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Fracture mechanics

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Abstract

In this article, after a brief description of some significant catastrophic structural failures going back to the period when iron and steel began to replace wood and masonry as the primary structural materials, the historical development of the field of fracture mechanics is discussed. First, the basic concepts of the field, the motivation underlying their development and the key contributions made in the area are briefly described. A summary of the key engineering applications of fracture mechanics, its methods, some recent important contributions and areas of future research is is then presented.

1. Introduction

In designing structural or machine components an important step is the identification of the most likely mode of failure and the application of a suitable failure criterion. *Fracture* characterized as the formation of new surfaces in the material is one such mode of mechanical failure. At the most basic level the essential feature of the process is breaking of interatomic bonds in the solid. From a macroscopic standpoint, however, fracture may be viewed as the rupture separation of the structural component into two or more pieces due to the propagation of cracks. In between the process involves the nucleation, growth and coalescence of microvoids and cracks in the material. Thus, in studying the fracture of solids ideally one would have to consider such widely diverse factors as the microscopic phenomena taking place at various length scales, and the macroscopic effects regarding the loading, environmental conditions, and the geometry of the medium. Due to this highly complex nature of the phenomenon, at the present time there seems to be no single *theory* dealing satisfactorily with all its relevant aspects. Quite naturally, then, the theories developed to study the fracture of solids tend to

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treat the subject generally from one of three points of view, namely microscopic or atomic, microstructural, and macroscopic or continuum mechanics.

From the standpoint of engineering applications, it has been the macroscopic theories based on the notions of continuum solid mechanics and classical thermodynamics that have provided the quantitative working tools in dealing with the fracture of structural materials. In the macroscopic approach to fracture, it is generally assumed that the material contains some flaws which may act as fracture nuclei and that the medium is a homogeneous continuum in the sense that the size of a dominant flaw is large in comparison with the characteristic microstructural dimension of the material. The problem is, then, to study the influence of the applied loads, the flaw geometry, environmental conditions and material behavior on the fracture process in the solid — a subject which has come to be known as *fracture mechanics*. The purpose of this article is to discuss, in general terms, the historical development of the field, the underlying motivation, the description of its basic concepts, engineering applications, and areas of future research.

2. Experience with brittle fracture

It is commonly known that surface notches and cracks would be very helpful in breaking certain materials. The practical and very useful applications of this knowledge in such areas as splitting wood, breaking and shaping rocks, and cleaving gems are perhaps as old as the human experience. With the progress made in metal working technology within the past three centuries, iron and steel gradually replaced wood and masonry as the primary structural materials. Despite their apparent overdesign, these metal structures and components did not always function satisfactorily and experienced occasional unexpected failures. Some of these accidents resulting from the catastrophic failure of a critical component in large structures and transportation and energy equipment have caused considerable loss of life and wide-spread property damage. A somewhat detailed description of some of these highly publicized catastrophic failures in oil and gas storage tanks, pressure vessels, turbine-generator rotors, steam boilers, pipelines, bridges, airplanes, railways and welded ships may be found in Volume 5 of Liebowitz (1968).

One of the earliest recorded brittle fractures in a major structure was the break of one of the main chains of the Montrose suspension bridge on March 19, 1830 in Great Britain. Since then, there have been a number of catastrophic bridge failures, including relatively recent cases of the Husselt Bridge over the Albert Canal in Belgium (1938), King's Bridge in Melbourne, Australia (1962) and Point Pleasant Bridge in West Virginia (1967). Through the 19th century, railway accidents due to fracture in axles, rails and wheels were relatively common. During the decade 1860–1870, the number of people who died in railway accidents in Great Britain alone was in the order of two hundred per year. Among typical storage tank failures, one may mention the molasses tank accident in Boston. On January 19 1919, ''without an instant's warning the top (of the tank) was blown into the air and the sides were burst apart'', collapsing a number of buildings and part of the elevated railway structure and causing 12 deaths and forty injuries. Another typical pressure boundary failure has been the fracture propagation in natural gas transmission lines. In this case, the crack growth usually initiates at surface defects and is aided by corrosion. Upon the rupture of net ligaments in the pipe wall, the crack propagates rapidly and attains a terminal velocity. The characteristic feature of the process is that, very often, it is accompanied by fire and, in the early designs, the terminal velocity of the running crack could be greater than the velocity of the decompression waves in the gas. For the materials used in recent designs, however, the operating conditions have been well onto the upper shelf. Despite this, occasionally long running cracks still occur, but at speeds less than that of the decompression waves.

