Problem 6.17

6.17 For the wide-flange beam with the loading shown, determine the largest load $P$ that can be applied, knowing that the maximum normal stress is 24 ksi and the largest shearing stress using the approximation $\tau_m = V/A_{web}$ is 14.5 ksi.

\[ \sum M_c = 0: -15R_a + 9P = 0 \]
\[ R_a = 0.6P \]

Draw shear and bending moment diagrams.

\[ |V|_{max} = 0.6P \quad |M|_{max} = 0.6P L_{AB} \]

\[ L_{AB} = 6 \text{ ft} = 72 \text{ in.} \]

Bending. For W24x104, $S = 258 \text{ in}^3$

\[ S = \frac{|M|_{max}}{6_{all}} = \frac{0.6P L_{AB}}{6_{all}} \]

\[ P = \frac{6_{all} S}{0.6 L_{AB}} = \frac{(24)(258)}{(0.6)(72)} = 143.3 \text{ kips} \]

Shear. $A_{web} = d t_w$

\[ = (24.06)(0.500) \]
\[ = 12.03 \text{ in}^2 \]

\[ \tau = \frac{|V|_{max}}{A_{web}} = \frac{0.6P}{A_{web}} \]

\[ P = \frac{\tau A_{web}}{0.6} = \frac{(14.5)(12.03)}{0.6} = 291 \text{ kips} \]

The smaller value of $P$ is the allowable value. $P = 143.3 \text{ kips}$
6.21 and 6.22 For the beam and loading shown, consider section n-n and determine the shearing stress at (a) point a, (b) point b.

- **Section n-n**
  - **Part** A (mm$^2$) | $y$ (mm) | $Ay$ (10$^3$ mm$^3$) | d (mm) | $Ad^2$ (10$^6$ mm$^4$) | $I$ (10$^6$ mm$^4$)
  - 1 3200 90 788 25 2.000 0.1067
  - 2 1600 40 64 -25 1.000 0.8533
  - 3 1600 40 64 -25 1.000 0.8533
  - **Σ** 6400 416 4.000 1.8133

- **Shear force diagram**
  - $V_{max} = 90$ kN

- **Moment of Inertia**
  \[ I = \sum A d^2 + \sum I = (4.000 + 1.8133) \times 10^6 \text{ mm}^4 = 5.8133 \times 10^6 \text{ mm}^4 = 5.8133 \times 10^{-6} \text{ m}^4 \]

- **Shear Stress (a)**
  \[ A = (80)(10) = 1600 \text{ mm}^2 \]
  \[ \bar{y} = 25 \text{ mm} \]
  \[ Q_a = A \bar{y} = 40 \times 10^3 \text{ mm}^3 = 40 \times 10^{-6} \text{ m}^3 \]
  \[ \tau_a = \frac{VQ_a}{It} = \frac{(90 \times 10^3)(40 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^3)} = 31.0 \times 10^6 \text{ Pa} = 31.0 \text{ MPa} \]

- **Shear Stress (b)**
  \[ A = (30)(20) = 600 \text{ mm}^2 \]
  \[ \bar{y} = 65 - 15 = 50 \text{ mm} \]
  \[ Q_b = A \bar{y} = 30 \times 10^3 \text{ mm}^3 = 30 \times 10^{-6} \text{ m}^3 \]
  \[ \tau_b = \frac{VQ_b}{It} = \frac{(90 \times 10^3)(30 \times 10^{-6})}{(5.8133 \times 10^{-6})(20 \times 10^3)} = 23.2 \times 10^6 \text{ Pa} = 23.2 \text{ MPa} \]
Problem 6.28

A beam having the cross section shown is subjected to a vertical shear \( V \). Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant \( k \) in the following expression for the maximum shearing stress

\[
\tau_{\text{max}} = k \frac{V}{A}
\]

where \( A \) is the cross-sectional area of the beam.

\[
A = \frac{1}{2} bh \quad I = \frac{1}{36} bh^3
\]

For a cut at location \( y \),

\[
A(y) = \frac{1}{2} \left( \frac{by}{h} \right) y = \frac{by^2}{2h}
\]

\[
\bar{y}(y) = \frac{2}{3} h - \frac{2}{3} y
\]

\[
Q(y) = A \bar{y} = \frac{by^2}{3} (h - y)
\]

\[
\tau(y) = \frac{VA}{It} = \frac{V \frac{by^2}{3} (h - y)}{\left( \frac{1}{36} bh^3 \right) \frac{by}{h}} = \frac{12 V y (h - y)}{bh^3} = \frac{12 V}{bh^3} (hy - y^2)
\]

