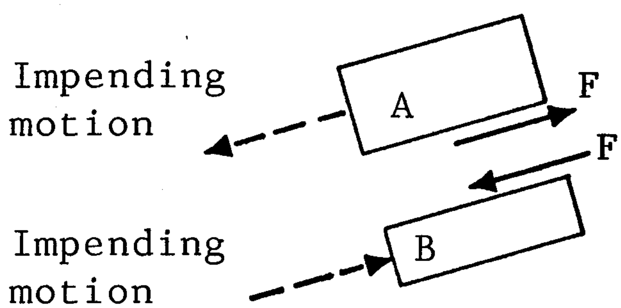
AD = x = 2.4 m

DB = 3.2 − 2.4 = 0.8 m





Given: $W_A = 50$ lb, $W_B = 25$ lb, $\mu_s = 0.15$ between the blocks, $\mu_s = 0$ between block B and incline. Determine the value of θ for which motion is impending.



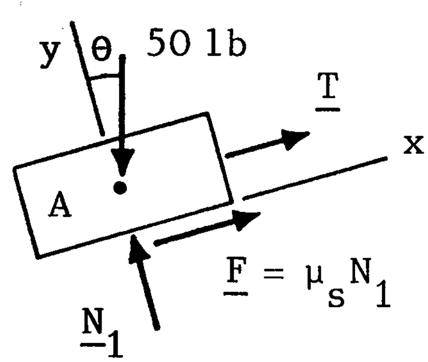
Impending Motion

Friction forces

$$F = \mu_s N$$

are directed as shown.

Free Body: Block A



$+\nearrow \Sigma F_y = 0$: $N_1 - 50\cos\theta = 0$ (1)

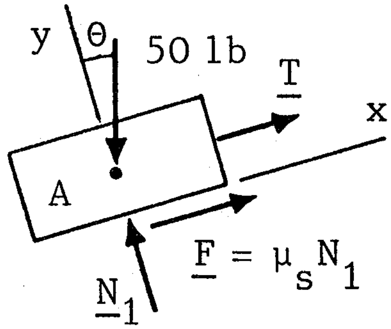
$+\nearrow \Sigma F_x = 0$: $T + \mu_s N_1 - 50\sin\theta = 0$ (2)

Substituting for N_1 from, (1) into (2):

$$T + 50\mu_s\cos\theta - 50\sin\theta = 0 \quad (3)$$

(continued)

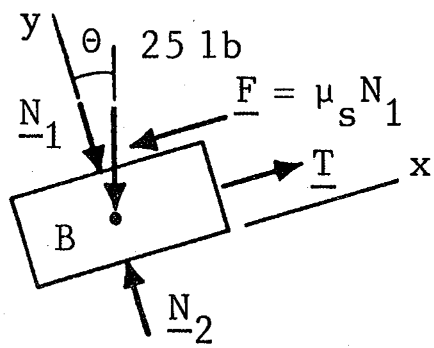
From the free body of block A we have found:



$$N_1 - 50\cos\theta = 0 \qquad (1)$$

$$T + 50\mu_s\cos\theta - 50\sin\theta = 0 \qquad (3)$$

Free Body: Block B



Note. $\mu_s = 0$ between block B and the incline.

$$+\nearrow \Sigma F_x = 0: \quad T - \mu_s N_1 - 25\sin\theta = 0 \qquad (4)$$

Substituting for N_1 from (1) into (4):

$$T - 50\mu_s\cos\theta - 25\sin\theta = 0 \qquad (5)$$

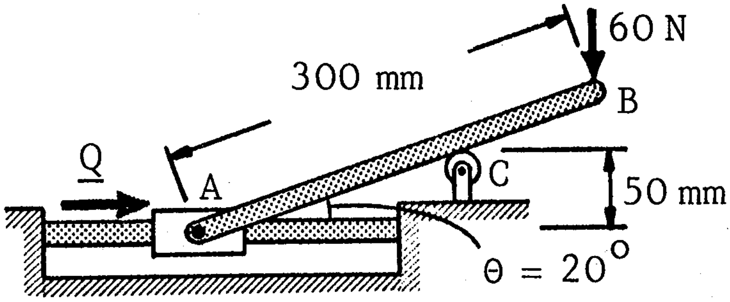
Subtracting (5) from (3):

$$100\mu_s\cos\theta - 25\sin\theta = 0$$

Given: $\mu_s = 0.15$

$$100(0.15)\cos\theta - 25\sin\theta = 0$$

$$\tan\theta = 0.6 \qquad \theta = 31.0^\circ \blacktriangleleft$$



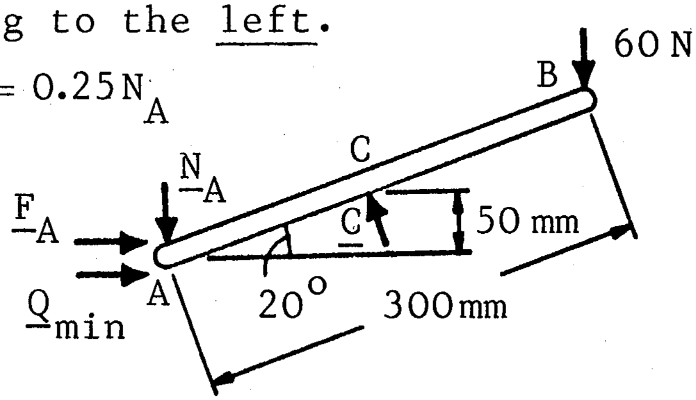
Given: $\mu_s = 0$ at C,

$\mu_s = 0.25$ between collar and horizontal rod.

Find: Range of values of Q for equilibrium.

Free Body: Rod AB. We first assume motion of collar A is impending to the left.

$F_A = \mu_s N_A = 0.25 N_A$



$AC = \dfrac{50\text{ mm}}{\sin 20^\circ}$

$+\circlearrowright \Sigma M_A = 0$: $(60\text{ N})(300\text{ mm})\cos 20^\circ - C\left(\dfrac{50\text{ mm}}{\sin 20^\circ}\right) = 0$

$C = (60\text{ N})(6)\cos 20^\circ \sin 20^\circ$ $\qquad C = 115.7\text{ N}$

$+\uparrow \Sigma F_y = 0$: $-N_A + C\cos 20^\circ - 60\text{ N} = 0$

$-N_A + (115.7\text{ N})(\cos 20^\circ) - 60\text{ N} = 0$

$N_A = +48.72\text{ N}$ $\qquad F_A = 0.25 N_A = 12.18\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0$: $Q_{min} + F_A - C\sin 20^\circ = 0$ (1)

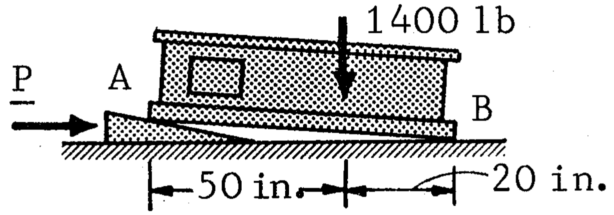
$Q_{min} + 12.18\text{ N} - (115.7\text{ N})\sin 20^\circ = 0$ $\qquad Q_{min} = 27.4\text{N}$ ◁

Assuming now motion of A is to the right, F_A is ← and Eq.(1) becomes:

$\xrightarrow{+} \Sigma F_x = 0$: $Q_{max} - F_A - C\sin 20^\circ = 0$

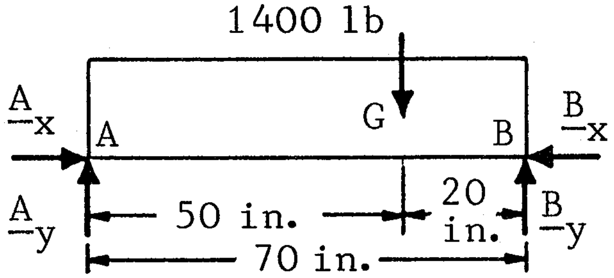
$Q_{max} - 12.18\text{ N} - (115.7\text{ N})\sin 20^\circ = 0$ $\qquad Q_{max} = 51.8\text{ N}$ ◁

Equilibrium maintained for: $27.4\text{ N} \le Q \le 51.8\text{ N}$ ◀



A 5° wedge is forced under the machine base at A. Knowing that $\mu_s = 0.20$ at all surfaces, determine (a) force $\underline{P}$ required to move the wedge, (b) whether the machine base will move.

Free Body: Machine Base



$+\circlearrowright \Sigma M_B = 0$:

$(1400\text{ lb})(20\text{ in.}) - A_y(70\text{ in.}) = 0$

$A_y = 400\text{ lb}$

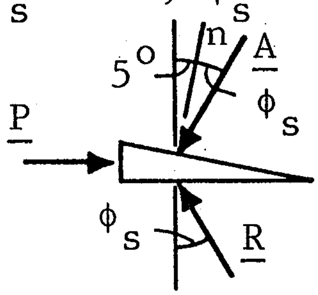
$+\uparrow \Sigma F_y = 0$: $A_y + B_y - 1400\text{ lb} = 0$

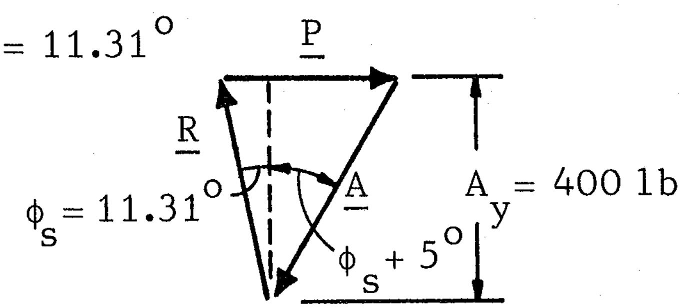
$400\text{ lb} + B_y - 1400\text{ lb} = 0$

$B_y = 1000\text{ lb}$

Free Body: Wedge. We assume that machine does not move and, therefore, that motion of wedge is impending relative to both machine base and floor.

$\mu_s = 0.20,\ \phi_s = \tan^{-1}0.20 = 11.31^\circ$





Force Triangle

(a) From force triangle, $P = A_y \tan\phi_s + A_y \tan(\phi_s + 5^\circ)$

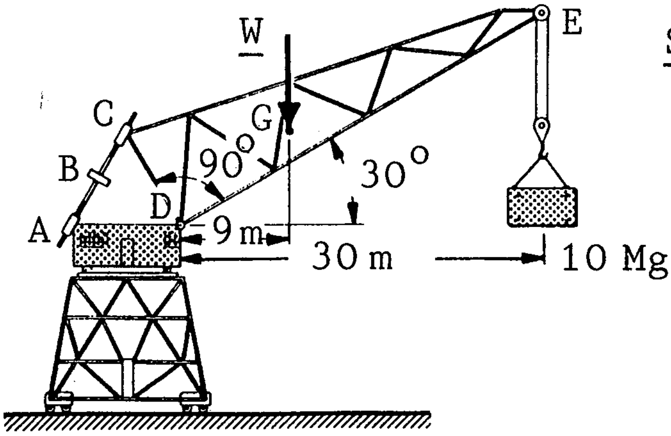
$P = (400\text{ lb})\tan 11.31^\circ + (400\text{ lb})\tan(11.31^\circ + 5^\circ) = 197.0\text{ lb}$

$\underline{P} = 197.0\text{ lb} \rightarrow$ ◀

(b) Total friction force available at A and B:

$F_m = \mu_s W = (0.20)(1400\text{ lb}) = 280\text{ lb}$

Since $P < F_m$, we check that machine will not move. ◀



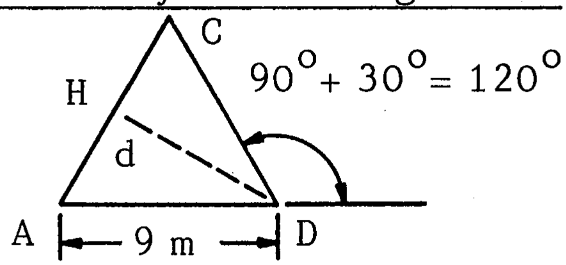
Screw ABC:

Double-threaded at each end, pitch = 16 mm
diameter = 200 mm
$\mu_s = 0.08$.

AD = CD = 9 m

The main features of a screw-luffing crane are shown. The position of the 6-Mg boom CDE is controlled by the screw ABC, which has a left-handed thread at A and a right-handed thread at C. Determine the magnitude of the couple which must be applied to the screw (a) to raise the boom, (b) to lower the boom.

