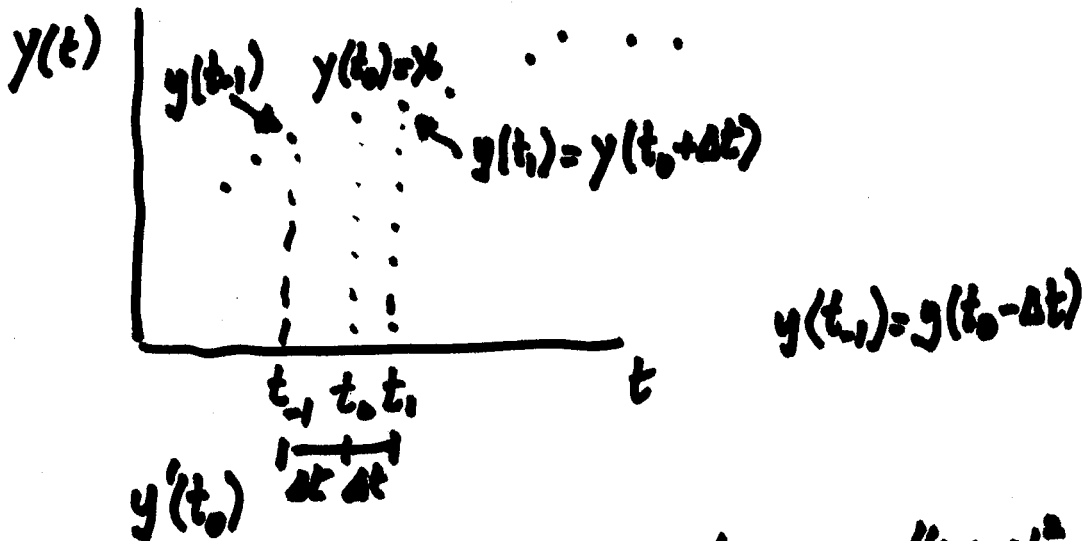


EGM 5346

COMP. ENG. ANALYSIS

DIFFERENTIATION



$$\begin{aligned}
 y_{-1} &= y(t_{-1}) = y(t_0 - \Delta t) = y(t_0) - y'(t_0)\Delta t + y''(t_0)\frac{\Delta t^2}{2!} - y'''(t_0)\frac{\Delta t^3}{3!} + \dots \\
 y_0 &= y(t_0) = y(t_0) \\
 y_1 &= y(t_1) = y(t_0 + \Delta t) = y(t_0) + y'(t_0)\Delta t + y''(t_0)\frac{\Delta t^2}{2!} + y'''(t_0)\frac{\Delta t^3}{3!} + \dots
 \end{aligned}$$

$$y_0 - y_{-1} = y'(t_0)\Delta t - y''(t_0)\frac{\Delta t^2}{2!} + y'''(t_0)\frac{\Delta t^3}{3!} + \dots$$

$$\begin{aligned}
 \frac{y_0 - y_{-1}}{\Delta t} &= y'(t_0) - y''(t_0)\frac{\Delta t}{2!} + y'''(t_0)\frac{\Delta t^2}{3!} + \dots \\
 &= y'(t_0) + O(\Delta t)
 \end{aligned}$$

1ST BACKWARD DIFFERENCE, $O(\Delta t)$

$$\begin{aligned}
 \frac{y_1 - y_0}{\Delta t} &= y'(t_0) + y''(t_0)\frac{\Delta t}{2!} + \dots \\
 &= y'(t_0) + O(\Delta t)
 \end{aligned}$$

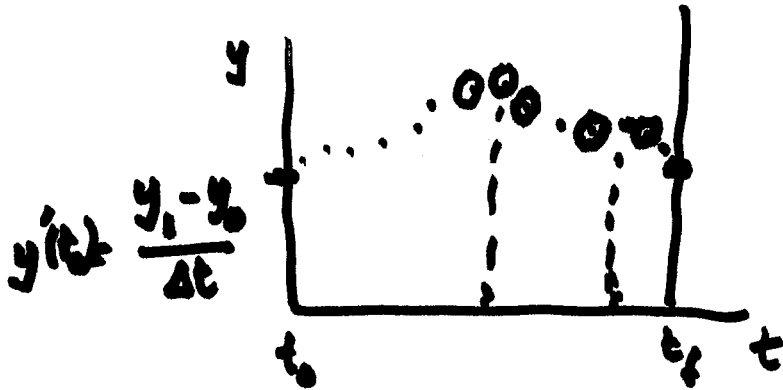
1ST FORWARD DIFFERENCE
 $O(\Delta t)$

$$y_1 - y_{-1} = 2y'(t_0)\Delta t + 2y'''(t_0)\frac{\Delta t^3}{3!}$$

$$\frac{y_1 - y_{-1}}{2\Delta t} = y'(t_0) + y'''(t_0)\frac{\Delta t^2}{3!}$$

$$= y'(t_0) + O(\Delta t^2)$$

CENTERED DIFFERENCE
 $O(\Delta t^2)$



$$\frac{y_1 - y_{-1}}{\Delta t} = y'(t_1)$$