Po(x)y" + P.(x)y' + P2(x)y = 0 about x=x. $\sum_{n=1}^{\infty} A_n(x-x_0)^n = y$ $P(x) \stackrel{?}{=} 0$ no ก II ส c1))) • O Type here to search Ū.

Tests used when Po(xo)=0

 $\sum A_n(x-x_o)^n = y$ $P_{n}(x) \stackrel{?}{=} 0$ 10 $\Rightarrow \frac{P_{i}}{P}(x-x_{o}) \stackrel{?}{=} \sum_{n=0}^{\infty} C_{n}(x-x_{o})$ P.(x.)=0 $\frac{P_z}{P}(x-x_0)^2 \stackrel{?}{=} \sum_{n=0}^{\infty} d_n(x-x_0)^n$ O Type here to search

Note that if either test fails then xo is an irregular singular point and the solution may or may not be in the form of a series as shown in the next view.

ZAn(x-xo) = y xo is ordinary pt 10 $= \frac{P_{i}}{P_{o}} (x-x_{o}) \stackrel{7}{=} \frac{\sum_{n=0}^{\infty} C_{n}(x-x_{o})}{\sum_{n=0}^{n} A_{o}(x-x_{o})}$ x_{o} is a singular pt $\frac{P_{z}}{P} (x-x_{o}) \stackrel{2}{=} \frac{\sum_{n=0}^{\infty} d_{n}(x-x_{o})}{\sum_{n=0}^{n} A_{n}(x-x_{o})}$ IF either test fails => Irregular singular pt & y + 2 An(x-x_) IF both tests pass => regular singular pt y= (x-x) 2 An(x-x) <u>โก || ว</u>ิเ 5)) O Type here to search

Coefficient of Lowest power of (x-xo)^r gives the indicial equation. Allows for solution for r.

 $(-x_0) = \sum d_n(x-x_0)$ test fails => Irregular singular pt & y = Z An(x-x_) tests pass => regular singular pt y= (x-x) 2 An(x-x) st power of (X-X) gives the indicial eq => Solve for r f₀ II 3ª 🗄 💽 🥅 🖾 🕅 🏹

$$T_{1} \neq T_{2} \quad and \left[T_{1} - T_{2}\right] \neq integer$$

$$then \quad y_{1}(x) = (x - x_{0})^{r} \left[1 + \sum_{n=1}^{\infty} A_{n}(x - x_{0})^{n}\right] \quad A_{n} = A_{n}(r)$$

$$y_{2}(x) = (x - x_{0})^{r} \left[1 + \sum_{n=1}^{\infty} B_{n}(x - x_{0})^{n}\right] \quad B_{n} = B_{n}(r)$$

$$y_{2}(x) = (x - x_{0})^{r} \left[1 + \sum_{n=1}^{\infty} B_{n}(x - x_{0})^{n}\right] \quad B_{n} = B_{n}(r)$$

 $y(x) = C_1 y_1 + C_2 y_2$ $r_1 \neq r_2$ but $|r_1 - r_2| = integer$ IF $y_{1}(x) = (x - x_{0})^{r_{1}} \left[1 + \sum_{n=1}^{\infty} A_{n}(x - x_{0})^{n} \right]$ $y_{z}(x) = A y_{z}(x) ln(x-x_{0}) + \overline{y}_{z}(x)$ $\tilde{y}_{2}(x) = (x - x_{0})^{r_{2}} \left[1 + \sum_{n=1}^{\infty} B_{n}(x - x_{0})^{n} \right]$ or $y_2(x) = \frac{\partial y_1}{\partial x}$ 0:15:59 5)) Type here to search

$$|F \quad F_{1} = F_{2}$$

$$y_{1}(x) = (x - x_{0})^{F_{1}} \left[(1 + \sum_{n=1}^{\infty} A_{n} (x - x_{0})^{n} \right]$$

$$\overline{y}(x) = (x - x_{0})^{F_{1}} \sum_{n=1}^{\infty} B_{n} (x - x_{0})^{n}$$

$$\overline{y}(x) = (x - x_{0})^{F_{1}} \sum_{n=1}^{\infty} B_{n} (x - x_{0})^{n}$$

$$\overline{y}_{2}(x) = y_{1}(x) \lambda_{n} (x - x_{0}) + \overline{y}(x)$$

Because of the structure of y2 you can use the partial derivative of y1 with respect to r1 to find y2 as in the previous case.