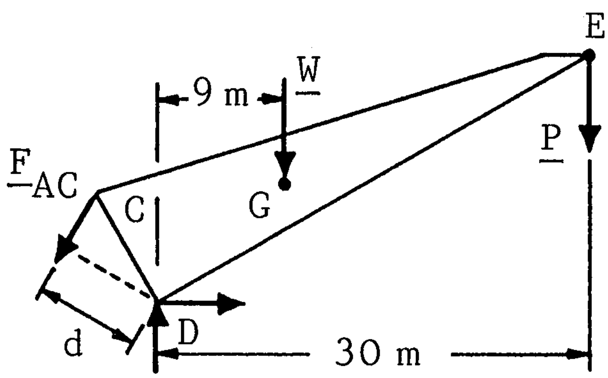
Geometry of Triangle ACD



Angle ACD = $180^\circ - 120^\circ = 60^\circ$, and AD = CD; thus Δ ACD is equilateral. We have

$d = DH = (9\,m)\cos 30^\circ = 7.7942\,m$

Free Body: Boom CDE



$W = (6\,Mg)(9.81\,m/s^2)$
$= 58.86\,kN$

$P = (10\,Mg)(9.81\,m/s^2)$
$= 98.1\,kN$

$+\circlearrowright \Sigma M_D = 0$:

$F_{AC}d - W(9\,m) - P(30\,m) = 0$

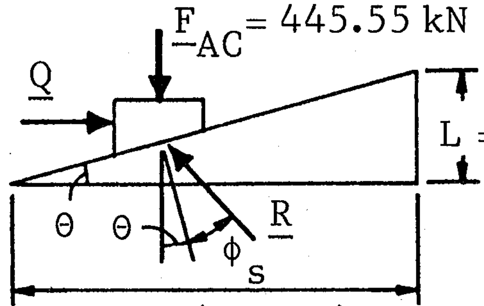
$$F_{AC}(7.7942\,m) - (58.86\,kN)(9\,m) - (98.1\,kN)(30\,m) = 0$$

$$F_{AC} = 445.55\,kN$$

(continued)

We have found the tension in screw ABC: $F_{AC} = 445.55$ kN

(a) To Raise the Boom.



Double-threaded screw.

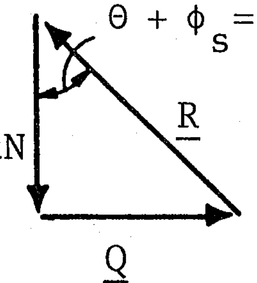
Lead = 2(pitch)

L = 2(16 mm) = 32 mm

$$\tan\theta = \frac{L}{2\pi r} = \frac{32}{200\pi} = 0.05093$$

$$\theta = 2.9155^\circ$$

$$\tan\phi_s = \mu_s = 0.08 \qquad \phi_s = 4.5739^\circ$$



$$\theta + \phi_s = 2.9155^\circ + 4.5739^\circ = 7.4894^\circ$$

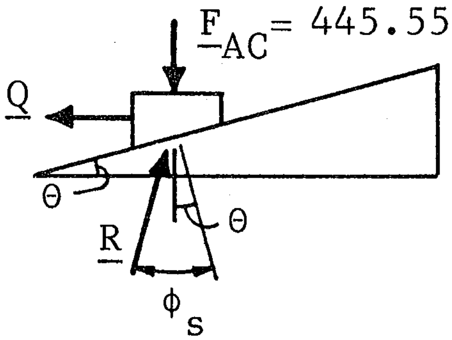
$$Q = (445.55\text{ kN})\tan 7.4894^\circ$$
$$= 58.57\text{ kN}$$

Since there are threads at both ends of the screw, torque is:

$$T = 2Qr = 2(58.57\text{ kN})(100\text{ mm})$$

$$T = 11.71\text{ kN·m}$$ ◀

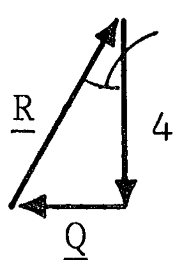
(b) To Lower the Boom.



We again have:

$$\theta = 2.9155^\circ$$

$$\phi_s = 4.5739^\circ$$

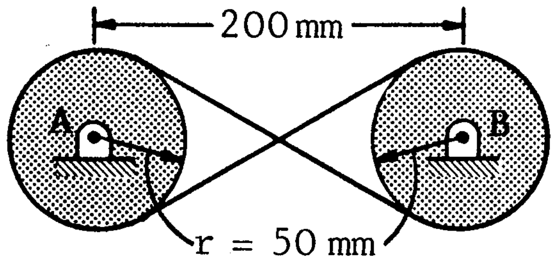


$$\phi_s - \theta = 4.5739^\circ - 2.9155^\circ = 1.6584^\circ$$

$$Q = (445.55\text{ kN})\tan 1.6584^\circ$$
$$= 12.90\text{ kN}$$

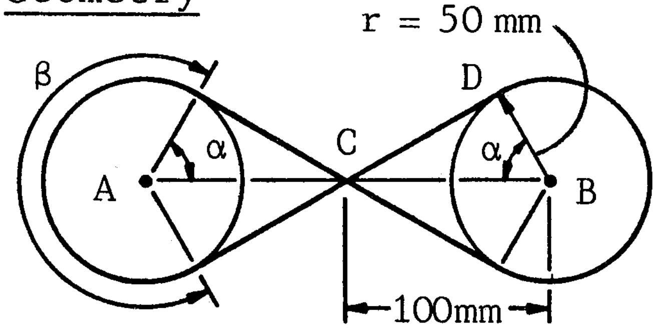
$$T = 2Qr = 2(12.90\text{ kN})(100\text{ mm})$$

$$T = 2.58\text{ kN·m}$$ ◀



A flat belt is looped around two pulleys in a figure 8. Determine the largest torque which can be transmitted if the allowable belt tension is 3 kN. $\mu_s = 0.30$

Geometry



Triangle BCD:

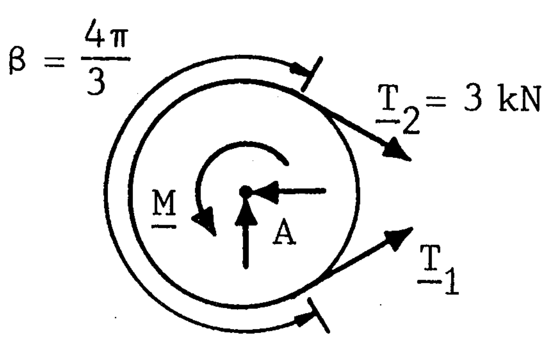
$\cos\alpha = \dfrac{50}{100}$ $\qquad \alpha = 60^\circ$

Thus:

$\beta = 360^\circ - 2\alpha = 360^\circ - 120^\circ$

$\beta = 240^\circ = \dfrac{4\pi}{3}$ rad

Free Body: Pulley A. Assume rotation is ↻. Then impending slipping of belt relative to pulley is ↺ and the tension forces $\mathbf{T}_1$ and $\mathbf{T}_2$ are as shown.



Eq.(8.14):

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{3\text{ kN}}{T_1} = e^{0.30\left(\frac{4\pi}{3}\right)} = 3.514$$

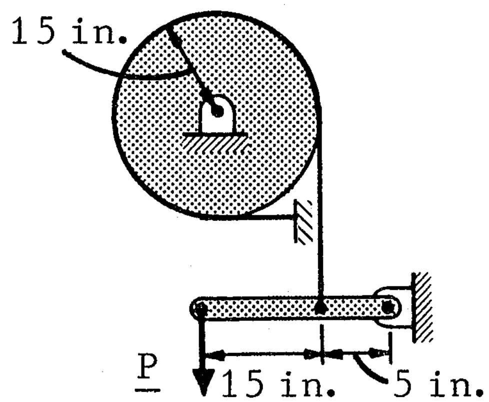
$$T_1 = \frac{3\text{ kN}}{3.514} \qquad T_1 = 0.854\text{ kN}$$

Torque

$+\circlearrowright\ \Sigma M_A = 0$: $\quad M - (T_2 - T_1)r = 0$

$M - (3\text{ kN} - 0.854\text{ kN})(0.05\text{ m}) = 0$

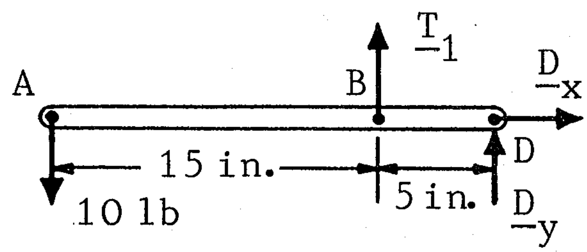
$M = 107.3\text{ N}\cdot\text{m}$ $\qquad$ Torque = 107.3 N•m ◀



A band brake is used to control the speed of a flywheel as shown. What torque should be applied to the flywheel to keep it rotating at a constant speed when P = 10 lb?

$\mu_s = 0.30 \qquad \mu_k = 0.25$

Free Body: Brake Lever

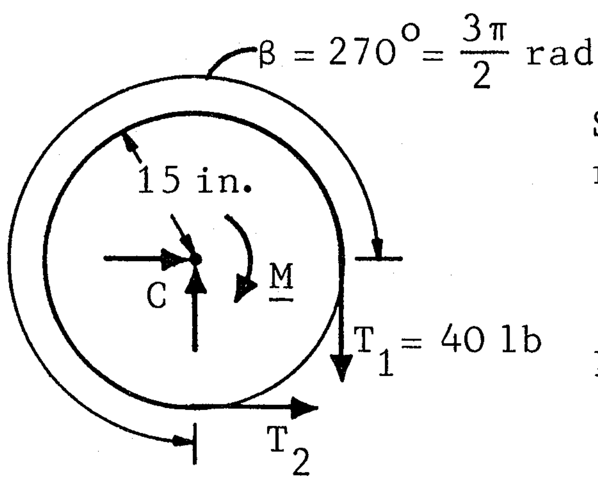


$+\circlearrowright \Sigma M_D = 0$

$(10\,\text{lb})(20\,\text{in.}) - T_1(5\,\text{in.}) = 0$

$T_1 = 40\,\text{lb}$

Free Body: Flywheel



Since flywheel moves relative to the belt

$\mu = \mu_k = 0.25$

From Eq.(8.14): $\dfrac{T_2}{T_1} = e^{\mu_k \beta}$

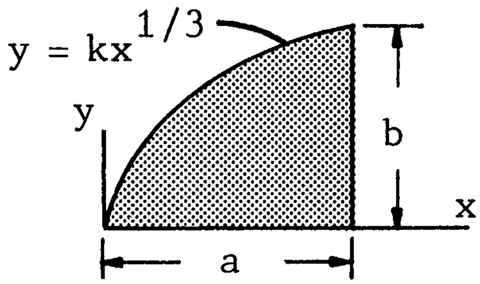
$T_2 = T_1 e^{\mu_k \beta} = (40\,\text{lb})e^{(0.25)\frac{3}{2}\pi}$

$= (40\,\text{lb})(3.2482) = 129.93\,\text{lb}$

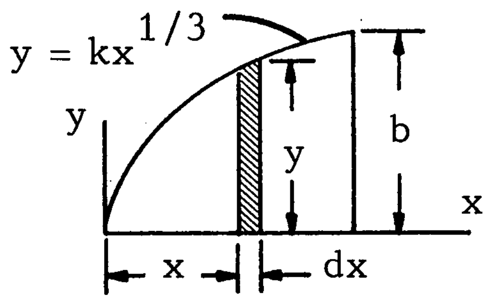
$+\circlearrowright \Sigma M_C = 0: \quad T_2(15\,\text{in.}) - T_1(15\,\text{in.}) - M = 0$

$(129.93\,\text{lb})(15\,\text{in.}) - (40\,\text{lb})(15\,\text{in.}) - M = 0$

$M = 1349\,\text{lb}\cdot\text{in} \qquad M = 112.4\,\text{lb}\cdot\text{ft}$ ◀



Determine the moment of inertia of the shaded area with respect to: (Prob. 9.2) the y axis, (Prob. 9.6) the x axis.



Value of k. For $x = a$, $y = b$

$$b = k a^{1/3} \qquad k = \frac{b}{a^{1/3}}$$

$$y = \frac{b}{a^{1/3}} x^{1/3}$$

Problem 9.2

$$dI_y = x^2\, dA = x^2 y\, dx = x^2 \frac{b}{a^{1/3}} x^{1/3}\, dx = \frac{b}{a^{1/3}} x^{7/3}\, dx$$

$$I_y = \int dI_y = \frac{b}{a^{1/3}} \int_0^a x^{7/3}\, dx = \frac{b}{a^{1/3}} \left| \frac{3}{10} x^{10/3} \right|_0^a = \frac{b}{a^{1/3}} \left(\frac{3}{10} a^{10/3} \right)$$

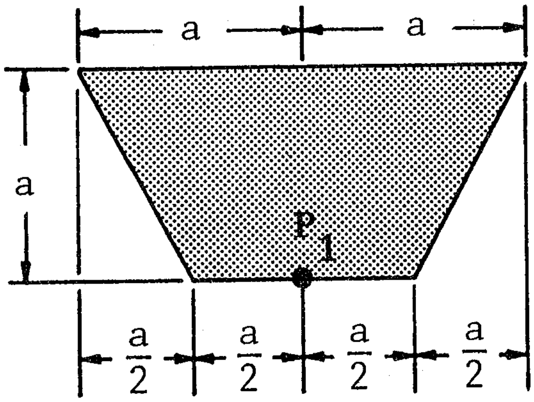
$$I_y = \frac{3}{10} a^3 b \quad ◀$$

Problem 9.6

$$dI_x = \frac{1}{3} y^3\, dx = \frac{1}{3} \left(\frac{b}{a^{1/3}} x^{1/3} \right)^3 dx = \frac{1}{3} \frac{b^3}{a} x\, dx$$

$$I_x = \int dI_x = \frac{1}{3} \frac{b^3}{a} \int_0^a x\, dx = \frac{1}{3} \frac{b^3}{a} \left| \frac{x^2}{2} \right|_0^a = \frac{1}{3} \frac{b^3}{a} \left(\frac{a^2}{2} \right)$$

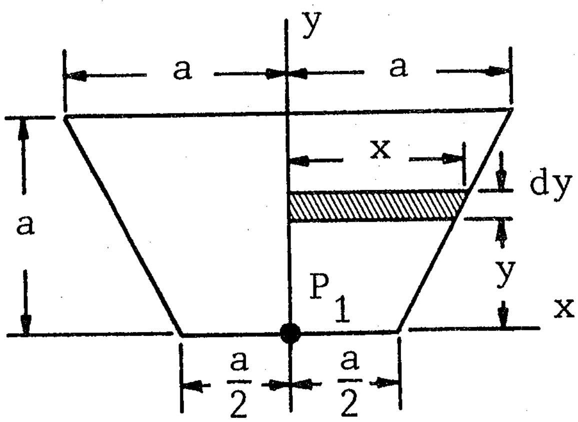
$$I_x = \frac{1}{6} a b^3 \quad ◀$$



Determine the polar moment of inertia and the polar radius of gyration of the trapezoid shown with respect to point P_1.

