Problem 2.10  The forces acting on the sailplane are represented by three vectors. The lift $\mathbf{L}$ and drag $\mathbf{D}$ are perpendicular, the magnitude of the weight $\mathbf{W}$ is 3500 N, and $\mathbf{W} + \mathbf{L} + \mathbf{D} = 0$. What are the magnitudes of the lift and drag?

Solution:  Draw the force triangle and then use the geometry plus

\[
\cos 25° = \frac{|L|}{|W|}
\]
\[
\sin 25° = \frac{|D|}{|W|}
\]

$|W| = 3500$ N

$|L| = 3500 \cos 25°$

$|D| = 3500 \sin 25°$

$|L| = 3170$ N

$|D| = 1480$ N
Problem 2.23  A fish exerts a 40-N force on the line that is represented by the vector $\mathbf{F}$. Express $\mathbf{F}$ in terms of components using the coordinate system shown.

Solution:

\[ F_x = |\mathbf{F}| \cos 60^\circ = (40)(0.5) = 20 \text{ (N)} \]

\[ F_y = -|\mathbf{F}| \sin 60^\circ = -(40)(0.866) = -34.6 \text{ (N)} \]

\[ \mathbf{F} = 20\hat{i} - 34.6\hat{j} \text{ (N)} \]
Problem 2.40  The hydraulic actuator $BC$ in Problem 2.39 exerts a 1.2-kN force $F$ on the joint at $C$ that is parallel to the actuator and points from $B$ toward $C$. Determine the components of $F$.

Solution:  From the solution to Problem 2.39,

$$e_{BC} = -0.78i + 0.625j$$

The vector $F$ is given by $F = |F|e_{BC}$

$$F = (1.2)(-0.78i + 0.625j) \text{ (k \cdot N)}$$

$$F = -937i + 750j \text{ (N)}$$
Problem 2.46  Four groups engage in a tug-of-war. The magnitudes of the forces exerted by groups $B$, $C$, and $D$ are $|F_B| = 800$ lb, $|F_C| = 1000$ lb, $|F_D| = 900$ lb. If the vector sum of the four forces equals zero, what are the magnitude of $F_A$ and the angle $\alpha$?

Solution:  The strategy is to use the angles and magnitudes to determine the force vector components, to solve for the unknown force $F_A$ and then take its magnitude. The force vectors are

\[ F_B = 800(\cos 110^\circ + \sin 110^\circ) = -273.6 \mathbf{i} + 751.75 \mathbf{j} \]
\[ F_C = 1000(\cos 30^\circ + \sin 30^\circ) = 866 \mathbf{i} + 500 \mathbf{j} \]
\[ F_D = 900(\cos(-20^\circ) + \sin(-20^\circ)) = 845.72 \mathbf{i} - 307.8 \mathbf{j} \]

\[ F_A = |F_A|(\cos(180 + \alpha) + \sin(180 + \alpha)) \]
\[ = |F_A|(-\cos \alpha - \sin \alpha) \]

The sum vanishes:

\[ F_A + F_B + F_C + F_D = l(1438.1 - |F_A| \cos \alpha) \]
\[ + j(944 - |F_A| \sin \alpha) = 0 \]

From which $F_A = 1438.1 \mathbf{i} + 944 \mathbf{j}$. The magnitude is

\[ |F_A| = \sqrt{(1438)^2 + (944)^2} = 1720 \text{ lb} \]

The angle is:

\[ \tan \alpha = \frac{944}{1438} = 0.6565, \text{ or } \alpha = 33.3^\circ \]
Problem 2.83  The distance from point \( O \) to point \( A \) is 20 ft. The straight line \( AB \) is parallel to the \( y \) axis, and point \( B \) is in the \( x-z \) plane. Express the vector \( \mathbf{r}_{OA} \) in terms of scalar components.

Strategy:  You can resolve \( \mathbf{r}_{OA} \) into a vector from \( O \) to \( B \) and a vector from \( B \) to \( A \). You can then resolve the vector form \( O \) to \( B \) into vector components parallel to the \( x \) and \( z \) axes. See Example 2.9.

Solution:  See Example 2.10. The length \( BA \) is, from the right triangle \( OAB \),

\[
|\mathbf{r}_{AB}| = |\mathbf{r}_{OA}| \sin 30^\circ = 20(0.5) = 10 \text{ ft.}
\]

Similarly, the length \( OB \) is

\[
|\mathbf{r}_{OB}| = |\mathbf{r}_{OA}| \cos 30^\circ = 20(0.866) = 17.32 \text{ ft}
\]

The vector \( \mathbf{r}_{OB} \) can be resolved into components along the axes by the right triangles \( OBP \) and \( OBQ \) and the condition that it lies in the \( x-z \) plane.

