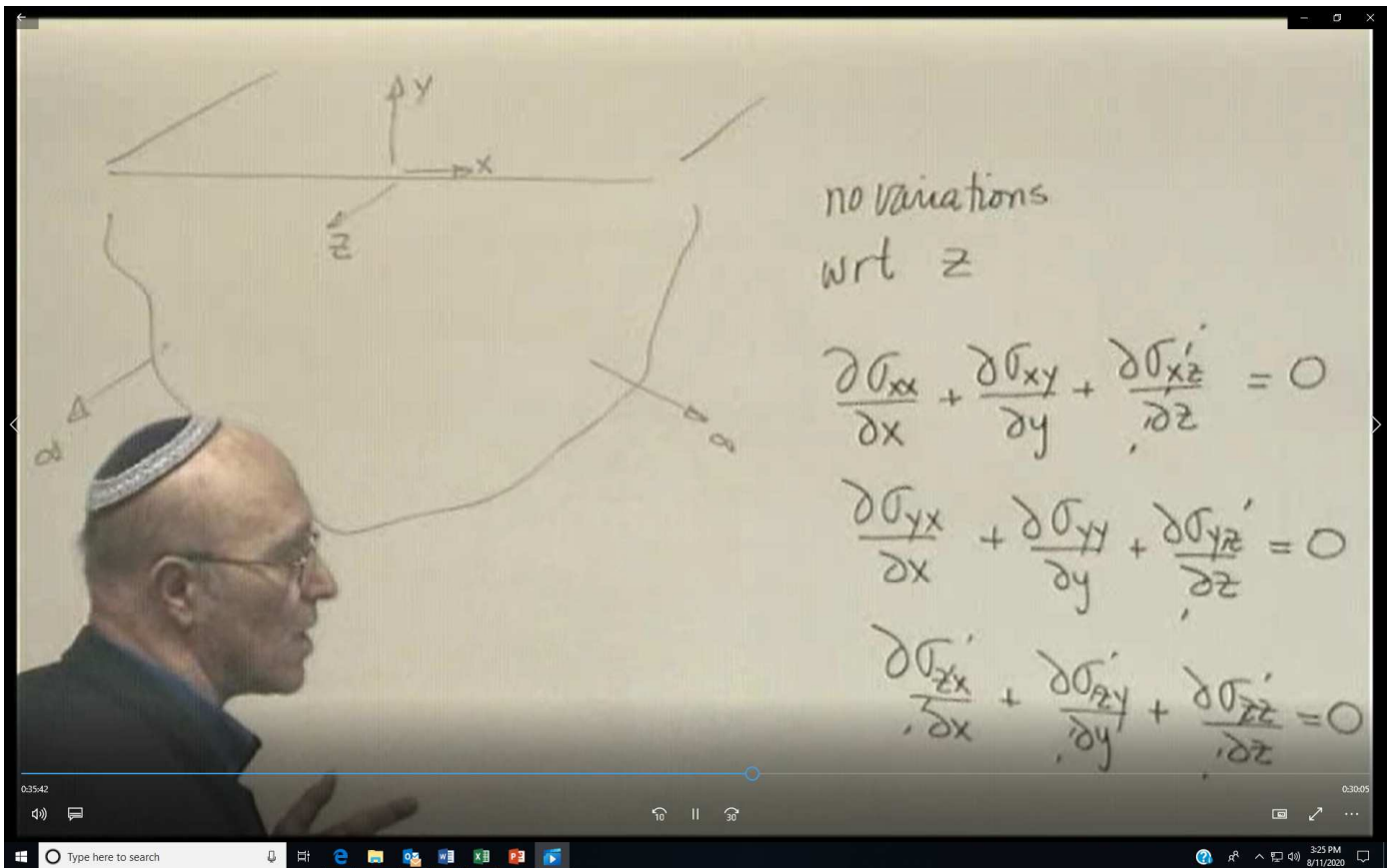


Preliminaries to using Fourier transform. Understanding definitions of stresses



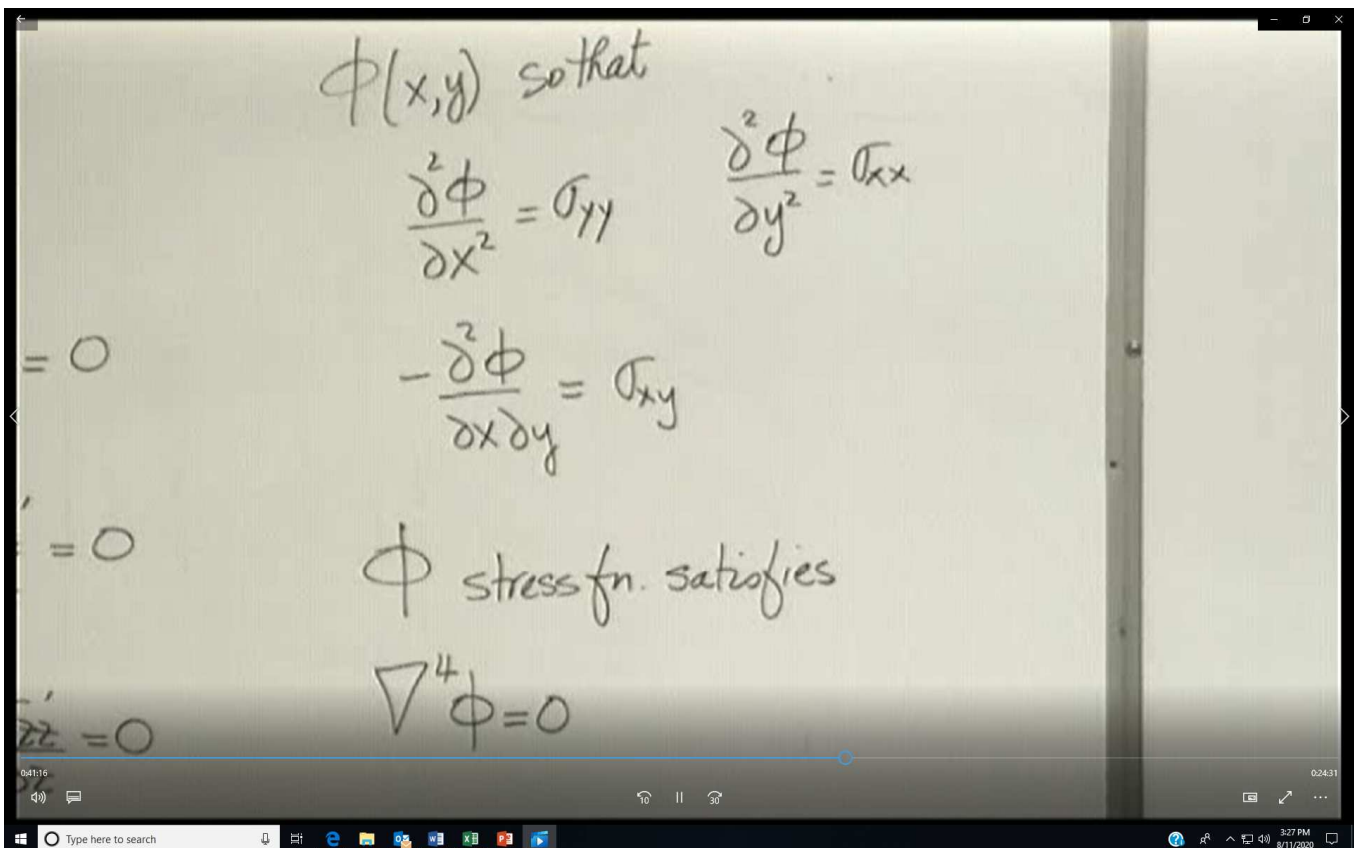
no variations  
wrt  $z$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

The equation that the stresses satisfy if we define a stress function is the biharmonic equation  $\nabla^4 \phi = 0$



$\phi(x,y)$  so that

$$\frac{\partial^2 \phi}{\partial x^2} = \sigma_{yy} \quad \frac{\partial^2 \phi}{\partial y^2} = \sigma_{xx}$$

$$-\frac{\partial^2 \phi}{\partial x \partial y} = \sigma_{xy}$$

$\phi$  stress fn. satisfies

$$\nabla^4 \phi = 0$$

Stress tensor has two unit vectors: one for direction of force component/one perpendicular to surface

no variations wrt  $z$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

$$\sigma = \sigma_{xx}e_xe_x + \sigma_{xy}e_xe_y + \sigma_{xz}e_xe_z + \dots$$

If we speak of a traction vector on a surface,  $\underline{t}_y$ , take dot product of stress tensor with normal to that surface.

$$\underline{t}_y = \sigma \cdot \underline{e}_y = \sigma_{xy}\underline{e}_x + \sigma_{yy}\underline{e}_y + \sigma_{zy}\underline{e}_z$$

$$\underline{e}_y \cdot \underline{t}_y = \sigma_{yy}$$

$$\underline{e}_y \cdot \sigma \cdot \underline{e}_y = \sigma_{yy}$$

On a surface there are 3 stresses. For problems in plane stress, where no variations with  $z$ ,  $\sigma_{yz}=0$ . To find a particular stress, e.g.,  $\sigma_{yz}$ , take the dot product of the stress tensor with the unit vectors  $e_y$  and  $e_z$

