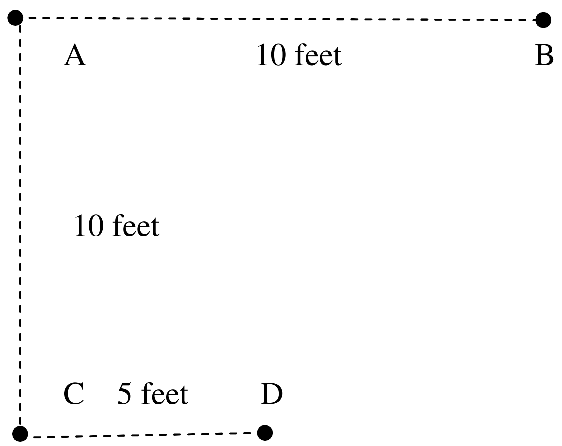
## *Stress–01: Stress increase at a point from several surface point loads.

(Revision: Aug-08)

**Point loads of 2000, 4000, and 6000 lbs act at points A, B and C respectively, as shown below. Determine the increase in vertical stress at a depth of 10 feet below point D.**




***Solution.***

**Using the Boussinesq (1883) table on page 202 for vertical point loads, the vertical increase in stress contributed by each at a depth *z* =10 feet is found by,**

$$\Delta p_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}\right\} = \frac{P}{z^2} I_1$$

| ***Increase in the load at:*** | ***P*** (lbs) | ***r*** (ft) | ***z*** (ft) | ***r/z*** | $I_1$ | $\Delta p$ (psf) |
|---|---|---|---|---|---|---|
| $\Delta p$ from A | 2,000 | $(10^2+5^2)^{1/2} = 11.18$ | 10 | 1.12 | 0.0626 | 1.25 |
| $\Delta p$ from B | 4,000 | $(10^2+5^2)^{1/2} = 11.18$ | 10 | 1.12 | 0.0626 | 2.50 |
| $\Delta p$ from C | 6,000 | 5 | 10 | 0.50 | 0.2733 | 16.40 |
| | | | | | **Total = *20.2 psf*** | |

***Therefore, the vertical stress increase at D from the three loads A, B and C is 20.2 psf.***

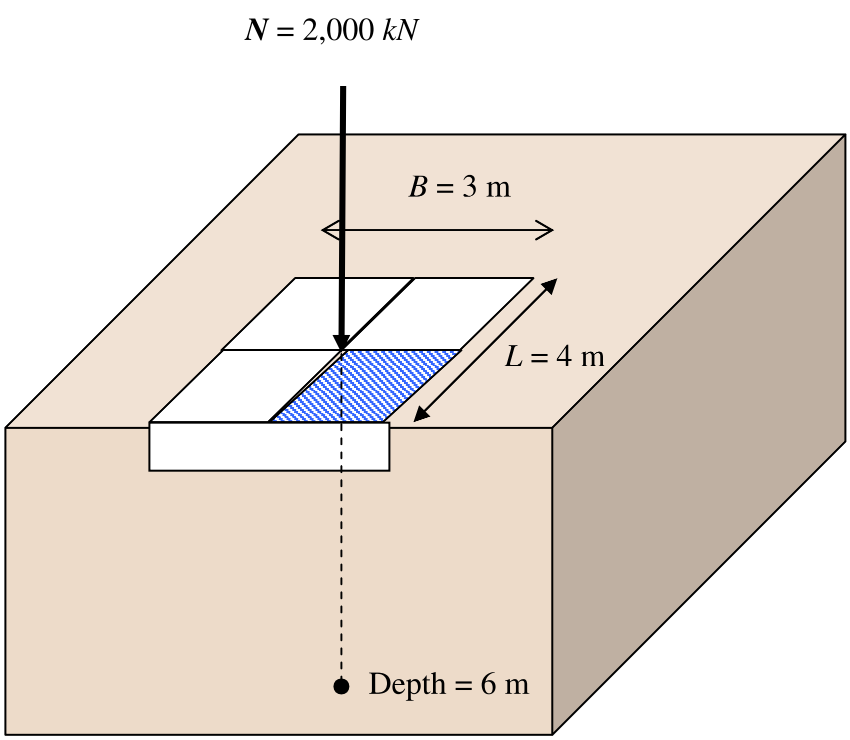
## ***Stress-02:*** *Find the stress under a rectangular footing.*

