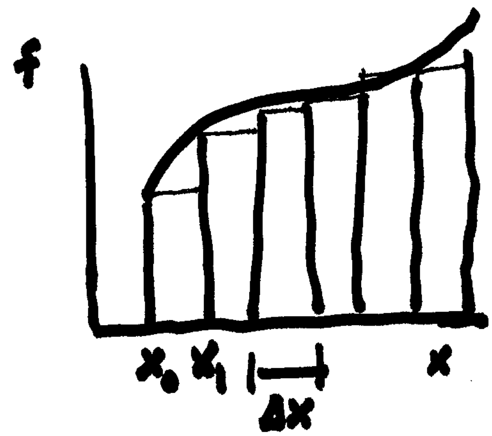
NUMERICAL SOLUTION OF FIRST ORDER
DIFF EQNS

$$\frac{dy}{dx} = f(x,y) \qquad y = y(x)$$
$$y(x = x_0) = y_0$$

$$\int_{y_0}^{y} d\bar{y} = \int_{x_0}^{x} \frac{dy}{dx} \cdot dx = \int_{x_0}^{x} f(\bar{x}, y)\, d\bar{x}$$

$$\bar{y}\Big|_{y_0}^{y} = y - y_0 = \int_{x_0}^{x} f(\bar{x}, y)\, d\bar{x}$$



EULER

$$y - y_0 = f(x_0, y_0)\Delta x$$

$$y_1 = y(x_1) \cong y_0 + f(x_0, y_0)\Delta x$$
$$x_1 = x_0 + \Delta x$$

$$y(x_{i+1}) \cong y_{i+1} = y_i + f(x_i, y_i)\Delta x$$
$$x_{i+1} = x_i + \Delta x$$

$$y(x_{i+1}) = y(x_i + \Delta x) = y(x_i) + \frac{dy}{dx}\Big|_{x_i} \cdot \Delta x + \frac{d^2y}{dx^2}\Big|_{x_i} \frac{\Delta x^2}{2!} + \dots$$
$$= y_i + f(x_i, y_i)\Delta x + O(\Delta x^2)$$

LE $O(\Delta x^2)$
GE $O(\Delta x)$

Given $x_0, \Delta x, f(x,y)$

$N = \frac{x_f - x_0}{\Delta x}$

DO I = 1, 2, ..., N
$y(I) = y(I-1) + f(x_I, y(I-1))\Delta x$
$x_I = x_{I-1} + \Delta x$
PRINT $x_I, y(I)$

$y(I-1) := y(I)$
$x_{I-1} := x_I$