$$\omega = \frac{1}{2}\left(\frac{\sigma}{\sigma_{yp}}\right)^2 a \quad \text{........(113)}$$

Substituting the upper limit of stress, $\sigma = 0.8\ \sigma_{yp}$, mentioned above, the relative size, w/a or r/a, for reasonable accuracy is about 0.3 from Eq 113. For this value of r/a, Eq 111 predicts a deviation of actual stresses from the field equations of about 20 per cent. Thus it appears that the zone of nonlinearity at a crack tip may be fairly sizable, that is, of the order of 0.3 of the crack length (and other planar dimensions such as net-section width), without grossly disturbing the usefulness of the elastic stress field approach. However, a more extensive evaluation of this limitation should be the subject of further research.

In addition to nonlinearity in the region of the crack tip, consideration of other conditions (such as anisotropic and viscous effects having cracks in the bond line between dissimilar materials, thermal stresses, couple stresses, inertial effects of moving cracks) and of all three modes of crack-tip stress fields, has led to positive results. The conclusion is that the current techniques of fracture mechanics may be extended to all of these areas, since similar types of crack-tip stress fields exist for them and the stress-intensity factor methods of assessing failure should apply equally well. At any rate, this conclusion should give full confidence that slight amounts of these effects do not invalidate the useful application of the concepts of fracture mechanics.

As a consequence of the above remarks, it is observed that the only real limitation of elastic stress analysis commences with the advent of sizable zones of nonlinearity that is, plasticity, at the crack tip. The current hope for extension of the applicability of fracture mechanics to such situations lies in developing a full analysis based on the theory of plasticity. This topic is a subject left for other discussions.

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APPENDIX I

THE WESTERGAARD METHOD OF STRESS ANALYSIS OF CRACKS

Any elementary text on the theory of elasticity gives a full development of the equations for plane extension. The equilibrium equations are:

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} &= 0 \\ \tau_{xy} &= \tau_{yx} \end{aligned}\right\} \quad \text{........(114)}$$

The strain-displacement relationships and Hooke's law lead to the compatability equation:

$$\nabla^2(\sigma_x + \sigma_y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = 0 \quad \text{..(115)}$$

The equilibrium equations (114) are automatically satisfied by defining an Airy stress