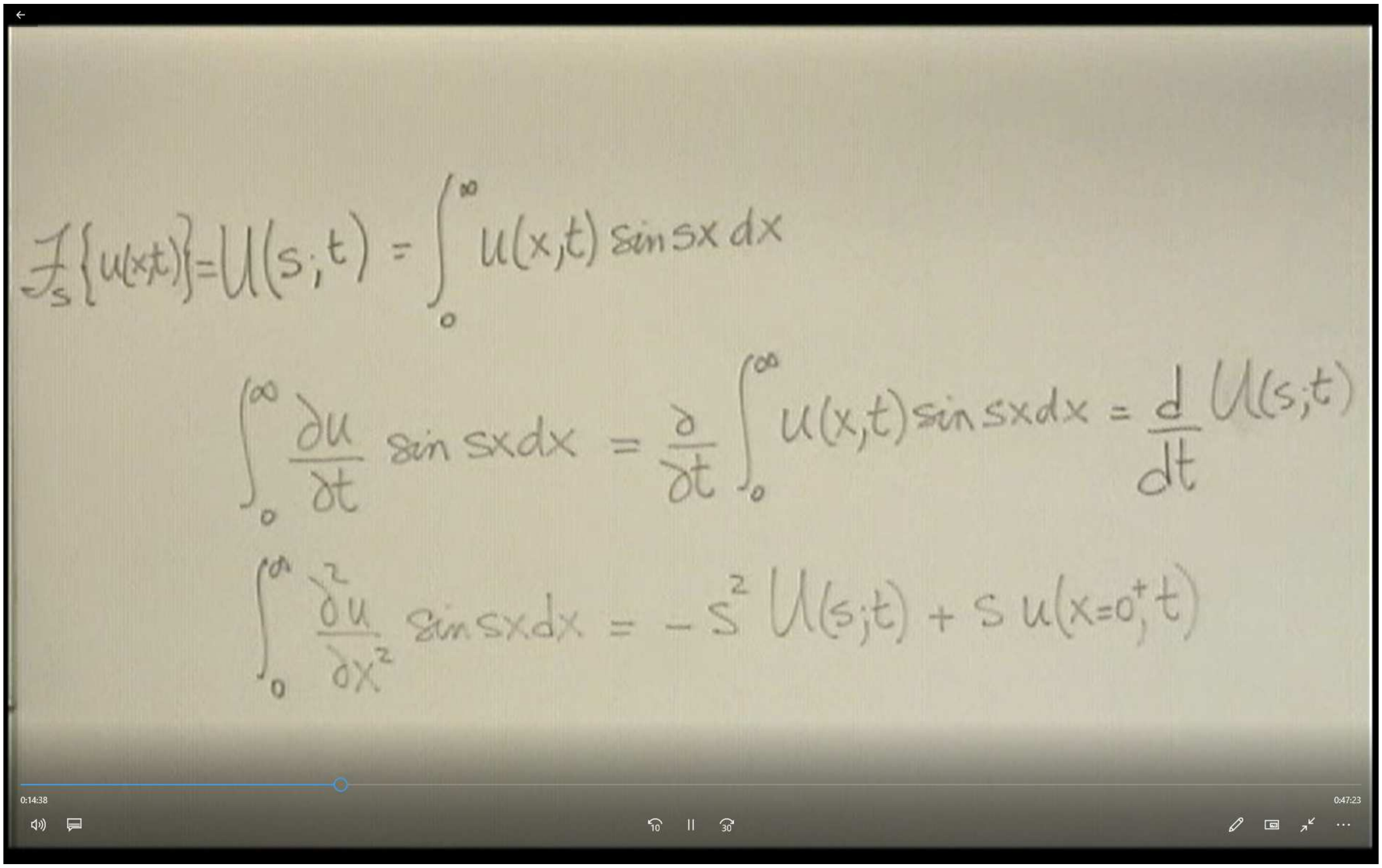
