Chapter 6:Mechanical Properties

Why mechanical properties?

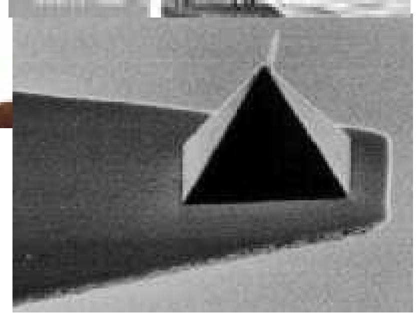
Need to design materials that can withstand applied load…

e.g. materials used in building bridges that can hold up automobiles, pedestrians…

materials for skyscrapers in the Windy City…

materials for space exploration…

NASA



materials for and designing MEMs and NEMs…

Space elevators?

ISSUES TO ADDRESS...

- Stress and strain: What are they and why are they used instead of load and deformation?
- Elastic behavior: When loads are small, how much deformation occurs? What materials deform least?
- Plastic behavior: At what point does permanent deformation occur? What materials are most resistant to permanent deformation?
- Toughness and ductility: What are they and how do we measure them?

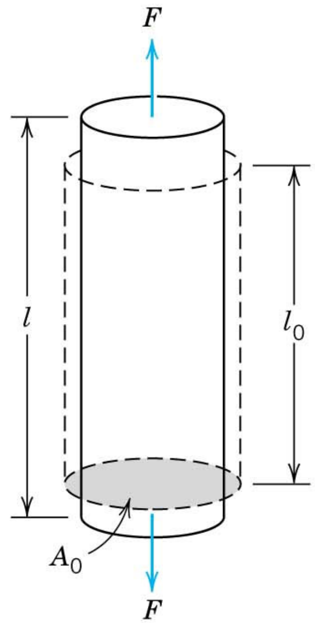
Stress and Strain

Stress: **Pressure due to applied load.**

tension, compression, shear, torsion, and their combination.

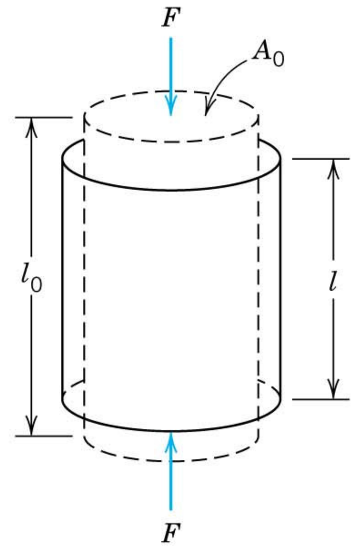
$$\boxed{stress \;=\sigma = \frac{force}{area}}$$

Strain: response of the material to stress (i.e. physical deformation such as elongation due to tension).



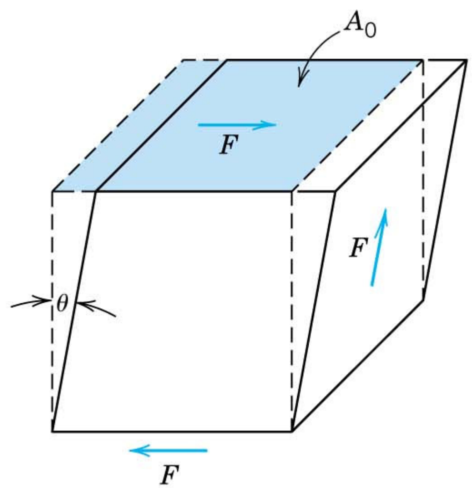
Tension

(a)



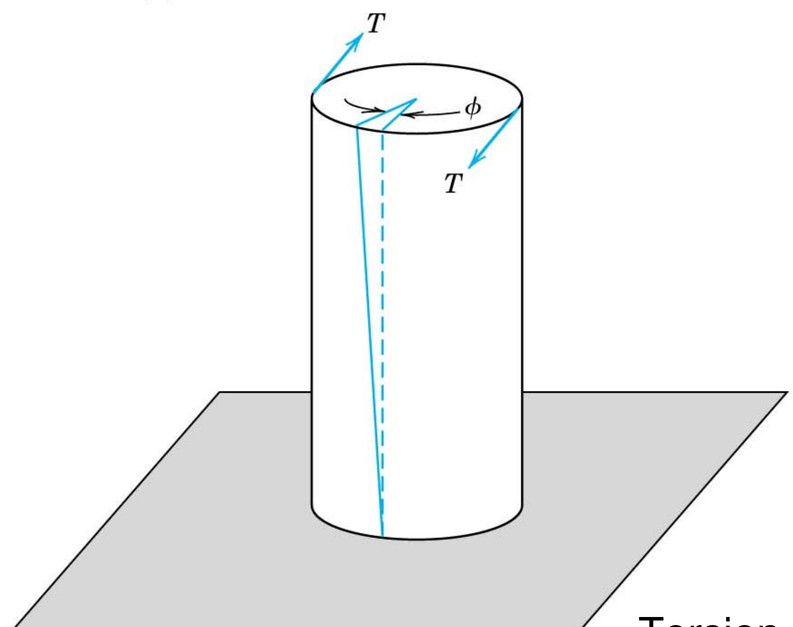
Compression

(b)



Shear

(c)

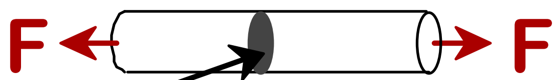


Torsion

(d)

COMMON STATES OF STRESS

- **Simple** tension: cable



A_o = cross sectional Area (when unloaded)

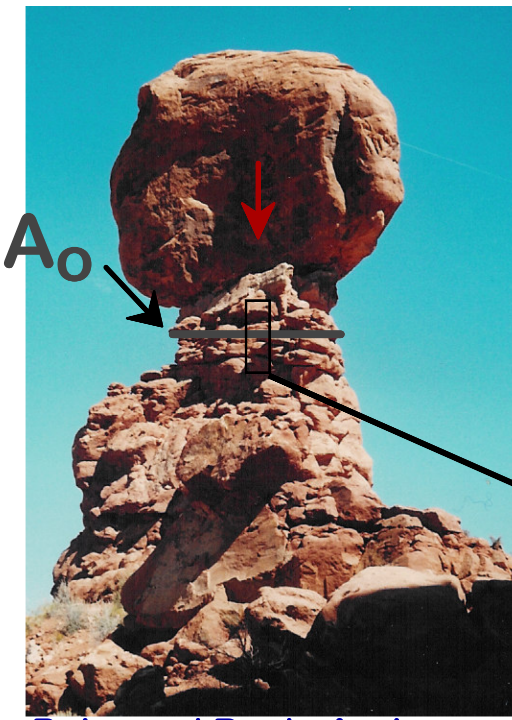
$$\sigma = \frac{F}{A_o}$$

Ski lift (photo courtesy P.M. Anderson)

From Callister 6e resource CD.

COMMON STATES OF STRESS

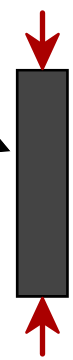
- Simple compression:



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$



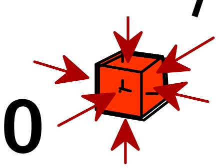
Note: compressive structure member (σ < 0 here).

