Appendix C

Special Fourier Transforms

$\mathcal{F}\{f\} = \int_{-\infty}^{\infty} f(x)e^{-i\alpha x}\,dx$

SPECIAL FOURIER TRANSFORM PAIRS

	$f(x)$	$F(\alpha)$
C-1	$\begin{cases} 1 & \lvert x\rvert < b \\ 0 & \lvert x\rvert > b \end{cases}$	$\dfrac{2\sin b\alpha}{\alpha}$
C-2	$\dfrac{1}{x^2+b^2}$	$\dfrac{\pi e^{-b\alpha}}{b}$
C-3	$\dfrac{x}{x^2+b^2}$	$-\dfrac{\pi i\alpha}{b}e^{-b\alpha}$
C-4	$f^{(n)}(x)$	$i^n\alpha^n F(\alpha)$
C-5	$x^n f(x)$	$i^n \dfrac{d^nF}{d\alpha^n}$
C-6	$f(bx)e^{itx}$	$\dfrac{1}{b}F\left(\dfrac{\alpha-t}{b}\right)$

$F_S(\alpha) = \int_0^\infty f(x) \sin \alpha x \, dx$

SPECIAL FOURIER SINE TRANSFORMS

	$f(x)$	$F_S(\alpha)$
C-21	$\begin{cases} 1 & 0 < x < b \\ 0 & x > b \end{cases}$	$\dfrac{1 - \cos b\alpha}{\alpha}$
C-22	x^{-1}	$\dfrac{\pi}{2}$
C-23	$\dfrac{x}{x^2 + b^2}$	$\dfrac{\pi}{2} e^{-b\alpha}$
C-24	e^{-bx}	$\dfrac{\alpha}{\alpha^2 + b^2}$
C-25	$x^{n-1} e^{-bx}$	$\dfrac{\Gamma(n) \sin (n \tan^{-1} \alpha/b)}{(\alpha^2 + b^2)^{n/2}}$
C-26	xe^{-bx^2}	$\dfrac{\sqrt{\pi}}{4b^{3/2}} \alpha e^{-\alpha^2/4b}$
C-27	$x^{-1/2}$	$\sqrt{\dfrac{\pi}{2\alpha}}$
C-28	x^{-n}	$\dfrac{\pi \alpha^{n-1} \csc (n\pi/2)}{2\,\Gamma(n)} \quad 0 < n < 2$
C-29	$\dfrac{\sin bx}{x}$	$\dfrac{1}{2} \ln \left(\dfrac{\alpha + b}{\alpha - b} \right)$
C-30	$\dfrac{\sin bx}{x^2}$	$\begin{cases} \pi\alpha/2 & \alpha < b \\ \pi b/2 & \alpha > b \end{cases}$
C-31	$\dfrac{\cos bx}{x}$	$\begin{cases} 0 & \alpha < b \\ \pi/4 & \alpha = b \\ \pi/2 & \alpha > b \end{cases}$
C-32	$\tan^{-1} (x/b)$	$\dfrac{\pi}{2\alpha} e^{-b\alpha}$
C-33	$\csc bx$	$\dfrac{\pi}{2b} \tanh \dfrac{\pi\alpha}{2b}$
C-34	$\dfrac{1}{e^{2x} - 1}$	$\dfrac{\pi}{4} \coth \left(\dfrac{\pi\alpha}{2} \right) - \dfrac{1}{2\alpha}$

$F_c\{f\} = \int_0^\infty f(x)\cos \alpha x \, dx$

SPECIAL FOURIER COSINE TRANSFORMS

	$f(x)$	$F_C(\alpha)$
C-7	$\begin{cases} 1 & 0 < x < b \\ 0 & x > b \end{cases}$	$\dfrac{\sin b\alpha}{\alpha}$
C-8	$\dfrac{1}{x^2 + b^2}$	$\dfrac{\pi e^{-b\alpha}}{2b}$
C-9	e^{-bx}	$\dfrac{b}{\alpha^2 + b^2}$
C-10	$x^{n-1} e^{-bx}$	$\dfrac{\Gamma(n) \cos (n \tan^{-1} \alpha/b)}{(\alpha^2 + b^2)^{n/2}}$
C-11	e^{-bx^2}	$\dfrac{1}{2}\sqrt{\dfrac{\pi}{b}}\, e^{-\alpha^2/4b}$
C-12	$x^{-1/2}$	$\sqrt{\dfrac{\pi}{2\alpha}}$
C-13	x^{-n}	$\dfrac{\pi \alpha^{n-1} \sec (n\pi/2)}{2\,\Gamma(n)}, \quad 0 < n < 1$
C-14	$\ln\left(\dfrac{x^2 + b^2}{x^2 + c^2}\right)$	$\dfrac{e^{-c\alpha} - e^{-b\alpha}}{\pi\alpha}$
C-15	$\dfrac{\sin bx}{x}$	$\begin{cases} \pi/2 & \alpha < b \\ \pi/4 & \alpha = b \\ 0 & \alpha > b \end{cases}$
C-16	$\sin bx^2$	$\sqrt{\dfrac{\pi}{8b}}\left(\cos\dfrac{\alpha^2}{4b} - \sin\dfrac{\alpha^2}{4b}\right)$
C-17	$\cos bx^2$	$\sqrt{\dfrac{\pi}{8b}}\left(\cos\dfrac{\alpha^2}{4b} + \sin\dfrac{\alpha^2}{4b}\right)$
C-18	$\operatorname{sech} bx$	$\dfrac{\pi}{2b} \operatorname{sech} \dfrac{\pi\alpha}{2b}$
C-19	$\dfrac{\cosh (\sqrt{\pi}\, x/2)}{\cosh (\sqrt{\pi}\, x)}$	$\sqrt{\dfrac{\pi}{2}}\, \dfrac{\cosh (\sqrt{\pi}\, \alpha/2)}{\cosh (\sqrt{\pi}\, \alpha)}$
C-20	$\dfrac{e^{-b\sqrt{x}}}{\sqrt{x}}$	$\sqrt{\dfrac{\pi}{2\alpha}}\{\cos (2b\sqrt{\alpha}) - \sin (2b\sqrt{\alpha})\}$

