

$$\frac{d}{dx}[Sy'] + [Q + \lambda^2 P]y = 0$$

$$\alpha y + \beta y' = 0 \quad @ x=a$$

$$\alpha y + \beta y' = 0 \quad @ x=b$$

Solution to S.L. equation $y(x; \lambda)$ ex. $J_0(\lambda r)$ must be bounded

" " S.L. equation + B.C. $y_n(x; \lambda_n)$ ex. $J_0(\frac{\omega_j}{c} r)$

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$$\frac{d}{dx}[Sy'] + [Q + \lambda^2 P]y = 0$$

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Solution to S.L. equation $y(x; \lambda)$ ex. $J_0(\lambda r)$ must be bounded

" " S.L. equation + B.C. $y_n(x; \lambda_n)$ ex. $J_0(\frac{\omega_j}{c} r)$

$$\int_a^b P(x) y_n^2(x) dx = \frac{1}{2\lambda_n} \left\{ y_n' \cdot S \cdot \frac{\partial y}{\partial \lambda} \Big|_a^b - y_n \cdot S \frac{\partial^2 y}{\partial x \partial \lambda} \Big|_a^b \right\}_{\lambda=\lambda_n}$$

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$$\begin{aligned}\alpha y + \beta y' &= 0 \quad @ x=a \\ \alpha y + \beta y' &= 0 \quad @ x=b\end{aligned}$$

Solution to S.L. equation $y(x; \lambda)$ must be bounded
 " S.L. equation + B.C. $y_n(x; \lambda_n)$ u. $J_0(\frac{\omega_i}{c}r)$

$$\int_a^b P(x) y_n^2(x) dx = \frac{1}{2\lambda_n} \left\{ y_n' \cdot S \cdot \frac{\partial y}{\partial \lambda} \Big|_a^b - y_n \cdot S \frac{\partial^2 y}{\partial x \partial \lambda} \Big|_a^b \right\}_{\lambda=\lambda_n}$$

$$\int_0^a r J_0^2\left(\frac{\omega_i}{c}r\right) dr = \frac{1}{2\left(\frac{\omega_i}{c}\right)} \left\{ \frac{\omega_i}{c} J_0'\left(\frac{\omega_i}{c}r\right) \cdot r J_0'(\lambda r) \cdot r \Big|_0^a - J_0\left(\frac{\omega_i}{c}r\right) \cdot r J_0'' \cdot r \cdot \lambda \Big|_0^a \right\}_{\lambda=\lambda_n}$$

THERE IS A MISSING TERM IN THE PREVIOUS SLIDE. THE LAST TERM SHOULD BE $J_0(\omega_i r/c) \cdot r \cdot [J_0''(\lambda r) \cdot r \cdot \lambda + J_0'(\lambda r) \cdot 1]$.
 When evaluated, that term gives zero. That term is in the next slide.

Solution to S.L. equation $y(x; \lambda)$ must be bounded
 " S.L. equation + B.C. $y_n(x; \lambda_n)$ u. $J_0(\frac{\omega_i}{c}r)$

$$\int_a^b P(x) y_n^2(x) dx = \frac{1}{2\lambda_n} \left\{ y_n' \cdot S \cdot \frac{\partial y}{\partial \lambda} \Big|_a^b - y_n \cdot S \frac{\partial^2 y}{\partial x \partial \lambda} \Big|_a^b \right\}_{\lambda=\lambda_n}$$

$$\int_0^a r J_0^2\left(\frac{\omega_i}{c}r\right) dr = \frac{1}{2\left(\frac{\omega_i}{c}\right)} \left\{ \frac{\omega_i}{c} J_0'\left(\frac{\omega_i}{c}r\right) \cdot r J_0'(\lambda r) \cdot r \Big|_0^a - J_0\left(\frac{\omega_i}{c}r\right) \cdot r \left[J_0'' \cdot r \cdot \lambda + J_0' \cdot 1 \right] \Big|_0^a \right\}_{\lambda=\lambda_n}$$

$$= \frac{1}{2\left(\frac{\omega_i}{c}\right)} \left\{ \frac{\omega_i}{c} a^2 \left(J_0'\left(\frac{\omega_i}{c}a\right) \right)^2 \right\}$$

An inhomogeneous PDE with inhomogeneous BCs can be split into several problems in which only one of the BCs is non-zero

This principle can also be extended to incorporate nonhomogeneous boundary conditions. To illustrate, consider the boundary value problem

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = F(x, y), \quad 0 < x < L, \quad 0 < y < L',$$

$$V(0, y) = g_1(y), \quad 0 < y < L',$$

$$V(L, y) = g_2(y), \quad 0 < y < L',$$

$$V(x, 0) = h_1(x), \quad 0 < x < L,$$

$$V(x, L') = h_2(x), \quad 0 < x < L,$$

for potential in the rectangle of Figure 3.1. The solution is the sum of the functions $V_1(x, y)$, $V_2(x, y)$, and $V_3(x, y)$, satisfying the PDEs in Figure 3.2 together with the

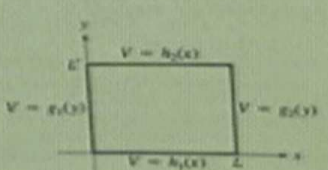


Figure 3.1

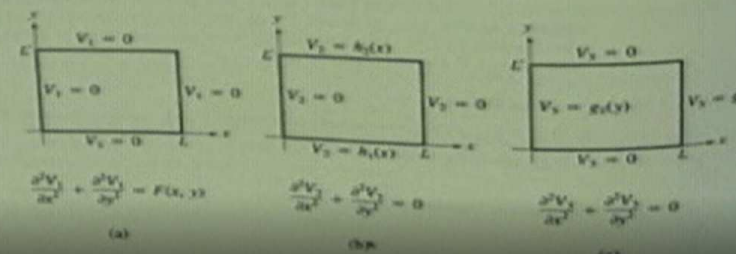


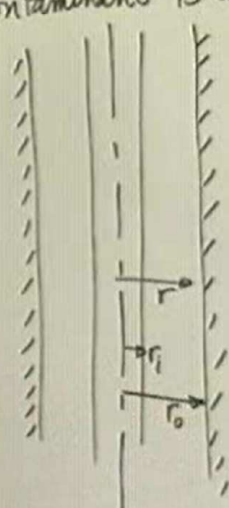
Figure 3.2

(a) $\frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial y^2} = F(x, y)$

(b) $\frac{\partial^2 V_2}{\partial x^2} + \frac{\partial^2 V_2}{\partial y^2} = 0$

(c) $\frac{\partial^2 V_3}{\partial x^2} + \frac{\partial^2 V_3}{\partial y^2} = 0$

Time History of Contaminant $c(r, t)$
in an annular region in which the
contaminant is always produced (source)



$$\frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) = \frac{r}{\alpha} \frac{\partial c}{\partial t} - rs$$

α diffusivity
 s source term

@ $r=r_0$ barrier block diffusion $\therefore \frac{\partial c}{\partial r} = 0$

Time History of Contaminant $C(r,t)$
in an annular region in which the
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$$\frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) = \frac{r}{\alpha} \frac{\partial C}{\partial t} - rs$$

α diffusivity

s source term

@ $r=r_o$ barrier block diffusion $\therefore \frac{\partial C}{\partial r} = 0$

@ $r=r_i$ $D \frac{\partial C}{\partial r} = h(C - C_\infty)$

@ $t=0$ $C = C_0$

h - convective transport coeff

C_∞ - fixed concentration of fluid passing the outer bore

D - diffusion coeff for contaminant in the solid

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