

I. ASTM model for K_{Ic} testing

A. designed to produce valid K_{Ic} results - How?

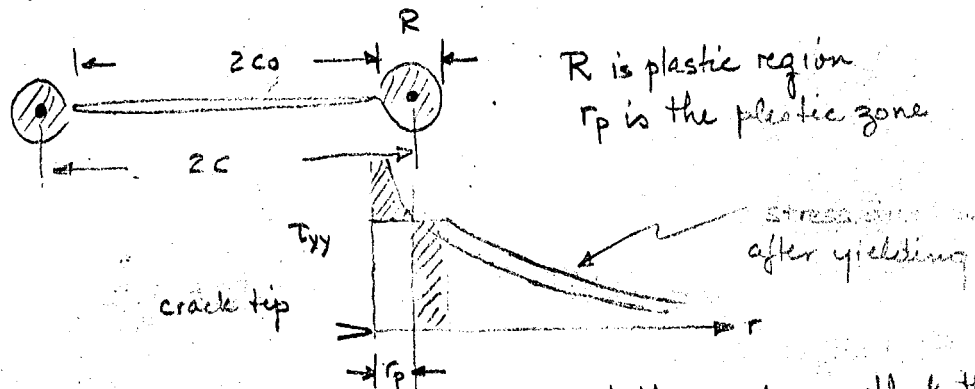
1. must meet $C_0 \geq 2.5 (K_{Ic}/\sigma_y)^2$
2. " " $B \geq 2.5 (K_{Ic}/\sigma_y)^2$; $W/B \approx 2$
3. starting crack length must be $0.45 - 0.55 W$ (width of specimen)
4. crack must be sharp and must be introduced via a fatigue crack starting from a V-notch
5. The fatigue crack must be introduced by low type cycling
6. A displacement gage will be used to accurately measure the relative displacement of two precisely located gages positions
7. Temperature and load rate requirements

B. Why these requirements -

1. $C_0 \geq 2.5 (K_{Ic}/\sigma_y)^2$. this is a requirement that is necessary and sufficient in order for LEFM to hold

Proof:

Consider a plate loaded in tension



- We assume that the stresses are redistributed ahead of the crack so that the load bearing capacity in front of the crack is unchanged when yielding occurs. We assume that the shaded areas under the graph are the same.

- Thus $2C = 2C_0 + 2r_p = 2C_0 + R$ is the effective length of the crack

- In plane strain mode I $R = \frac{1}{6\pi} (K_I/\sigma_y)^2$

$$\text{and } C = C_0 + \frac{1}{12\pi} (K_I/\sigma_y)^2$$

$(p^{\alpha})^{\beta} = 0 \quad \forall \alpha + \beta \geq t$

if the stress $\sigma \uparrow$ $K_I \uparrow$ also $K_I \uparrow$ due to the plastic zone correction.

$$\text{Thus } K_I = \sigma \sqrt{\pi c_0} \left\{ 1 - \frac{1}{12} \left(\frac{\sigma}{\sigma_y} \right)^2 \right\}^{-1/2} \quad (1)$$

- In a test as $\sigma \rightarrow \sigma_y$, $K_I \rightarrow K_{Ic}$

- If σ reaches σ_y before $K_I = K_{Ic}$ we get yielding and by our elastic-plastic model r_p (and R) $\rightarrow \infty$. Hence we violate the LEFM assumption of small scale yielding

- We want $K_I = K_{Ic}$ before $\sigma = \sigma_y$. Thus let $K_I = K_{Ic}$ in (1) and solve for the crack length $2c_0$

$$2c_0 = \frac{2}{\pi} \left(\frac{K_{Ic}}{\sigma_y} \right)^2 \left\{ \left(\frac{\sigma}{\sigma_y} \right)^2 - \frac{1}{12} \right\} \quad K_I = K_{Ic}$$

This will cause unstable crack growth

- The crack length that produces yielding is when $\sigma_y = \sigma$

$$\text{or } 2c_0 = \frac{11}{12} \cdot \frac{2}{\pi} \left(\frac{K_{Ic}}{\sigma_y} \right)^2 \sim \frac{1}{2} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$$

if $\sigma > \sigma_y$ then $2c_0 < \frac{1}{2} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$ unacceptable

$\sigma < \sigma_y$ then $2c_0 > \frac{1}{2} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$ or $c_0 > \frac{1}{4} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$

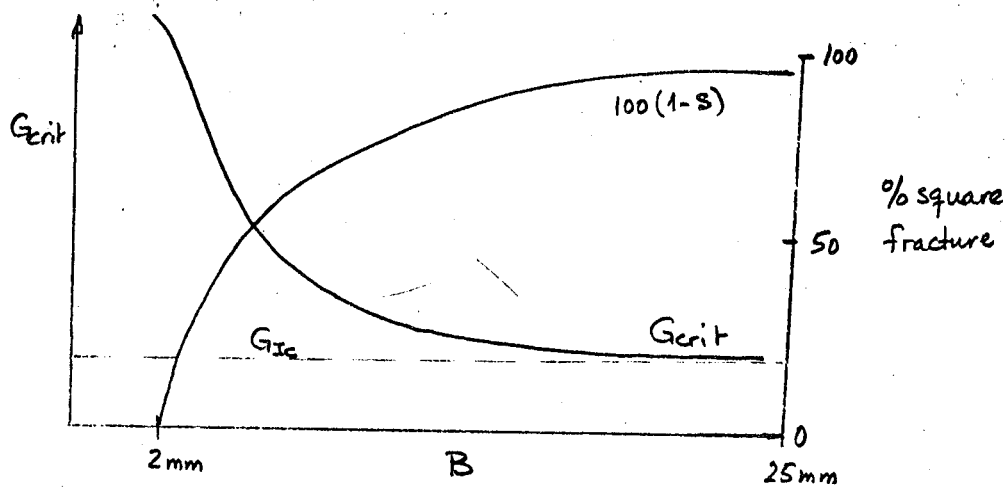
- Because we want to make adequate measurements of K_{Ic}

we want $c_0 \gg \frac{1}{4} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$

Stawley and Brown suggested that $c_0 \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$

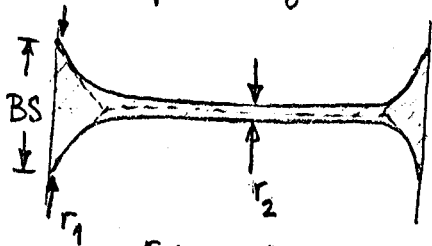
and this is accepted as the standard.

Now $G_I = \frac{1}{B} \frac{dW}{da} = K(1-S) + \frac{BS^2\theta}{2}$. Note that S is picked so that $BS = \text{constant as crack length}$
 Thus as $B \rightarrow \infty$ $S \rightarrow 0$ and $G_I \rightarrow K$.



