02- Phases of Soils Relationships

***01. Convert from metric units to *SI* and *US* units.**
***02. Compaction checked via the voids ratio.**
***03. Find the moisture w when the soil is fully saturated.**
***04. Identify the wrong moisture data.**
***05. Dry unit weight and degree of saturation.**
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***15. The apparent cheapest soil is not!**
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***17. How many truck-loads are needed for a housing project?**
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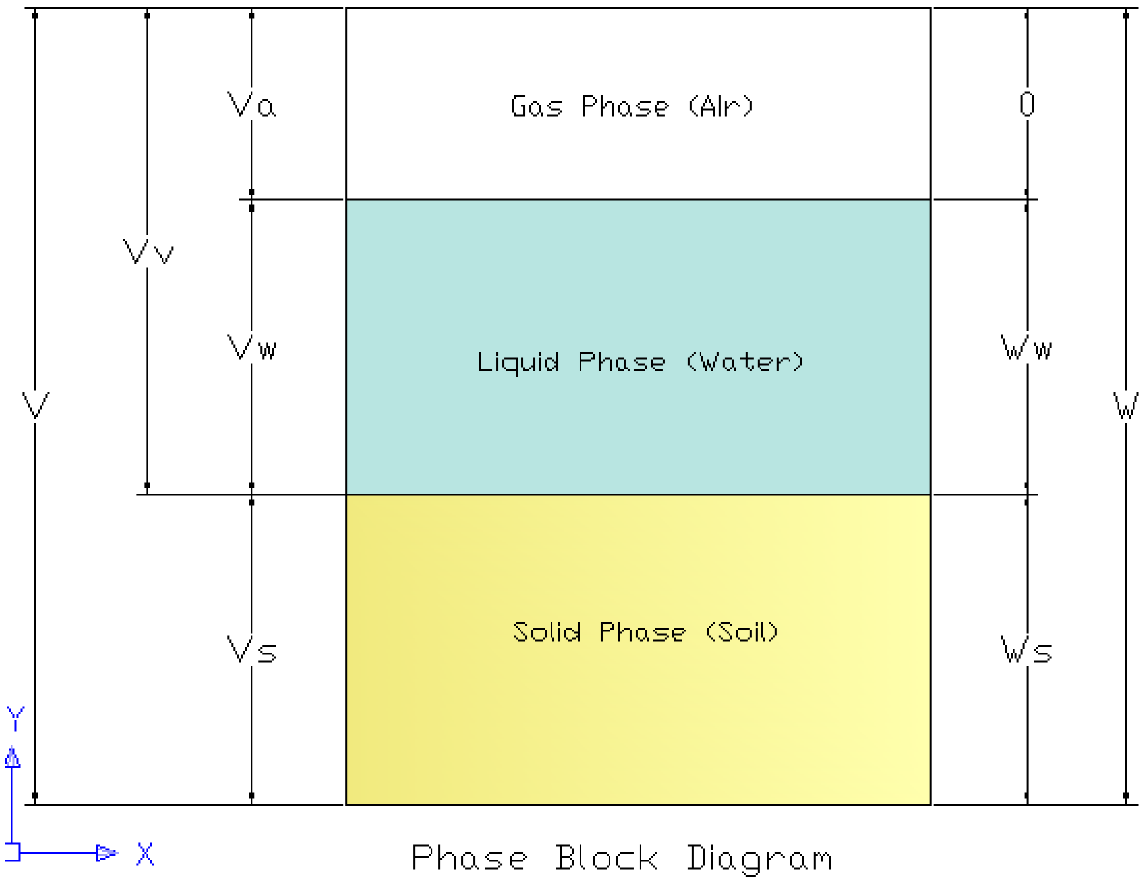
****19. Use a matrix to find the missing data.**
****20. Find the voids ratio of "muck" (a highly organic soil).**

Symbols for the Phases of Soils

e → *Voids ratio.*
G_S → *Specific gravity of the solids of a soil.*
n → *Porosity.*
S → *Degree of saturation.*
V → *Total volume (solids + water + air).*
V_a → *Volume of air.*
V_V → *Volume of voids (water + air).*
V_S → *Volume of solids.*
V_W → *Volume of water.*
w → *Water content (also known as the moisture content).*
W_S → *Weight of solids.*
W_W → *Weight of water.*
γ → *Unit weight of the soil.*
γ_d → *Dry unit weight of the soil.*
γ_b → *Buoyant unit weight of the soil (same as $\gamma' = \gamma_{sat} - \gamma_w$).*
γ_{SAT} → *Unit weight of a saturated soil.*
γ_W → *Unit weight of water.*

Basic Concepts and Formulas for the Phases of Soils.



(A) Volumetric Relationships:

1. - ***Voids ratio e***

$$e = \frac{V_V}{V_S}$$

ranges from 0 to infinity.

Typical values of sands are: very dense 0.4 to very loose 1.0
Typical values for clays are: firm 0.3 to very soft 1.5.

2. - ***Porosity n***

$$n = \frac{V_V}{V}(100\%)$$ ranges from 0% to 100%.

The porosity provides a measure of the ***permeability*** of a soil.

The interrelationship of the voids ratio and porosity are given by,

$$e = \frac{n}{1-n} \quad and \quad n = \frac{e}{1+e}$$

3. - **_Saturation S_** $$S = \frac{V_W}{V_V} x100\%$$ ranges from 0% to 100%.

(B) Weight Relationships:

4. - **_Water content w_** $$w = \frac{W_W}{W_S} x100\%$$

Values range from 0% to over 500%; also known as ***moisture content***.

5. – **_Unit weight of a soil γ_** $$\gamma = \frac{W}{V} = \frac{W_S + W_W}{V_S + V_W + V_A}$$

The unit weight may range from being dry to being saturated.

Some engineers use "***bulk density ρ***" to refer to the ratio of mass of the solids and water contained in a unit volume (in Mg/m^3). Note that,

$$\gamma = \frac{W}{V} = \rho g = \frac{m}{V} g \quad which\ is\ the\ equivalent\ of\ F = ma.$$

6. - **_Dry unit weight γ_d_** $$\gamma_d = \frac{W_S}{V} = \frac{\gamma}{1+w}$$

The soil is perfectly dry (its moisture is zero).

7. - ***The unit weight of water*** γ_w

$$\gamma_w = \frac{W_W}{V_W} \quad where\ \gamma = \rho g\ (F = ma)$$

$$\gamma_w = 62.4\ pcf\ = 1\ g/ml = 1\ kg/liter = 9.81\ kN/m^3$$

Note that the above is for fresh water. Salt water is 64 pcf, etc.

