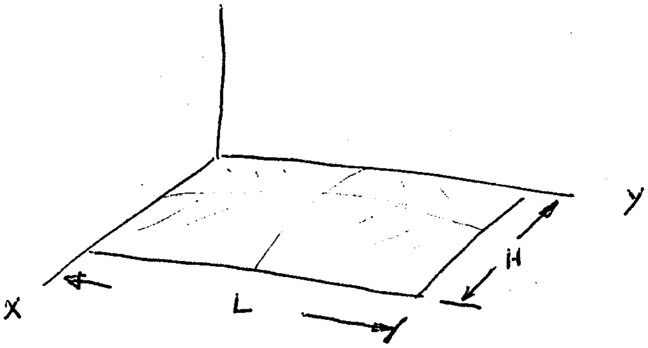
LET'S LOOK AT VIBS OF RECTANGULAR MEMBRANE



$$\nabla^2 W - \frac{1}{c^2}\frac{\partial^2 W}{\partial t^2} = 0$$

with $W(x, y=0, t) = W(x, y=L, t) = 0$

$W(x=0, y, t) = W(x=H, y, t) = 0$

Homogeneous 1st kind (Neumann)

AGAIN LET $W(x,y,t) = w(x,y)T(t)$ $\qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

$\therefore\ T\nabla^2 w - \frac{1}{c^2} w\ddot{T} = 0 \qquad \div$ BY wT

AND $\dfrac{c^2\nabla^2 w}{w} = \dfrac{\ddot{T}}{T} = -\omega^2$

$\Rightarrow \ddot{T} + \omega^2 T = 0 \quad \Rightarrow T = C_1 \sin\omega t + C_2 \cos\omega t$

$\Rightarrow \nabla^2 w + \left(\frac{\omega}{c}\right)^2 w = 0 \qquad$ LET $\lambda = \frac{\omega}{c}$

NOW LET $w(x,y) = X(x)Y(y)$

$\therefore\ \nabla^2 w + \lambda^2 w = X''Y + XY'' + \lambda^2 XY = 0 \qquad \div$ by XY

$$\Rightarrow \left(\frac{X''}{X} + \lambda^2\right) = -\left(\frac{Y''}{Y}\right) = \alpha^2$$

$\therefore\ Y'' + \alpha^2 Y = 0 \quad \Rightarrow \quad Y = A\sin\alpha y + B\cos\alpha y$

$X'' + (\lambda^2 - \alpha^2)X = 0 \Rightarrow X = C\sin\beta x + D\cos\beta x \qquad \beta = \sqrt{\lambda^2 - \alpha^2}$

$\Rightarrow$ Now $W(x, y=0, t) = w(x, y=0)T(t) = X(x)Y(0)T(t) = 0$ $\quad$ FOR ALL $x, t \Rightarrow Y(0) = 0$

$W(x, y=L, t) = w(x, y=L)T(t) = X(x)Y(L)T(t) = 0$ $\quad$ " " " $\Rightarrow Y(L) = 0$

$\Rightarrow$ FROM $Y(0) = 0$ & $Y(L) = 0 \Rightarrow B = 0$ & $Y(L) = A\sin\alpha L = 0 \Rightarrow \alpha L = n\pi$

$\therefore\ Y(y) = A\sin\frac{n\pi}{L}y \qquad$ A WILL DEPEND ON n & SO WILL $Y(y)$

FROM $W(x=0,y,t) = w(x=0,y)T(t) = X(0)Y(y)T(t) = 0$ FOR ALL y & $t \Rightarrow X(0)=0$

$W(x=H,y,t) = w(x=H,y)T(t) = X(H)Y(y)T(t)=0$ FOR ALL y & $t \Rightarrow X(H)=0$

$\Rightarrow X(0)$ & $X(H)=0 \Rightarrow D=0$ & $\beta H = m\pi$ $\quad (C\sin\beta H = 0)$

$\therefore \quad X(x) = C\sin\frac{m\pi}{H}x \qquad$ (C WILL DEPEND ON m & SO WILL X)

$\therefore \quad w(x,y) = \overbrace{A_n C_m}\sin\frac{n\pi}{L}y\sin\frac{m\pi}{H}x = E_{nm}\sin\frac{n\pi}{L}y\sin\frac{m\pi x}{H}$

since $\alpha = \frac{n\pi}{L}$ & $\beta = \sqrt{\lambda^2-\alpha^2} = \frac{m\pi}{H} \Rightarrow \lambda = \sqrt{\beta^2+\alpha^2} = \sqrt{\frac{n^2\pi^2}{L^2}+\frac{m^2\pi^2}{H^2}}$

- note that n,m start at 1, WHY?
- IF $n=0 \Rightarrow \alpha=0 \Rightarrow -\frac{Y''}{Y}=0$ or $Y = Ay+B$

since $Y(0)=Y(L)=0$ FROM BC's $\quad \left.\begin{matrix} Y(0)=0 \Rightarrow B=0 \\ Y(L)=0 \Rightarrow A=0 \end{matrix}\right\} \Rightarrow \underset{\text{TRIVIAL SOLUTION}}{Y(y)=0}$

$\Rightarrow$ NOTE MUST ALWAYS CHECK $n=0$ CASE BY GOING BACK TO DIFF. EQ. & USING BCS

$\Rightarrow W(x=0,y,t)=0 \Rightarrow X(0)=0$ ONLY WORKS FOR <u>ZERO</u> BC's

$W(x=0,y,t)=5 \not\Rightarrow X(0)=5$

LOWEST FREQUENCY OF VIBRATION $\omega_{mn} = \lambda_{mn}\cdot c = c\pi\sqrt{\frac{n^2}{L^2}+\frac{m^2}{H^2}}\Bigg|_{m=n=1} = c\pi\sqrt{\frac{1}{L^2}+\frac{1}{H^2}}$

SO $W(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[A_{nm}\sin\omega_{nm}t + B_{nm}\cos\omega_{nm}t\right]\sin\frac{n\pi y}{L}\sin\frac{m\pi x}{H}$

~~and~~ where $\omega_{nm} = c\pi\left[\frac{n^2}{L^2}+\frac{m^2}{H^2}\right]^{1/2}$; $\omega_{11} = c\pi\left[\frac{1}{L^2}+\frac{1}{H^2}\right]$