Solution. Selecting x and y axes with origin at P_1, we have $J_1 = I_x + I_y$

Differential Element. We use the element shown to compute half of the area and half of the moments of inertia of the trapezoid.



$$x = \frac{a}{2} + \frac{a}{2}\frac{y}{a} = \frac{1}{2}(a + y)$$

$$dA = x\,dy = \frac{1}{2}(a + y)\,dy$$

Area.

$$\frac{1}{2}A = \int dA = \frac{1}{2}\int_0^a (a + y)\,dy = \frac{1}{2}\left|ay + \frac{1}{2}y^2\right|_0^a = \frac{1}{2}\left(a^2 + \frac{1}{2}a^2\right) = \frac{3}{4}a^2$$

$$A = \frac{3}{2}a^2 \quad \triangleleft$$

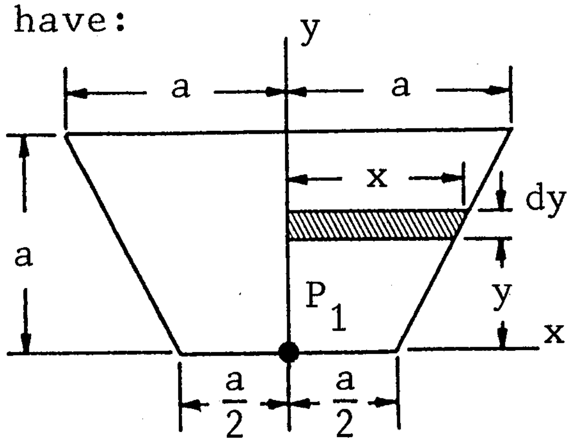
Moment of Inertia I_x

$$\frac{1}{2}I_x = \int y^2 dA = \frac{1}{2}\int_0^a (ay^2 + y^3)\,dy = \frac{1}{2}\left|\frac{1}{3}ay^3 + \frac{1}{4}y^4\right|_0^a$$

$$= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right)a^4 = \frac{7}{24}a^4 \qquad I_x = \frac{7}{12}a^4 \quad \triangleleft$$

(continued)

We have:



$$x = \frac{1}{2}(a + y)$$

$$A = \frac{3}{2}a^2 \quad \triangleleft$$

$$I_x = \frac{7}{12}a^4 \quad \triangleleft$$

Moment of Inertia I_y

$$\frac{1}{2}I_y = \int \frac{1}{3}x^3 dy = \frac{1}{3}\int_0^a \left[\frac{1}{2}(a+y)\right]^3 dy = \frac{1}{24}\int_0^a (a+y)^3 dy$$

$$= \frac{1}{24}\left|\frac{1}{4}(a+y)^4\right|_0^a = \frac{1}{96}\left[(2a)^4 - a^4\right] = \frac{15}{96}a^4 = \frac{5}{32}a^4$$

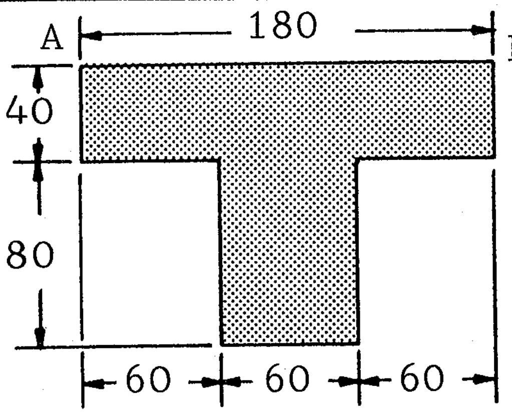
$$I_y = \frac{5}{16}a^4 \quad \triangleleft$$

Polar Moment of Inertia

$$J_1 = I_x + I_y = \frac{7}{12}a^4 + \frac{5}{16}a^4 \qquad J_1 = \frac{43}{48}a^4 \quad \blacktriangleleft$$

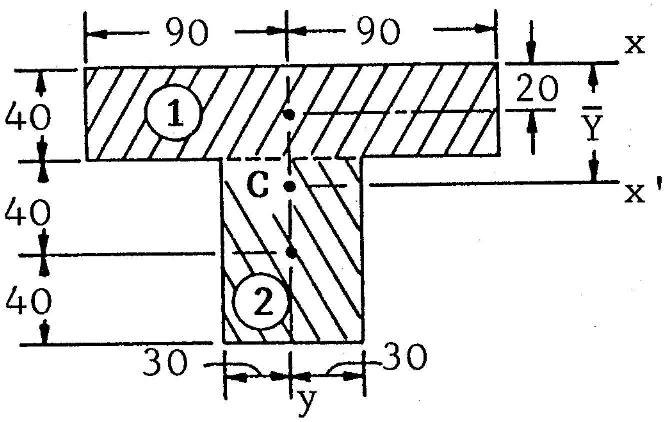
Polar Radius of Gyration

$$k_1^2 = \frac{J_1}{A} = \frac{\frac{43}{48}a^4}{\frac{3}{2}a^2} = \frac{43}{72}a^2 \qquad k_1 = a\sqrt{\frac{43}{72}} \quad \blacktriangleleft$$



Determine $\bar{I}_x$ and $\bar{I}_y$ with respect to centroidal axes parallel and perpendicular to side AB.

Dimensions in mm

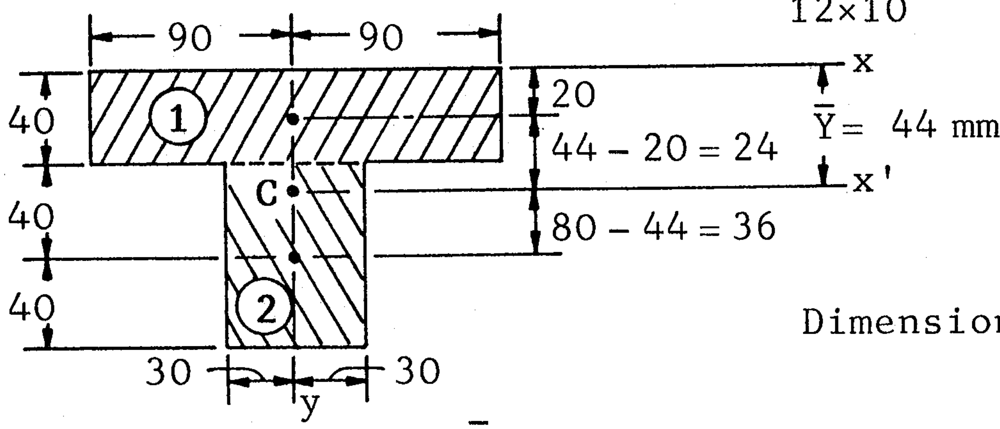


Centroid $\bar{X} = 0$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{(40\times180)(20) + (80\times60)(80)}{(40\times180) + (80\times60)}$$

$$= \frac{528\times10^3}{12\times10^3} \qquad \bar{Y} = 44 \text{ mm}$$



Dimensions in mm

Moment of Inertia $\bar{I}_{x'}$

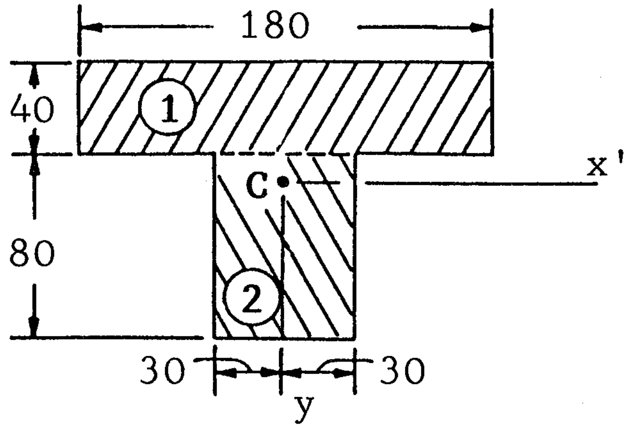
Section ①: $I_{x'} = \frac{1}{12}(180)(40)^3 + (180)(40)(24)^2 = 5.11\times10^6 \text{ mm}^4$

Section ②: $I_{x'} = \frac{1}{12}(60)(80)^3 + (60)(80)(36)^2 = 8.78\times10^6 \text{ mm}^4$

Entire Area: $\bar{I}_{x'} = 5.11\times10^6 + 8.78\times10^6$

$$\bar{I}_{x'} = 13.89\times10^6 \text{ mm}^4$$ ◀

(continued)



We have

$$\bar{I}_{x'} = 13.89\times10^6 \text{ mm}^4$$

Dimensions in mm

Moment of Inertia $\bar{I}_y$

Section (1): $I_y = \frac{1}{12}(40)(180)^3 = 19.44\times10^6 \text{ mm}^4$

Section (2): $I_y = \frac{1}{12}(80)(60)^3 = 1.44\times10^6 \text{ mm}^4$

Entire Area: $\bar{I}_y = 19.44\times10^6 + 1.44\times10^6$

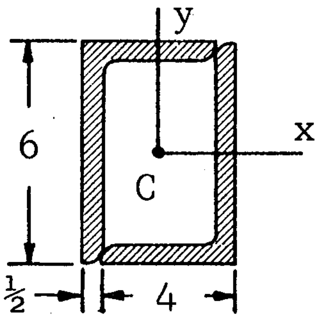
$$\bar{I}_y = 20.88\times10^6 \text{ mm}^4$$

Radii of Gyration

$A = (40\times180) + (80\times60) = 12\times10^3 \text{mm}^2$

$\bar{k}_{x'}^2 = \frac{\bar{I}_{x'}}{A} = \frac{13.89\times10^6 \text{ mm}^4}{12\times10^3 \text{ mm}^2}$ $\qquad \bar{k}_{x'} = 34.02 \text{ mm}$

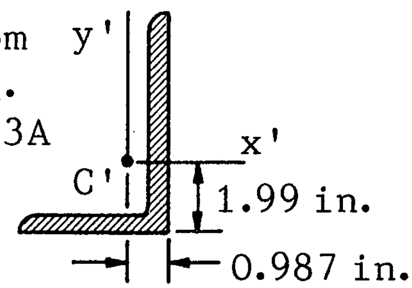
$\bar{k}_y = \frac{\bar{I}_y}{A} = \frac{20.88\times10^6 \text{ mm}^4}{12\times10^3 \text{ mm}^2}$ $\qquad \bar{k}_y = 41.71 \text{ mm}$



Two $6\times4\times\frac{1}{2}$ in. angles are welded together to form the section shown. Determine the moments of interia and radii of gyration of the section with respect to the centroidal axes shown.