(Revision: Aug-08)

**Determine the vertical stress increase in a point at a depth of 6 m below the center of the invert of a newly built spread footing, 3 m by 4 m in area, placed on the ground surface carrying a columnar axial load of $N = 2,000$ kN.**

***Solution:***

**The Boussinesq solution for a rectangular loaded area only admits finding stresses below a corner of the loaded area. Therefore, the footing must be cut so that the load is at a "corner" (shown as the quarter of the area), where the reduced footing dimensions for the shaded area are $B_1 = 1.5$ m and $L_1 = 2.0$ m.**




$$m = \frac{B_1}{z} = \frac{1.5m}{6.0m} = 0.25 \quad and \quad n = \frac{L_1}{z} = \frac{2.0m}{6.0m} = 0.33$$

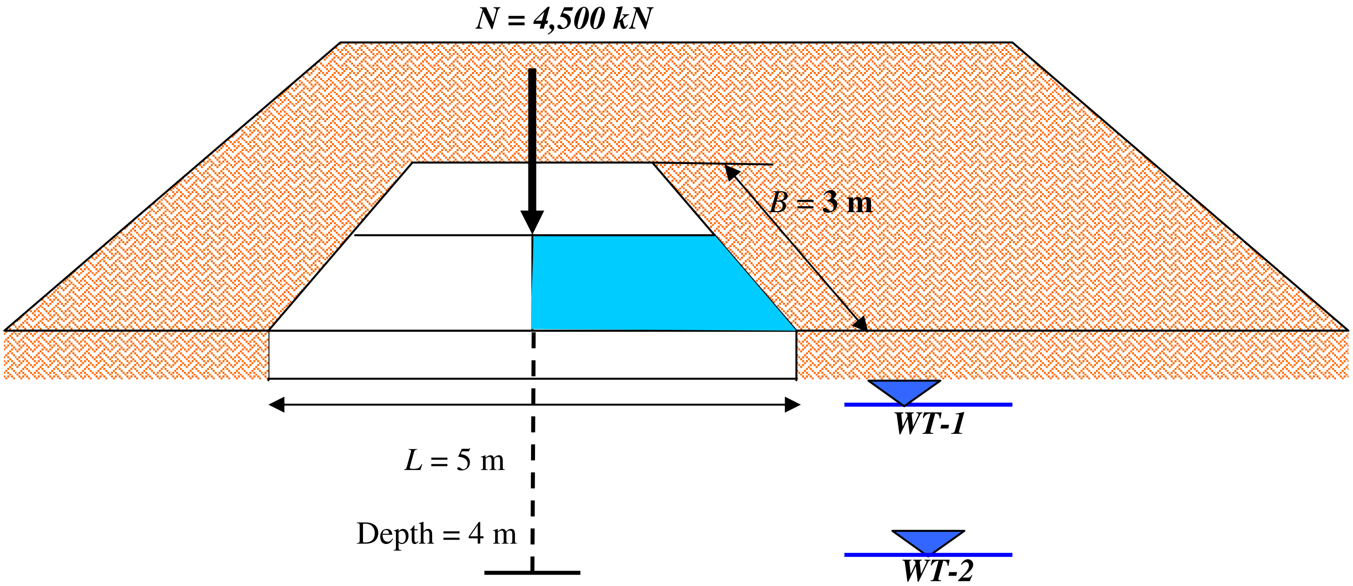
Use the table and extrapolate and find $I_4 = 0.0345$

$$\Delta q_z = q_o\,(4\,I_4) = \left(\frac{N}{BL}\right)(4\,I_4) = \left(\frac{2,000\,kN}{(3\,m)(4\,m)}\right)(4)(0.0345) = 23\ kN/m^2$$

