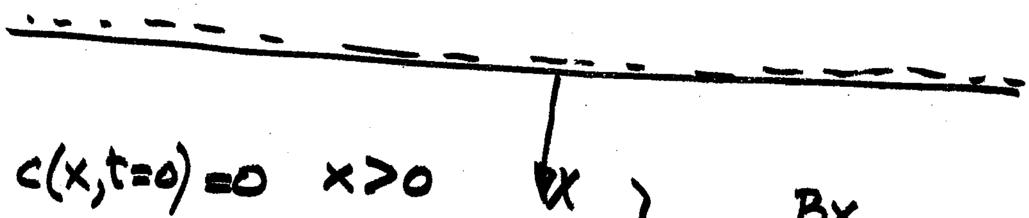
$$\alpha \frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t}$$



initially $c(x,t=0)=0 \quad x>0$

for $x \to \infty \quad c(x,t) \to 0$

$$\Bigg\} \; \eta = \frac{Bx}{t^m}$$

$$\int_0^\infty c(x,t)\,dx = Q$$

$$c = A f(\eta)$$

$$\int_0^\infty c(x,t)\,dx = \int_0^\infty A f(\eta)\,d\eta \cdot \frac{t^m}{B} = Q$$

impossible to satisfy

choose

$$c = A t^n f(\eta)$$

$$\int_0^\infty c(x,t)\,dx = \int_0^\infty A t^n f(\eta) \cdot d\eta \frac{t^m}{B} = Q \implies n = -m$$

$$\frac{\partial c}{\partial x} = \frac{\partial c}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = A t^n f' \cdot \frac{B}{t^m} \; ; \quad \frac{\partial^2 c}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial c}{\partial x}\right) = \frac{\partial}{\partial \eta}\left(\frac{\partial c}{\partial x}\right) \cdot \frac{\partial \eta}{\partial x}$$

$$= A t^n f'' \cdot \frac{B^2}{t^{m \cdot 2}}$$

$$\frac{\partial c}{\partial t} = A n t^{n-1} f + A t^n f' \cdot \left(-\frac{mBx}{t^{m+1}}\right)$$

$$= A n t^{n-1} f + A t^n f' \left(-\frac{m\eta}{t}\right)$$

$$\alpha \frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t} : \quad \alpha A t^n \frac{B^2}{t^{2m}} f'' = A n t^{n-1} f + A t^{n-1} f' \cdot (-m\eta)$$

$$\alpha A t^{-3m} B^2 f'' = -A m t^{-m-1} \left[f + \eta f' \right]$$

for "t" to disappear $-3m = -m-1 \qquad m = \tfrac{1}{2} = -n$

$$\alpha B^2 f'' + \tfrac{1}{2}\left[f + \eta f'\right] = 0$$

$$2\alpha B^2 f'' + \eta f' + f = 0 \quad \Rightarrow \quad B = \frac{1}{\sqrt{2\alpha}}$$

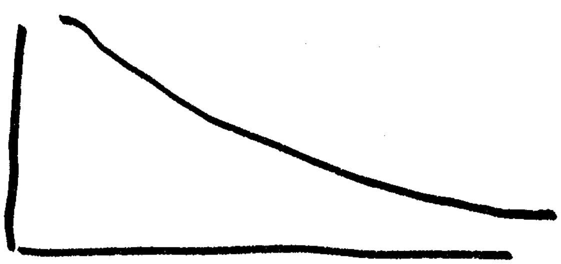
$$\Rightarrow \quad f'' + \underbrace{\eta f' + f} = 0$$

$$f'' + (\eta f)' = 0 \quad \Rightarrow \quad f' + \eta f = C_1$$

$$\left.\begin{array}{l} c(x,t) \to 0 \text{ as } x \to \infty \\ c(x, t=0) = 0 \end{array}\right\} \eta = \infty \qquad \eta = \frac{Bx}{t^m}$$

$c = A t^n f(\eta)$ since $c \to 0$ as $\eta \to \infty$ irrespective of t $\Rightarrow \underline{f(\eta \to \infty) \to 0}$

f



η

$$\Rightarrow \quad f' \to 0 \text{ as } \eta \to \infty$$

$f' + \eta f = C_1 \Rightarrow C_1 = 0$ since f' & $f \to 0$ as $\eta \to \infty$

$$\Rightarrow \quad f' + \eta f = 0 \quad \Rightarrow \quad \frac{df}{f} = -\eta\, d\eta$$

$$\ln f = -\eta^2/2 + \ln C_2$$

$$f = C_2 e^{-\eta^2/2}$$

$$c = At^n f(\eta) \qquad n = -m = -\tfrac{1}{2}$$

$$\eta = \frac{Bx}{t^m} \cdot = \frac{x}{\sqrt{2\alpha t}}$$

$$f = C_2 e^{-\eta^2/2}$$

$$\int_0^\infty c(x,t)\,dx = \frac{A}{B}\int_0^\infty f(\eta)\,d\eta = Q$$

choose $f(\eta=0) = 1 \Rightarrow C_2 = 1 \Rightarrow f = e^{-\eta^2/2}$

$$\int_0^\infty c(x,t)\,dx = \frac{A}{B}\int_0^\infty e^{-\eta^2/2}\,d\eta = Q$$

$$= \sqrt{2\alpha}\cdot A \underbrace{\int_0^\infty e^{-z^2}\,dz}_{\frac{\sqrt{\pi}}{2}} \cdot \sqrt{2} = Q$$

$$A = \frac{Q}{\sqrt{\pi\alpha}}$$

$$c = \frac{Q}{\sqrt{\pi\alpha t}}\, e^{-\frac{x^2}{4\alpha t}}$$

- Do 2.1 & 2.2

Exercises:

2.1 The temperature field $T(x,t)$ in a semi-infinite slab with a constant heat flux is described by

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad ; \quad T(x,0) = T_i$$

$$T(x,t) \rightarrow T_i \quad \text{as} \quad x \rightarrow \infty \quad ; \quad -k \frac{\partial T}{\partial x} = q \quad \text{at} \quad x = 0$$

Solve for the temperature field for $x \geq 0$, $t \geq 0$.

2.2 The temperature field in the thermal boundary layer that grows within a hydrodynamic boundary layer at a step in wall temperature is described by

$$\alpha \frac{\partial^2 T}{\partial y^2} = \beta y \frac{\partial T}{\partial x} \quad ; \quad T(0,y) = T_\infty \quad y > 0$$

$$T(x,y) \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \quad ; \quad T(x,0) = T_w \; ;$$

Solve for the temperature field for $x \geq 0$, $y \geq 0$.

2.3 A device for measuring the velocity gradient in flows is shown in the figure. It consists of a heated plate at the wall, over which a thermal boundary layer grows. As long as the thermal boundary layer is confined to the region where the flow velocity u is linear $(u = \beta y)$, the problem is described by

$$\alpha \frac{\partial^2 T}{\partial y^2} = \beta y \frac{\partial T}{\partial x} \quad ; \quad T(0,y) = T_\infty \quad y > 0$$

$$T(x,y) \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \quad ; \quad -k \frac{\partial T}{\partial y} = q \quad \text{at} \quad y = 0$$

Derive an expression relating the <u>local</u> wall temperature, $T_w(x)$, to the flow parameters and x . Evaluate any constants in this expression. Hint: Γ .