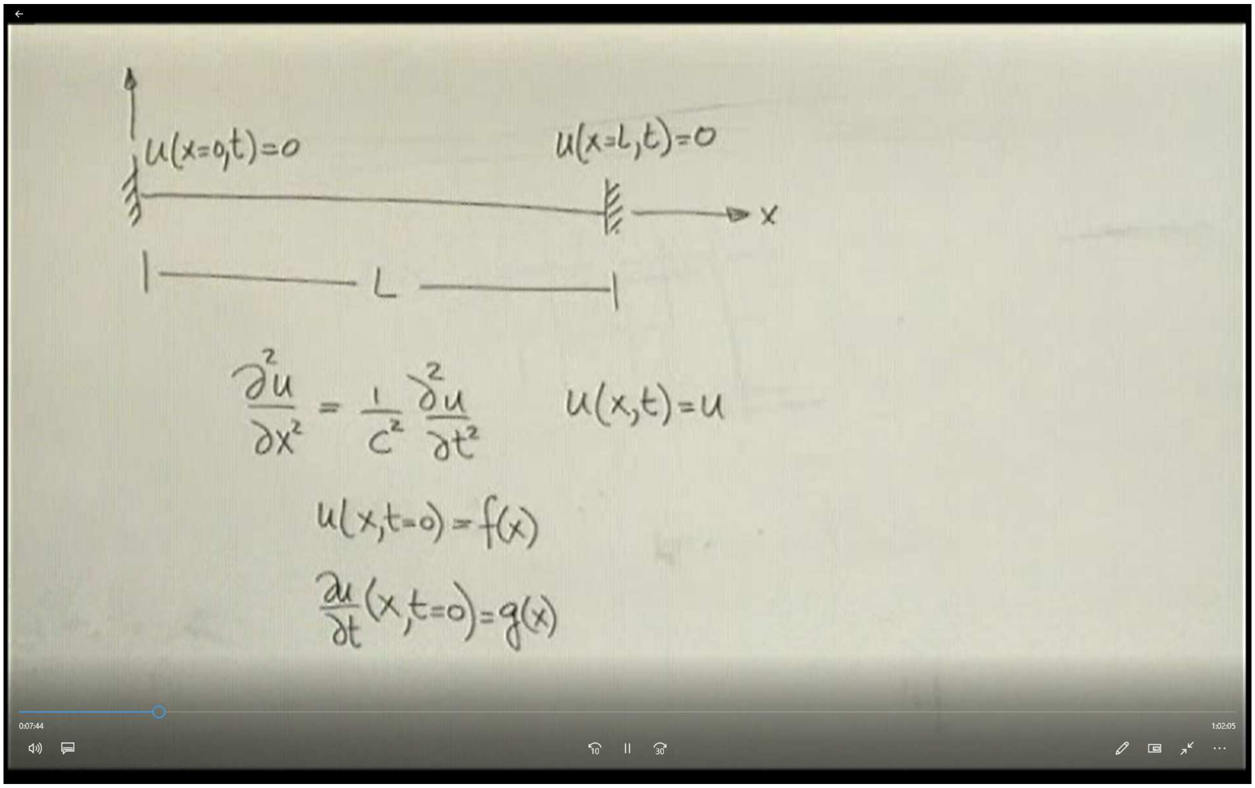
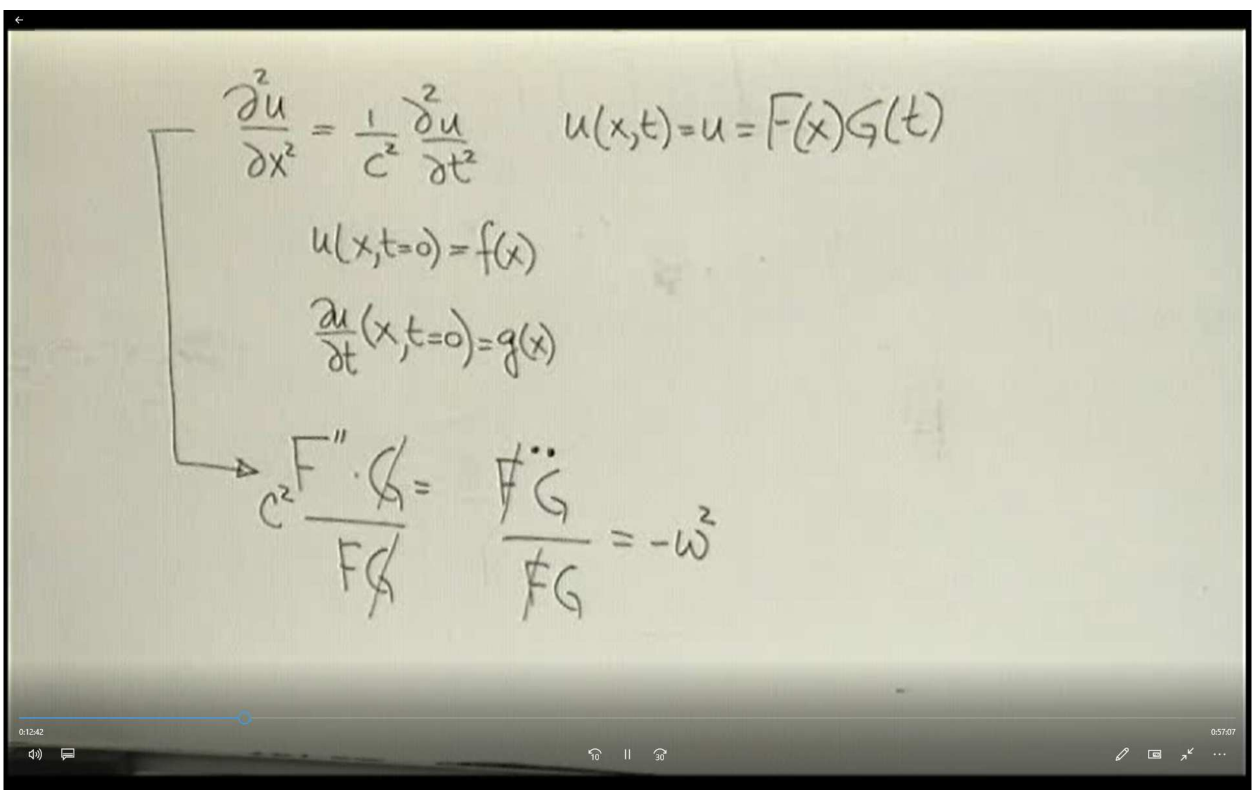
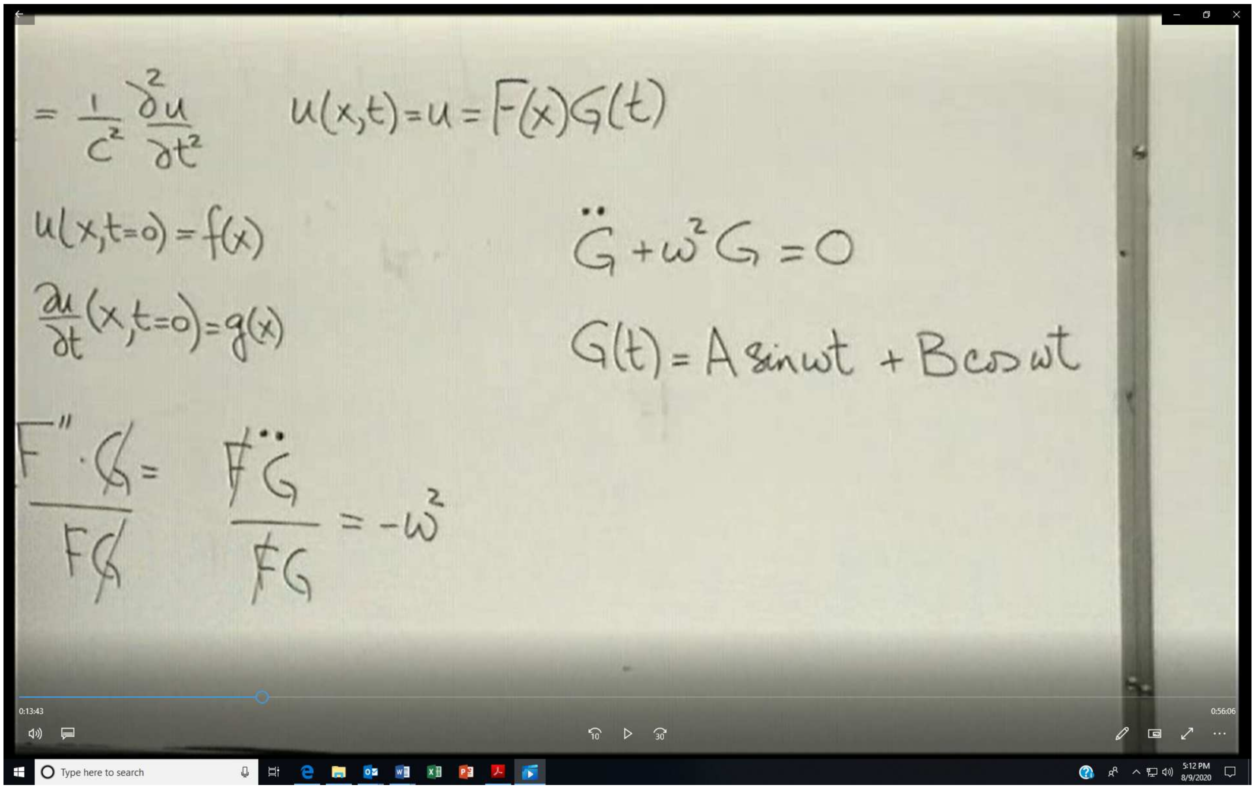
