Florida International University

Department of Electrical and Computer Engineering

Digital Filters

A Practical Guide to Design Digital IIR Bandpass Chebyshev Filters

Author: Pablo Gomez, Ph.D.

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Objective

The purpose of this paper is to develop a practical method to design Digital IIR Chebyshev Filters. This document explains all the required steps using a specific filter design that is calculated, realized and verified with a program written in MATLAB®.

Filter Specifications

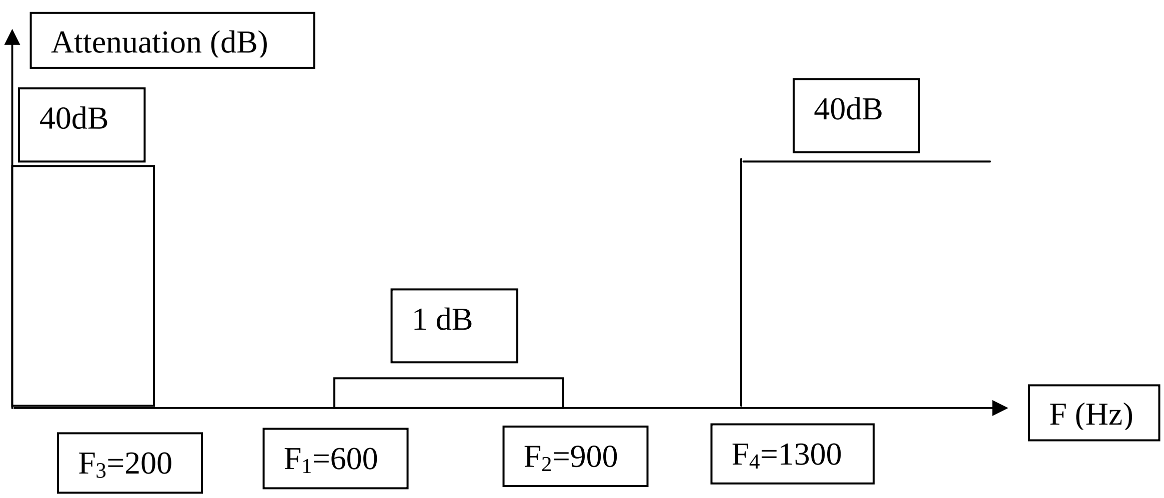
A Chebyshev Digital Bandpass Filter is to be designed to meet the following specifications:

a) A 1dB ripple in the frequency range of 600 to 900 Hz
b) A sampling frequency of 3000 Hz
c) A maximum gain of –40dB for 0 <= f <= 200 Hz

Solution

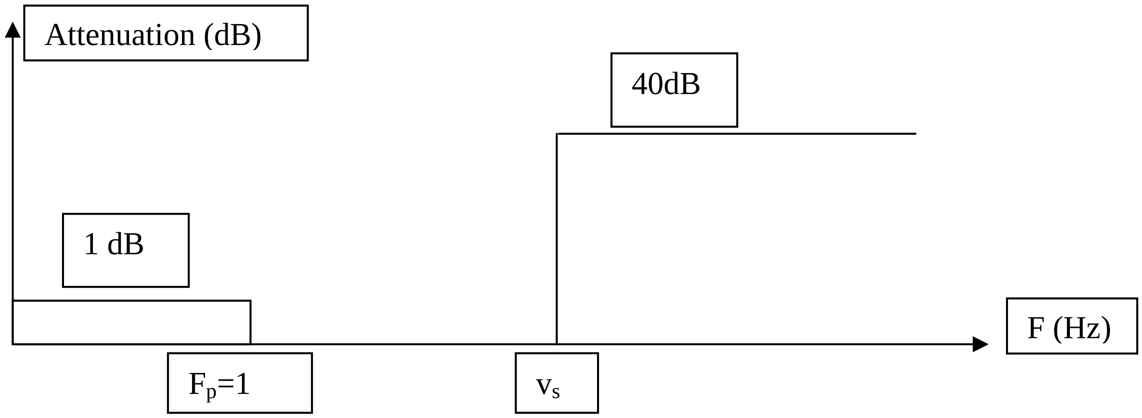
Step 1.

We assume that the filter is symmetric, that is, the filter should have an attenuation of 40dB for frequencies >= 1300 Hz. Therefore, the ideal magnitude response sketch is as follows:



Step 2.