Table of Fourier Transform Pairs

Function, f(t)	Fourier Transform, F(ω)
Definition of Inverse Fourier Transform $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega$	*Definition of Fourier Transform* $F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}\,dt$
$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
$f(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}F(\frac{\omega}{\alpha})$
$F(t)$	$2\pi f(-\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$

Fourier Transform Table

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$F(t)$	$\widehat{F}(\omega)$	Notes	(0)
$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	Definition.	(1)
$\frac{1}{2\pi}\int_{-\infty}^{\infty}\widehat{f}(\omega)e^{i\omega t}\,d\omega$	$\widehat{f}(\omega)$	Inversion formula.	(2)
$\widehat{f}(-t)$	$2\pi f(\omega)$	Duality property.	(3)
$e^{-at}u(t)$	$\frac{1}{a+i\omega}$	a constant, $\Re(a)>0$	(4)
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	a constant, $\Re(a)>0$	(5)
$\beta(t)=\begin{cases}1, & \text{if } \lvert t\rvert<1,\\ 0, & \text{if } \lvert t\rvert>1\end{cases}$	$2\,\mathrm{sinc}(\omega)=2\frac{\sin(\omega)}{\omega}$	Boxcar in time.	(6)
$\frac{1}{\pi}\,\mathrm{sinc}(t)$	$\beta(\omega)$	Boxcar in frequency.	(7)
$f'(t)$	$i\omega\widehat{f}(\omega)$	Derivative in time.	(8)
$f''(t)$	$(i\omega)^2\widehat{f}(\omega)$	Higher derivatives similar.	(9)
$tf(t)$	$i\frac{d}{d\omega}\widehat{f}(\omega)$	Derivative in frequency.	(10)
$t^2f(t)$	$i^2\frac{d^2}{d\omega^2}\widehat{f}(\omega)$	Higher derivatives similar.	(11)
$e^{i\omega_0 t}f(t)$	$\widehat{f}(\omega-\omega_0)$	Modulation property.	(12)
$f\left(\frac{t-t_0}{k}\right)$	$ke^{-i\omega t_0}\widehat{f}(k\omega)$	Time shift and squeeze.	(13)
$(f*g)(t)$	$\widehat{f}(\omega)\widehat{g}(\omega)$	Convolution in time.	(14)
$u(t)=\begin{cases}0, & \text{if } t<0\\ 1, & \text{if } t>0\end{cases}$	$\frac{1}{i\omega}+\pi\delta(\omega)$	Heaviside step function.	(15)
$\delta(t-t_0)f(t)$	$e^{-i\omega t_0}f(t_0)$	Assumes f continuous at t_0.	(16)
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	Useful for $\sin(\omega_0 t)$, $\cos(\omega_0 t)$.	(17)

Convolution: $$(f*g)(t)=\int_{-\infty}^{\infty} f(t-u)g(u)\,du=\int_{-\infty}^{\infty} f(u)g(t-u)\,du.$$

Parseval: $$\int_{-\infty}^{\infty}\lvert f(t)\rvert^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\lvert\widehat{f}(\omega)\right\rvert^2\,d\omega.$$

$j\frac{1}{\pi t}$	$\mathrm{sgn}(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$	$2\pi\sum_{n=-\infty}^{\infty} F_n\delta(\omega-n\omega_0)$
$rect(\frac{t}{\tau})$	$\tau Sa(\frac{\omega\tau}{2})$
$\frac{B}{2\pi}Sa(\frac{Bt}{2})$	$rect(\frac{\omega}{B})$
$tri(t)$	$Sa^2(\frac{\omega}{2})$
$A\cos(\frac{\pi t}{2\tau})rect(\frac{t}{2\tau})$	$\frac{A\pi}{\tau}\frac{\cos(\omega\tau)}{(\pi/2\tau)^2-\omega^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)\cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$
$u(t)\sin(\omega_0 t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega^2}{\omega_0^2-\omega^2}$
$u(t)e^{-\alpha t}\cos(\omega_0 t)$	$\frac{(\alpha+j\omega)}{\omega_0^2+(\alpha+j\omega)^2}$

$u(t)e^{-\alpha t}\sin(\omega_0 t)$	$\frac{\omega_0}{\omega_0^2+(\alpha+j\omega)^2}$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+\omega^2}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\, e^{-\sigma^2\omega^2/2}$
$u(t)e^{-\alpha t}$	$\frac{1}{\alpha+j\omega}$
$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha+j\omega)^2}$

- ***Trigonometric Fourier Series***

$$f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos(\omega_0 nt)+b_n\sin(\omega_0 nt))$$

where

$$a_0=\frac{1}{T}\int_0^T f(t)dt\ ,\ a_n=\frac{2}{T}\int_0^T f(t)\cos(\omega_0 nt)dt\ \text{, and}$$

$$b_n=\frac{2}{T}\int_0^T f(t)\sin(\omega_0 nt)dt$$

- ***Complex Exponential Fourier Series***

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{j\omega nt}\ \text{, where}\quad F_n=\frac{1}{T}\int_0^T f(t)e^{-j\omega_0 nt}dt$$

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Properties and table of Fourier transforms