## *Stress-03: *The effect of the WT on the stress below a rectangular footing.*

(Revision: Aug-08)

**Find the effective stress increase in the soil at a depth of 4 m below the footing, and then find the increase in the stress due to a drop of the *WT* from originally 1 m below the footing to 5 m below the footing.**




***Solution:***

$$q = \frac{N}{A} = \frac{4,500\,kN}{15\,m^2} = 300\frac{kN}{m^2} \quad m = \frac{B}{z} = \frac{1.5\,m}{4\,m} = 0.375 \; and \; n = \frac{L}{z} = \frac{2.5\,m}{4\,m} = 0.625$$

$$\therefore \quad I_4 = 0.076$$

$a$) The total stress increase from the footing is,

$$\Delta p_o = q(4I_4) = (300)(4)(0.076) = 91.2\; kN/m^2$$

and the effective stress when the $WT$ is 1 m below the footing is,

$$\Delta p_o^{'} = \Delta p_o - u = (91.2) - (3m)(9.8) = 61.8\; kN/m^2$$

$b$) When the WT drops from -1 m to -5 m below the footing, the effective stress is identical to the total stress. Therefore the effective stress increase is,

$\Delta p_o = 91.2\; kN/m^2$ which is a 48% increase in stress.

## *Stress–04: *Finding the stress outside the footing area.*

(Revision: Aug-08)

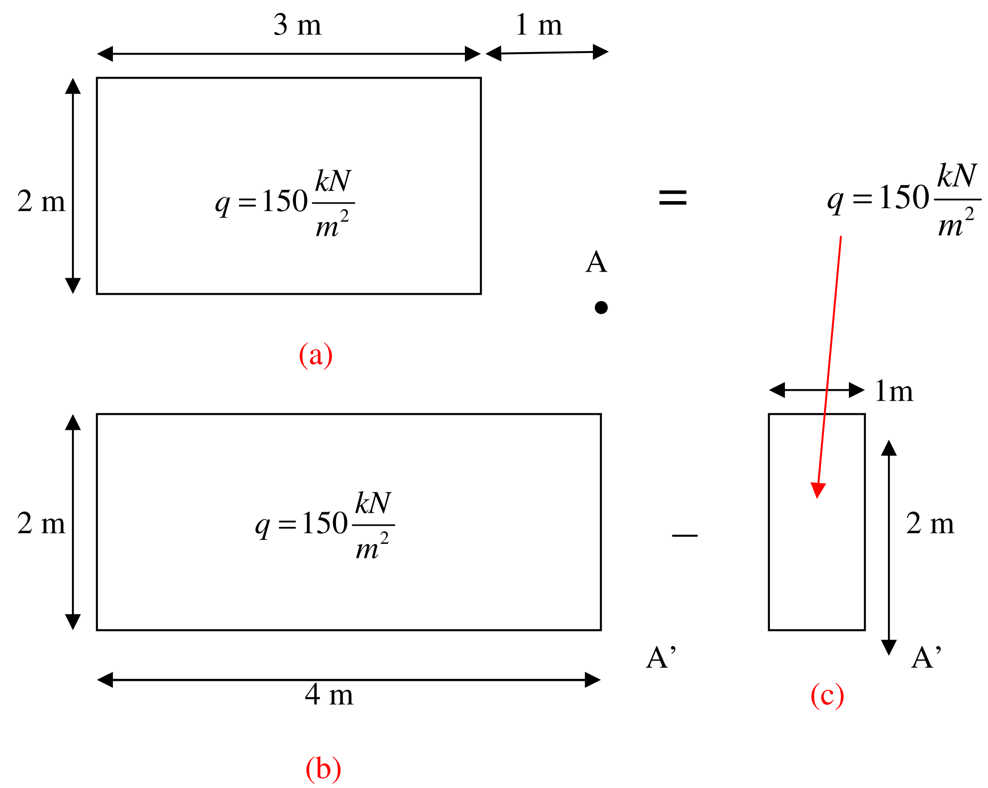
**Find the vertical stress increase $\Delta p$ below the point A at a depth $z$ = 4 m.**

***Solution:***

The stress increase, $\Delta p$, can be written as : $\Delta p = \Delta p_1 - \Delta p_2$

where $\Delta p_1$ = stress increase due to the loaded area shown in (b).