8. - ***Saturated unit weight of a soil*** γ_{sat} $$\gamma_{SAT} = \frac{W_S + W_W}{V_S + V_W + 0}$$

9. - ***Buoyant unit weight of a soil*** γ_b $$\gamma_b = \gamma' = \gamma_{SAT} - \gamma_w$$

10. - ***Specific gravity of the solids of a soil*** G_s $$G_S = \frac{\gamma_S}{\gamma_w}$$

Typical Specific Gravities of Minerals in Soils and Rocks		
Mineral	***Chemical Composition***	***Absolute specific gravity*** G_s
Anhydrite	$CaSO_4$	2.90
Barites	$BaSO_4$	4.50
Aragonite	$CaCO_3$	2.94
Calcite	$CaCO_3$	2.80 – 2.90
Chlorite		2.60 to 3.0
Dolostone (Dolomite)	$CaMgCO_3$	2.87
Orthoclase feldspar		2.56
Plagioclase feldspar	$KALSi_3O_8$	2.62 - 2.76
Gypsum	$CaSO_4\ 2H_2O$	2.32
Hematite	Fe_2O_3	4.30 to 5.20
Kaolinite clay family	$Al_4Si_4O_{10}(OH)_8$	2.60- 2.63
Illite clay family	$K_yAl_4Si_{8-y}O_{20}(OH)_4$	2.80
Montmorillonite clay family	$Al_4Si_8O_{20}(OH)_4 \cdot nH_2O$	2.65 to 2.80
Biotite (mica)		3.00 to 3.10
Muscovite (mica)		2.80 to 2.90
Limonite	Fe_3O_4	3.80
Magnetite	Fe_3O_4	5.18
Lead	Pb	11.34
Quartz (silica)	SiO_2	2.65
Talc		2.70
Peat	Organic	1.0 or less

Diatomaceous earth	Skeletons of plants	2.00

Other useful formulas dealing with phase relationships:

$$Se = wG_S$$

$$e = \frac{\gamma_s}{\gamma_{dry}} - 1$$

Unit weight relationships:

$$\gamma = \frac{(1+w)G_S\gamma_w}{1+e} = \frac{(G_S + Se)\gamma_w}{1+e} = \frac{(1+w)G_S\gamma_w}{1+\frac{wG_S}{S}} = G_S\gamma_w(1-n)(1+w)$$

Saturated unit weights:

$$\gamma_{SAT} = \frac{(G_S+e)\gamma_w}{1+e} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$$

$$\gamma_{SAT} = \gamma_d + n\gamma_w = \left[(1-n)G_s + n\right]\gamma_w = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$$

$$\gamma_{SAT} = \gamma' + \gamma_w$$

Dry unit weights:

$$\gamma_d = \frac{\gamma}{1+w} = G_s\gamma_w(1-n) = \frac{G_S\gamma_w}{1+e} = \frac{eS\gamma_w}{(1+e)w} = \frac{eG_s\gamma_w}{(S+wG_s)}$$

$$\gamma_d = \gamma_{SAT} - n\gamma_w = \gamma_{SAT} - \left(\frac{e}{1+e}\right)\gamma_w$$

*Phases of soils-01: Convert from metric units to SI and US units.

(Revision: Aug.-09)

A cohesive soil sample was taken from an *SPT* and returned to the laboratory in a glass jar. It had a mass of 140.5 grams. The sample was then placed in a container of *V* = 500 cm^3 and 423 cm^3 of water were added to fill the container. From these data, what was the unit weight of the soil in *kN/m^3* and *pcf*?

Solution.

Notice that the 140.5 grams is identified as a mass. The ratio of mass to volume is a density ρ,

$$\rho = \frac{m}{V} = \frac{140.5\ g}{(500-423)cm^3} = 1.82\frac{g}{cm^3}$$

$$\gamma = \rho g = \left(1.82\frac{g}{cm^3}\right)\left(\frac{1\ kg}{10^3\ g}\right)\left(9.806\frac{m}{\sec^2}\right)\left(\frac{1\ kN}{10^3\ N}\right)\left(\frac{10^2\ cm}{1\ m}\right)^3 = 17.9\frac{kN}{m^3}\ \ (SI\ units)$$

$$\gamma = \left(17.9\frac{kN}{m^3}\right)\left(\frac{1000N}{1\ kN}\right)\left(\frac{0.2248\ lbs_f}{1\ N}\right)\left(\frac{1\ m^3}{35.3\ ft^3}\right) = 114\ pcf\ \ (US\ units)$$

*Phases of soils–02: Compaction checked via the voids ratio e.

(Revision: Sept.-08)

A contractor has compacted the base course for a new road and found that the mean value of the test samples shows w = 14.6%, G_S = 2.81, and γ = 18.2 kN/m^3. The specifications require that $e \leq 0.80$.

Has the contractor complied with the specifications?

Solution:

$$\gamma = \frac{G_S \gamma_W (1+w)}{1+e} \quad \therefore \quad 1+e = \frac{G_S \gamma_W (1+w)}{\gamma}$$

$$1+e = \frac{2.81\left(9.81 \frac{kN}{m^3}\right)(1+0.146)}{18.2 \frac{kN}{m^3}} = 1.74$$

$$e = 1.74 - 1 = 0.74$$

$$\therefore \quad e = 0.74 < 0.80 \quad \textit{Yes, the contractor has complied.}$$

*Phases of soils–03: What is the moisture when the soil is fully saturated?

(Revision: Aug.-09)

(1) Show that at saturation the moisture (water content) is $w_{sat} = \frac{(n\gamma_W)}{(\gamma_{sat} - n\gamma_W)}$.

(2) Show that at saturation the moisture (water) content is $w_{sat} = \gamma_w \left(\frac{1}{\gamma_d} - \frac{1}{\gamma_S} \right)$

Solution:

(1) In a fully saturated soil the relation, $Se = wG_S$ becomes simply $e = wG_s$

because $S = 1$ $\quad or \quad G_S = \frac{e}{w_{sat}} = \frac{n}{w_{sat}(1-n)}$

$$but \quad \gamma_{sat} = \gamma_w \left[(1-n) G_S + n \right]$$

$$rearranging \quad \frac{\gamma_{sat}}{\gamma_w} = \left[(1-n) G_S + n \right] = \left[(1-n) \frac{n}{w_{sat}(1-n)} + n \right] = \frac{n}{w_{sat}} + n$$

$$or \quad \frac{\gamma_{sat}}{\gamma_w} - n = \frac{n}{w_{sat}} \quad therefore \quad w_{sat} = \frac{n\gamma_w}{\gamma_{sat} - n\gamma_w}$$