From data for Aluminum
 7075-T6
 $K \sim 200 \text{ KJ/m}^2$
 $\theta \sim 20 \text{ KJ/m}^2$

Look at the plastic zone and superpose the model of Krafft:



$$BS \gg r_{\text{plane stress}} \sim r_p = \frac{1}{2\pi} \left(\frac{K_{IC}}{\sigma_y} \right)^2 \quad (*)$$

since $r_1 > r_2$ (plane strain). If $(*)$ is true

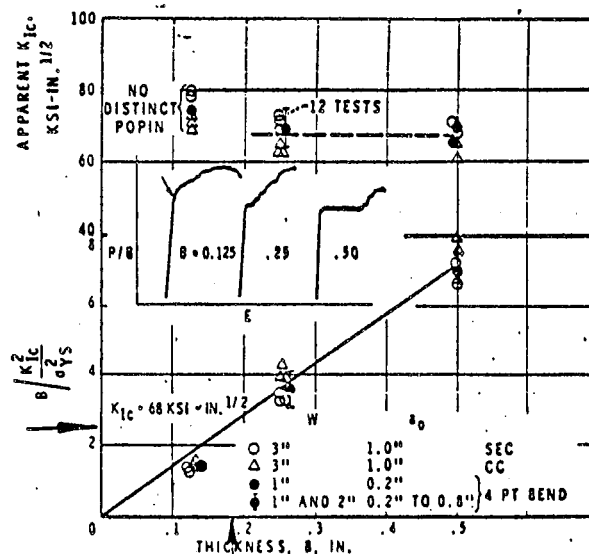
then plain strain conditions will extend over most of the cross-section and we will have

essentially mode I fracture.

hence $B \gtrsim \left(\frac{K_{IC}}{\sigma_y} \right)^2$. To determine the real equation,

tests were done on many types of metals and here are some of the results.

Example: Maraging Steel $\sigma_y = 259 \text{ KSI}$



Specimen Size Requirements

We have argued that to limit yielding we must make large samples with long cracks. Thus $K_I \rightarrow K_{Ic}$ before $\sigma \rightarrow \sigma_y$. From our analysis we expect

$$C_o \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$$

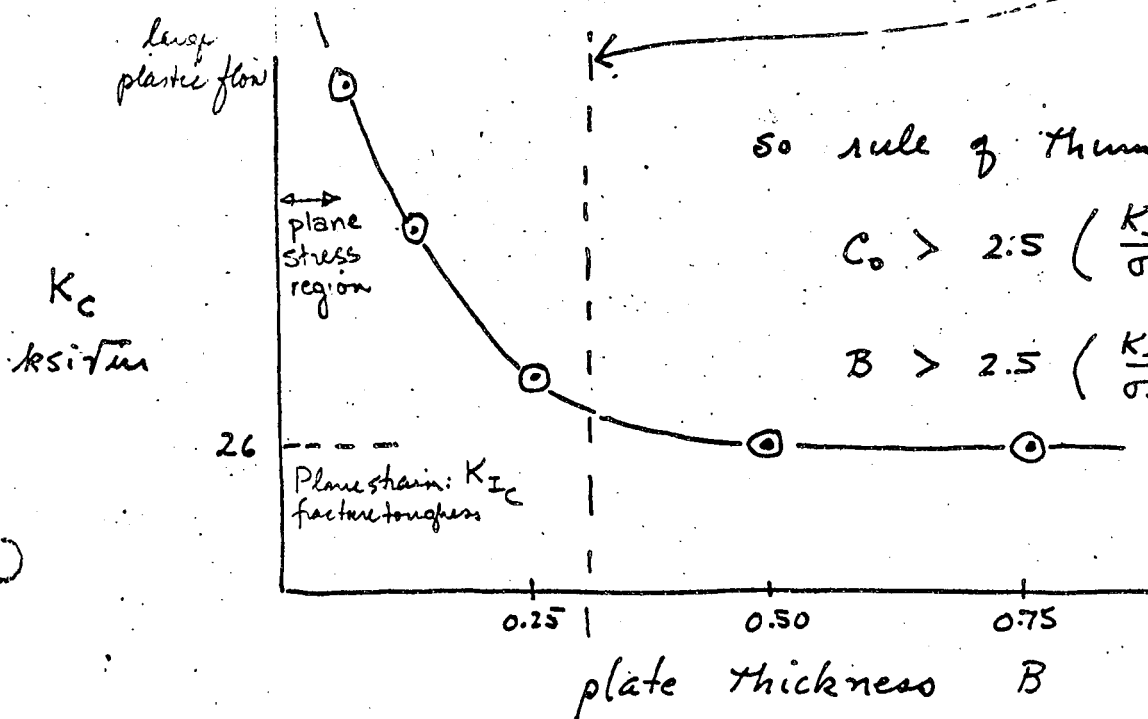
This needs to be checked. Also, How thick must sample be for plane strain conditions?

Consider 7075-T6 (MSE 202C experiment).

$$\sigma_y = 75 \text{ Ksi}$$

$$K_{Ic} = 26 \text{ Ksi}\sqrt{\text{in}}$$

$$2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2 = 0.3 \text{ in}$$

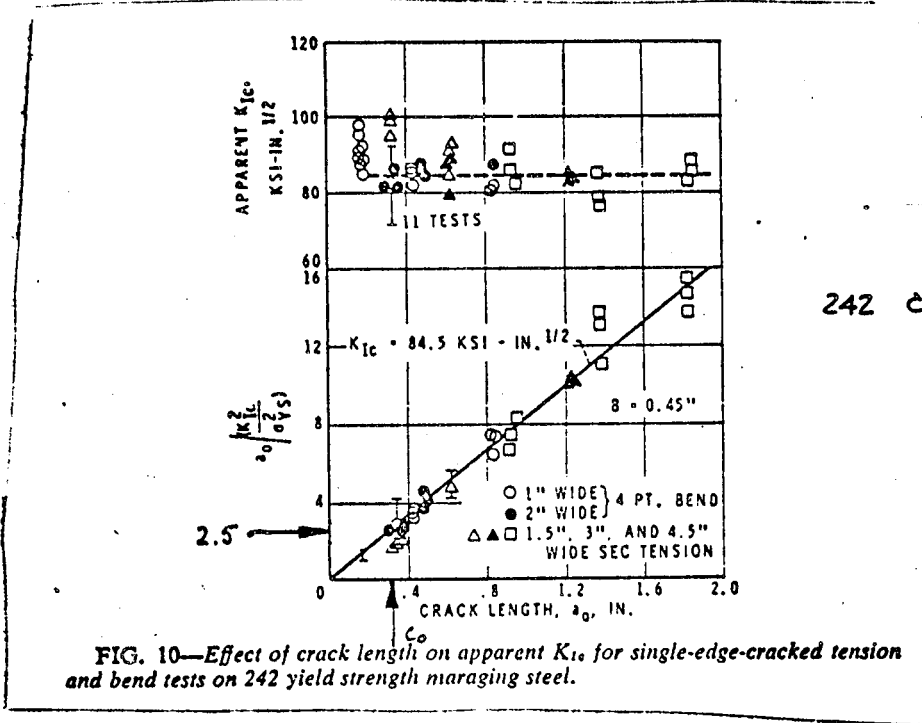


Specimen Size Requirements

we have already discussed the fact that there is a critical thickness to achieve plane strain conditions and to measure K_{Ic} . Also, there is a critical crack length. Generally speaking, there are no reliable techniques for predicting the critical crack length or critical thickness. One simply measures K_{Ic} to determine if it is independent of crack length or thickness.