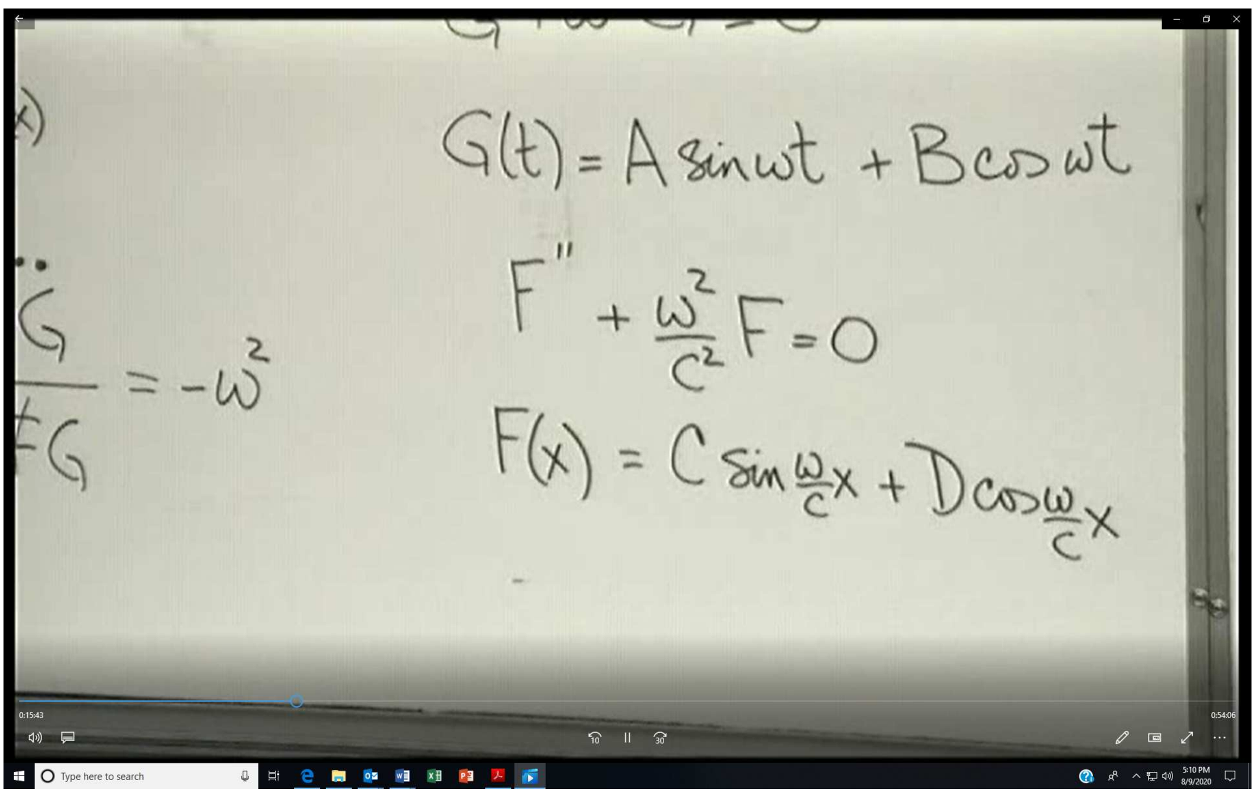
Solution of the vibrating string fixed at both ends using SOV