(2) Again, in a fully saturated soil, $w_{sat} = \frac{e}{G_S} = \frac{V_V}{V_S} \frac{\gamma_w}{\gamma_S} = \frac{V_V}{V_S} \frac{\gamma_w}{1} \frac{V_S}{W_S} = \frac{\gamma_w V_V}{W_S}$

$$\therefore \quad w_{sat} = \frac{\gamma_w V_V}{W_S} = \gamma_w \left(\frac{V_V}{W_S} \right) = \gamma_w \left(\frac{V_V + V_S - V_S}{W_S} \right) = \gamma_w \left(\frac{V_V + V_S}{W_S} - \frac{V_S}{W_S} \right)$$

$$or \quad w_{sat} = \gamma_w \left(\frac{1}{\gamma_d} - \frac{1}{\gamma_S} \right)$$

*Phases of soils–04: *Identifying the wrong moisture data.*

(Revision: Aug.-09)

A geotechnical laboratory reported these results of five samples taken from a single boring. Determine which are not correctly reported, if any.
Sample #1: w = 30%, γ_d = 14.9 kN/m^3, γ_s = 27 kN/m^3; clay.
Sample #2: w = 20%, γ_d = 18 kN/m^3, γ_s = 27 kN/m^3; silt.
Sample #3: w = 10%, γ_d = 16 kN/m^3, γ_s = 26 kN/m^3; sand.
Sample #4: w = 22%, γ_d = 17.3 kN/m^3, γ_s = 28 kN/m^3; silt.
Sample #5: w = 22%, γ_d = 18 kN/m^3, γ_s = 27 kN/m^3; silt.

Solution:

The water content is in error if it is reported greater than the saturated moisture.

That is, if S appears to be greater that 100%.

First, find the w_{sat}

$$w_{sat} = \frac{e}{G_S} = \frac{V_V}{V_S}\frac{\gamma_w}{\gamma_S} = \frac{V_V}{V_S}\frac{\gamma_w}{1}\frac{V_S}{W_S} = \frac{\gamma_w V_V}{W_S} = \gamma_w\left(\frac{V_V}{W_S}\right) = \gamma_w\left(\frac{V_V + V_S - V_S}{W_S}\right)$$

$$w_{sat} = \gamma_w\left(\frac{V_V + V_S}{W_S} - \frac{V_S}{W_S}\right) = \gamma_w\left(\frac{1}{\gamma_d} - \frac{1}{\gamma_S}\right)$$

$$\therefore \quad \text{The moisture } w \le w_{SAT} = \gamma_w\left(\frac{1}{\gamma_d} - \frac{1}{\gamma_S}\right)$$

1) $w_{SAT} = \left(9.81\, kN/m^3\right)\left(\frac{1}{14.9} - \frac{1}{27}\right) = 30\% = w = 30\%\ GOOD$

2) $w_{SAT} = \left(9.81\, kN/m^3\right)\left(\frac{1}{18} - \frac{1}{27}\right) = 18.5\% < w = 20\%\ WRONG$

3) $w_{SAT} = \left(9.81\, kN/m^3\right)\left(\frac{1}{16} - \frac{1}{26}\right) = 24\% > w = 10\%\ GOOD$

4) $w_{SAT} = \left(9.81\, kN/m^3\right)\left(\frac{1}{17.3} - \frac{1}{28}\right) = 22.1\% > w = 22\%\ GOOD$

5) $w_{SAT} = \left(9.81\, kN/m^3\right)\left(\frac{1}{18} - \frac{1}{27}\right) = 18.5\% < w = 22\%\ WRONG$

*Phases of soils–05: Dry unit weight and degree of saturation.

(Revision: Aug.-09)

During compaction, the engineer needs to know the relationship between the saturation of the soil and the ensuing dry density. As a rule, the maximum dry density is in the range of 70% to 85% saturation. Express the theoretical dry unit weight as a function of the saturation *S*.

Solution:

The two basic equations of phase relationships are,

$$\gamma_d = \frac{G_S \gamma_W}{1+e} \qquad (1) \quad and$$

$$Se = wG_S \qquad (2)$$

Solving for *e* in equation (2), and substituting that value into (1), yields

$$\gamma_d = \frac{G_S \gamma_W}{1+\frac{wG_S}{S}} \quad or \quad \gamma_d = \frac{SG_S \gamma_W}{S + wG_S}$$

*Phases of soils–06: Increasing the saturation of a soil.

(Revision: Aug.-09)

A soil sample has a unit weight of 105.7 pcf and a saturation of 50%. When its saturation is increased to 75%, its unit weight raises to 112.7 pcf.

Determine the voids ratio *e* and the specific gravity G_s of this soil.

Solution:

$$\gamma = \frac{\gamma_W \left(G_S + Se\right)}{1+e}$$

$$\therefore \quad 105.7 \; pcf = \frac{62.4(G_S + 0.50e)}{1+e} \qquad (1)$$

$$and \quad 112.7 \; pcf = \frac{62.4(G_S + 0.75e)}{1+e} \qquad (2)$$

Solving explicitly for G_s in equation (1),

$$G_s = \frac{\left(105.7\right)\left(1+e\right)}{62.4} - 0.50e$$

Replace G_s in equation (2) with the above relation from (1),

$$\therefore \quad \left(112.7\right)\left(1+e\right) = \left(105.7\right)\left(1+e\right) + \left(62.4\right)\left(0.25e\right)$$

$$\therefore \quad \underline{e = 0.814 \quad and \quad G_S = 2.67}$$

*Phases of soils–07: How much water is added to a truck to offset losses?

(Revision: Sept.-09)

The weight of an empty truck is 3,000 lbs. It is filled with 1,000 lbs of soil. The original moisture of the soil was 12%, but during transportation the moisture dropped to 11.5%.

1) How much water (in gallons) did the soil lose during transportation?
2) How much water (in gallons) must be added to the truck to arrive at the site with a 15% moisture?

