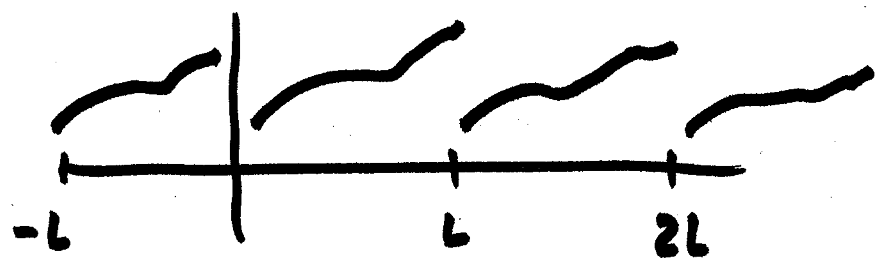
FOURIER SERIES

$$f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi x}{L} + a_2 \cos\frac{2\pi x}{L} + \cdots + b_1 \sin\frac{\pi x}{L} + b_2 \sin\frac{2\pi x}{L} + \cdots$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx \qquad n=0,1,2,\ldots$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx \qquad n=0,1,2..$$



FOURIER SINE TRANSFORM

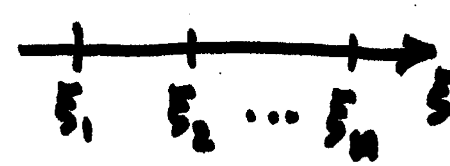
$-f(x) = f(-x)$ for $x \in (-L, L)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L} \qquad b_n = \frac{2}{L}\int_0^L f(\bar{x})\sin\frac{n\pi\bar{x}}{L}\,d\bar{x}$$

LOOK AT WHAT HAPPENS AS $L \to \infty$

$$f(x) = \sum_{n=0}^{\infty} b_n \sin\frac{n\pi x}{L}\,\Delta n \qquad \Delta n = n+1-n$$

LET $\xi_n = \frac{n\pi}{L}$; $\Delta\xi_n = \frac{\pi}{L}\Delta n$



$$f(x) = \sum_{n=0}^{\infty} b_n \sin\frac{n\pi x}{L}\,\Delta n = \frac{L}{\pi}\sum_{n=0}^{\infty} b_n \sin\overset{\xi_n x}{\left(\frac{n\pi x}{L}\right)}\Delta\xi_n$$

$$b_n = \frac{2}{L}\int_0^L f(s)\sin\xi_n s\,ds$$

$$f(x) = \frac{2}{L}\cdot\frac{L}{\pi}\sum_{n=0}^{\infty}\left\{\int_0^L f(s)\sin\xi_n s\,ds\right\}\sin\xi_n x\,\Delta\xi_n$$

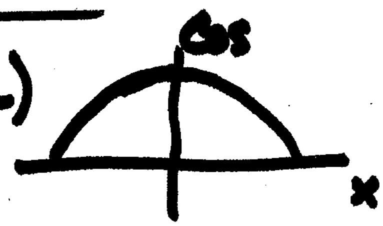
L's CANCEL & TAKE limit as $L\to\infty$

$$f(x) = \frac{2}{\pi}\int_0^{\infty} d\xi\left\{\int_0^{\infty} f(s)\sin\xi_0 s\,ds\right\}\sin\xi x$$

$$\text{LET}\quad \begin{cases} F(\xi) = \displaystyle\int_0^{\infty} f(s)\sin\xi s\,ds \\ f(x) = \displaystyle\frac{2}{\pi}\int_0^{\infty} F(\xi)\sin\xi x\,d\xi \end{cases}$$

FOR ODD $f(x)$

IF $f(x) = f(-x)$ FOR $x\in(-L,L)$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

as $L\to\infty$

$$\begin{cases} F(\xi) = \displaystyle\int_0^{\infty} f(s)\cos\xi s\,ds \\ f(x) = \displaystyle\frac{2}{\pi}\int_0^{\infty} F(\xi)\cos\xi x\,d\xi \end{cases}$$

FOR EVEN $f(x)$