COMMON STATES OF STRESS

- **Hydrostatic compression:**

Fish under water

(photo courtesy P.M. Anderson)

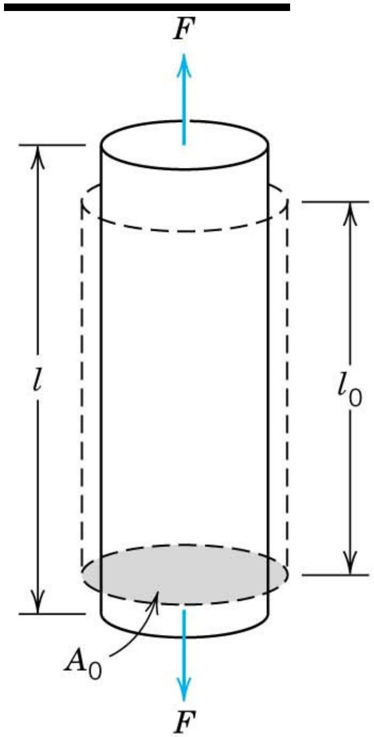


$\sigma_h < 0$

From Callister 6e resource CD.

Tension and Compression

Tension



Engineering stress = $\sigma = \dfrac{F}{A_o}$

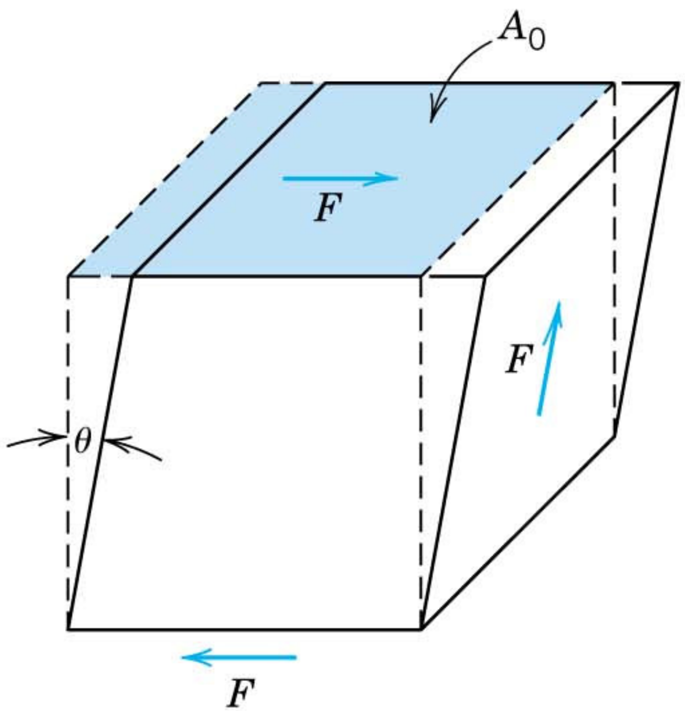
Engineering strain = $\varepsilon = \dfrac{l_i - l_o}{l_o} = \dfrac{\Delta l}{l_o}$

A_o = original cross sectional area
l_i = instantaneous length
l_o = original length
Note: strain is unitless.

Compression

Same as tension but in the opposite direction (stress and strain defined in the same manner).
By convention, stress and strain are negative for compression.

Shear

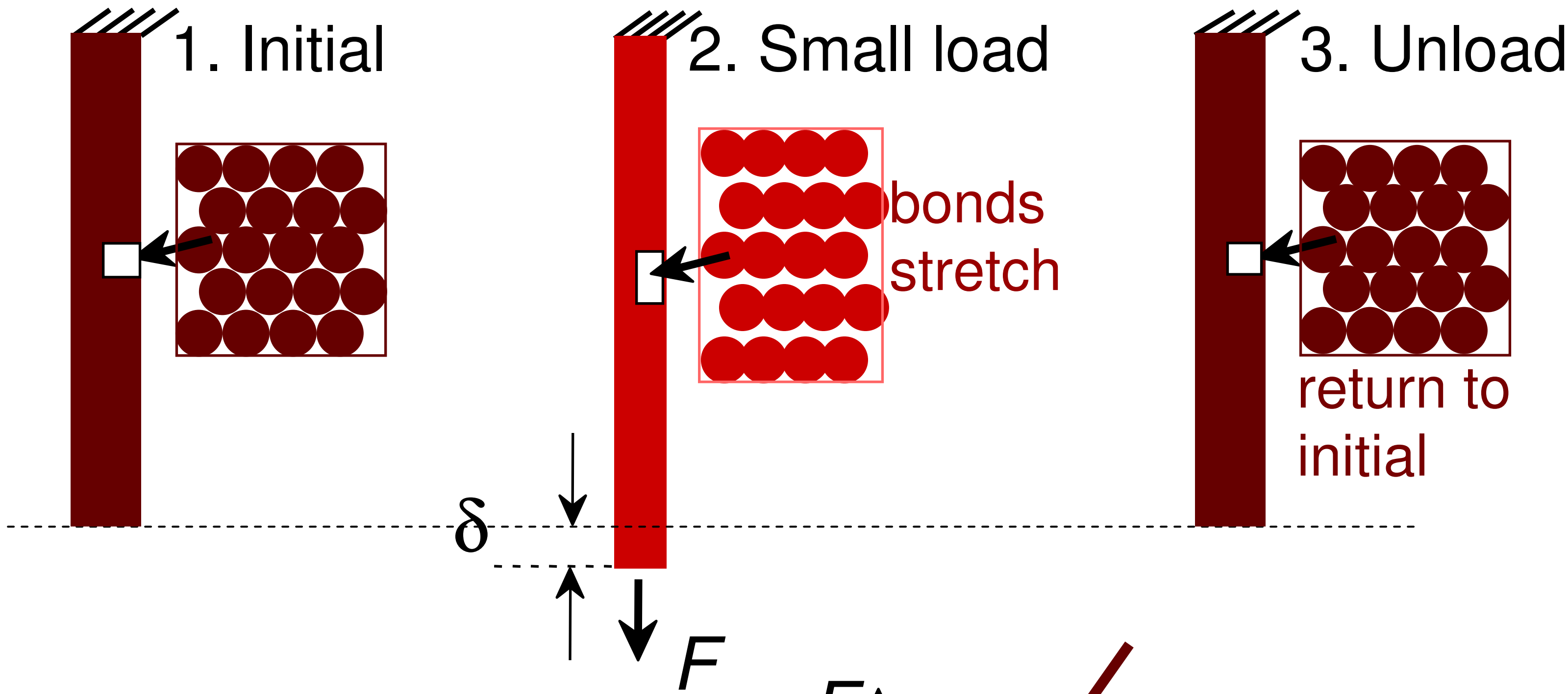


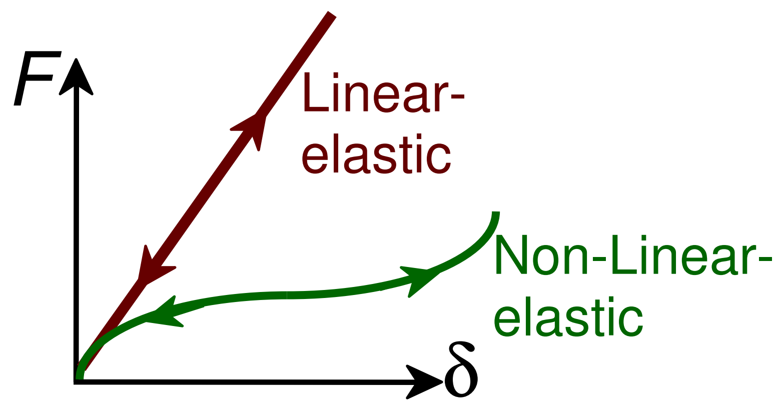
Pure shear stress = $\tau = \dfrac{F}{A_o}$

Pure shear strain = $\gamma = \tan\theta$

Strain is always dimensionless.