Handwritten notes on a surface with stresses and equilibrium equations. The top part shows a diagram of a surface with a normal vector  $\underline{n} = e_y$  and a position vector  $\underline{r}$ . The stress components are given as  $\sigma_{xy}(x, y=0) = g(x)$  and  $\sigma_{yy}(x, y=0) = f(x)$ . Below this, two diagrams show stress distributions  $\sigma_{yy}$  and  $\sigma_{xy}$  on a surface. The stress tensor is written as:

$$\underline{\sigma} = \sigma_{xx} \underline{e}_x \underline{e}_x + \sigma_{xy} \underline{e}_x \underline{e}_y + \sigma_{xz} \underline{e}_x \underline{e}_z + \dots + \sigma_{zz} \underline{e}_z \underline{e}_z$$

To the right, the equilibrium equations are listed, with the note "no variations wrt  $z$ ":

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

Einsteinian notation says that if two elements have the same subscript, it represents a summation

Handwritten notes on stress functions and equilibrium equations. The top part shows the note "no variations wrt  $z$ ". The stress components are given as  $\sigma_{xx} = \frac{\partial^2 \phi}{\partial x^2}$ ,  $\sigma_{yy} = \frac{\partial^2 \phi}{\partial y^2}$ , and  $\sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$ . The stress tensor is written as:

$$\underline{\sigma} = \sigma_{ij} \underline{e}_i \underline{e}_j = \sum_{i=1}^3 \sum_{j=1}^3 \sigma_{ij} \underline{e}_i \underline{e}_j$$

To the right, the stress function  $\phi(x, y)$  is introduced, and the equilibrium equations are listed, with the note "no variations wrt  $z$ ":

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

The stress function  $\phi$  satisfies the biharmonic equation:

$$\nabla^4 \phi = 0$$

as  $|r| \rightarrow \infty$  all stresses must be bounded