$(1-x^2)y'' - 2xy' + 2y = 0$ near $x=\infty$. let $x = \frac{1}{t}$; $\frac{d}{dx} = -t^2\frac{d}{dt}$; $\frac{d^2}{dx^2} = t^4\frac{d^2}{dt^2} + 2t^3\frac{d}{dt}$

then $\left(1-\frac{1}{t^2}\right)\left[t^4Y''+2t^3Y'\right] - \frac{2}{t}\left[-t^2\frac{dY}{dt}\right] + 2Y = 0$

or

$$\frac{1}{t^2}(t^2-1)\left[t^4Y''+2t^3Y'\right] + 2tY' + 2Y = 0$$

$$t^2(t^2-1)Y'' + 2t^3Y' + 2Y = 0$$

$P_0(t) = t^2(t^2-1)$

$P_1(t) = 2t^3$ $\qquad P_0(t=0) = 0 \Rightarrow t=0$ $(x=\infty)$ is a singular pt

$P_2(t) = 2$ $\qquad \frac{P_1(t)}{P_0(t)}\cdot t = \frac{2t^3\cdot t}{t^2(t^2-1)} = \frac{2t^2}{t^2-1}$ $\quad \lim_{t\to 0} = \frac{0}{-1} = 0$ ✓

$$\frac{P_2(t)}{P_0(t)}\cdot t^2 = \frac{2t^2}{t^2(t^2-1)} = \frac{2}{t^2-1} \quad \lim_{t\to 0} = \frac{2}{-1} = -2 \checkmark$$ regular singular

let $Y(t) = \sum_{n=0}^{\infty} A_n t^{n+r}$ $\quad Y'(t) = \sum_{n=0}^{\infty} A_n (n+r) t^{n+r-1}$ $\quad Y''(t) = \sum_{n=0}^{\infty} A_n (n+r)(n+r-1) t^{n+r-2}$

$$(t^4-t^2)Y'' + 2t^3Y' + 2Y = \overset{①}{\sum_{n=0}^{\infty} A_n(n+r)(n+r-1)t^{n+r+2}} - \overset{③}{\sum_{n=0}^{\infty} A_n(n+r)(n+r-1)t^{n+r}} + \overset{②}{2\sum_{n=0}^{\infty} A_n(n+r)t^{[illegible]}}$$

$$+ \overset{④}{2\sum_{n=0}^{\infty} A_n t^{n+r}} = 0$$

$$(①+②)+(③+④) = \sum_{n=0}^{\infty} A_n t^{n+r+2}\left[\underbrace{(n+r)(n+r-1)+2(n+r)}_{(n+r)(n+r+1)}\right] - \sum_{n=0}^{\infty} A_n t^{n+r}\left[(n+r)(n+r-1)-2\right] = 0$$

now let $n+2 = m$ & $n = m-2$

$$\sum_{n=0}^{\infty} A_n t^{n+r+2}(n+r)(n+r+1) = \sum_{m=2} A_{m-2} t^{m+r}(m+r-2)(m+r-1)$$

and since n & m are arbitrary indices

$$\left\langle \sum_{n=2}^{\infty}\left\{A_{n-2}(n+r-2)(n+r-1)\right\} - \left\{\sum_{n=0}^{\infty} A_n\left[(n+r)(n+r-1)-2\right]\right\}\right\rangle t^{n+r} = 0 = \sum_{n=0}^{\infty} 0\cdot t^{n+r}$$

and $-A_0[(0+r)(0+r-1)-2]t^r - A_1[(1+r)(r)-2]t^{r+1} + \sum_{n=2}^{\infty} A_{n-2}(n+r-2)(n+r-1) - A_n[(n+r)(n+r-1)-2]\, t^{n+r} = 0 = \sum_{n=0}^{\infty} 0\, t^{n+r}$

so at $n=0$: $-A_0[(0+r)(0+r-1)-2] = 0 \Rightarrow$ either $A_0 = 0$ or $-r(r-1)+2 = +r^2-r-2 = 0 \Rightarrow r=2, r=-1$

if $r=2$ & $r=-1$, then at $n=+1$ $-A_1[(1+r)(r)-2] = 0$ but $r(r+1)-2 \neq 0 \Rightarrow A_1 \equiv 0$

for any n: $A_n[(n+r)(n+r-1)-2] - [(n+r-2)(n+r-1)]A_{n-2} = 0$ or $\boxed{A_n = A_{n-2}\frac{(n+r-2)(n+r-1)}{(n+r)(n+r-1)-2}}$

Recursion Formula and all odd [illegible]

use this formula with $r=2$ to find $y_1(t)$

since $|r_1 - r_2|$ = integer you know how to get second formula