6.9 through 6.12 For the beam and loading shown, consider section n-n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.

\[ V = 45 \text{ kips} \]

**Table**

<table>
<thead>
<tr>
<th>Part</th>
<th>( A (\text{in}^2) )</th>
<th>( d (\text{in}) )</th>
<th>( A\bar{d}^2 (\text{in}^4) )</th>
<th>( \bar{I} (\text{in}^4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flange</td>
<td>6.00</td>
<td>4.7</td>
<td>132.54</td>
<td>0.18</td>
</tr>
<tr>
<td>Web</td>
<td>3.30</td>
<td>0</td>
<td>0</td>
<td>21.296</td>
</tr>
<tr>
<td>Flange</td>
<td>6.00</td>
<td>4.7</td>
<td>132.54</td>
<td>0.18</td>
</tr>
<tr>
<td>[ \Sigma ]</td>
<td></td>
<td></td>
<td>265.08</td>
<td>21.656</td>
</tr>
</tbody>
</table>

\[ I = \Sigma A\bar{d}^2 + \Sigma \bar{I} \]
\[ = 265.08 + 21.656 \]
\[ = 286.736 \text{ in}^4 \]

\[ Q = A_1\bar{y}_1 + A_2\bar{y}_2 \]
\[ = (6.00)(4.7) + (0.375)(4.4)(2.2) = 31.83 \text{ in}^3 \]

\[ t = 0.375 \text{ in.} \]

\[ \gamma_{\text{max}} = \frac{VQ}{It} = \frac{(45)(31.83)}{(286.736)(0.375)} = 13.32 \text{ ksi} \]

\[ Q_a = A_1\bar{y}_1 + A_3\bar{y}_3 \]
\[ = (6.00)(4.7) + (0.375)(0.4)(4.4 + 4.0) \]
\[ = 28.83 \text{ in}^3 \]

\[ t = 0.375 \text{ in.} \]

\[ \gamma_a = \frac{VQ_a}{It} = \frac{(45)(28.83)}{(286.736)(0.375)} = 12.07 \text{ ksi} \]
6.9 through 6.12 For the beam and loading shown, consider section n-n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.

By symmetry \( R_A = R_B \)

\[ +1 \sum F_y = 0: \quad R_A + R_B - 10 - 10 = 0 \]

\[ R_A = R_B = 10 \text{ kips} \]

From the shear diagram, \( V = 10 \text{ kips at n-n} \).

\[ I = \frac{1}{12} b h^3 - \frac{1}{12} b h^3 \]

\[ = \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4 \]

\[ Q = A_1 y_1 + A_2 y_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1) \]

\[ = 4.625 \text{ in}^3 \]

\[ t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.} \]

\[ \tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17 \text{ ksi} \]

(b) \[ Q = A \bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3 \]

\[ t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.} \]

\[ \tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40 \text{ ksi} \]
Problem 6.18

For the wide-flange beam with the loading shown, determine the largest \( P \) that can be applied, knowing that the maximum normal stress is 160 MPa and the largest shearing stress using the approximation \( \tau_m = V/A_{\text{web}} \) is 100 MPa.

\[
+3 \sum M = 0 = -3.6 R_A + 3.0 P + 2.4 P + 1.8 P = 0
\]

\[ R_A = 2P \]

Draw shear and bending moment diagrams.

\[ M_B = 2P L_{AB} \quad M_C = M_D = 3P L_{AB} \]

\[ |V|_{\text{max}} = 2P \quad |M|_{\text{max}} = 3P L_{AB} \]

**Bending.** For \( W360 \times 122 \)

\[
S = 2010 \times 10^3 = 2010 \times 10^6
\]

\[
\frac{|M|_{\text{max}}}{G_{\text{all}}} = \frac{3PL_{AB}}{6_{\text{all}}} = S
\]

\[
P = \frac{G_{\text{all}} S}{3L_{AB}} = \frac{(160 \times 10^6)(2010 \times 10^3)}{(3)(0.6)} = 178.7 \times 10^3
\]

