*Mohr-01: *Find the angle of internal friction φ of a soil.*

(Revision: Jan-09)

The horizontal stress on a soil particle at a depth of 130 ft is ⅓ of its vertical stress. If the average unit weight of the intervening soil mass above is 116.4 pcf, and the shear stress on the soil particle's failure plane is 30 psi, what is the soil's average angle of friction?

Solution:

The vertical stress is the major principal stress σ_1 and the horizontal stress is the minor principal stress σ_3. The shear stress is related to these two principal stresses via the angle θ, which is the angle to the failure plane with respect to the principal stress σ_1. These are related by the equation,

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

From this relation, the angle of internal friction is found from θ,

$$\varphi = 2\theta - 90°$$

Therefore,

$$\sigma_1 = \gamma h = \left(0.116.4 \frac{lb}{ft^3} \right) (130 ft) \left(\frac{1 ft^2}{144 in^2} \right) = 105 psi$$

$$\sigma_3 = \frac{1}{3} \sigma_1 = (0.33)(105 psi) = 35 psi$$

$$From \quad \tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \quad we\ have \quad 30 psi = \frac{105 psi - 35 psi}{2} \sin 2\theta$$

$$\therefore 2\theta = 120°$$

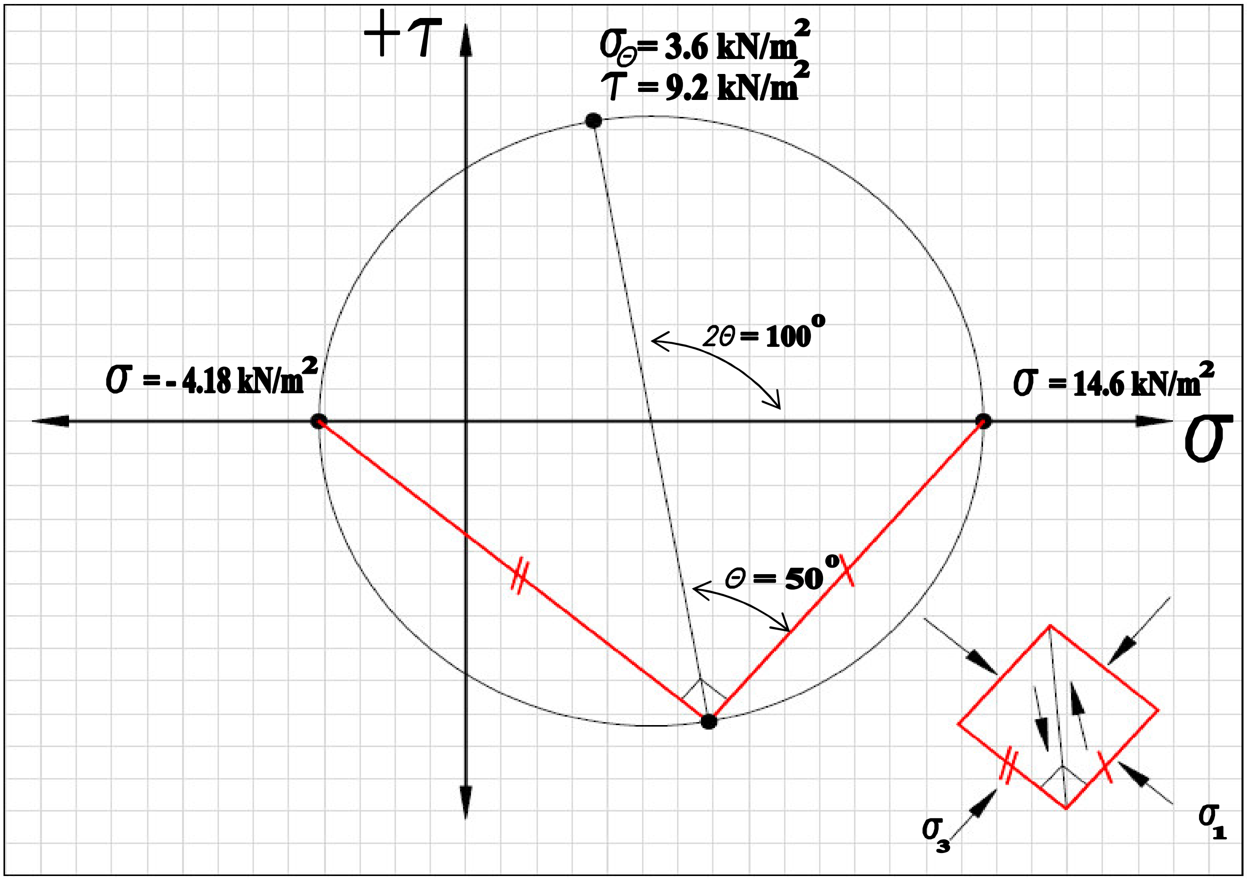
$$\therefore \varphi = 2\theta - 90° = (120°) - 90° = 30°$$

