# *20 - Mat Foundations*

**01: The ultimate bearing capacity of a mat in a pure clay soil.**
**02: The ultimate bearing capacity of a mat in a granular soil.**
**03: Find the depth $D_f$ for a fully compensated mat.**
**04: Find the consolidation settlement of mat foundation.**
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**06: Mat foundation for a large transfer girder.**
**07: Design a mat foundation for a small office building.**

## ***Mat Foundations–01:** Ultimate bearing capacity in a pure cohesive soil.*

(Revision: Sept-08)

**Determine the ultimate bearing capacity of a mat foundation measuring 45 feet long by 30 feet wide placed 6.5 feet below the surface and resting upon a saturated clay stratum with $c_u$ = 1,950 lb/ft$^2$ and $\varphi$ = 0º.**

***Solution:***

Mat foundations in purely cohesive soils have the following ultimate bearing capacity:

$$q_{\text{ult(net)}} = 5.14\, c_u \left(1+\frac{0.195B}{L}\right)\left(1+\frac{0.4D_f}{B}\right)$$

$$q_{\text{ult(net)}} = 5.14(1.95\ ksf)\left[1+\frac{0.195(30\ ft)}{(45\ ft)}\right]\left[1+\frac{0.40(6.5\ ft)}{(30\ ft)}\right] = 12\ ksf$$

## *Mat Foundations–02: Ultimate bearing capacity in a granular soil.

(Revision: Sept-08)

**What will be the net allowable bearing capacity of a mat foundation 15 m long by 10 m wide, embedded 2 m into a dry sand stratum with a corrected SPT to 55% efficiency $N_{55}$ = 10? It is desired that the allowable settlement is $\Delta H_{all}$ = 30 mm.**

***Solution:***

**The allowable bearing capacity of a mat foundation in granular soils was proposed by Meyerhof (with a Factor of Safety of 3) to be based on the SPT corrected to a 55% efficiency as,**

$$q_{all} = 12.5\, N_{55}[1+\frac{0.33 D_f}{B}]\,[\frac{\Delta H_{all}}{25.4\, mm}] = 12.5\, N_{55}[1+\frac{0.33(2\, ft)}{(10\, ft)}]\,[\frac{30\, mm}{25.4\, mm}] = 151\, kN/m^2$$

An alternate formula is,

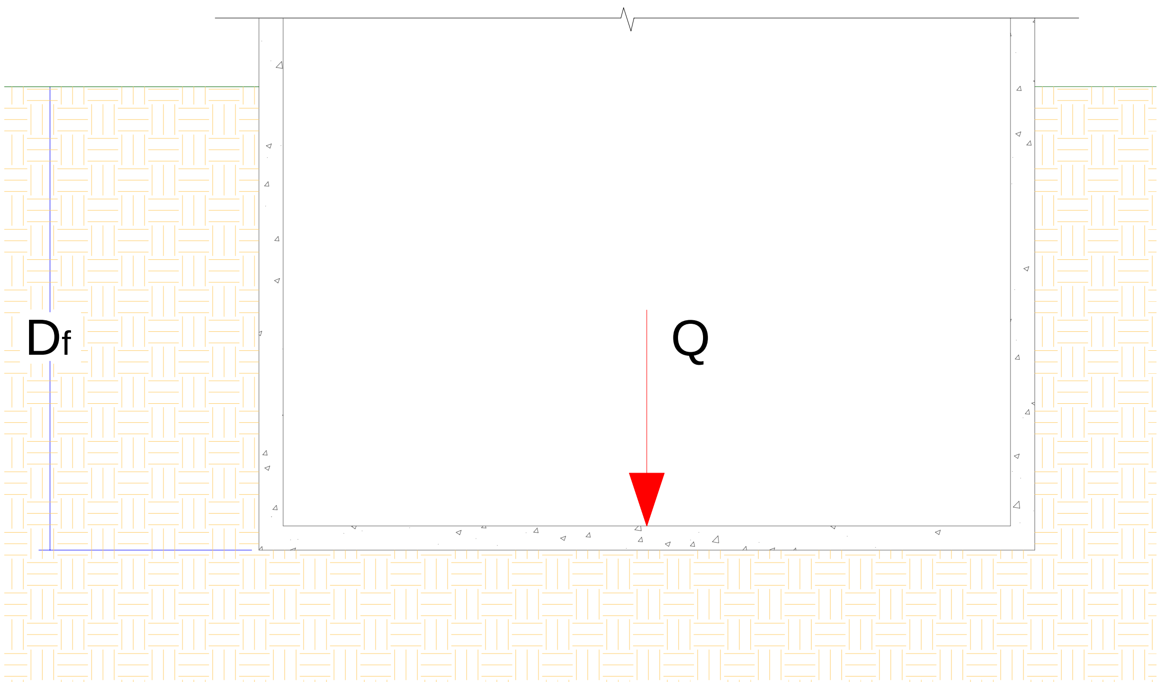
$$q_{all} = 15.9\, N_{55}\,[\frac{\Delta H_{all}}{25.4\, mm}] = 15.9\, N_{55}\,[\frac{30\, mm}{25.4\, mm}] = 188\, kN/m^2$$

