- PHYSICAL PROCESSES NORMALLY VARY WITH TIME AND LOCATION
- TO UNDERSTAND THESE PROCESSES
  - IT WOULD BE NICE TO KNOW HOW THEY VARY IN TIME & SPACE
  - WHAT DRIVES THESE PROCESSES (HOW PROCESSES DEPEND ON SYSTEM PARAM
  - WHERE THESE PROCESSES WILL BE AT SOME FUTURE TIME OR WHAT WILL HAPPEN AT SOME FUTURE LOCATION
- IT TURNS OUT THAT EQNS THAT DESCRIBE THESE PROCESSES ARE GENERALLY DIFFERENTIAL EQUATIONS
- WHEN THESE PROCESSES DEPEND ON THE VARIATION OF TWO OR MORE QUANTITIES, THEN THESE PROCESSES ARE GOVERNED BY PARTIAL DIFFERENTIAL EQUATIONS
- MANY PROCESSES IN NATURE ARE DESCRIBED BY 2nd ORDER P.D.E.
- EXAMPLES VIBRATIONS OF A ROD (LONGITUDINAL VIBRATIONS)

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$c = \sqrt{E/\rho}$ BAR VELOCITY

u - LONGITUDINAL DISPLACEMENT

HEAT TRANSFER

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$\frac{k}{c\rho} = \alpha$ THERMAL DIFFUSIVITY

T TEMPERATURE

POTENTIAL FLOW

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \nabla^2 \phi = 0$$

LAPLACE'S EQN

ϕ VELOCITY POTENTIAL $\quad \underset{\sim}{V} = \nabla \phi$

$v = \frac{\partial \phi}{\partial y} \quad u = \frac{\partial \phi}{\partial x}$

STEADY STATE INCOMPRESSIBLE

CONTINUITY EQN IS $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

IN THESE CASES u, T, ϕ ARE FIELD VARIABLE OR DEPENDENT VARIABLE

WHILE x, y, t SPACE COORDINATES OR TIME ARE INDEPENDENT VARIABLES

PARTIAL DERIV IS DEFINED BY

$$\frac{\partial}{\partial x} u(x,t) = \lim_{\Delta x \to 0} \frac{u(x+\Delta x, t) - u(x,t)}{\Delta x}$$

- THESE THREE EQNS CAN DESCRIBE MANY SYSTEMS IN MECH. ENG.
  - WAVE EQUATION ARISES FROM ACOUSTICS, VIBRATIONS, SHALLOW-WATER WAVE THEORY
  - HEAT EQN ARISES IN HEAT TRANSFER & ONE-DIMENSIONAL DIFFUSION PROBLEMS
  - LAPLACE'S EQN ARISES IN POTENTIAL FLOW THEORY, STEADY STATE HEAT TRANSFER, TORSION OF A BAR, STEADY STATE VIB. OF A MEMBRANE

- DO ALL THREE HAVE ANY COMMON IDEAS?
- HOW CAN THEY BE SOLVED? WHAT METHODS EXIST TO SOLVE THE EQNS
- HOW CAN I DERIVE THE MATHEMATICAL EQN?
- WHAT IS A WELL POSED PROBLEM - CAN I FIND A UNIQUE SOLUTION

- CHARACTERIZATION & CLASSIFICATION
  - MOST GENERAL 2^{nd} ORDER PDE OF A FN $u(x,y)$

$$\varphi\,(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0 \quad \text{i.e., } u_{xx} = \frac{\partial^2 u}{\partial x^2}$$

- φ: LINEAR WITH RESPECT TO HIGHEST DERIVATIVE WHEN

$$a\,u_{xx} + b\,u_{xy} + c\,u_{yy} + F(x, y, u, u_x, u_y) = 0$$

- HERE a, b, c are fns of x, y only.
- IF $F(x, y, u, u_x, u_y)$ quasilinear

IF $F(x, y, u, u_x, u_y) = d\,u_x + e\,u_y + fu + g$ LINEAR

IF $g = 0$ THEN IT IS HOMOGENEOUS