Elastic Deformation

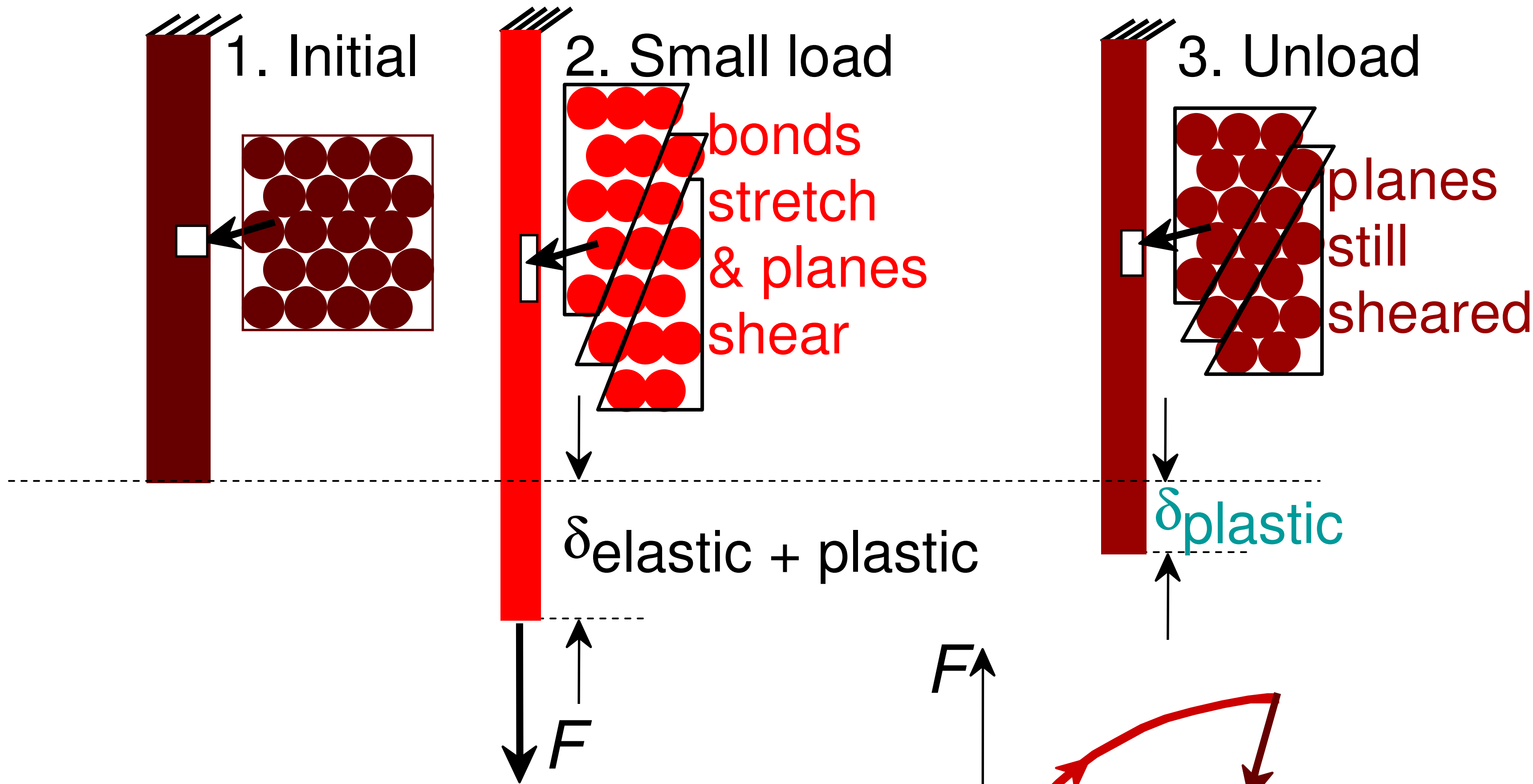




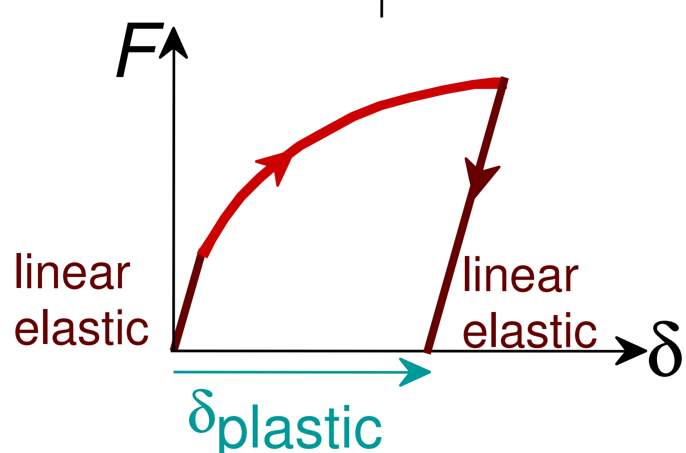
Elastic means reversible!

-a non-permanent deformation where the material completely recovers to its original state upon release of the applied stress.