Since $q_{all} = 151\, kN/m^2$ is the smaller of the two, choose this one for the answer.

## *Mat Foundations–03: *Find the depth $D_f$ for a fully compensated mat.**

(Revision: Sept-08)

**The mat shown below is 30 m wide by 40 m long. The live and dead load on the mat is 200 *MN*. Find the depth $D_f$ for a fully compensated foundation placed upon a soft clay with a unit weight $\gamma = 18.75\ kN/m^3$.**




***Solution:***

The net soil pressure $q$ under the mat is the load from the building $Q$ over the entire mat, minus the weight of the soil excavated $D_f\gamma$,

$$q = \frac{Q}{A} - D_f\gamma$$

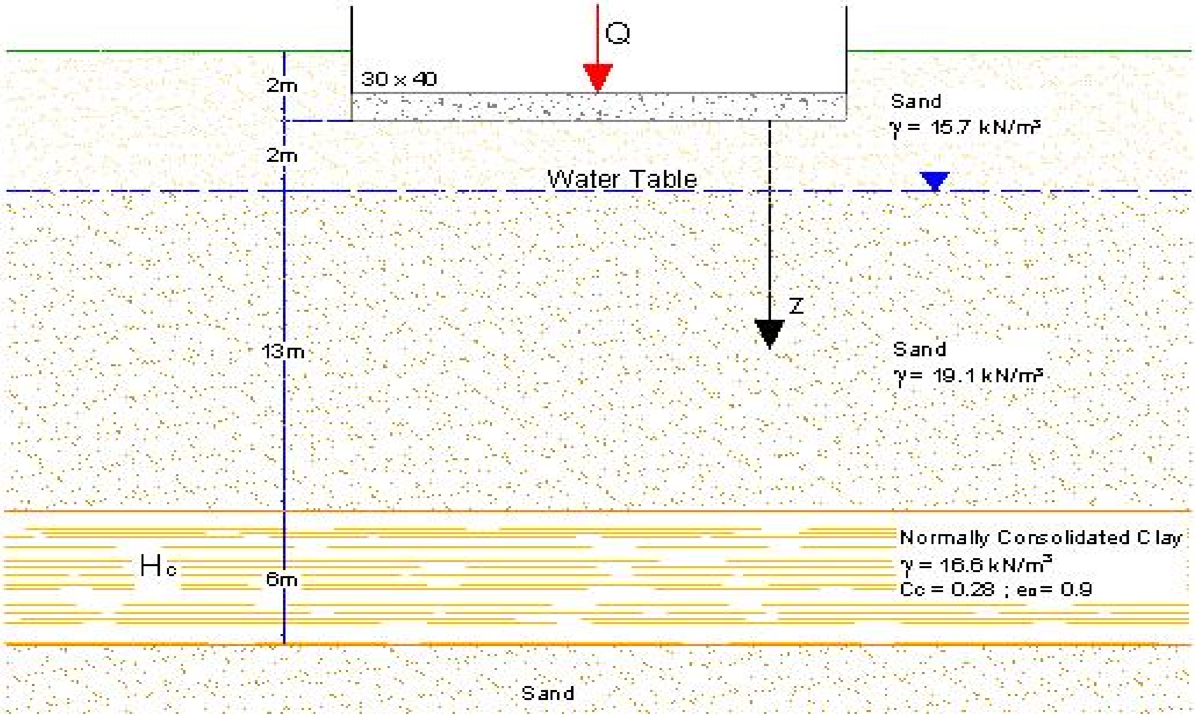
When the mat is fully compensated, the weight of the soil $W$ excavated is equal to the weight of the newly imposed building $Q$, in other words $q = 0$ and therefore,