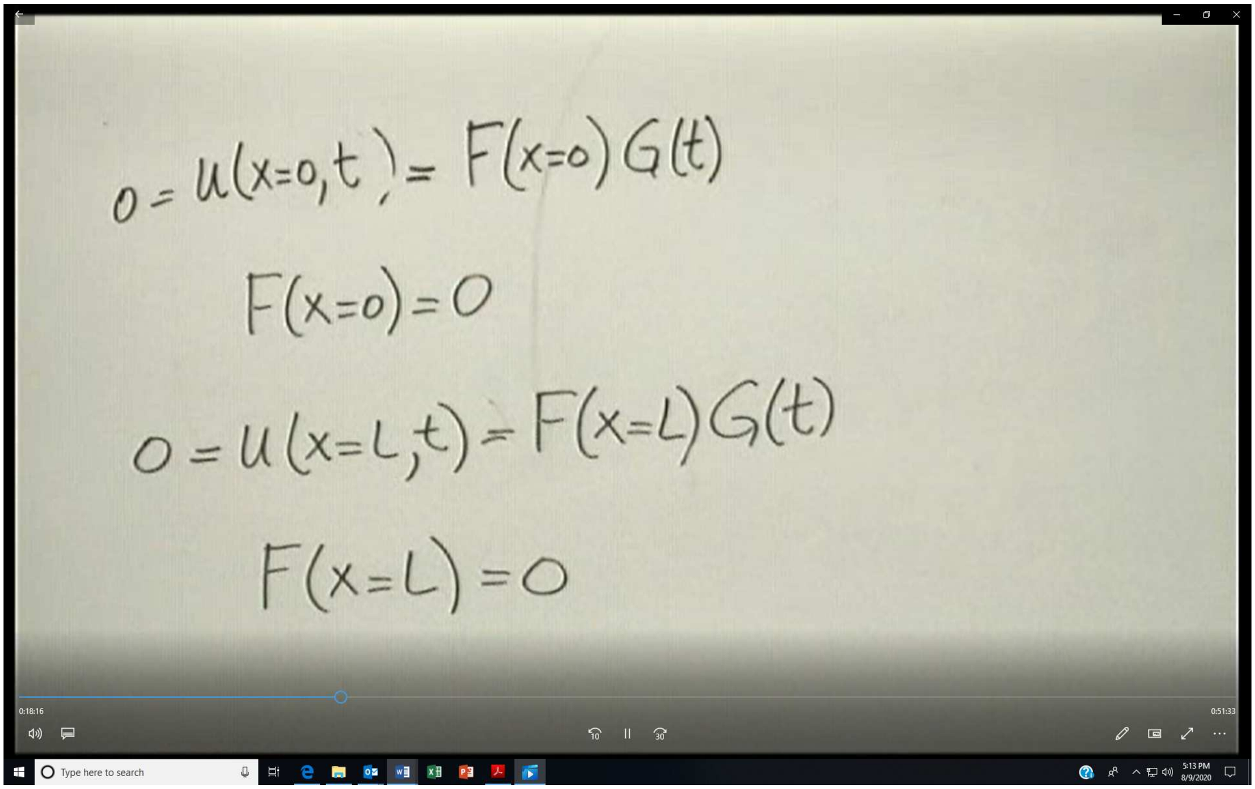
Assume u(x,t)=F(x)G(t). Put into the PDE and bring terms that involve x to one side and terms involving t to the other side. C^2 is brought to the x side so that the t side produces a simple equation