**Shear.** \( A_{\text{web}} = d t_w = (363)(13.0) = 4.719 \times 10^3 \text{ mm}^2 = 4.719 \times 10^3 \]

\[
\tau = \frac{|V|_{\text{max}}}{A_{\text{web}}} = \frac{2P}{A_{\text{web}}}
\]

\[
P = \frac{\tau A_{\text{web}}}{2} = \frac{(100 \times 10^6)(4.719 \times 10^3)}{2} = 236 \times 10^3
\]

The smaller value of \( P \) is the allowable one. \( P = 178.7 \text{ kN} \)
Problem 6.22

6.21 and 6.22 For the beam and loading shown, consider section \( n-n \) and determine the shearing stress at (a) point \( a \), (b) point \( b \).

\[ R_A = R_B = 12 \, \text{kips} \]

Draw shear diagram.

\[ V = 12 \, \text{kips} \]

Determine section properties.

<table>
<thead>
<tr>
<th>Part</th>
<th>( A (\text{in}^2) )</th>
<th>( \bar{y} (\text{in}) )</th>
<th>( A\bar{y} (\text{in}^3) )</th>
<th>( d (\text{in}) )</th>
<th>( Ad^2 (\text{in}^4) )</th>
<th>( I (\text{in}^4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>16</td>
<td>2</td>
<td>16</td>
<td>5.333</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>1</td>
<td>8</td>
<td>-1</td>
<td>8</td>
<td>2.667</td>
</tr>
<tr>
<td>Σ</td>
<td>12</td>
<td>24</td>
<td>24</td>
<td></td>
<td>8.000</td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \, \text{in.} \]

\[ I = \Sigma Ad^2 + \Sigma I = 32 \, \text{in}^4 \]

(a) \[ A = 1 \, \text{in}^2 \quad \bar{y} = 3.5 \, \text{in.} \quad Q_a = A\bar{y} = 3.5 \, \text{in}^3 \]

\[ t = 1 \, \text{in.} \]

\[ \tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} = 1.3125 \, \text{ksi} \]

(b) \[ A = 2 \, \text{in}^2 \quad \bar{y} = 3 \, \text{in.} \quad Q_b = A\bar{y} = 6 \, \text{in}^3 \]

\[ t = 1 \, \text{in.} \]

\[ \tau_b = \frac{VQ_b}{It} = \frac{(12)(6)}{(32)(1)} = 2.25 \, \text{ksi} \]
6.25 through 6.28 A beam having the cross section shown is subjected to a vertical shear \( V \). Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant \( k \) in the following expression for the maximum shearing stress

\[
\tau_{\text{max}} = k \frac{V}{A}
\]

where \( A \) is the cross-sectional area of the beam.

\[
A = 2 \left( \frac{1}{2}bh \right) = bh \\
I = 2 \left( \frac{1}{6}bh^3 \right) = \frac{b}{6}h^3
\]

For a cut at location \( y \), where \( y \leq h \)

\[
A(y) = \frac{1}{2} \left( \frac{by}{h} \right)y = \frac{by^2}{2h}
\]

\[
\bar{y}(y) = h - \frac{2}{3}y
\]

\[
Q(y) = A \bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}
\]

\[
t(y) = \frac{by}{h}
\]

\[
\tau(y) = \frac{VQ}{It} = V \frac{6}{bh^2} \cdot \frac{h}{by} \cdot \frac{by^2}{2} - \frac{by^3}{3h} = \frac{V}{bh} \left[ 3(\frac{y}{h}) - 2(\frac{y^2}{h}) \right]
\]

(a) To find the location of maximum of \( \tau \), set \( \frac{d\tau}{dy} = 0 \).

\[
\frac{d\tau}{dy} = \frac{V}{bh^2} \left[ 3 - 4 \frac{y_m}{h} \right] = 0 \\
\frac{y_m}{h} = \frac{3}{4} \quad \text{i.e.} \quad \pm \frac{3}{4}h \text{ from neutral axis.}
\]