Dimensions in inches

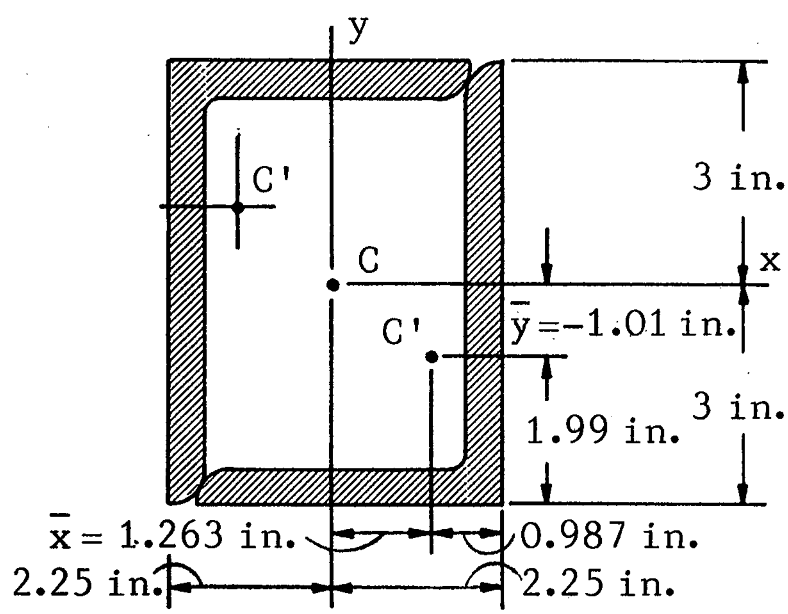
From Fig. 9.13A



$A = 4.75\ in^2$

$\bar{I}_{x'} = 17.4\ in^4$

$\bar{I}_{y'} = 6.27\ in^4$



$$\bar{I}_x = 2\left[\bar{I}_{x'} + A\bar{y}^2\right] = 2\left[17.4\ in^4 + (4.75\ in^2)(1.01\ in.)^2\right] = 44.49\ in^4$$

Total Area $= 2(4.75) = 9.5\ in^2$

$\bar{I}_x = 44.5\ in^4$ ◀

$$\bar{k}_x^{\,2} = \frac{\bar{I}_x}{\text{Area}} = \frac{44.49\ in^4}{9.5\ in^2} = 4.683\ in^2$$

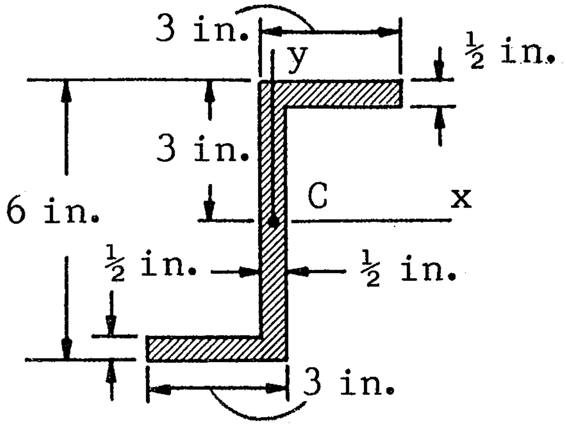
$\bar{k}_x = 2.16\ in.$ ◀

$$\bar{I}_y = 2\left[\bar{I}_{y'} + A\bar{x}^2\right] = 2\left[6.27\ in^4 + (4.75 in^2)(1.263\ in.)^2\right] = 27.69\ in^4$$

$\bar{I}_y = 27.7\ in^4$ ◀

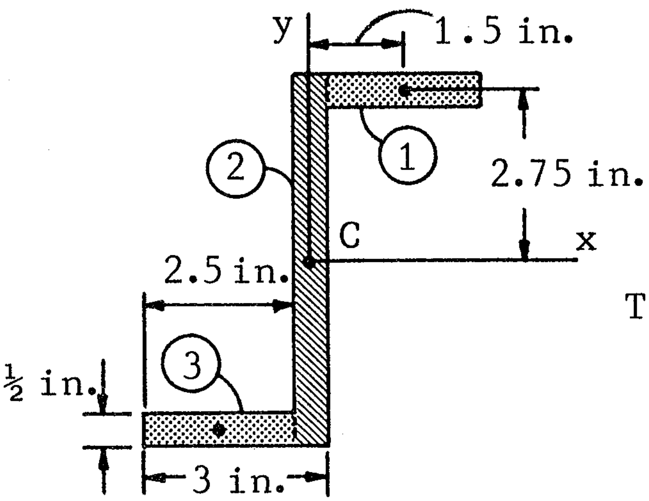
$$\bar{k}_y^{\,2} = \frac{\bar{I}_y}{\text{Area}} = \frac{27.69\ in^4}{9.5\ in^2} = 2.915\ in^2$$

$\bar{k}_y = 1.707\ in.$ ◀



Determine the product of inertia of the area shown with respect to the x and y axes.

Solution. Divide area into three rectangles.



For each rectangle: (by symmetry)

$$\bar{I}_{x'y'} = 0$$

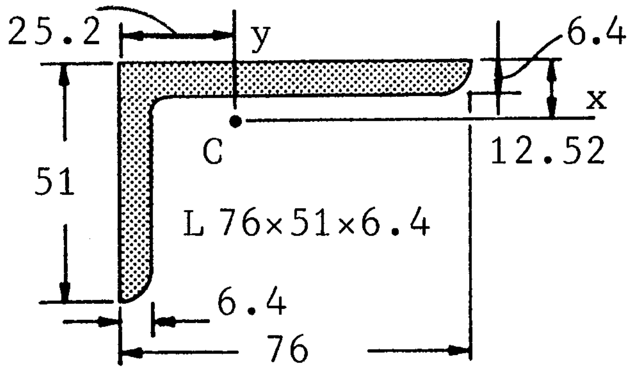
Thus: $I_{xy} = \Sigma(\bar{I}_{x'y'} + \bar{x}\bar{y}A)$

$$I_{xy} = \Sigma\bar{x}\bar{y}A$$

Rectangle	Area, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\bar{y}A$, in^4
1	(2.5)(0.5) = 1.25	+1.5	+2.75	+5.156
2	(6)(0.5) = 3	0	0	0
3	(2.5)(0.5) = 1.25	−1.5	−2.75	+5.156

$$\Sigma\bar{x}\bar{y}A = +10.312 \text{ in}^4$$

$$I_{xy} = +10.31 \text{ in}^4$$ ◀



Determine the moments of inertia and product of inertia with respect to new axes obtained by rotating the x and y axes through 30° clockwise.

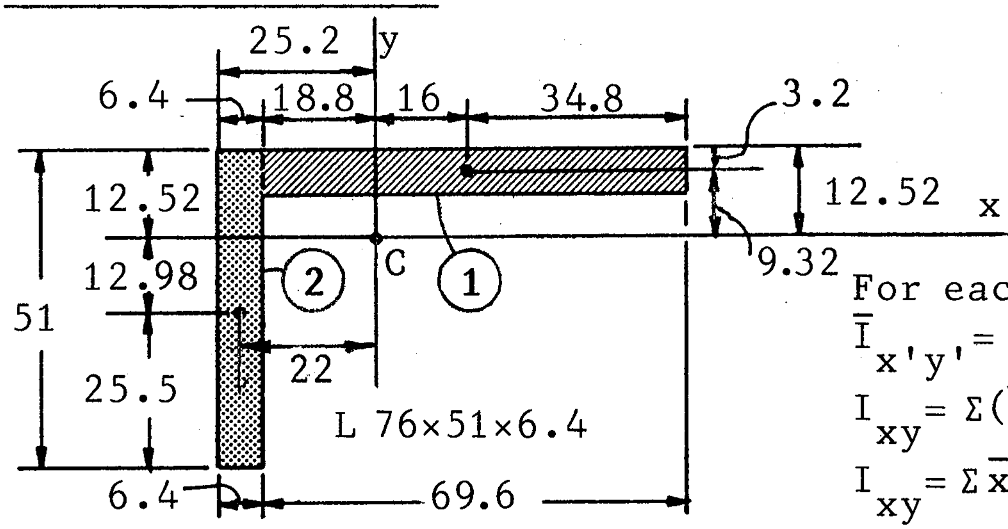
Dimensions in mm

Moments of Inertia. For the axes shown here we use Fig. 9.13B and record:

$$I_x = 0.163\times10^6\ \text{mm}^4 = 163\times10^3\ \text{mm}^4 \quad \triangleleft$$

$$I_y = 0.454\times10^6\ \text{mm}^4 = 454\times10^3\ \text{mm}^4 \quad \triangleleft$$

Product of Inertia. Divide area into two rectangles.



For each rectangle:

$$\bar{I}_{x'y'} = 0$$

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + \bar{x}\bar{y}A)$$

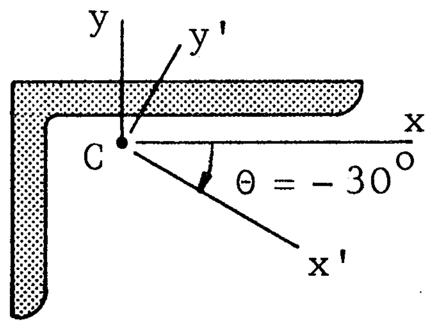
$$I_{xy} = \Sigma\bar{x}\bar{y}A$$

Rectangle	Area, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\bar{y}A$, mm^4
1	(6.4)(69.6) = 445.4	+16.0	+9.32	$+66.42\times10^3$
2	(6.4)(51) = 326.4	−22.0	−12.98	$+93.21\times10^3$

$$\Sigma\bar{x}\bar{y}A = +159.63\times10^3\ \text{mm}^4$$

$$I_{xy} = +159.6\times10^3\ \text{mm}^4 \quad \triangleleft$$

(continued)



We have found:

$$I_x = 163\times10^3\ mm^4$$
$$I_y = 454\times10^3\ mm^4$$
$$I_{xy} = +159.6\times10^3\ mm^4$$

For x', y' axes:

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{1}{2}(163 + 454)10^3 + \frac{1}{2}(163 - 454)10^3\cos(-60^\circ) - 159.6\times10^3\sin(-60^\circ)$$

$$= 308.5\times10^3 - 72.75\times10^3 + 138.2\times10^3$$

$$= 373.9\times10^3\ mm^4 \qquad I_{x'} = 374\times10^3\ mm^4 \blacktriangleleft$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= 308.5\times10^3 + 72.75\times10^3 - 138.2\times10^3$$

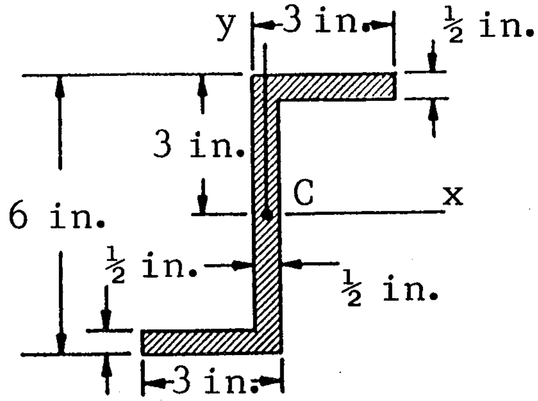
$$= 243.1\times10^3\ mm^4 \qquad I_{y'} = 243\times10^3\ mm^4 \blacktriangleleft$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{1}{2}(163 - 454)10^3\sin(-60^\circ) + 159.6\times10^6\cos(-60^\circ)$$

$$= +126.0\times10^3 + 79.80\times10^3$$

$$= +205.8\times10^3\ mm^4 \qquad I_{x'y'} = +206\times10^3\ mm^4 \blacktriangleleft$$

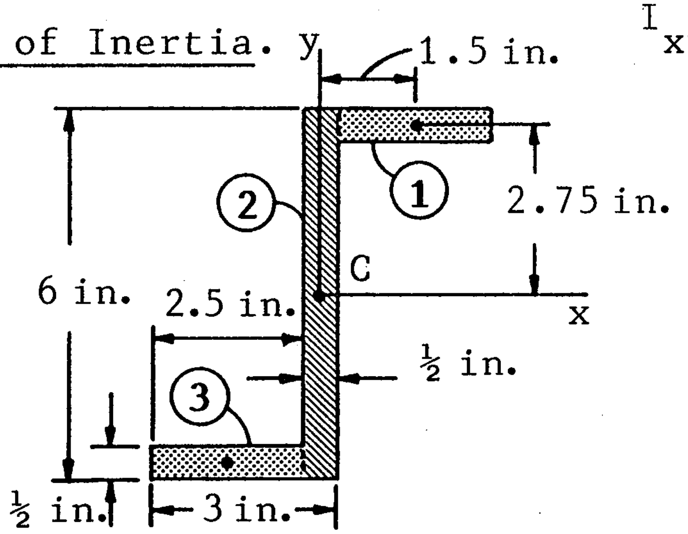


Determine the moments and product of inertia of the area shown with respect to new axes obtained by rotating the x and y axes through 30° counterclockwise.

