Solution of BVP's can use both shooting & FD

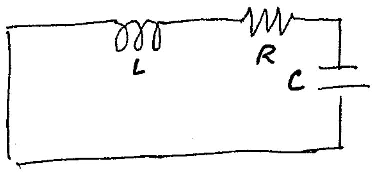
FD to linear ODE's lead to tridiag systems

FD to non linear ODE " " an iterative method

- advantage (no need to convert to a set of 1st order ODEs)
- accuracy depends on mesh size (smaller mesh - more eqns).
- shooting method can be used for linear & non linear ODEs
  - easy to implement since we can use IVP method (RK, adams etc)
  - no guarante of convergence. when it converges, they converge faster than FD

STIFF ODES ARISE

- WHEN SYSTEM IS MODELED BY DIFF. EQS WHOSE EIGENVALUES VARY WIDELY
  - CONTROL SYSTEMS
  - SYSTEMS w/ FLUID BOUNDARY LAYERS
  - ELECTRICAL CIRCUITS
- CAUSE PROBLEMS OF STABILITY & ACCURACY



$\frac{R}{L} = 10{,}001 \ \frac{1}{sec}$

$\frac{1}{LC} = 10{,}000 \ \frac{1}{sec^2}$

Consider circuits in which

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

with $v(0) = 1$ volt & $\frac{dv}{dt}(0) = -1$ volt/sec

Solution will be $v = Ce^{st}$ & $\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)Ce^{st} = 0 \Rightarrow s^2 + 10{,}001s + 10{,}000 = 0$

and $v = ae^{-t} + be^{-10000t}$

- EV are -1 & -10000 i.e. $(s+10000)(s+1) = 0$

- IC give that $a = 1$ $b = 0$ $\Rightarrow v = e^{-t}$

however : 5^{th} order Runge Kutta or Adams Moulton blows up (10^{293} volts in 0.2 sec) using $\Delta t = .001$
- unstable at $\Delta t = 5 \times 10^{-4}$
- using $\Delta t = 1 \times 10^{-4}$ sec works ie Δt is governed by EV which is larger ie $|s| = 10000$

- must use larger EV to determine Δt ie $e^{-st} \sim e^{-1}$ & $|st| \cong 1$ or $t = \frac{1}{|s|}$

Total Error in Solution of D.E.

depends on

1) Round off error (due to significant digits ~~[illegible]~~ error)

2) Truncation ~~[illegible]~~ error due to representation of y' by an approximate formula (stepsize error)

3) e_{n+1} depends on errors that existed before hand at some step i.e. $e_{n+1} = f(e_n)$

- Double Precision can combat round off error.
- RK or RKF or formulas that have ~~[illegible]~~ high α $O(h^{\alpha})$ can take care of truncation errors.
- stable methods can reduce effects of truncation errors.

SELECTING A NUMERICAL INTEGRATION METHOD

- IF RANGE OF INTEGRATION MODERATE (COMPUTING TIME NOT A FACTOR) & NO NEED FOR STEP SIZE CONTROL <u>USE R-K</u>
- IF MANY INTEGRATIONS MUST BE DONE (TIME IS IMPORTANT) BUT ACCURACY NOT REQUIRED USE <u>MODIFIED EULER</u>
- IF RANGE OF INTEGRATION LARGE $\Rightarrow$ COMPUTING TIME IMPORTANT USE HIGH ORDER ACCURACY <u>LIKE ADAMS-MOULTON</u>
- IF COMPUTING TIME IS NOT IMPORTANT BUT NEED ACCURACY <u>USE RKSFX5</u>