Hence,

\[
\mathbf{r}_{OB} = |\mathbf{r}_{OB}|(i \cos 30^\circ + j \cos 90^\circ + k \cos 60^\circ)
\]

or

\[
\mathbf{r}_{OB} = 15i + 0j + 8.66k.
\]

The vector \( \mathbf{r}_{BA} \) can be resolved into components from the condition that it is parallel to the \( y \)-axis. This vector is

\[
\mathbf{r}_{BA} = |\mathbf{r}_{BA}|(i \cos 90^\circ + j \cos 0^\circ + k \cos 90^\circ) = 0i + 10j + 0k.
\]

The vector \( \mathbf{r}_{OA} \) is given by \( \mathbf{r}_{OA} = \mathbf{r}_{OB} + \mathbf{r}_{BA} \), from which

\[
\mathbf{r}_{OA} = 15i + 10j + 8.66k \text{ (ft)}
\]
Problem 2.96  The cable $AB$ exerts a 32-lb force $T$ on the collar at $A$. Express $T$ in terms of scalar components.

Solution:  The coordinates of point $B$ are $B (0, 7, 4)$. The vector position of $B$ is $\mathbf{r}_{OB} = 0\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$.

The vector from point $A$ to point $B$ is given by

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA}.$$   

From Problem 2.95, $\mathbf{r}_{OA} = 2.67\mathbf{i} + 2.33\mathbf{j} + 2.67\mathbf{k}$. Thus

$$\mathbf{r}_{AB} = (0 - 2.67)\mathbf{i} + (7 - 2.33)\mathbf{j} + (4 - 2.67)\mathbf{k}$$

$$\mathbf{r}_{AB} = -2.67\mathbf{i} + 4.67\mathbf{j} + 1.33\mathbf{k}.$$   

The magnitude is

$$|\mathbf{r}_{AB}| = \sqrt{(-2.67)^2 + 4.67^2 + 1.33^2} = 5.54 \text{ ft}.$$   

The unit vector pointing from $A$ to $B$ is

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -0.4819\mathbf{i} + 0.8429\mathbf{j} + 0.2401\mathbf{k}.$$   

The force $T$ is given by

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{u}_{AB} = 32\mathbf{u}_{AB} = -15.4\mathbf{i} + 27.0\mathbf{j} + 7.7\mathbf{k} (\text{lb})$$
Problem 2.105  The magnitudes $|U| = 10$ and $|V| = 20$.

(a) Use the definition of the dot product to determine $U \cdot V$.

(b) Use Eq. (2.23) to obtain $U \cdot V$.

**Solution:**

The definition of the dot product (Eq. (2.18)) is

$$U \cdot V = |U||V| \cos \theta.$$ Thus

$$U \cdot V = (10)(20) \cos(45° - 30°) = 193.2$$

- components of $U$ and $V$ are

$$10(i \cos 45° + j \sin 45°) = 7.07i + 7.07j$$

$$20(i \cos 30° + j \sin 30°) = 17.32i + 10j$$

Eq. (2.23) $U \cdot V = (7.07)(17.32) + (7.07)(10) = 193.2$
Problem 2.120  In Problem 2.119, what is the vector component of \( \mathbf{F} \) parallel to the surface?

Solution: From the solution to Problem 2.119,

\[ \mathbf{F} = -0.123\mathbf{i} + 0.123\mathbf{j} - 0.0984\mathbf{k} \text{ (lb)} \] and

\[ \mathbf{F}_{\text{NORMAL}} = 0\mathbf{i} + 0.0927\mathbf{j} + 0.0232\mathbf{k} \text{ (lb)} \]

The component parallel to the surface and the component normal to the surface add to give \( \mathbf{F}(\mathbf{F} = \mathbf{F}_{\text{NORMAL}} + \mathbf{F}_{\text{parallel}}) \).

Thus

\[ \mathbf{F}_{\text{parallel}} = \mathbf{F} - \mathbf{F}_{\text{NORMAL}}. \]

Substituting, we get

\[ \mathbf{F}_{\text{parallel}} = -0.1231\mathbf{i} + 0.0304\mathbf{j} - 0.1216\mathbf{k} \text{ lb} \]
Problem 2.127  Determine the cross product $r \times F$ of the position vector $r = 4i - 12j + 3k$ (m) and the force $F = 16i - 22j - 10k$ (N).

Solution:

$$r \times F = \begin{vmatrix} i & j & k \\ 4 & -12 & 3 \\ 16 & -22 & -10 \end{vmatrix}$$

$$r \times F = (120 - (-66))i + (48 - (-40))j + (-88 - (-192))k \text{ (N-m)}$$

$$r \times F = 186i + 88j + 104k \text{ (N-m)}$$