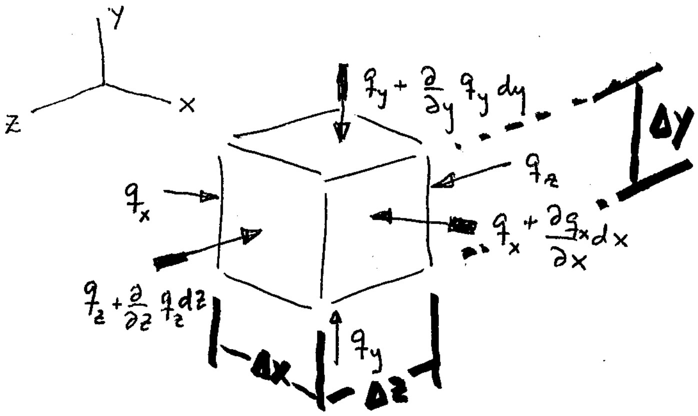
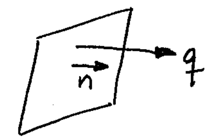


Fourier Heat Conduction law

$$q = -kA\frac{\partial T}{\partial n}$$



k — heat conduction coeff

A = area $\perp$ to heat flow

n = normal to area

$$-\left(\frac{\partial q_x}{\partial x}dx + \frac{\partial q_y}{\partial y}dy + \frac{\partial q_z}{\partial z}dz\right) = mc_v\frac{\partial T}{\partial t}$$

$$+\frac{\partial}{\partial x}k_xA_x\frac{\partial T}{\partial x}dx + \frac{\partial}{\partial y}k_yA_y\frac{\partial T}{\partial y}dy + \frac{\partial}{\partial z}k_zA_z\frac{\partial T}{\partial z}dz = \rho\, d\forall\, c_v\frac{\partial T}{\partial t}$$

; IF $A_x = A_y = A_z = A$ & $A_x dx = A_y dy = A_z dz = d\forall$

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)d\forall + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)d\forall + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)d\forall = \rho c_v d\forall\frac{\partial T}{\partial t}$$

thus $\nabla\cdot(k\nabla T) = \rho c_v\frac{\partial T}{\partial t}$ when $\nabla = \frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{k}$

if k = const $\Rightarrow$ $k\nabla^2 T = \rho c_v\frac{\partial T}{\partial t}$ c_v - specific heat of matter at const. volume

or $\nabla^2 T = \frac{\rho c_v}{k}\frac{\partial T}{\partial t}$

$\nabla^2 T = \alpha\frac{\partial T}{\partial t}$ $\alpha = \frac{\rho c_v}{k}$ = thermal diffusivity

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now if $T \neq$ fn of t $\nabla^2 T = 0$ steady state (Laplace Eq)

$T \neq$ fn of $\cancel{z}, y$ $\frac{\partial^2 T}{\partial x^2} = \alpha\frac{\partial T}{\partial t}$ one - D heat eqn.

$$\frac{\partial^2 T}{\partial x^2} - \alpha\frac{\partial T}{\partial t} = au_{xx} + bu_{xt} + cu_{tt} + du_x + eu_t + fu + g = 0$$

here $a = 1$ $b = c = d = 0$ $e = -\alpha$ $f = g = 0$

$\therefore$ $b^2 - 4ac = 0$ parabolic when $\frac{\partial^2 T}{\partial x^2} = \alpha\frac{\partial T}{\partial t}$