*Mohr-01: *Simple transformation from principal to general stress state.*

(Revision: Feb, 2009)

A soil particle is found to be subjected to a maximum stress of 14.6 kN/m^2, and a minimum stress of – 4.18 kN/m^2. Find the σ and τ on the plane of θ = 50° with respect to the major principal stresses, and also find $\tau_{max.}$

(a) The graphical solution,



(b) The calculated solution,

$$\sigma_\theta = \left(\frac{\sigma_1+\sigma_3}{2}\right) + \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta = \left(\frac{14.6-4.18}{2}\right) + \left(\frac{14.6+4.18}{2}\right)\cos 2(50°) = 3.6\frac{kN}{m^2}$$

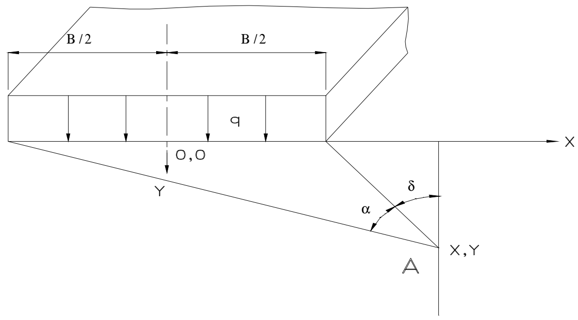
$$\tau_\theta = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\theta = \left(\frac{14.6+4.18}{2}\right)\sin 2(50°) = 9.2\frac{kN}{m^2}$$

$$\tau_{\text{maximum}} = \left(\frac{\sigma_1-\sigma_3}{2}\right) = 9.4\frac{kN}{m^2}$$

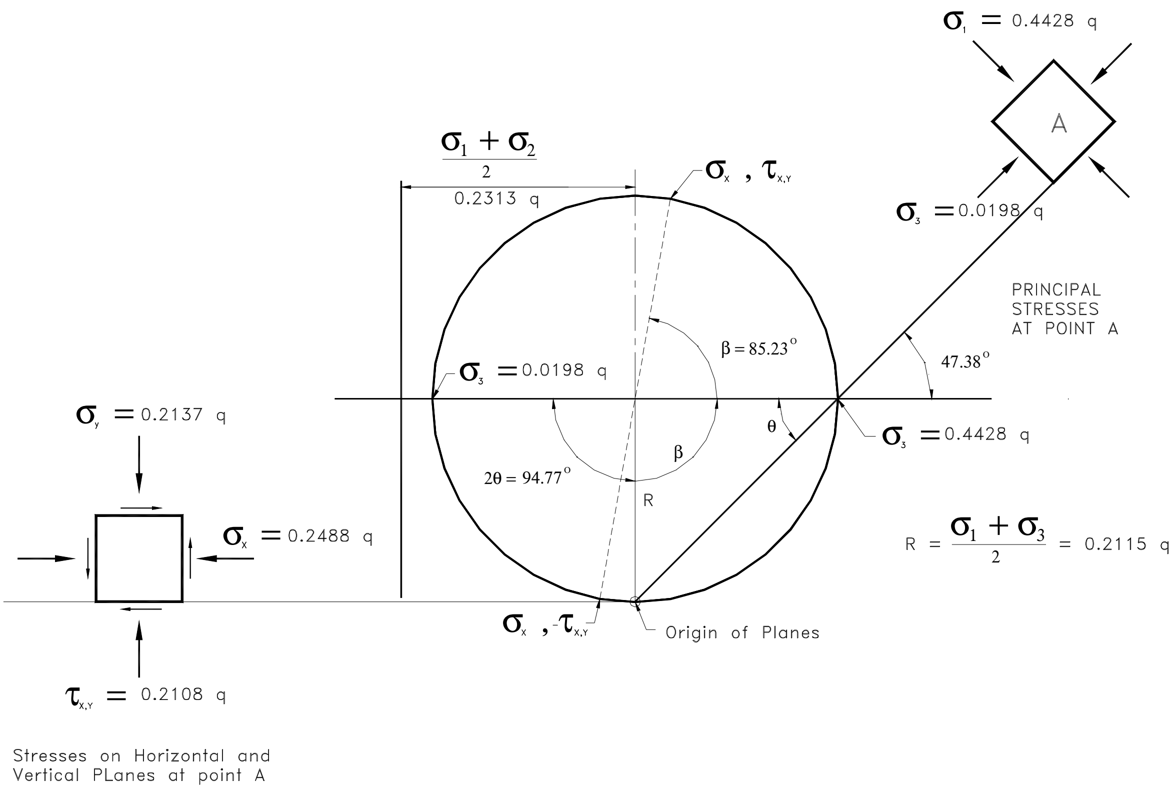
*Mohr–03: *Using Mohr for stresses under a pavement.*