Transform the Bandpass Filter specifications to an equivalent normalized Lowpass filter. Then, obtain *N*, the order of the Chebyshev filter.



$$\upsilon_s = \frac{f_4 - f_3}{f_2 - f_1} = \frac{1300-200}{900-600} = \frac{1100}{300} = 3.666$$

So we have, $f_p = 1.0,\ f_s = 3.666,\ A_{\min} = 40dB,\ A_{\max} = 1.0dB$

To calculate the order of the Chebyshev filter we use the following equations:

$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1} = \sqrt{10^{0.1} - 1} = 0.5088$$

$$N = \frac{\cosh^{-1}(\sqrt{10^{A_{\min}/10} - 1} / \varepsilon)}{\cosh^{-1}(f_s / f_p)}$$

$$N = \frac{\cosh^{-1}(\sqrt{10^{40/10} - 1} / 0.5088)}{\cosh^{-1}(3.666/1.0)}$$

$$N = \frac{\cosh^{-1}(\sqrt{10^4 - 1} / 0.5088)}{\cosh^{-1}(3.666)} = 3.027 \cong 3$$

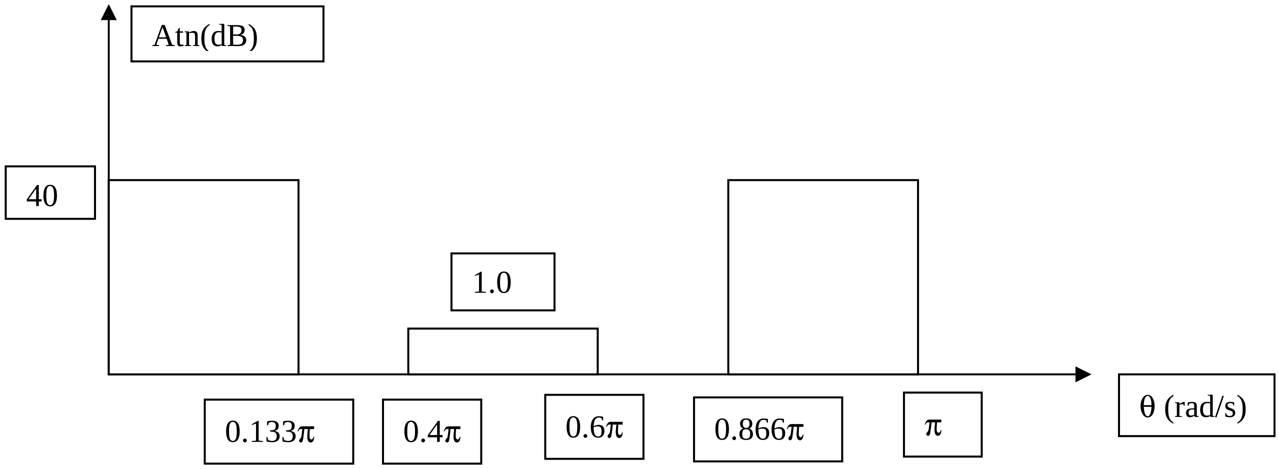
The general rule is to choose the next integer number. In this case, we are going to try *N*=3 since the result is very close to it. If the performance of the filter meets the specifications we are saving 1 order; otherwise, the order will be *N*=4 and the new performance will be calculated and verified.

Step 3.

The filter will be specified using digital frequencies as follows:

$$f_s = 3000Hz \rightarrow f_s / 2 = 1500Hz \rightarrow \pi$$

$$\theta_1 = \frac{600}{1500}\pi = 0.4\pi \,, \theta_2 = \frac{900}{1500}\pi = 0.6\pi \,, \theta_3 = \frac{200}{1500}\pi = 0.133\pi \,, \theta_4 = \frac{1300}{1500}\pi = 0.866\pi$$



Step 4.

An equivalent Digital Prototype Lowpass Chebyshev filter is obtained from the standard table for N=3, ripple=1dB, ε=0.5088. The Transfer Function is:

$$H_{LP_N}(z) = \frac{0.491(1+z)^3}{3.717z^3 - 1.277z^2 + 2.247z - 0.759}$$

Step 5.