(a) To find location of maximum of \( \tau \), set \( \frac{d\tau}{dy} = 0 \).

\[
\frac{d\tau}{dy} = \frac{12 V}{bh^3} (h - 2y_m) = 0 \quad y_m = \frac{1}{2} h \quad \text{i.e. at mid-height}
\]

(b) \( \tau_m = \frac{12 V}{bh^3} (hy_m - y_m^2) = \frac{12 V}{bh^3} \left[ \frac{1}{2} h^2 - (\frac{1}{2} h)^2 \right] = \frac{3V}{bh^2} = \frac{3}{2} \bar{\tau} \)

\[
k = \frac{3}{2} = 1.500
\]
Problem 6.37

6.37 and 6.38 The extruded beam shown has a uniform wall thickness of \( \frac{1}{8} \) in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.

\[
I = \frac{1}{12} (2.50)(2.50)^3 - \frac{1}{12} (2.125)(2.25)^3 = 1.2382 \text{ in}^4
\]

Add symmetric points c', b', and a'.

\[
Q_{c'} = 0
\]

\[
Q_d = (0.125)(1.125)(\frac{1.125}{2}) = 0.07910 \text{ in}^3 \quad t_d = 0.125 \text{ in.}
\]

\[
Q_c = Q_{c'} + (0.125)^2(1.1875) = 0.09765 \text{ in}^3 \quad t_c = 0.25 \text{ in.}
\]

\[
Q_b = Q_c + (2 \times 1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3 \quad t_b = 0.25 \text{ in.}
\]

\[
Q_a = Q_b + (2 \times 0.125)(1.25)(\frac{1.25}{2}) = 0.60839 \text{ in}^3 \quad t_a = 0.25 \text{ in.}
\]

\[
\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60839)}{(1.2382)(0.25)} = 3.93 \text{ ksi}
\]

\[
\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi}
\]

\[
\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi}
\]

\[
\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.125)} = 1.02 \text{ ksi}
\]

\[
\tau_e = \frac{VQ_e}{It_e} = 0
\]
Problem 6.92

6.92 For the beam and loading shown, consider section n-n and determine the shearing stress at (a) point a, (b) the shearing stress at point b.

[Diagram of beam and loading]

\[ R_A = R_B = 25 \text{ kips} \]

At section n-n \[ V = 25 \text{ kips} \]

Locate centroid and calculate moment of inertia.

<table>
<thead>
<tr>
<th>Part</th>
<th>( A ) (in²)</th>
<th>( \bar{y} ) (in)</th>
<th>( A\bar{y} ) (in³)</th>
<th>( d ) (in)</th>
<th>( A\bar{d}^2 ) (in⁵)</th>
<th>( I ) (in⁶)</th>
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<tr>
<td>1</td>
<td>4.875</td>
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<td>33.52</td>
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<td>2</td>
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<td>( \Sigma )</td>
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<td>72.94</td>
<td>56.66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.} \]

\[ I = \Sigma A\bar{d}^2 + \Sigma I = 56.66 + 47.86 = 83.42 \text{ in}^6 \]

(a) \[ Q_a = A\bar{y} = \left(\frac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3 \]

\[ t = \frac{3}{4} = 0.75 \text{ in.} \]

\[ \tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} = 1.745 \text{ ksi} \]

(b) \[ Q_b = A\bar{y} = \left(\frac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3 \]

\[ t = 0.75 \text{ in.} \]

\[ \tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} = 2.82 \text{ ksi} \]
Problem 6.95

Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (a) at A, (b) at B.

\[ I = \frac{1}{12} (120)(200)^3 - \frac{1}{12} (80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4 \]
\[ = 52.693 \times 10^6 \text{ m}^4 \]

(a) \[ Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3 \]
\[ = 144 \times 10^{-6} \text{ m}^3 \]
\[ t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m} \]
\[ \tau_A = \frac{VQ_A}{I t_A} = \frac{(3.5 \times 10^3)(144 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} \]
\[ = 239 \times 10^3 \text{ Pa} = 239 \text{ kPa} \]

(b) \[ Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 \]
\[ = 216 \times 10^{-6} \text{ m}^3 \]
\[ t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m} \]
\[ \tau_B = \frac{VQ_B}{I t_B} = \frac{(3.5 \times 10^3)(216 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} \]
\[ = 359 \times 10^3 \text{ Pa} = 359 \text{ kPa} \]