(Revision: Jan-09)

Equations for the principal stresses in the elastic half-space shown below for a uniformly loaded strip footing are as follows: $\sigma_1 = q/\pi(\alpha + \sin\alpha)$ and $\sigma_3 = q/\pi(\alpha - \sin\alpha)$



The direction of the major principal stress bisects the angle α. Calculate the vertical stress σ_y, the horizontal stress σ_x, and τ_{xy} at point A if x = 0.75B and y = 0.5B using Mohr's diagram.



$\alpha + \delta = \arctan 2.5 = 68.20°$

$\delta = \arctan 0.5 = 26.57°$

$68.20° - 26.57° = 41.63°$

$2\theta + \beta = 180°$

$\beta = 85°$

if q = 432.5 kPa then

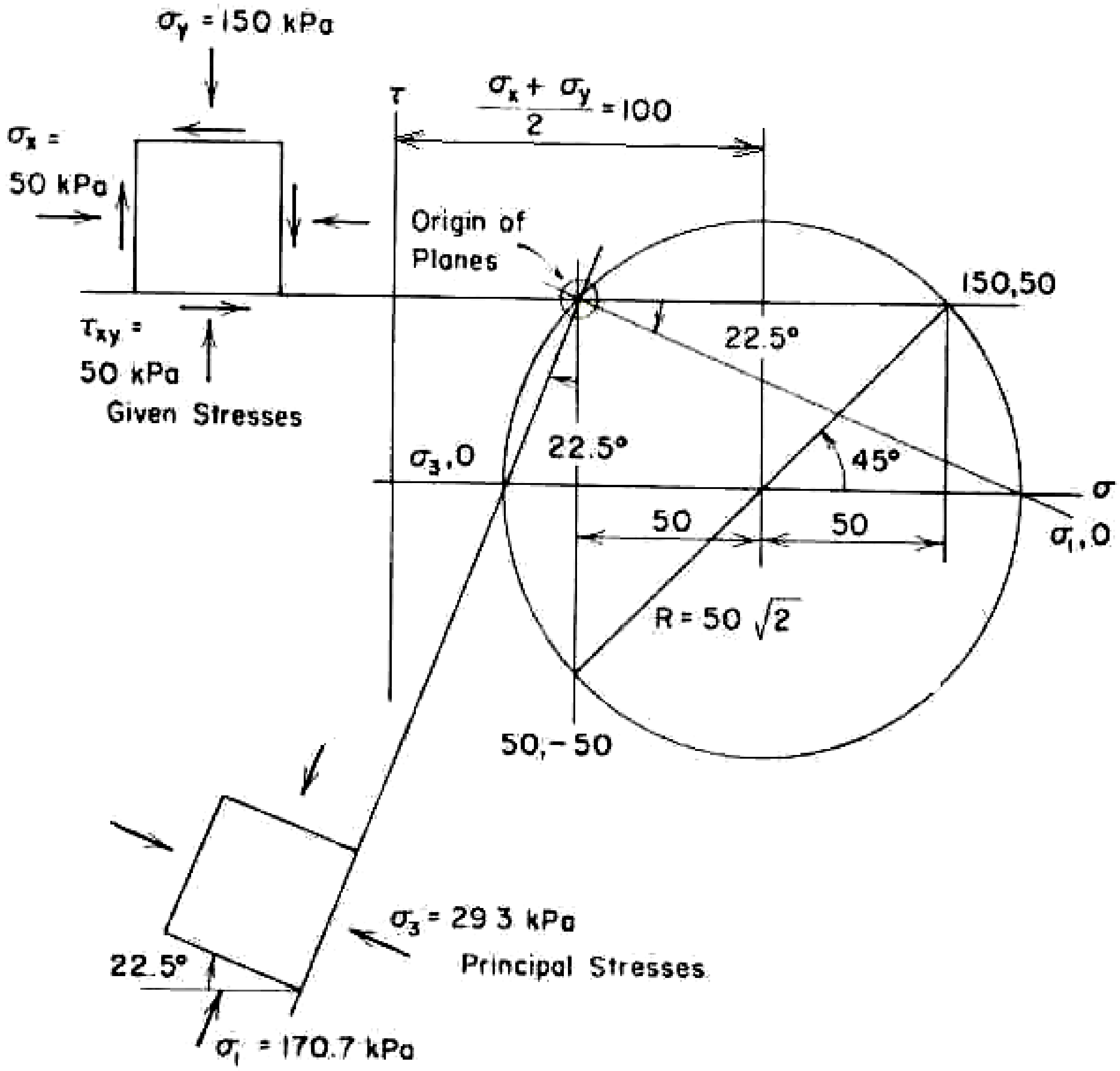
$\sigma_1 = (\sigma_x + \sigma_y)/2 + R = 100 + 70.7 = 170.7$ kPa