We now need to perform the frequency transformations to obtain the transfer function of the equivalent Bandpass Filter based on the Lowpass prototype of the previous step. This is done using the transformation:

$$H_{BP}(z) = H_{LP_N}(z)\Big|_{z = -\frac{z^2 - \frac{2\alpha\beta}{\beta+1}z + \frac{\beta-1}{\beta+1}}{1 - \frac{2\alpha\beta}{\beta+1}z + \frac{\beta-1}{\beta+1}z^2}}$$

The values for α and β are obtained with the following equations:

$$\alpha = \frac{\cos(\frac{\theta_{un}}{2} + \frac{\theta_{\ln}}{2})}{\cos(\frac{\theta_{un}}{2} - \frac{\theta_{\ln}}{2})} = \frac{\cos(\frac{0.6\pi}{2} + \frac{0.4\pi}{2})}{\cos(\frac{0.6\pi}{2} - \frac{0.4\pi}{2})} = \frac{\cos(\pi/2)}{\cos(0.1\pi)} = 0$$

$$\beta = \tan(\theta_c / 2)\cot(\frac{\theta_{un}}{2} - \frac{\theta_{\ln}}{2})$$ for a digital prototype filter $\theta_c = \pi / 2$, therefore,

$$\beta = \frac{\tan(\frac{\pi/2}{2})}{\tan(\frac{0.6\pi}{2} - \frac{0.4\pi}{2})} = \frac{\tan(0.25\pi)}{\tan(0.1\pi)} = 3.077$$

Substituting α and β we obtain:

$$H_{BP}(z) = H_{LP_N}(z)|_{z=-\frac{z^2+\frac{\beta-1}{\beta+1}}{1+\frac{\beta-1}{\beta+1}z^2}}$$

$$H_{BP}(z) = H_{LP_N}(z)|_{z=-\frac{z^2+0.5094}{1+0.5094z^2}}$$

Lets make the constant k=0.5094 to simplify the substitution.

$$H_{BP}(z) = \frac{0.491\left(-\frac{z^2+k}{1+kz^2}+1\right)^3}{3.717\left(-\frac{z^2+k}{1+kz^2}\right)^3 - 1.277\left(-\frac{z^2+k}{1+kz^2}\right)^2 + 2.247\left(-\frac{z^2+k}{1+kz^2}\right) - 0.759}$$

Taking $(1+kz^2)^3$ as a common denominator we have

$$H_{BP}(z) = \frac{-0.491\left((1-k)z^2 + (k-1)\right)^3}{-3.717(z^2+k)^3 - 1.277(z^2+k)^2(1+kz^2) - 2.247(z^2+k)(1+kz^2)^2 - 0.759(1+kz^2)^3}$$

If we express $H_{BP}(z)$ as a fraction, that is $H_{BP}(z) = \frac{N(z)}{D(z)}$ and with some further algebraic manipulation we obtain the following transfer function,

$N(z) = 0.0579z^6 - 0.1738z^4 + 0.1738z^2 - 0.0579$ and
$D(z) = 5.05z^6 + 10.7969z^4 + 8.9365z^2 + 2.7262$

Frequency Amplitude Response Plot

The Amplitude Response was obtained with the following MATLAB program:

```
% Transfer Function Numerator Coefficients:
b=[-0.0579 0 0.1738 0 -0.1738 0 0.0579];

% Transfer Function Denominator Coefficients:
a=[2.7262 0 8.9364 0 10.7969 0 5.05];

%Obtain Impulse Response and Frequency Vectors (512 points, Sampling
Frequency=3000Hz)
[h,f]=freqz(b,a,512,3000);

% Plot the Amplitud Response in dB
plot(f,20*log10(abs(h)));
grid on;
title('Chebyshev Digital Bandpass Filter (1dB ripple N=3)');
xlabel('Frequency (Hz)');
ylabel('Amplitude Response - dB');

%Plot the Passband Details (from point 170 thru point 340):
plot(f(170:340),20*log10(abs(h(170:340))));
```