Crack Length Effects

Example: Maraging Steel. $\sigma_y = 242 \text{ KSI}$

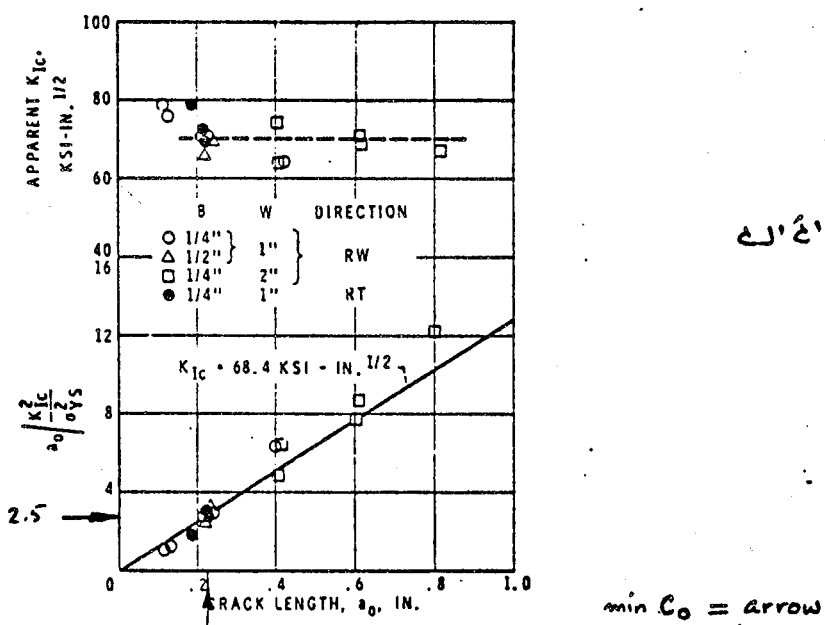


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Conclusion:

$$\left(\frac{c_0}{K_{Ic}^2 / \sigma_y^2} \right) \geq 2.5 \quad \text{hence} \quad c_0 \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$$

Example: Maraging Steel $\sigma_y = 259 \text{ ksi}$

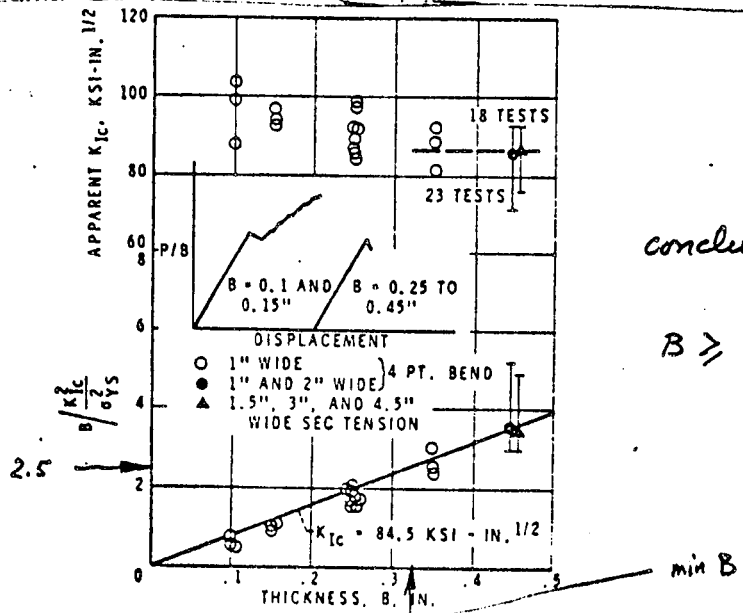


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conclusion $C_o \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$

Plate Thickness Effects

Example: Maraging Steel $\sigma_y = 242 \text{ ksi}$



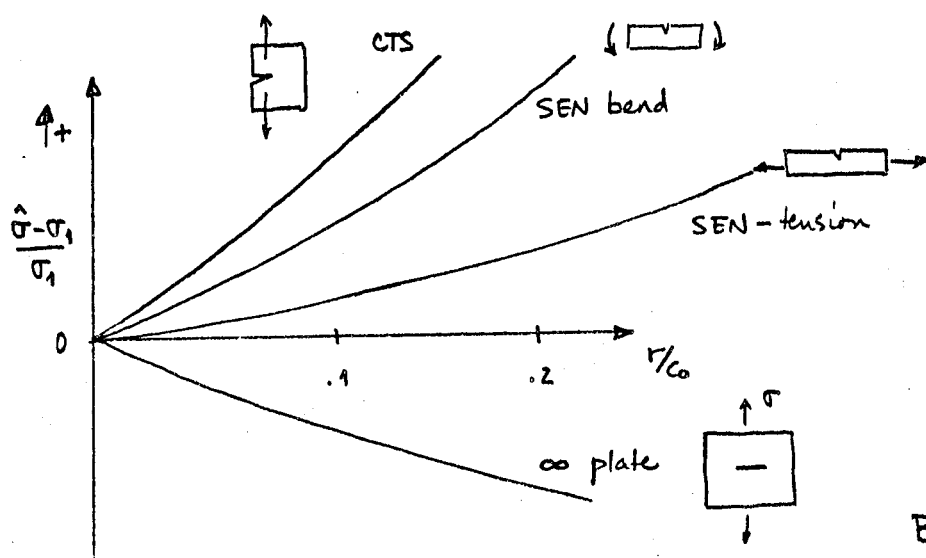
conclusion: $B \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$

- 2b. The ratio of W/B is picked to be $\cong 2$ so that the stress state ahead of the crack tip is not affected by the free surfaces of the specimen.

Proof: Before we answer this question we must answer the questions

- 1) how long should the initial crack length be?
- 2) what is the effect of the width, W , of the specimen?

- 1) To determine the length of the crack we look at the stress $\hat{\sigma} = \frac{K_I}{\sqrt{2\pi r}} + \dots$ where $\hat{\sigma}$ is the 1 term expansion for the opening stress that cause the crack to propagate, r is the distance ahead of the crack tip in the direction of propagation, and $K_I = \sigma \sqrt{\pi c_0}$ (σ being the far field stress). For different test pieces, if we plot $(\hat{\sigma} - \sigma_1)/\sigma_1$ where σ_1 is the actual stress, then we find that



at $r/c_0 = .02$: Wilson found (1966)

CTS: 7% error
SEN-B: 6% error
SEN-T: 2% error
 ∞ PL: -1.5% error

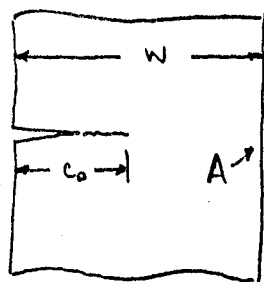
why .02? This is a value judgment and we want to describe what happens in front of the crack by one parameter and accurately.