Solution:

For a moisture content $w = 12\%$,

$$w = \frac{W_W}{W_S} = 0.12 \quad but\ W = 1{,}000 lbs = W_S + W_W = \frac{W_W}{0.12} + W_W = 9.33 W_W$$

$$\therefore W_W = \frac{1{,}000 lbs}{9.33} = 107\ lb \quad and\ W_S = \frac{W_W}{0.12} = \frac{107.2 lb}{0.12} = 893\ lb$$

For the lowered moisture content of $w = 11.5\%$,

$$w = \frac{W_W}{W_S} = 0.115 \quad but\ W = 1{,}000 lbs = W_S + W_W = \frac{W_W}{0.115} + W_W = 9.696 W_W$$

$$\therefore W_W = \frac{1{,}000 lbs}{9.696} = 103\ lb$$

$\therefore$ the truck lost 4 lbs of water = $(4lb)\left(\frac{1 ft^3}{62.4 lb}\right)\left(\frac{7.48 gallons}{1 ft^3}\right) = 0.48 gallons$

To arrive at the site with a moisture content of $w = 15\%$,

$$w = \frac{W_W}{W_S} = 0.15 \quad \therefore W_W = 0.15 W_S = (0.15)(893 lb) = 134 lb$$

The truck load needs to add $134 - 103 = 31\ lbs$ of water $\equiv$ $3.71 gallons$

*Phases of soils–08: *Find γ_d, n, S and W_w.*

(Revision: Aug.-09)

The moist unit weight of a soil is 16.5 kN/m^3. Given that the w = 15% and G_s = 2.70, find:
a) Dry unit weight γ_d,
b) The porosity n,
c) The degree of saturation S, and
d) The mass of water in kg_m/m^3 that must be added to reach full saturation.

Solution:

$$a) \quad \gamma_d = \frac{\gamma}{(1+w)} = \frac{16.5}{(1+0.15)} = 14.3 \ \frac{kN}{m^3}$$

$b)$ From the table of useful relationships,

$$\gamma_d = \frac{G_s \gamma_w}{1+e} \quad \therefore \quad 1+e = \frac{G_s \gamma_w}{\gamma_d} = \frac{(2.70)(9.81)}{(14.3)} = 1.85 \quad \therefore \quad e = 0.85$$

$$n = \frac{e}{1+e} = \frac{0.85}{1+0.85}(100\%) = 46\%$$

$$c) \quad \text{Since } Se = wG_s \quad \therefore S = \frac{wG_s}{e} = \frac{(0.15)(2.70)}{(0.85)}(100) = 48\%$$

$$d) \quad \gamma_{sat} = \frac{(G_S + e)\,\gamma_w}{1+e} = \frac{(2.70+0.85)(9.81)}{1+0.85} = 18.8 \ \frac{kN}{m^3}$$

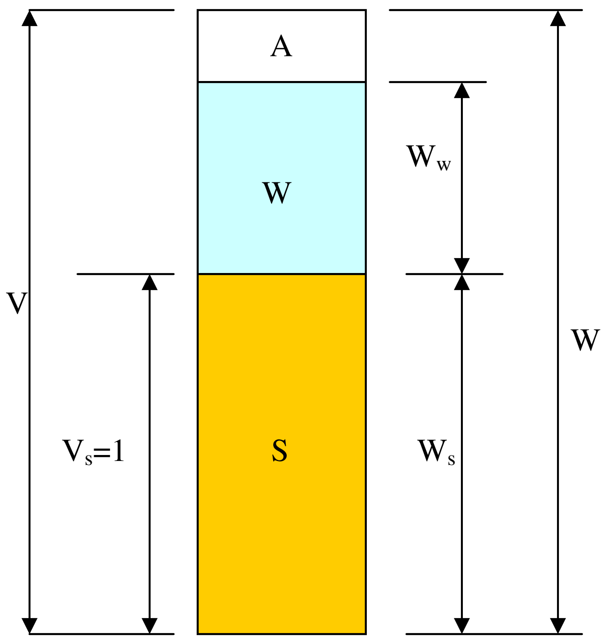
The water to be added can be found from the relation $\gamma = \rho g$

$$\therefore \rho(\text{mass of water}) = \frac{\gamma}{g} = \left[\frac{(18.8 - 16.5)\,kN/m^3}{9.81\,kg\text{-}m/s^2}\right]\left(\frac{1{,}000\ N}{1\ kN}\right)\left(\frac{9.81 kg\text{-}m/s^2}{N}\right) = 2{,}340 \ \frac{kg_m}{m^3}$$

*Phases of soils–09: Use the block diagram to find the degree of saturation.

(Revision: Aug.-09)

A soil has an "in-situ" (in-place) voids ratio $e_o = 1.87,\ w_N = 60\%,\ and\ G_S = 2.75$**. What are the γ_{moist} and *S*? (Note: All soils are really "moist" except when dry, that is when *w* = 0%).**



Solution: Set V_S = 1 m^3 (Note: this problem could also be solved by setting $V = 1.0\ m^3$).

$$\therefore \quad e_o = \frac{V_V}{V_S} = \frac{1.87}{1} = 1.87 \quad \therefore \quad V = V_S + V_V = 1 + 1.87 = 2.87\ m^3$$

The "natural" water content is $w_N = \frac{W_w}{W_s} = 0.60 \quad \therefore \quad W_w = 0.60 W_s$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\frac{W_s}{V_s}}{\gamma_w} \ \therefore \ W_s = V_s\left(G_S \gamma_w\right) = \left(1\,m^3\right)\left(2.75\right)\left(9.81\,kN/m^3\right) = \ 26.98\ kN$$

$$W_w = 0.60\left(W_s\right) = \left(0.60\right)\left(26.98\right) \qquad = \ 16.18\ kN$$

$$W = W_s + W_w = 26.98 + 16.19 \qquad = \ 43.16\ kN$$

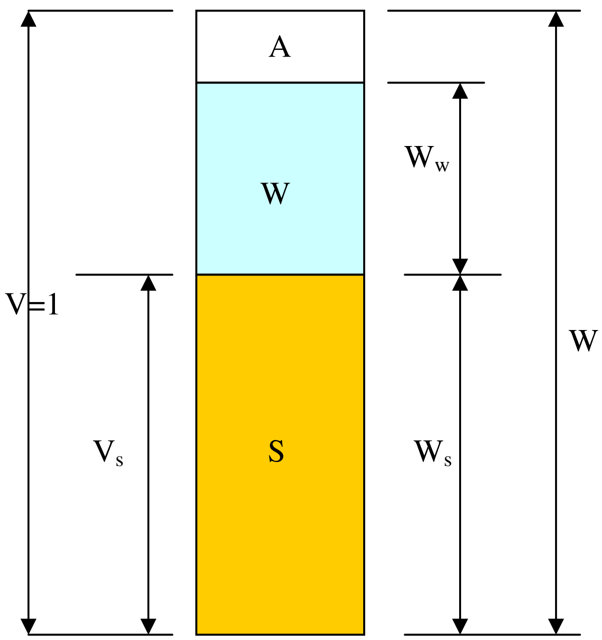
$$\therefore \quad \gamma_{moist} = \frac{W}{V} = \frac{43.16\,kN}{2.87\,m^3} = 15.0\ \frac{kN}{m^3}$$

$$\therefore \quad S = \frac{V_w}{V_V} = \frac{\frac{W_w}{\gamma_w}}{V_V} = \frac{\left(\frac{16.19}{9.81}\right)}{1.87} = 88.2\%$$

*Phases of soils–10: Same as Phases-09 but setting the total volume V=1 m³.

