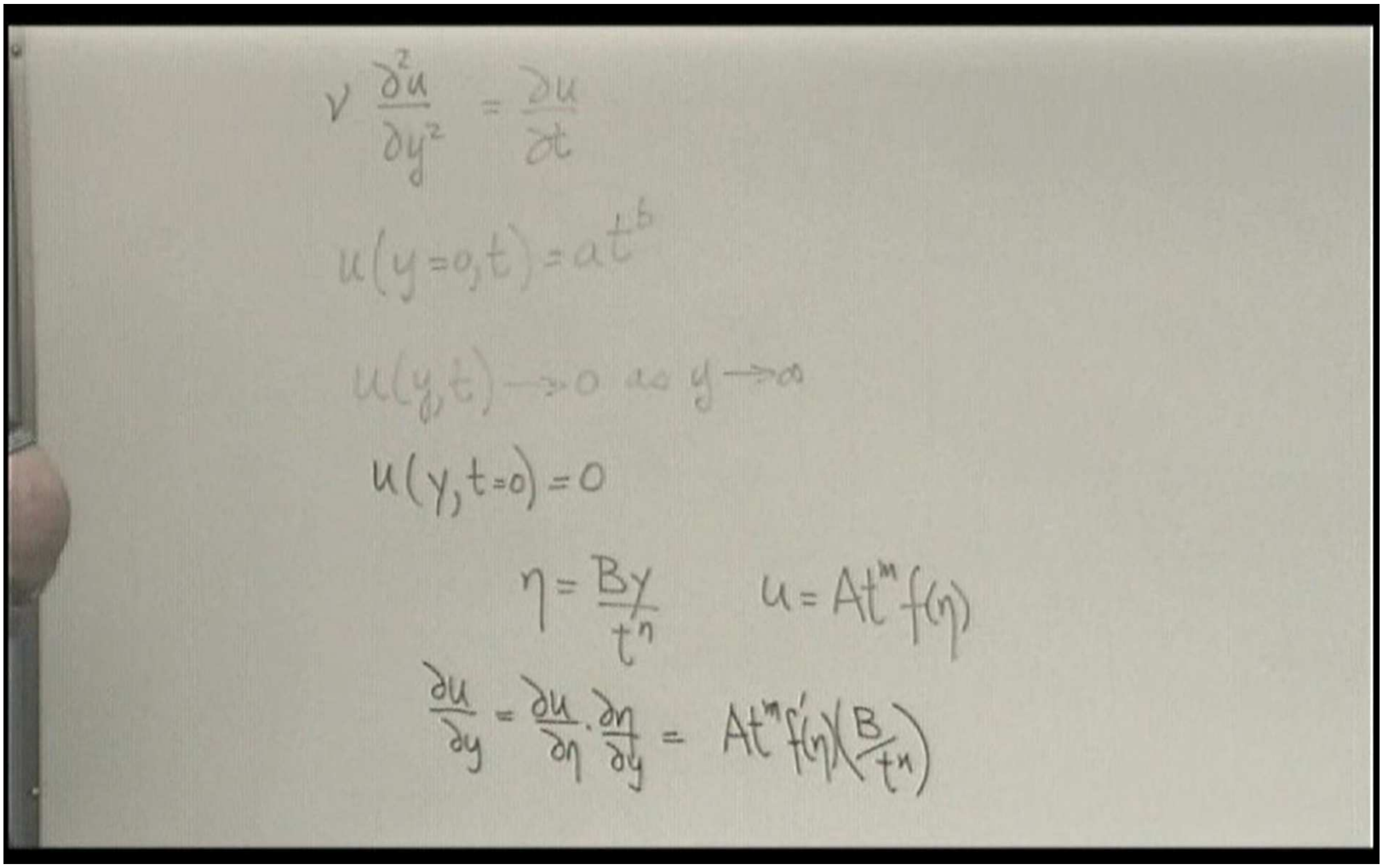
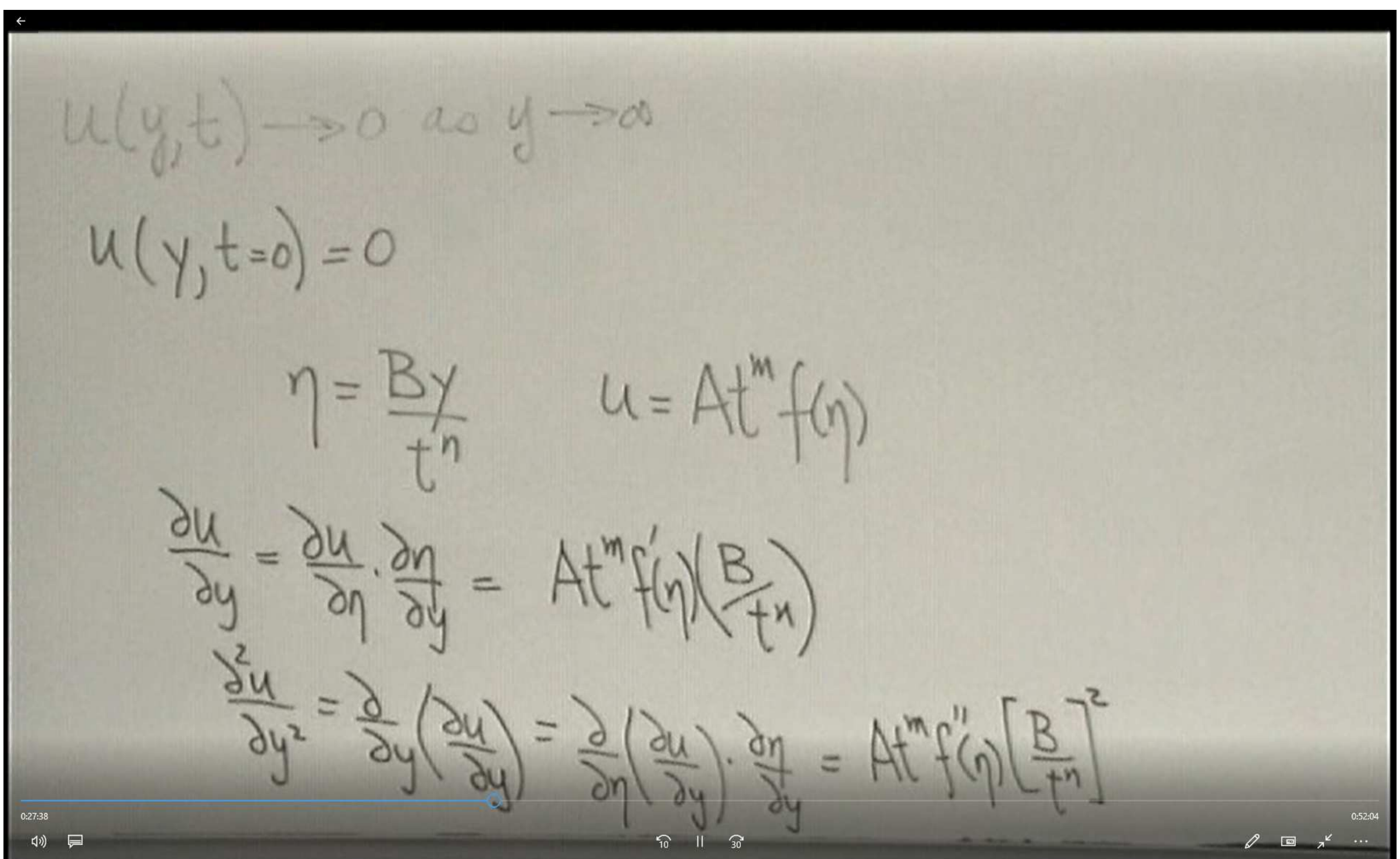
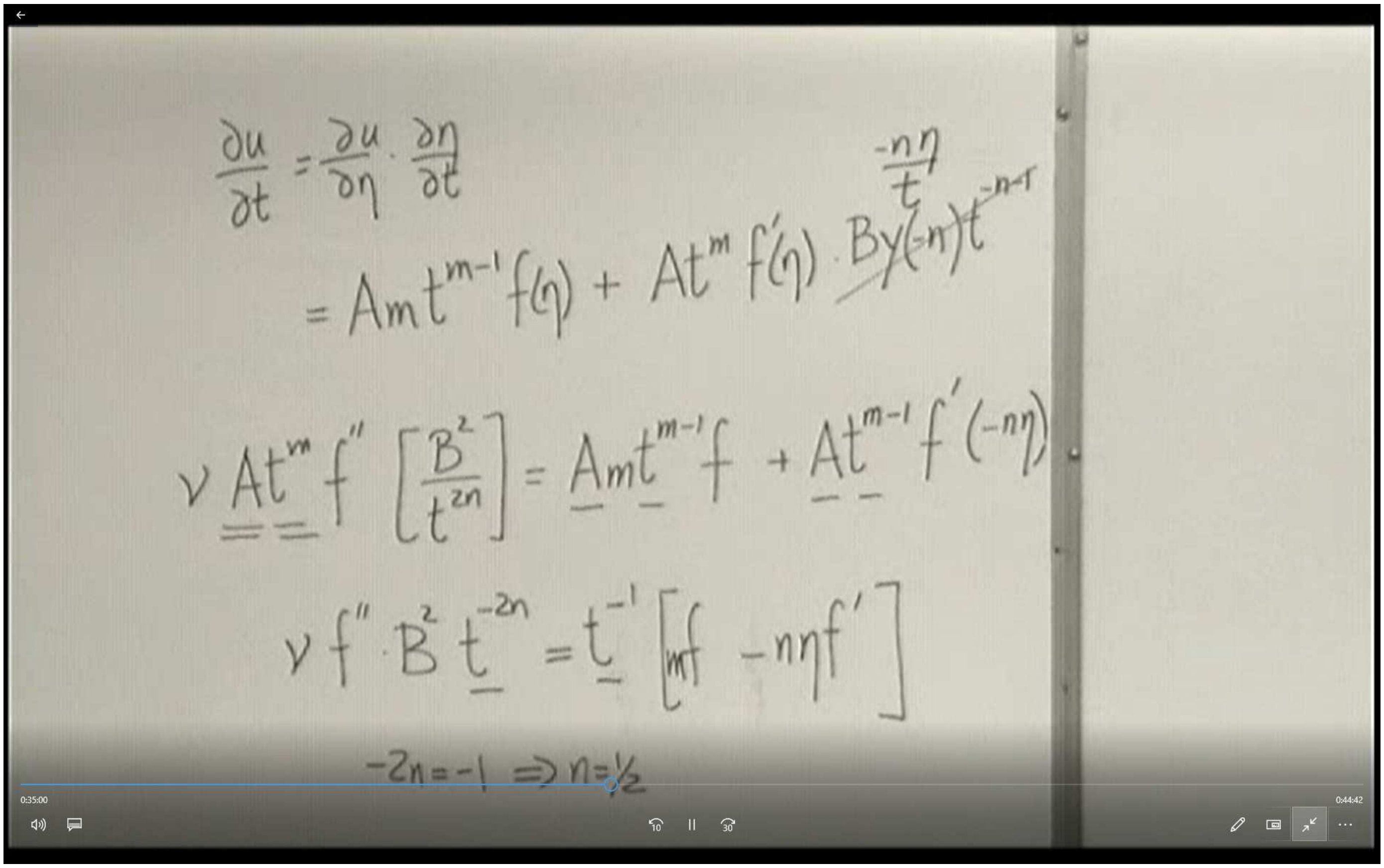
Solving self similar solution using non-constant boundary condition



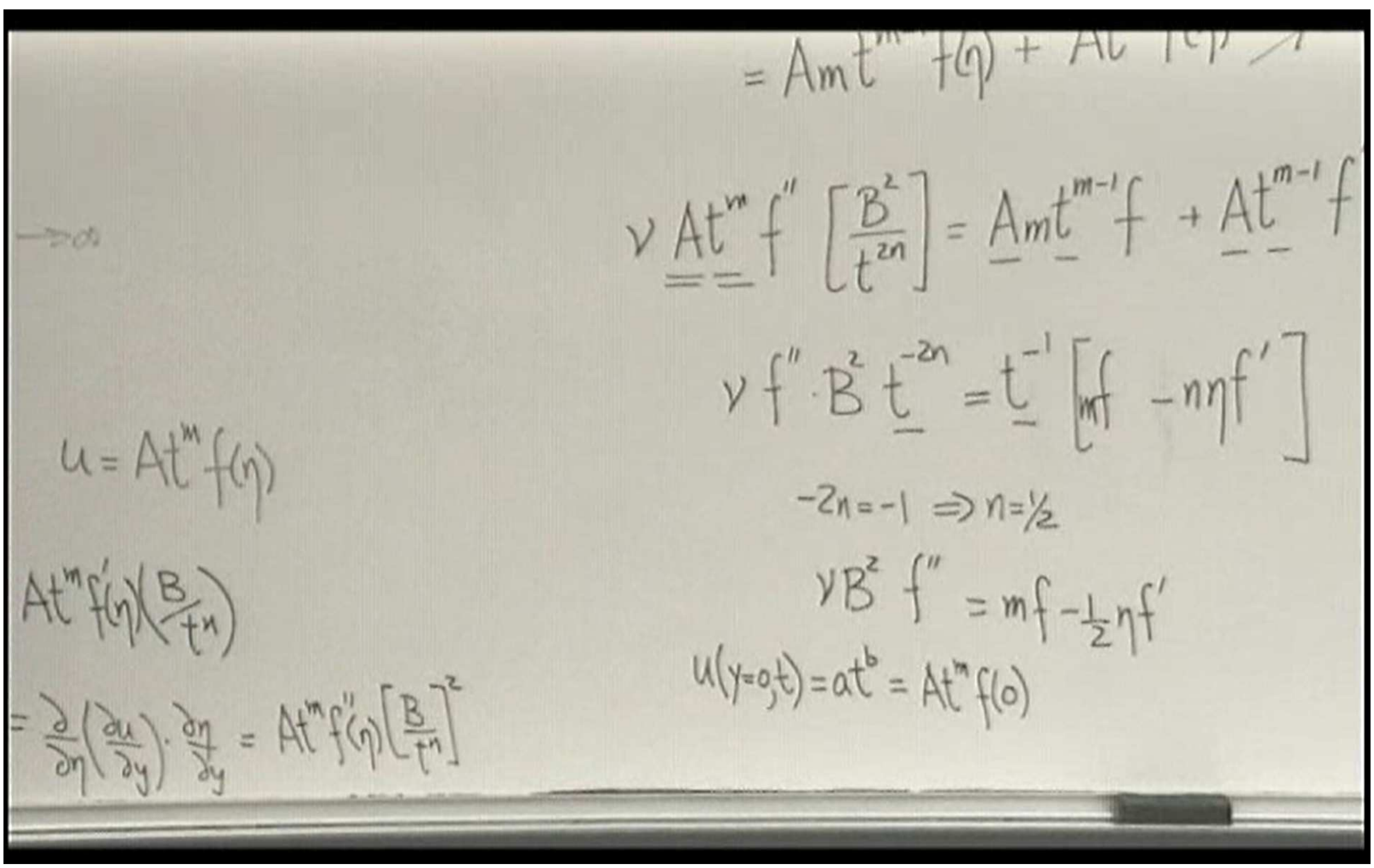
Defining the self similar parameter as usual and