	$f(t)$		$F(\omega)=\hat{f}(\omega)$
F1.	$f(t)$		$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$
F2.	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega$		$F(\omega)$
F3.	$af(t)+bg(t)$		$aF(\omega)+bG(\omega)$
F4.	$f(at)$	($a\neq 0$ real)	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
F5.	$f(-t)$		$F(-\omega)$
F6.	$\overline{f(t)}$		$\overline{F(-\omega)}$
F7.	$f(t-T)$	(T real)	$e^{-i\omega T}F(\omega)$
F8.	$e^{i\Omega t}f(t)$	(Ω real)	$F(\omega-\Omega)$
F8a.	$f(t)\cos\Omega t$		$\frac{1}{2}(F(\omega-\Omega)+F(\omega+\Omega))$
F8b.	$f(t)\sin\Omega t$		$\frac{1}{2i}(F(\omega-\Omega)-F(\omega+\Omega))$
F9.	$F(t)$		$2\pi f(-\omega)$
F10.	$\left(\frac{d}{dt}\right)^n f(t)$		$(i\omega)^n F(\omega)$
F11.	$(-it)^n f(t)$		$\left(\frac{d}{d\omega}\right)^n F(\omega)$
F12.	$\int_{-\infty}^{t} f(\tau)d\tau$		$\frac{F(\omega)}{i\omega}+\pi F(0)\delta(\omega)$
F13.	$f*g(t)=\int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$		$F(\omega)G(\omega)$
F14.	$f(t)g(t)$		$\frac{1}{2\pi}F*G(\omega)$
F15.	$\delta(t)$		1
F16.	$\delta^{(n)}(t)$		$(i\omega)^n$
F17.	$\delta^{(n)}(t-T)$		$(i\omega)^n e^{-i\omega T}$ $(n=0,1,2,\ldots)$
F18.	1		$2\pi\delta(\omega)$
F19.	t^n		$2\pi i^n\delta^{(n)}(\omega)$ $(n=1,2,3,\ldots)$
F20.	t^{-n}		$\frac{\pi(-i)^n}{(n-1)!}\omega^{n-1}\operatorname{sgn}\omega$ $(n=1,2,3,\ldots)$

(A-1)

Table of Fourier sine transform

	$f(x),\ x>0$		$F_s(\beta),\ \beta>0$
$F_s1.$	$\begin{cases}1, & x<a\\ 0, & x>a\end{cases}$	$(a>0)$	$\dfrac{1-\cos a\beta}{\beta}$
$F_s2.$	e^{-ax}	$(a>0)$	$\dfrac{\beta}{a^2+\beta^2}$
$F_s3.$	xe^{-ax^2}	$(a>0)$	$\sqrt{\dfrac{\pi}{a}}\,\dfrac{\beta}{4a}\,e^{-\beta^2/4a}$
$F_s4.$	x^{a-1}	$(-1<a<1)$	$\Gamma(a)\beta^{-a}\sin\dfrac{a\pi}{2}$
$F_s5.$	$\cos ax^2$		$\sqrt{\dfrac{\pi}{2a}}\left[\sin\dfrac{\beta^2}{4a}C\left(\dfrac{\beta}{\sqrt{2\pi a}}\right)-\cos\dfrac{\beta^2}{4a}S\left(\dfrac{\beta}{\sqrt{2\pi a}}\right)\right]$
$F_s6.$	$\sin ax^2$		$\sqrt{\dfrac{\pi}{2a}}\left[\cos\dfrac{\beta^2}{4a}C\left(\dfrac{\beta}{\sqrt{2\pi a}}\right)+\sin\dfrac{\beta^2}{4a}S\left(\dfrac{\beta}{\sqrt{2\pi a}}\right)\right]$

Fourier transforms in higher dimensions

Two-dimensional Fourier transforms

Fourier transform $F(u,v)=\displaystyle\iint_{\mathbf{R}^2} f(x,y)e^{-i(ux+vy)}\,dxdy$

Inversion formula $f(x,y)=\dfrac{1}{(2\pi)^2}\displaystyle\iint_{\mathbf{R}^2} F(u,v)e^{i(ux+vy)}\,dudv$

Parseval's formulas $\displaystyle\iint_{\mathbf{R}^2} f(x,y)\,\overline{g(x,y)}\,dxdy=\dfrac{1}{(2\pi)^2}\iint_{\mathbf{R}^2} F(u,v)\,\overline{G(u,v)}\,dudv$

$$\iint_{\mathbf{R}^2} |f(x,y)|^2\,dxdy=\frac{1}{(2\pi)^2}\iint_{\mathbf{R}^2} |F(u,v)|^2\,dudv$$

A-4

	$f(t)$	$F(\omega)$
F21.	$\theta(t)=H(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$	$\frac{1}{i\omega}+\pi\delta(\omega)$
F22.	$t^n\theta(t)$	$\frac{n!}{(i\omega)^{n+1}}+\pi i^n\delta^{(n)}(\omega)\ (n=1,2,3,\dots)$
F23.	$\operatorname{sgn} t=2\theta(t)-1=\begin{cases}1, & t>0\\ -1, & t<0\end{cases}$	$\frac{2}{i\omega}$
F24.	$t^n \operatorname{sgn} t$	$\frac{2n!}{(i\omega)^{n+1}}$
F25.	$\lvert t\rvert=t \operatorname{sgn} t$	$-\frac{2}{\omega^2}$
F26.	$\lvert t\rvert^{2n-1}$	$2(-1)^n\frac{(2n-1)!}{\omega^{2n}}\quad (n=1,2,3,\dots)$
F27.	$\lvert t\rvert^{2n}=t^{2n}$	$2\pi(-1)^n\delta^{(2n)}(\omega)\quad (n=1,2,3,\dots)$
F28.	$\lvert t\rvert^{p-1}$	$\frac{2\Gamma(p)\cos\frac{p\pi}{2}}{\lvert\omega\rvert^p}\quad (p\neq\text{integer})$
F29.	$\lvert t\rvert^{p-1}\operatorname{sgn} t$	$\frac{-2i\Gamma(p)\sin\frac{p\pi}{2}\operatorname{sgn}\omega}{\lvert\omega\rvert^p}\quad (p\neq\text{integer})$
F30.	$e^{-at}\theta(t)=\begin{cases}e^{-at}, & t>0\\ 0, & t<0\end{cases}$	$\frac{1}{a+i\omega}\quad (a>0)$
F31.	$e^{at}(1-\theta(t))=e^{at}\theta(-t)=\begin{cases}0, & t>0\\ e^{at}, & t<0\end{cases}$	$\frac{1}{a-i\omega}\quad (a>0)$
F32.	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}\quad (a>0)$
F33.	$e^{-a\lvert t\rvert}\operatorname{sgn} t$	$-\frac{2i\omega}{a^2+\omega^2}\quad (a>0)$
F34.	$te^{-a\lvert t\rvert}$	$-\frac{4ia\omega}{(a^2+\omega^2)^2}\quad (a>0)$
F35.	$\lvert t\rvert e^{-a\lvert t\rvert}$	$\frac{2(a^2-\omega^2)}{(a^2+\omega^2)^2}\quad (a>0)$