Product of Inertia. From Prob. 9.60, we have

$$I_{xy} = +10.31 \text{ in}^4 \quad \triangleleft$$

Moments of Inertia.



$$I_x = 2(I_x)_1 + (I_x)_2 =$$

$$= 2\left[\frac{1}{12}(2.5)(0.5)^3 + (2.5)(0.5)(2.75)^2\right] + \frac{1}{12}(0.5)(6)^3$$

$$= 2(0.026 + 9.453) + 9.00$$

$$I_x = 27.96 \text{ in}^4 \quad \triangleleft$$

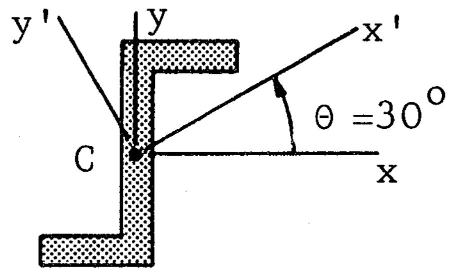
$$I_y = 2(I_y)_1 + (I_y)_2 =$$

$$= 2\left[\frac{1}{12}(0.5)(2.5)^3 + (2.5)(0.5)(1.5)^2\right] + \frac{1}{12}(6)(0.5)^3$$

$$= 2(0.651 + 2.813) + 0.062$$

$$I_y = 6.99 \text{ in}^4 \quad \triangleleft$$

(continued)



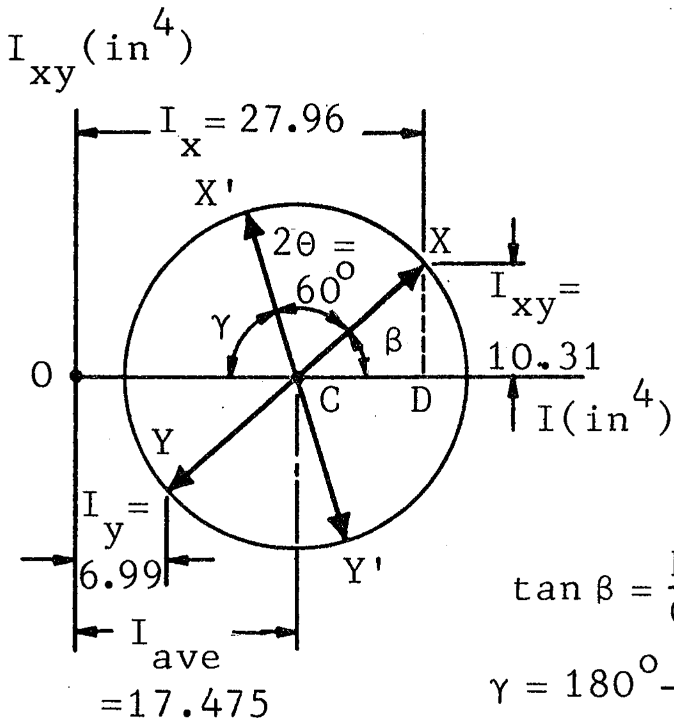
We have found:

$I_x = 27.96\ \text{in}^4$ ◁

$I_y = 6.99\ \text{in}^4$ ◁

$I_{xy} = +10.31\ \text{in}^4$ ◁

<u>Mohr's Circle.</u> The diameter XY is defined by X(27.96, +10.31) and Y(6.99, −10.31)



$$I_{ave} = \frac{1}{2}(27.96 + 6.99)$$
$$= 17.475\ \text{in}^4$$

$$CD = (27.96 - 17.475)$$
$$= 10.485\ \text{in}^4$$

$$R = \sqrt{(CD)^2 + (DX)^2}$$
$$= \sqrt{(10.485)^2 + (10.31)^2}$$
$$= 14.705\ \text{in}^4$$

$$\tan\beta = \frac{DX}{CD} = \frac{10.31}{10.485} \qquad \beta = 44.5^\circ$$

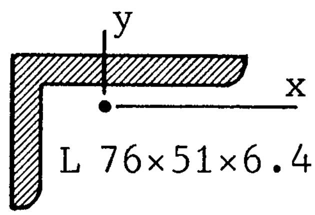
$$\gamma = 180^\circ - 60^\circ - 44.5^\circ \qquad \gamma = 75.5^\circ$$

$$I_{x'} = I_{ave} - R\cos\gamma = 17.475 - (14.705)\cos 75.5^\circ$$
$$= 17.475 - 3.682 = 13.793\ \text{in}^4 \qquad I_{x'} = 13.79\ \text{in}^4$$ ◀

$$I_{y'} = I_{ave} + R\cos\gamma$$
$$= 17.475 + 3.682 = 21.157\ \text{in}^4 \qquad I_{y'} = 21.16\ \text{in}^4$$ ◀

$$I_{x'y'} = +R\sin\gamma = (14.705)\sin 75.5^\circ = 14.237\ \text{in}^4$$

$$I_{x'y'} = +14.24\ \text{in}^4$$ ◀



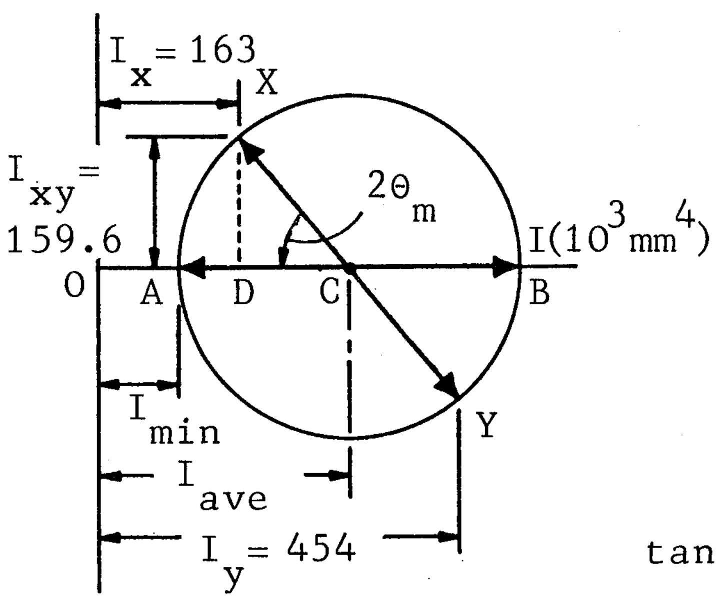
For the angle shown, determine the principal axes and the principal moments of inertia. From Fig. 9.13:

$$I_x = 163\times10^3\ \text{mm}^4, \quad I_y = 454\times10^3\ \text{mm}^4$$

From Prob. 9.66: $I_{xy} = +159.6\times10^3\ \text{mm}^4$

Mohr's Circle. The diameter is defined by

$$X(163\times10^3,\ 159.6\times10^3)\ \text{and}\ Y(454\times10^3,\ -159.6\times10^3)$$



$$I_{ave} = \frac{1}{2}(163 + 454)10^3 = 308.5\times10^3\ \text{mm}^4$$

$$CD = (308.5 - 163)10^3 = 145.5\times10^3\ \text{mm}^4$$

$$R = \sqrt{(CD)^2 + (DX)^2} = \sqrt{(145.5)^2 + (159.6)^2}\ 10^3 = 216\times10^3\ \text{mm}^4$$

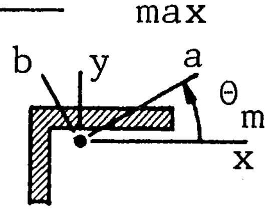
$$\tan 2\theta_m = \frac{DX}{CD} = \frac{159.6}{145.5}$$

$$2\theta_m = 47.6^\circ \qquad \theta_m = 23.8^\circ ↺ \blacktriangleleft$$

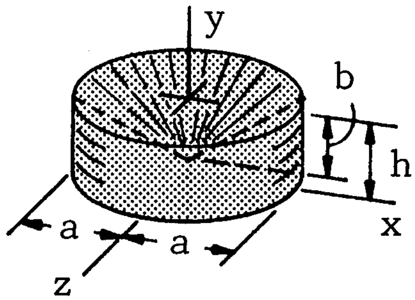
At A: $I_{min} = I_{ave} - R = (308.5 - 216)10^3 = 92.5\times10^3\ \text{mm}^4$

$$I_{min} = 92.5\times10^3\ \text{mm}^4 \blacktriangleleft$$

At B: $I_{max} = I_{ave} + R = (308.5 + 216)10^3 = 524.5\times10^3\ \text{mm}^4$

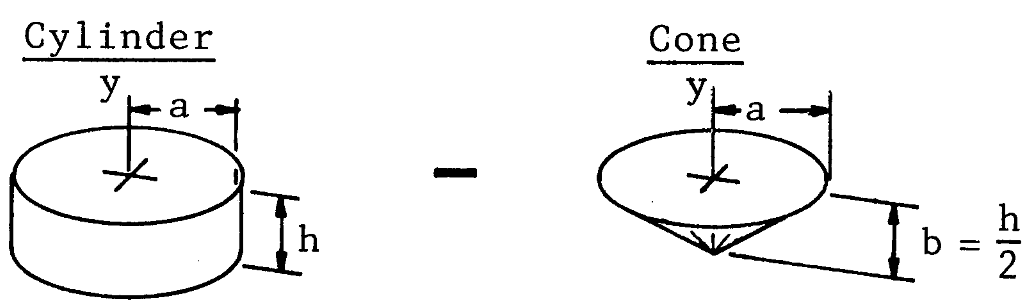


$$I_{max} = 524\times10^3\ \text{mm}^4 \blacktriangleleft$$



A machine part is formed by machining a conical surface into a circular cylinder. For $b = \frac{1}{2}h$, determine I_y and k_y for the machine part.

The machine part is seen to consist of:



Mass: $m_{cyl} = \rho\pi a^2 h$ $\qquad m_{cone} = \frac{1}{3}\rho\pi a^2 \frac{h}{2}$

I_y: $I_{cyl} = \frac{1}{2} m_{cyl} a^2$ $\qquad I_{cone} = \frac{3}{10} m_{cone} a^2$

$= \frac{1}{2}\rho\pi a^4 h$ $\qquad = \frac{3}{10}\left(\frac{1}{6}\rho\pi a^4 h\right)$

Machine Element

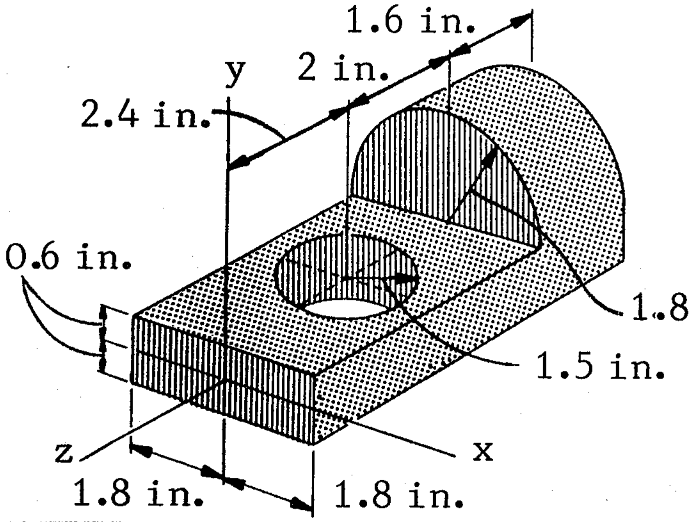
$$m = m_{cyl} - m_{cone} = \rho\pi a^2 h - \frac{1}{6}\rho\pi a^2 h = \frac{5}{6}\rho\pi a^2 h$$

$$I_y = I_{cyl} - I_{cone} = \frac{1}{2}\rho\pi a^4 h - \frac{1}{20}\rho\pi a^4 h = \frac{9}{20}\rho\pi a^4 h$$

Introducing the total mass m,

$$I_y = \frac{9}{20} a^2 \left(\frac{6}{5}\right)\left(\frac{5}{6}\rho\pi a^2 h\right) = \frac{54}{100} a^2 m \qquad I_y = \frac{27}{50} m a^2 \blacktriangleleft$$

$$k_y^2 = \frac{I}{m} = \frac{27}{50} a^2 \qquad k_y = 0.735 a \blacktriangleleft$$

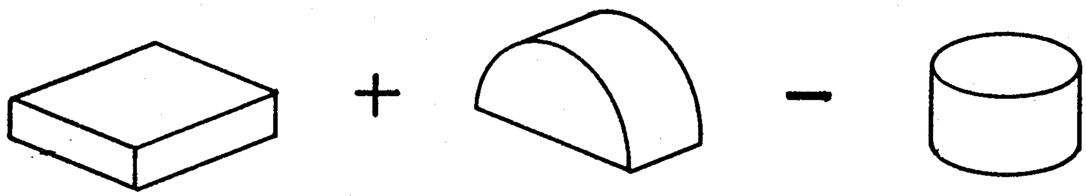


For the steel machine element shown, determine I_x.

$$\gamma = 490\ \text{lb/ft}^3$$

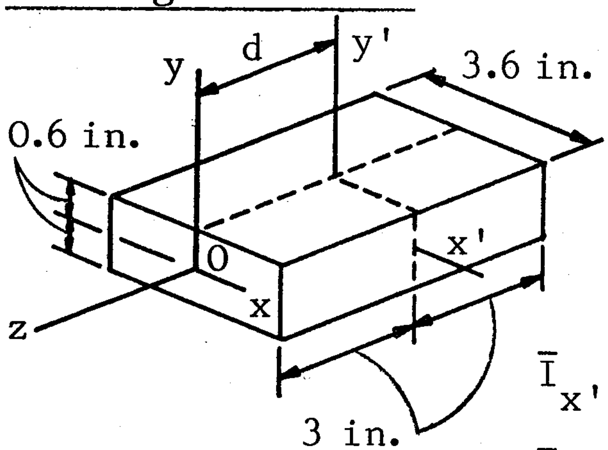
$$\rho = \frac{\gamma}{g} = \frac{490}{32.2}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}^4}$$

We observe that the machine element consists of:



Rectangular Prism + Half Cylinder − Cylinder

Rectangular Prism



$$V = (1.2\ \text{in.})(6\ \text{in.})(3.6\ \text{in.})$$

$$= 25.92\ \text{in}^3 = 0.015\ \text{ft}^3$$

$$m = \rho V = (0.015\ \text{ft}^3)\rho$$

$$\bar{I}_{x'} = \frac{m}{12}(a^2 + b^2)$$

$$\bar{I}_{x'} = \frac{(0.015\ \text{ft}^3)\rho}{12}\left[\left(\frac{1.2}{12}\text{ft}\right)^2 + \left(\frac{6}{12}\text{ft}\right)^2\right]$$