$$P_{0}(x) y'' + P_{1}(x) y' + P_{2}(x) y = 0$$

$$xy''_{1} + y'_{2} - y = 0 \quad about x=0$$

$$P_{0}(x) = x \quad \Rightarrow P_{0}(0) = 0 \Rightarrow sumplan pt$$

$$P_{0}(x - x) = \frac{1}{x} x = 1 = \frac{a}{2} c_{n}(x + x) = \frac{b}{2} c_{n}x^{n}, c_{n} = 1, c_{n} = c_{n} = 0$$

$$P_{0}(x - x) = \frac{1}{x} (x - x)^{2} = -x = \frac{a}{2} d_{n}(x - x)^{n} = \frac{a}{2} d_{n}x^{n} \Rightarrow d_{n} = 0, d_{1} = -1, d_{2} = \dots = d_{n} = 0$$

 $\sum_{n=0}^{\infty} (n+r)A_n x^{n+r-1} \left[n+r-1+1 \right] - \sum_{n=0}^{\infty} A_n x^{n+r} = 0$ $\sum_{n=0}^{\infty} (n+r)^{2} A_{n} \chi^{n+r-1} - \sum_{n=0}^{\infty} A_{n} \chi^{n+r} = 0$ n=o n=0 let n-1= m m+1=n $\sum_{n+r+1}^{\infty} A_n \chi^{m+r} - \sum_{n+1}^{\infty} A_n \chi^{n+r} = 0$ M=-1 5)) O Type here to search

lit n-1=mm+1=1 $\sum_{m+r+1}^{\infty} A_{m+1} \times^{m+r} - \sum_{n=0}^{\infty} A_n \times^{n+r} = 0$ $r^{2}A_{0}\chi^{r-1} + \sum_{n=0}^{\infty} \left[(n+r+1)^{2}A_{n+1} - A_{n} \right] \chi^{n+r} = 0 = 0 \cdot \chi$ Indicial ean r'A.=O fio || 30 O Type

Recursion formula comes from the coefficient of the general term being zero.

1=0 $(n+r+i)^{2} A_{n+i} - A_{n} X^{n+r} = 0 = 0 \cdot x^{n+r}$ (n+r+1) An+1 - An= O Recursive Formula O Type here to search J 🗄 💽 🔚 02 x 🛛 🏹

Rewrite terms for An+1 in terms of An and then use the formula to regress the result back to an expression involving Ao. Now write solution in terms of the general value r using the information from the recursion formula

 $y_{1}(x) = x^{r} A_{0} \left[1 + \frac{x}{(r+1)^{2}} + \frac{x^{2}}{(r+1)^{2}(r+2)^{2}} + \frac{x^{2}}{(r+1)^{2}(r+2)^{2}(r+$ for r=0 $y_1 = A_0 \sum_{n=0}^{\infty} \frac{x^n}{(n!)^2}$ yz = [=1,=0 1:01:28 fi II 38 O Type here to search

$$\begin{pmatrix} (+1)^{n} & (+1)^{n} & (+2)^{n} & (+2)^{$$

Having found the form, now let r=0, in this case, to find $\partial \overline{y} / \partial r$.

If xo is not zero then the form of $y_2(x)$ would be $(x-xo)^r \ln (x-xo) \overline{y} + (x-xo)^r \partial \overline{y} / \partial r$