Plastic Deformation (Metals)

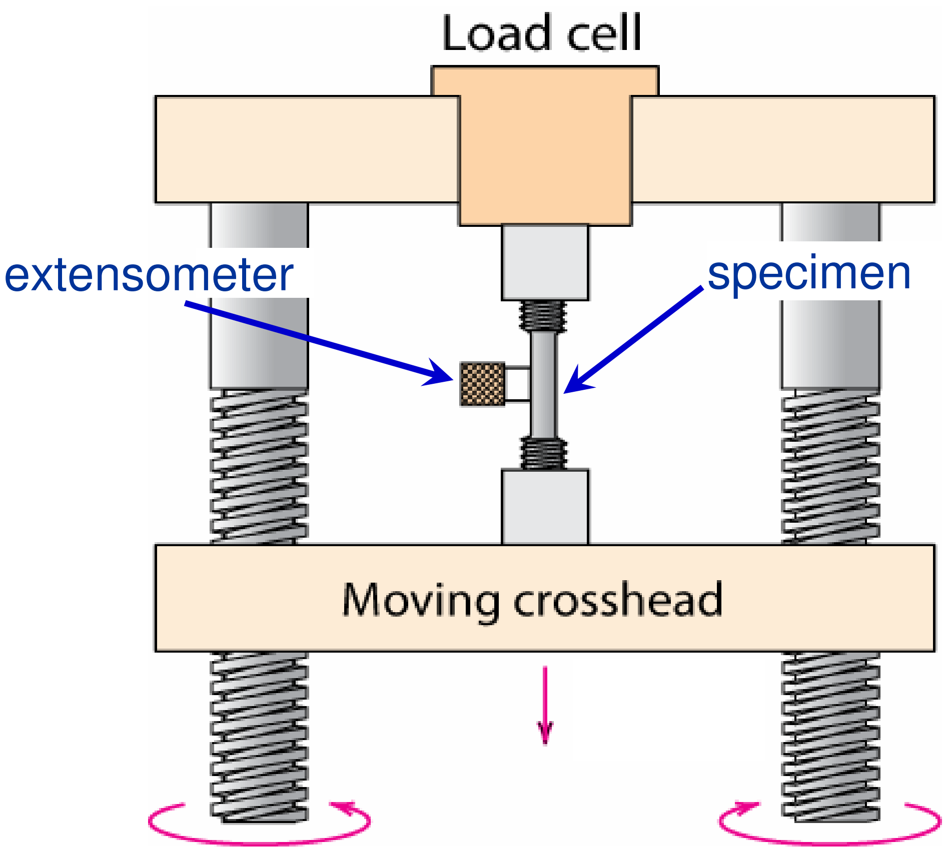


Plastic means permanent!



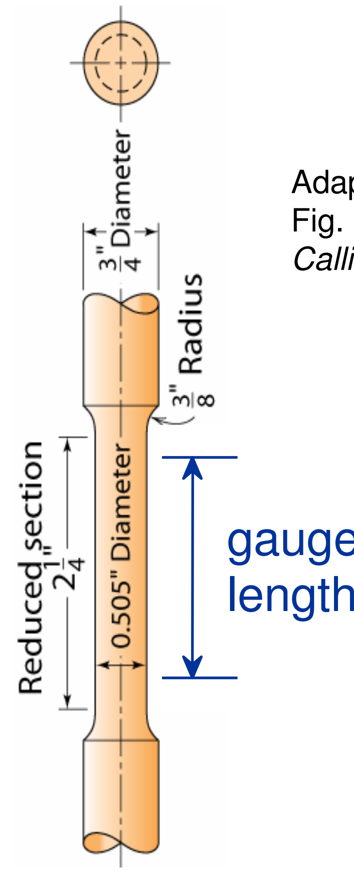
Stress-Strain Testing

- **Typical tensile test machine**



Adapted from Fig. 6.3, *Callister 7e.* (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

- **Typical tensile specimen**



Adapted from Fig. 6.2, *Callister 7e.*

Linear Elastic Properties

- **Modulus of Elasticity, *E*:**
  (also known as Young's modulus)

- **Hooke's Law:**

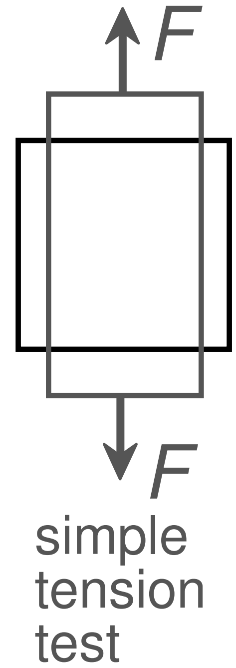
$$\sigma = E\,\varepsilon$$

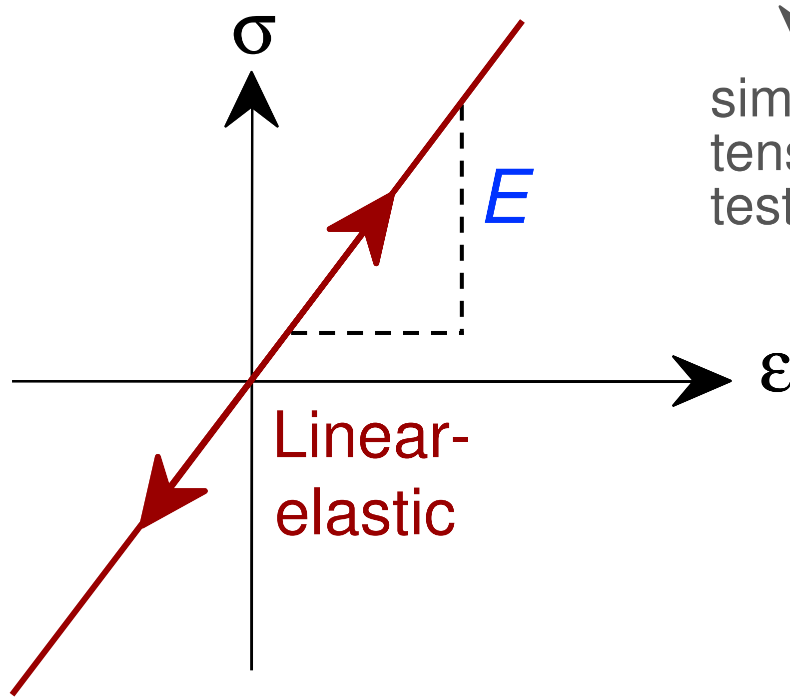
stress

strain

Modulus of elasticity
(Young's modulus)

Measure of material's resistance to elastic deformation (stiffness).

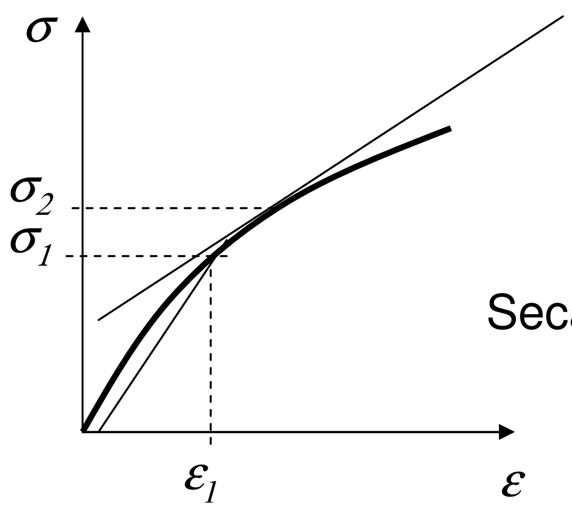




For metals, typically E ~ 45 – 400 GPa

Note: some materials do not have linear elastic region (e.g. cast iron, concrete, many polymers…)