$\Delta p_2$ = stress increase due to the loaded area shown in (c).




For the loaded area shown in (b):

$$m = \frac{B}{Z} = \frac{2}{4} = 0.5 \quad and \quad n = \frac{L}{Z} = \frac{4}{4} = 1.0$$

$$\Delta p_1 = qI_4 = (150)(0.12) = 18\ kN/m^2$$

Similarly, for the loaded area show in (c):

$$m = \frac{B}{Z} = \frac{1}{4} = 0.25 \quad and \quad n = \frac{L}{Z} = \frac{2}{4} = 0.5$$

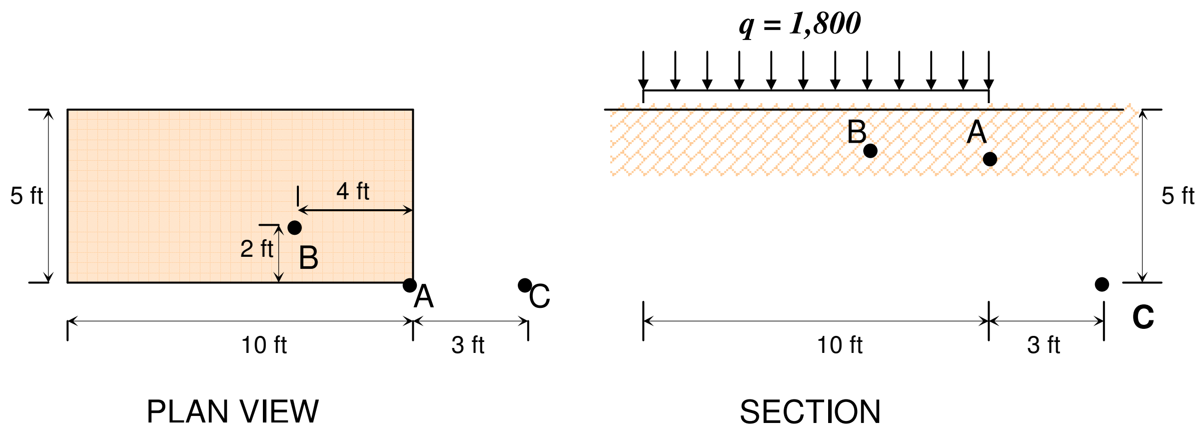
$$\Delta p_2 = (150)(0.048) = 7.2\ kN/m^2$$

Therefore, $\Delta p = \Delta p_1 - \Delta p_2 = 18 - 7.2 = 10.8\ kN/m^2$

## *Stress-05: *Stress below a footing at different points.*

(Revision: Sept.-08)

**A clay sanitary pipe is located at a point C below the footing shown below. Determine the increase in the vertical stress Δp at the depth of the pipe, which is z = 5 feet below the footing invert, and 3 feet away from its edge. The footing has a uniformly distributed load *q* = 1,800 psf.**




***Solution:***

**For the expanded 5' x 13' area,**

$$m = \frac{B}{Z} = \frac{5}{5} = 1 \quad and \quad n = \frac{L}{Z} = \frac{13}{5} = 2.6 \quad \text{therefore, } I_4 = 0.200$$

**For virtual 3' x 5' area**

$$m = \frac{B}{Z} = \frac{3}{5} = 0.6 \quad and \quad n = \frac{L}{Z} = \frac{5}{5} = 1 \quad \text{therefore, } I_4 = 0.136$$

