FINITE DIFFERENCE FORMS

Given x_0 & Δx $\quad f(x)$

$f'(x_0)$

$$f(x_0+\Delta x) = f(x_1) = f(x_0) + f'(x_0)\Delta x + \underline{f''(x_0)\frac{\Delta x^2}{2!}} + \dots$$

Forward difference formula

$$\frac{f(x_1)-f(x_0)}{\Delta x} = f'(x_0) + f''(x_0)\frac{\Delta x}{2!} + \dots$$

$$O(\Delta x)$$

$$f(x_0-\Delta x) = f(x_{-1}) = f(x_0) - f'(x_0)\Delta x + f''(x_0)\frac{\Delta x^2}{2!} + \dots$$

Backward Difference formula

$$\frac{f(x_0)-f(x_{-1})}{\Delta x} = f'(x_0) - f''(x_0)\cdot\frac{\Delta x}{2!} + \dots$$

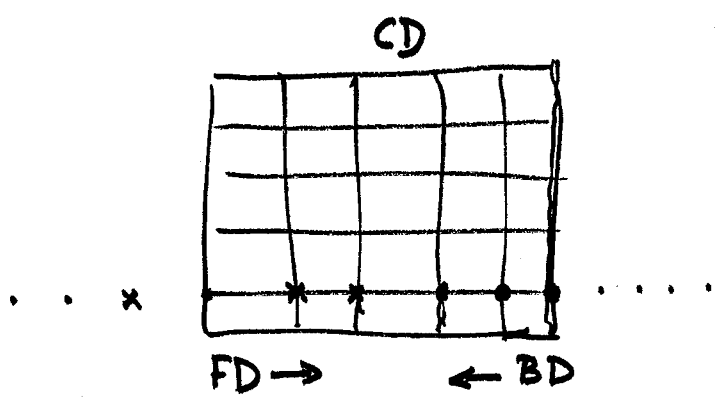
$$O(\Delta x)$$

$$f(x_1) + f(x_{-1}) = 2f'(x_0)\Delta x + 2f'''(x_0)\frac{\Delta x^3}{3!} + \text{h.o.t..}$$

Centered Difference method

$$\frac{f(x_1)-f(x_{-1})}{2\Delta x} = f'(x_0) + 2f'''(x_0)\frac{\Delta x^2}{2!} + \text{h.o.t.}$$

$$O(\Delta x^2)$$



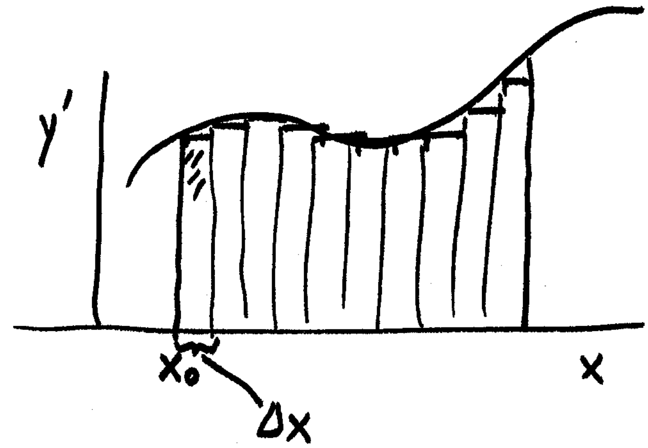
SOLUTION OF ODEs

$$y' = \frac{dy}{dx} = f(x,y)$$

$$y(x=x_0) = y_0$$

$$\int_{y_0}^{y} d\bar{y} = \int_{x_0}^{x} f(\bar{x},y)\,d\bar{x}$$

$$y = y_0 + \int_{x_0}^{x} f(\bar{x},y)\,d\bar{x}$$



Euler's Method

$$y = y_0 + f(x_0,y_0)\Delta x = y_0 + y'\Big|_{x=x_0} \Delta x \quad \underbrace{+ y''\frac{\Delta x^2}{2!} + \dots}_{\text{error}}$$

$\Delta x = x_1 - x_0$ then $y \cong y_1$

$x_0 := x_1$

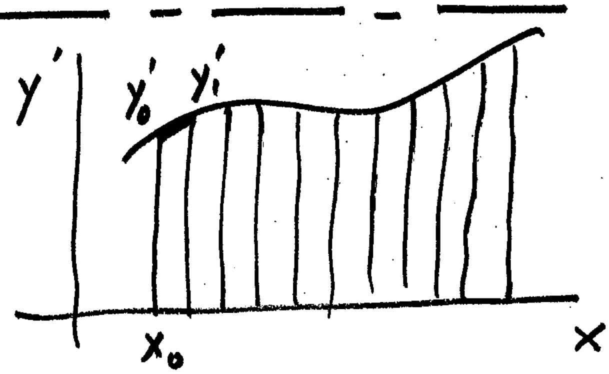
$y_0 := y_1$

error $O(\Delta x^2)$ per step

Local error = per step error $O(\Delta x^2)$

global error $O(\Delta x)$

Modified Euler's Method.



$$y_{1_c} = y_0 + \frac{y_0' + y_1'}{2}\Delta x$$

$y_0' = f(x_0,y_0)$

$y_1' = f(x_1,y_1)$ problem

$y_{1_P} \cong y_0 + f(x_0,y_0)\Delta x$

$x_1 = x_0 + \Delta x$

$y_0' = y'\big|_{x=x_0}$

$y_1' = y'\big|_{x=x_0+\Delta x = x_1}$

L.E. $O(\Delta x^3)$

G.E. $O(\Delta x^2)$

Taylor's Algorithm of order k

$$y_1 = y_0 + \Delta x \, T_k(x_0, y_0) \qquad y_1 \neq y(x_1)$$

$$T_k(x,y) = f(x,y) + \frac{\Delta x}{2!} f'(x,y) + \frac{\Delta x^2}{3!} f''(x,y) + \dots + \frac{\Delta x^{k-1}}{k!} f^{(k-1)}(x,y)$$

$k=1 \quad T_1(x,y) = f(x,y)$ Euler's Method

$k=2 \quad T_2(x,y) = f(x,y) + \frac{\Delta x}{2!} f'(x,y)$ Modified Euler

$$f'(x,y) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot y' = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot f$$

APP 7. PG 143-144

APP8 PG 144-145