

For problem 2 on pg 284 of Trim

$$\mathcal{F}_s\{u(x,t)\} = U(s,t) = \int_0^{\infty} u(x,t) \sin sx \, dx$$

$$\int_0^{\infty} \frac{\partial u}{\partial t} \sin sx \, dx = \frac{\partial}{\partial t} \int_0^{\infty} u(x,t) \sin sx \, dx = \frac{d}{dt} U(s,t)$$

$$\int_0^{\infty} \frac{\partial^2 u}{\partial x^2} \sin sx \, dx = -s^2 U(s,t) + s u(x=0^+, t)$$

For problem 5 on pg 284 Trim

$$\mathcal{F}_c\{u(x,t)\} = U(s,t) = \int_0^{\infty} u(x,t) \cos sx \, dx \quad ; \quad \mathcal{F}_c\{g(x,t)\} = G(s,t) = \int_0^{\infty} g(x,t) \cos sx \, dx$$

$$\int_0^{\infty} \frac{\partial u}{\partial t} \cos sx \, dx = \frac{\partial}{\partial t} \int_0^{\infty} u(x,t) \cos sx \, dx = \frac{d}{dt} U(s,t)$$

$$\int_0^{\infty} \frac{\partial^2 u}{\partial x^2} \cos sx \, dx = -s^2 U(s,t) + s \frac{\partial u}{\partial x}(x=0^+, t)$$

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + \frac{k}{\kappa} g(x,t)$$

$$\frac{d}{dt} U(s,t) = k \left[ -s^2 U(s,t) + s \frac{\partial u}{\partial x}(x=0^+, t) \right] + \frac{k}{\kappa} G(s,t)$$

$$\cos s x dx = -s U(s; t) + s \frac{\partial U}{\partial x}(x=0; t)$$

$$\frac{\partial U}{\partial t} = k \frac{\partial^2 U}{\partial x^2} + \frac{k}{\kappa} g(x, t)$$

$$\frac{d}{dt} U(s; t) = k \left[ -s^2 U(s; t) + s \frac{\partial U}{\partial x}(x=0; t) \right] + \frac{k}{\kappa} G(s; t)$$

$$\frac{d}{dt} U + ks^2 U = ks \cdot \left( -\frac{1}{\kappa} f_1(t) \right) + \frac{k}{\kappa} G(s; t)$$

Solving using the integrating factor

$$\mu(t) = e^{ks^2 t}$$

$$U(s; t) = \frac{1}{\mu(t)} \int_0^t \mu(\bar{t}) \left[ -\frac{ks}{\kappa} f_1(\bar{t}) + \frac{k}{\kappa} G(s; \bar{t}) \right] d\bar{t}$$

$$+ \frac{1}{\mu(t)} \cdot C$$

$$U(s,t) = \frac{1}{\mu(t)} \left[ \mu(t) \left[ \frac{1}{k} \frac{\partial U}{\partial t} + \frac{1}{k} \frac{\partial U}{\partial x} \right] + \frac{1}{\mu(t)} \cdot C \right]$$

$$+ \frac{1}{\mu(t)} \cdot C$$

IC:  $u(x,t=0) = f(x) \Rightarrow U(s,t=0) = F(s)$

where  $F(s) = \int_0^\infty f(x) \cos sx dx$

$$U(s,t=0) = F(s) = e^{-ks^2 t} \int_0^\infty e^{ks^2 \bar{t}} \left[ -\frac{ks}{k} f(\bar{t}) + \frac{k}{k} G(s,\bar{t}) \right] d\bar{t} + C e^{-ks^2 t}$$

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$$C = F(s) - \int_0^\infty e^{ks^2 \bar{t}} \left[ \right] d\bar{t}$$

$$U(s,t) = F(s) e^{-ks^2 t} + \int_0^t \left[ \right] - \int_0^0 \left[ \right] + \int_0^t \left[ \right] + \int_0^0 \left[ \right]$$

$$U(s,t) = F(s) e^{-ks^2 t} + \frac{1}{\mu(t)} \int_0^t e^{ks^2 \bar{t}} \left[ -\frac{ks}{k} f(\bar{t}) + \frac{k}{k} G(s,\bar{t}) \right] d\bar{t}$$

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