The related considerations of fracture control and crack arrest techniques have relevance to all pressurized containers.

The structural failures that attracted the attention of mechanics and materials communities most were two systematic events in the 1940s and 1950s, namely the failures of welded ships and commercial jet airplanes. Starting in 1943, over 4000 'Liberty' type all-welded cargo ships and 530 T2 type tankers were built in various shipyards in the United States and Canada. Of these, over 1200 experienced brittle fracture of the hull, 233 of which were catastrophic and lost to service and 16 broke in half. Subsequent research performed in the United States and Great Britain, mostly by the materials community, identified notch brittleness as one of the major causes of these fractures. The concept of notch brittleness was, of course, not new and was known since 1885. What came out of these studies was, however, the concept of brittle–ductile transition temperature. Because of the presence of cyclic loading combined with occasional peak loads, low temperatures and a highly corrosive environment, marine vessels are particularly prone to brittle fracture and the catastrophic failures continue to occur. Indeed, a very recent news report states that:

> jolting sailors awake from their sleep, the freighter *Flare*, en route from Rotterdam to Montreal, broke in half for unknown reasons on Friday (January 16 1998) and sank off the south coast of Newfoundland. Rescuers managed to save 4 crew members and retrieved the bodies of 15 others. Six are still missing in icy waters. (*New York Times*)

The second important incident was the loss of the Comet I airplanes, the world's first jet-propelled passenger aircraft, built by de Havilland in Great Britain. The design of these aircraft started in 1946, the prototype flew in 1949 and the first commercial flight took place in May 1952. On January 10 1954, the first aircraft entering the passenger service disintegrated in the air at approximately 30,000 feet and crashed into the Mediterranean Sea after 1286 pressurized flights. On January 11 1954, the Comets were removed from service and, after some modifications, they resumed service on March 23 1954. Shortly after, on April 8 1954, another Comet disintegrated in the air at 35,000 feet and crashed into the Mediterranean near Naples. On April 12 1954, the certificate of airworthiness for the Comets was withdrawn. To determine the cause of the accidents, an aircraft which had accumulated 1230 flights was subjected to cyclic loading under water simulating pressurized flights and to 33% overload at approximately 1000 cycle intervals. It was under one of these proof tests, after 1830 further pressurizations in the test facility, that the cabin failed. The fracture initiated at the corner of a passenger window. Examination of the failure indicated evidence of fatigue. Further investigation of the first failed aircraft recovered near Elba confirmed that the primary cause of the accident was pressure cabin failure due to fatigue (Liebowitz, 1968).

In all these accidents resulting from brittle fracture, it appears that in each case the then existing design rules were fully followed and yet catastrophic failures continued to occur. It was, therefore, becoming very difficult to attribute the causes of the failures to 'material defects' only, as was routinely done prior to the 1940s. Thus, the large scale ship failures during the 1940s and the failures in a highly critical industry of jet transportation in the 1950s were responsible not only for the recognition of brittle fracture as a serious problem but, to a great extent, for the extensive research that followed to find its causes and to develop methods for its control.

3. The development of the field

The early strength theories of solids were based on maximum stress. However, it appears that the so-called 'size effect', which plays a rather important role in fracture, was known before the introduction of the concept of stress to study the strength of solids. In one of his sketch books, Leonardo da Vinci describes his experiments on breaking iron wires and how the weight required to break the wire increases as its length is cut in half in successive tests (Timoshenko, 1953). Similar results were observed by Lloyd and by Le Blanc in the 1830s, again on iron bars and wires. In 1858, Karmarsch gave an empirical expression for the load bearing capacity of metal wires which had the form $\sigma_u = A + (B/d)$, where A and B are constants, d is the wire diameter and σ_u is the breaking stress. The trend was substantially verified later by Griffith's experiments on glass fibers (Griffith, 1921). Early studies showed that the strength was also dependent on surface quality and, particularly, on surface notches. This was observed by Wöhler in his fatigue studies in the 1860s and by Kommers in 1912. These and similar investigations showed that surface polishing could increase the strength of the machined specimens by as much as 20 to 50 percent.

