to this form. Noting that Eq 131 may be substituted into Eqs 125, and using polar coordinates, that is,

$$\zeta = re^{i\theta} \qquad (132)$$

the crack-tip stress field is:

$$\left.\begin{aligned} \sigma_x &= \frac{K_I}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \\ \sigma_y &= \frac{K_I}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \\ \tau_{xy} &= \frac{K_I}{(2\pi r)^{1/2}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \end{aligned}\right\} \qquad (133)$$

where, from Eq 131,

$$K_I = \lim_{|\zeta|\to 0} (2\pi\zeta)^{1/2} Z_I \qquad (134)$$

The strain in the y-direction can be written in terms of displacements and stresses by Hooke's law, or

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) \qquad (135)$$

For plane strain, Hooke's law ($\epsilon_z = 0$) also leads to

$$\sigma_z = \nu(\sigma_x + \sigma_y) \qquad (136)$$

Substituting Eqs 125 and 136 into Eq 135 and integrating lead to

$$v = \frac{1+\nu}{E}[2(1-\nu)\,\mathrm{Im}\,\bar{Z}_I - y\,\mathrm{Re}\,Z_I] \qquad (137)$$

Similarly, consideration for ϵ_x gives

$$u = \frac{1+\nu}{E}[(1-2\nu)\,\mathrm{Re}\,\bar{Z}_I - y\,\mathrm{Im}\,Z_I] \qquad (138)$$

Substituting Eqs 131 and 132 into Eqs 137 and 138 and noting $E = 2G(1+\nu)$ lead to

$$\left.\begin{aligned} u &= \frac{K_I}{G}(r/2\pi)^{1/2}\cos\frac{\theta}{2}\left[1 - 2\nu + \sin^2\frac{\theta}{2}\right] \\ v &= \frac{K_I}{G}(r/2\pi)^{1/2}\sin\frac{\theta}{2}\left[2 - 2\nu - \cos^2\frac{\theta}{2}\right] \end{aligned}\right\} \qquad (139)$$

(for plane strain, $w = 0$)

Equations 133, 134, 136, and 139 are the resulting crack-tip stress and displacement field, that is, Eqs 1 and 13, for the first mode.

Second Mode:

Instead of choosing the Airy stress function as in Eq 124, it is equally permissible to choose the form,

$$\Phi_{II} = -y\,\mathrm{Re}\,\bar{Z}_{II} \qquad (140)$$

Repeating all of the operations from Eqs 124–139 and again making use of Eqs 114–123 lead to:

$$\left.\begin{aligned} \sigma_x &= 2\,\mathrm{Im}\,Z_{II} + y\,\mathrm{Re}\,Z_{II}' \\ \sigma_y &= -y\,\mathrm{Re}\,Z_{II}' \\ \tau_{xy} &= \mathrm{Re}\,Z_{II} - y\,\mathrm{Im}\,Z_{II}' \end{aligned}\right\} \qquad (141)$$

and

$$\left.\begin{aligned} u &= \frac{1+\nu}{E}\cdot[2(1-\nu)\,\mathrm{Im}\,\bar{Z}_{II} + y\,\mathrm{Re}\,Z_{II}] \\ v &= \frac{1+\nu}{E}\cdot[-(1-2\nu)\,\mathrm{Re}\,\bar{Z}_{II} - y\,\mathrm{Im}\,Z_{II}] \end{aligned}\right\} \qquad (142)$$

and in the neighborhood of a crack tip, that is, $|\zeta| \to 0$,

$$Z_{II}\big|_{|\zeta|\to 0} = \frac{K_{II}}{(2\pi\zeta)^{1/2}} \qquad (143)$$

or

$$K_{II} = \lim_{|\zeta|\to 0} (2\pi\zeta)^{1/2} Z_{II} \qquad (144)$$

In addition, near the crack tip, substitution of Eq 143 into Eqs 141 and 142 leads to:

$$\left.\begin{aligned} \sigma_x &= \frac{-K_{II}}{(2\pi r)^{1/2}}\sin\frac{\theta}{2}\cdot\left[2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right] \\ \sigma_y &= \frac{K_{II}}{(2\pi r)^{1/2}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} \\ \tau_{xy} &= \frac{K_{II}}{(2\pi r)^{1/2}}\cos\frac{\theta}{2}\cdot\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \end{aligned}\right\} \qquad (145)$$