(b) \[
\tau(y_m) = \frac{V}{bh} \left[ 3 \left( \frac{3}{4} \right) - 2 \left( \frac{3}{4} \right)^2 \right] = \frac{9}{8} \frac{V}{bh} = 1.125 \frac{V}{A} \quad k = 1.125
Problem 6.38

6.37 and 6.38 The extruded beam shown has a uniform wall thickness of \( \frac{1}{8} \) in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.

\[ I = \frac{1}{12} (2.50)(2.50)^3 - \frac{1}{12} (2.125)(2.25)^3 = 1.2382 \text{ in}^4 \]

\[ t = 0.125 \text{ in. at all sections.} \]

\[ V = 2 \text{ kips} \]

\[ Q_a = 0 \quad \tau_a = \frac{VQ_a}{It} = 0 \]

\[ Q_b = (0.125)(1.25 \times \frac{1.25}{2}) = 0.09766 \text{ in}^3 \]

\[ \tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} = 1.26 \text{ ksi} \]

\[ Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in}^3 \]

\[ \tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} = 3.30 \text{ ksi} \]

\[ Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929 \]

\[ \tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} = 6.84 \text{ ksi} \]

\[ Q_e = Q_d + (0.125)(1.125 \times \frac{1.125}{2}) = 0.60839 \]

\[ \tau_e = \frac{VQ_e}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} = 7.86 \text{ ksi} \]
Problem 6.41

6.41 An extruded beam has the cross section shown and a uniform wall thickness \( t = 0.20 \) in. Knowing that a given vertical shear \( V \) causes a maximum shearing stress \( \tau = 9 \) ksi, determine the shearing stress at the four points indicated.

\[
Q_a = (0.2)(0.5)(0.5 - 0.25) = 0.125 \text{ in}^3
\]

\[
Q_b = (0.2)(0.5)(0.3 + 0.25) = 0.055 \text{ in}^3
\]

\[
Q_c = Q_a + Q_b + (1.4)(0.2)(0.9) = 0.432 \text{ in}^3
\]

\[
Q_d = 2Q_a + 2Q_b + (3.0)(0.2)(0.9) = 0.900 \text{ in}^2
\]

\[
Q_m = Q_d + (0.2)(0.8)(0.4) = 0.964 \text{ in}^3
\]

\[
\tau = \frac{VQ}{It}
\]

Since \( V, I, \) and \( t \) are constant, \( \tau \) is proportional to \( Q \).

\[
\frac{\tau_a}{0.125} = \frac{\tau_b}{0.055} = \frac{\tau_c}{0.432} = \frac{\tau_d}{0.900} = \frac{\tau_m}{0.964} = \frac{9}{0.964}
\]

\[
\tau_a = 1.167 \text{ ksi}; \quad \tau_b = 0.513 \text{ ksi}; \quad \tau_c = 4.03 \text{ ksi}; \quad \tau_d = 8.40 \text{ ksi}
\]
6.94 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (a) at A, (b) at B. (Given: \( I_x = 1.504 \times 10^9 \text{ mm}^4 \).)

\[
\begin{align*}
I_x &= 1.504 \times 10^9 \text{ mm}^4 = 1504 \times 10^{-6} \text{ m}^4 \\
S_A &= 60 \text{ mm} = 0.060 \text{ m} \\
S_B &= 25 \text{ mm} = 0.025 \text{ m}
\end{align*}
\]

(a) \( Q_A = Q_1 = A_1 \bar{y}_1 = (50 \times 100) \times 150 = 750 \times 10^3 \text{ mm}^3 \)

\[
F_A = q_f A_S = \frac{VQ_1 S_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}} = 239 \text{ N}
\]

(b) \( Q_2 = A_2 \bar{y}_2 = (300) \times 50 \times 175 = 2625 \times 10^3 \text{ mm}^3 \)

\( Q_B = 2Q_1 + Q_2 = 4125 \times 10^3 \text{ mm}^3 \)

\[
F_B = q_f A_S = \frac{VQ_2 S_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}} = 549 \text{ N}
\]