3.1. Wieghardt's Work

It is against this background that Griffith became interested in the strength of solids. However, before discussing the key contributions made by Griffith, it is worth mentioning a previous study by Wieghardt (1907, 1995) which is particularly important from the solid mechanics standpoint. In a remarkable, but little-known, article, Wieghardt essentially provided the solution for a linear elastic wedge subjected to an arbitrary concentrated force P applied to one of the wedge boundaries. The solution includes the analysis of the asymptotic behavior of the stress state near the wedge apex and the special case of the crack problem in considerable detail. This appears to be the first elasticity solution in which the existence of stress singularity was recognized, its correct form of $r^{-\alpha}$ was obtained (r being the distance from the wedge apex), and the dependence of α on the wedge angle and on the symmetry of loading was demonstrated[1]. In the crack problem, after obtaining the solution, the leading terms having the form $1/\sqrt{r}$ in the asymptotic expansion were separated and the correct angular distributions were given. Wieghardt then went on to state that:

> we will now use these equations (for the stresses) to provide answers to more questions one might pose regarding the *strength* of our crack against the action of the force P. One may ask: given the strength parameters of our elastic material, what is the magnitude of P necessary for material fracture? And, furthermore, at which place and in which direction will the fracture initiate?

Thus, since he did not question the validity of the maximum stress criterion for fracture, Wieghardt was faced with a paradox. At the crack tip, the stress becomes infinite for any arbitrarily small P and yet the experience shows that the material fractures only if P is raised to a critical value. He tried to resolve the paradox by stating, that:

> since an elastic material does not rupture at a single point but rather fractures over a small section, one might argue that cracking occurs not due to *specific* stresses (or deformations) but due to a *resultant* over a small section.

Since the stresses are integrable, the resultant would always be finite. Thus, after essentially side-stepping the first question, he observes that, if it occurs at all, the fracture will initiate at the crack tip. He then proceeds to examine the direction of fracture initiation by using the then accepted hypotheses of *maximum shear stress* or *maximum tensile stress*. Thus, Wieghardt's work leaves the impression that

[1] Under symmetric loading, $\alpha=\alpha_1$, $0 \leq \alpha_1 \leq 1/2$ for $\pi \leq \theta_o \leq 2\pi$ and, under skewsymmetric loading, $\alpha=\alpha_2$, $0 \leq \alpha_2 \leq 1/2$ for $1.43\pi \leq \theta_o \leq 2\pi$, where θ_o is the total wedge angle. For a given θ_o, α_1 is always greater than α_2.

the fracture criterion would consist of the comparison of an average stress in a 'small section' around the crack tip with the theoretical strength of the solid[2].

3.2. Griffith's work

The starting point of Griffith's studies was the then current knowledge based on ample observations in glass and metal wires, rods, and plates that there is an approximately two orders of magnitude difference between theoretical strength and bulk strength of solids, and his conclusion, based again on observations, that various forms of imperfections, defects and scratches are primarily responsible for this discrepancy. The obvious approach would then be to calculate the correct values of the maximum stresses around these defects and compare them with the theoretical strength of the material[3]. This Griffith did by simulating the defects with an elliptical hole, the solution for which was previously given by Inglis. The results showed that the calculated maximum stress is independent of the absolute size of the flaw and depends only on the ratio of the semiaxes of the ellipse. These findings were in apparent conflict with the test results and led Griffith to conclude that 'maximum stress' may not be an appropriate strength criterion and an alternative theory was needed (Griffith, 1921, 1924).

The basic concept underlying Griffith's new theory was that, similar to liquids, solids possess surface energy and, in order to propagate a crack (or increase its surface area), the corresponding surface energy must be compensated through the externally added or internally released energy. For a linear elastic solid, this input energy which is needed to extend the crack may be calculated from the solution of the corresponding crack problem. Using Inglis's solution for a uniformly loaded plate with an elliptical hole, Griffith calculated the increase in strain energy and, from the energy balance, obtained the stress corresponding to fracture as

$$\sigma = \left(\frac{2\gamma E^*}{\pi a}\right)^{1/2},$$

where $E^* = E$ for plane stress and $E^* = E/(1-\nu^2)$ for plane strain conditions, γ is the specific surface energy and a is the half crack length. Thus, regarding the fracture of brittle solids, Griffith's major contributions were that he was able to resolve the infinite stress paradox recognized earlier by Wieghardt and to show that the fracture stress is dependent on the flaw size through the expression $\sigma = m/\sqrt{a}$, where m is a material constant. He also verified this expression by performing some carefully designed experiments on pressurized glass tubes and spherical bulbs containing cracks of various sizes.