$$D_f = \frac{Q}{A\gamma} = \frac{[200\text{x}10^3\ kN]}{[(30\ m)(40\ m)(18.75\ kN/m^3)]} = 9\text{ m}$$

## *Mat Foundations–04: The consolidation settlement of a mat foundation.

(Revision: Sept-08)

**The mat foundation shown below is 30 m wide by 40 m long. The total dead plus live load on the mat is 200 *MN*. Estimate the consolidation settlement at the center of the foundation; $C_c$ and $e_o$ of the normally consolidated clay are 0.28 and 0.9 respectively.**




***Solution:***

The net load per unit area $q$ is,

$$q = \frac{Q}{A} - D_f\gamma = \frac{(200{,}000 kips)}{(30m)(40m)} - (2m)(15.7\ kN/m^3) = 135\ kN/m^2$$

The pressure at mid-clay (depth of 18 m below the mat) is found via Boussinesq as,

$$m = \frac{z}{B} = \frac{18m}{30m} = 0.6 \quad n = \frac{z}{B} = \frac{18m}{40m} = 0.45 \quad \text{and} \quad \frac{L}{B} = \frac{40m}{30m} = 1.33 \therefore \Delta p_m = 0.66q$$

$$\Delta p_m = 0.66q = (0.66)(135\ kN/m^2) = 89\ kN/m^2$$

The in-situ stress at mid-clay layer before the mat foundation is built is,

$$p_o = (4m)(15.7kN/m^3) + (13m)(19.1-9.81)(kN/m^3) + (3m)(18.6-9.81)(kN/m^3) = 210kN/m^2$$

The consolidation (plastic) settlement is,

$$\Delta H = \frac{C_c H}{1+e_o}\log_{10}\left[\frac{p_o + \Delta p_m}{p_o}\right] = \frac{(0.28)(6{,}000mm)}{1+(0.9)}\log_{10}\left[\frac{(210)+(89)}{(210)}\right] = 136\ mm$$

## *Mat Foundations–05: *Settlement of a rigid mat.*

(Revision: Sept-08)

**A building is to be supported by a rigid reinforced concrete mat foundation, whose dimensions are 20 m wide by 50 m long. The load on the mat is to be uniformly distributed with a magnitude of 65 kPa. The mat rests on a deep deposit of saturated clay with an approximate un-drained Young's modulus $E_u$ = 40 *MPa* and a Poisson ratio $v$ = 0.4. Estimate the immediate settlement at the center and corner of the mat.**

***Solution:***

**Since the mat foundation is stiff, use the rigid factor $C_s$ is found from the *L x B* ratio,**

$\frac{L}{B} = \frac{50}{20}$ **= 2.5, which by interpolation in the chart provides a $C_s$ = 1.20.**

