EGM 5346

4/7/99

DIFFERENCE EQNS : example $y' = 5 - 5y = f(x,y)$

$$y_i' = 5 - 5y_i \Rightarrow y_i' + 5y_i = 5$$

$$y_{i_h} = \mathcal{C} e^{-5x_i}$$

$$y_{ip} = D \; ; \; 5D = 5 ; \; D = 1$$

$$\frac{y_{i+1} - y_i}{\Delta x} = 5 - 5y_i$$

$$y_{i+1} + y_i(5\Delta x - 1) = 5\Delta x$$

$$y_{i_p} = C = y_{i+1_p} \Rightarrow C = 1 = y_{ip}$$

HOMOG: $y_{i+1} + y_i(5\Delta x - 1) = 0$

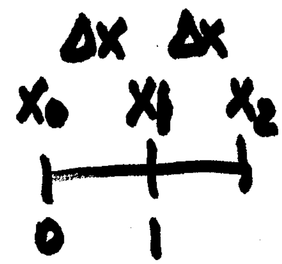
$$y_i = \mathcal{C}\beta^i \Rightarrow \mathcal{C}\beta^i[\beta + (5\Delta x - 1)] = 0$$

$$\beta = (1 - 5\Delta x)$$

$$y_i = \mathcal{C}(1 - 5\Delta x)^i$$

$$e = \lim_{\varepsilon \to 0} (1 + \varepsilon)^{1/\varepsilon}$$

$$\frac{x_i}{i} = \Delta x$$



$$\underline{y_i = \mathscr{C}\left(1 - 5\frac{x_i}{i}\right)^{\frac{-i}{5x_i} \cdot -5x_i}}$$

x_i is fixed
$\Delta x \to 0 \Rightarrow i \to \infty$

$$= \mathscr{C} e^{-5x_i} \quad \textit{when } i \to \infty$$

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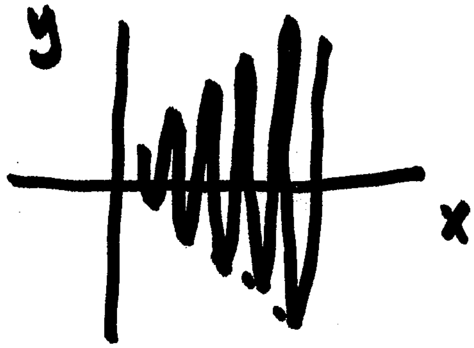
$$\sqrt{1+\epsilon} \cong 1 + \tfrac{1}{2}\epsilon$$



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$$y'' = f(x,y)$$

$$\left.\begin{aligned} y' &= p \\ y'' = p' &= f(x,y,p) \end{aligned}\right] \quad \begin{aligned} y' &= g(x,y,p) \\ p' &= h(x,y,p) \end{aligned}$$

$$y' \to x' \qquad g(x,y,p) = p$$
$$p' \to y' \qquad h(x,y,p) = g(t,x,y)$$

$$y' = 5 - 5y_i$$

$$y' = \frac{y_{i+1} - y_{i-1}}{2\Delta x} = 5 - 5y_i$$

$$y_{i+1} + 10\Delta x\, y_i - y_{i-1} = 10\Delta x$$

$$\text{if } y_{i_p} = C = y_{i+1_p} = y_{i-1_p} \cdots \quad y_{i_p} = C = 1$$

$$y_i = \mathcal{C}\beta^i$$

$$\mathcal{C}\beta^{i-1}\left[\beta^2 + 10\Delta x\,\beta - 1\right] = 0$$

$$\beta = \frac{-10\Delta x \pm \sqrt{(10\Delta x)^2 - 4(1)(-1)}}{2}$$

using $\sqrt{1+\epsilon} \cong 1 + \frac{\epsilon}{2} + \text{h.o.t.}$

$$= -5\Delta x \pm \sqrt{1 + (5\Delta x)^2}$$

$$= \pm\left(1 + \frac{25}{2}\Delta x^2\right) \mp 5\Delta x$$

$$= \pm 1 \mp 5\Delta x + \text{h.o.t. in } \Delta x^2$$

$$y_i = \mathcal{C}_1\beta_1^i + \mathcal{C}_2\beta_2^i = \mathcal{C}_1(1+5\Delta x)^i(-1)^i + \mathcal{C}_2(1-5\Delta x)^i$$

$$y_i \longrightarrow \mathcal{C}_1 e^{+5x_i}(-1)^i + \mathcal{C}_2 e^{-5x_i}$$