Soil Dynamics

Lecture 10

A Review of some Basic Concepts of Vibrations

Question #01.

For a single-degree of freedom system, the natural period *T* is,

I. The inverse of the natural frequency,
II. Equivalent to the linear natural frequency,
III. Expressed in Hertz (Hz).

Answers:

- **I**
- **II**
- **III**
- **I and II**

Answer to Question #01.

a) I only.

For a single-degree-of-freedom system, the natural period *T* is the time in which the system completes one cycle of oscillation. The natural period *T* is the inverse of the natural frequency, which is expressed as,

$$T = 1 / f$$

Question #02.

What is the spectral acceleration of a single-degree-of-freedom system?

A. **It is the minimum acceleration experienced by the system in response to a perturbation.**

B. **It is the average acceleration experienced by the system in response to a perturbation.**

C. **It is the maximum acceleration experienced by the system in response to a perturbation.**

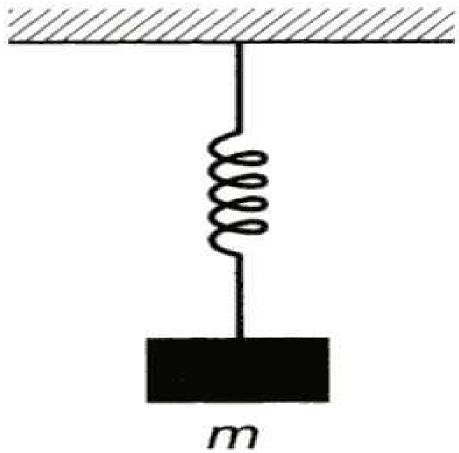
D. **It is the absolute acceleration experienced by the system in response to a perturbation.**

Answer to Question #02.

C. **The maximum vibration of a single-degree-of-freedom system is measured in terms of acceleration, velocity or displacement. The maximum acceleration experienced by the system in response to a disturbing force is known as the *spectral acceleration*.**

Question #03.

For the following harmonic oscillator, how is the stiffness best described?



A. The stiffness is best described as the force acting on the ideal linear spring.

B. The stiffness is best described as the force deflecting the spring a distance of 1 unit.

C. The stiffness is best described as the magnitude of the spring deflection.

D. The stiffness is best described as the reciprocal of deflection.

Answer to Question #03.

B. Hooke's law relates the amount of deflection of an ideal linear spring to the force applied to it.

It is expressed as $F = k\,x$

The stiffness *k* is the force that deflects the spring a distance of 1 unit.

Question #04.

When does the natural period of a building coincide with an earthquake's period?

A. They coincide when the natural frequency is at its maximum.

B. They coincide when the acceleration is at its maximum.

C. They coincide when the displacement is at its maximum.

D. All the above are right.

Answer to Question #04.

B. When the building's period coincides with an earthquake's period, resonance occurs.

At resonance the acceleration response is at its maximum.

Question #05.

The period of a structure is correlated with the,

- **I. The mass of the structure,**
- **II. The stiffness of the structure.**

Answers:

A. I only.

B. II only.

C. I and II.

D. None of the above.

Answer to Question #05.

C. $$\boldsymbol{T} = 2\pi\sqrt{\frac{\boldsymbol{m}}{\boldsymbol{k}}}$$

As the weight of a structure increases, the period of vibration becomes longer.

For example, steel or reinforced concrete structures tend to have higher periods of vibrations compared with timber structures.

The stiffness of a structure inversely affects its period of vibration. That is, the stiffer the structure, the shorter the period of vibration.

Question #06.

What is the maximum possible earthquake motion at a site?

A. It is the motion intensity with a 10% probability of being exceeded in a 100-year time period.

B. It is based on presently available data and knowledge of the site.

C. It is the maximum level of earthquake ground motion expected at the site.

D. All the above are true.

Answer to Question #06.

D.

The maximum possible ("considered") earthquake is defined in the California Building Code (the CBC) in its Section 1655, Appendix, Division IV, as the maximum level of earthquake ground shaking that may be expected at the building site within the known geological framework.

In seismic zones 3 and 4, this intensity may be taken as the level of earthquake ground motion that has a 10% probability of being exceeded in 100 years.

Question #07.

Consider the natural period *T* and the acceleration a of a single-degree-of-freedom system.

When the mass *m* of the system increases, how are *T* and *a* affected?

