O.D.E. REVIEW

- ORDER OF DIFF. EQ. = HIGHEST DERIV.

$$y^{IV} + 2y'' = 0 \qquad y^{IV} = \frac{d^4y}{dx^4}$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^2 w}{\partial x^2} = 0 \qquad w = w(x,y)$$

- IF COEFFS OF DERIVATIVES ARE CONST. $\Rightarrow$ CONSTANT COEFF DIFF. EQ.

$$u''(t) + cu'(t) + m\,u(t) = 0 \qquad m, c \text{ CONST.}$$

- IF FN IS FN OF ONE INDEP. VARIABLE ie $y = y(x)$

$\Rightarrow$ ORDINARY DIFF. EQ.

$$a(x)y'' + b(x)y' + c(x)y = 0 \qquad y = y(x)$$

- LINEAR IF Power [redacted] OF y or its derivatives are $y^1, (y')^1$ etc only

$$a(x)\,y'' + b(x)y' + c(x)y = 0$$

- NON LINEAR IF products of y & derivs or powers of y

$$a(x)\,y'' + b(x)\,yy' = 0$$

$$a(x)\,y'' + b(x)(y')^2 + c(x) = 0$$

- HOMOGENEOUS IF TERM THAT DOESN'T INVOLVE y or derivs $= 0$

$$a(x)y'' + b(x)\,y' + c(x)\,y + d(x) = 0 \qquad \text{HOMOG IF } d(x) = 0$$

- FIRST ORDER ODE : FIND IN ANY O.D.E. BOOK.

$$a(x)y' + b(x)y = c(x) \qquad \div \text{ by } a(x)$$

$$\Rightarrow \quad y' + p(x)\,y = h(x)$$

- SOLUTION use integrating factor $\mu(x) = e^{\int^x p(t)dt}$

$$y(x) = \frac{1}{\mu(x)} \int^x \mu(t)h(t)\,dt + \frac{\text{const}}{\mu(x)}$$

- need one condition $y = y_0$ at $x = x_0$ to define const

- WHAT IF IT IS HARD TO FIND SOLUTION BY CLOSED FORM

- PICARD'S METHOD Good for $|x|<a$ if y & $\partial f/\partial y$ are continuous in $|x|\le a$ $|y|\le b$

WRITE $y' = f(x,y)$ with $y(x=x_0) = y_0$

$$\Rightarrow \quad y - y_0 = \int_{x_0}^{x} f(\bar{x},y)\,d\bar{x} \qquad \text{when we integrate}$$

Picard says: define sequence $y_0, y_1, y_2, \dots y_n \to y$ in limit

$$y_1 = y_0 + \int_{x_0}^{x} f(\bar{x},y_0)\,d\bar{x}$$
$$y_2 = y_0 + \int_{x_0}^{x} f(\bar{x},y_1)\,d\bar{x}$$
$$\vdots$$
$$y_n = y_0 + \int_{x_0}^{x} f(\bar{x},y_{n-1})\,d\bar{x}$$

- unsatisfactory on practical reasons it is hard to integrate

- example $y' = x - y$ $\quad y(x=0) = 1$ $\quad y_0 = 1$ $\quad x_0 = 0$

$f(x,y) = x - y$

$$y_1 = y_0 + \int_{x_0}^{x} f(\bar{x},y_0)\,d\bar{x} = 1 + \int_0^x (\bar{x}-1)\,d\bar{x}$$

$$= 1 + \frac{(\bar{x}-1)^2}{2}\Big|_0^x = 1 + \frac{(x-1)^2}{2} - \frac{(-1)^2}{2} = 1 + \frac{x^2}{2} - x = 1 - x + \frac{x^2}{2}$$

$$y_2 = y_0 + \int_{x_0}^{x} f(\bar{x},y_1)\,d\bar{x} = 1 + \int_0^x \left[\bar{x} - \left(1 - \bar{x} + \frac{\bar{x}^2}{2}\right)\right]d\bar{x}$$

$$= 1 + \left[-\bar{x} + \frac{\bar{x}^2}{\cancel{2}} - \frac{\bar{x}^3}{6}\right]_0^{x} = 1 - x + \frac{x^2}{\cancel{2}} - \frac{x^3}{6}$$

- actual: $\mu(x) = e^{\int^x 1\,dt} = e^x$ $\qquad y' + y = x \qquad p(x) = 1 \quad h(x) =$

$$y(x) = \frac{1}{e^x}\int^x e^t\,t\,dt + \frac{C}{e^x}$$
$$= \frac{1}{e^x}\left[te^t - e^t\right]^x + C/e^x = x - 1 + Ce^{-x},$$

when $x=0$ $y=1$ $\Rightarrow C = 2$

AT $x=0 \quad y=1 \quad \Rightarrow \quad C=2$

$$y(x) = x-1+2e^{-x}$$
$$= x-1+2\left[1-x+\frac{x^2}{2}-\frac{x^3}{3!}+\dots\right]$$
$$= 1-x+x^2-\frac{x^3}{3}+\dots$$

y_1 error in last term

y_2 error in last term

but sequence tends to y.

<u>HW</u> Use Picard's method to find the solution to $y' - x^2 y = x$ with $y(x=0)=1$ at $x=.2$ (ie $y(x=.2)=?$) Solution must be accurate to 5 decimal places

- Picard's method is good but unwieldy