Define **secant modulus** and **tangent modulus.**

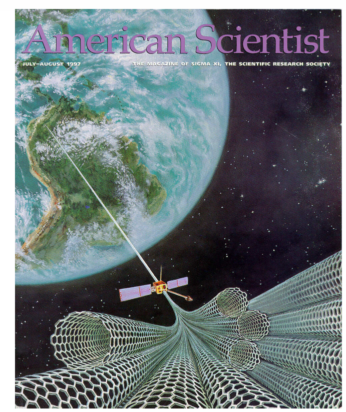


Tangent modulus = slope of the tangent line

Secant modulus = $\dfrac{\Delta\sigma}{\Delta\varepsilon} = \dfrac{\sigma_1}{\varepsilon_1}$

Table 6.1 Room-Temperature Elastic and Shear Moduli, and Poisson's Ratio for Various Metal Alloys

	Modulus of Elasticity		Shear Modulus		
Metal Alloy	*GPa*	*10^6 psi*	*GPa*	*10^6 psi*	*Poisson's Ratio*
Aluminum	69	10	25	3.6	0.33
Brass	97	14	37	5.4	0.34
Copper	110	16	46	6.7	0.34
Magnesium	45	6.5	17	2.5	0.29
Nickel	207	30	76	11.0	0.31
Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

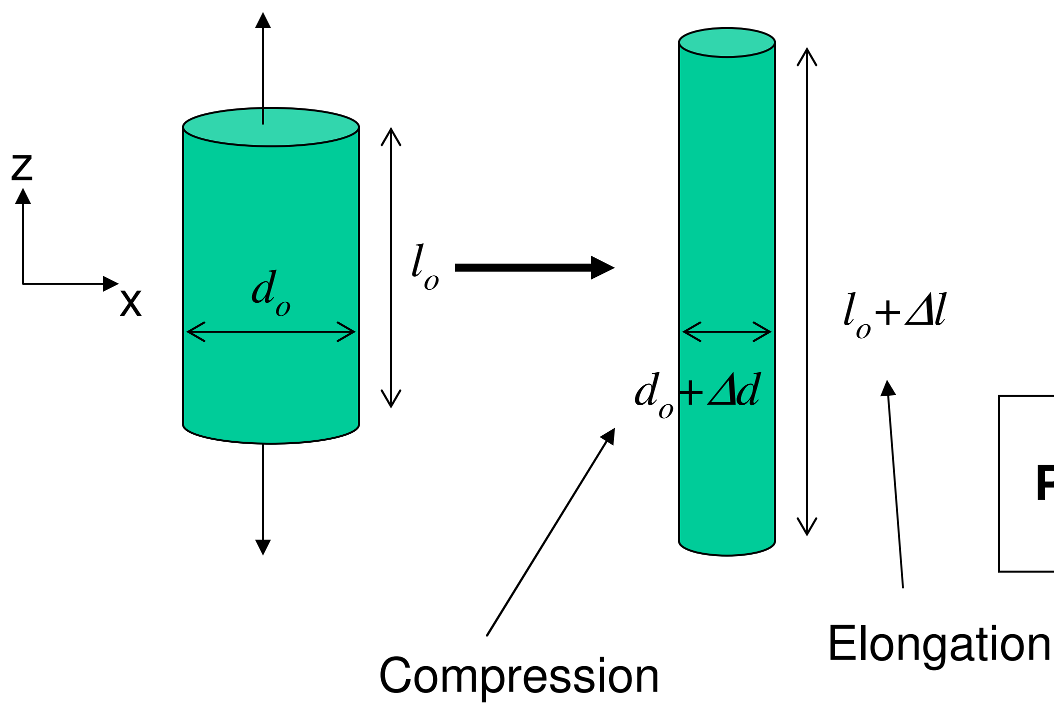


Silicon (single crystal)	120 - 190 (depends on crystallographic direction)
Glass (pyrex)	70
SiC (fused or sintered)	207 - 483
Graphite (molded)	~12
High modulus C-fiber	400
Carbon Nanotubes	**~1000**

If we normalize to density: ~20 times that of steel wire
Density normalized strength is ~56X that of steel wire

Poisson Ratio

So far, we've considered stress only along one dimension…



Along z: tension $\varepsilon_z = \dfrac{\Delta l}{l_o}$

Along x: compression $\varepsilon_x = \dfrac{\Delta d}{d_o}$

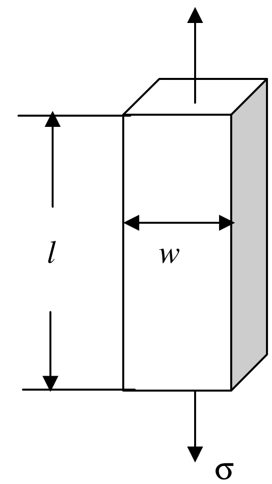
Isotropic x and y: $\varepsilon_y = \varepsilon_x$

Poisson ratio = $\upsilon = -\dfrac{\varepsilon_x}{\varepsilon_z} = -\dfrac{\varepsilon_y}{\varepsilon_z}$

Relation between elastic and shear moduli: $E = 2G(1+\nu)$

Poisson Ratio

- *Poisson Ratio has a range* $-1 \le \nu \le 1/2$



Look at extremes

- No change in aspect ratio: $\Delta w / w = \Delta \ell / \ell$

$$\nu = -\frac{\Delta w / w}{\Delta \ell / \ell} = -1$$

- Volume (V = AL) remains constant: ΔV =0 or $l\Delta A = - A\,\Delta l$

Hence, $\Delta V = (l\,\Delta A + A\,\Delta l) = 0$.

In terms of width, $A = w^2$,

and $\Delta A = w^2 - (w+\Delta w)^2 = 2w\,\Delta w + \Delta w^2$

then $\Delta A/A = 2\,\Delta w/w + \Delta w^2/w^2$

in the limit of small changes

$\Delta A/A = 2\,\Delta w/w$

then

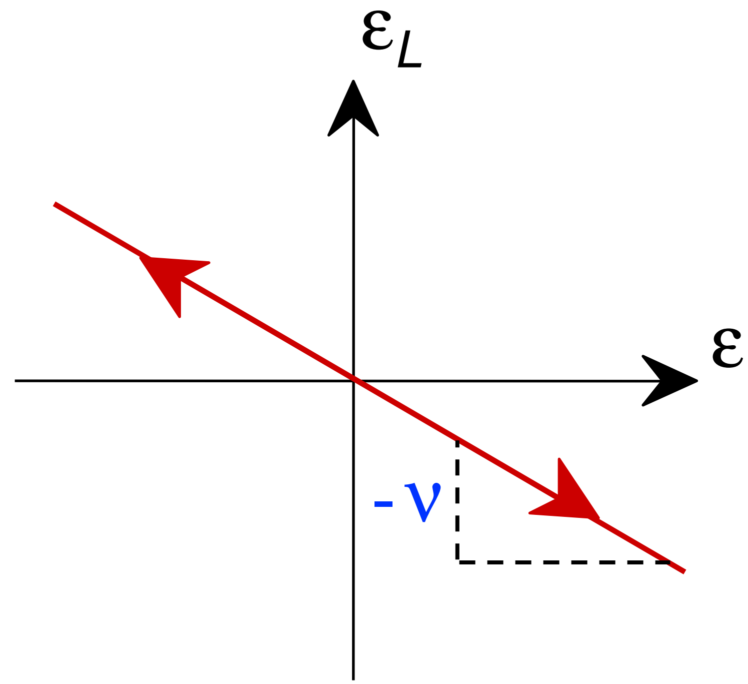
$2\,\Delta w/w = -\Delta l/l$

$$\nu = -\frac{\Delta w / w}{\Delta \ell / \ell} = -\frac{(-\frac{1}{2}\Delta \ell / \ell)}{\Delta \ell / \ell} = 1/2$$