Basically this implies that

$$r_p (\text{plane strain}) = \frac{1}{3} \left(\frac{K_{Ic}^2}{2\pi \sigma_y^2} \right) \leq 0.02 c_0$$

$$\text{thus } c_0 \geq 50 r_p$$

- 2) To determine how wide the specimen must be, we must look at $(W - c_0)$ and insure that $(W - c_0)$ is not too small, compared to the radius of the plastic zone at fracture. If $(W - c_0)$ is too small then the free surface (at A) affects your results.



(1965) Smith looked at the solution of an infinite set of parallel cracks, each of length $2c_0$ and the distance between their centers being $2W$ and looked at what happens when each tip spreads plastically.

He found that

$$2r_p = c_0 \left\{ \frac{2W}{\pi c_0} \sin^{-1} \left[\sin \left(\frac{\pi c_0}{2W} \right) \sec \left(\frac{\pi \sigma}{2\sigma_y} \right) \right] - 1 \right\}$$

and that the distance W does indeed play an important role. For the specimen whose $W/c_0 = 2$, an error of 25% in the size of the plastic zone can be found but an error in the value of K_{Ic} is less than 1%. Hence the standard has been to take $(W - c_0) = c_0$ (or $W/c_0 = 2$).

Previously we had shown that $B \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2 = 5\pi r_p$ since $r_p = \frac{1}{2\pi} \left(\frac{K_{Ic}}{\sigma_y} \right)^2$ in plane stress. Now r_p in plane strain $= \frac{1}{3} r_p$ (plane stress)

thus $B \geq 15\pi r_p$ (plane strain) $\approx 47 r_p$

Note that since $B \approx c_0$, then $W/c_0 \approx W/B \approx 2$; hence the standard

$W/B \approx 2$. (This also implies that $c_0 \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$).

Thus $c_0 \approx B = W/2$. (Really $.45W \leq c_0 \leq .55W$ by experimental experience).

Note that we have also shown point number 3 in the above proof.

There are several models used for testing

1. The Center Cracked Plate
2. The Double Edge Cracked Plate
3. The Single Edge Cracked Plate (Tension SEN)
- * 4. The Single Edge Cracked Bend Specimen (3 point loaded)
- * 5. The Compact Tension Test Specimen (CTT)
- * 6. The C-shaped Specimen (used to represent K_{Ic} testing of cylinders and thick bars).

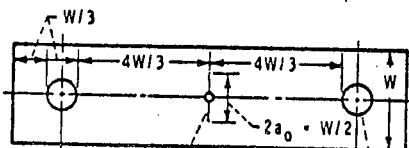
* represents the recommended test specimens

If the dimensions of the standard test specimens lead to impractical dimensions, alternative sizes are allowed. For bend specimens, the thickness $.25W \leq B \leq W$; The CTT may have dimensions $.25W \leq B \leq 0.5W$ (also true for the C shaped specimen).

On the next few pages are given the specimen and its dimensions
Note the ASTM E-399 - it is the "standard" for K_{Ic} testing.

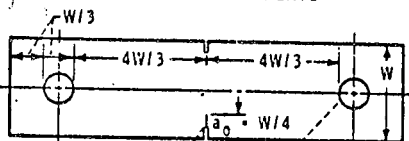
Sample shapes

PREFERRED RANGE OF THICKNESS $W/2$ TO $W/4$



SEE FIG. 45

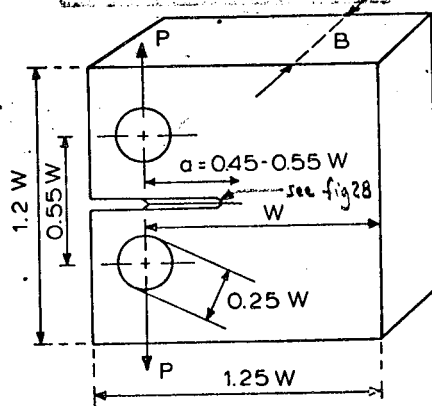
CENTER CRACKED PLATE



SEE FIG. 28

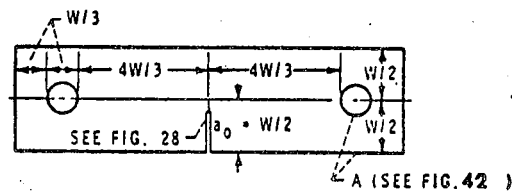
DOUBLE EDGE CRACKED PLATE

FIG. 42—Proportions for center- and double-edge-cracked plate specimens. A-surfaces must be symmetric to specimen centerline within $W/1000$.

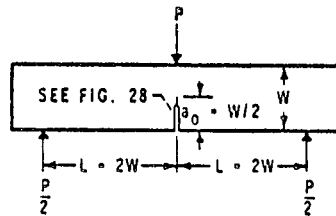


compact tension test - CTT

PREFERRED RANGE OF THICKNESS $W/2$ TO $W/4$



SINGLE EDGE CRACKED PLATE (TENSION)



SINGLE EDGE CRACKED BEND SPECIMEN
(THREE POINT LOADED)

FIG. 43—Proportions for single-edge-cracked tension and bend specimens.

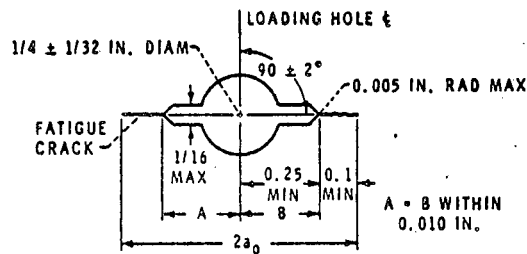


FIG. 45—Fatigue crack starter for center-cracked plate specimens.

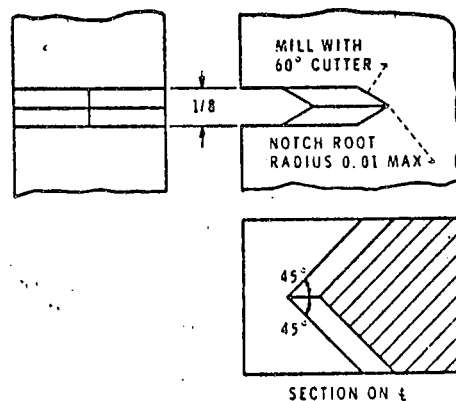
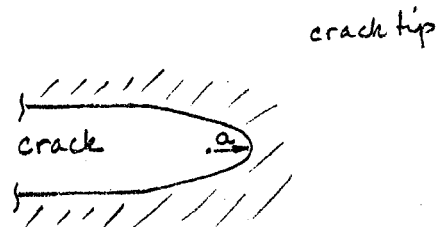
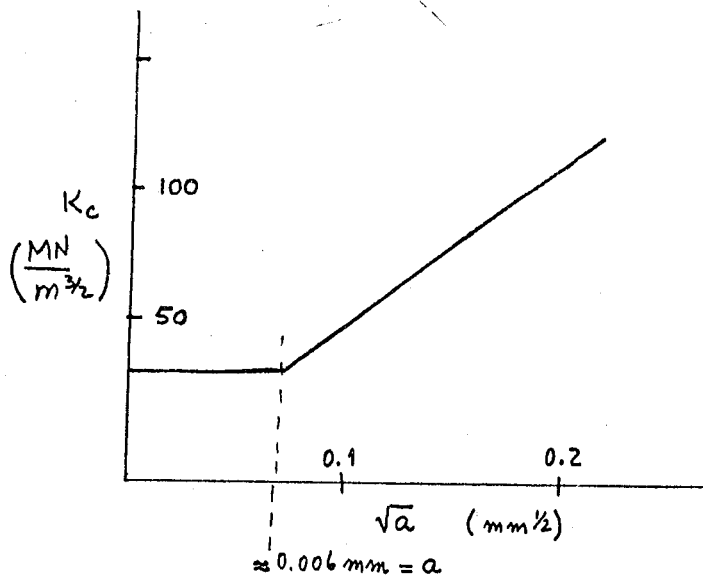


FIG. 28—Chevron notch for edge-crack plate specimens.