A-2

	$f(t)$	$F(\omega)$
F36.	e^{-at^2}	$\sqrt{\frac{\pi}{a}}\,e^{-\omega^2/4a}\quad (a>0)$
F37.	$\frac{1}{\sqrt{4\pi a}}\,e^{-t^2/4a}$	$e^{-a\omega^2}\quad (a>0)$
F38.	$\frac{i}{2(n-1)!}\,(it)^{n-1}e^{ict}\operatorname{sgn} t$	$\frac{1}{(\omega-c)^n}\quad (c \text{ real}, n=1,2,3,\dots)$
F39.	$\frac{1}{(n-1)!}\,t^{n-1}e^{-at}\theta(t)$	$\frac{1}{(a+i\omega)^n}\quad (a>0, n=1,2,3,\dots)$
F40.	$\frac{(-1)^{n-1}}{(n-1)!}\,t^{n-1}e^{at}(1-\theta(t))$	$\frac{1}{(a-i\omega)^n}\quad (a>0, n=1,2,3,\dots)$
F41a.	$\frac{1}{2a}\,e^{-a\lvert t\rvert}$	$\frac{1}{\omega^2+a^2}\quad (a>0)$
F41b.	$\frac{1}{t^2+a^2}$	$\frac{\pi}{a}\,e^{-a\lvert\omega\rvert}$
F42a.	$\frac{i}{2}\,e^{-a\lvert t\rvert}\operatorname{sgn} t$	$\frac{\omega}{\omega^2+a^2}\quad (a>0)$
F42b.	$\frac{t}{t^2+a^2}$	$-i\pi e^{-a\lvert\omega\rvert}\operatorname{sgn}\omega$
F43.	$\frac{1}{2a}\,e^{-a\lvert t\rvert+ict}(iak\operatorname{sgn} t+b+kc)$	$\frac{k\omega+b}{(\omega-c)^2+a^2}\quad (a>0, c \text{ real})$
F44.	$\frac{1}{4a^3}\,e^{-a\lvert t\rvert+ict}[ia^2kt+(b+kc)a\lvert t\rvert+b+kc]$	$\frac{k\omega+b}{[(\omega-c)^2+a^2]^2}\quad (a>0, c \text{ real})$
F45.	$e^{i\Omega t}$	$2\pi\delta(\omega-\Omega)$
F46.	$\cos\Omega t$	$\pi[\delta(\omega+\Omega)+\delta(\omega-\Omega)]$
F47.	$\sin\Omega t$	$i\pi[\delta(\omega+\Omega)-\delta(\omega-\Omega)]$
F48.	$-\frac{1}{2a}\sin at\operatorname{sgn} t$	$\frac{1}{\omega^2-a^2}\quad (a>0)$
F49.	$\frac{i}{2}\cos at\operatorname{sgn} t$	$\frac{\omega}{\omega^2-a^2}\quad (a>0)$
F50.	$\theta(t+a)-\theta(t-a)=\begin{cases}1, & \lvert t\rvert<a\\ 0, & \lvert t\rvert>a\end{cases}$	$\frac{2\sin a\omega}{\omega}$

$F(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ixy} f(y)\,dy$	$f(y) = \int_{-\infty}^{+\infty} e^{ixy} F(x)\,dx$
$\sqrt{a^2+x^2}^{\,\nu} K_\nu(b\sqrt{a^2+x^2})$	$\sqrt{2\pi}\, a\, a^\nu b^\nu \frac{K_{\nu+1/2}(a\sqrt{b^2+y^2})}{\sqrt{b^2+y^2}^{\,\nu+1/2}}$
$\frac{H_\nu^{(2)}(b\sqrt{a^2+x^2})}{\sqrt{a^2+x^2}^{\,\nu}}$	$i\,2\sqrt{\frac{2a}{\pi}}\, a^{-\nu} b^{-\nu} \sqrt{y^2-b^2}^{\,\nu-1/2} K_{\nu-1/2}(a\sqrt{y^2-b^2})$
$e^{-x^2/2} He_n(x\sqrt{2})$	$\sqrt{2\pi}\, i^n e^{-y^2/2} He_n(y\sqrt{2})$
$e^{-x^2/2} He_n(x) He_m(x)$	$\sqrt{2\pi}\, n!\,(iy)^{m-n} e^{-y^2/2} L_n^{(m-n)}(y^2)$

§ 2. The Laplace-transformation.

The theorems formulated in this section are concerned without exception with the simplest and most important case in which the Laplace-transformation is used; we state two relations between a function $f(t)$ and a function $g(p)$, where the real and imaginary parts of $f(t)$ satisfy the conditions formulated in the preliminary remarks of this chapter, and where moreover:

1a) $f(t)$ is defined for all real values of t; $f(t) = 0$ for $t < 0$.

1b) there exists a real constant c_0, such that the integral

$$\int_0^\infty e^{-c_0 t} |f(t)|\, dt$$

exists.

2a) $g(p)$ is an analytic function of the complex variable $p = \sigma + i\tau$, which is one-valued and regular in an entire half-plane $\sigma \geqq c_0$.