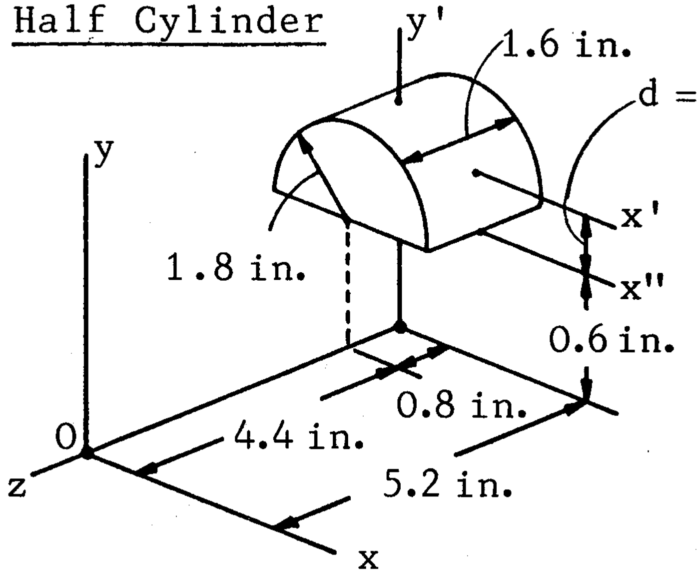
$$\bar{I}_{x'} = (0.325\times10^{-3}\ \text{ft}^5)\rho$$

$$I_x = \bar{I}_{x'} + md^2 = 0.325\times10^{-3}\rho + (0.015\rho)\left(\frac{3}{12}\right)^2$$

$$= 0.325\times10^{-3}\rho + 0.9375\times10^{-3}\rho \qquad I_x = (1.2625\times10^{-3}\text{ft}^5)\rho \ \triangleleft$$

(continued)

Half Cylinder



$$d = \frac{4r}{3\pi} = \frac{4(1.8)}{3\pi} = 0.7639 \text{ in.}$$

$$V = \frac{\pi}{2}(1.6)(1.8)^2 = 8.143 \text{ in}^3$$

$$= 4.712\times10^{-3} \text{ ft}^3$$

$$m = \rho V = 4.712\times10^{-3}\rho$$

$$I_{x''} = \frac{m}{12}(3r^2 + L^2)$$

$$I_{x''} = \frac{4.712\times10^{-3}\rho}{12}\left[3\left(\frac{1.8}{12}\right)^2 + \left(\frac{1.6}{12}\right)^2\right]$$

$$I_{x''} = 33.49\times10^{-6}\rho$$

$$I_{x''} = \bar{I}_{x'} + md^2$$

$$33.49\times10^{-6}\rho = \bar{I}_{x'} + (4.712\times10^{-3}\rho)\left(\frac{0.7639}{12}\right)^2$$

$$\bar{I}_{x'} = 14.39\times10^{-6}\rho$$

$I_x = \bar{I}_{x'} + ma^2$ (a = distance between x and x' axes)

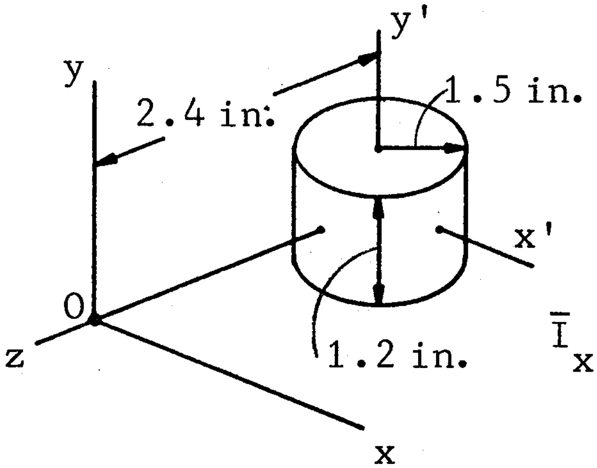
$$I_x = 14.39\times10^{-6}\rho + (4.712\times10^{-3}\rho)\left[\left(\frac{5.2}{12}\right)^2 + \left(\frac{0.6 + 0.7639}{12}\right)^2\right]$$

$$= 14.39\times10^{-6}\rho + (4.712\times10^{-3}\rho)(0.2007)$$

$$I_x = (0.9601\times10^{-3}\text{ft}^5)\rho \quad \triangleleft$$

(continued)

Cylinder. (To be subtracted)



$$m = \rho V = \rho\pi\left(\frac{1.5}{12}\right)^2\left(\frac{1.2}{12}\right)$$

$$= 4.909\times10^{-3}\rho$$

$$\bar{I}_{x'} = \frac{m}{12}(3r^2 + L^2)$$

$$\bar{I}_{x'} = \frac{4.909\times10^{-3}\rho}{12}\left[3\left(\frac{1.5}{12}\right)^2 + \left(\frac{1.2}{12}\right)^2\right]$$

$$\bar{I}_{x'} = (23.27\times10^{-6})\rho$$

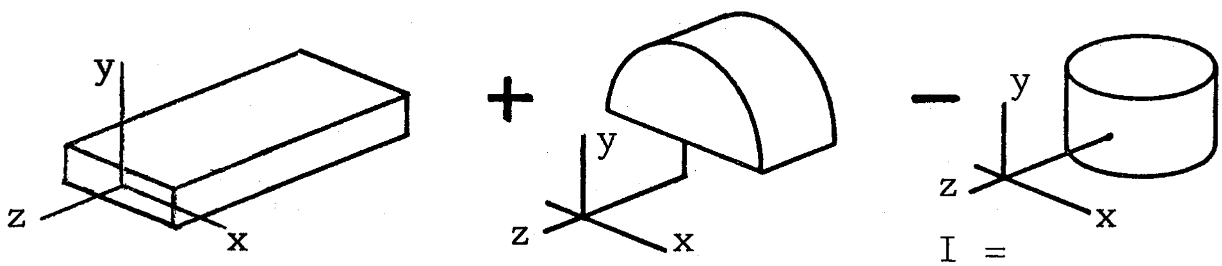
$$I_x = \bar{I}_{x'} + md^2$$

$$= 23.27\times10^{-6}\rho + (4.909\times10^{-3}\rho)\left(\frac{2.4}{12}\right)^2$$

$$I_x = (0.2196\times10^{-3}\,\text{ft}^5)\rho \quad \triangleleft$$

Entire Machine Element

Rectangular Prism | Half Cylinder | Cylinder

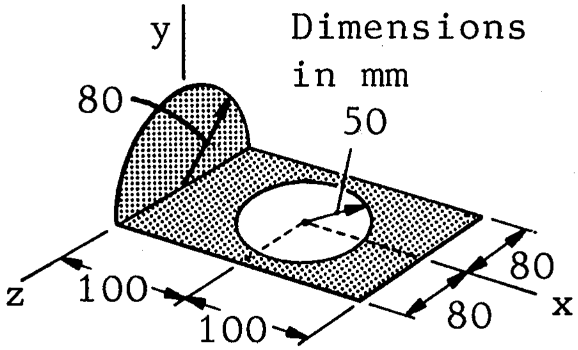


$I_x = (1.2625\times10^{-3}\,\text{ft}^5)\rho$ $\quad I_x = (0.9601\times10^{-3}\,\text{ft}^5)\rho$ $\quad I_x = (0.2196\times10^{-3}\,\text{ft}^5)\rho$

$$I_x = (1.2625 + 0.9601 - 0.2196)10^{-3}\rho = (2.003\times10^{-3}\,\text{ft}^5)\rho$$

$$= (2.003\times10^{-3}\,\text{ft}^5)\left(\frac{490}{32.2}\ \frac{\text{lb}\cdot\text{s}^2}{\text{ft}^4}\right) = 30.48\times10^{-3}\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_x = 30.5\times10^{-3}\,\text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad \blacktriangleleft$$



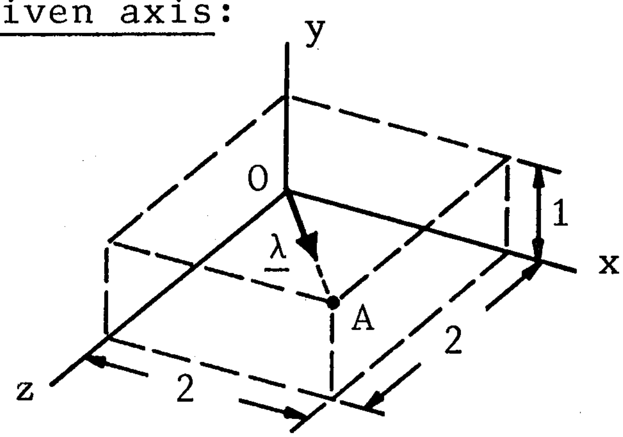
This machine part was considered in Sample Prob. 9.13. Determine I with respect to an axis through the origin characterized by the unit vector: $\underline{\lambda} = \frac{2}{3}\underline{i} + \frac{1}{3}\underline{j} + \frac{2}{3}\underline{k}$.

From Sample Prob. 9.13. (Page 375.)

$$I_x = 3.00\times10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$I_y = 13.28\times10^{-3}\ \text{kg}\cdot\text{m}^2$$
$$I_z = 11.29\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

All products of inertia are zero.

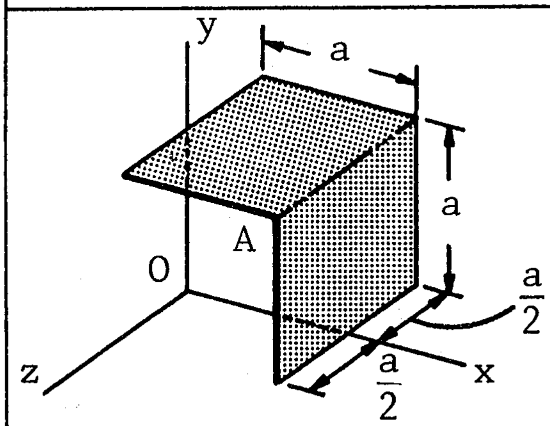
For given axis:



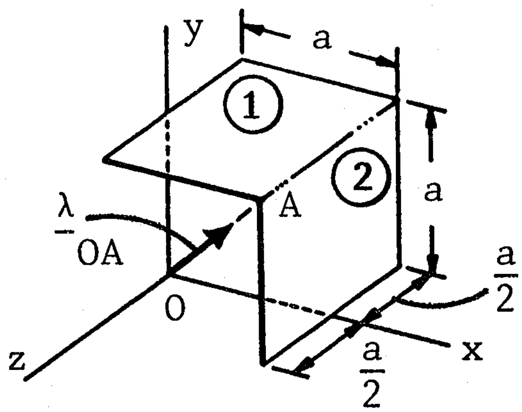
$$\lambda_x = \frac{2}{3}$$
$$\lambda_y = \frac{1}{3}$$
$$\lambda_z = \frac{2}{3}$$

Substituting into Eq.(9.46),

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2$$
$$= (3.00\times10^{-3})(\tfrac{2}{3})^2 + (13.28\times10^{-3})(\tfrac{1}{3})^2 + (11.29\times10^{-3})(\tfrac{2}{3})^2$$
$$= 7.827\times10^{-3}\ \text{kg}\cdot\text{m}^2 \qquad I_{OA} = 7.83\times10^{-3}\ \text{kg}\cdot\text{m}^2$$



The thin bent plate is of uniform density and total mass m. Determine its mass moment of inertia with respect to a line joining the origin O and point A.