A. **_T_ increases and _a_ increases.**

B. **_T_ increases and _a_ decreases.**

C. **_T_ decreases and _a_ increases.**

D. **_T_ decreases and _a_ decreases.**

Answer to Question #07.

B. $$T = 2\pi\sqrt{\frac{m}{k}}$$

Increasing the mass of a single-degree-of-freedom system will result in a longer period of vibration.

From Newton's second law $F = m\,a$ **it is seen that the response of the larger mass to the same force will result in a lower acceleration.**

Question #08.

If *k* is the stiffness and Δ is the deflection of a system, which of the following relationships describe best the rigidity *R*?

A. 1 / *k*

B. *k* Δ

C. Δ

D. 1 / Δ

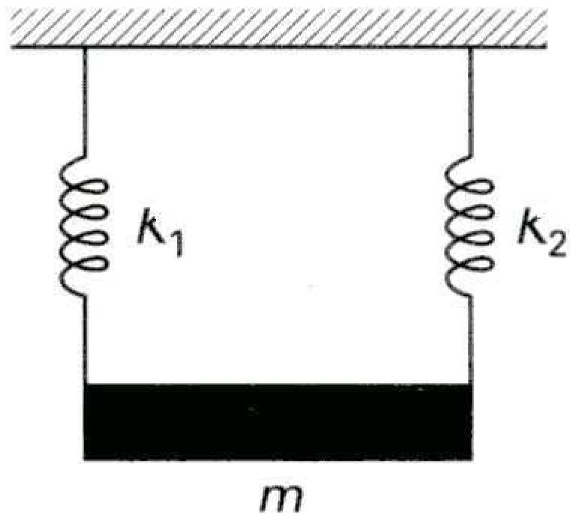
Answer to Question #08.

D.

The rigidity *R* is the reciprocal of the deflection Δ.

Question #09.

A mass *m* hangs from two "ideal" springs; what is the total composite spring constant for the system?



A. $k_1 + k_2$

B. $k_1 k_2$

C. $1 / k_1 + 1 / k_2$

D. $(k_1 k_2) / k_1 + k_2$

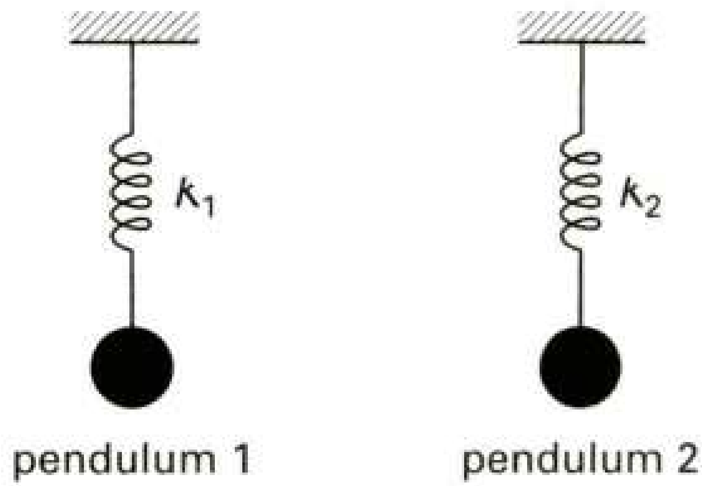
Answer to Question #09.

A.

For parallel springs, the composite stiffness is the sum of each individual spring stiffness.

Question #10.

Each of the following pendulums have equal masses and are hanging from ideal springs. The periods of vibration for these pendulums are 1.73 s and 3.0 s respectively. What is the stiffness of the second pendulum's spring?



A. $k_1 / 3$

B. $k_1 / 2$

C. k_1

D. $1.33\ k_1$

Answer to Question #10.

A.

$$\boldsymbol{T}_1 = 2\pi\sqrt{\frac{\boldsymbol{m}_1}{\boldsymbol{k}_1}} \quad \therefore \quad \boldsymbol{T}_1^2 = 4\pi^2\left(\frac{\boldsymbol{m}_1}{\boldsymbol{k}_1}\right) \quad \therefore \quad \boldsymbol{T}_1^2\boldsymbol{k}_1 = 4\pi^2\boldsymbol{m}_1$$