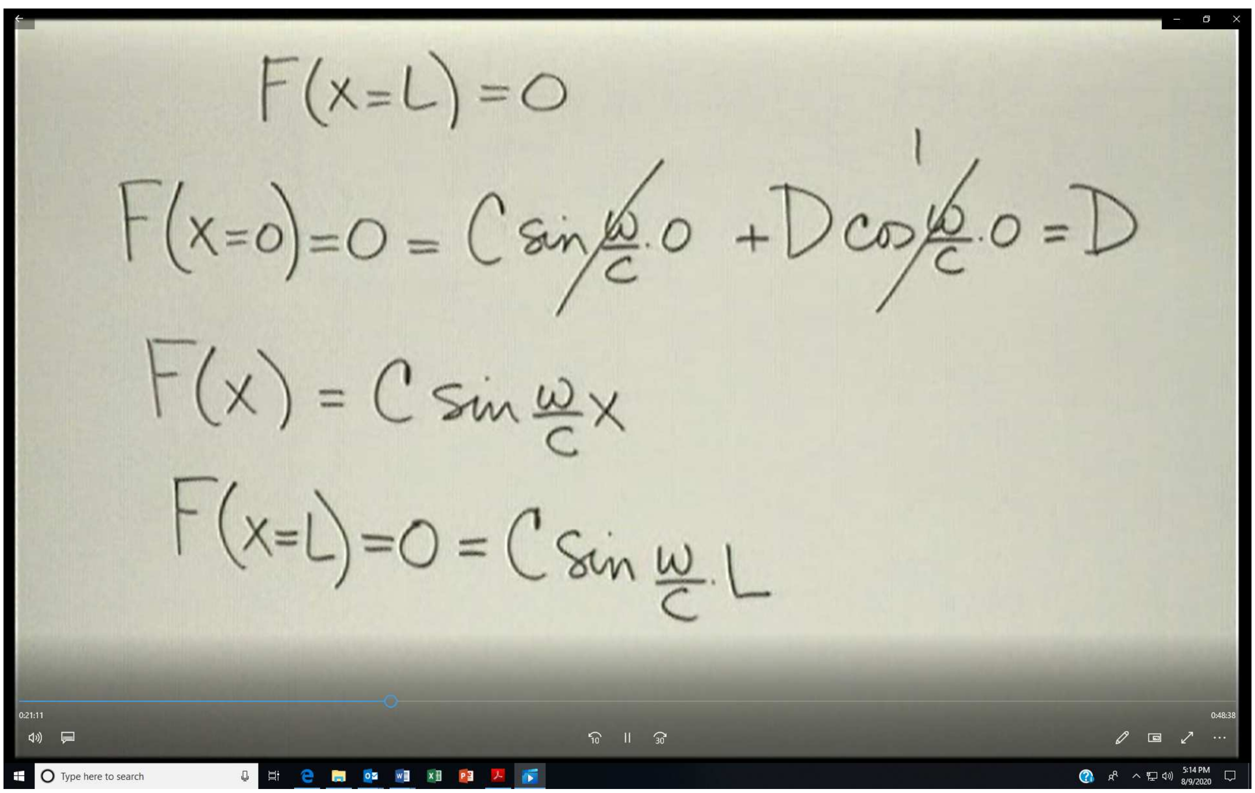


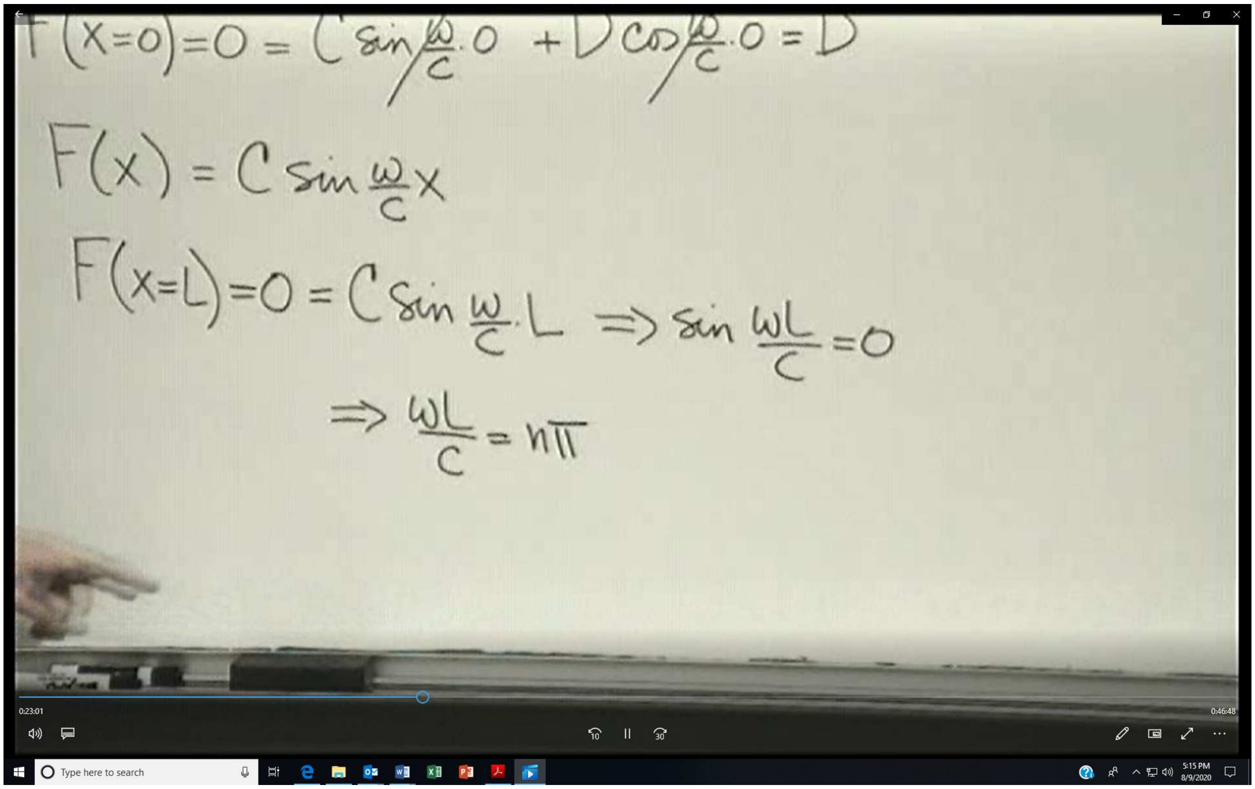


From the BCs we find:

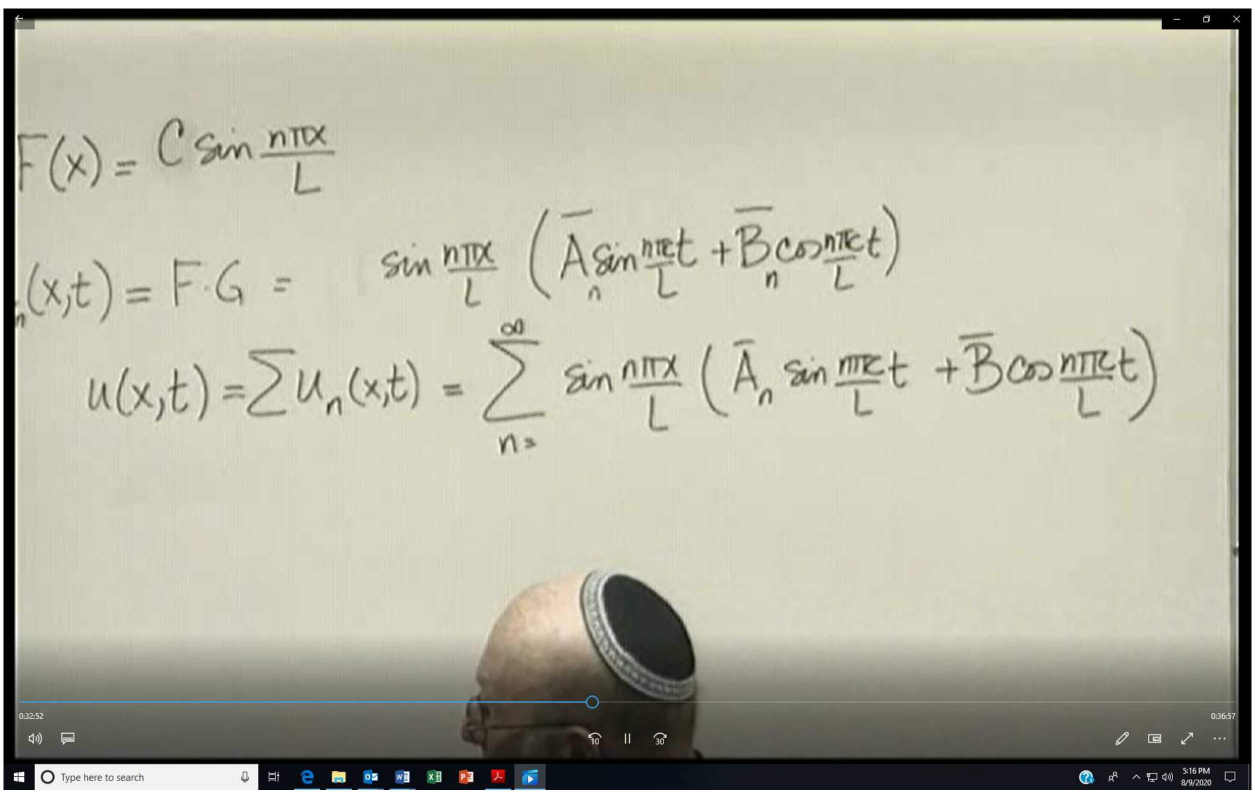


Applying these conditions to F(x) at x=L, leads to either C=0 or sin() =0. C=0 leads to a trivial solution. We are looking for nontrivial solutions.

