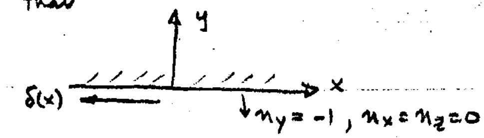
2. again let $\phi(x,y) = \int_{-\infty}^{\infty} e^{-i\lambda x}\{A+By\}e^{-|\lambda|y}\,d\lambda$

we note that



$\therefore\ T_y = \sigma_{yj}n_j = -\sigma_{yy} = 0 \qquad T_x = -\delta(x) = n_j\sigma_{xj} = -\sigma_{xy}$ thus σ_{xy} must be to the left.

Now $\frac{\partial\phi}{\partial x} = \int_{-\infty}^{\infty}(-i\lambda)e^{-i\lambda x}\{A+By\}e^{-|\lambda|y}d\lambda$ and $\frac{\partial^2\phi}{\partial x^2} = \sigma_{yy} = \int_{-\infty}^{\infty} -\lambda^2 e^{-i\lambda x}\{A+By\}e^{-|\lambda|y}d\lambda$

since $\sigma_{yy}\big|_{y=0} = 0 \Rightarrow 0 = \int_{-\infty}^{\infty} -\lambda^2 e^{-i\lambda x}A\,d\lambda$. It can be shown that $\int_{-\infty}^{\infty}\lambda^2 e^{-i\lambda x}d\lambda \neq 0 \therefore \underline{A\equiv 0}$

Now $-\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x}\right) = +\sigma_{xy} = i\int_{-\infty}^{\infty}\lambda e^{-i\lambda x}Be^{-|\lambda|y}\{1-|\lambda|y\}d\lambda$

But since $\sigma_{xy} = \delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\lambda x}d\lambda = i\int_{-\infty}^{\infty}\lambda e^{-i\lambda x}B\,d\lambda \Rightarrow Bi\lambda = \frac{1}{2\pi}$ or $\underline{B = \frac{1}{2\pi i\lambda}}$

hence $\phi(x,y) = \int_{-\infty}^{\infty} e^{-i\lambda x}\frac{y}{2\pi i\lambda}e^{-|\lambda|y}d\lambda$; using all this we have

$\sigma_{yy} = \frac{\partial^2\phi}{\partial x^2} = \int_{-\infty}^{\infty} \frac{-\lambda y}{2\pi i}e^{-i\lambda x}e^{-|\lambda|y}d\lambda$; $\sigma_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\lambda x}e^{-|\lambda|y}\{1-|\lambda|y\}d\lambda$

Now since $\frac{\partial\phi}{\partial y} = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{e^{-i\lambda x}}{\lambda}[1-|\lambda|y]e^{-|\lambda|y}d\lambda$ thus $\frac{\partial^2\phi}{\partial y^2} = \frac{1}{2\pi i}\int_{-\infty}^{\infty} -\frac{|\lambda|}{\lambda}(2-|\lambda|y)e^{-i\lambda x-|\lambda|y}d\lambda$

using the even/odd argument we then obtain:

a. $\sigma_{yy} = \frac{1}{2\pi i}\int_{-\infty}^{\infty}-\lambda(-i\sin\lambda x)\,y\,e^{-|\lambda|y}d\lambda = \frac{y}{\pi}\int_0^\infty \lambda\sin\lambda x\, e^{-\lambda y}d\lambda = \frac{y}{\pi}\frac{-\partial}{\partial x}\left(\int_0^\infty\cos\lambda x\, e^{-\lambda y}d\lambda\right)$

$= \frac{-y}{\pi}\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) = \frac{-y}{\pi}\left[\frac{-2yx}{(x^2+y^2)^2}\right] = \underline{\frac{2}{\pi}\frac{y^2x}{(x^2+y^2)^2} = \sigma_{yy}}$

b. $\sigma_{xy} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\cos\lambda x\, e^{-|\lambda|y}\{1-|\lambda|y\}d\lambda = \frac{1}{\pi}\int_0^\infty\cos\lambda x\, e^{-\lambda y}[1-\lambda y]d\lambda = \frac{1}{\pi}\left[\frac{y}{x^2+y^2}\right]+\frac{y}{\pi}\frac{\partial}{\partial y}\left(\int_0^\infty\cos\lambda x\, e^{-\lambda y}d\lambda\right)$