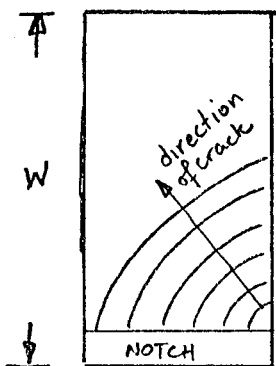
4#5. Crack must be sharp and must be introduced via a fatigue crack starting from a V-notch and the fatigue crack must be introduced by low type cycling. (low K_{max}).

Proof: it has been found through testing of high strength steel and other materials that K_{crit} was an increasing function of root radius of the crack, if the radius was above some "cut off" radius.

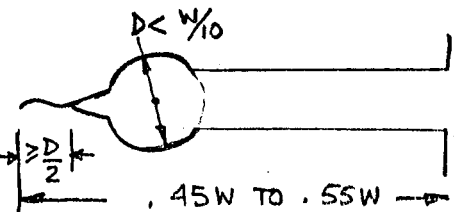
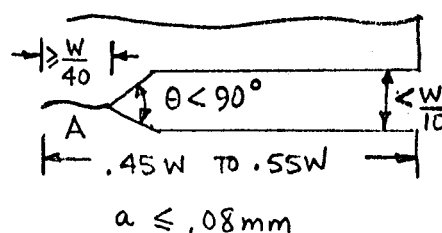
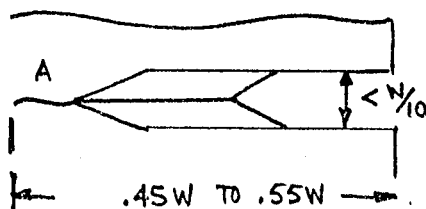


Thus in order to obtain a constant level of K_c , one must introduce sharp cracks by means of fatigue. However one must be careful about how one introduces the crack.

If the crack is introduced as a notch in a thick member, then the fatigue



crack will propagate from a corner and will result in an irreproducible curved crack-front and this is not suitable for a standard test. We then have to modify the introduced crack so that the fatigue crack propagates in a relatively straight crack front. What is done is to introduce a Chevron Notch, or a straight-through notch or a slot ending in a drilled hole.



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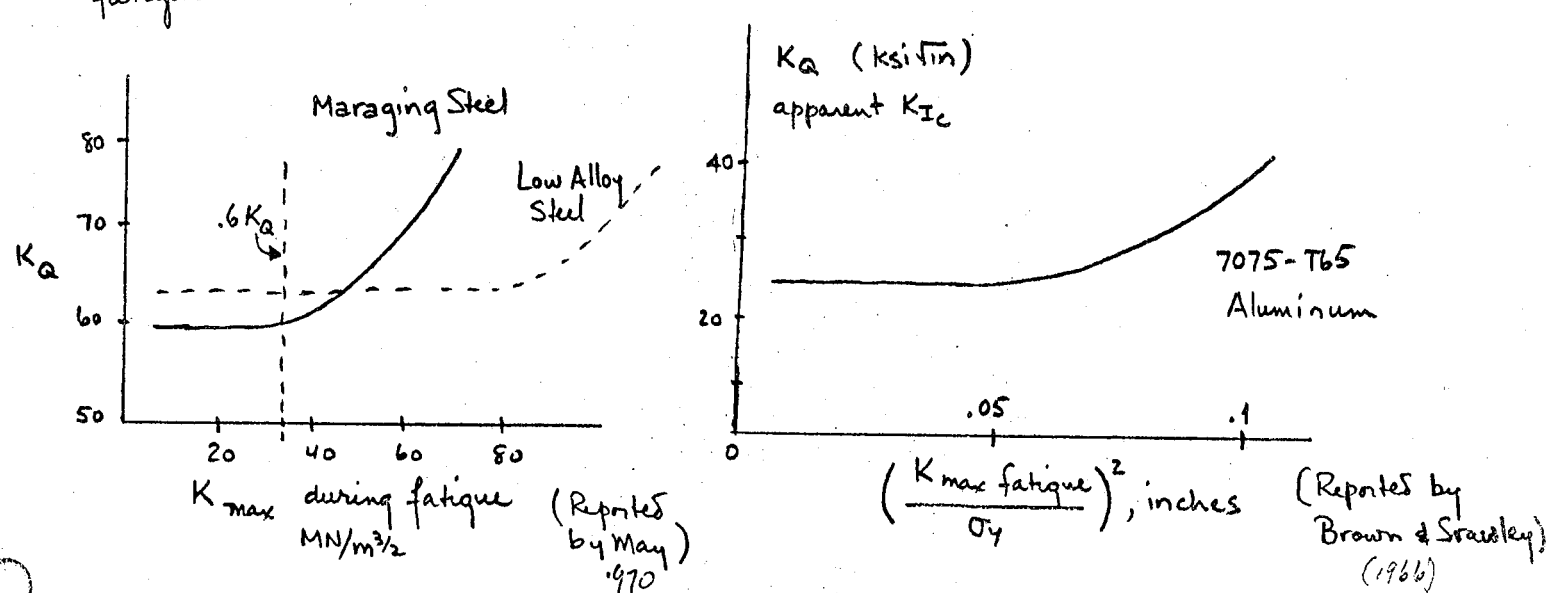


- c_1, c_2 must be $< (1.15)a$ for a valid test
- a_1, a_2, a_3 " " $< (1.10)a$ ——— " ———

ג.ב.: אתה יכול לבדוק את הפרטים האלה רק אחרי שזית את הניסיון.

By using a chevron notch you have the additional advantage that the fatigue crack starts almost immediately upon cycling. You need between 10^4 - 10^6 cycles.

In order to ensure that a sharp notch will form there are several rules that you have to observe, the most important is that the maximum stress intensity during cycling of the last 2.5% of fatigue must not exceed $0.6K_{IC}(=K_{IC})$. The reason for this is that K_{IC} grows with the maximum stress intensity during fatigue.



Finally, the plane of the crack should not twist or tilt by more than $\pm 10^\circ$. If it does, that implies mixed mode fracture and the test is invalid.



6. A displacement gage must measure accurately of the displacement across the "mouth" of the crack in order to satisfy the condition $P_{max}/P_Q < 1.1$ (where P_{max} = load at failure and P_Q is the 'pop-in' load).

