$$\frac{d}{dx}[Sy'] + [Q + \lambda^2 P]\, y = 0$$

$\alpha y + \beta y' = 0$ @ $x = a$

$\alpha y + \beta y' = 0$ @ $x = b$

Solution to S.L. equation $y(x;\lambda)$ — ex. $J_0(\lambda r)$ must be bounded

" " S.L. equation + B.C. $y_n(x;\lambda_n)$ ex. $J_0(\frac{w_i}{c} r)$

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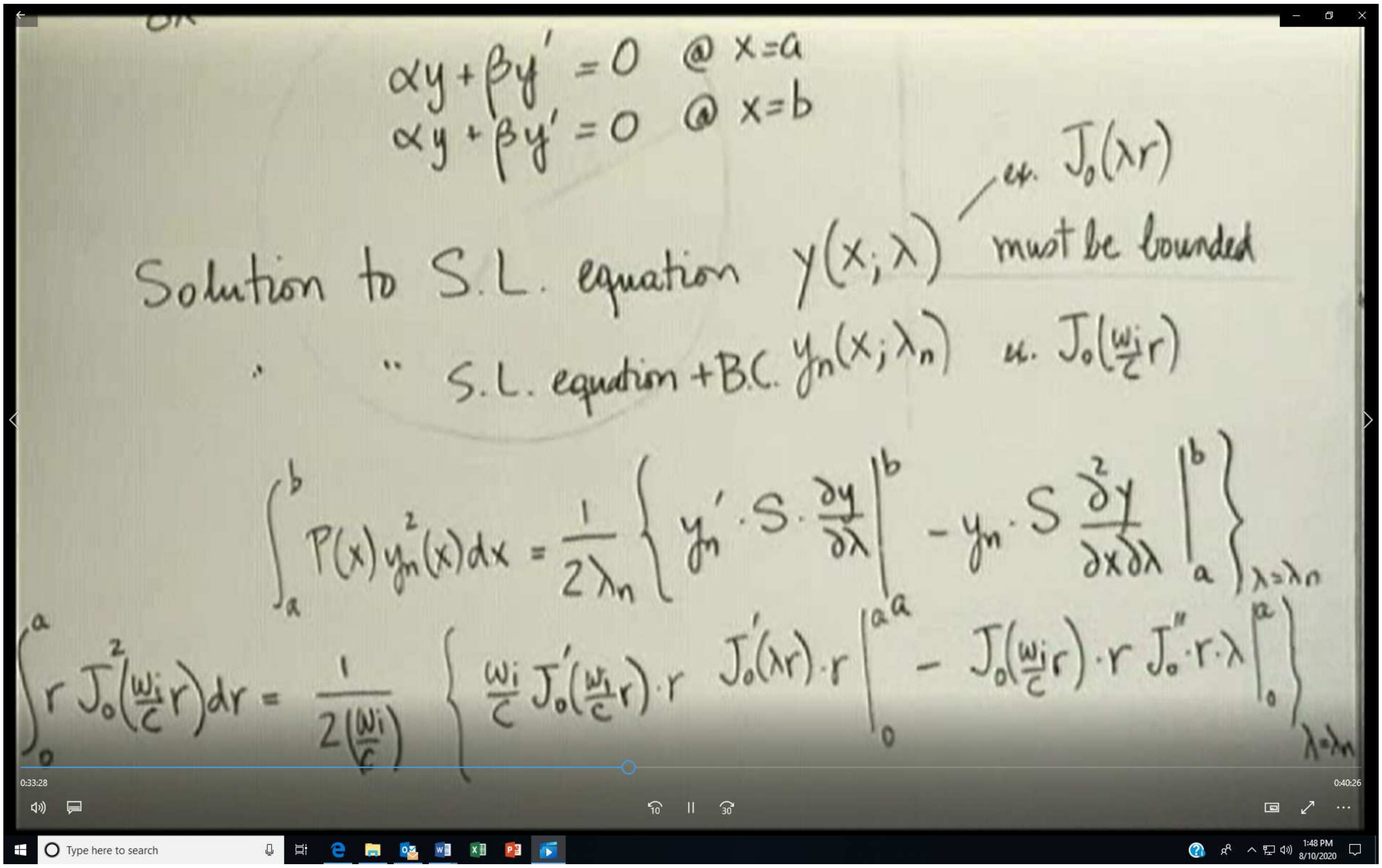
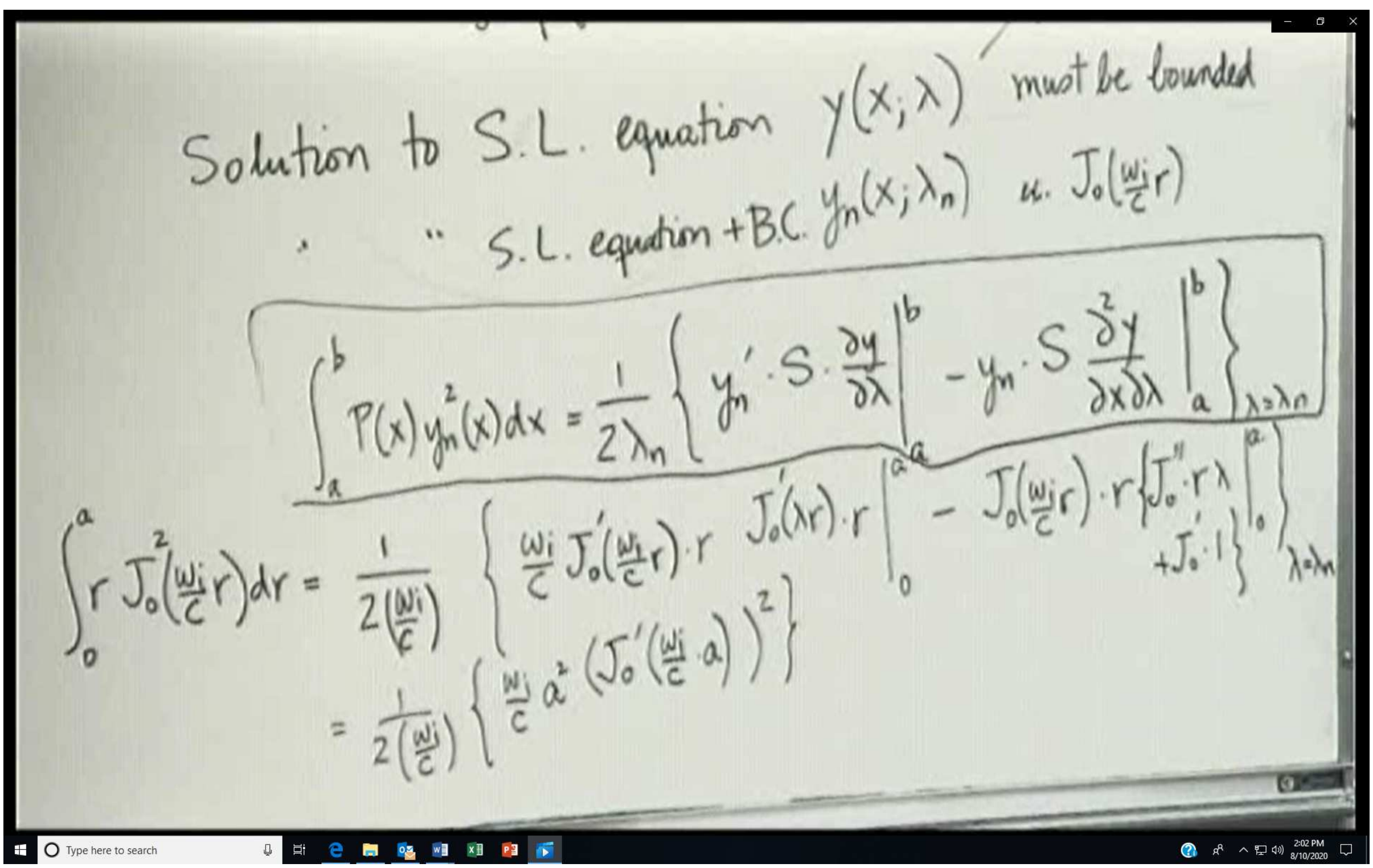
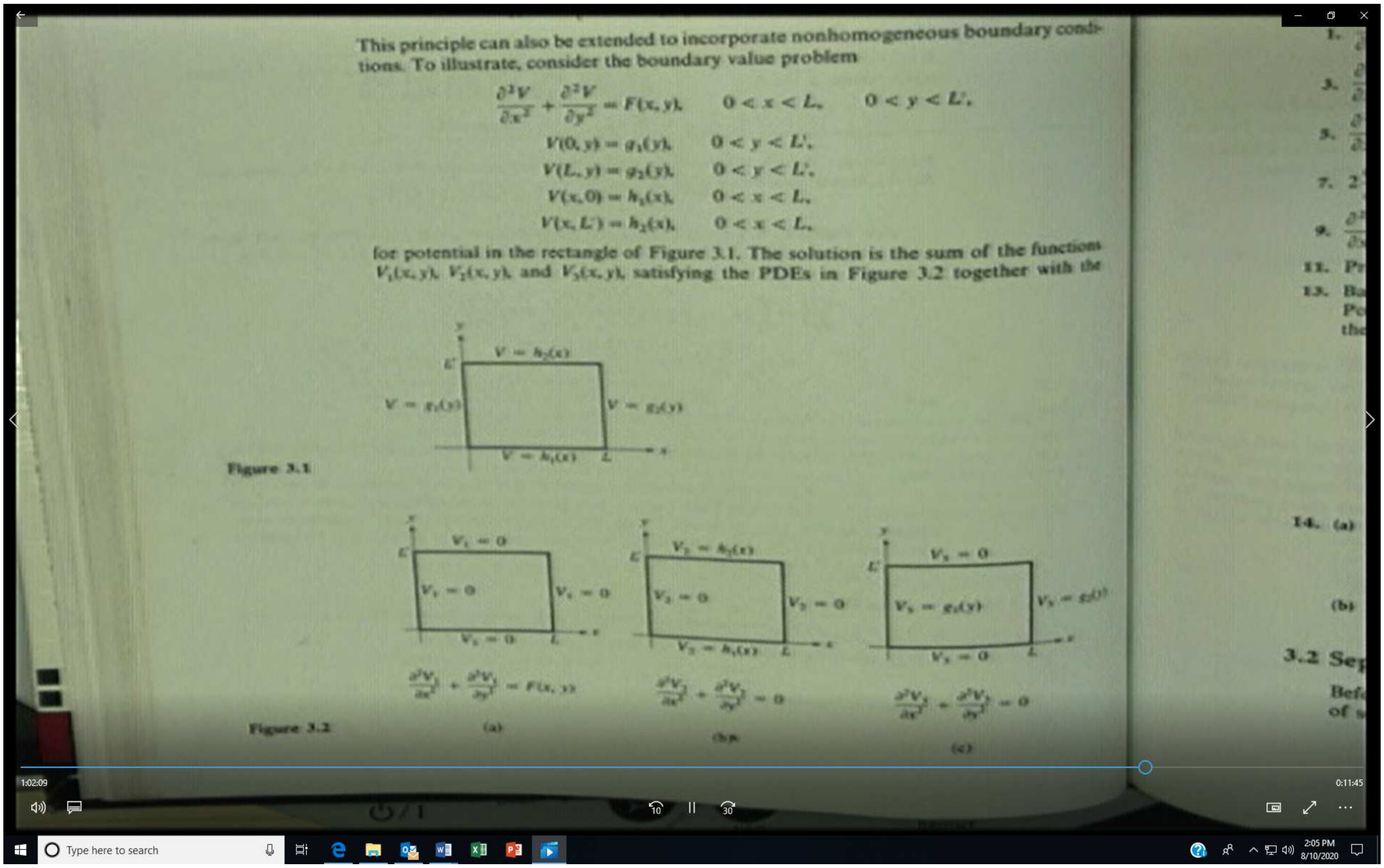
$$\int_a^b P(x)\, y_n^2(x)\, dx = \frac{1}{2\lambda_n}\left\{ y_n' \cdot S \cdot \frac{\partial y}{\partial \lambda}\Big|_a^b - y_n \cdot S \frac{\partial^2 y}{\partial x \partial \lambda}\Big|_a^b \right\}_{\lambda=\lambda_n}$$