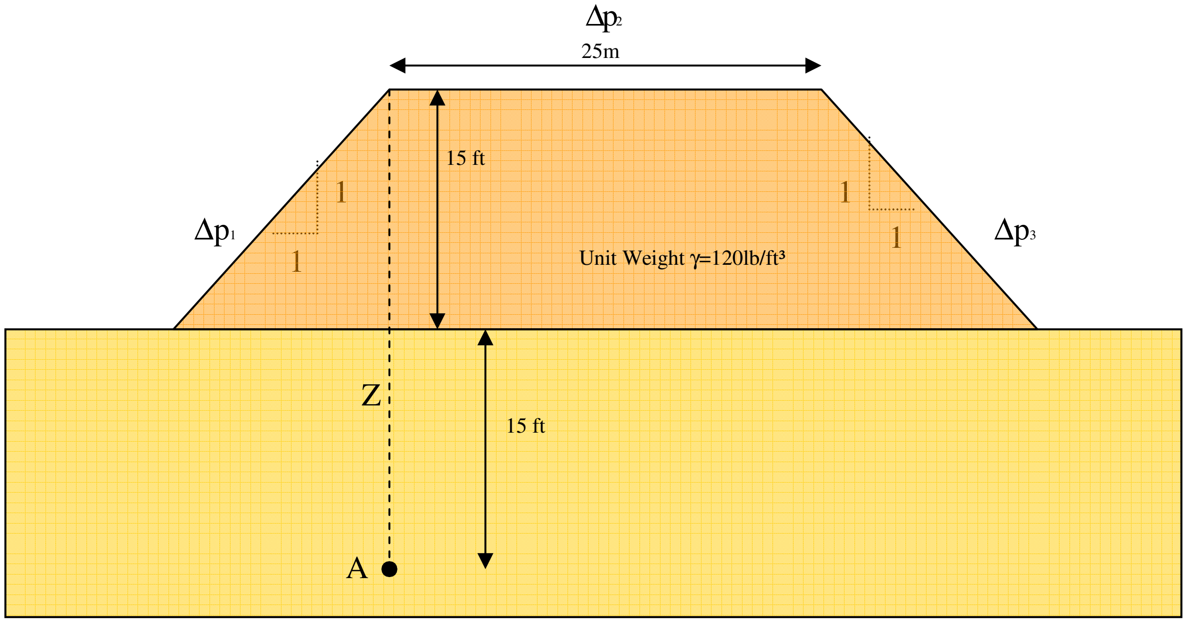
**The increase in stress at point *C* from the footing is therefore,**

$$\Delta p = q(I_4 - I_4^{'}) = \left(1,800 \ \frac{lb}{ft^2}\right)(0.200 - 0.136) = 115 \ psf$$

## *Stress-06: Stress increase from a surcharge load of limited width.

(Revision: Aug-08)

**Calculate the stress increase at the point *A* due to the new road embankment.**




**The contribution from the central portion of the fill is $\Delta p_2$, whereas the contribution from the left and right hand slopes are $\Delta p_1$ and $\Delta p_3$ respectively. Using Boussinesq,**

$$\Delta p_1 \Rightarrow \frac{2x_1}{B_1} = \frac{2(15')}{15'} = 2 \qquad \frac{2z}{B_1} = \frac{2(15')}{15'} = 2 \qquad \frac{\Delta p_1}{q} = 0.25$$

$$\therefore \Delta p_1 = (0.25)(15')\left(120\frac{lb}{ft^3}\right) = 450\ psf$$

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$$\Delta p_2 \Rightarrow \frac{2x_2}{B_2} = \frac{2(-12.5')}{25'} = -1 \qquad \frac{2z}{B_2} = \frac{2(15')}{25'} = 1.2 \qquad \frac{\Delta p_2}{q} = 0.47$$

$$\therefore \Delta p_2 = (0.47)(15')\left(120\frac{lb}{ft^3}\right) = 846\ psf$$

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$$\Delta p_3 \Rightarrow \frac{2x_3}{B_3} = \frac{2(40')}{15'} = 5.3 \qquad \frac{2z}{B_3} = \frac{2(15')}{15'} = 2 \qquad \frac{\Delta p_3}{q} = 0.02$$