$= \frac{1}{\pi}\left(\frac{y}{x^2+y^2}\right)+\frac{y}{\pi}\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right) = \underline{\frac{2}{\pi}\frac{x^2y}{(x^2+y^2)^2} = \sigma_{xy}}$

c. $\sigma_{xx} = \frac{1}{2\pi i}\int_{-\infty}^{\infty} -\frac{i\sin\lambda x}{\lambda}\left\{-|\lambda|e^{-|\lambda|y}[2-|\lambda|y]\right\}d\lambda = \frac{1}{\pi}\int_0^\infty \sin\lambda x\, e^{-\lambda y}(2-\lambda y)d\lambda$

$= \frac{2}{\pi}\left[\frac{x}{x^2+y^2}\right] + \frac{y}{\pi}\frac{\partial}{\partial x}\left[\int_0^\infty\cos\lambda x\, e^{-\lambda y}d\lambda\right] = \frac{2}{\pi}\left[\frac{x}{x^2+y^2}\right]+\frac{y}{\pi}\frac{\partial}{\partial x}\left[\frac{y}{x^2+y^2}\right] = \frac{2}{\pi}\frac{x^3}{(x^2+y^2)^2}$

or $\underline{\sigma_{xx} = \frac{2}{\pi}\frac{x^3}{(x^2+y^2)^2}}$

d. To obtain ϕ :

$\frac{\partial^2\phi}{\partial x^2} = \sigma_{yy} = \frac{2}{\pi}\frac{y^2x}{(x^2+y^2)^2} \Rightarrow \frac{\partial\phi}{\partial x} = -\frac{y^2}{\pi}\frac{1}{x^2+y^2}+\hat f_1(y) \Rightarrow \phi = \frac{-y}{\pi}\arctan\frac{x}{y}+x\hat f_1(y)+\hat f_2(y)$

$= y/\pi \arctan y/x + x\hat f_1(y)+\hat f_2(y)$

$$\frac{\partial^2\phi}{\partial x\,\partial y} = -\sigma_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x}\right) = -\frac{2yx^2}{\pi(x^2+y^2)^2} + \hat{f}_1' \Rightarrow \hat{f}_1'(y)=0 \text{ or } \hat{f}_1'(y) = c_1$$

$$\frac{\partial\phi}{\partial y} = \frac{1}{\pi}\arctan y/x + \frac{y}{\pi}\frac{x}{x^2+y^2} + \hat{f}_2' \;;\; \frac{\partial^2\phi}{\partial y^2} = \frac{2x^3}{\pi(x^2+y^2)^2} + \hat{f}_2''(y) = \sigma_{yy} \Rightarrow \hat{f}_2''(y)=0 \text{ or } \hat{f}_2 = c_2 y + c_3$$

$$\therefore \phi(x,y) = \frac{y}{\pi}\arctan y/x + c_1 x + c_2 y + c_3$$ same argument on c_1, c_2, c_3 as problem 1.

3a For principal stresses in a plane strain problem $\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$: Define λ_i to be the principal stresses $\therefore \sigma\cdot n = \lambda n$ or $\det\begin{pmatrix}\sigma_{xx}-\lambda & \sigma_{xy} & 0\\ \sigma_{xy} & \sigma_{yy}-\lambda & 0\\ 0 & 0 & \sigma_{zz}-\lambda\end{pmatrix} = 0$ $\therefore$ $\lambda_3 = \sigma_{zz}$ and

$$\lambda_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$ after the plug in and the algebra

$$\therefore \nu(\sigma_{xx}+\sigma_{yy}) = \sigma_{zz} = \frac{-2\nu y}{(x^2+y^2)} = \lambda_3 \qquad \lambda_1 = 0 \qquad \lambda_2 = \frac{-2y}{\pi(x^2+y^2)}$$

Since $y>0$ and we assume $0<\nu<1$ the stresses are ordered

$(\lambda_1, \lambda_3, \lambda_2)$ in decreasing tension (from left to right)

3b. again we obtain for plane strain $\lambda_3 = \sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$ and

$$\lambda_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

$$\therefore \nu(\sigma_{xx}+\sigma_{yy}) = \sigma_{zz} = \lambda_3 = \frac{2\nu x}{\pi(x^2+y^2)} \qquad \lambda_1 = 0 \;,\; \lambda_2 = \frac{2x}{\pi(x^2+y^2)}$$

for $x>0$ and assuming $0<\nu<1$ the stresses are ordered $(\lambda_2, \lambda_3, \lambda_1)$ in decreasing tension (from left to right). for $x<0$ the stresses are ordered $(\lambda_1, \lambda_3, \lambda_2)$ in decreasing tension (from left to right)