THERE IS A MISSING TERM IN THE PREVIOUS SLIDE. tHE LAST TERM SHOULD BE Jo(wir/c)*r*[Jo”(λr) *r*λ+Jo’(λr)*1]. When evaluated, that term gives zero. That term is in the next slide.



An inhomogeneous PDE with inhomogeneous BCs can be split into several problems in which only one of the BCs is non-zero

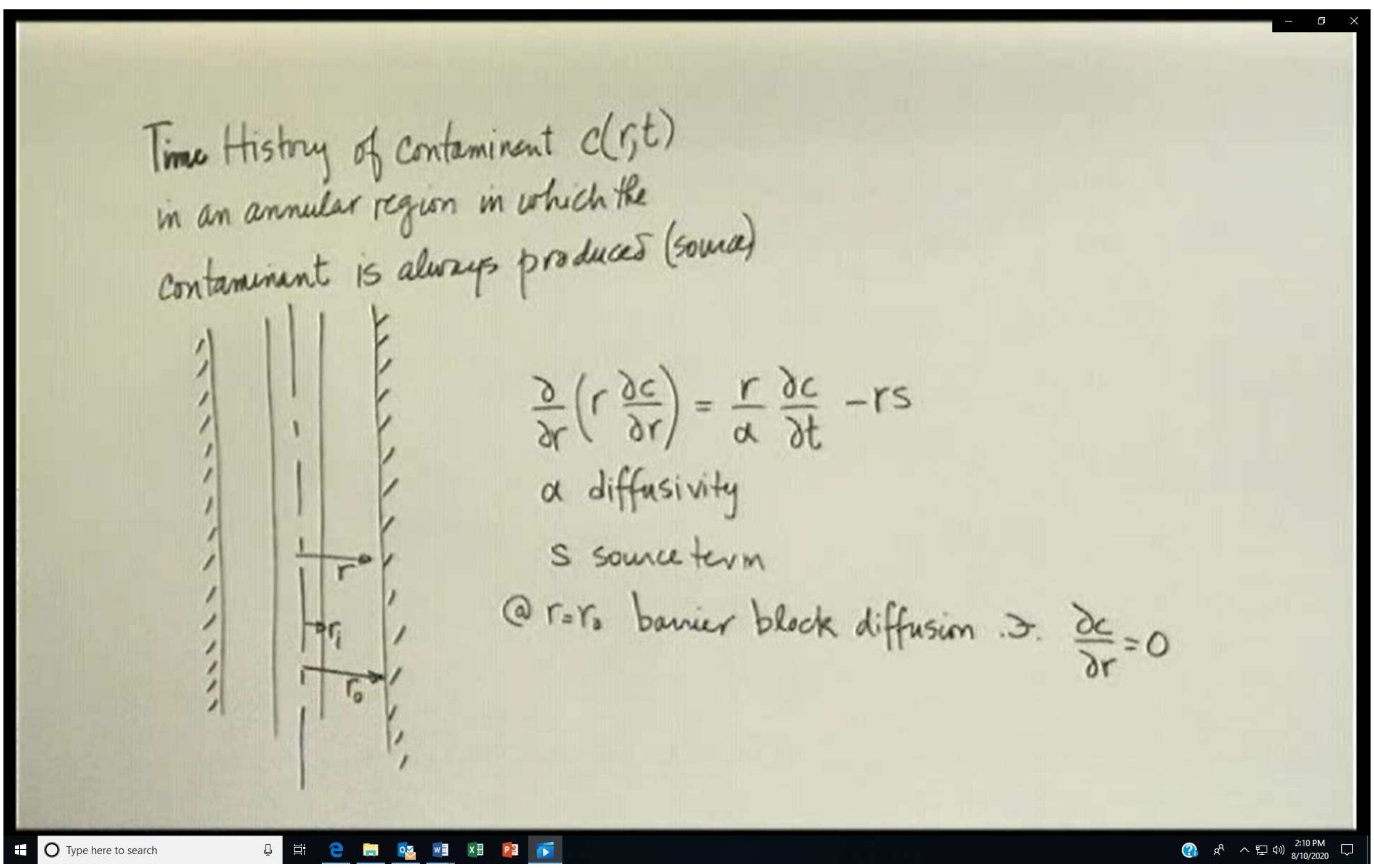


This principle can also be extended to incorporate nonhomogeneous boundary conditions. To illustrate, consider the boundary value problem

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=F(x,y),\qquad 0<x<L,\qquad 0<y<L',$$

$$V(0,y)=g_1(y),\qquad 0<y<L',$$
$$V(L,y)=g_2(y),\qquad 0<y<L',$$
$$V(x,0)=h_1(x),\qquad 0<x<L,$$
$$V(x,L')=h_2(x),\qquad 0<x<L,$$

for potential in the rectangle of Figure 3.1. The solution is the sum of the functions $V_1(x,y)$, $V_2(x,y)$, and $V_3(x,y)$ satisfying the PDEs in Figure 3.2 together with the



Time History of Contaminent $c(r,t)$ in an annular region in which the contaminant is always produced (source)

$$\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)=\frac{r}{\alpha}\frac{\partial c}{\partial t}-rs$$

α diffusivity

s source term

@ $r=r_o$ barrier block diffusion $\therefore$ $\frac{\partial c}{\partial r}=0$

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@ $r=r_i$ $D\frac{\partial c}{\partial r} = h(c-c_\infty)$

@ $t=0$ $c=c_0$

h - convective transport coeff

c_∞ - fixed concentration of fluid passing the center bore

D - diffusion coeff for contaminant in the solid