**One of the possible equations for immediate settlement $\Delta H_i$ is this one,**

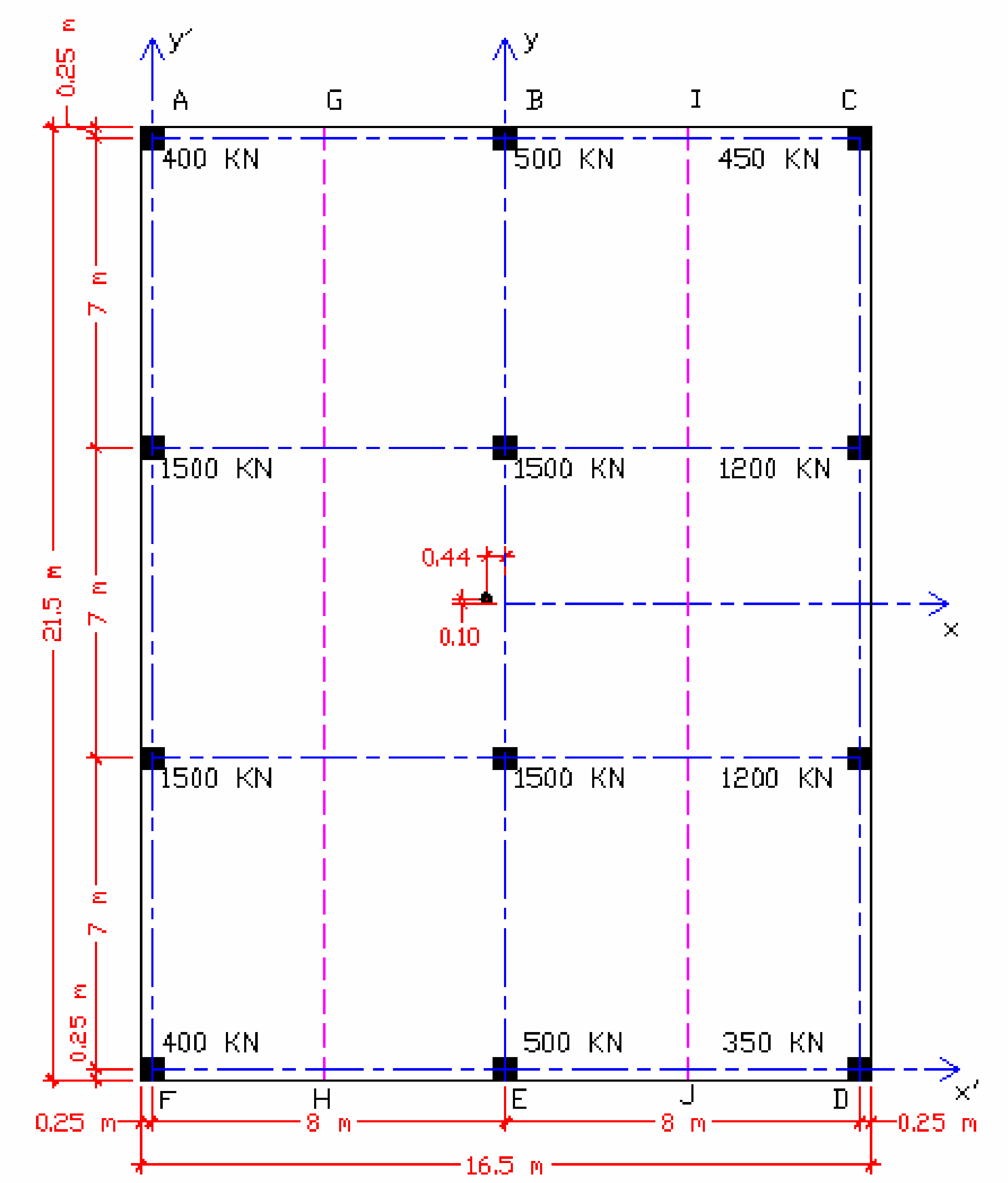
$$\Delta H_i = C_s \, q \, B \left(\frac{1 - \mu^2}{E_u}\right) = (1.20)(65 \text{ kN/m}^2)(20 \text{ m}) \left[\frac{1 - (0.4)^2}{40 \text{x} 10^3 \; kN/m^2}\right] = \mathbf{0.033 \text{ m}}$$