$\sigma_3 = (\sigma_x + \sigma_y)/2 - R = 100 - 70.7 = 29.3$ kPa

*Mohr–04: *Find the principal stresses and their orientation.*

(Revision: Jan-09)

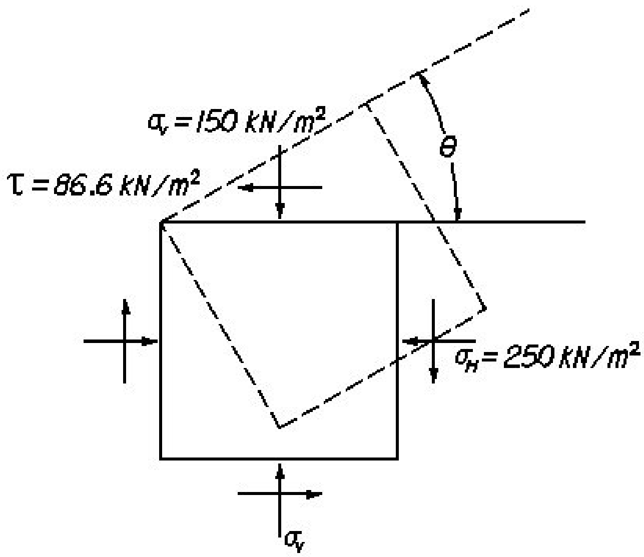
Given the general stresses at a point in a soil, determine the principal stresses and show them on a properly oriented element.



*Mohr–05: *Find the angle of internal friction φ.*

(Revision: Jan-09)

A sample of clean sand was retrieved from 7 m below the surface. The sample had been under a vertical load of 150 kN/m^2, a horizontal load of 250 kN/m^2, and a shear stress of 86.6 kN/m^2. If the angle θ between the vertical stress and the principal stress is 60°, what is the angle of internal friction ϕ of this sample?

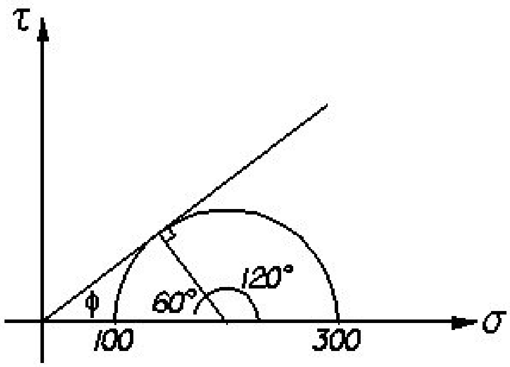


$$\sigma_1 = \frac{\sigma_v + \sigma_h}{2} \pm \sqrt{\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + (\tau)^2}$$

$$\sigma_1 = \frac{150+250}{2} \pm \sqrt{\left(\frac{150-250}{2}\right)^2 + (86.6)^2}$$

$$\sigma_1 = 200 + 100 = \boxed{300 \text{ kN/m}^2}$$

$$\therefore \sigma_3 = 200 - 100 = \boxed{100 \text{ kN/m}^2}$$



$$\sin\phi = \frac{\frac{1}{2}(\sigma_1 - \sigma_3)}{\frac{1}{2}(\sigma_1 + \sigma_3)} = \frac{300 - 100}{300 + 100}$$

$$\sin\phi = 0.5$$

$$\phi = \sin^{-1}(0.5)$$

$$\boxed{\phi = 30^\circ}$$