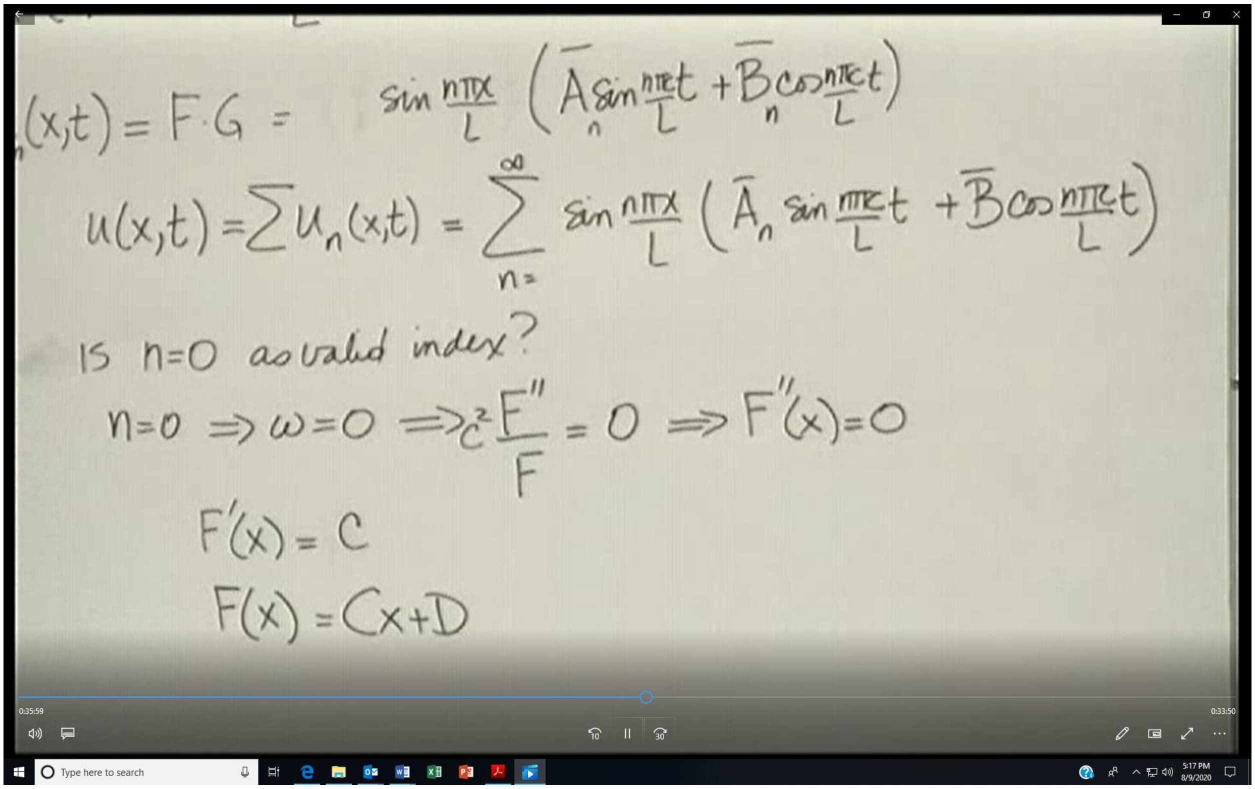


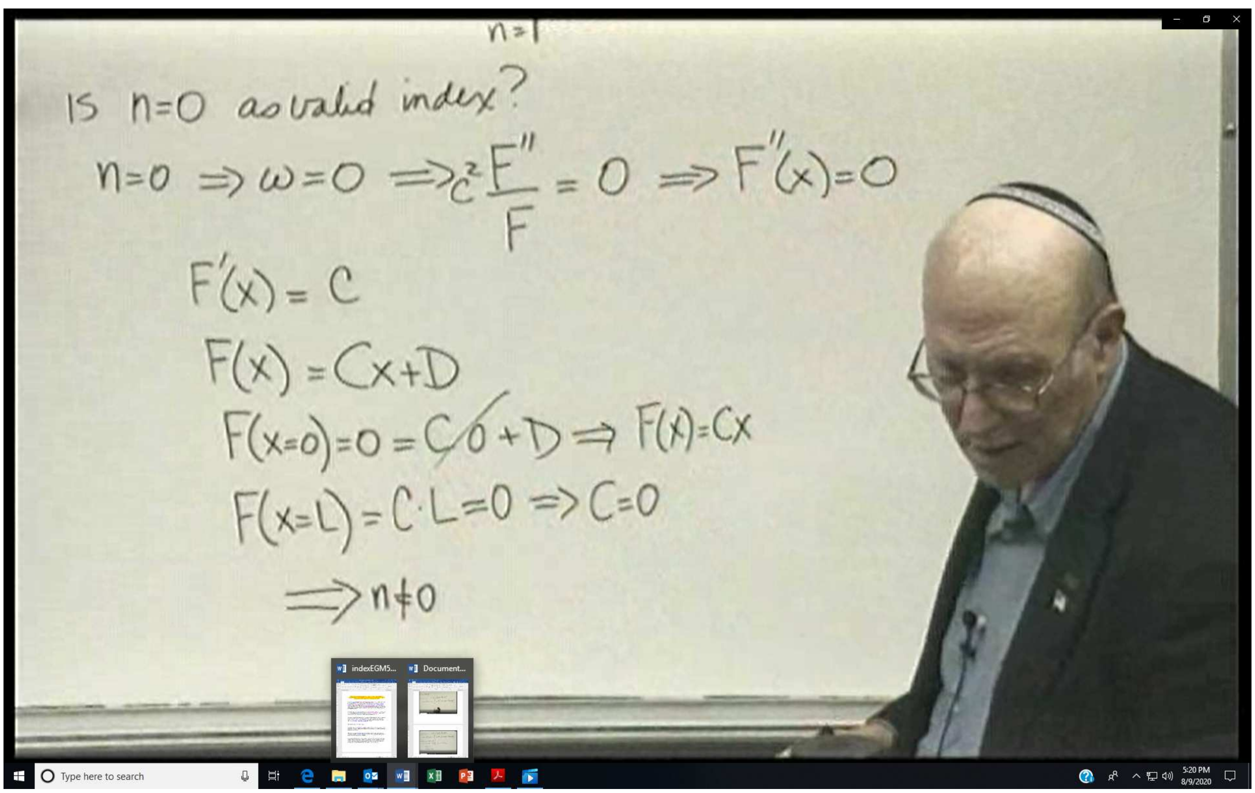


Now solve for omega and put back into the equation

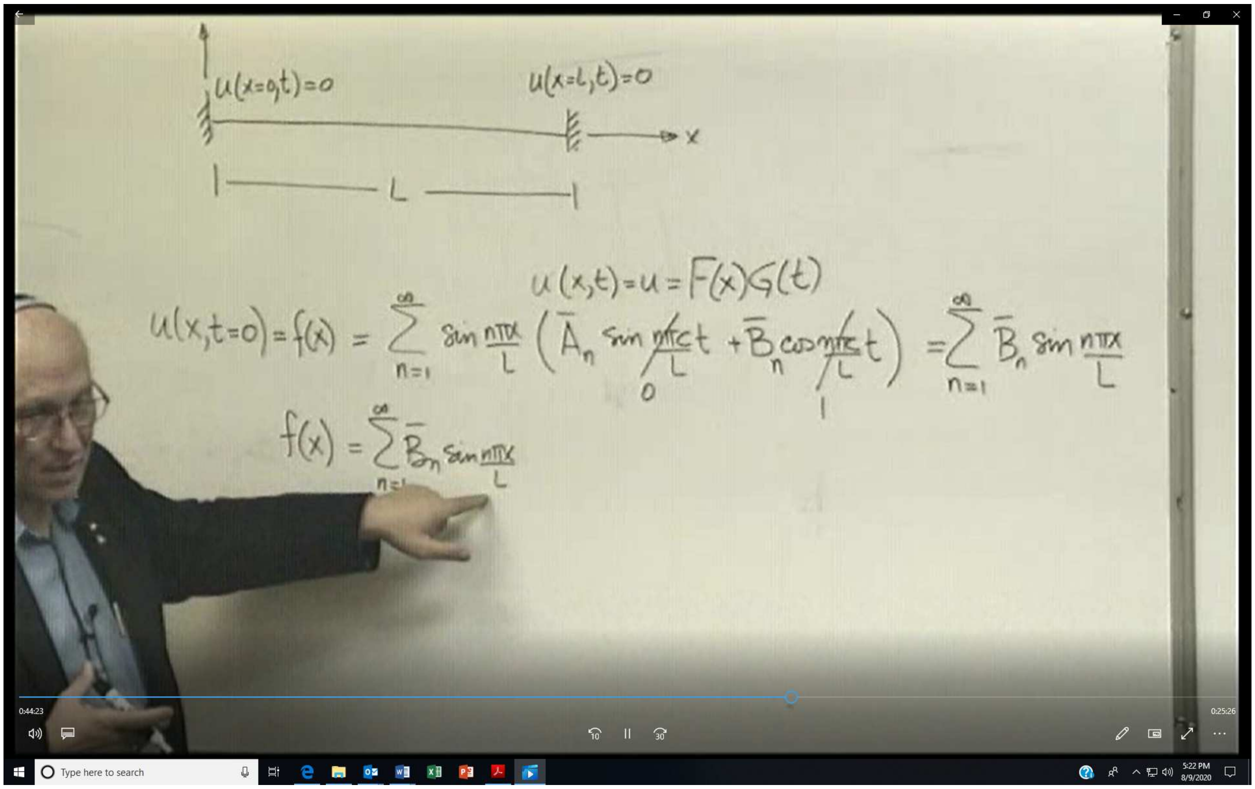


The question is “What is the starting value of the index n?”