(Revision: Aug.-09)

A soil has an "in-situ" (in-place) voids ratio $e_o = 1.87,\ w_N = 60\%,\ and\ G_S = 2.75$. What are the γ_{moist} and S? (Note: All soils are really "moist" except when dry, that is when $w = 0\%$).



Solution: **Set $V = 1$ m³ (instead of $V_s = 1$ m³ used in Phases-09).**

$$but\ e_o = \frac{V_V}{V_S} = 1.87 \therefore\ but\ V = 1\,m^3 = V_S + V_V = V_S + 1.87V_S = 2.87V_S$$

$$\therefore V_S = 0.348\ and\ V_V = 0.652$$

$$\text{The "natural" water content is} \quad w_N = \frac{W_w}{W_s} = 0.60 \ \therefore \quad W_w = 0.60W_s$$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\frac{W_s}{V_s}}{\gamma_w} \ \therefore\ W_s = V_s\left(G_S\gamma_w\right) = \left(0.348\ m^3\right)\left(2.75\right)\left(9.81\ kN/m^3\right) = 9.39\ kN$$

$$W_w = 0.60\left(W_s\right) = \left(0.60\right)\left(9.39\right) = \underline{5.63\ kN}$$

$$W = W_s + W_w = 9.39 + 5.63 = 15.02\ kN$$

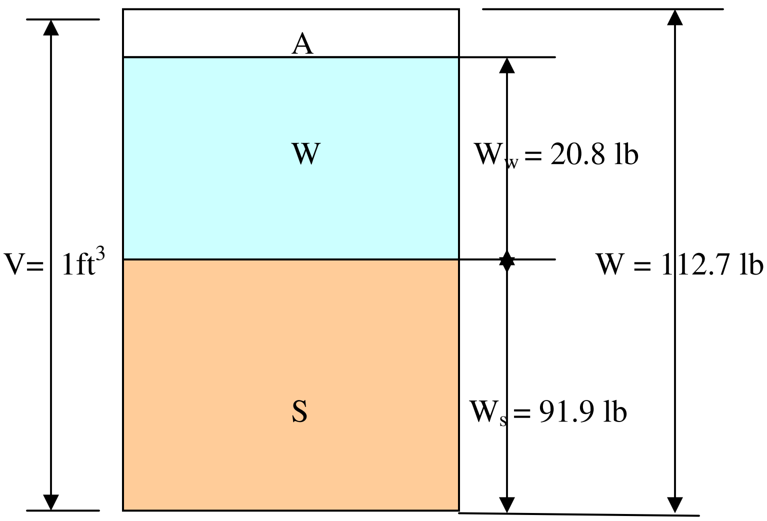
$$\therefore \quad \gamma_{moist} = \frac{W}{V} = \frac{15.0\,kN}{1\,m^3} = 15.0\ \frac{kN}{m^3}$$

$$\therefore \quad S = \frac{V_w}{V_V} = \frac{\frac{W_w}{\gamma_w}}{V_V} = \frac{\left(\frac{5.63}{9.8}\right)}{0.652} = 88.1\%$$

*Phases of soils–11: Same as Phases-08 but using a block diagram.

(Revision: Aug.-09)

A soil sample has a unit weight of 105.7 pcf and a water content of 50%. When its saturation is increased to 75 %, its unit weight raises to 112.7 pcf. Determine the voids ratio e and the specific gravity G_s of the soil. (NB: This is the same problem as Phase–06, but solved with a block diagram).



Solution:

$$\text{Set } V = 1 \, ft^3$$

$$\gamma_2 - \gamma_1 = 112.7 - 105.7 = 7.0 \quad lbs \quad are \; 25\% \; of \; water$$

$$\therefore \quad 21.0 \quad lbs \quad are \; 75\% \; of \; water$$

$$\therefore \quad W_S = 112.7 - 20.8 = 91.9 \; lb$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{20.8 \; lb}{62.4 \; pcf} = 0.333 \; ft^3$$

$$V_a = \frac{1}{3} V_w = 0.111 \; ft^3 \quad \therefore \quad V_v = V_a + V_w = 0.111 + 0.333 = 0.444$$

$$\therefore \quad V_s = 1 - V_v = 1 - 0.444 = 0.556$$

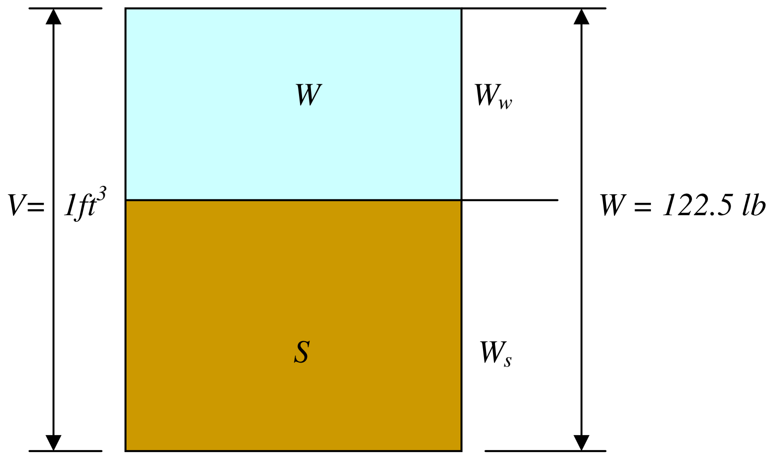
$$e = \frac{V_V}{V_S} = \frac{0.444}{0.556} = 0.80$$

$$and \quad G_S = \frac{\gamma_S}{\gamma_w} = \frac{W_S}{V_S \gamma_w} = \frac{91.9 \; lb}{(0.556)(62.4)} = 2.65$$

*Phases of soils–12: Block diagram for a saturated soil.

(Revision: Aug.-09)

A saturated soil sample has a unit weight of 122.5 pcf and G_s = 2.70. Find γ_{dry}, *e*, *n*, and *w*.