Bent plate consists of two thin square plates each of mass $\frac{m}{2}$

Moments of Inertia

$$I_x = (I_x)_1 + (I_x)_2 = \frac{1}{12}\left(\frac{m}{2}\right)a^2 + \frac{m}{2}a^2 + \frac{1}{12}\left(\frac{m}{2}\right)\left(a^2 + a^2\right) + \frac{m}{2}\left(\frac{a}{2}\right)^2$$

$$I_x = \frac{3}{4}ma^2 \quad \triangleleft$$

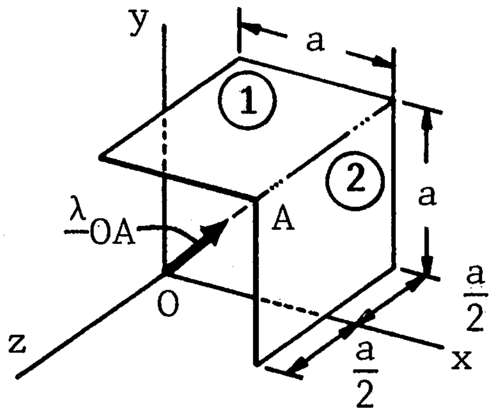
$$I_y = \frac{1}{12}\left(\frac{m}{2}\right)(a^2 + a^2) + \frac{m}{2}\left(\frac{a}{2}\right)^2 + \frac{1}{12}\left(\frac{m}{2}\right)a^2 + \frac{m}{2}a^2$$

$$I_y = \frac{3}{4}ma^2 \quad \triangleleft$$

$$I_z = \frac{1}{12}\left(\frac{m}{2}\right)a^2 + \frac{m}{2}\left[a^2 + \left(\frac{a}{2}\right)^2\right] + \frac{1}{12}\left(\frac{m}{2}\right)a^2 + \frac{m}{2}\left[a^2 + \left(\frac{a}{2}\right)^2\right]$$

$$I_z = \frac{4}{3}ma^2 \quad \triangleleft$$

(continued)



We have:

$$I_x = \frac{3}{4} m a^2 \quad \triangleleft$$

$$I_y = \frac{3}{4} m a^2 \quad \triangleleft$$

$$I_z = \frac{4}{3} m a^2 \quad \triangleleft$$

Products of Inertia. For each square plate $\bar{I}_{x'y'} = 0$

$$I_{xy} = \frac{m}{2}\left(+\frac{a}{2}\right)\left(+a\right) + \frac{m}{2}\left(+a\right)\left(+\frac{a}{2}\right) \qquad I_{xy} = +\frac{1}{2} m a^2 \quad \triangleleft$$

By symmetry: $\qquad I_{yz} = I_{zx} = 0 \quad \triangleleft$

Unit Vector along OA

$$(OA)^2 = a^2 + a^2 + \left(\frac{a}{2}\right)^2 \qquad OA = \frac{3}{2} a$$

$$\boldsymbol{\lambda}_{OA} = \frac{\overrightarrow{OA}}{OA} = \frac{a\mathbf{i} + a\mathbf{j} + \frac{a}{2}\mathbf{k}}{\frac{3}{2}a} \qquad \boldsymbol{\lambda}_{OA} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$$

Substituting into Eq.(9.46),

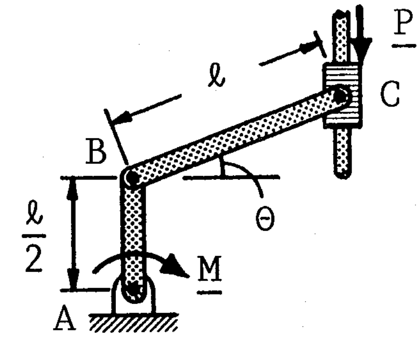
$$I_{OA} = I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \frac{3}{4}ma^2\left(\frac{2}{3}\right)^2 + \frac{3}{4}ma^2\left(\frac{2}{3}\right)^2 + \frac{4}{3}ma^2\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{2}ma^2\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) - 0 - 0$$

$$= ma^2\left(\frac{1}{3} + \frac{1}{3} + \frac{4}{27} - \frac{4}{9}\right) = \frac{10}{27} m a^2$$

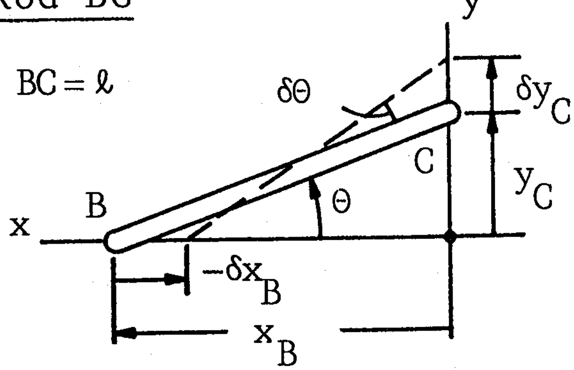
$$I_{OA} = \frac{10}{27} m a^2 \quad \blacktriangleleft$$

BEER & JOHNSTON VECTOR MECHANICS FOR ENGINEERS 4/e	STATICS	Problem 10.14 Page 400



Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage in the position shown.

Rod BC



For the coordinate system shown:

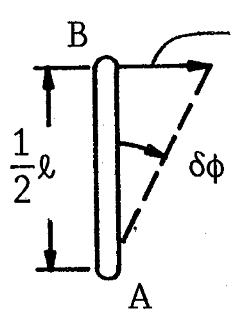
$$x_B = \ell \cos\theta$$

$$\delta x_B = -\ell \sin\theta\ \delta\theta$$

$$y_C = \ell \sin\theta$$

$$\delta y_C = \ell \cos\theta\ \delta\theta$$ ◁

Crank AB



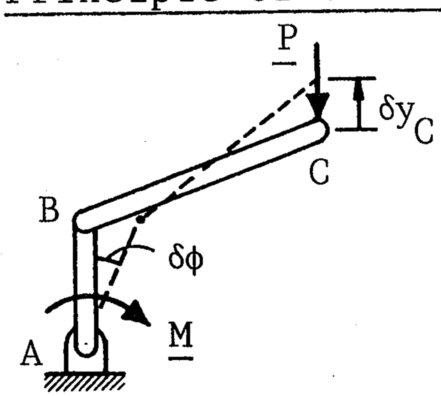
$$-\delta x_B = \ell \sin\theta\ \delta\theta$$

$$\frac{1}{2}\ell\,\delta\phi = -\delta x_B$$

$$= \ell \sin\theta\ \delta\theta$$

$$\delta\phi = 2\sin\theta\ \delta\theta$$ ◁

Principle of Virtual Work

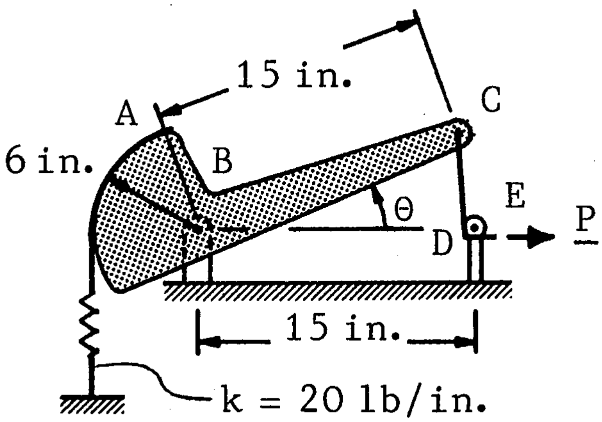


$$\delta U = 0:$$

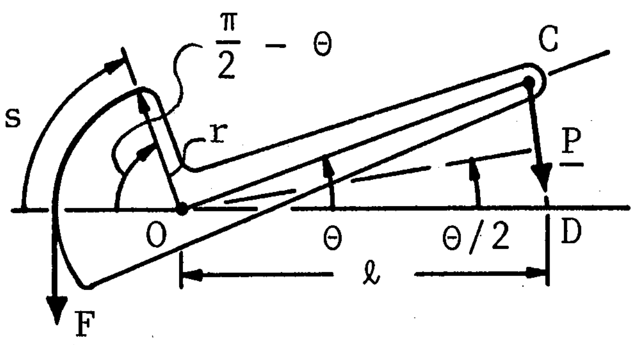
$$-P\,\delta y_C + M\,\delta\phi = 0$$

$$-P(\ell \cos\theta\ \delta\theta) + M(2\sin\theta\ \delta\theta) = 0$$

$$M = \frac{1}{2}P\ell\cot\theta$$ ◀



The spring is unstretched when $\theta = 90^\circ$. Determine the value of θ corresponding to equilibrium for P = 60 lb.



Spring. (θ in radians)

$$s = r\left(\frac{\pi}{2} - \theta\right)$$

$$\delta s = -r\,\delta\theta \quad \triangleleft$$

$$F = ks = kr\left(\frac{\pi}{2} - \theta\right)$$

Triangle COD is isosceles: $CD = 2\ell \sin\frac{\theta}{2}$

$$\delta(CD) = 2\ell\cos\frac{\theta}{2}\left(\frac{1}{2}\delta\theta\right) \qquad \delta(CD) = \ell\cos\frac{\theta}{2}\,\delta\theta \quad \triangleleft$$

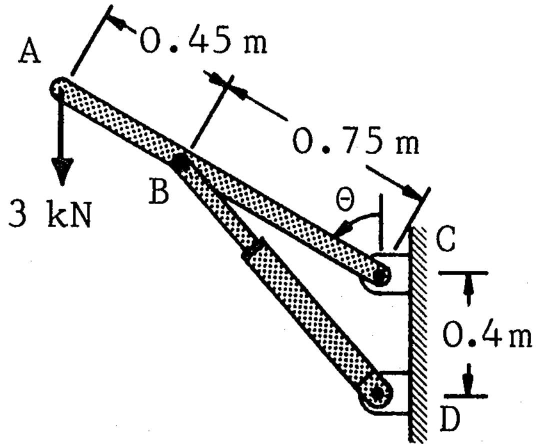
Principle of Virtual Work. Since F tends to decrease s and P tends to decrease CD, we have:

$\delta U = 0$: $\quad -F\delta s - P\,\delta(CD) = 0$

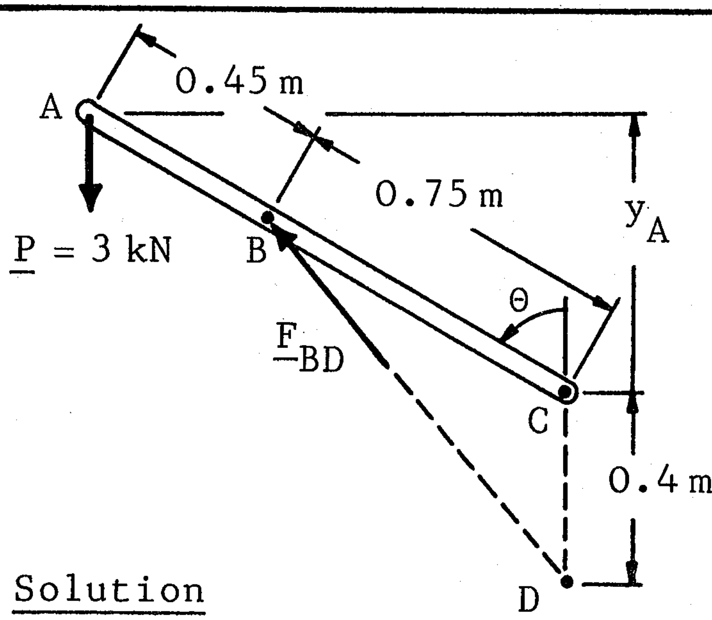
$$-kr\left(\frac{\pi}{2} - \theta\right)(-r\,\delta\theta) - P\left(\ell\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$\frac{\frac{\pi}{2} - \theta}{\cos\frac{\theta}{2}} = \frac{P\ell}{kr^2} = \frac{(60\text{ lb})(15\text{ in.})}{(20\text{ lb/in.})(6\text{ in.})^2} = 1.25$$

Solve by trial: $\theta = 0.33868$ rad $\qquad \theta = 19.4^\circ$ ◀



For the loading shown, (a) express the force exerted by the hydraulic cylinder on pin B as a function of the length BD, (b) determine the largest value of θ if the maximum force exerted by the cylinder on B is 12.5 kN.



Solution

Origin for y at C: $y_A = (1.2\,\text{m})\cos\theta \qquad \delta y_A = -1.2\sin\theta\,\delta\theta$ ◁

Length BD. Law of cosines:

$$(BD)^2 = (BC)^2 + (CD)^2 - 2(BC)(CD)\cos(180^\circ - \theta)$$

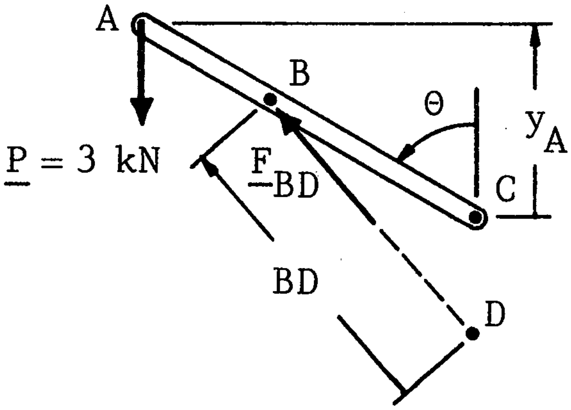
$$= (0.75)^2 + (0.4)^2 + 2(0.75)(0.4)\cos\theta$$

$$(BD)^2 = 0.7225 + 0.6\cos\theta \qquad (1)$$

Differentiating: $2(BD)\,\delta(BD) = -0.6\sin\theta\,\delta\theta$

$$\delta(BD) = -\frac{0.3\sin\theta}{BD}\,\delta\theta \qquad (2)$$

(continued)



We have:

$$\delta y_A = -1.2 \sin\theta \, \delta\theta$$

$$\delta(BD) = -\frac{0.3 \sin\theta}{BD} \, \delta\theta$$

Principle of Virtual Work. Since $\underline{P}$ tends to decrease y_A and $\underline{F}_{BD}$ tends to increase BD, we write

