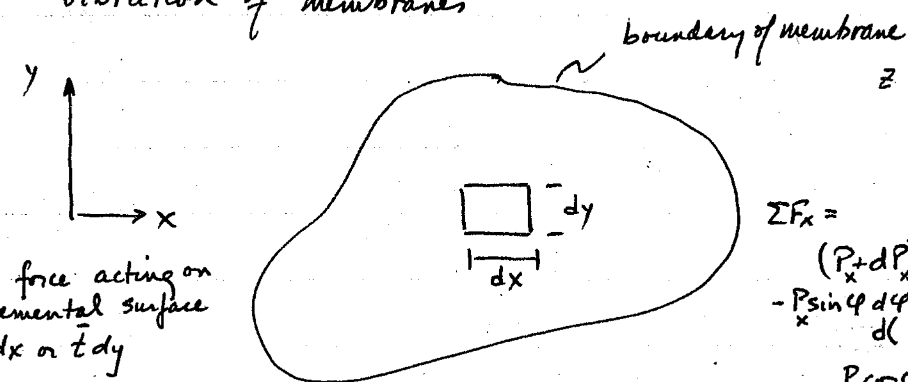
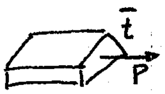
if E and A are not functions of x and $f(x,t)=0$

$$u_{xx} - \frac{1}{c^2} u_{tt} = 0 \qquad c = \sqrt{E/\rho} \text{ is the bar velocity}$$

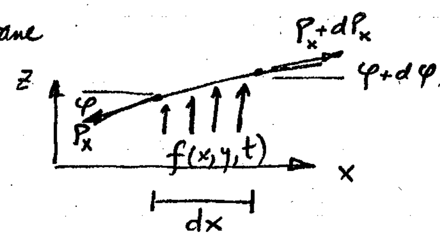
if $f(x,t) \neq 0$ we have an inhomogeneous problem: example vertical rod where weight cannot be neglected

- vibration of membranes





P force acting on elemental surface $\bar{t}dx$ or $\bar{t}dy$



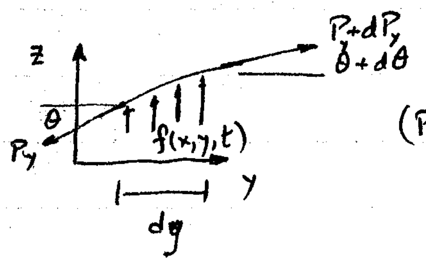
$\Sigma F_x =$

$$(P_x + dP_x)\cos(\varphi + d\varphi) - P_x \cos\varphi = 0$$

$$-P_x \sin\varphi \, d\varphi + dP_x \cos\varphi = 0 \quad \text{TO FIRST ORDER}$$

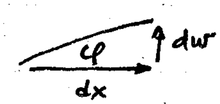
$$d(P_x \cos\varphi) = 0$$

$$P_x \cos\varphi = \text{const. or fn of } y$$

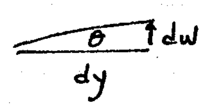


$$(P_y + dP_y)\cos(\theta + d\theta) - P_y \cos\theta = 0 = \Sigma F_y$$

$$d(P_y \cos\theta) = 0 \Rightarrow P_y \cos\theta = \text{const or fn of } x$$



$$\cos\varphi = \frac{dx}{\sqrt{dx^2 + dw^2}} \cong 1 \quad \text{if } \left|\frac{\partial w}{\partial x}\right| \ll 1$$



$$\cos\theta = \frac{dy}{\sqrt{dy^2 + dw^2}} \cong 1 \quad \text{if } \left|\frac{\partial w}{\partial y}\right| \ll 1$$

$\Rightarrow$ P_x is essentially constant everywhere
P_y " " " " $\Big\}$ $P_x = P_y = P$

$$\Sigma F_z = m \cdot \text{accel} = \rho \frac{\partial^2 w}{\partial t^2} dy dx \cdot \bar{t}$$

$$f(x,y,t)dxdy + (P_y + dP_y)\sin(\theta + d\theta) - P_y \sin\theta + (P_x + dP_x)\sin(\varphi + d\varphi) - P_x \sin\varphi = \Sigma F_z$$

$$d(P_y \sin\theta) + d(P_x \sin\varphi) + f(x,y,t)dxdy = \Sigma F_z$$

$$\frac{d}{dy} \underbrace{(P_y \sin\theta)}_{\bar{P}\cdot\bar{t}dx} dy + \frac{d}{dx} \underbrace{(P_x \sin\varphi)}_{\bar{P}\cdot\bar{t}dy} dx + f(x,y,t)\, dxdy = \Sigma F_z$$

$$\sin\theta \sim \frac{\partial w}{\partial y} \qquad \sin\varphi \sim \frac{\partial w}{\partial x}$$