**Since the mat is assumed to be rigid, the surface settlement at both the center and at the corners of the mat, are the same, which is *33 mm*.**

## ****Mat Foundations–06: *Design a small mat for an office building.***

(Revision: May-09)

**A small office building with the column loads shown below is founded 3 m deep into a sand stratum with a unit weight of 18 $kN/m^3$. The foundation is the mat shown below. All the columns are 0.5 m x 0.5 m. The concrete strength is $f'_c$ = 20.7 $MN/m^2$ and the steel yield strength is $f_y$ = 413.7 $MN/m^2$. Determine their reinforcement requirements in the y-direction only.**



***Solution:***

**Step 1: Find the soil pressures, the location of the soil reaction's resultant and the eccentricities in the x and y directions.**

$\text{The service load} = (400\times 2)+(500\times 2)+450+350+(1500\times 4)+(1200\times 2)=11,000\,kN$
The moments of inertia of the mat in the x and y-directions are,

$$I_X = \frac{xy^3}{12} = \frac{16.5\times 21.5^3}{12} = 13,670 \;\; m^4$$

$$I_Y = \frac{x^3 y}{12} = \frac{16.5^3 x 21.5}{12} = 8,048 \;\; m^4$$

To find the eccentricity in ***x*** and ***y*** directions, take moments about the axes.

For the eccentricity about the y'-axis, take $\sum M_{y'} = 0$

(11,000)x' = (8 m)(500+1500+1500+500)+(16 m)(450+1200+1200+350)
x' = 7.56 m, which translated to the mat's centroid gives $e_x = \frac{16.5}{2} - 7.56 = 0.69 \;\; m$

For the eccentricity about the x'-axis, take $\sum M_{x'} = 0$

(11,000)y' = (7 m)(1500+1500+1200) + (14 m)(1500+1500+1200) + (21)(400+500+450)
y' = 10.60 m, translated to the mat's centroid gives $e_y = \frac{21.5}{2} - 10.60 = -0.15 \;\; m$

**Step 2. Find the soil reaction pressures.**

Let us factor the applied loads: 1.7(Service Loads) = 1.7(11,000 kN) = **18,700 kN**
The two eccentricities $e_x$ and $e_y$ create moments about the centroid. The soil reaction is no longer uniform, and varies linearly between the columns. These moments are:

$M_x = R\, e_y$ = (18,700 kN)(0.15 m) = 2,805 kN-m

$M_y = R\, e_x$ = (18,700 kN)(0.69 m) = 12,903 kN-m

The soil reaction pressure at any point under the mat is found from the relation:

$$q=\frac{R}{A}\pm\frac{M_y x}{I_y}\pm\frac{M_x y}{I_x}=\frac{18{,}700}{16.5\times 21.5}\pm\frac{12{,}903(x)}{8{,}048}\pm\frac{2{,}805(y)}{13{,}665}$$

Therefore, $q = 52.7 \pm 1.6x \pm 0.21y \quad kN/m^2$

**Step 3. Using the equation for *q*, prepare a table of its value at points A through J.**

| POINT | R/A ($kN/m^2$) | x (m) | 1.6x (m) | y (m) | 0.21y (m) | q ($kN/m^2$) |
|---|---|---|---|---|---|---|
| A | 52.7 | -8 | -12.8 | 10.5 | 2.21 | 42.11 |
| B | 52.7 | 0 | 0 | 10.5 | 2.21 | 54.91 |
| C | 52.7 | 8 | 12.8 | 10.5 | 2.21 | 67.71 |
| D | 52.7 | 8 | -8.16 | -10.5 | -2.21 | 63.29 |
| E | 52.7 | 0 | 0 | -10.5 | -2.21 | 50.50 |
| F | 52.7 | -8 | -12.8 | -10.5 | -2.21 | 37.7 |
| G | 52.7 | -4 | -6.4 | 10.5 | 2.21 | 48.51 |
| H | 52.7 | -4 | -6.4 | -10.5 | -2.21 | 44.10 |
| I | 52.7 | 4 | 6.4 | 10.5 | 2.21 | 61.31 |
| J | 52.7 | 4 | 6.4 | -10.5 | -2.21 | 56.9 |