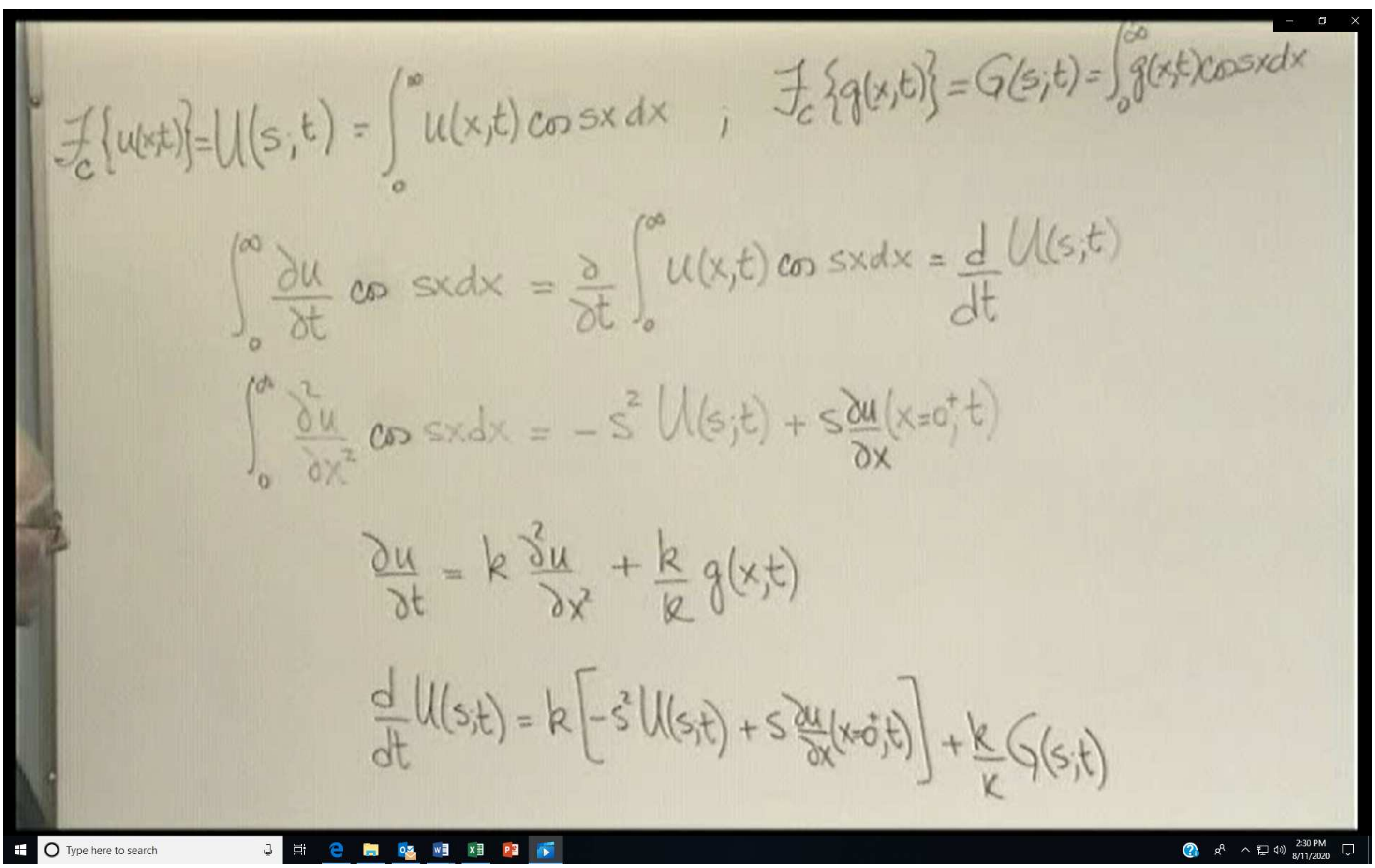
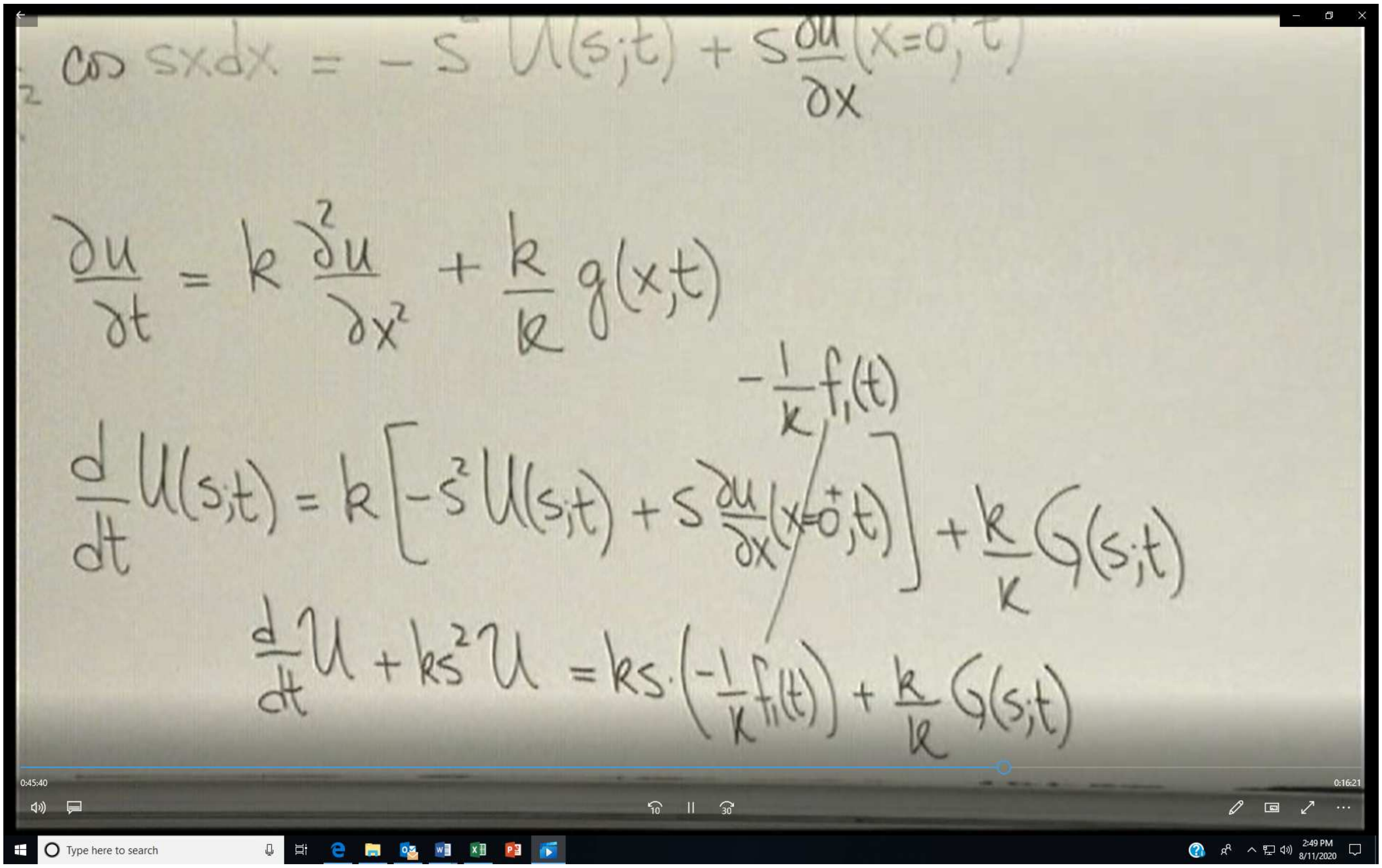
For problem 2 on pg 284 of Trim