generalizing function of u to have the form of the BC at y=0



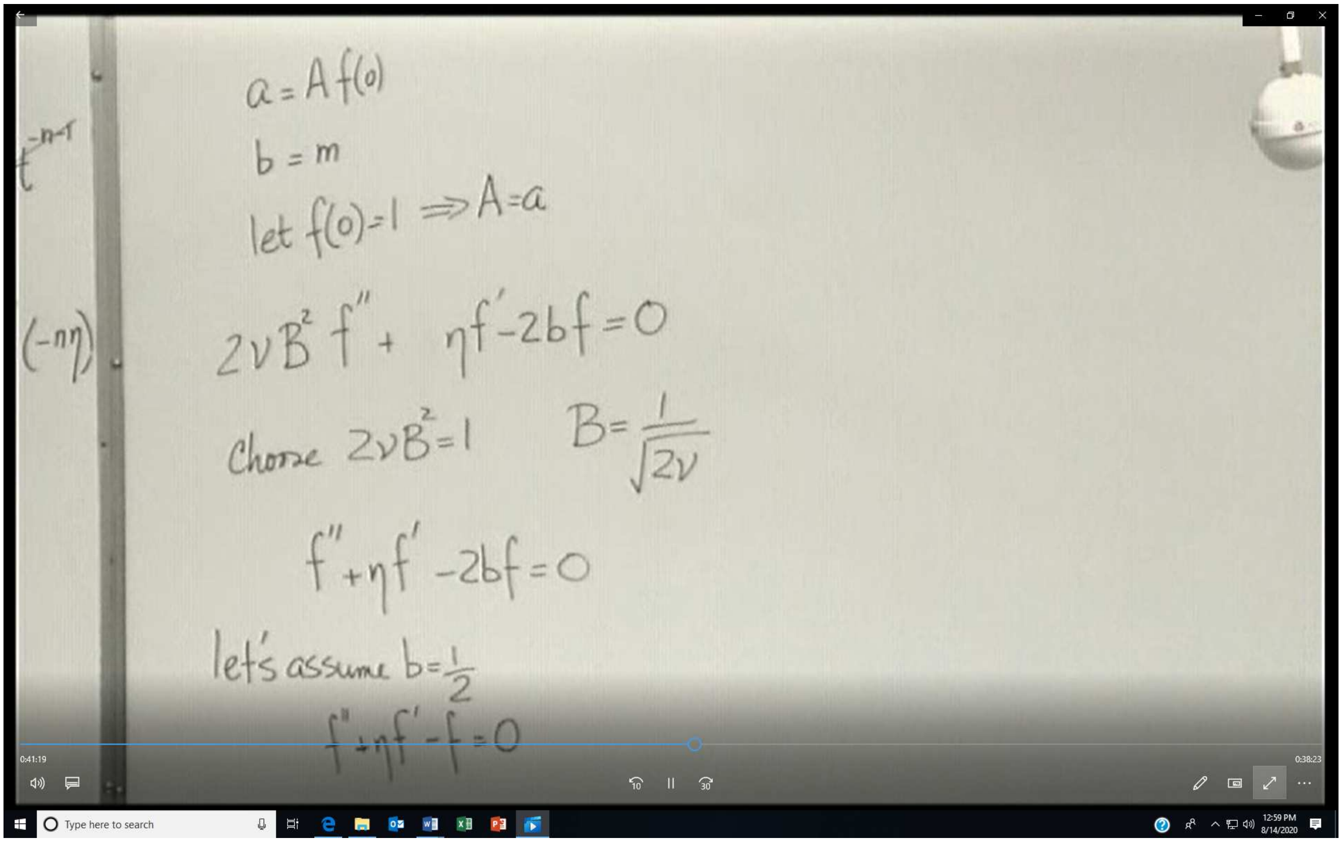
Want to minimize the number of variables, meaning convert the y term to –n*η/t. Now put what you have into original PDE. Here we replace terms involving y and t, if possible, with terms involving η only:



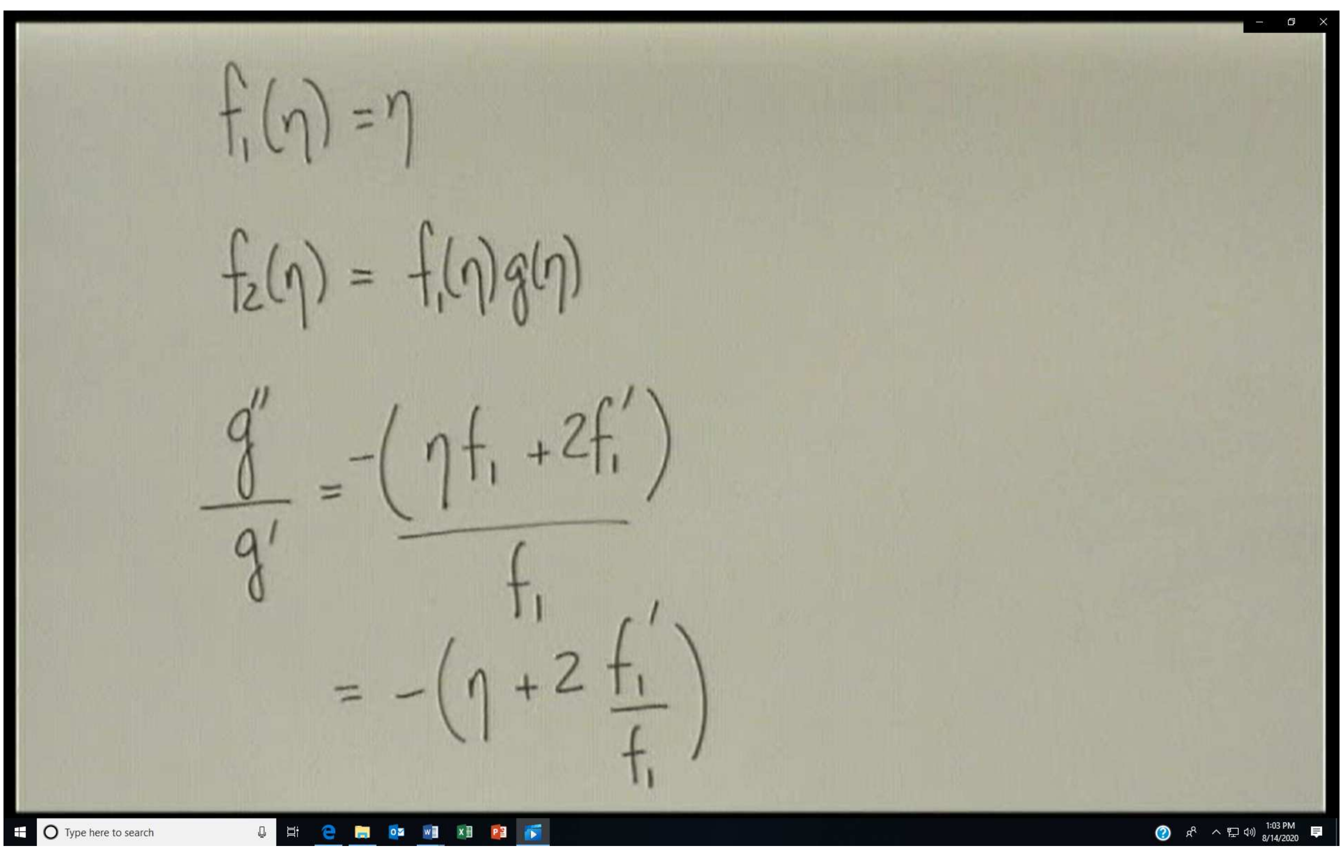
Plugging what we have back into the PDE and ensuring the ODE for f does not have t or y in the equation



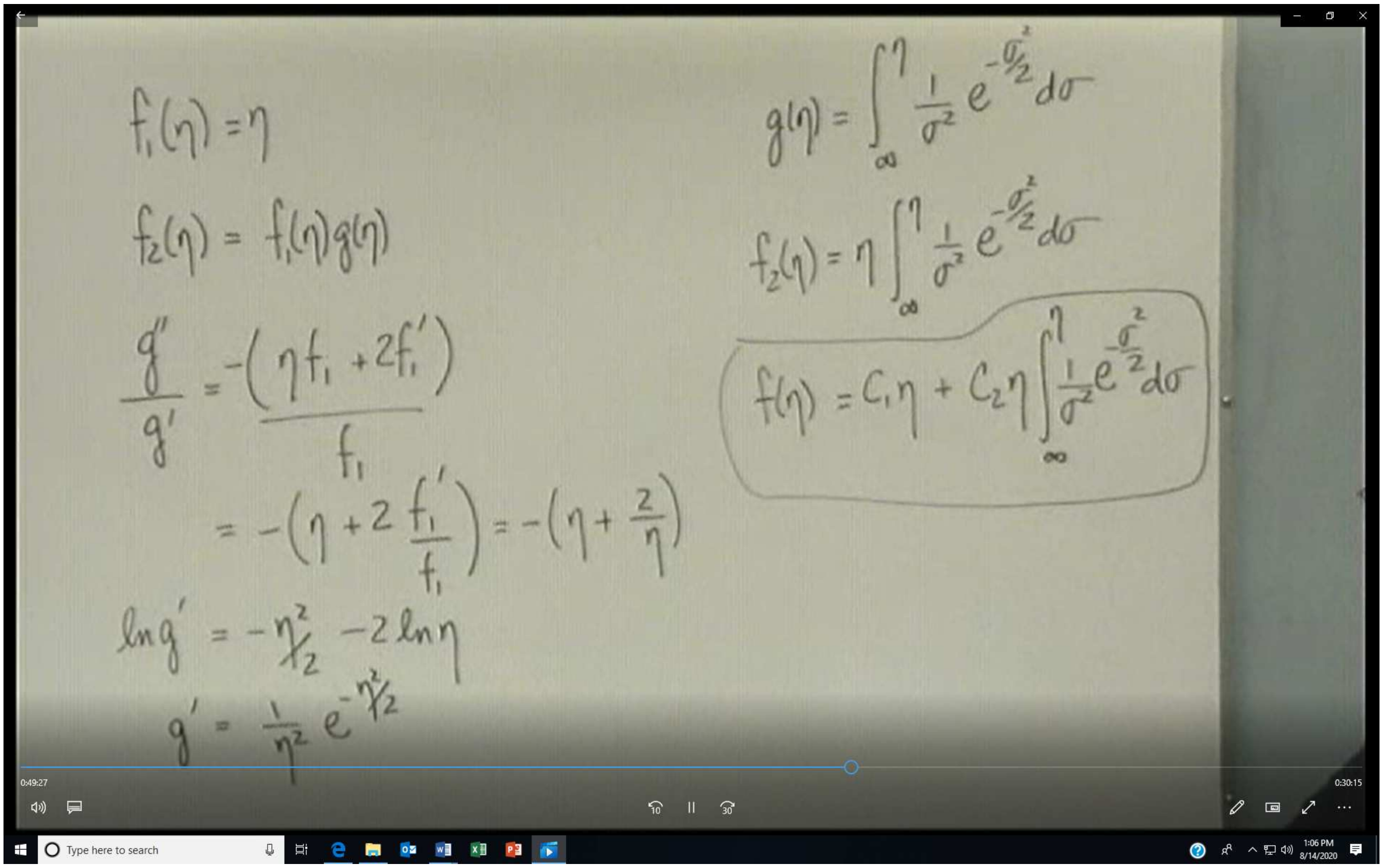
From the BC at y=0 we get the screen below. Now choose f(0) in the simplest form so not to complicate matters:



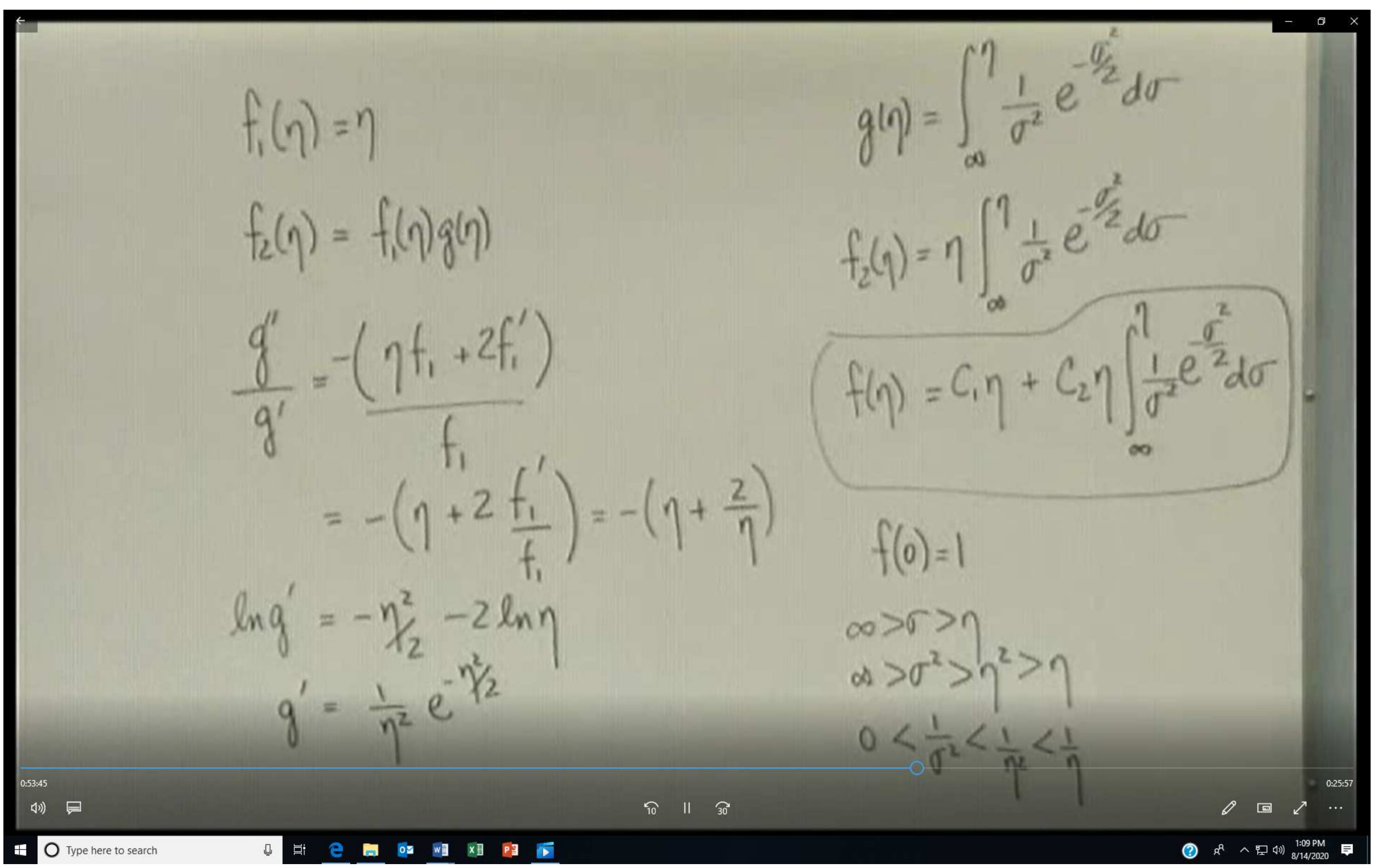
Here we have chosen the value of B to simplify the differential equation. In this particular case we look at b=1/2. We can see that f(η)=η is a solution of the ODE. We can build the second solution using