3.3. The stress intensity factors and Irwin's work

Griffith's work was largely ignored by the engineering community until the early 1950s. The reasons for this appear to be (a) in the actual structural materials the level of energy needed to cause fracture is orders of magnitude higher than the corresponding surface energy, and (b) in structural materials there are always some inelastic deformations around the crack front that would make the assumption of linear elastic medium with infinite stresses at the crack tip highly unrealistic.

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[2] Wieghardt's work along with Kolosoff's solution of plane crack problems obtained by using complex potentials (1914) still remains relatively unknown. The journal in which Wieghardt's article appeared ceased publication in 1922. Upon the publication of the English translation of Muskhelishvili's book in 1953, the method of complex potentials became, of course, quite well-known and subsequently was very widely and effectively used in solving crack problems.

[3] The theoretical strength may be estimated as $\sigma_c = (E\gamma/c)^{1/2}$ where E, γ and c are, respectively, the Young's modulus, the surface energy and the lattice parameter. More refined calculations seem to indicate that $\sigma_c = (1/4\text{–}1/13)E$.

In the early 1950s, the energy balance theory was also reconsidered by researchers who were primarily interested in catastrophic failure of large scale metallic structures. The X-ray studies conducted earlier by Orowan (1948) showed that even in materials fracturing in a 'purely brittle' manner, there was evidence of extensive plastic deformation on the crack surfaces. This led Irwin (1948) and Orowan independently to conclude that the plastic work γ_p at the crack front must also be considered as dissipative energy and the surface energy γ in Griffith's model must be replaced by $\gamma+\gamma_p$. Orowan also estimated that for typical metals, $\gamma_p \cong 10^3\gamma$. What followed during the next few years was solid mechanics research perhaps at its best: a combination of relatively simple analyses, physically sound conjectures and some careful experiments — most of the important contributions coming from Irwin and his coworkers. In the early 1950s, Irwin first made the observation that in a fracturing elastic solid, if the characteristic size of the energy dissipation or plastic zone around the crack tip is very small compared to the crack size, then it is reasonable to assume that the energy flowing into the crack tip fracture zone will come from the elastic bulk of the solid and, therefore, will not be critically dependent on the details of the stress state very near the crack tip. Consequently, one may conclude that the stress state in the elastic bulk of the medium will not differ from a purely elastic crack solution to any significant extent. The importance of this seemingly simple observation cannot be overemphasized and lies in the fact that one may now be justified in using a purely elastic solution to calculate the rate of energy available for fracture.

Irwin's second important contribution was the development of a universal method for calculating the rate of energy available to fracture the solid. For this, the continuum elasticity solution for each crack geometry and loading condition is needed. Griffith's work showed only too well how difficult this task could be. During this period, the timely analytical results appear to be Sneddon's work on plane and axisymmetric crack problems. By using Westergaard's solution for plane problems and by solving the penny-shaped crack problem, Sneddon (1946) obtained the correct asymptotic behavior of the stress field near the crack tip and showed that the results for the two cases differ only by a numerical factor of $2/\pi$. For the penny-shaped crack, he also obtained the correct expression of Griffith's energy balance equation. However, Sneddon seemed to have failed to recognize the universal nature of the crack tip stress field by stating (incorrectly) that $\sigma_{\theta\theta}$ which appears in the axisymmetric problem has no analog in the plane problem. As later pointed out by Irwin (1957), asymptotically, the stress state around the border of penny-shaped crack is one of plane strain and the identification of the 'numerical factor' observed by Sneddon is a key factor in generalizing the results. One of Irwin's major contributions, thus, was his recognition of the universal nature of asymptotic stress and displacement fields around the crack front in a linear elastic solid. He observed that the symmetric crack solutions given by Sneddon and Westergaard may be generalized to include the asymptotic expressions for all crack problems and, for a small distance r from the crack tip, the stresses may be expressed as:

$$\sigma_{ij} \simeq \left(\frac{K}{\sqrt{2\pi r}}\right) f_{ij}(\theta),$$

where f_{ij} are the functions previously obtained by Wieghardt, Westergaard and Sneddon for specific crack geometries and loading conditions. Irwin called the coefficient of the singular term, K, the *stress intensity factor*. Note that with universal functions $f_{ij}(\theta)$ known and independent of the crack geometry and loading conditions, K fully characterizes the asymptotic stress and displacement fields around the crack front.

