and, for plane strain,

$$\left.\begin{aligned} u &= \frac{K_{II}}{G}(r/2\pi)^{1/2}\sin\frac{\theta}{2} \cdot \left[2 - 2\nu + \cos^2\frac{\theta}{2}\right] \\ v &= \frac{K_{II}}{G}(r/2\pi)^{1/2}\cos\frac{\theta}{2} \cdot \left[1 - 2\nu + \sin^2\frac{\theta}{2}\right] \end{aligned}\right\} \ldots (146)$$

These results are reflected in Eqs 2 and 14, for the second mode.

The first and second modes may be superimposed, since

$$\Phi = \Phi_I + \Phi_{II} \ldots\ldots\ldots (147)$$

is a perfectly permissible Airy stress function, in which case stress and displacement components should simply be added to each other.

Third Mode:

The plane (two-dimensional) problem of pure shear may be specified by:

$$u = 0, \quad v = 0, \quad w = w(x,y) \ldots (148)$$

The strain-displacement equations and Hooke's law give (105)

$$\left.\begin{aligned} \gamma_{xz} &= \frac{\partial w}{\partial x} = \frac{\tau_{xz}}{G} \\ \gamma_{yz} &= \frac{\partial w}{\partial y} = \frac{\tau_{yz}}{G} \end{aligned}\right\} \ldots\ldots\ldots (149)$$

The stress components, σ_x, σ_y, σ_z, and τ_{xy}, all vanish so the equilibrium equations become

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \ldots\ldots\ldots (150)$$

which, when combined with Eqs 149, gives

$$\nabla^2 w = 0 \ldots\ldots\ldots\ldots (151)$$

Choosing

$$w = \frac{1}{G}\,\mathrm{Im}\, Z_{III} \ldots\ldots\ldots (152)$$

leads to

$$\left.\begin{aligned} \tau_{xz} &= \mathrm{Im}\, Z_{III}' \\ \tau_{yz} &= \mathrm{Re}\, Z_{III}' \end{aligned}\right\} \ldots\ldots\ldots (153)$$

The stress function, Z_{III}, for a crack along the negative y-axis to the origin, takes the form near the crack tip

$$Z_{III}\big|_{|\zeta|\to 0} = \frac{K_{III}}{(2\pi\zeta)^{1/2}} \ldots\ldots (154)$$

Consequently,

$$K_{III} = \lim_{|\zeta|\to 0} (2\pi\zeta)^{1/2} Z_{III} \ldots\ldots (155)$$

Moreover, substituting Eq 154 into Eqs 152 and 153 and using Eq 132 lead to

$$\left.\begin{aligned} \tau_{xz} &= -\frac{K_{III}}{(2\pi r)^{1/2}}\sin\frac{\theta}{2} \\ \tau_{yz} &= \frac{K_{III}}{(2\pi r)^{1/2}}\cos\frac{\theta}{2} \end{aligned}\right\} \ldots\ldots (156)$$

and

$$w = \frac{K_{III}}{G}(2r/\pi)^{1/2}\sin\frac{\theta}{2} \ldots\ldots (157)$$

These results are reflected in Eqs 3 and 15 for the third mode.

APPENDIX II

A HANDBOOK OF BASIC SOLUTIONS FOR STRESS-INTENSITY FACTORS AND OTHER FORMULAS

The results to be presented for stress-intensity factors will conform with their definition as implied by Eqs 1–3, 48, 81, and 94. References which contain further results and details will be listed for the readers convenience.

A selection of solutions for stress-intensity factors, in addition to those already listed, will be chosen on the basis of their generality. Since superposition may be used, that is, addition of the stress-intensity factors for each mode, the results which lend themselves