\$ (1) show for Et = Up St = Up or = ajeiej=ZZajeiej Einskensen Notahon - Stay = Oxy when subscript appears turce it means 2 plane strain: no z dependence V4 \$=0 Dr + Dr = 0 stress equil equil 0:06:27 ļ り 

Look at problems with no z dependence

d(x,y) stress t 1=1=1 T = Tyleiej=ZZTijeiej 2 = Jy1 Einsteinian Notation - 20 = Oxy when subscript appears twice it meens 2 plane strain: no z dependence  $t_{n} = \sigma \underline{n} = \sigma_{xy} e_{x} + \sigma_{yy} e_{y} + \sigma_{y} e_{z}$   $= \sigma_{xy} \neq \sigma_{yy}$   $= \sigma_{xy} \neq \sigma_{yy}$  $\nabla^4 \phi = 0$ 0:08:26 り ļ Ø Type here to search へ口の Q. wE

let Jyy (y=0) = f(x) = au + Zaicom x + Zbian now if periodic of period L choose  $\phi(x,y) = e^{-i\lambda x} g(y)$  complex imbedding  $e^{i\theta} = cos\theta + isin\theta$  $\nabla^{\mu} \phi = \frac{\partial^{4} \phi}{\partial x^{4}} + 2 \frac{\partial^{4} \phi}{\partial x^{2}} \frac{\partial^{4} \phi}{\partial y^{2}} = 0$ =  $\left[\lambda^{4}g - 2\lambda^{2}g'' + g''\right]e^{-i\lambda x} = 0$ O Type here to search

if periodic of period L Choose  $\phi(x,y) = e^{-i\lambda x} g(y)$   $e^{i\theta} = co\theta + i \sin \theta$  $\nabla^{\mu} \phi = \frac{\partial^{4} \phi}{\partial x^{4}} + 2 \frac{\partial^{4} \phi}{\partial x^{2}} + \frac{\partial^{4} \phi}{\partial y^{4}} = 0$ =  $\left[\lambda^{4}g - 2\lambda^{2}g'' + g^{N}\right]e^{-i\lambda x} = 0$ 1F g(y) = est 8 [X - 2x2 + 54]=0  $(\lambda^2 - S^2)^2 = 0 = S = \pm \lambda + \pm \lambda$ 

 $\oint_{\lambda}(x,y) = e^{i\lambda x} \left[ Ae^{\lambda y} + Bye^{\lambda y} + Ce^{-\lambda y} + Dye^{-\lambda y} \right]$  $\int_{-\infty}^{\infty} e^{-i\lambda y} u(y) dy = U(\lambda)$  $\frac{1}{2\pi}\int_{\infty}^{\infty}e^{i\lambda y}U(\lambda)d\lambda=u(y)$  $\phi(x,y) = e^{i\lambda x} \left[ Ce^{-\lambda y} + Dye^{-\lambda y} \right]$ 0.27:34 5)) Type here to search

 $\Phi_{\lambda}(x,y) = e^{i\lambda x} \left[ Ae^{\lambda y} + Bye^{\lambda y} + Ce^{-\lambda y} + Dye^{-\lambda y} \right]$ omplex imbedding =0 \$\phi\_x(x,y) = e^{i \lambda x} [Ce Hy + Dye Hy]  $\phi(x,y) = \int_{a}^{\infty} dx \, e^{i\lambda x} \left[ C e^{i\lambda y} + Dy e^{i\lambda y} \right]$ S==+>,=> Type here to se

(xy = g(x) my=0  $\frac{\partial \phi}{\partial x} = J_{\gamma\gamma}(\gamma = 0) = f(x)$  $\sigma_{xy} = -\frac{\delta_{xy}}{\delta_{xy}} = \int_{-i\lambda_{yy}}^{\infty} dx$  $=\frac{2}{\partial x^{2}}\int_{-\infty}^{\infty}d\lambda e^{-i\lambda x}\left[Ce^{-i\lambda y}+Dye^{-i\lambda y}\right]$ =  $\int_{a}^{a} \frac{\delta^{2}}{\lambda x^{2}} \left\{ e^{i\lambda x} \left[ (e^{i\lambda y} + Dy e^{i\lambda y}) d\lambda \right] \right\}$ (xy (my=0) = g(x) =  $\frac{\delta \phi}{\delta x^2} = -\int x^2 e^{i\lambda y} \left[ C e^{i\lambda y} + D y e^{i\lambda y} \right] d\lambda$  $\frac{\partial \varphi}{\partial x^2}\Big|_{x=0} = \sigma_{yy} = f(x) = -\int_{a}^{x} e^{i\lambda x} C d\lambda$ 0:44:16 (() 10 II 30 O Type here to search

let Jyy(y=0) = f choose \$\phi(  $\begin{aligned}
(T_{xy} = g(x) & my = 0 \\
(T_{xy} = -\frac{3\phi}{3xy} = \int_{-i\lambda}^{\infty} -i\lambda x \left[ C(-1\lambda) e^{-i\lambda y} + \frac{1}{2} e^{-i\lambda y} - \frac{1}{2} e^{-i\lambda y} \right] d\lambda
\end{aligned}$  $\sigma_{xy}(my=0) = g(x) = \int_{-i\lambda}^{\infty} e^{i\lambda x} \left[ C(-|\lambda|) + D \right] d\lambda$  $\nabla^{\mu} \phi =$ IF g(y)