$\delta U = 0$: $\quad -P \, \delta y_A + F_{BD} \, \delta(BD) = 0$

$$-P(-1.2 \sin\theta \, \delta\theta) + F_{BD}\left(-\frac{0.3 \sin\theta}{BD} \, \delta\theta\right) = 0$$

$$F_{BD} = 4(BD)P$$

(a) For $P = 3\text{ kN}$, $\quad F_{BD} = 4(BD)3 \qquad F_{BD} = 12(BD)$

(b) For $(F_{BD})_{max} = 12.5\text{ kN}$, $\qquad 12.5 = 12(BD)$

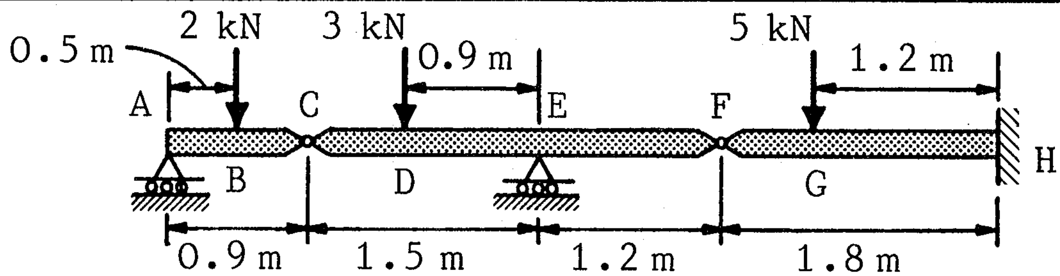
$$BD = 1.04167\text{ m}$$

We recall Eq.(1):

$$(BD)^2 = 0.7225 + 0.6 \cos\theta$$

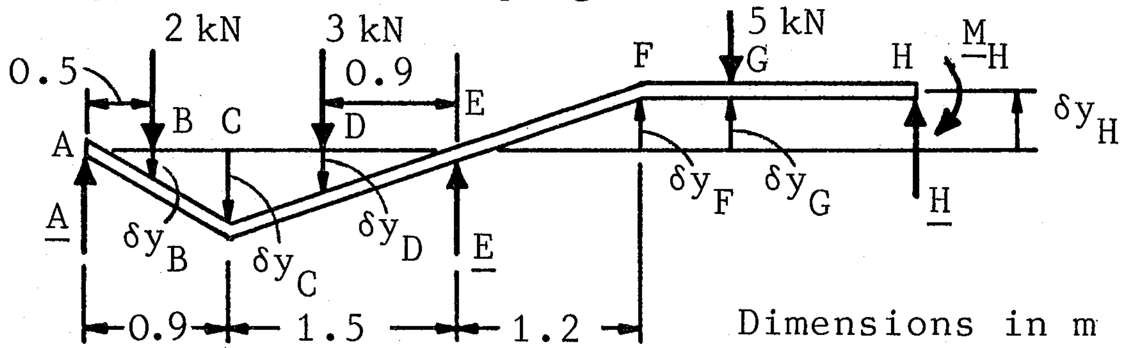
$$(1.04167)^2 = 0.7225 + 0.6 \cos\theta$$

$\cos\theta = 0.6043 \qquad \theta = 52.8^\circ$



Determine seperately the force and couple representing the reaction at H.

Force at H. We give a vertical virtual displacement δy_H to point H while keeping FH horizontal.



Geometry $\quad \delta y_F = \delta y_G = \delta y_H$

$$\delta y_D = \frac{0.9}{1.2}\,\delta y_F = 0.75\,\delta y_H$$

$$\delta y_C = \frac{1.5}{0.9}\,\delta y_D = \frac{1.5}{0.9}(0.75\,\delta y_H) = 1.25\,\delta y_H$$

$$\delta y_B = \frac{0.5}{0.9}\,\delta y_C = \frac{0.5}{0.9}(1.25\,\delta y_H) = 0.6944\,\delta y_H$$

Principle of Virtual Work

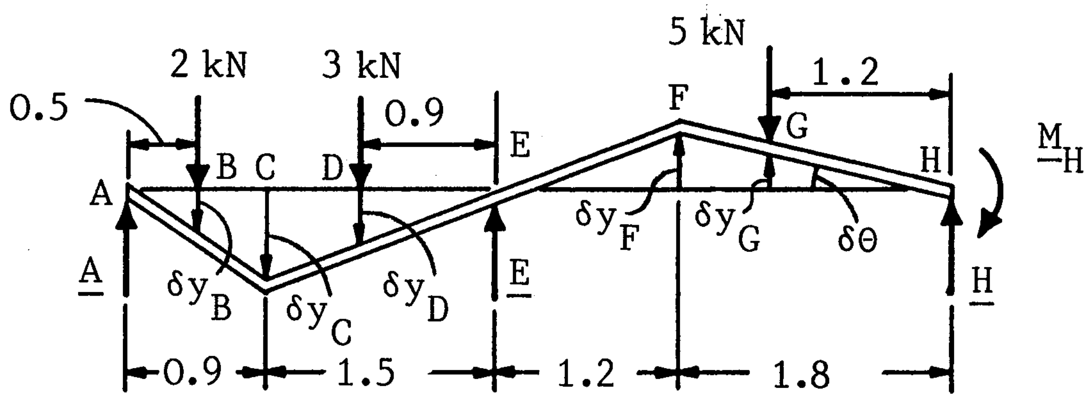
$$\delta U = 0: \quad (2\text{ kN})\,\delta y_B + (3\text{ kN})\,\delta y_D - (5\text{ kN})\,\delta y_G + H\,\delta y_H = 0$$

$$2(0.6944\,\delta y_H) + 3(0.75\,\delta y_H) - 5\,\delta y_H + H\,\delta y_H = 0$$

$H = +1.361$ kN $\qquad$ **H** = 1.361 kN ↑ ◀

(continued)

Couple at H. We rotate FH through $\delta\theta$ about H.



Geometry

$\delta y_G = 1.2\ \delta\theta$

$\delta y_F = 1.8\ \delta\theta$

$\delta y_D = \frac{0.9}{1.2}\ \delta y_F = \frac{0.9}{1.2}(1.8\ \delta\theta) = 1.35\ \delta\theta$

$\delta y_C = \frac{1.5}{0.9}\ \delta y_D = \frac{1.5}{0.9}(1.35\ \delta\theta) = 2.25\ \delta\theta$

$\delta y_B = \frac{0.5}{0.9}\ \delta y_C = \frac{0.5}{0.9}(2.25\ \delta\theta) = 1.25\ \delta\theta$

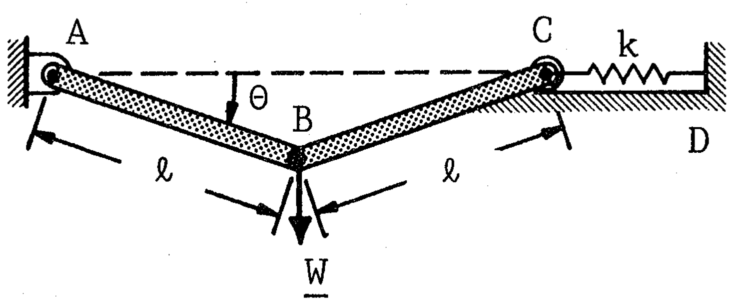
Principle of Virtual Work

$\delta U = 0$: $(2\text{ kN})\ \delta y_B + (3\text{ kN})\ \delta y_D - (5\text{ kN})\ \delta y_G + M_H\ \delta\theta = 0$

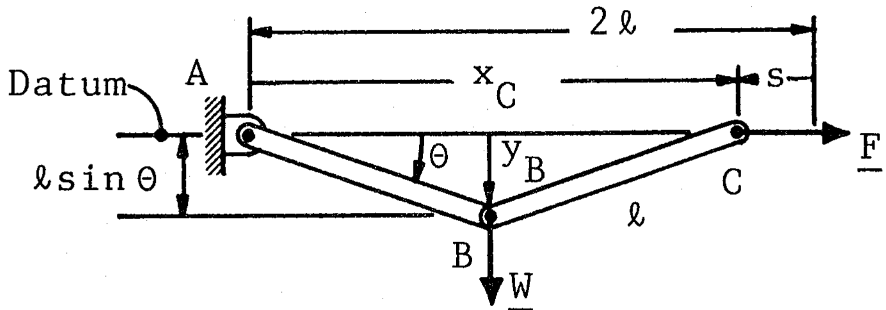
$2(1.25\ \delta\theta) + 3(1.35\ \delta\theta) - 5(1.2\ \delta\theta) + M_H\ \delta\theta = 0$

$M_H = -0.550\text{ kN·m}$

$\mathbf{M}_H = 550\text{ N·m}$ ↻ ◀



When AB and BC are horizontal the spring in unstretched. Determine the value of θ for equilibrium for: W = 900 N, ℓ = 225 mm, k = 2 kN/m



Spring. $x_C = 2\ell\cos\theta$ $s = 2\ell - 2\ell\cos\theta$

Potential Energy: $V_e = \frac{1}{2}ks^2 = \frac{1}{2}k(2\ell - 2\ell\cos\theta)^2$ ◁

Weight. Potential Energy: $V_g = -Wy_B = -W\ell\sin\theta$ ◁

Total Potential Energy

$$V = V_e + V_g = 2k\ell^2(1 - \cos\theta)^2 - W\ell\sin\theta$$

Position of Equilibrium

$$\frac{dV}{d\theta} = 0: \quad 2k\ell^2[2(1 - \cos\theta)\sin\theta] - W\ell\cos\theta = 0$$

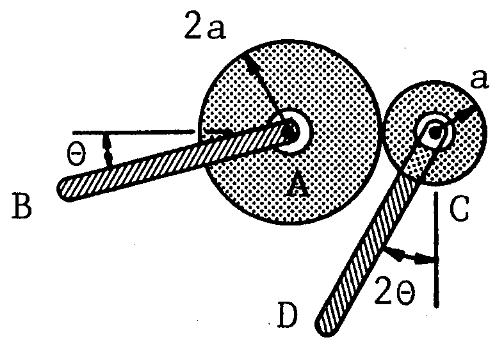
$$(1 - \cos\theta)\tan\theta = \frac{W}{4k\ell}$$

Substitute given data:

$$(1 - \cos\theta)\tan\theta = \frac{900\text{ N}}{4(2000\text{ N/m})(0.225\text{ m})}$$

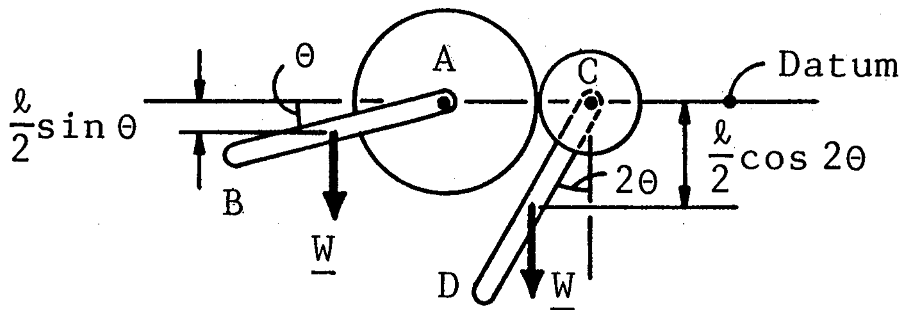
$$(1 - \cos\theta)\tan\theta = 0.5$$

Solve by trial: $\theta = 52.23^\circ$ $\theta = 52.2^\circ$ ◀



Two uniform rods, each of length ℓ and weight $W = 3$ lb are attached to gears as shown. (Gear teeth not shown here.) Determine the positions of equilibrium of the system and state whether the equilibrium is stable, unstable or neutral.

Potential Energy



$$V = -W\left(\frac{\ell}{2}\sin\Theta\right) - W\left(\frac{\ell}{2}\cos 2\Theta\right)$$

$$V = \frac{1}{2}W\ell(-\sin\Theta - \cos 2\Theta) \qquad (1)$$

Positions of Equilibrium

$$\frac{dV}{d\Theta} = \frac{1}{2}W\ell(-\cos\Theta + 2\sin 2\Theta) \qquad (2)$$

$$= \frac{1}{2}W\ell(-\cos\Theta + 4\sin\Theta\cos\Theta)$$

$\dfrac{dV}{d\Theta} = 0$: $\quad \frac{1}{2}W\ell\cos\Theta(-1 + 4\sin\Theta) = 0$

$\cos\Theta = 0 \qquad \Theta = -90^\circ$ and $\Theta = 90^\circ$ ◀

and, $-1 + 4\sin\Theta = 0$

$\Theta = \sin^{-1}\frac{1}{4} \qquad \Theta = 14.5^\circ$ and $\Theta = 165.5^\circ$ ◀

(continued)

Positions of Equilibrium

$\theta = -90^\circ, \quad \theta = 90^\circ, \quad \theta = 14.5^\circ, \quad \theta = 165.5^\circ$

Stability of Equilibrium

Eq.(2): $$\frac{dV}{d\theta} = \frac{1}{2}W\ell(-\cos\theta + 2\sin 2\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{1}{2}W\ell(\sin\theta + 4\cos 2\theta)$$

θ	$\frac{d^2V}{d\theta^2}$	stability
-90°	$\frac{1}{2}W\ell[\sin(-90^\circ) + 4\cos(-180^\circ)] = -2.5W\ell < 0$	unstable ◀
14.5°	$\frac{1}{2}W\ell(\sin 14.5^\circ + 4\cos 29.0^\circ) = +1.874W\ell > 0$	stable ◀
90°	$\frac{1}{2}W\ell(\sin 90^\circ + 4\cos 180^\circ) = -1.5W\ell < 0$	unstable ◀
165.5°	$\frac{1}{2}W\ell(\sin 165.5^\circ + 4\cos 331^\circ = +1.874W\ell > 0$	stable ◀