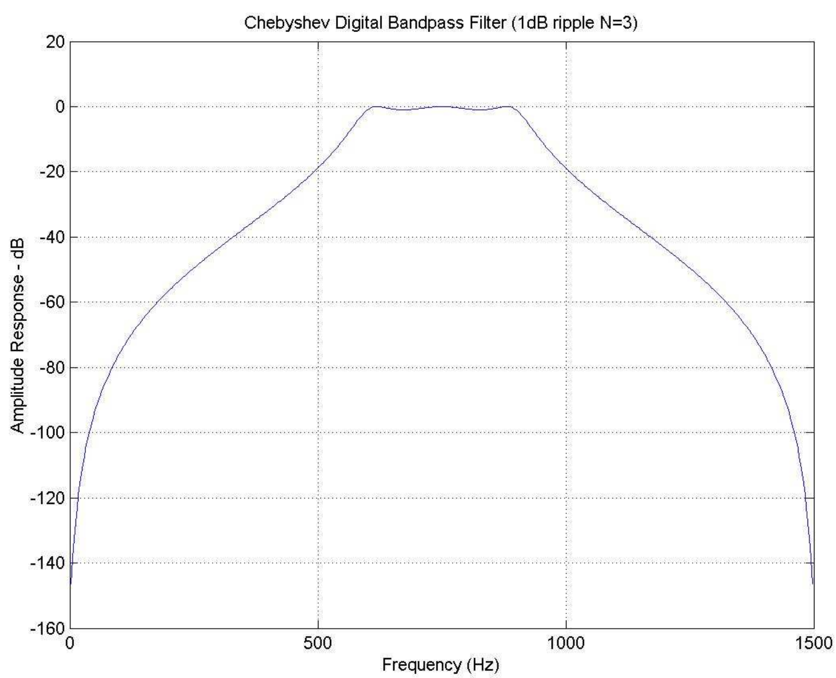


Figure 1. Amplitude Frequency Response

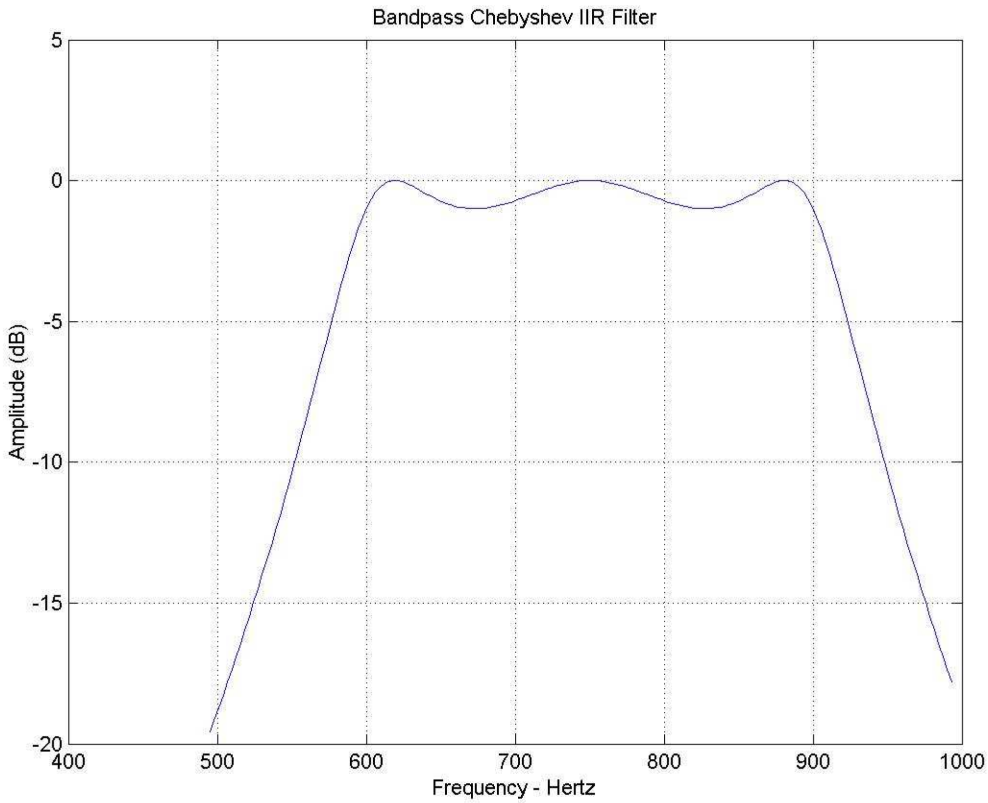


Figure 2. Amplitude Frequency Response – Bandpass Details

Results and Conclusion

Looking at the plots in Figures 1 and 2, we conclude that the performance specifications of the filter have been fulfilled.

The required attenuation at 200Hz and 1300Hz has been greatly exceeded. The requirement was 40dB and the attenuation obtained at 200Hz was 60dB and 57dB at 1300Hz.

In the passband, between 600 and 900 Hz, the resulting ripple is exactly 1dB.