To calculate the energy available for fracture, Irwin then interpreted the fixed-grip strain energy release rate involving the entire solid in terms of the rate of crack closure energy, which can be calculated by using the local asymptotic expressions for the crack surface displacement and the corresponding cleavage stress only. For the symmetric loading of a planar crack, he then evaluated the energy release rate as K^2/E^* and designated it by **G**. Subsequently, by using Westergaard's solution he showed that the stresses and displacements in the close neighborhood of the smooth internal boundary of a planar crack in a linearly elastic solid under most general loading conditions may be expressed in terms of three stress intensity factors K_{I}, K_{II} and K_{III} (and three sets of related universal angular functions) associated with the symmetric opening, in-plane shear and antiplane shear modes of deformation, respectively. He also evaluated the corresponding strain energy release rates $\mathbf{G}_{\mathrm{I}}$, $\mathbf{G}_{\mathrm{II}}$ and $\mathbf{G}_{\mathrm{III}}$ and their sum **G** for the general co-planar crack growth problem.

To make the energy balance theory, as modified by Irwin, an effective design tool, the concept of a realistic single-parameter characterization of the material's resistance to fracture also needed to be developed. This required the additional assumption that, in solids fracturing in a brittle manner, the size and shape of the fracture process (or the energy dissipation) zone remain essentially constant as the crack propagates, leading to the hypothesis that in such solids the energy needed to create a unit fracture surface is a material constant. Irwin designated this fracture resistance energy as the *fracture toughness*, $\mathbf{G}_{\mathrm{IC}}$. Thus, by the middle of the 1950s, nearly all the pieces of a new field called *fracture mechanics* were in place and, by the late 1950s, the field was ready for rapid expansion.

In addition to the loss of Comets during this period, the aerospace industry suffered a number of serious structural failures that appeared to be brittle in nature and caused a great deal of concern. These fracture failures included that of landing gears, propellers, high strength bolts, plastic canopies, rocket motor cases, propellant tanks and aircraft fuselages. In its early stages, the rapid development of fracture mechanics was greatly helped by three major factors. The first was the willingness of the aerospace industry (led by the Boeing Company) to support research in fracture mechanics and to apply some of its results in designing civilian and military equipment. The second was the enthusiastic response shown by the solid mechanics community to undertake theoretical and experimental research in this new field. The third was the recognition of the potential and importance of the field by the federal funding agencies.

4. Linear elastic methods

During the 1950s and early 1960s, the immediate concern in applications was brittle fracture and fatigue crack growth. It turned out that the stress intensity factor could be used as an extremely effective correlation parameter to model both of these phenomena. Since the stress intensity factor is a product of linear elasticity solutions of crack problems, the initial efforts in fracture mechanics research were concentrated on developing and adapting methods for solving such problems. A fairly thorough description of the methods used in solving elastic crack problems may be found in Erdogan (1978) and various articles in Liebowitz (1968), Sih (1973) and Atluri (1986).

There are two major methods of solving the mixed boundary value problems in elasticity that arise from the formulation of the crack problems, namely the complex function theory and the integral transforms. The complex potentials were first introduced by Goursat to represent the biharmonic function. Their applications to problems in elasticity were, however, developed in detail by a school of Georgian mathematicians led by Muskhelishvili. The method is nearly indispensable for the examination of the singular nature of stresses at the crack tips, particularly in problems involving bonded dissimilar materials. However, the shortcoming of the method is that it is restricted to two-dimensional problems.

There are a number of techniques used in the solution of two-dimensional crack problems which are also based on the complex function theory. Among these, one may mention conformal mapping, Laurent series expansion, boundary collocation method and certain applications of the Wiener–Hopf method.

The method of integral transforms is one of the most widely used techniques in the formulation of the boundary value problems in mechanics. Depending on the crack geometry and the coordinate system, the most often used transforms are Fourier, Mellin and Hankel transforms. In simpler cases, the problem can very often be reduced to an Abel's equation and solved directly. In many cases, however, it may be necessary to reduce the resulting dual series or dual integral equations to singular integral equations, use the complex function theory to obtain the fundamental function and solve the problem numerically by taking advantage of the properties of the related orthogonal polynomials (Erdogan, 1978). Among other analytical methods used for solving linear elastic crack problems, one may also mention the method of eigenfunction expansion and the alternating method.

Theoretical methods are essential for solving crack problems for two main reasons. First, they provide the correct form of singularities and asymptotic results that may be needed to analyze and interpret the experimental results and to use for improving the accuracy of purely numerical solutions. Secondly, they provide accurate solutions for relatively simple part/crack geometries and for certain idealized material behavior that could be used as benchmarks for numerical and approximate procedures. However, in practical applications, the geometry of the medium is seldom simple and realistic material models seldom lead to analytically tractable formulations. It is therefore necessary to develop purely numerical methods that can accommodate complicated part/crack geometries and material models. The *finite element method*, which is another major contribution of the solid mechanics community to the science and art of engineering within the past fifty years, appears to be ideal for this purpose and is widely used in fracture mechanics (see, for example, Atluri, 1986 and Owen and Fawkes, 1983).