2b) The function $|g(p)|$ is for $\sigma > c_0$ dominated by a function $G(\tau)$ with the property

$$|g(\sigma + i\tau)| \leqq G(\tau) \qquad \text{for } \sigma > c_0$$

such that the integral

$$\int_{-\infty}^{+\infty} G(\tau)\, d\tau$$

exists; it suffices that if this condition be satisfied not by $g(p)$ itself, but only by a function

$$g_0(p) = g(p) - \frac{a_1}{p^{\nu_1}} - \frac{a_2}{p^{\nu_2}} \cdots - \frac{a_n}{p^{\nu_n}}$$

where $a_1, \ldots, a_n$;
positive real part; o
pret the integral $\int_{-\infty}^{+\infty}$
Now we have the
conditions (1a), (1
relations

($\mathfrak{L}$) $g(p) =$

holds, where $c \geqq c_0$
hold between $g(p)$
The conditions c
the relations ($\mathfrak{L}$); fro
the first relation in (
tion in ($\mathfrak{L}$) follows f(
If the relation (
"original function" a

$g(p) = \mathfrak{L} f(t)$,

and calls $g(p)$ the "
One should note

since $f(t) = 0$ for t

In what follows t
make it possible to
the Laplace-transfor
Furthermore, a num
out; in these nothing
the range of values
can choose $c_0 > 0$ l
can easily be obtain
The parameters
be chosen real and p
The letter n alw

	$f(t)$	$F(\omega)$
F51.	$[\theta(t+a)-\theta(t-a)]\operatorname{sgn} t=\begin{cases}1, & 0<t<a\\ -1, & -a<t<0\\ 0, & \|t\|>a\end{cases}$	$\dfrac{4\sin^2\frac{a\omega}{2}}{i\omega}$
F52.	$[\theta(t+a)-\theta(t-a)]e^{i\Omega t}=\begin{cases}e^{i\Omega t}, & \|t\|<a\\ 0, & \|t\|>a\end{cases}$	$\dfrac{2\sin a(\Omega-\omega)}{\Omega-\omega}$
F53.	$\dfrac{\sin\Omega t}{\pi t}$	$\theta(\omega+\Omega)-\theta(\omega-\Omega)=\begin{cases}1, & \|\omega\|<\Omega\\ 0, & \|\omega\|>\Omega\end{cases}$
F54.	$\sin at^2$	$\sqrt{\dfrac{\pi}{a}}\cos\left(\dfrac{\omega^2}{4a}+\dfrac{\pi}{4}\right)\quad(a>0)$
F55.	$\cos at^2$	$\sqrt{\dfrac{\pi}{a}}\cos\left(\dfrac{\omega^2}{4a}-\dfrac{\pi}{4}\right)\quad(a>0)$
F56.	$h_n(t)$	$i^{-n}\sqrt{2\pi}\,h_n(\omega)$
F57.	$l_n(t)\theta(t)$	$\left(i\omega-\frac{1}{2}\right)^n\Big/\left(i\omega+\frac{1}{2}\right)^{n+1}$
F58.	$\begin{cases}(a^2-t^2)^{-1/2}, & \|t\|<a\\ 0, & \|t\|>a\end{cases}$	$\pi J_0(a\omega)$
F59.	$\begin{cases}t(a^2-t^2)^{-1/2}, & \|t\|<a\\ 0, & \|t\|>a\end{cases}$	$-ia\pi J_1(a\omega)$
F60.	$\dfrac{1}{\cosh t}$	$\dfrac{\pi}{\cosh\frac{\pi\omega}{2}}$
F61.	$\dfrac{1}{\sinh t}$	$-i\pi\tanh\dfrac{\pi\omega}{2}$
F62.	$\dfrac{\sinh at}{\sinh bt}\quad(0<a<b)$	$\dfrac{\pi\sin(a\pi/b)}{b\cosh(\omega\pi/b)+b\cos(a\pi/b)}$
F63.	$\dfrac{\cosh at}{\cosh bt}\quad(0<a<b)$	$\dfrac{2\pi\cos(a\pi/2b)\cosh(\omega\pi/2b)}{b\cosh(\omega\pi/b)+b\cos(a\pi/b)}$
F64.	$\dfrac{1}{2\sqrt{2}a^3}e^{-a\|t\|/\sqrt{2}}\left(\cos\dfrac{at}{\sqrt{2}}+\sin\dfrac{a\|t\|}{\sqrt{2}}\right)$	$\dfrac{1}{\omega^4+a^4}\quad(a>0)$

A-3

Cosine and sine transforms

$$F_c(\beta)=\int_0^\infty f(x)\cos\beta x\,dx \qquad f(x)=\frac{2}{\pi}\int_0^\infty F_c(\beta)\cos\beta x\,d\beta$$

$$F_s(\beta)=\int_0^\infty f(x)\sin\beta x\,dx \qquad f(x)=\frac{2}{\pi}\int_0^\infty F_s(\beta)\sin\beta x\,d\beta$$

Plancherel's formulas

$$\int_0^\infty |f(x)|^2dx=\frac{2}{\pi}\int_0^\infty |F_c(\beta)|^2d\beta=\frac{2}{\pi}\int_0^\infty |F_s(\beta)|^2d\beta$$

Relations between Fourier transforms

If $F(\beta)$ is the Fourier transform of $f(x)$, $-\infty<x<\infty$, then

$f(x)$ even $\Rightarrow F(\beta)=\;2F_c(\beta)$

$f(x)$ odd $\Rightarrow F(\beta)=-2iF_s(\beta)$

$\mathcal{F}_c\{f\} = \int_0^\infty f(x)\cos\beta x\,dx$

Table of Fourier cosine transform

	$f(x)$, $x>0$		$F_c(\beta)$, $\beta>0$
$F_c1.$	$\begin{cases}1, & x<a\\ 0, & x>a\end{cases}$	$(a>0)$	$\dfrac{\sin a\beta}{\beta}$
$F_c2.$	e^{-ax}	$(a>0)$	$\dfrac{a}{a^2+\beta^2}$
$F_c3.$	e^{-ax^2}	$(a>0)$	$\dfrac{1}{2}\sqrt{\dfrac{\pi}{a}}\,e^{-\beta^2/4a}$
$F_c4.$	x^{a-1}	$(0<a<1)$	$\Gamma(a)\beta^{-a}\cos\dfrac{a\pi}{2}$
$F_c5.$	$\cos ax^2$		$\dfrac{1}{2}\sqrt{\dfrac{\pi}{a}}\cos\left(\dfrac{\beta^2}{4a}-\dfrac{\pi}{4}\right)$
$F_c6.$	$\sin ax^2$		$\dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}\cos\left(\dfrac{\beta^2}{4a}+\dfrac{\pi}{4}\right)$