Solution:

$$V = V_S + V_w = \frac{1}{\gamma_w}\left(\frac{W_S}{G_S} + W_w\right) \qquad (1)$$

$$W = W_S + W_w = 122.5\ lb \qquad (2)$$

Combining equations (1) and (2) yields $1 = \frac{1}{(62.4\ pcf)}\left(\frac{122.5 - W_w}{2.70} + W_w\right)$

$$\therefore W_w = 27.0\ lb \quad \therefore \quad V_w = \frac{W_w}{\gamma_w} = \frac{27.0\ lb}{62.4\ pcf} = 0.433\ ft^3$$

$$\therefore W_S = 95.5\ lb \quad \therefore \quad V_S = \frac{W_S}{G_S \gamma_w} = \frac{95.5\ lb}{(2.70)(62.4\ pcf)} = 0.567\ ft^3$$

$$\therefore \qquad \gamma_{dry} = \frac{W_S}{V} = \frac{95.5\ lb}{1\ ft^3} = 95.5\ pcf$$

$$\therefore \qquad e = \frac{V_V}{V_S} = \frac{0.433}{0.567} = 0.764$$

$$\therefore \qquad n = \frac{V_V}{V} = \frac{0.433}{1} = 0.433 \qquad n = 43.3\%$$

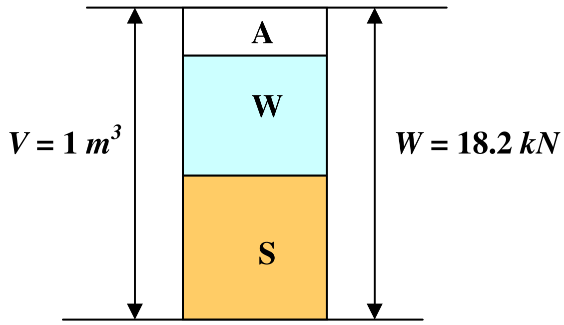
$$\therefore \qquad w = \frac{W_w}{W_S} = \frac{27}{95.5} = 0.283 \qquad w = 28.3\%$$

*Phases of soils–13: Find the weight of water needed for saturation.

(Revision: Aug.-09)

Determine the weight of water (in *kN*) that must be added to a cubic meter of soil to attain a 95 % degree of saturation, if the dry unit weight is 17.5 *kN/m³*, its moisture is 4%, the specific gravity of solids is 2.65 and the soil is entirely made up of a clean quartz sand.

Solution:



$$\gamma_d = 17.5\frac{kN}{m^3} = \frac{\gamma}{1+w} = \frac{\gamma}{1+0.04} \quad \therefore \quad \gamma = 18.2\frac{kN}{m^3}$$

$$W = 18.2 = W_S + W_w = W_S + wW_S = (1.04)W_S$$

$$\therefore \quad W_S = 17.5\ kN, \quad and \quad W_w = 0.70\ kN$$

$$V_S = \frac{W_S}{\gamma_S} = \frac{17.5\ kN}{G_S\gamma_w} = \frac{17.5\ kN}{(2.65)(9.81\ kN/m^3)} = 0.673\ m^3$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.70\ kN}{(9.81\ kN/m^3)} = 0.07\ m^3 \quad \therefore \quad V_a = V - V_s - V_w = 0.257\ m^3$$

$$e = \frac{V_V}{V_S} = \frac{0.07+0.257}{0.673} = 0.49$$

$$The\ existing\ S = \frac{wG_S}{e} = \frac{(0.04)(2.65)}{0.49}(100) = 21.6\%$$

We require a $S = 95\%$, therefore,

$$w = \frac{Se}{G_S} = \frac{(0.95)(0.49)}{2.65} = 0.17$$

$$W_w = wW_S = (0.17)(17.5\ kN) = 2.98\ kN$$

$$already\ have\ W_w = 0.70\ kN$$

$$\therefore \quad must\ add\ water = 2.28\ kN$$

*Phases of soils–14: Identify the wrong piece of data.

(Revision: Aug.-09)

A project engineer receives a laboratory report with tests performed on marine marl (calcareous silt). The engineer suspects that one of the measurements is in error. Are the engineer's suspicions correct? If so, which one of these values is wrong, and what should be its correct value?

$$Given \quad \gamma = unit\ weight\ of\ sample = 18.4\ \frac{kN}{m^3}$$

$$\gamma_S = unit\ weight\ of\ solids = 26.1\ \frac{kN}{m^3}$$

$$w = water\ content = 40\%$$

$$e = voids\ ratio = 1.12$$

$$S = degree\ of\ saturation = 95\%$$

Solution:

Check the accuracy of 4 out of 5 of the variables using,

$$Se = wG_S \quad \therefore \quad Se = (0.95)(1.12) = 1.06$$

$$wG_S = (w)\frac{\gamma_S}{\gamma_w} = (0.4)\frac{26.1}{9.81} = 1.06 \quad \therefore \quad \text{Therefore, these four are correct.}$$

The only possibly incorrect value is γ. Assume that $V = 1$ m^3.

$$V = 1\ m^3 = V_a + V_w + V_S \qquad (1)$$

$$but \quad e = \frac{V_V}{V_S} = 1.12 \quad \therefore \quad 0 = -V_a - V_w + 1.12V_S \quad (2)$$

$$\therefore \ V_S = 0.472\ m^3,\ V_V = 0.528\ m^3 \ \ but \ \ V_w = 0.95V_V = 0.502\ m^3$$

$$\therefore \ V_a = 0.026\ m^3$$

$$\therefore W_S = \gamma_S V_S = \left(26.1\frac{kN}{m^3}\right)\left(0.472\,m^3\right) = 12.3\ kN$$

$$W_w = wW_S = (0.40)\left(12.3\frac{kN}{m^3}\right) = 4.9\ kN$$

$$W = 12.3\ kN + 4.9\ kN = 17.2\ kN$$

Therefore, the actual unit weight of the soil is,

$$\therefore \gamma = \frac{W}{V} = \frac{17.2\ kN}{1\ m^3} = 17.2\ \frac{kN}{m^3} \neq 18.4\ \frac{kN}{m^3}$$

*Phases of soils–15: The apparent cheapest soil is not!

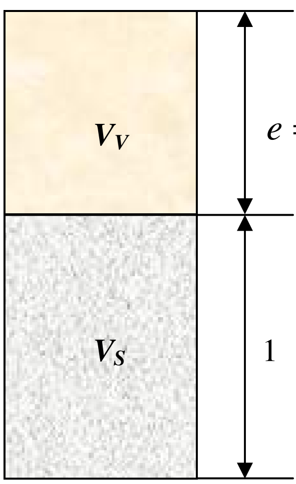
(Revision: Aug.-09)

You are the Project Engineer of a large earth dam project that has a volume of 5x10^6 yd^3 of select fill, compacted such that the final voids ratio in the dam is 0.80. Your boss, the Project Manager delegates to you the important decision of buying the earth fill from one of three suppliers. Which one of the three suppliers is the most economical, and how much will you save?