Poisson's ratio, ν

- **Poisson's ratio, ν:**

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$



metals: $\nu \sim 0.33$
ceramics: $\nu \sim 0.25$
polymers: $\nu \sim 0.40$

Units:
E: [GPa] or [psi]
ν: dimensionless

$-\nu > 0.50$ density increases

$-\nu < 0.50$ density decreases (voids form)

Poisson Ratio: materials specific

Metals:	Ir	W	Ni	Cu	Al	Ag	Au
	0.26	0.29	0.31	0.34	0.34	0.38	0.42

generic value ~ 1/3

Solid Argon: 0.25

Covalent Solids:	Si	Ge	Al_2O_3	TiC	
	0.27	0.28	0.23	0.19	generic value ~ 1/4

Ionic Solids: MgO 0.19

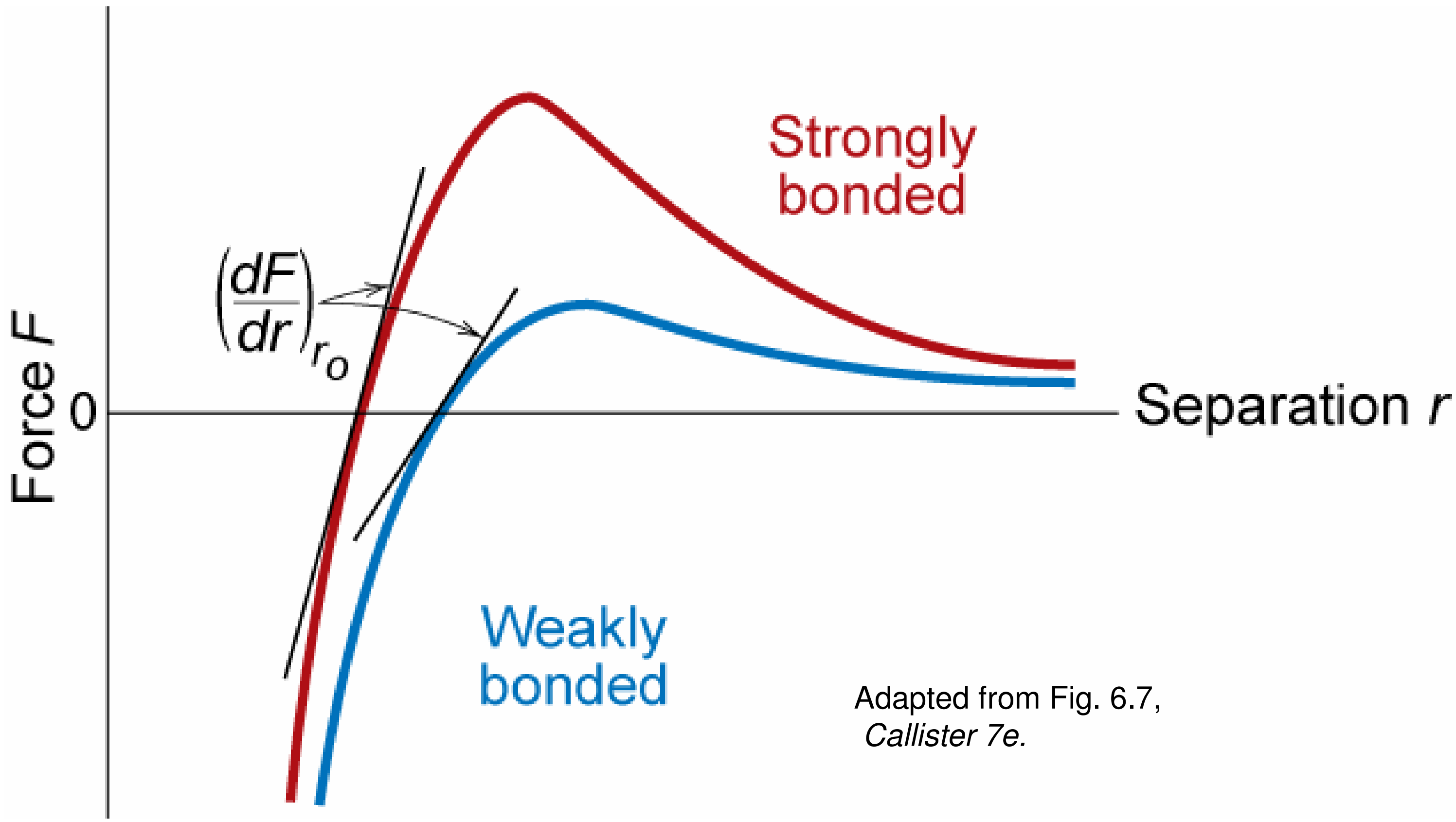
Silica Glass: 0.20

Polymers: Network (Bakelite) 0.49 Chain (PE) 0.40

Elastomer: Hard Rubber (Ebonite) 0.39 (Natural) 0.49

Mechanical Properties

- Slope of stress strain plot (which is proportional to the elastic modulus) depends on bond strength of metal

