$a \frac{\partial^2 u}{\partial x^2} + b \frac{\partial u}{\partial x \partial y} + c \frac{\partial^2 u}{\partial y^2} + F(x, y, u, u, u, u, u) = 0$ => hyperbolic PDE 6-4ac >0 = 0 => parabolic PDE <0 => elliptic PDE

Reminiscent of conic sections

6-4ac >0 => hyperbolic PDE = O => parabolic PDE <0 => elliptic PDE $ax^2+bxy+cy^2+dx+ey+f=0$ B-4ac>0 hyperbola =0 parabola Lo ellipse

Linear PDE since all are functions of x and y only, then

AUxx + BUxy + Cuyy + Dux + Euy + Fu+G=0 A, B, C, D, E, F, G are fins of X & y only IF B-4AC>0 $u_{\xi\xi} - u_{\eta\eta} + \tilde{b}_{,}u_{\xi} + \tilde{b}_{,}u_{\eta} + \tilde{c}u + \tilde{f} = 0$ X=5+1 B=3-1 $u_{\alpha\beta} - \overline{b}_i u_{\alpha} + \overline{b}_2 u_{\beta} + \overline{c} u + \overline{f} = 0$

All tilde variables and all barred variables are functions of ξ and η only for all the cases we discuss

• $u_{\alpha\beta} - \overline{b}_i u_{\alpha} + \overline{b}_z u_{\beta} + \overline{c}u + \overline{f} = 0$ IF B2-4AC=0 PARABOLIC PDE $u_{55} + \tilde{b}_{1}u_{5} + \tilde{b}_{2}u_{1} + \tilde{c}u + \tilde{f} = 0$ IF B2-4AC <O ELLIPTIC TYPE $u_{\overline{s}\overline{s}} + u_{\eta\eta} + \overline{b}_{,}u_{\overline{s}} + \overline{b}_{,}u_{\eta} + \widetilde{c}u + \overline{f} = 0$

Example problem

1 1 34xx+104xy+34yy=0 $B^2 - 4AC = 10^2 - 4(3)(3) = 64 > 0$ dy = B ± B2-4AC

 $\frac{dy}{dx} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{10 \pm 8}{6} = 3, \pm$ $\frac{dy}{dx} = 3 \implies y = 3x + C$ $\frac{dy}{dx} = 3 \implies y = 3x = \xi$ $\frac{dy}{dx} = \frac{1}{3} \implies y = \frac{1}{3}x + C$ と-シン=リ Type here to search $U_{\mathbf{X}} = U_{\mathbf{\xi}} \cdot \mathbf{\xi}_{\mathbf{X}} + U_{\mathbf{\eta}} \cdot \mathbf{\eta}_{\mathbf{X}}$ $u_{x} = u_{\overline{z}} \cdot (-3) + u_{\eta} (-\frac{1}{3})$ $U_{y} = U_{\xi} \cdot \xi_{y} + U_{\eta} \cdot \eta_{y}$ = Uz 1 + Un 1

$$U_{Y} = U_{\xi} \cdot S_{\xi} + u_{\eta} + H_{\eta} \cdot H_{\eta} + H_{\eta$$

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 $U_{xy} = \frac{\partial}{\partial y}(u_x) = \frac{\partial}{\partial x}(u_y)$ $= \frac{\partial}{\partial y}(\frac{\partial}{\partial y} + \frac{\partial}{\partial y}(\frac{\partial}{\partial y}) +$

 $= \frac{1}{25} () \cdot 5_{y} + \frac{1}{20} () \cdot 7_{y}$ = $\frac{1}{25} () \cdot 5_{y} + \frac{1}{20} () \cdot 7_{y}$ = $\frac{1}{25} [-3u_{s} - \frac{1}{3}u_{\eta}] \cdot 1 + \frac{1}{20} [-3u_{s} - \frac{1}{3}u_{$ = - 3UEE - 19 UEN - 3UM O Type here to search w ×⊞

Take the expressions and put them in the original equation to get

7 7 34xx+104xy+34yy=0 27455 + 6 Uzy + 5 Uzy - 30455 - 13 Uzy - 13 +3455 +6457 + 3497 =0

34xx + 104xy + 34yy = 0 27455 +6457 + 3 497 -30455 - 13 457 - 13 497 +3455 +6457 + 3497 =0 4-3X=5 $\left(-\frac{64}{3}\right)u_{\text{sg}} = 0$ y-3×=7 UEN = O

This is the canonical form for the example problem.

Now suppose we add a $2u_x$ to the original equation

A = A = A = A = 0 $3u_{xx} + 10u_{xy} + 3u_{yy} + 2u_x = 0$ $U_x =$ 27455 + 64 37 + 3 un - 30455 - 13 un - 13 un +3455+6457+3497+2[-345-347]=0 Uy= $(-\frac{64}{3})u_{g\eta} - 6u_g - \frac{2}{3}u_\eta = 0$ Uxx O Type here to search

By making the first coefficient equal to one we get

34xx+104xy+34yy+24x=0 27485 + 6457 + 5 4ng - 30455 - 13 459 - 13 497 +3455+6459+3492+2[-345-349]=0 (-64) UEN -645-347=0 Usy + 9 Us + - Uy = 0 oz w∎ x∄

 $6U_{\xi} - \frac{2}{3}U_{\eta} = 0$ $f = \frac{1}{32} u_{5} + \frac{1}{32} u_{1} = 0$ Usy=0 $u_{\xi} = f(\xi)$ $u = \left(f_{1}(\xi)d\xi + g(\eta) = f_{2}(\xi) + g(\eta)\right)$ 📄 🔯 🖬 🗱 🛐

If we had the first problem, integrating along the characteristics given

Creating the reduced canonical forms (if coeffs of first derivatives are constant)

let $U(\xi, \eta) = e^{\lambda \xi + \mu \eta} U(\xi, \eta)$ = $e^{\lambda \xi + \mu \eta} \cdot \lambda U + e^{\lambda \xi + \mu \eta}$ ×П 02 w PE

+ 4 $u_{\eta} = e^{\chi_{\xi} + \mu \eta} \left[\mu \sigma + \sigma_{\eta} \right]$ $U_{sy} = \frac{\partial}{\partial s} [u_{y}] = \lambda e^{\lambda s + \mu \eta} [\mu v + v_{y}]$ + UNE + UNE exexpen + [UV = + Un =] e x = + un = enstand [unit + hung + ung + ung ×П w

When we apply to the second problem



By choosing the coefficients of the first derivative terms to be equal to zero, we solve for λ and μ

Choose $\mu = -\frac{4}{32}$ } $\frac{2}{32} \frac{2}{32} \frac{2}{32} \frac{2}{32} \frac{2}{32} \frac{1}{32} \frac{1}{3} \frac{1}$ UEN -9 U=0

We can divide by the exponential since it is not equal to zero. What is left is called the reduced canonical form.

If this problem had boundary and initial conditions on u, if you plan to use the reduced canonical form, then you much convert the boundary and initial conditions to conditions on v. This means that the BCs and the ICs may become very complicated.