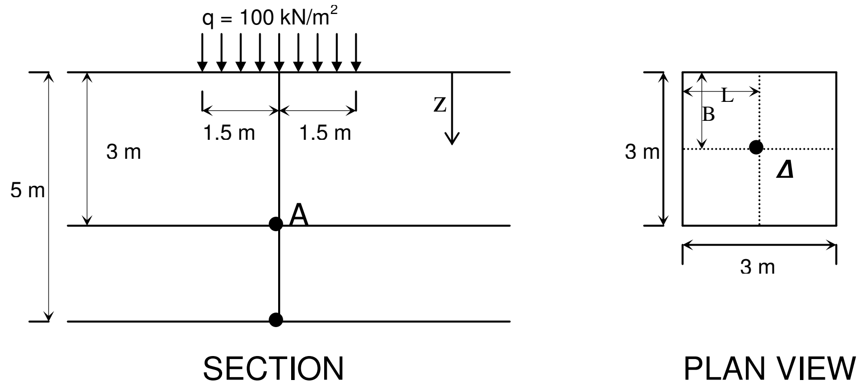
$$\therefore \Delta p_3 = (0.02)(15')\left(120\frac{lb}{ft^3}\right) = 306\ psf$$

$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3 = 450 + 846 + 36 = 1,332\ psf$$

## *Stress-07: Finding a stress increase from a surface load of limited width.

(Revision: Aug-08)

**Determine the average stress increase below the center of the loaded area, between z = 3 m and z = 5 m.**




***Solution:***

The stress increase between the required depths (below the corner of each rectangular area) can be given as:

$$\Delta p_{avg(H_2/H_1)} = q\left[\frac{(H_2)(I_{4(H_2)}) - (H_1)(I_{4(H_1)})}{H_2 - H_1}\right] = 100\left[\frac{(5)(I_{4(H_2)}) - (3)(I_{4(H_1)})}{5-3}\right]$$

For $I_{4(H2)}$: $m' = B / H_2 = 1.5 / 5 = 0.3$

$n' = L / H_2 = 1.5 / 5 = 0.3$

For $m' = n' = 0.3$, $I_{4(H2)} = 0.038$

For $I_{4(H1)}$: $m' = B / H_1 = 1.5 / 3 = 0.5$

$n' = L / H_1 = 1.5 / 3 = 0.5$

For $m' = n' = 0.5$, $I_{4(H1)} = 0.086$

Therefore:

$$\Delta p_{av(H2/H1)} = 100 \times \frac{(5)(0.038) - (3)(0.086)}{5-3} = 3.4 \text{ kN/m}^2$$

**The stress increase between z = 3 m and z = 5 m below the center of the load area is equal to:**

$$4\Delta p_{avg(H_2/H_1)} = (4)(3.4) = 13.6 \;\; kN/m^2$$

## **Stress-08: *Stress increase as a function of depth.***

(Revision: Aug-08)

**The vertical stress $\sigma_v$ in a soil at any depth below the surface can be estimated as a function of the soil unit weight γ by the equation,**

$$\sigma_v = \int_0^Z \gamma(\sigma_v)\,dz = \int_0^{100} (95 + 0.0007\sigma_v)\,dz$$

**If a particular stratum has a function γ = 95 + 0.0007 $\sigma_v$ , where $\gamma$ is in pcf and $\sigma_v$ is in psf, find the vertical stress at a depth of 100 feet below the surface.**

***Solution:***

Rearranging, and integrating by parts,

$$\int_0^{\sigma_v} \frac{d\sigma_v}{(95 + 0.0007\sigma_V)} = \int_0^{100} dz$$

$$\frac{1}{0.0007}\ln\left(95 + 0.0007\sigma_V\right)_0^{\sigma_V} = z_0^{100}$$

$$\sigma_v = 135,800\left(e^{0.0007z} - 1\right)_0^{100} = 9,840\ psf$$

At Z = 100 $\quad \sigma_v = 135,800(1.0725 - 1)$

$$\therefore \quad \sigma_v = 9,840\ psf$$