methods learned at start of semester, namely variation of parameters where f2=f1*g, and g is an unknown function of η



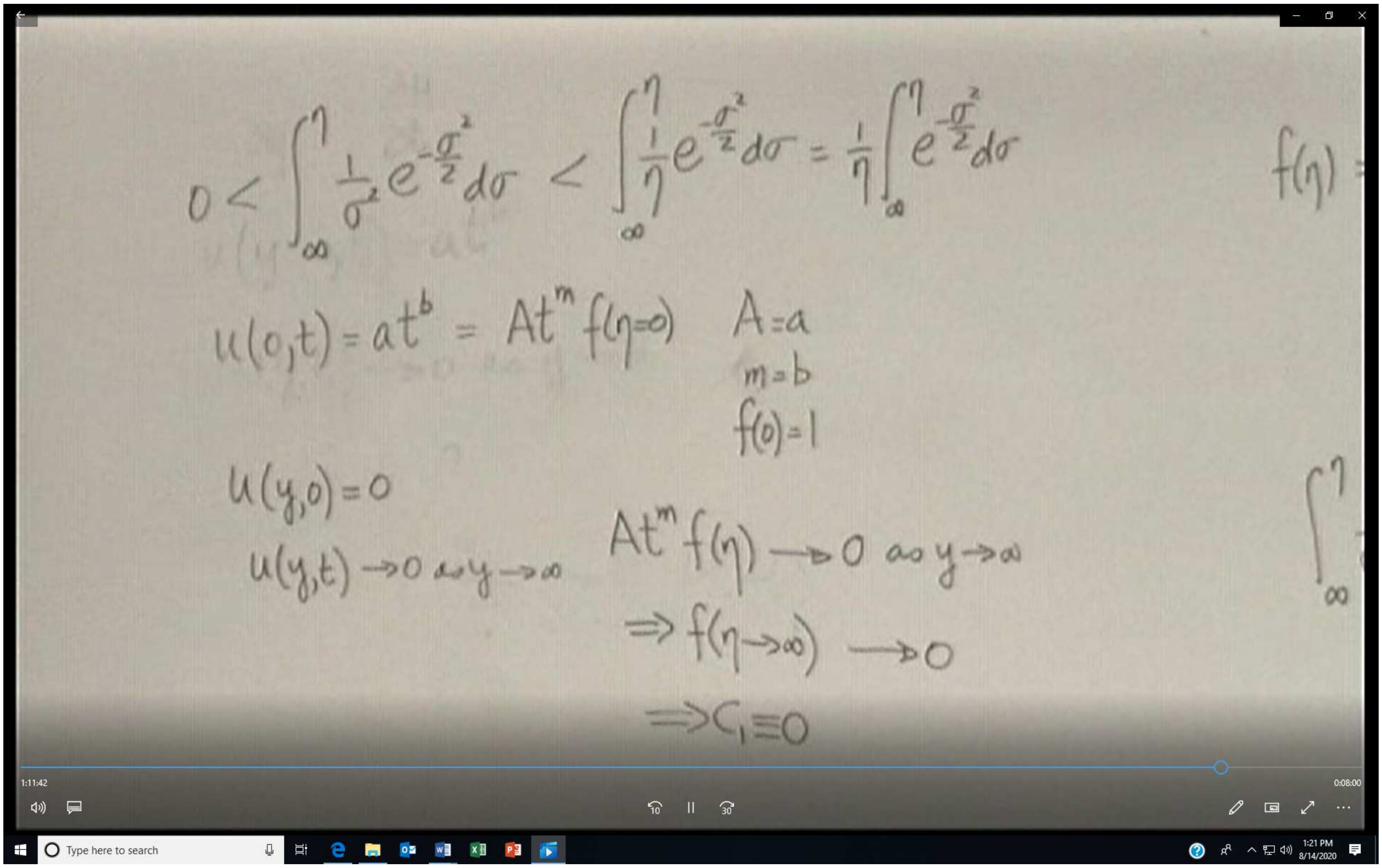
We can get the diff eqn that g satisfies and solve for g. Next build f2=f1 *g. Lastly build f =c1*f1+c2*f2