Some Useful Mathematical Relationships

$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$2\cos^2(x) = 1 + \cos(2x)$
$2\sin^2(x) = 1 - \cos(2x)$
$\cos^2(x) + \sin^2(x) = 1$
$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$
$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$
$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$

Useful Integrals

$\int \cos(x)dx$	$\sin(x)$
$\int \sin(x)dx$	$-\cos(x)$
$\int x\cos(x)dx$	$\cos(x) + x\sin(x)$
$\int x\sin(x)dx$	$\sin(x) - x\cos(x)$
$\int x^2\cos(x)dx$	$2x\cos(x) + (x^2 - 2)\sin(x)$
$\int x^2\sin(x)dx$	$2x\sin(x) - (x^2 - 2)\cos(x)$
$\int e^{\alpha x}dx$	$\frac{e^{\alpha x}}{a}$
$\int xe^{\alpha x}dx$	$e^{\alpha x}\left[\frac{x}{a} - \frac{1}{a^2}\right]$
$\int x^2e^{\alpha x}dx$	$e^{\alpha x}\left[\frac{x^2}{a} - \frac{2x}{a^2} - \frac{2}{a^3}\right]$
$\int \frac{dx}{\alpha + \beta x}$	$\frac{1}{\beta}\ln\lvert\alpha + \beta x\rvert$
$\int \frac{dx}{\alpha^2 + \beta^2x^2}$	$\frac{1}{\alpha\beta}\tan^{-1}(\frac{\beta x}{\alpha})$

$F(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-ixy}f(y)\,dy$	$f(y)=\int_{-\infty}^{+\infty}e^{ixy}F(x)\,dx$
$\frac{1}{\sqrt{\lvert x\rvert}}$	$\sqrt{\frac{2\pi}{\lvert y\rvert}}$
$\operatorname{sgn} x\,\frac{1}{\sqrt{\lvert x\rvert}}$	$i\operatorname{sgn} y\sqrt{\frac{2\pi}{\lvert y\rvert}}$
$\frac{\sin a x}{x}$	$\begin{cases}\pi & \text{for } \lvert y\rvert \\ 0 & ,, \ \lvert y\rvert\end{cases}$
$\frac{\sin^2 a x}{x^2}$	$\begin{cases}\pi\left(a-\frac{y}{2}\right) & \text{for } \lvert y\rvert< \\ 0 & ,, \ \lvert y\rvert>\end{cases}$
$\left.\begin{matrix} e^{i\omega x} & \text{for } p<x<q \\ 0 & ,, \ x<p;\ x>q\end{matrix}\right\}$	$i\,\frac{e^{ip(\omega+y)}-e^{iq(\omega+y)}}{\omega+y}$
$\left.\begin{matrix} e^{-\beta x}e^{i\omega x} & \text{for } x>0 \\ 0 & ,, \ x<0\end{matrix}\right\}$	$\frac{i}{\omega+y+i\beta}$
$\frac{1}{\mathfrak{Cof}\,a x}$	$\frac{\pi}{a}\,\frac{1}{\mathfrak{Cof}\left(\frac{\pi y}{2a}\right)}$
$e^{-\lambda x^2}$ $\quad\mathrm{Re}(\lambda)>0$	$\sqrt{\frac{\pi}{\lambda}}\,e^{-y^2/4\lambda}$ $\quad\mathrm{Re}(\lambda)>$ $\mathrm{Re}(\sqrt{\lambda})>$
$\cos a x^2$	$\sqrt{\frac{\pi}{a}}\cos\left(\frac{y^2}{4a}-\frac{\pi}{4}\right)$
$\sin a x^2$	$\sqrt{\frac{\pi}{a}}\cos\left(\frac{y^2}{4a}+\frac{\pi}{4}\right)$
$\frac{1}{\sqrt{x^2+a^2}}$	$2K_0(a\lvert y\rvert)$
$\operatorname{sgn} x\sqrt{\frac{1}{x^2+a^2}}$	$\pi i\operatorname{sgn} y\,[J_0(iay)+i\,\mathbf{H}_0(iay)]$
$\frac{1}{x^2+a^2}$	$\frac{\pi}{a}e^{-a\lvert y\rvert}$
$\operatorname{sgn} x\,\frac{1}{x^2+a^2}$	$\frac{i}{a}\,[e^{-ay}\,\mathrm{Ei}(ay)-e^{ay}\,\mathrm{Ei}(-ay)]$
$\frac{e^{-a\lvert x\rvert}}{\sqrt{\lvert x\rvert}}$	$\sqrt{2\pi}\,\frac{\sqrt{\sqrt{a^2+y^2}+a}}{\sqrt{a^2+y^2}}$
$\operatorname{sgn} x\,\frac{e^{-a\lvert x\rvert}}{\sqrt{\lvert x\rvert}}$	$i\sqrt{2\pi}\operatorname{sgn} y\,\frac{\sqrt{\sqrt{a^2+y^2}-a}}{\sqrt{a^2+y^2}}$