$$\boldsymbol{T}_2 = 2\pi\sqrt{\frac{\boldsymbol{m}_2}{\boldsymbol{k}_2}} \quad \therefore \quad \boldsymbol{T}_2^2 = 4\pi^2\left(\frac{\boldsymbol{m}_2}{\boldsymbol{k}_2}\right) \quad \therefore \quad \boldsymbol{T}_2^2\boldsymbol{k}_2 = 4\pi^2\boldsymbol{m}_2$$

however, since $\boldsymbol{m}_1 = \boldsymbol{m}_2$

$$\boldsymbol{T}_1^2\boldsymbol{k}_1 = \boldsymbol{T}_2^2\boldsymbol{k}_2$$

$$(1.73)^2\,\boldsymbol{k}_1 = (3.0)^2\,\boldsymbol{k}_2$$

$$\therefore \quad \boldsymbol{k}_2 = \frac{3}{9}\boldsymbol{k}_1 = \frac{1}{3}\boldsymbol{k}_1$$

Question #11.

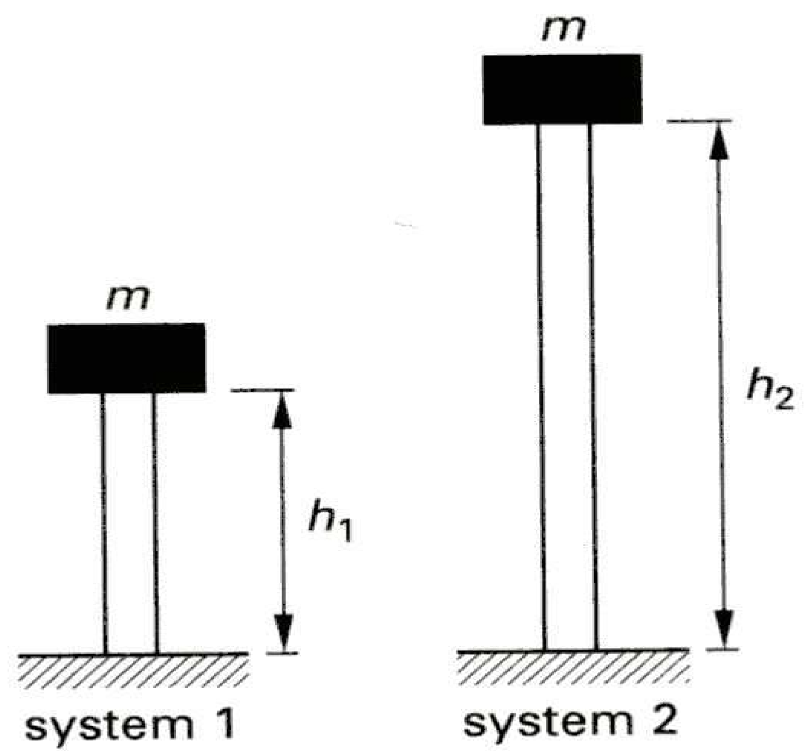
This problem is based on the wooden model we use to play in class, except that we will consider two blocks, instead of three. Each of the steel strips supports a block of wood with an ideal mass *m*. The steel strip columns are fixed at the bottom and are free to displace at the top. The height of the second column is twice the height of the first column. Assume the same modulus of elasticity E and moment of inertia I for both columns. The systems have natural periods of vibrations T1 and T2 respectively. Neglecting the weight of the columns, what is the natural period of vibration for the second system?

A. 0.5 T_1

B. 2.0 T_1

C. 3.0 T_1

D. 4.0 T_1



Answer to Question #11.

C. Both columns are modelled as cantilever beams,

$$k_1 = \frac{3E_1I_1}{h_1^3} \quad \therefore \quad 3E_1I_1 = k_1h_1^3$$

$$k_2 = \frac{3E_2I_2}{h_2^3} \quad \therefore \quad 3E_2I_2 = k_2h_2^3$$

however, it was given that $h_2 = 2h_1$, $E_1 = E_2$ and $I_1 = I_2$

$$k_1h_1^3 = k_2h_2^3 \quad \therefore \quad k_1h_1^3 = k_2\left(2h_1\right)^3 \quad \therefore \quad k_1 = 8k_2 \quad \text{equation (1)}$$

$$T_1 = 2\pi\sqrt{\frac{m_1}{k_1}} \quad \therefore \quad T_1^2 = 4\pi\left(\frac{m_1}{k_1}\right) \quad \therefore \quad T_1^2k_1 = 4\pi m_1$$

$$T_2 = 2\pi\sqrt{\frac{m_2}{k_2}} \quad \therefore \quad T_2^2 = 4\pi\left(\frac{m_2}{k_2}\right) \quad \therefore \quad T_2^2k_2 = 4\pi m_2$$