Below is the design of the strain gage recommended by J.E. Srawley to measure the crack opening displacement. The strain gages (T_1, T_2, C_1, C_2) are connected by means of a wheatstone bridge, so that the displacement versus load can be recorded automatically. The gages T_1, T_2 will be in tension; C_1 and C_2 will be in compression. When the test is complete, you will be able to compute the crack length from the relative displacement of the two crack faces.

Instrumentation

measurement of crack face displacements.

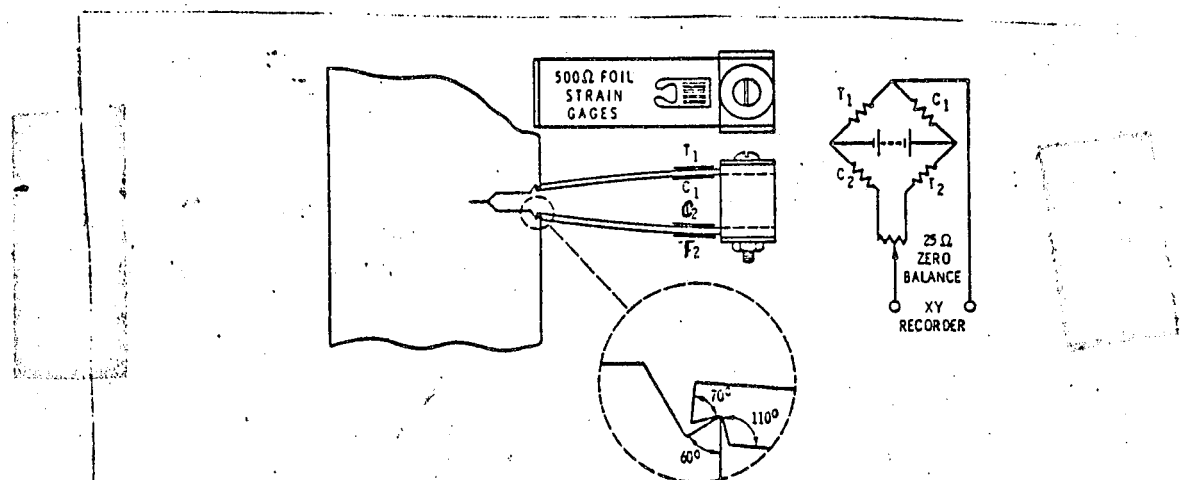


FIG. 19—Double cantilever beam gage and method of mounting on crack-notched specimen for displacement measurement (designed by J. E. Srawley).

This graph will then allow you to obtain the load versus crack length, and finally after using the relationship

$$K = \sigma \sqrt{\pi c_0} f_1(c_0/W) = \sigma \sqrt{W} f_2(c_0/W)$$

you will be able to obtain the value of K_Q .

Let's look at how this is done. First, the strain gages will record graphs that look like those on the following page

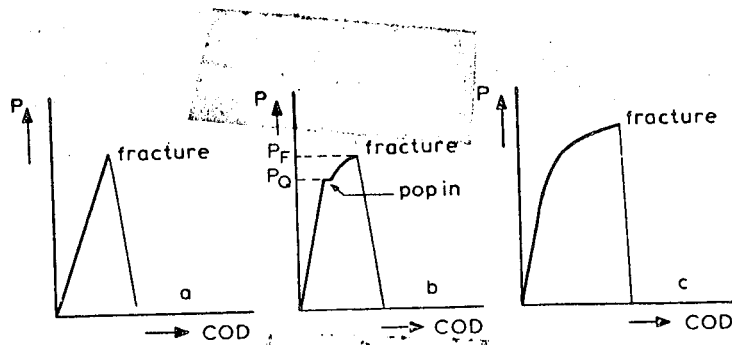


Figure 7.9. Test records

The first is the ideal case; here there is no plasticity. COD stands for crack opening displacement. Here total fracture occurs upon reaching K_{Ic} . In many cases rapid crack extension occurs at a load P_Q ; either the load remains constant or there is a small drop in load, but the crack is arrested and the load can be further increased to P_F (or P_{max}) where total fracture occurs. The displacement due to the increase in P above P_Q is due to non linear elastic effects and small plasticity. To determine K_Q , the value of P_Q must be used. Here if $P_F/P_Q > 1.1$ the test is invalid. Another reason for crack arrest after pop in can be due to the growth of the shear lips.

In the third case, the crack grows to P_F via non linearity and plasticity. Here we define P_Q as follows:

- Draw the line tangent to the original record - find the slope of the line
- Next, take 98% of the slope and draw that straight line
- where that line crosses the original record we define P_Q
- However, if there is a load which is larger than the value found by the intersection of the 98% slope and the original record, then this larger load is defined to be P_Q .

The inequality $P_{max}/P_Q > 1.1$ replaced a more complicated method of finding where plasticity began on the P -COD record. (For those interested see Broek Pgs 178-180 or Knott Pgs 138-144). The disadvantage of the old method was the fact that the region of plasticity was very difficult to measure with any accuracy on the P -COD record, and the method could be more restrictive with respect to some cases of fracture. Even though $P_{max}/P_Q > 1.1$ is a subjective restriction, it is easier to measure.

It is possible to compute the crack length from the measured displacements across the crack faces.

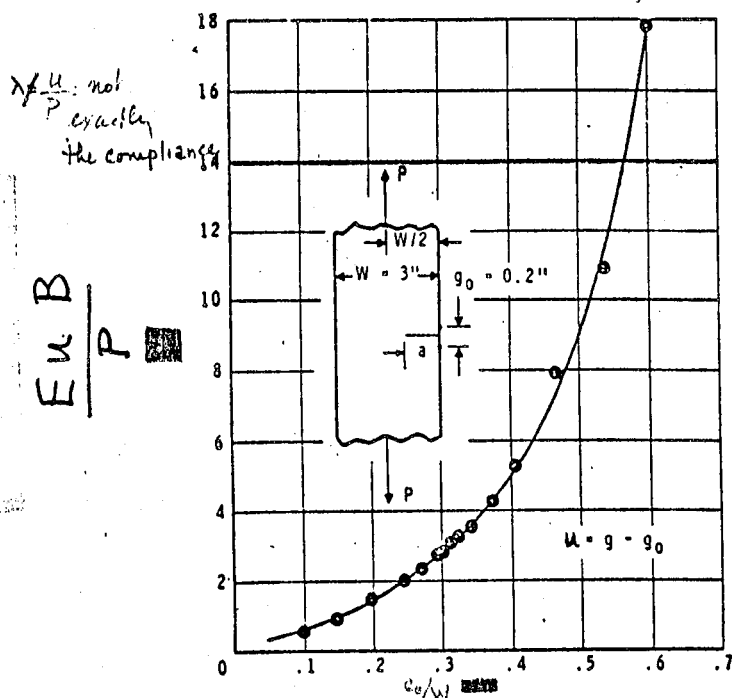
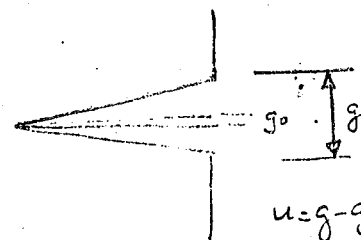


FIG. 22—Calibration curve for converting displacement measurements to crack length single-edge-crack tension specimens.



gives a more sensitive measurement of crack running; however $u \neq P$ are not actually related by $u = \lambda P$ since u is not measured where P is applied.