## Transform of the delta function. First we speak of properties of delta function



Normal to surface is in –y direction=-e<sub>y</sub>. So stress tensor•  $\underline{n} = -\sigma_{yy} = -f(x) = -\delta(x)$ 

XG Txy  $=\frac{2}{\partial x^2}\int_{-\infty}^{\infty}d\lambda e^{-i\lambda x}\left[Ce^{-i\lambda y}+Dye^{-i\lambda y}\right]$ =  $\int_{a}^{a} \frac{\lambda^{2}}{\lambda^{2}} \left\{ e^{i\lambda x} \left[ \left( e^{i\lambda y} + Dy e^{i\lambda y} \right) d\lambda \right] \right\}$  $\frac{\delta \Phi}{\delta x^2} = -\int_{x^2}^{\infty} \left[ C e^{-ix/y} + Dy e^{ix/y} \right] dx$  $\frac{\delta \frac{1}{2}}{\delta x^2}\Big|_{y=0} = \sigma_{yy} = f(x) = -\int_{-\infty}^{\infty} \int_{-\infty}^{x} \frac{e^{i\lambda x}}{c} C d\lambda$   $\sigma_{yy} = -f(x) = -\delta(x)$ 0-53-24 ļ fo ⊳ 30° (1)) O Type here to search

 $\delta(x) = \int_{x}^{\infty} \lambda e^{-i\lambda x} C d\lambda$ + Dye hly => F(x2C) = 8(x)  $\begin{aligned}
 & \mathcal{T}_{xy} = \mathcal{O}_{yy} \\
 & \delta(x) = -f(x) = \mathcal{T}_{yy}
 \end{aligned}$  $\lambda C = \frac{1}{2\pi} \int_{0}^{\infty} \delta(x) e^{i\lambda x} dx = \frac{1}{2\pi} \int_{0}^{\infty} \delta(x) e^{i\lambda x} dx$ +Dye My dh  $C(\lambda) = \frac{1}{2\pi \lambda^2}$  $\sigma_{xy} = \int -i\lambda e^{i\lambda x} \left\{ D - |\lambda| C \right\} d\lambda$ Cd IF 0xy=0 => D-1x/C =0 D=1x1 O Type here to search U 🗄 🧁 🥽 🔯 🖬 🌈 ヘ に (小) ENG 8:07 PM 単

With the definitions of C and D which are functions of  $\lambda$ , we can get expressions for all the stresses. Also  $\lambda^2/|\lambda| = |\lambda|$ 

 $\sigma_{yy} = \int_{\lambda}^{\infty} \frac{1}{2\pi\lambda} \left[ \frac{1}{2\pi\lambda} e^{-\lambda y} + \frac{1}{2\pi\lambda} y e^{-\lambda y} \right] d\lambda$  $G_{\gamma\gamma} = \frac{1}{2\pi} \int_{0}^{\infty} e^{i\lambda x} e^{i\lambda y} \left[ 1 + y |\lambda| \right] d\lambda$ Type here to search 

Since the integral of an even function produces an odd function. Evaluation of an odd function between +c and –c gives twice the value of the function between 0 and +c. The expression involving the cosine should have a "2" in the numerator.

Now look at the first of the cosine integrals and use the laplace transform definition of the cosine function.



Now for the second cosine integral and realize that since we are integrating with respect to  $\lambda$ , y acts like a constant

 $\begin{cases} f(x) = -\delta(x) \\ g(x) = 0 \end{cases} = \begin{cases} C = \frac{1}{2\pi\lambda^{2}} \end{cases}$  $\sigma_{TT} = \frac{1}{\pi} \left[ \frac{y}{x^2 + y^2} - y \frac{2}{2y} \left( \frac{y}{x^2 + y^2} \right) \right]$  $D = \frac{1}{2\pi h}$  $G_{\gamma\gamma} = \int_{-\infty}^{\infty} \chi e^{i\lambda x} \left[ \frac{1}{2\pi \lambda^{2}} e^{-i\lambda ly} + \frac{1}{2\pi |\lambda|} y e^{-i\lambda ly} \right] d\lambda$  $\sigma_{\gamma\gamma} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{e^{i\lambda x}}{e^{i\lambda y}} \left[ \frac{e^{i\lambda y}}{1 + y|\lambda|} d\lambda = \frac{1}{2\pi} \int_{0}^{\infty} \frac{e^{i\lambda y}}{e^{i\lambda y}} \frac{(1 + \lambda y)d\lambda}{(1 + \lambda y)d\lambda}$  $\int_{-\infty}^{\infty} dx dx = \int_{-\infty}^{\infty} dx = \int_{-\infty}^{\infty$ y (" NONXE" JAN = - 4 2 (7) 1:20:38 5)) ļ <u>โด II ว</u>ี Type here to search 🗄 🤰 🔚 🔯 📓

The y derivative of the first cosine integral will give you the methodology to get the second cosine integral.  $\sigma_{yy}$  as shown above is correct.

- 1. As an exercise find the remaining two stresses  $\sigma_{xx}$  and  $\sigma_{xy}\,$  for this problem.
- 2. and then try the case where on the boundary y=0  $\sigma_{yy} = f(x) = 0$  and the delta function  $\delta(x)$  is pointing to the right as mentioned in the video.

Use the same ideas we developed in class with the traction  $\underline{t_n}$  to determine  $\sigma_{xy}$  in terms of  $\delta(x).$ 

3. Now determine the complete expression for  $\sigma_{xy}$  for this new problem in the same way as we did for  $\sigma_{yy}$  during the class.