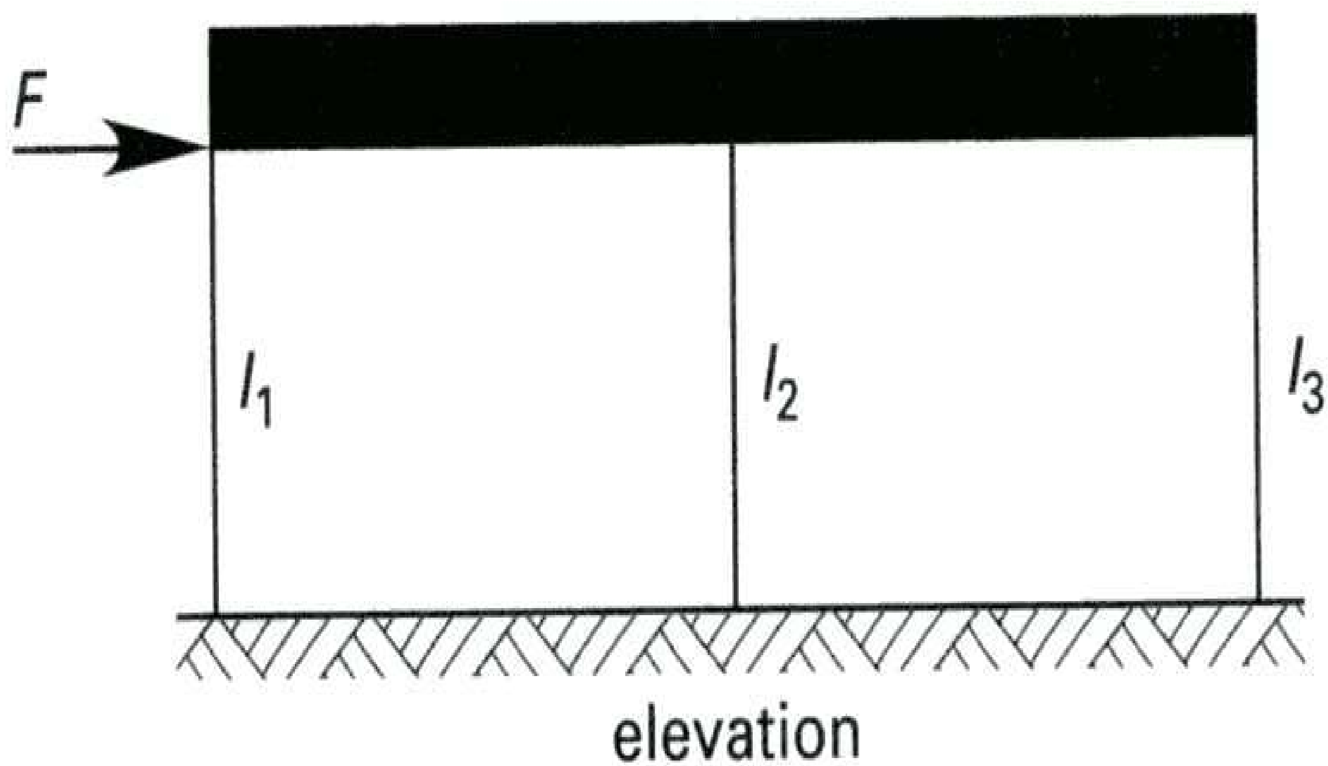
since $m_1 = m_2$

$T_1^2k_1 = T_2^2k_2$ equation (2). Substitute equation (1) into (2),

$$T_1^2\left(8k_2\right) = T_2^2k_2 \quad \therefore \quad T_2^2 = 8T_1^2 \quad \textit{or} \quad \underline{T_2 \approx 3.0\, T_1}$$

Question #12.

A force is acting at the top of a building frame. The supporting columns are of equal height and are fixed at the base. The modulus of elasticity *E* is the same for each column. The top plate is rigid. The ratio of the moments of inertia are $I_1 = 0.33\ I_2 = 0.165\ I_3$. What is the shear distribution to the first column?



A. 0.10 *F*

B. 0.20 *F*

C. 0.67 *F*

D. 0.33 *F*

Answer to Question #12.