Supplier A	**Sells fill at \$ 5.28/ yd^3 with**	**e = 0.90**
Supplier B	**Sells fill at \$ 3.91/ yd^3 with**	**e = 2.00**
Supplier C	**Sells fill at \$ 5.19/ yd^3 with**	**e = 1.60**

Solution:

Without considering the voids ratio, it would appear that Supplier B is cheaper than Supplier A by \$1.37 per yd^3.



$$e = 0.8 = \frac{V_V}{V_S} = \frac{V_V}{1} \quad \therefore \quad V_V = 0.8$$

Therefore: To put 1yd^3 of solids in the dam you would need 1.8 yd^3 of soil.
For 1yd^3 of solids from A you would need 1.9 yd^3 of fill.
For 1yd^3 of solids from B you would need 3.0 yd^3 of fill.
For 1yd^3 of solids from C you would need 2.6 yd^3 of fill.

The cost of the select fill from each supplier is (rounding off the numbers):

$$A = \frac{1.9}{1.8}(5)\left(10^6\ yd^3\right)\left(\frac{5.28\$}{yd^3}\right) \approx \$\ 27,900,000$$

$$B = \frac{3.0}{1.8}(5)\left(10^6\ yd^3\right)\left(\frac{3.91\$}{yd^3}\right) \approx \$\ 32,600,000$$

$$C = \frac{2.6}{1.8}(5)\left(10^6\ yd^3\right)\left(\frac{5.19\$}{yd^3}\right) \approx \$\ 37,500,000$$

Therefore Supplier *A* is the cheapest by about \$ 4.7 Million compared to Supplier B.

*Phases of soils–16: Number of truck loads.

(Revision: Aug.-09)

Based on the previous problem's (*Phases–15*) data, if the fill dumped into the truck has an *e* = 1.2, how many truck loads will you need to fill the dam? Assume that each truck carries 10 yd³ of soil.

Solution:

$$Set\ \ V_S = 1 \quad \therefore e = \frac{V_V}{V_S} = \frac{V_V}{1} = V_V = 1.2$$

which means that there is 1 yd^3 of solids per 1.2 yd^3 of voids,

Or 2.2 yd^3 of soil for each 1 yd^3 of solids.

Or 10 yd^3 of soil in a truck will have x yd^3 of solids.

$\therefore \quad x = 4.54\ \text{yd}^3$ of solids per truck trip.

The required volume of solids in the dam is,

$$V_{solids} = \frac{\left(5x10^6\ yd^3 of\ soil\right)\left(1\ yd^3 of\ solids\right)}{1.8\ \ yd^3 of\ soil} = 2.8x10^6\ yd^3\ of\ solids$$

Therefore, (rounding off)

$$Number\ of\ Truck - trips = \frac{\left(2.78x10^6\ yd^3 of\ solids\right)}{\left(4.54\ \ yd^3 of\ solids / truck - trip\right)} = 613,000$$

*Phases of soils–17: How many truck loads are needed for a project?

(Revision: Aug.-09)

You are the Project Engineer for a development company that is building 600 housing units surrounding four lakes. Since the natural ground is low, you will use the limestone excavated from the lakes to fill the land to build roads and housing pads. Your estimated fill requirements are 700,000 m^3, with a dry density equivalent to a voids ratio $e = 0.46$.

The "in-situ" limestone extracted from the lakes has an $e = 0.39$, whereas the limestone dumped into the trucks has an $e = 0.71$. How many truckloads will you need, if each truck carries 10 m^3?

Solution:

In the finished pads and roadways, the compacted fill (solids + voids) is found by,

assuming $V_S = 1\,m^3 \;\therefore\; e = \dfrac{V_V}{V_S} = \dfrac{V_V}{1} = V_V = 0.46\,m^3$

The required 700,000 m^3 of fill have 1.46 m^3 of soil per each 1 m^3 of solids

Therefore, the 700,000 m^3 of fill (solids+voids) have 479,400 m^3 of solids

Each truck carries 1.71 m^3 of fill per each 1 m^3 solids

In order for the trucks to carry 479,400 m^3 of solids they must carry 820,000 m^3 of fill

Since each truck carries 10 m^3 of fill,

$\therefore$ The number of truck-loads = $\dfrac{820{,}000\,m^3}{10\,m^3}$ = 82,000 truck-loads.

*Phases of soils–18: Choose the cheapest fill supplier.

(Revised: Aug.-09)

A large housing development requires the purchase and placement of the fill estimated to be 200,000 cubic yards of lime-rock compacted at 95% Standard Proctor with an *OMC* of 10%. Two lime-rock suppliers offer to fill your order: Company A has a borrow material with an in-situ γ = 115 pcf, w = 25%, G_S = 2.70; Standard Proctor yields a maximum γ_d = 112 pcf; at a cost of \$0.20/yd^3 to excavate, and \$ 0.30/yd^3 to haul. Company B has a borrow material with an in-situ γ = 120 pcf, w = 20%, G_S = 2.70; Standard Proctor yields a maximum γ_d = 115 pcf; a cost of \$0.22/yd^3 to excavate, and \$ 0.38/yd^3 to haul.

a) What volume would you need from company A?
b) What volume would you need from company B?
c) Which would be the cheaper supplier?

Solution:

(1) The key idea: **1 *yd*^{3} of solids from the borrow pit supplies 1 *yd*^{3} of solids in the fill.**

(2) Pit *A*: W_S = 92 lb, W_W = 23 lb ⟶ V_W = 0.369 ft^3, V_S = 0.546 ft^3, V_a = 0.085 ft^3

$$e = \frac{V_V}{V_S} = \frac{0.454}{0.546} = 0.83 \;\therefore\; 1.83\ yd^3 \text{ of soil contains } 1.0\ yd^3 \text{ of solids.}$$

Pit *B*: W_S = 100 lb, W_W = 20 lb, V_W = 0.321 ft^3, V_S = 0.594 ft^3, V_a = 0.08 ft^3

$$e = \frac{V_V}{V_S} = \frac{0.401}{0.594} = 0.68 \;\therefore\; 1.68\ yd^3 \text{ of soil contains } 1.0\ yd^3 \text{ of solids.}$$

(3) Material needed for fill from company A:

$$\gamma = 0.95\gamma_d(1+w) = 0.95(112)(1+0.10) = 117\ pcf \;\therefore\; W_S = 106.4\ lb, \quad W_w = 10.6\ lb$$

$$e = \frac{V_V}{V_S} = \frac{0.37}{0.63} = 0.59 \;\therefore\; 1.59\ yd^3 \text{ of soil contains } 1.0\ yd^3 \text{ of solids}$$

$$\therefore \text{Site A requires } \frac{200{,}000\ yd^3 \text{ of fill}}{1.59} = 125{,}800\ yd^3 \text{ of solids}$$