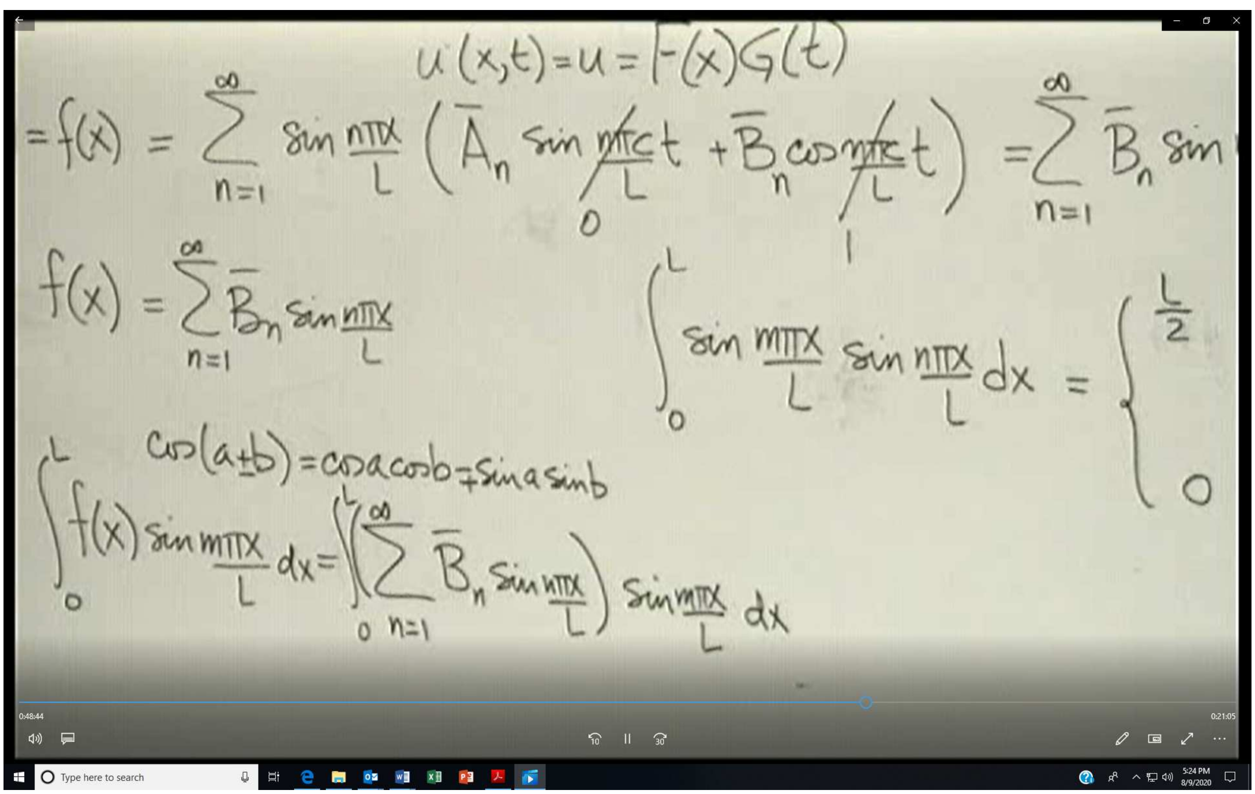




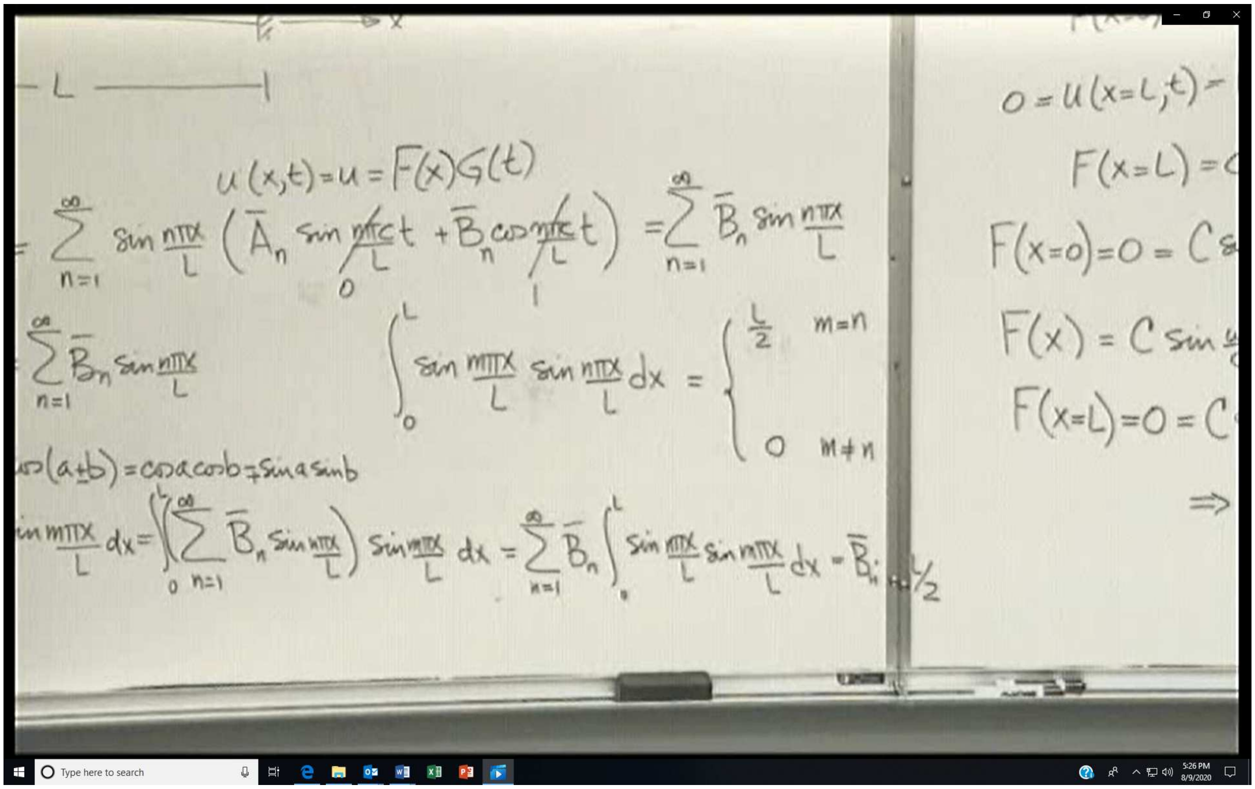
So we see that n=0 leads to the solution F(x)=0, which is a trivial solution. Now we begin applying the initial conditions to the solution u(x,t) that we found.