A. Since the columns are rigid, the force is distributed to the columns in proportional to their rigidities or stiffnesses. The composite stiffness is the sum of the individual column stiffnesses.

$$V_1 = F\left(\frac{k_1}{k_1+k_2+k_3}\right) \quad \text{equation (1)}$$

$$\textit{where } k_1 = \frac{12E_1I_1}{h_1^3} \quad \textit{and} \quad k_2 = \frac{12E_2I_2}{h_2^3} \quad \textit{and} \quad k_3 = \frac{12E_3I_3}{h_3^3}$$

however, it is given that $E_1 = E_2 = E_3$ and $h_1 = h_2 = h_3$

$$\therefore \quad k_T = k_1 + k_2 + k_3 = \left(\frac{12E_1}{h_1^3}\right)\left(I_1 + I_2 + I_3\right) \quad \text{equation (2)}$$

Substituting equation (2) into (1),

$$V_1 = F\left(\frac{I_1}{I_1+I_2+I_3}\right) = F\left(\frac{I_1}{I_T}\right) \quad \text{but} \quad I_1 = 0.33I_2 \quad \text{and} \quad I_2 = 0.167I_3$$

$$\therefore \quad I_T = I_1 + 3I_1 + 6I_1 = 10I_1$$

$$\therefore \quad V_1 = F\left(\frac{I_1}{10I_1}\right) = \underline{0.10F}$$

Question #13.

How is the term damping described?

I. Damping is the ratio of one cycle's amplitude to the subsequent cycle.
II. Damping is the dynamic magnification factor.
III. Damping is the dissipation of energy from an oscillating system.

Answers:

A. I only.
B. III only.
C. I and III.
D. II and III.

Answer to Question #13.

B.

When a system is set into oscillatory motion, it will continuously move until the dissipation of energy, primarily through friction, will eventually cause the system to reach a motionless equilibrium position. This dissipation of energy is called *damping*.

Question #14.

How is the term *flexibility* defined?

A. Stiffness.
B. The reciprocal of stiffness.
C. Rigidity.
D. Static deflection.

Answer to Question #14.

B.

Flexibility **is the total deflection of a structural system when a unit lateral force is applied. Flexibility can also de defined as the inverse of stiffness.**

Question #15.

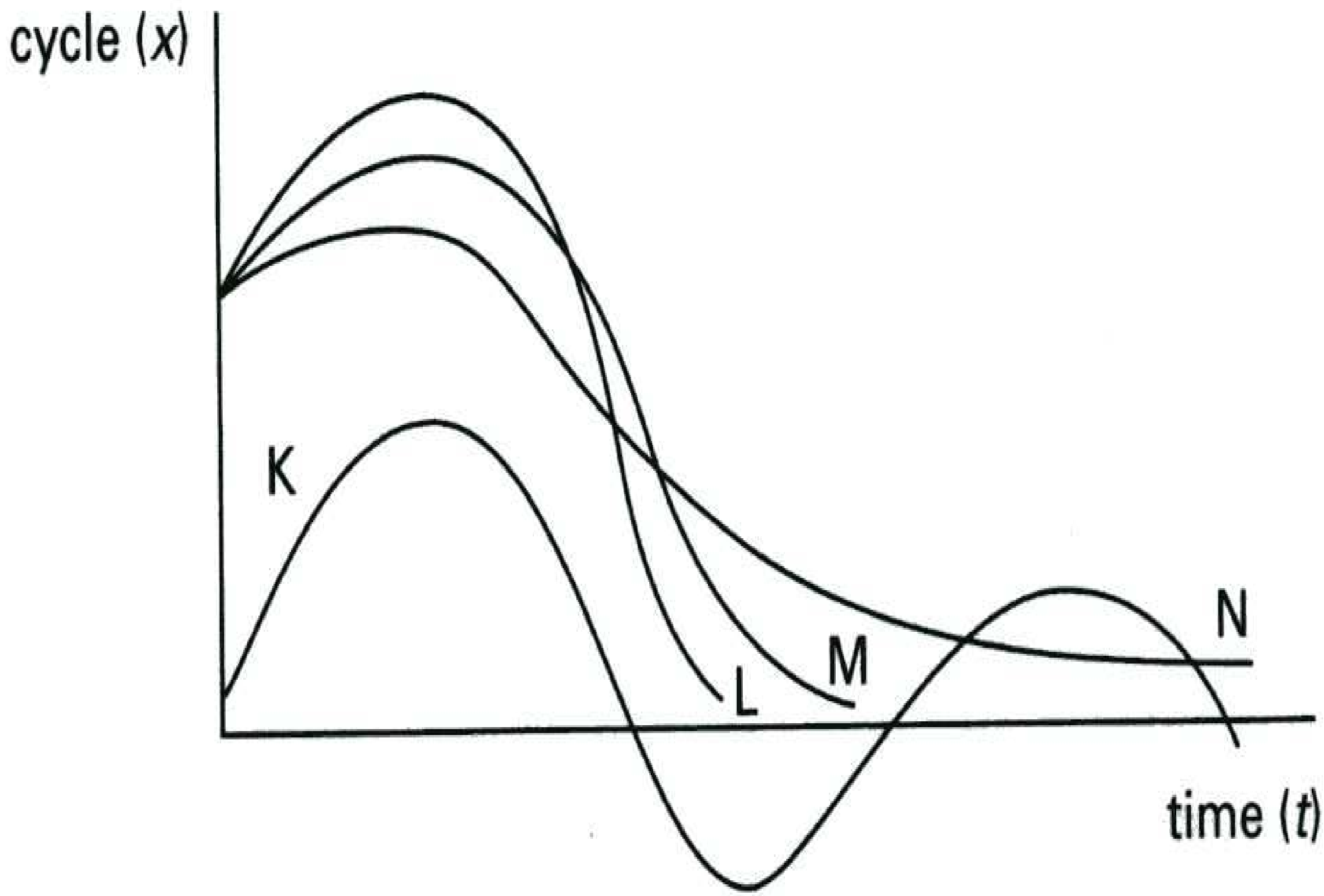
Choose the spectra below that best represents moderate damping.