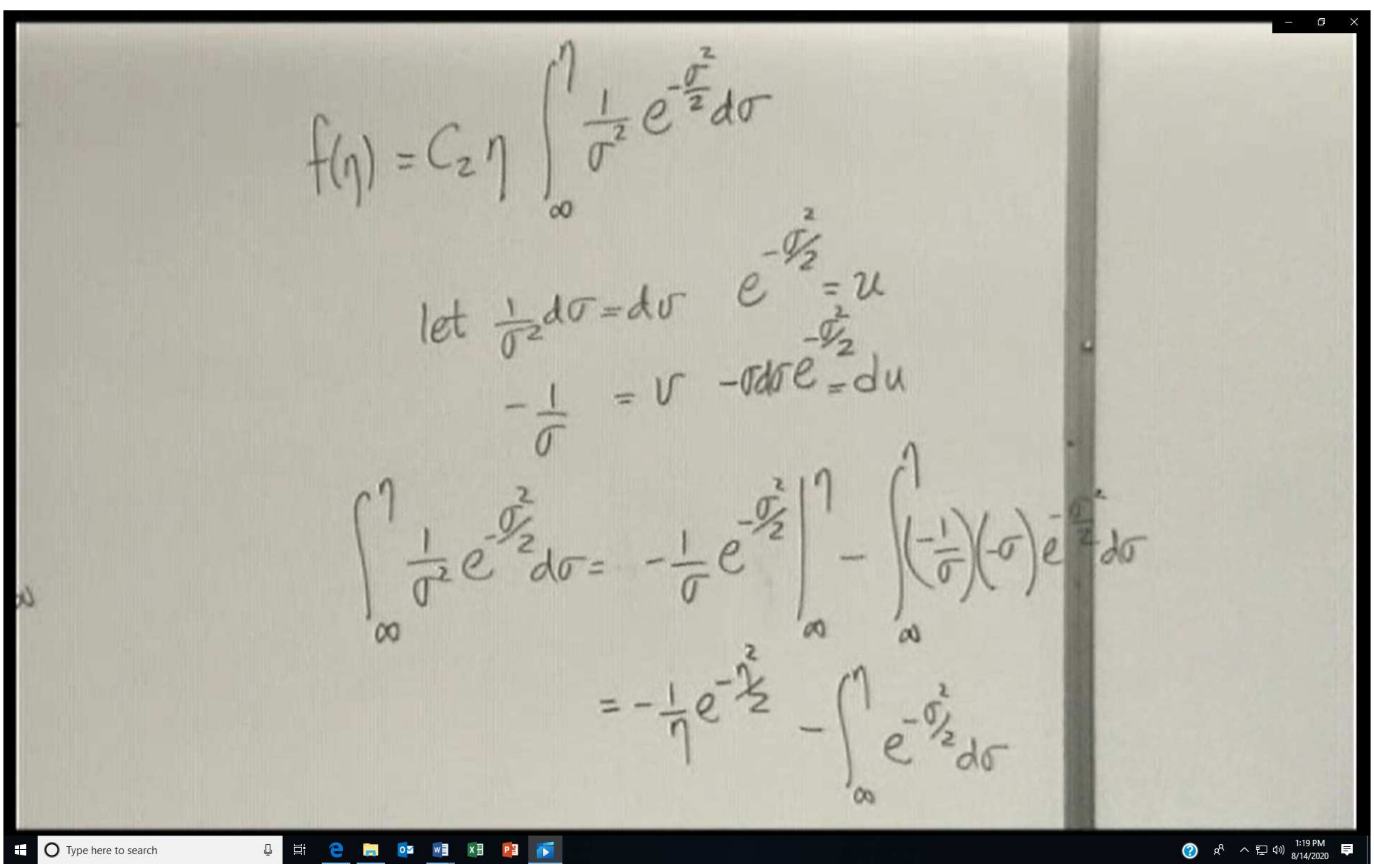
Now applying information we know,

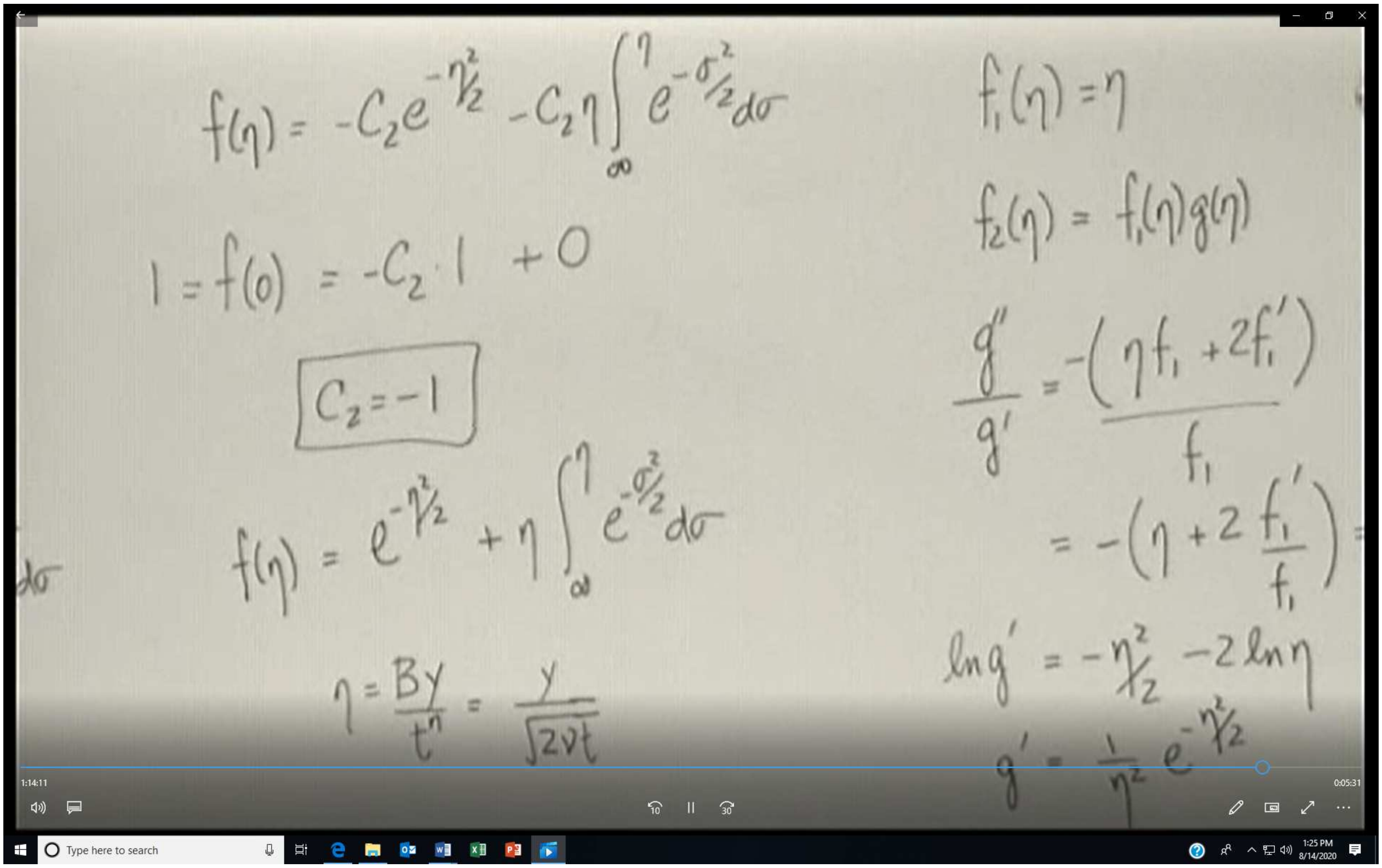


From our IC and the far field BC implies f→0. The first term C1* η will blow up; so for bounded solutions C1=0

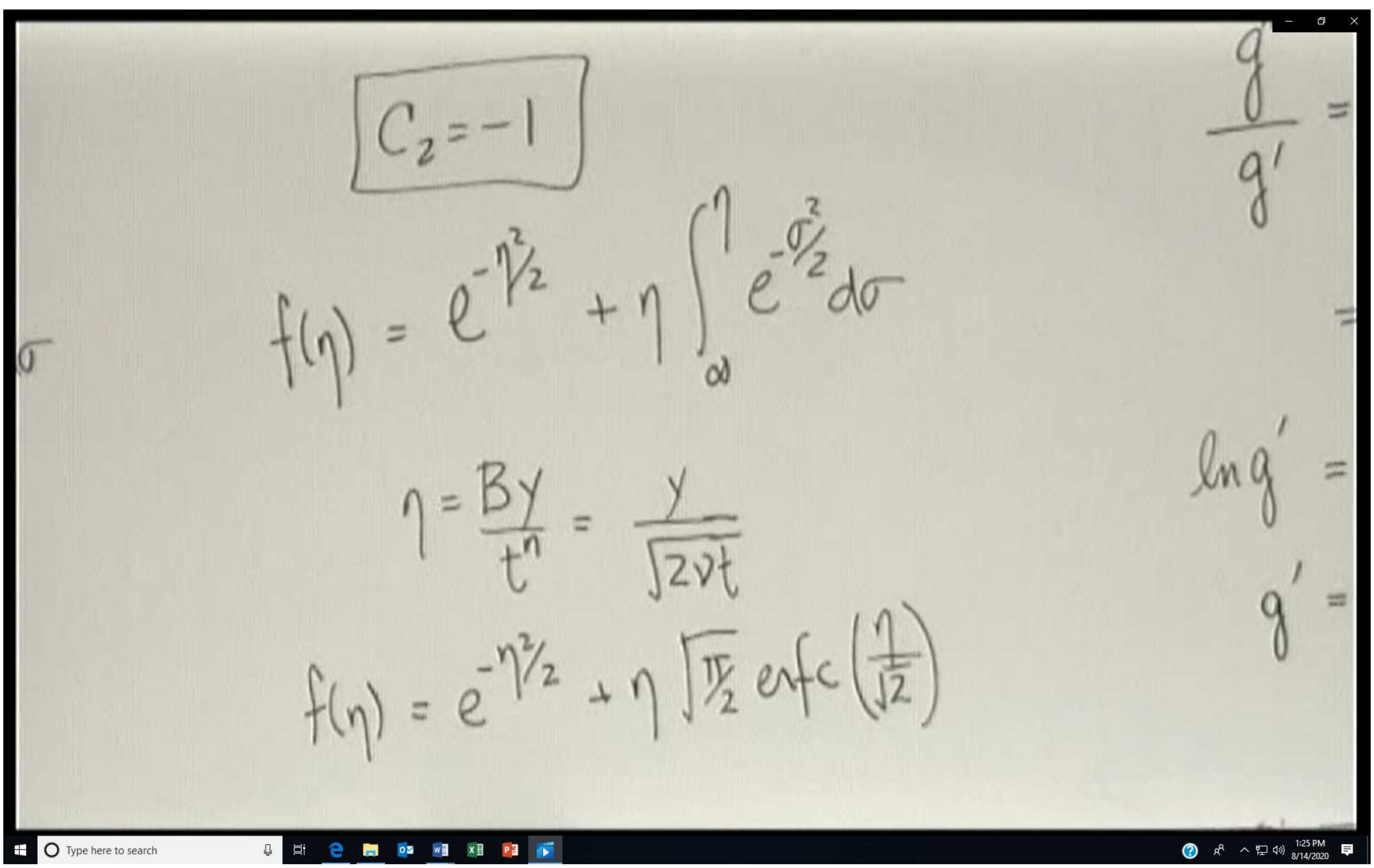


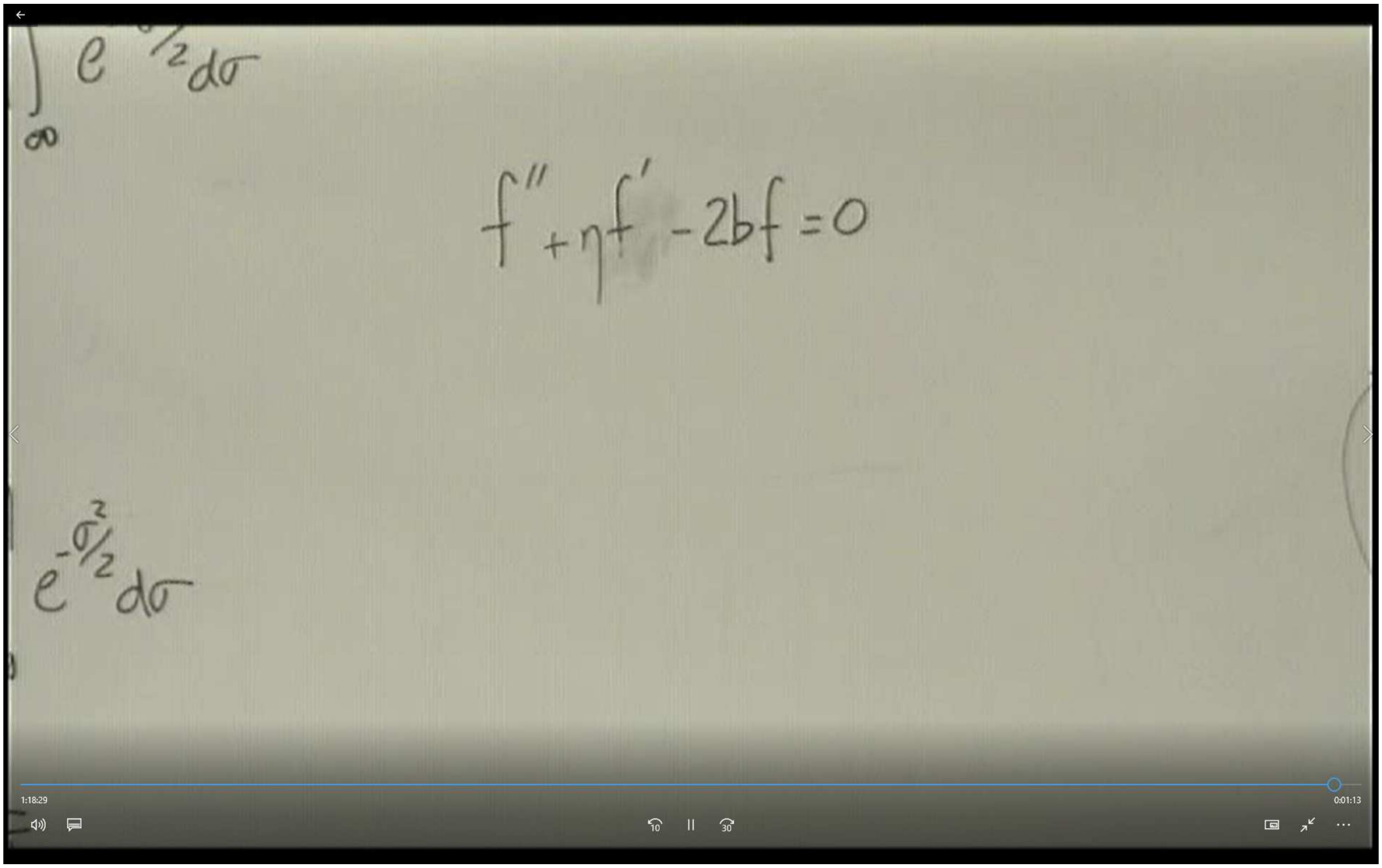
So the solution leads to





Since we know η in terms of B and n, we define η. Using the definition of the erfc (·) we finally get





We started with this equation and then looked at the specific case where b=1/2. If b is not ½, then you would have to find solutions, possibly using series solutions in the form $f(\eta) = \Sigma A_i \eta^i$ in order to find the two solution functions $f(\eta)$, that satisfy the condition that $f(0)=1$ and $f(\eta \rightarrow \infty) \rightarrow \infty$ and those solutions must reduce to what we found when b=1/2.