Note: $\mathfrak{Cof} \equiv \cosh$

$F(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-ixy}f(y)\,dy$	$f(y)=\int_{-\infty}^{+\infty}e^{ixy}F(x)\,dx$
$\frac{\cos a x}{\sqrt{\lvert x\rvert}}$	$\sqrt{\frac{\pi}{2}}\left(\frac{1}{\sqrt{\lvert y-a\rvert}}+\frac{1}{\sqrt{\lvert y+a\rvert}}\right)$
$\frac{\sin a x}{\sqrt{\lvert x\rvert}}$	$\sqrt{\frac{\pi}{2}}\left(\frac{1}{\sqrt{\lvert y-a\rvert}}-\frac{1}{\sqrt{\lvert y+a\rvert}}\right)$
$\frac{\sin a x^2}{x}$	$-i\pi\left[S\left(\frac{y}{\sqrt{2\pi a}}\right)-C\left(\frac{y}{\sqrt{2\pi a}}\right)\right]$
$\frac{\sin a x^2}{x^2}$	$\pi y\left[S\left(\frac{y}{\sqrt{2\pi a}}\right)-C\left(\frac{y}{\sqrt{2\pi a}}\right)\right]+2\sqrt{\pi a}\sin\left(\frac{y^2}{4a}+\frac{\pi}{4}\right)$
$\operatorname{sgn} x\cos a x^2$	$i\sqrt{\frac{2\pi}{a}}\left\{\sin\left(\frac{y^2}{4a}\right)C\left(\frac{y}{\sqrt{2\pi a}}\right)-\cos\left(\frac{y^2}{4a}\right)S\left(\frac{y}{\sqrt{2\pi a}}\right)\right\}$
$\operatorname{sgn} x\sin a x^2$	$i\sqrt{\frac{2\pi}{a}}\left\{\cos\left(\frac{y^2}{4a}\right)C\left(\frac{y}{\sqrt{2\pi a}}\right)+\sin\left(\frac{y^2}{4a}\right)S\left(\frac{y}{\sqrt{2\pi a}}\right)\right\}$
$\frac{\sin^2 a x}{x}$	$\begin{cases}\operatorname{sgn} y\, i\,\frac{\pi}{2} & \text{for } \lvert y\rvert<2a \\ 0 & ,, \ \lvert y\rvert>2a\end{cases}$
$\frac{\mathfrak{Cof}\,a x}{\mathfrak{Cof}\,\pi x}$	$\frac{2\cos\frac{a}{2}\,\mathfrak{Cof}\,\frac{y}{2}}{\cos a+\mathfrak{Cof}\,y}$ $\quad a<\pi$
$\left.\begin{matrix} \frac{\sin b\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}} & \text{for } \lvert x\rvert<a \\ -\frac{e^{-b\sqrt{x^2-a^2}}}{\sqrt{x^2-a^2}} & ,, \ \lvert x\rvert>a\end{matrix}\right\}$	$\pi N_0(a\sqrt{b^2+y^2})$
$\frac{e^{-a/\sqrt{\lvert x\rvert}}}{\sqrt{\lvert x\rvert}}$	$\sqrt{\frac{2\pi}{\lvert y\rvert}}\left(\cos\sqrt{2a\lvert y\rvert}-\sin\sqrt{2a\lvert y\rvert}\right)$
$\operatorname{sgn} x\,\frac{e^{-a/\sqrt{\lvert x\rvert}}}{\sqrt{\lvert x\rvert}}$	$i\operatorname{sgn} y\sqrt{\frac{2\pi}{\lvert y\rvert}}\left(\cos\sqrt{2a\lvert y\rvert}+\sin\sqrt{2a\lvert y\rvert}\right)$
$\left.\begin{matrix} \frac{1}{\sqrt{a^2-x^2}} & \text{for } \lvert x\rvert<a \\ 0 & ,, \ \lvert x\rvert>a\end{matrix}\right\}$	$\pi J_0(a y)$
$\left.\begin{matrix} 0 & \text{for } \lvert x\rvert<a \\ \frac{\operatorname{sgn} x}{\sqrt{x^2-a^2}} & ,, \ \lvert x\rvert>a\end{matrix}\right\}$	$\pi i\operatorname{sgn} y\,J_0(a y)$

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$F(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ixy} f(y)\,dy$	$f(y)=\int_{-\infty}^{+\infty} e^{ixy} F(x)\,dx$
$\left.\begin{matrix} 0 & \text{for } \lvert x\rvert < a \\ -\frac{1}{\sqrt{x^2-a^2}} & ,, \ \lvert x\rvert > a \end{matrix}\right\}$	$\pi N_0(a\lvert y\rvert)$
$\left.\begin{matrix} \frac{\arcsin\left(\frac{x}{a}\right)}{\sqrt{a^2-x^2}} & \text{for } \lvert x\rvert < a \\ \frac{\ln\left[\frac{x}{a}-\sqrt{\left(\frac{x}{a}\right)^2-1}\right]}{\sqrt{x^2-a^2}} & ,, \ \lvert x\rvert > a \end{matrix}\right\}$	$\operatorname{sgn} y\, N_0(a\lvert y\rvert)\, i\,\frac{\pi^2}{2}$
$\frac{1}{2}\,\frac{1}{(x^2+a^2)^{\nu+1/2}}$ $\quad(\mathrm{Re}\,\nu > -\frac{1}{2})$	$\left(\frac{\lvert y\rvert}{2a}\right)^{\nu}\frac{\Gamma(\frac{1}{2})}{\Gamma(\nu+\frac{1}{2})}K_\nu(a\lvert y\rvert)$ $\quad(\mathrm{Re}\,\nu >$
$\frac{\mathfrak{Ar}\,\mathfrak{Sin}\left(\frac{x}{a}\right)}{\sqrt{x^2+a^2}}$	$\pi i \operatorname{sgn} y\, K_0(a\lvert y\rvert)$
$e^{ia^2x^2}$	$\begin{cases} \frac{2}{3a}\sqrt{\frac{y}{a}}\,K_{1/3}\left(\frac{2y}{3a}\sqrt{\frac{y}{3a}}\right) & \text{for } y > \\ \frac{2\pi}{3a}\sqrt{\frac{\lvert y\rvert}{3a}}\left[J_{1/3}\left(\frac{2\lvert y\rvert}{3a}\sqrt{\frac{\lvert y\rvert}{3a}}\right)+J_{-1/3}\left(\frac{2\lvert y\rvert}{3a}\sqrt{\frac{\lvert y\rvert}{3a}}\right.\right. & \text{for } y < \end{cases}$
$\frac{\cos\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}}$	$\begin{cases} 2K_0\left(a\sqrt{y^2-b^2}\right) & \text{for } \lvert y\rvert > \\ -\pi N_0\left(a\sqrt{b^2-y^2}\right) & ,, \ \lvert y\rvert < \end{cases}$
$\frac{\sin\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}}$	$\begin{cases} 0 & \text{for } \lvert y\rvert > \\ \pi J_0\left(a\sqrt{b^2-y^2}\right) & ,, \ \lvert y\rvert < \end{cases}$
$\frac{e^{-b\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}$	$2K_0\left(a\sqrt{b^2+y^2}\right)$
$\frac{e^{ib\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}$	$\pi i H_0^1\left(a\sqrt{b^2-y^2}\right)$
$\left.\begin{matrix} \frac{\cos\left(b\sqrt{a^2-x^2}\right)}{\sqrt{a^2-x^2}} & \text{for } \lvert x\rvert < a \\ 0 & ,, \ \lvert x\rvert > a \end{matrix}\right\}$	$\pi J_0\left(a\sqrt{b^2+y^2}\right)$
$\left.\begin{matrix} -\frac{\sin\left(b\sqrt{x^2-a^2}\right)}{\sqrt{x^2-a^2}} & \text{for } \lvert x\rvert > a \\ +\frac{e^{-b\sqrt{a^2-x^2}}}{\sqrt{a^2-x^2}} & ,, \ \lvert x\rvert < a \end{matrix}\right\}$	$\begin{cases} \pi J_0\left(a\sqrt{y^2-b^2}\right) & \text{for } \lvert y\rvert > \\ 0 & ,, \ \lvert y\rvert < \end{cases}$

