$$y_{i+1_c} = y_{i-1} + \frac{\Delta x}{3}\left(f_{i+1} + 4 f_i + f_{i-1}\right)$$

$$f_i \cong \lambda y_i$$

$$y_{i+1} = y_{i-1} + \frac{\lambda \Delta x}{3}\left[y_{i+1} + 4 y_i + y_{i-1}\right]$$

$$y_{i+1}\left[1 - \frac{\lambda\Delta x}{3}\right] - 4 y_i \frac{\lambda \Delta x}{3} - y_{i-1}\left[1 + \frac{\lambda \Delta x}{3}\right] = 0$$

$$y_i = \mathcal{C}\beta^i$$

$$\mathcal{C}\beta^{i-1}\left[\beta^2\left(1 - \frac{\lambda\Delta x}{3}\right) - 4\beta\frac{\lambda\Delta x}{3} - \left(1 + \frac{\lambda\Delta x}{3}\right)\right] = 0$$

$$\beta = \frac{\frac{4\lambda\Delta x}{3} \pm \sqrt{\frac{16\lambda^2\Delta x^2}{9} + 4\left(1 - \frac{\lambda\Delta x}{3}\right)\left(1 + \frac{\lambda\Delta x}{3}\right)}}{2\left(1 - \frac{\lambda\Delta x}{3}\right)}$$

$$= \frac{\frac{4\lambda\Delta x}{3} \pm 2\sqrt{\cancel{4}^{1} + \frac{\cancel{12}^{3}\lambda^2\Delta x^2}{9}}}{2\left(1 - \frac{\lambda\Delta x}{3}\right)}$$

$$= \frac{\frac{2\lambda\Delta x}{3} \pm \sqrt{1 + \frac{\lambda^2\Delta x^2}{3}}}{1 - \frac{\lambda\Delta x}{3}}$$

$$\sqrt{1+\epsilon} = 1 + \frac{1}{2}\epsilon \qquad \frac{\frac{2\lambda\Delta x}{3} \pm \left(1 + \frac{1}{6}\lambda^2\Delta x^2\right)}{1 - \frac{\lambda\Delta x}{3}}$$

$$\frac{1}{1-\epsilon} = 1 + \epsilon + \epsilon^2 + \dots + \epsilon^n$$

$$\beta = \left[\frac{2\lambda\Delta x}{3} \pm \left(1 + \frac{1}{6}\Delta x^2\lambda^2\right)\right]\left[1 + \frac{\lambda\Delta x}{3}\right]$$

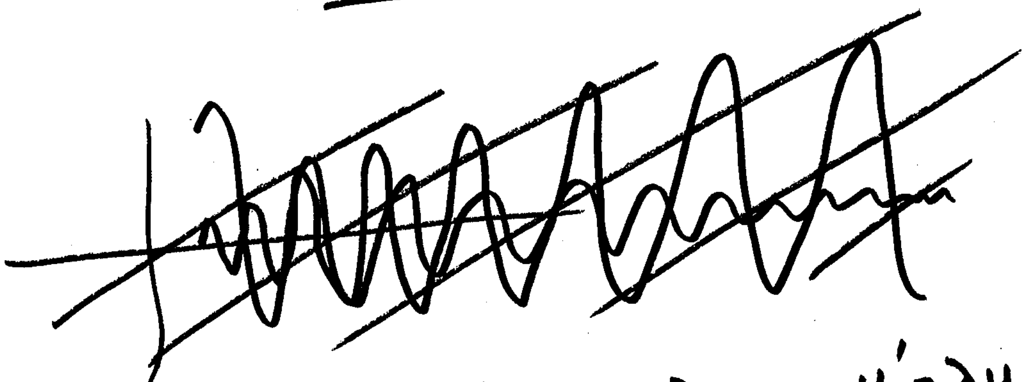
$$= \pm 1 + \frac{2\lambda\Delta x}{3} \pm \frac{\lambda\Delta x}{3} + \text{h.o.t. in } \Delta x^2$$

$$= 1 + \lambda\Delta x$$

$$-1 \mp \frac{\lambda\Delta x}{3}$$

$$y_i = \mathcal{C}_1\beta_1^i + \mathcal{C}_2\beta_2^i = \mathcal{C}_1\left[1 + \lambda\Delta x\right]^i + \mathcal{C}_2\left[1 - \frac{\lambda\Delta x}{3}\right]^i(-1)^i$$

$$\lim_{\Delta x \to 0} y_i \to \underline{\underline{\mathcal{C}_1 e^{+\lambda x_i}}} + \underline{\mathcal{C}_2 e^{-\frac{\lambda x_i}{3}}(-1)^i}$$



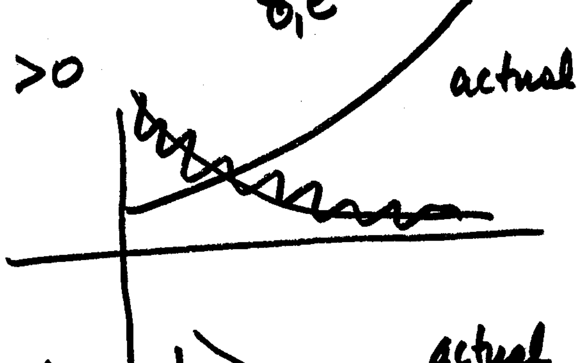
$$y' = f(x,y) = \lambda y \qquad y' = \lambda y$$

$$y = \mathcal{C}e^{\lambda x}$$

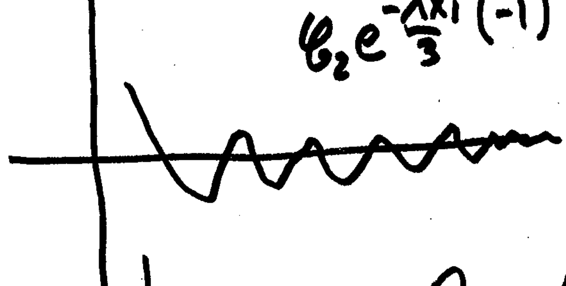
$\mathcal{C}_1 e^{\lambda x_i}$

for $\lambda > 0$

actual



$\mathcal{C}_2 e^{-\frac{\lambda x_i}{3}}(-1)^i$



for $\lambda < 0$

actual