**Step 4. Determine the effective depth *d* and the thickness *T* of the mat.**

**a) Check a critical edge column (for example, one of the 1.5 MN at the left edge):**

U = factored column load = 1.2(1.0) + 1.6(0.5) = 2 MN or 1.7(1.5) =2.55 MN
$b_0$ = critical perimeter = 2(0.5 m + d/2) + (0.5 + d) = (1 + d) + 0.5 + d = 1.5 + 2d

Using $\varphi V_C \geq V_u$ (from ACI 318-05) and $f_c'$ = 20.7 MN/m$^2$ (3 ksi),

$$\left[\phi(0.34)\sqrt{f'_c}\right](b_o d) = (0.85)(0.34)\sqrt{20.7}\times(1.5+2d)(d) \geq U = 2\text{ MN}$$

$d^2 + 0.75\,d - 0.76 \geq 0$

$$d=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-0.75\pm\sqrt{(0.75)^2-4(1)(-0.76)}}{2(1)}=0.57\,m$$

**b) Check the largest corner column (the 0.45 MN at top right corner):**

d = 0.36 m (This does not control).

**c) Check the most critical internal column (the 1.5 MN):**

$\boldsymbol{b_0} = 4(0.5 + \boldsymbol{d}) = 2 + 4\boldsymbol{d}$

$$0.85(0.34)(\sqrt{20.7})(2+4d)(d)=2MN$$
$$5.26d^2+2.63d-2=0$$
$$d=0.415m$$

$\therefore$ use $\boldsymbol{d}$ = 23 *inches* or <u>*585 mm*</u>

and $\boldsymbol{T}$ = 23+3+1 = <u>*27 in*</u> or <u>*686 cm*</u>

**Step 5. Find the average soil reaction for each strip:**

Strip AGHF (W = 4.25 m)

$$q_1 = \frac{q_A + q_G}{2} = \frac{42.11 + 48.5}{2} = 45.31 \ \frac{kN}{m^2}$$

$$q_2 = \frac{q_H + q_F}{2} = \frac{44.1 + 37.7}{2} = 41 \ \frac{kN}{m^2}$$

Strip GIJH (W = 8 m)

$$q_1 = \frac{48.51 + 54.91 + 61.31}{3} = 54.91 \ \frac{kN}{m^2}$$

$$q_2 = \frac{56.9 + 50.5 + 44.1}{3} = 50.5 \ \frac{kN}{m^2}$$

Strip ICDJ (W = 4.25 m)