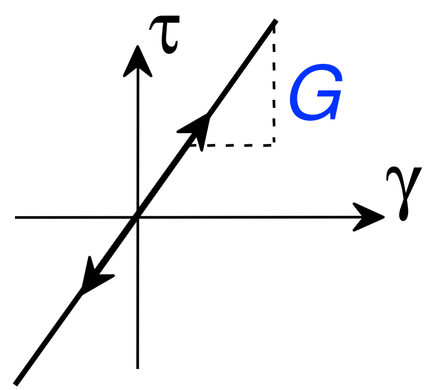


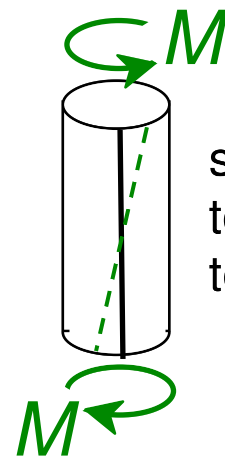
Adapted from Fig. 6.7, *Callister 7e.*

Other Elastic Properties

- Elastic **Shear modulus, *G*:**

$$\tau = G\gamma$$

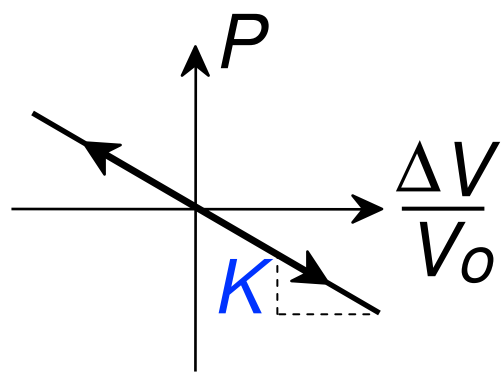


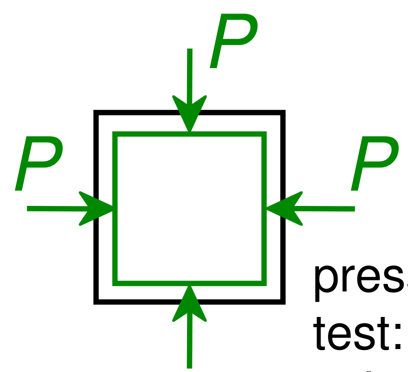


simple torsion test

- Elastic **Bulk modulus, K:**

$$P = -K\frac{\Delta V}{V_o}$$



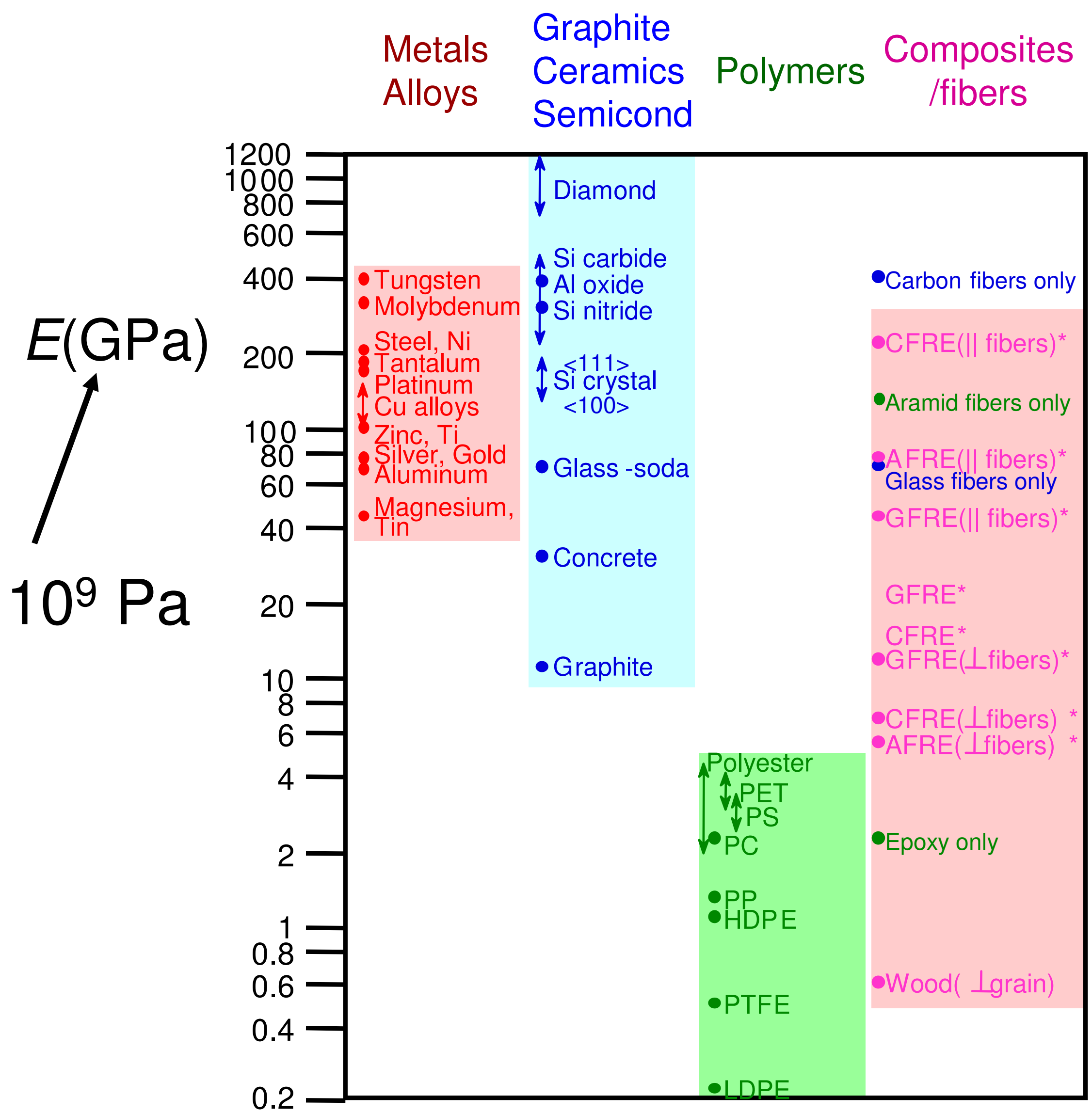


pressure test: Init. vol $= V_o$. Vol chg. $= \Delta V$

- **Special relations for isotropic materials:**

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Young’s Moduli: Comparison



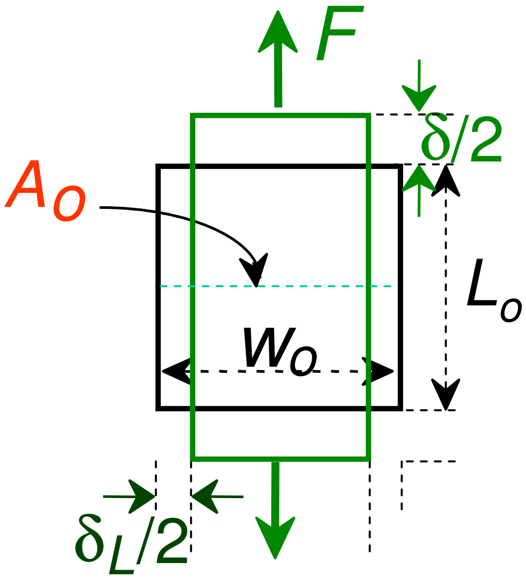
Based on data in Table B2, *Callister 7e*.