One may also note that in certain problems, such experimental techniques as photoelasticity, moire interferometry and the method of caustics may prove to be very effective in estimating the stress intensity factors. In particular, the three-dimensional photoelasticity (using frozen fringe technique) was shown to be very useful in studying cracked structural components with relatively complex geometries (for review and references, see Kobayashi, 1973, 1975).

5. Elastic–plastic fracture mechanics

Under operating temperatures and relatively high loads, most engineering materials exhibit some form of inelastic behavior. The main feature of the fracture process in such materials is that the characteristic length parameter of the inelastic region around the crack front where the energy dissipation takes place is generally of the same order of magnitude as the crack size. In elastic–plastic materials, the size of the plastic zone itself is very heavily dependent on the constraint conditions along the crack front. Thus, in relatively 'thick' specimens, the interior region would be under plane strain conditions, whereas, near the surfaces, due to lack of constraints, there would be a transition to plane stress conditions, accompanied by greater plastic zone size and higher resistance to fracture. One would then observe a low energy flat fracture in the interior and shear lip formation and high energy ductile fracture near the surfaces. Very often, the terminology $\mathbf{G}_{\mathrm{IC}}$ used for 'fracture toughness' in practice implies plane strain value of $\mathbf{G}_{\mathrm{c}}$, the critical strain energy release rate. The latter is rather heavily dependent on specimen constraints, particularly on thickness. Furthermore, the size of the plastic region around the crack front varies with growing crack size, generally increasing with the increasing crack size. This implies that a material's resistance to fracture would also increase with increasing crack size.

5.1. The R-curve

For ductile crack propagation, it appears that two essential conditions need to be satisfied. The first is a local condition needed for crack growth initiation. The mechanism for crack initiation may be the nucleation, growth and coalescence of voids or microcracks, or simply decohesion at the crack tip. For this, in a small region ahead of the crack front, the mechanical conditions must reach a critical state regardless of what goes on in the surrounding plastic region. The initiation of crack growth can thus be characterized by a single strength parameter such as *critical crack tip opening displacement*, *critical crack opening angle* or *work of separation*. On the other hand, for further crack growth or unstable fracture, the condition of global energy balance must be satisfied. Since the material's resistance to ductile fracture is highly dependent on the crack size and the geometry of the medium, this second condition cannot be characterized by a single strength parameter.

The first important attempt at modeling the ductile fracture process, too, seems to have been made by Irwin and his associates. By recognizing the increase in fracture resistance with growing crack size, they introduced the concept of the *crack extension resistance curve* (R-curve), which consists of the plot of total energy dissipation rate (including the work of separation) as a function of the crack size. The fracture resistance is represented by energy ($\mathbf{G}_R$, R, J_R) or sometimes by the equivalent stress intensity factor K_R. The R-curve is thus a continuously distributed parameter characterization of ductile fracture growth. By also including the crack driving force $\mathbf{G}$ or K in the same plot, the concept provides an effective tool to examine the processes of slow stable crack growth and fracture instability.

5.2. The J-integral

Even though it is based on a very sound physical concept, initially the R-curve was not widely used in applications. The main reasons for this are that it is generally dependent on the geometry of the component and the corresponding crack driving force is difficult to calculate. However, starting in the early 1970s, practicing engineers began to use the R-curve technique rather extensively. There seems to be a number of factors influencing this trend. It was during this period that *fracture mechanics* was becoming not only an interesting field to study but also an acceptable and effective design tool. There were some highly critical safety issues involving nuclear pressure vessels and other pressurized containers that needed a closer examination by a more rational technique. There were also some experimental studies indicating that, at least for some materials, the R-curve may indeed be the universal signature of the material's ductile fracture resistance. Finally, there was the very timely appearance of Rice's work on the so-called J-integral.