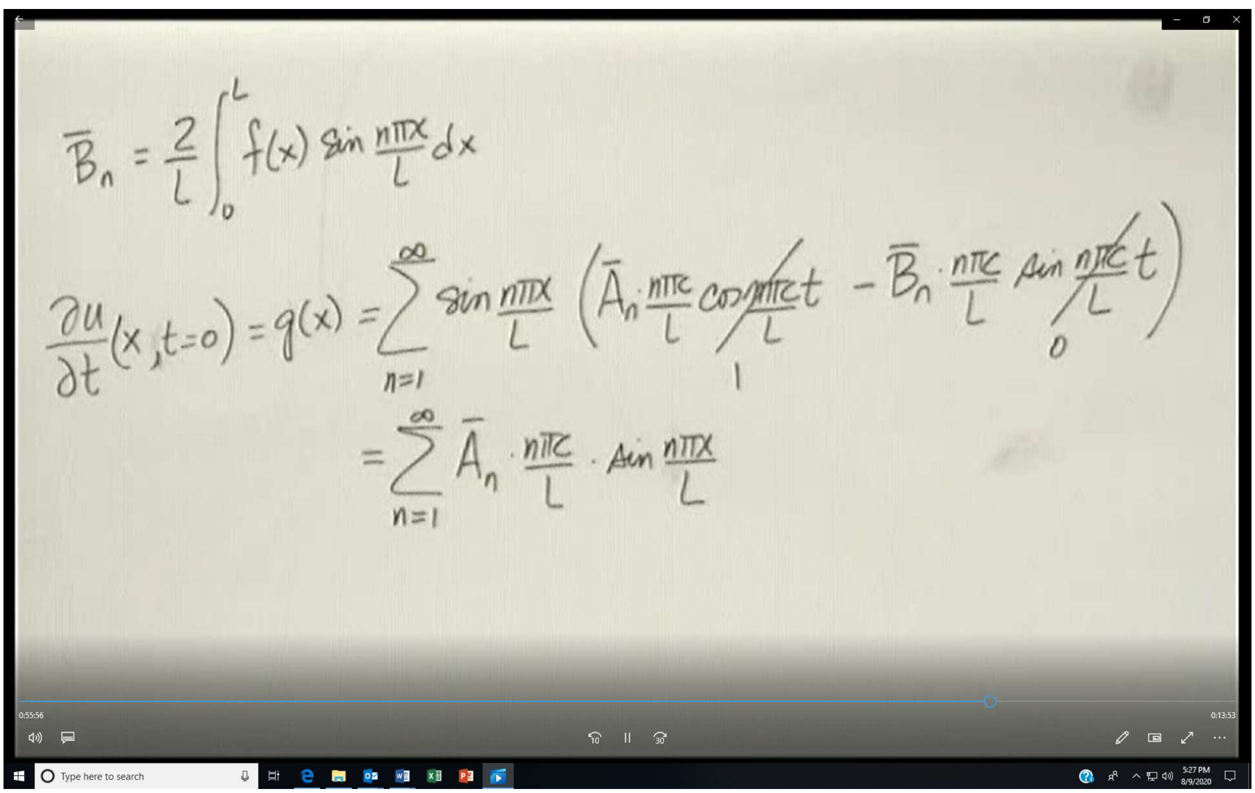
By multiplying both sides by the sin (m*pi*x/L) and integrating, we will find that the results depends on m and n