$$q_1 = \frac{61.31 + 67.71}{2} = 64.51 \ \frac{kN}{m^2}$$

$$q_2 = \frac{63.29 + 56.9}{2} = 60.1 \ \frac{kN}{m^2}$$

Soil reaction AGHF = ½ (45.31+41)(4.25)(21.5) = 3943 *kN*

Soil reaction GIJH = ½ (54.91+50.5)(8)(21.5) = 9065 *kN*

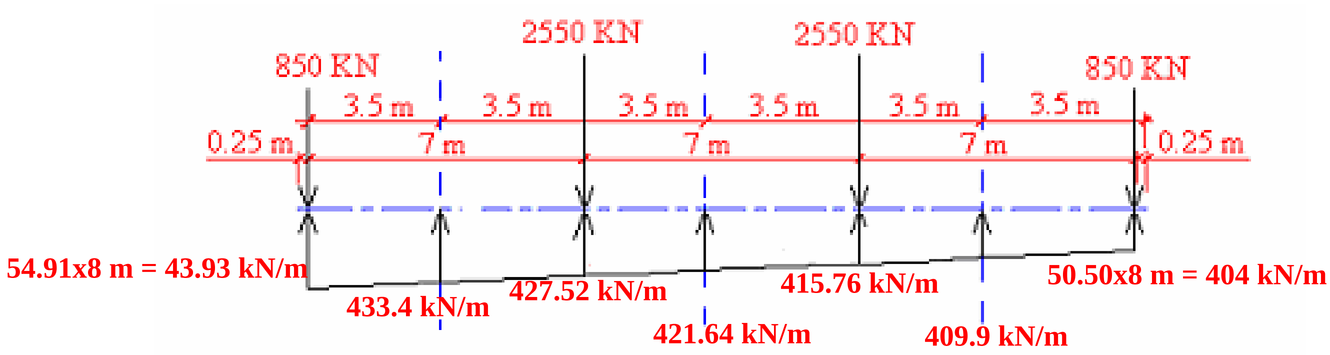
Soil reaction ICDH = ½ (64.51+60.1)(4.25)(21.5) = 5693 *kN*

$\sum \boldsymbol{F_y}$ = 3943 + 9065 + 56903 = 18,700 *kN*

Strip GIJH.

$Q_1$ = 1.7(500) = 850 *kN*
$Q_2$ = 1.7(1500) = 2550 *kN*

$Q_3 = 1.7(1500) = 2550\ kN$
$Q_4 = 1.7(500) = 850\ kN$




**Step 6. Find the maximum positive moments for each span at midpoints a, b & c.**

$\sum M_a = 0 \qquad \sum M_A = (850 \times 3.5\ m) - \frac{(439.3 + 433.4)}{2}\frac{(3.5^2)}{2} + M_A = 0$

$\therefore M_A = -302.36 \quad kN\text{-}m/m$

$\sum M_b = 0 \qquad \sum M_b = (850 \times 10.5m) + (2550 \times 3.5m) - \frac{(439.3 + 415.76)}{2}\frac{(10.5^2)}{2} + M_b = 0$

$\therefore M_b = 5718 \quad kN\text{-}m/m$

**Step 7. Calculate maximum negative moment at d, column B, see page 449:**

$\sum M_d = (850 \times 7m) - \frac{(439.3 + 427.52)}{2}\frac{(7^2)}{2} + M_d = 0$

$\therefore M_d = 4668.5 \ kN\text{-}m/m$

**Step 8. Design the strip for flexure:**

$d = 23$ *inches = 685 mm*, $f`_c = 3\ ksi\ (20.7\ MN/m^2),\ f_y = 60\ ksi\ (413.7\ MN/m^2)$

$a = \frac{A_s f_y}{0.85 f`_c b} \qquad A_S = \frac{M_u}{\phi \cdot f_y (d - a/2)}$

say $\boldsymbol{M_u}$ = 5718 *kN-m*/ 8 *m* = 715 *kN-m/m* = 161 *k-ft/ft*

say $\boldsymbol{a}$ = 3.3 *in* , $\boldsymbol{A_s}$ required = 1.68 $in^2/ft$

Try #9 @ 6" o.c. $A_s = (1)(12/6) = 2\ in^2 > 1.68\ in^2$

∴ ***Use #9 @ 6" bottom***, $A_s$ required = 1.68 $in^2$

$\rho_{min} = 200/60000 = 0.0033$ $A_{s\text{-}min} = 0.0033(23)(12) = 0.91\ \text{in}^2 < 1.68\ in^2$ Good

$A_{s\text{-}min}$ Bottom → 1.68 $in^2$
Top → 1.3 $in^2$

**Negative moment:**

$M_u = 4668.5 / 8\ m = 584\ KN\text{-}m/m = 131.6\ k\text{-}ft/ft$

Say $a = 2.54\ in$, $A_s$ required = 1.3 $in^2/ft$

Try #9 @ 9" o.c. $A_s = (1)(12/9) = 1.33\ in^2 > 1.30\ in^2$ Good

∴ ***Use #9 @ 9" top.***

**Use top and bottom reinforcing throughout the mat in the y-direction.**

**Step 9. Sketch the mat's cross-sections and reinforcement.**