Once you have measured the load P_a , use a graph like this to give you the crack length. (Note that you will probably have to calibrate your test set-up first before doing any tests that can be used for official data). Next use your crack length to width ratio (c_0/W) in the proper formula for the test you are doing to find $f(c_0/W)$. Sawley defines K_a as follows:

For the SEN Bend specimen

$$K_a = P_a S / B W^{3/2} \cdot f(c_0/W) \quad \text{where} \quad P_a S / B W^{3/2} = \sigma \sqrt{W}$$

$$f(\zeta = c_0/W) = 3\zeta^{1/2} [1.99 - 5(1-\zeta)(2.15 - 3.93\zeta + 2.7\zeta^2)] / [2(1+2\zeta)(1-\zeta)^{3/2}]$$

For the CTT

$$K_a = P_a / B W^{1/2} \cdot f(c_0/W) \quad \text{where} \quad P_a / B W^{1/2} = \sigma \sqrt{W}$$

$$f(\zeta = c_0/W) = (2+\zeta) [0.886 + 4.64\zeta - 13.32\zeta^2 + 14.72\zeta^3 - 5.6\zeta^4] / (1-\zeta)^{3/2}$$

For the C-Shaped specimen, see the ASTM E-399 Standard.

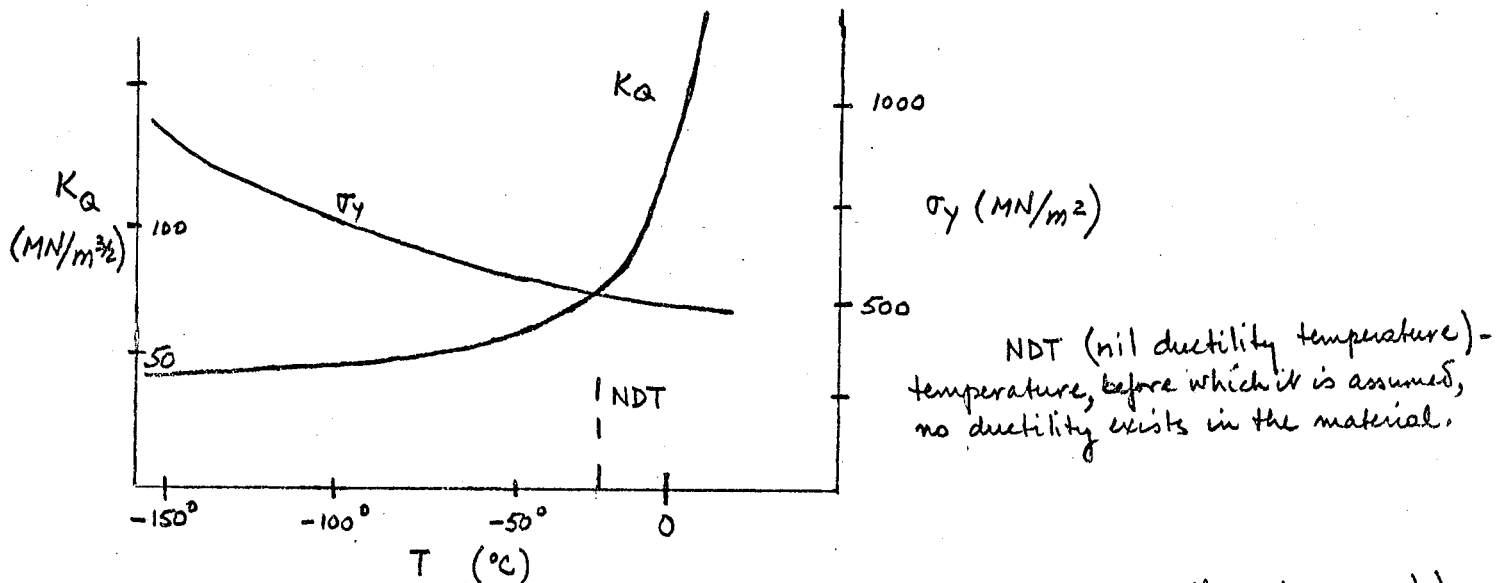
7. Temperature and Loading Requirements for fatigue loading

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a. If fatigue fracture is conducted at temperature T_1 and the actual test is done at temperature T_2 then $K_{max} \leq .6 (\sigma_{y1}/\sigma_{y2}) K_Q$

Proof:

From tests performed on low alloy steel by Wessel⁽¹⁹⁶⁹⁾, it was found that the variation in σ_y versus temperature and Fracture Toughness versus temperature looked like this for the CTT specimen:



As the temperature drops the material becomes more brittle, the stress needed to cause yielding and plasticity increases, but the value of K_{Ic} (or K_Q) decreases. As the temperature increases the material tends to become more "plastic"; hence the value of σ_y must decrease. As the material yields more readily, the amount of energy needed to cause ^{brittle} fracture increases, and this causes K_{Ic} to increase also.

$$\text{if for the same thickness } B \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2 \Rightarrow \left(\frac{K_{Ic}}{\sigma_y} \right)_{T_1}^2 = \left(\frac{K_{Ic}}{\sigma_y} \right)_{T_2}^2$$

Thus $(K_{Ic})_{T_1} = \left(\frac{\sigma_{yT_1}}{\sigma_{yT_2}} \right) (K_{Ic})_{T_2}$. If we test the material at T_2 ,

then $(K_{Ic})_{T_2} \equiv K_Q$. If we fatigued the material at T_1 , then

$$K_{max} \leq .6 (K_{Ic})_{T_1} \text{ Hence } K_{max} \leq .6 \left(\frac{\sigma_{yT_1}}{\sigma_{yT_2}} \right) K_Q$$

- From this we can also see the following:

a plate with small thickness will show plane stress behavior and high toughness at room temperature. However at low temperatures, the material has a higher yield stress and thus causes the plastic zone ($r_p \sim (\frac{K_{Ic}}{\sigma_y})^2$) to be smaller.

The material may then fracture either in plain strain or in plane strain / plane stress, due to the dependence of σ_y on Temperature.

- The trend shown by low alloy steel also can be found for other materials and is correct in general. There may exist materials for which this is not true.

b. According to ASTM 399 the fatigue crack is loaded cyclically with a ratio of minimum to maximum stress between -1 and +0.1 (ie S_{min}/S_{max})
As has been shown (or will be shown to you), Paris, Gomez and Anderson showed that

$$\frac{dc}{dN} = f(K_{max}, R) \quad \text{where } c \text{ is the crack length; } N \text{ is the cycles}$$

K_{max} is max K during cycle, $R = S_{min}/S_{max}$

- Schijve found that when $R < 0$, $\frac{dc}{dN} = f(K_{max})$ only (based on the data for 7075-T6 Aluminium Alloy) and $\frac{dc}{dN}$ increases as K_{max} increases.