A. K

B. L

C. M

D. N



Answer to Question #15.

A.

The spectral curve K represents an oscillating system that takes many cycles before reaching the motionless state of equilibrium.

Systems with small or moderate amounts of damping are called *underdamped*.

The spectral curves L, M and N are *critically damped* or *overdamped*.

Question #16.

Consider a system with several masses and many modes.

Is it true that,

I. the characteristic shape of each mode is unique;
II. all the modes have the same natural frequency;
III. the fundamental mode has the longest period.

Answers:

A. I only

B. II only

C. I and III

D. II and III

Answer to Question #16.

C.

For a multiple-degree-of-freedom system, the oscillation of the system is a combination of the oscillations of the several lumped masses. Each mode of oscillation has as many modes as there are lumped masses. Each mode of oscillation has its own shape and natural frequency. The first, or fundamental, mode has the longest period. That is, the smallest frequency.

Question #17.

Consider a system with lumped masses and many modes, which of the following statements is correct?

A. Higher modes have lower frequencies.

B. Higher modes have higher frequencies.

C. Higher modes have longer periods.

D. Lower modes have shorter periods.

Answer to Question #17.

B.

For a multiple-degree-of-freedom system, at higher modes the periods are smaller (that is, frequencies are longest). The lower modes have the longest periods (that is, the lowest frequencies).

Question #18.

Compared with buildings with few stories, high-rise buildings have which of the following?

A. **High-rise buildings have higher frequencies.**

B. **High-rise buildings have longer periods.**

C. **High-rise buildings have higher accelerations.**

D. **High-rise buildings have higher stiffness.**

Answer to Question #18.

B.

High-rise buildings have greater flexibility.

In general, the height of these buildings influences the period of vibration. The taller the building, the longer the period of vibration.

Question #19.

Consider the ratio of a building's acceleration to the ground acceleration during an earthquake. Which of the following are true?

A. The ratio depends on the building's period.

B. The ratio is equal to 1.0 for an infinitely stiff building.

C. The ratio is equal to 1.0 for buildings with zero natural period.

D. Building accelerations are typically lower than ground accelerations.

<u>Answer to Question #19.</u>

C.

The ground acceleration is an acceleration to which a building responds, while the building acceleration is a function of its dynamic characteristics, its mass *m* and its stiffness *k*.

The response of the building to the ground acceleration depends on the characteristics of both the earthquake and the structure, including the building period.

For infinitely stiff buildings and buildings with zero natural periods, the ground acceleration and the building acceleration are identical. Therefore, the ratio is 1.0. However, in practice, the building acceleration is higher than the ground acceleration.