The theoretical basis of the J-integral may be found in certain conservation laws previously studied by Eshelby and Günther. Also, physical concepts similar to that of the J-integral regarding the energy release rate were described earlier by Sanders and Cherepanov. However, for two-dimensional notch and crack problems in nonlinear elastic solids, Rice (1968) seems to have developed the J-integral independently. Rice showed that in a general nonlinear elastic solid, the line integral

$$\int_{\Gamma}\left(W\,\mathrm{d}y-\mathbf{T}\cdot\frac{\partial\mathbf{u}}{\partial x}\,\mathrm{d}s\right)$$

encircling the crack tip is path independent and its value J represents the energy release rate for coplanar crack growth, that is $J = \mathrm{d}(U-V)/\mathrm{d}a$, where U is the external work, V is the strain energy, a is the crack length, W is the strain energy density, $\mathbf{T}$ is the traction vector and $\mathbf{u}$ the displacement vector on Γ. Since nonlinear elasticity is equivalent to the deformation theory of plasticity (provided there is no unloading) and since the path integral can be calculated in a straightforward manner by using, for example, the finite element technique, J gradually became an attractive alternative to $\mathbf{G}$ or K in studying elastic–plastic fracture.

It was also at this time that Hutchinson and Rice and Rosengren worked out the details of the singular behavior of stresses, strains and deformations around the crack tip for a particular nonlinear solid (Ramberg–Osgood strain hardening material) and showed that J is a measure of the strength of this so-called HRR singularity. Even though the concept of J is not valid in the plastic zone near the crack tip where loading is nonproportional, J would give a reasonably good approximation to the energy release rate, provided the characteristic size of the plastic zone with nonproportional loading is small compared to the size of the J-controlled region. This is somewhat analogous to the fracture process zone and K-controlled region in linear elastic fracture mechanics and provides further analytical justification for selectively using J as the measure of crack driving force in certain elastic–plastic fracture problems.

6. Engineering applications

Prior to the 1960s, the primary failure criteria used in mechanical design were tensile strength for brittle fracture, Mises or Tresca criterion for yielding, energy absorption or toughness for impact and various versions of the Wöhler diagram for fatigue. Since, in most cases, the loading has a cyclic component, fatigue has always been a major consideration in design. Thus, until the 1960s, the universally accepted criterion for design was the *safe life* criterion which required that time for crack initiation be longer than the expected operational life of the structure. In bulky components, nearly 95% of the life is consumed by the initiation of a detectable crack. Therefore, despite the heavy reliance on safety factors necessitated by large scale variabilities in observations, the technique is still used rather widely in conventional design. However, in some very highly critical application such as (civilian and military) aircraft components, nuclear pressure vessels, steel bridges and certain microelectronics devices, the existence of initial defects that may form the nuclei of fatigue cracking is practically unavoidable. The assumption then must be that the 'crack' is always there and is growing; it becomes detectable only when it reaches a certain size. Consequently, in these structures a very significant part of the service life is consumed by subcritical crack propagation. It turns out that by using the stress intensity factor as the load parameter, the subcritical crack growth rate can be correlated much more tightly and hence, the service life can be estimated with much less uncertainty.

In the 1970s, these consideration led to an entirely new design philosophy based on *damage tolerance evaluation* concept (FAA Handbook, 1993). The objective of damage tolerance analysis is to insure that, at any given time, the remaining crack growth life is greater than the expected accumulation of service loads. This can be achieved by developing an inspection program that detects and monitors the initiation and propagation of cracks due to fatigue, corrosion and possible accidents. The inspection intervals must be determined in such a way that they provide ample opportunity for corrective action. The entire procedure is based on a fracture mechanics approach, is highly structured (mostly through the design and maintenance regulations issued by the FAA, USAF and NRC, and standards and codes designed by the ASTM and ASME), and very widely used. In fact, fracture mechanics plays a vital role in the design of every critical structural or machine component in which durability and reliability are important issues. It has also become a valuable tool for material scientists and engineers to guide their efforts in developing materials with improved mechanical properties.

7. Areas of active research and research needs

7.1. Composites, interfaces, graded materials

Composite materials generally consist of two dissimilar phases having a particulate, layered or fiber or filament-reinforced structure. In applications, the constituents are invariably metals, polymers and ceramics. In most cases, the material is processed in such a way that the medium may be modeled as an isotropic or anisotropic homogeneous continuum. In some cases, however, the properties of the medium may be intentionally graded in a continuous or piecewise homogeneous manner. Fracture-related failures in composites may generally be considered at two levels. First, there is the phase of formation and propagation of small cracks. In this phase microstructure, particularly interfaces, play a major role, the crack size is of the order of the local characteristic dimension of the material and from the standpoint of fracture mechanics, the medium may be modeled as a piecewise homogeneous continuum with various crack geometries. Very often, the problem is mixed mode and three-dimensional with cracks intersecting or growing subcritically along the interfaces under thermomechanical and residual stresses.

