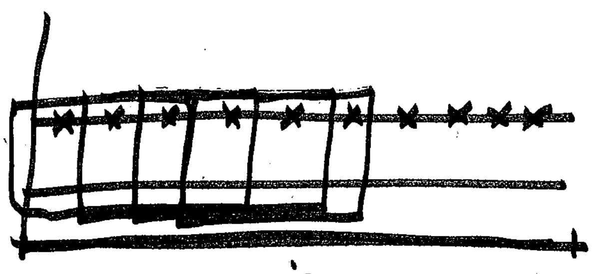
CRANK-NICHOLSON 2-D

$$\frac{u_{ijk+1} - u_{ijk}}{\Delta t} = \frac{\alpha}{2}\left\{ \nabla^2 u\Big|_{ijk} + \nabla^2 u\Big|_{ijk+1} \right\}$$

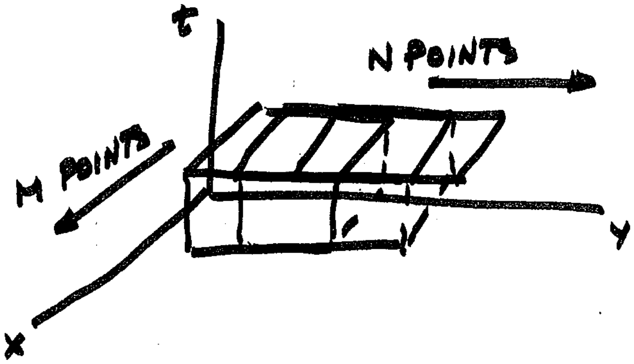
$$= \frac{\alpha}{2}\left\{ \frac{u_{i+1jk} - 2u_{ijk} + u_{i-1jk}}{\Delta x^2} + \frac{u_{ij+1k} - 2u_{ijk} + u_{ij-1k}}{\Delta y^2} + \frac{u_{i+1jk+1} - 2u_{ijk+1} + u_{i-1jk+1}}{\Delta x^2} + \frac{u_{ij+1k+1} - 2u_{ijk+1} + u_{ij-1k+1}}{\Delta y^2} \right\}$$

$O(\Delta t^2, \Delta x^2, \Delta y^2)$

C-N 1D



M POINTS ⟶ M-1 eqs to be solved



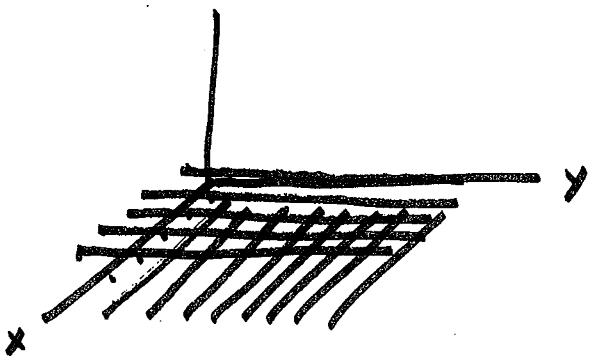
$(M-1)(N-1)$ points

Peaceman & Rashford in 1955 Alternating Direction Implicit Method.
(ADI)

ADI method solves the CN equs in two steps

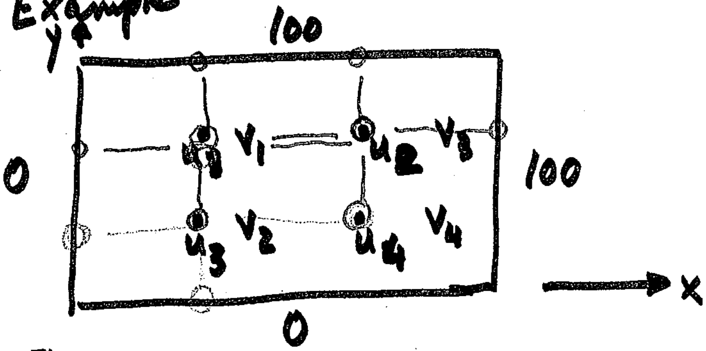
$t = \Delta t$ 1. Assume x is known & y is unknown

$t = 2\Delta t$ 2. " y is known & x is unknown



method works well for rectangular regions & every other time step is accurate

Example



$i \rightarrow x$

$j \rightarrow y$

$k \rightarrow t$

known

$$R\,u^{k}_{i+1\,j} + (1-2R)u^{k}_{ij} + R u^{k}_{i-1\,j} = -R\,u^{k+1}_{ij+1} + (1+2R)u^{k+1}_{ij} - R u^{k+1}_{ij-1}$$

$$-R\,u^{k+2}_{i+1\,j} + (1+2R)u^{k+2}_{ij} - R\,u^{k+2}_{i-1\,j} = R u^{k+1}_{ij+1} + (1-2R)u^{k+1}_{ij} + R u^{k+1}_{ij-1}$$

known

UNKNOWN KNOWN

@ k+1 level

@ v_1 $\quad -R\cdot 100 + (1+2R)v_1 - R v_2 = R u_2 + (1-2R)u_1 + 0$

@ v_3 $\quad -R\cdot 100 + (1+2R)v_3 - R v_4 = R\,100 + (1-2R)u_2 + R u_1$

@ v_2 $\quad -R\,v_1 + (1+2R)v_2 - R\cdot 0 = R\cdot u_4 + (1-2R)u_3 + R\cdot 0$

@ v_4 $\quad -R\,v_3 + (1+2R)v_4 - R\cdot 0 = R\cdot 100 + (1-2R)u_4 + R\cdot u_3$

UNKNOWN KNOWN

@ k+2

@ u_1 $\quad -R u_2 + (1+2R)u_1 - R\cdot 0 = R\cdot 100 + (1-2R)v_1 + R v_2$

@ u_2 $\quad -R\,100 + (1+2R)u_2 - R u_1 = R\cdot 100 + (1-2R)v_3 + R v_4$

@ u_3 $\quad -R u_4 + (1+2R)u_3 - R\cdot 0 = R v_1 + (1-2R)v_2 + R\cdot 0$

@ u_4 $\quad -R\cdot 100 + (1+2R)u_4 - R u_3 = R v_3 + (1-2R)v_4 + R\cdot 0$

READ § 8.2 in chapter 8

Do # 28, 29, 34 10/20 THURS + L.E.T.