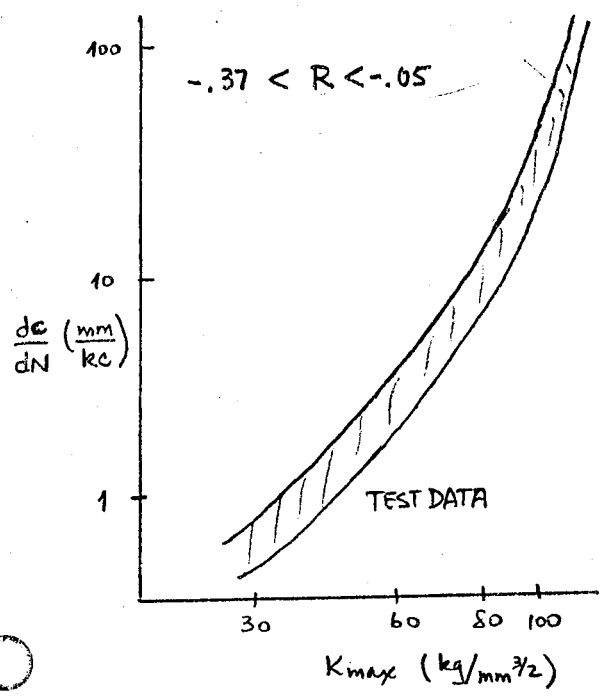
- This is a reason that you are required to keep $K_{max} \leq .6 K_Q$ for the last 2½ % of crack growth.

- The thickness also plays an important part here also. As the crack begins to propagate from the notch, it always starts as a tensile mode crack $\perp$ to the sheet surface. As the crack grows the plastic zone increases and plane stress develops. This in turn causes the shear lips to grow. As the crack continues to grow in plane stress, the crack would transition to a shear mode fracture forming either the slant or V slant shapes discussed earlier. The shear lips would then tend to slow down crack growth because their radius of plasticity is larger (hence we have a type of ductile crack growth in the material) and it would take the crack more cycles to grow.

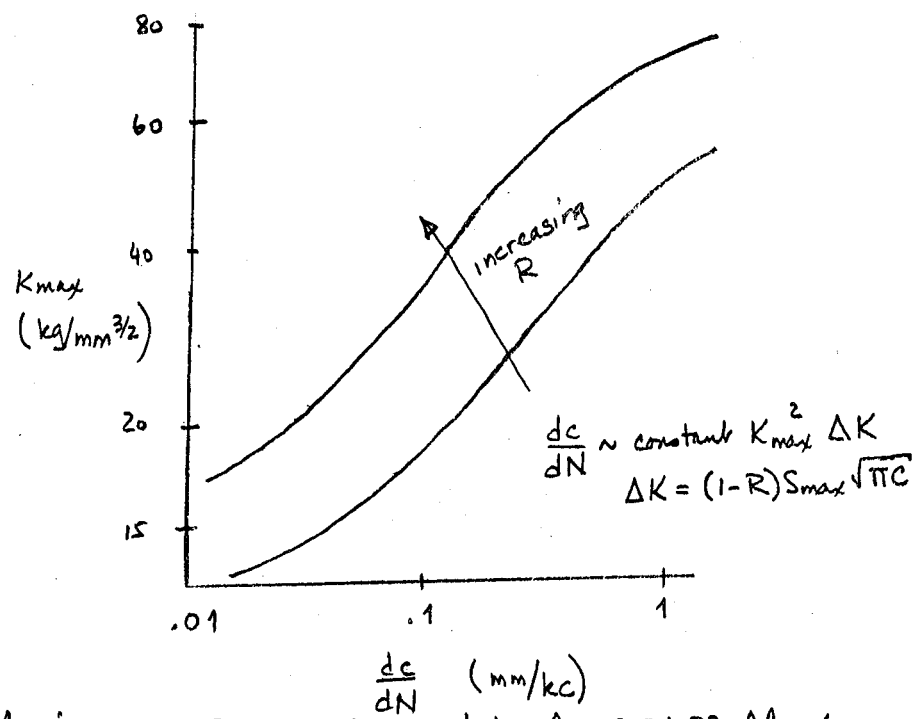
- In plain strain, the shear lips are confined to the surface and the radius of plasticity over a majority of the crack surface is smaller. Therefore the fracture is more brittle and thus requires a lower number of cycles to grow the crack. This is why you only need 10^4 to 10^6 cycles.

For example in 2024-T3 Aluminium an initial crack of 3mm grows to 10 mm in about 10^4 cycles for a thickness of 4mm, whereas it takes

As R increases above zero, as expected, the K_{max} value obtained for the same crack growth rate increases. This is due to the fact that the amplitude of the stresses and the mean stresses experienced by the crack are higher and K_{max} is higher $\left\{ \frac{dc}{dN} \sim f([1-R] K_{max}) \right\}$; if $\frac{dc}{dN}$ is constant and R increases $1-R$ decreases and causes K_{max} to increase $\}$.



Schijve - data for 7075-T6 Aluminum



Broek & Schijve data for 2024-T3 Aluminum

c. Humidity - According to the work of Meyn, Bradshaw and others, it was found that humidity also affects the rate of crack growth. For Aluminum Alloys it was found that growth rates were lower in dry environments than in normal wet environments. Achter found the opposite to be true for other materials. This effect can be explained by the corrosive action of the environment and is time-dependent, just as fatigue crack growth is time dependent. This is probably the reason you are required to report the relative humidity for the test.

d. Load rate for the test

The ASTM requires that moderate increase in the load is to be used when the fracture toughness is carried out. The reason for this is that K_{IC} can be affected by the load rate. Results of Radon and Turner on semi-heat treated steels, and the University of Illinois tests on Mild Steel, as reported by Krafft show that K_{IC} is a decreasing function of load rate.

of load rate; however this turns out not to be the case for the high strength Titanium alloy (6Al-4V). For this alloy, K_{Ic} is an increasing function of load rate, what is true for both cases is that for high load rates K_{Ic} increases with load rate ~~for both~~. This is basically an adiabatic effect (i.e. no heat dissipation).

- For high load rates, the deformation ahead of the crack occurs so quickly that the heat generated cannot be dissipated to the rest of the specimen. Thus the temperature ahead of the crack increases substantially, also increasing the plasticity (hence the ductility) ahead of the crack. As shown before, this implies that for the plain strain radius to ~~remain the same~~ ^{increase} ($r_p \sim \left(\frac{K_{Ic}}{\sigma_y}\right)^2$), that implies that K_{Ic} must also increase. Thus in order to minimize this effect, the load rate must be low to moderate, as is required in the test. The report by Krafft and Irwin on Crack-Velocity Considerations given in ASTM STP 381 goes into more details of this effect.

In summary, we have discussed

- The thickness requirements for a good K_{Ic} test;
- How thickness is related to the width of the specimen;
- How crack length is related to the width;
- How the initial crack is to be introduced into the specimen;
- How temperature and load rate affect the value of K_{Ic}
- What must be measured and how this is an indication of a valid test.