In the second phase of the fracture process, the crack size is generally very large in comparison with the characteristic dimensions of the constituents and, as a consequence, the composite medium may be modeled as an homogeneous continuum with a macroscopic crack. The global thermomechanical properties of the medium are obtained by using various homogenization techniques and an appropriate subcritical crack growth model, and energy balance criteria are used in fracture analysis (see Dvorak, this volume and Hutchinson and Suo, 1992 for review and references).

Some of the favorable properties of composites may be enhanced by grading their volume fractions and consequently their thermomechanical properties. Among the potentially important applications of these so-called *functionally graded materials* (FGMs), one may mention thermal barrier coatings, wear and corrosion-resistant coatings, interfacial zones for increasing the bonding strength and reducing the residual stresses and thermoelectric materials for increasing the figure of merit in direct energy conversion systems. In FGMs, the composite medium is processed in such a way that the material properties are continuous functions of the thickness coordinate. The analytical consequence of this is the elimination of such anomalous behavior as non-square-root and complex crack tip singularities associated with piecewise homogeneous materials (Erdogan, 1995). Following are some of the research needs:

- Three-dimensional corner singularities in bonded dissimilar materials.
- Singularities in interface cracks intersecting stress-free boundaries.
- Determination of local residual stresses in bonded anisotropic solids and their effect on crack initiation.
- Three-dimensional periodic surface cracking and crack propagation in coatings.
- The effect of temperature dependence of the thermomechanical parameters in layered materials undergoing thermal cycling and thermal shock.
- The effect of material and geometric nonlinearities on spallation.
- Crack tip singularities in inelastic graded materials.
- Crack tip behavior in graded materials — additional nonsingular terms.
- Developing methods for fracture characterization of FGMs at room and elevated temperatures.

7.2. Three-dimensional problems

Despite the fact that quite considerable progress has been made in three-dimensional crack problems since the 1950s, there are still some unresolved issues that require further study:

- A clean asymptotic study of the stress field for a plane crack intersecting a stress-free surface at an arbitrary angle.
- Kinking of a three-dimensional crack under mixed mode loading.

- Full field solution of part-through and through crack problems in relatively thin-walled plate and shell structures in plastic range (net ligament instability, flat-to-shear transition).

7.3. Dynamic effects, NDE

Dynamic effects become significant in two groups of crack problems, namely solids with a stationary crack under impact loading, and solids with a crack propagating at a sufficiently high velocity. In the former, the asymptotic stress field near the crack tip is identical to that of the corresponding elastostatic crack problem and there is generally an overshoot in the stress intensity factor. In the latter, the asymptotic crack tip behavior is dependent on the instantaneous crack velocity only (see the article by Freund in Atluri, 1986 and Rosakis, this volume, for review and references). An important application of elastodynamics is in the area of nondestructive evaluation (NDE) of structural components, which generally involves the determination of the type, size, location and orientation of flaws in the medium from ultrasonic and acoustic emission test results (Achenbach, this volume). Some of the research needs are:

- The effect of component geometry — free and fixed boundaries.
- Three-dimensional effects.
- Effect of material anisotropy and material nonlinearity.
- Characterization methods — dynamic effects.
- Asymptotic analysis of and criterion for dynamic crack branching and crack arrest.
- Branching of dynamically growing interface cracks and its effect on fracture resistance.
- Wave propagation in FGMs and spallation fracture.

Other areas of active research and critical issues relating to fracture mechanics are computational methods, experimental methods (Knauss this volume; Rosakis this volume), subcritical crack growth due to corrosion, corrosion fatigue, creep and fatigue (Ritchie, this volume), and probabilistic considerations (Harlow and Wei, 1993).

8. Conclusions

It is in the nature of human endeavor that technology always looks forward and never waits for science to provide the fundamental solutions. Whether or not they have the necessary physically sound tools, the engineers have and will continue to design and build. Fracture is one critical area in which the solid mechanics community has provided the fundamental concepts as well as the working tools and has maintained its leadership in the development of the subject from the very beginning. However, it should be observed that despite the considerable progress made during the past fifty years, the field is still very much in development. Many of the unresolved fundamental issues such as environmental degradation, oxidation and high temperature behavior are multidisciplinary in nature and require understanding and treatment of the subject from the viewpoints of material science and electrochemistry as well as solid mechanics.

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