Note: Sin = sinh

$F(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ixy} f(y)\,dy$	$f(y)=\int_{-\infty}^{+\infty} e^{ixy} F(x)\,dx$
$\left.\begin{matrix} \operatorname{sgn} x\,\frac{\cos\left(b\sqrt{x^2-a^2}\right)}{\sqrt{x^2-a^2}}; & \lvert x\rvert > a \\ 0; & \lvert x\rvert < a \end{matrix}\right\}$	$\begin{cases} \pi i \operatorname{sgn} y\, J_0\left(a\sqrt{y^2-b^2}\right) & \text{for } \lvert y\rvert > b \\ 0 & ,, \ \lvert y\rvert < b \end{cases}$
$\left.\begin{matrix} \frac{\mathfrak{Cos}\left(b\sqrt{a^2-x^2}\right)}{\sqrt{a^2-x^2}} & \text{for } \lvert x\rvert < a \\ 0 & ,, \ \lvert x\rvert > a \end{matrix}\right\}$	$\pi J_0\left(a\sqrt{y^2-b^2}\right)$
$\frac{\sin\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}}\times$ $\times\left[\mathrm{Ci}\left(b\sqrt{a^2+x^2}+bx\right)+\mathrm{Ci}\left(b\sqrt{a^2+x^2}-bx\right)\right]-$ $-\frac{\cos\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}}\times$ $\times\left[\mathrm{Si}\left(b\sqrt{a^2+x^2}+bx\right)+\mathrm{Si}\left(b\sqrt{a^2+x^2}-bx\right)\right]$	$\begin{matrix} \pi^2 N_0\left(a\sqrt{b^2-y^2}\right) & \text{for } \lvert y\rvert < b \\ 0 & ,, \ \lvert y\rvert > b \end{matrix}$
$\frac{\cos\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}}\times$ $\left[\pi-\mathrm{Si}\left(b\sqrt{a^2+x^2}+bx\right)-\mathrm{Si}\left(b\sqrt{a^2+x^2}-bx\right)\right]+$ $+\frac{\sin\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}}\times$ $\left[\mathrm{Ci}\left(b\sqrt{a^2+x^2}+bx\right)+\mathrm{Ci}\left(b\sqrt{a^2+x^2}-bx\right)\right]$	$\begin{cases} 2\pi K_0\left(a\sqrt{y^2-b^2}\right) & \text{for } \lvert y\rvert > b \\ 0 & ,, \ \lvert y\rvert < b \end{cases}$
$\left.\begin{matrix} P_n(x) & \text{for } \lvert x\rvert < 1 \\ 0 & ,, \ \lvert x\rvert > 1 \end{matrix}\right\}$	$2i^n\sqrt{\frac{\pi}{2y}}\,J_{n+1/2}(y)$
$\left.\begin{matrix} \frac{J_\nu\left(b\sqrt{a^2-x^2}\right)}{\sqrt{a^2-x^2}} & \text{for } \lvert x\rvert < a \\ 0 & ,, \ \lvert x\rvert > a \end{matrix}\right\}$	$\pi J_{\nu/2}\left[\frac{a}{2}\left(\sqrt{b^2+y^2}-y\right)\right]\times$ $\times J_{\nu/2}\left[\frac{a}{2}\left(\sqrt{b^2+y^2}+y\right)\right]$
$\frac{J_\nu\left(b\sqrt{a^2+x^2}\right)}{\sqrt{a^2+x^2}^{\,\nu}}$	$\begin{cases} \sqrt{2\pi a}\,a^{-\nu}b^{-\nu}\sqrt{b^2-y^2}^{\,\nu-1/2}J_{\nu-1/2}\left(a\sqrt{b^2-y^2}\right) & \text{for } \lvert y\rvert < b \\ 0 & ,, \ \lvert y\rvert > b \end{cases}$
$\left.\begin{matrix} \sqrt{a^2-x^2}^{\,\nu}J_\nu\left(b\sqrt{a^2-x^2}\right) & \text{for } \lvert x\rvert < a \\ 0 & ,, \ \lvert x\rvert > a \end{matrix}\right\}$	$\sqrt{2\pi a}\,a^\nu b^\nu\,\frac{J_{\nu+1/2}\left(a\sqrt{b^2+y^2}\right)}{\sqrt{b^2+y^2}^{\,\nu+1/2}}$