Material needed for fill from company B:

$$\gamma = 0.95\gamma_d(1+w) = 0.95(115)(1+0.10) = 120\ pcf \;\therefore\; W_S = 109.1\ lb, \quad W_w = 10.9\ lb$$

$$e = \frac{V_V}{V_S} = \frac{0.35}{0.65} = 0.54 \;\therefore\; 1.54\ yd^3 \text{ of soil contains } 1.0\ yd^3 \text{ of solids}$$

$$\therefore \text{Site B requires } \frac{200{,}000\ yd^3 \text{ of fill}}{1.54} = 130{,}000\ yd^3 \text{ of solids}$$

(4) a) Cost of using Company A:

$$Cost_A = (125{,}800\ yd^3)(1.83)\left(\frac{\$0.50}{yd^3}\right) = \$\,115{,}100$$

b) Cost of using Company B:

$$Cost_B = (130{,}000\ yd^3)(1.68)\left(\frac{\$0.60}{yd^3}\right) = \$\,131{,}100$$

Using Company A will save about \$ 16,000.

*Phases of soils–19: Use a matrix to the find the missing data.

(Revision: Aug.-09)

A contractor obtains prices for 34,000 yd^3 of compacted "borrow" material from three pits: Pit #3 is $11,000 cheaper than Pit #2 and $39,000 cheaper than Pit #1. The fill must be compacted down to a voids ratio of 0.7. Pit #1 costs $ 6.00/yd^3 and Pit #3 costs $ 5.50/yd^3. Pits #2 and #3 reported their voids ratios as 0.88 and 0.95 respectively. Use a matrix to find,

a) **The missing unit cost C_2 for Pit #2;**
b) **The missing voids ratio e for Pit #1;**
c) **The missing volume of fill V required from each pit; and**
d) **The amount paid by the contractor for each pit.**

***Solution*:**

A summary of the data provided is herein shown in matrix form,

Item	Borrow Sites			Project
	Pit-1	Pit-2	Pit-3	Site
Unit Cost ($\$/yd^3$)	6.00	C_2	5.50	
Voids ratio e	e_1	0.88	0.95	0.70
Volume (yd^3)	V_1	V_2	V_3	34,000
Total Cost ($)	$TC_1 = TC_2 + 28{,}000$	TC_2	$TC_3 = TC_2 - \$11{,}000$	

The volume of solids V_s contained in the total volume of fill V = 34,000 yd^3 can be found from,

$$V = V_V + V_S = 0.7V_S + V_S = V_S(0.7+1) = 34,000\ yd^3 \ \therefore\ V_S = \frac{34,000}{1.7} = 20,000\ yd^3\ of\ solids$$

At Pit #3, $\frac{V_3}{V_S} = 1 + e_3 \ \therefore\ V_3 = V_S(1+e_3) = (20,000\, yd^3)(1+0.95) = 39,000\, yd^3\ of\ soil$

The total cost of Pit #3 is $TC_3 = (39,000\, yd^3)(\$\,5.50/yd^3) = \$\,214,500$

At Pit #2: $\frac{V_2}{V_S} = 1 + e_2 \ \therefore V_2 = V_S(1+e_2) = (20,000\, yd^3)(1+0.88) = 37,600\, yd^3\ of\ soil$

But, the total cost of Pit #2 is $TC_2 - \$\,11,000 = TC_3 = \$\,214,500 \quad \therefore\ TC_2 = \$\,225,500$

The unit cost of Pit #2 $C_2 = \frac{TC_2}{V_2} = \frac{\$\,225,500}{37,600\, yd^3} = \$\,6.00/yd^3$

At Pit #1: $V_1 = \frac{TC_1}{\$\,6.00/yd^3} = \frac{TC_2 + 28,000}{\$\,6.00/yd^3} = \frac{225,500+28,000}{\$\,6.00/yd^3} = 42,250\, yd^3\ of\ soil$

But, $V_1 = V_S(1+e_1) = (20,000\, yd^3)(1+e_1) = 42,250\, yd^3 \quad \therefore \quad e_1 = 1.11$

Phases of soils–20: *Find the voids ratio of "muck" (a highly organic soil).*

(Revision: Aug.-09)

You have been retained by a local municipality to prepare a study of their "muck" soils. Assume that you know the dry unit weight of the material (solids) γ_{sm} and the dry unit weight of the organic solids γ_{so}. What is the unit weight γ_s of the combined dry organic mineral soil whose organic content is M_0? (The organic content is the percentage by weight of the dry organic constituent of the total dry weight of the sample for a given volume.) What is the voids ratio e of this soil if it is known that its water content is w and its degree of saturation is *S*?

Solution:

Set $W_s = 1$ unit and $\gamma_s = \dfrac{W_s}{V_s} = \dfrac{1}{(V_{so} + V_{sm})}$

(a) Assume $M_o = W_o$ for a unit weight of the ***dry soil***

Therefore $1 - M_o = W_m$

$$\frac{M_o}{\gamma_{so}} = \text{volume of organic } V_{so} \text{ solids}$$

$$\frac{(1 - M_o)}{\gamma_{sm}} = \text{volume of mineral } V_{sm} \text{ solids}$$

The total unit weight is the weight of a unit volume.

$$\text{Therefore } \gamma_S = \frac{1}{\left(\dfrac{M_o}{\gamma_{so}} + \dfrac{(1 - M_o)}{\gamma_{sm}}\right)} = \gamma_{so}\left[\frac{\gamma_{sm}}{M_o(\gamma_{sm} - \gamma_{so}) + \gamma_{so})}\right]$$

$$\text{(b)}\quad e = \frac{V_v}{V_s} = \frac{\left(\dfrac{\text{volume of water}}{S}\right)}{V_s} = \frac{\left(\dfrac{\text{weight of water}}{\gamma_w S}\right)}{V_s} = \frac{\left(\dfrac{w(\text{weight of solids})}{\gamma_w S}\right)}{Vs}$$

$$\text{Therefore}\quad e = \frac{\left(\dfrac{w}{\gamma_w S(1)}\right)}{\left(\dfrac{M_o}{\gamma_{so}} + \dfrac{(1 - M_o)}{\gamma_{sm}}\right)} = \frac{w\gamma_{sm}\gamma_{so}}{\gamma_w S\left[M_o(\gamma_{sm} - \gamma_{so}) + \gamma_{so}\right]}$$