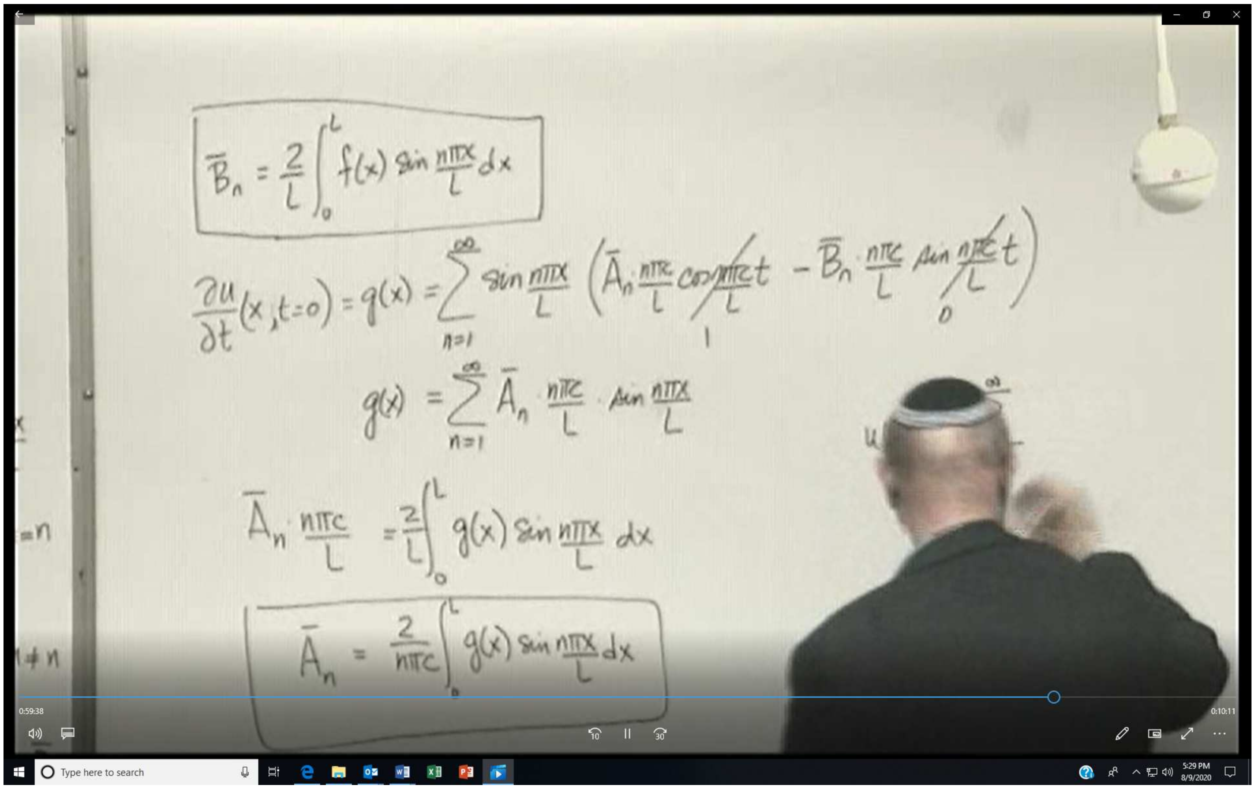


The only non zero term that is left is when m=n

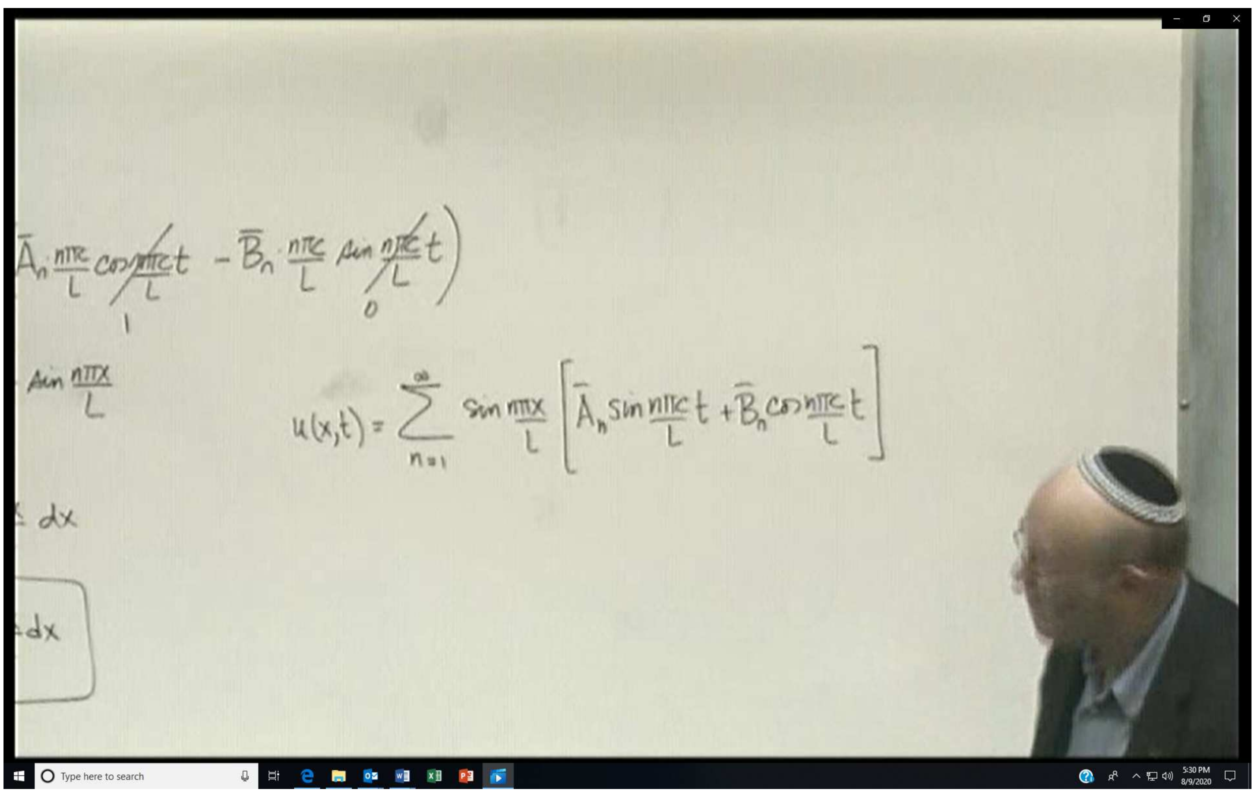


By using the second initial condition you can get a similar result to find the An

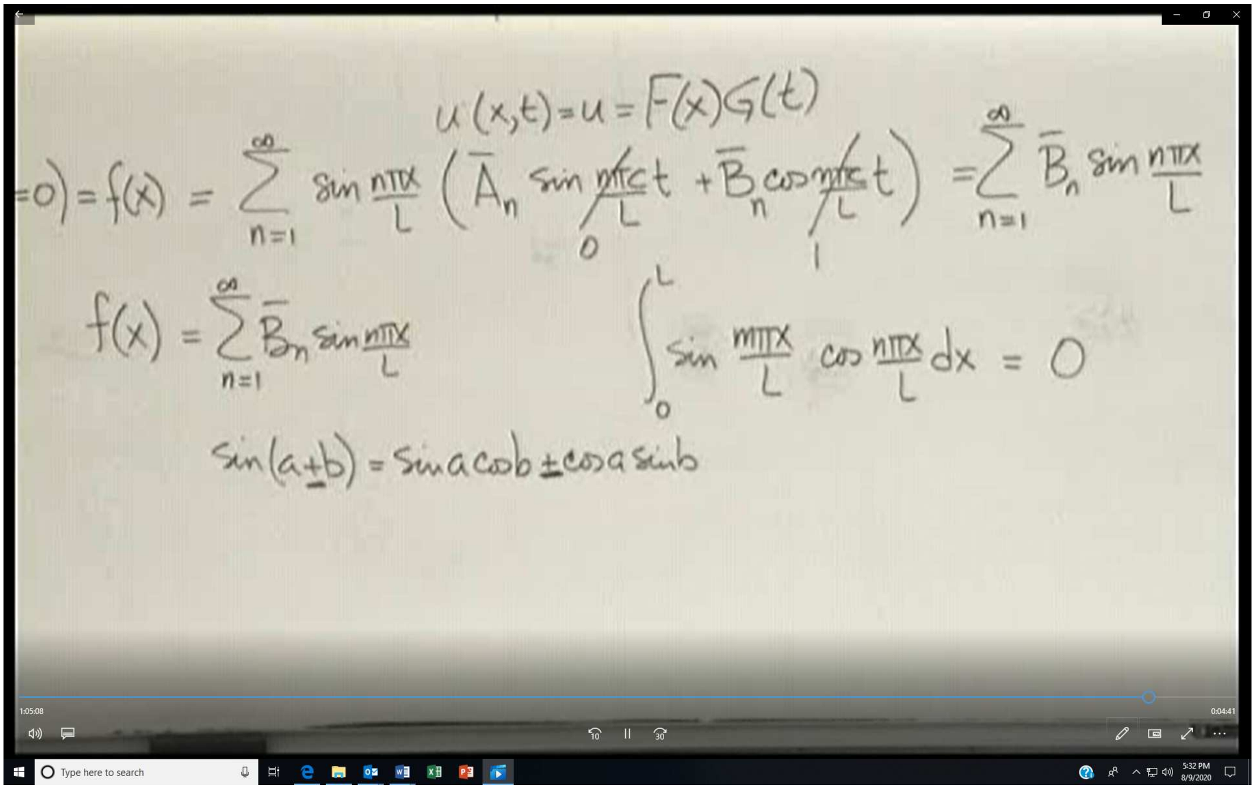




So our final solution is as shown below with the definitions for An and Bn shown above.



Thus the overall solution is an infinite series which is a function of n.



You can use the trigonometric identities to show that the integral of the sin *cos will be equal to zero irrespective of what m and n are.