Question #20.

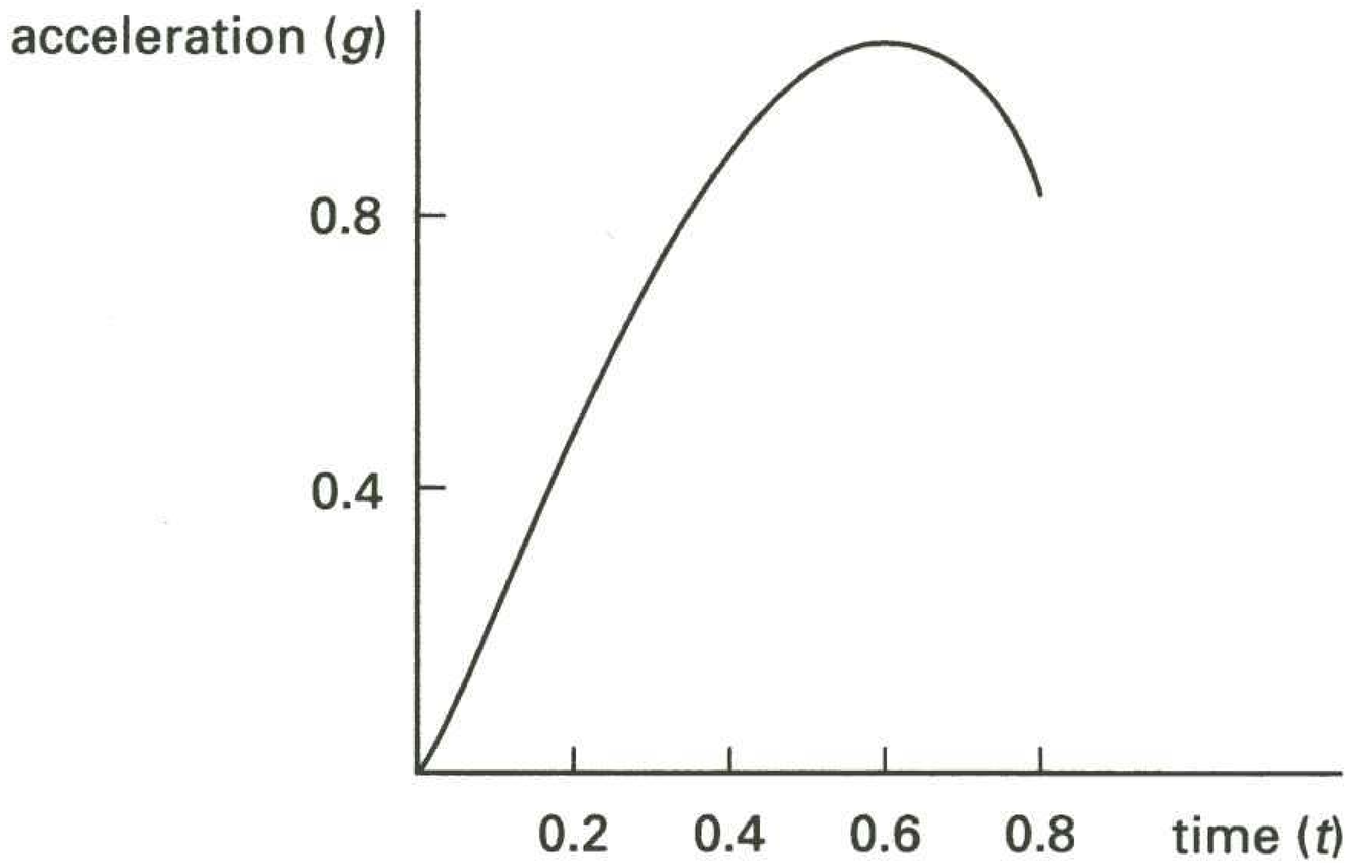
In the response spectra shown below, what is the base shear? Assume that $W = 712$ kN and $T = 0.3$ seconds.

A. 180 kN

B. 320 kN

C. 500 kN

D. 640 kN



Answer to Question #20.

C.

Notice that for a period *T* = 0.3 sec, the spectral acceleration is about 0.7 g.

$$F = ma = \left(\frac{W}{g}\right)a = \left(\frac{W}{g}\right)(0.7g) = 0.7W = (0.7)(712\ kN) \approx \underline{500\ kN}$$