For problem 5 on pg 284 Trim





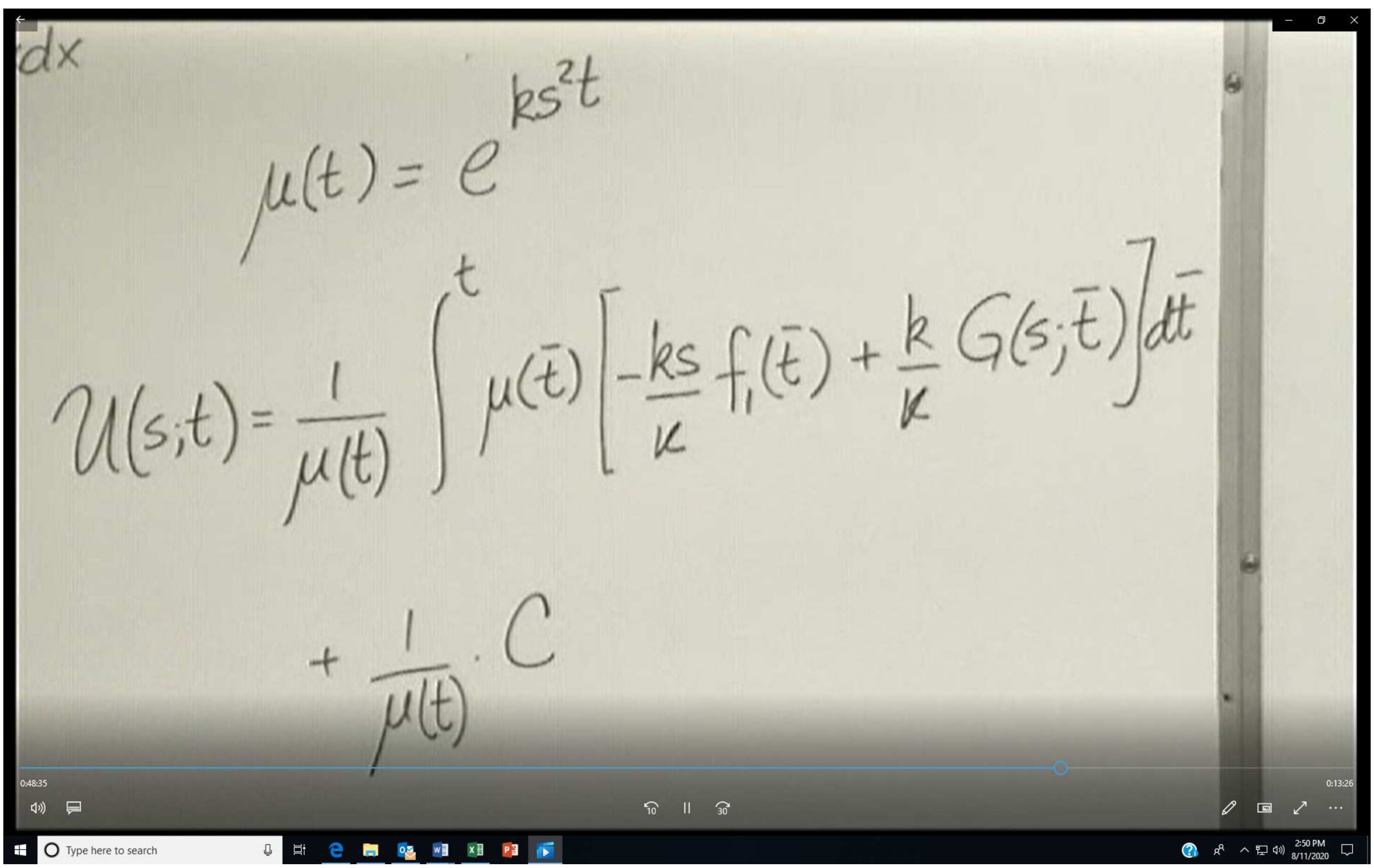
$$\cos sx\,dx = -s^2\, U(s;t) + s\frac{\partial u}{\partial x}(x=0,t)$$

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \frac{k}{K} g(x;t)$$

$$\frac{d}{dt}U(s;t) = k\left[-s^2 U(s;t) + s\frac{\partial u}{\partial x}(x=0,t)\right] + \frac{k}{K}G(s;t) \qquad \frac{\partial u}{\partial x}(x=0,t) = -\frac{1}{K}f_1(t)$$

$$\frac{d}{dt}U + ks^2 U = ks\cdot\left(-\frac{1}{K}f_1(t)\right) + \frac{k}{K}G(s;t)$$

Solving using the integrating factor



$$\mu(t) = e^{ks^2 t}$$

$$U(s;t) = \frac{1}{\mu(t)}\int^t \mu(\bar{t})\left[-\frac{ks}{K}f_1(\bar{t}) + \frac{k}{K}G(s;\bar{t})\right]d\bar{t} + \frac{1}{\mu(t)}\cdot C$$

$$\mathcal{U}(s,t) = \frac{1}{\mu(t)} \int \mu(\ldots)\left[\frac{\ldots}{\kappa} \ldots \frac{\ldots}{k} \ldots \right]$$

$$+ \frac{1}{\mu(t)} \cdot C$$

IC: $u(x,t=0) = f(x) \Longrightarrow \mathcal{U}(s,t=0) = F(s)$

where $F(s) = \int_0^\infty f(x) \cos sx \, dx$

$$\mathcal{U}(s,t=0) = F(s) = e^{-ks^2t} \int^0 e^{ks^2\bar{t}} \left[-\frac{ks}{\kappa} f_1(\bar{t}) + \frac{k}{\kappa} G(s,\bar{t}) \right] d\bar{t} + C e^{-ks^2t}$$

$$C = F(s) - \int^0 e^{+ks^2\bar{t}} \left[\qquad \right] d\bar{t}$$

$$\mathcal{U}(s,t) = F(s) e^{-ks^2t} + \cdot \int^t [\quad] - \cdot \int^0 [\quad]$$

$$\cdot \int^t [\quad] + \cdot \int_0 [\quad]$$

$$\mathcal{U}(s,t) = F(s) e^{-ks^2t} + \frac{1}{\mu(t)} \int_0^t e^{ks^2\bar{t}} \left[-\frac{ks}{\kappa} f_1(\bar{t}) + \frac{k}{\kappa} G(s,\bar{t}) \right] d\bar{t}$$