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Useful Linear Elastic Relationships

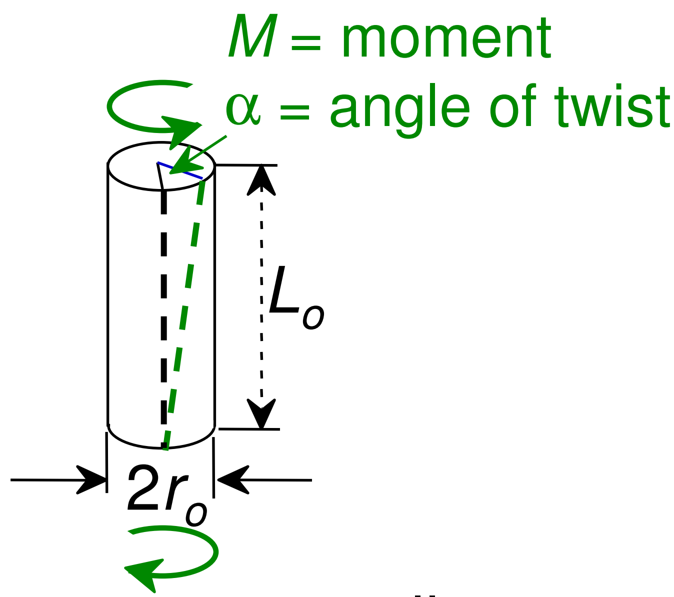
- **Simple tension:**

$$\delta = \frac{FL_o}{EA_o} \qquad \delta_L = -\nu \frac{Fw_o}{EA_o}$$



- **Simple torsion:**

$$\alpha = \frac{2ML_o}{\pi r_o^4 G}$$

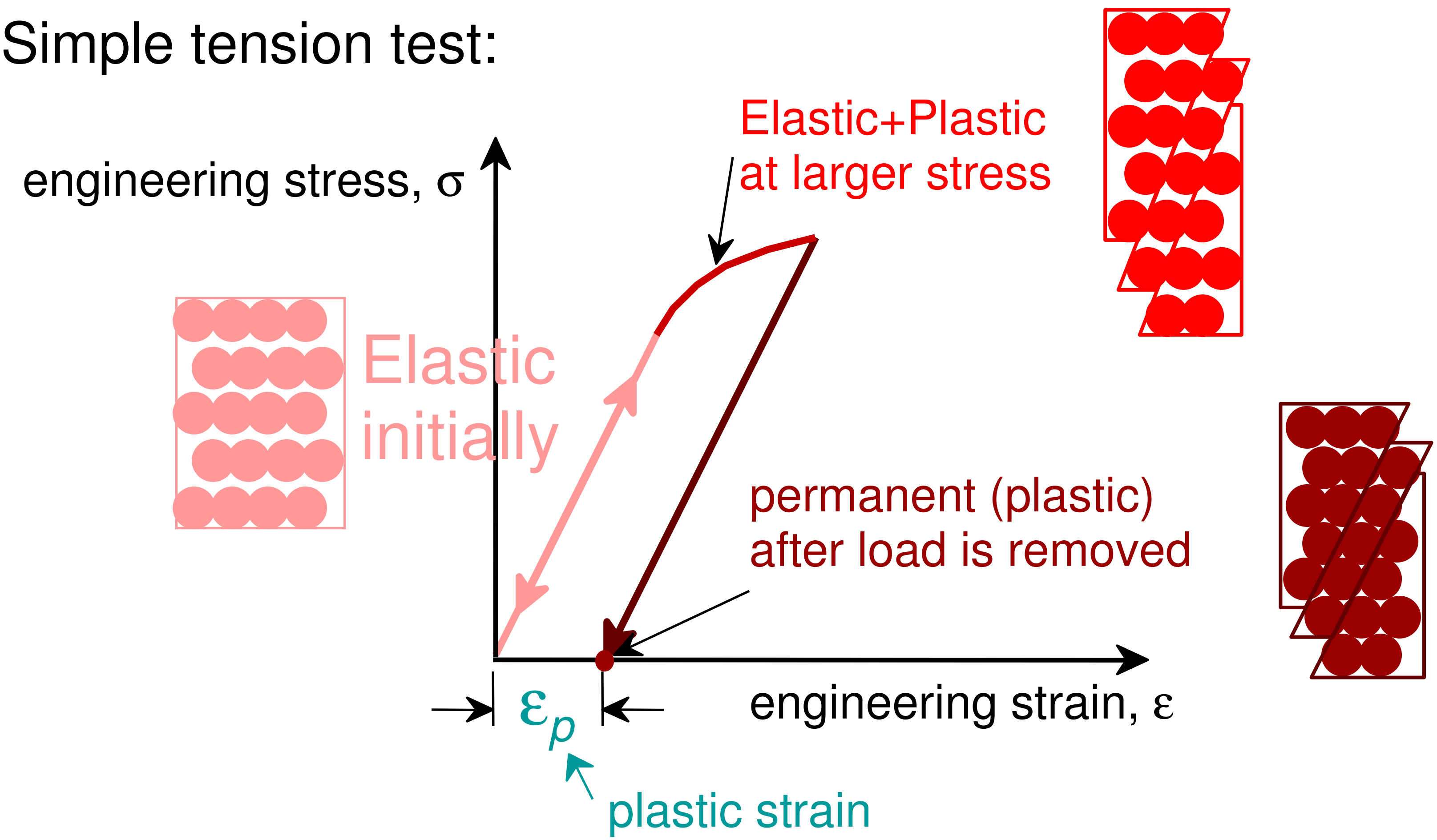


- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

Plastic (Permanent) Deformation

Adapted from Fig. 6.10 (a), *Callister 7e.*

- Simple tension test:



•A permanent deformation (usually considered for $T<T_m/3$).
•Atoms break bonds and form new ones.
•In metals, plastic deformation occurs typically at strain ≥0.005.

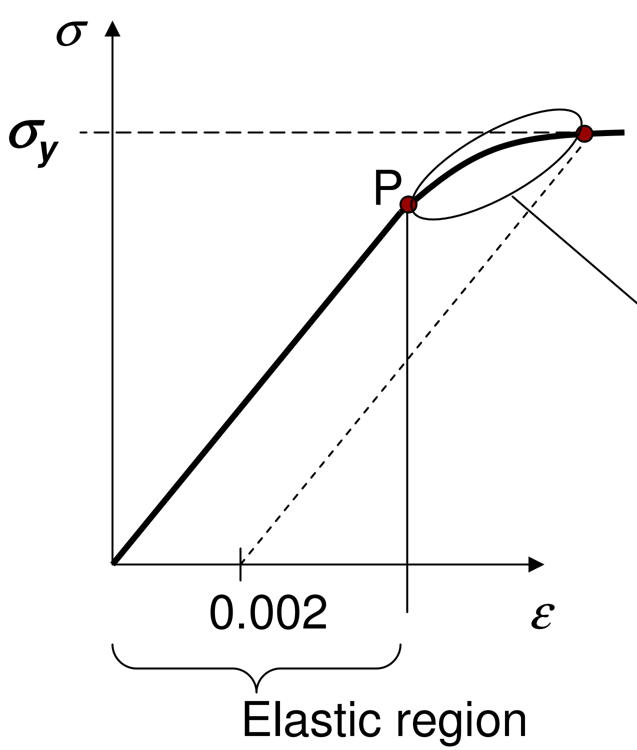
Tensile properties

A. <u>Yield strength</u> (σ_y): the strength required to produce a very slight yet specified amount of plastic deformation.

What is the specified amount of strain?

Strain offset method

1. Start at 0.002 strain (for most metals).
2. Draw a line parallel to the linear region.
3. σ_y = where the dotted line crosses the stress-strain curve.



P = proportional limit (beginning of deviation from linear behavior.

Mixed elastic-plastic behavior

For materials with nonlinear elastic region: σ_y is defined as stress required to produce specific amount of strain (e.g. ~0.005 for most metals).

Tensile properties

<u>Yield point phenomenon</u> occurs when elastic-plastic transition is well-defined and abrupt.

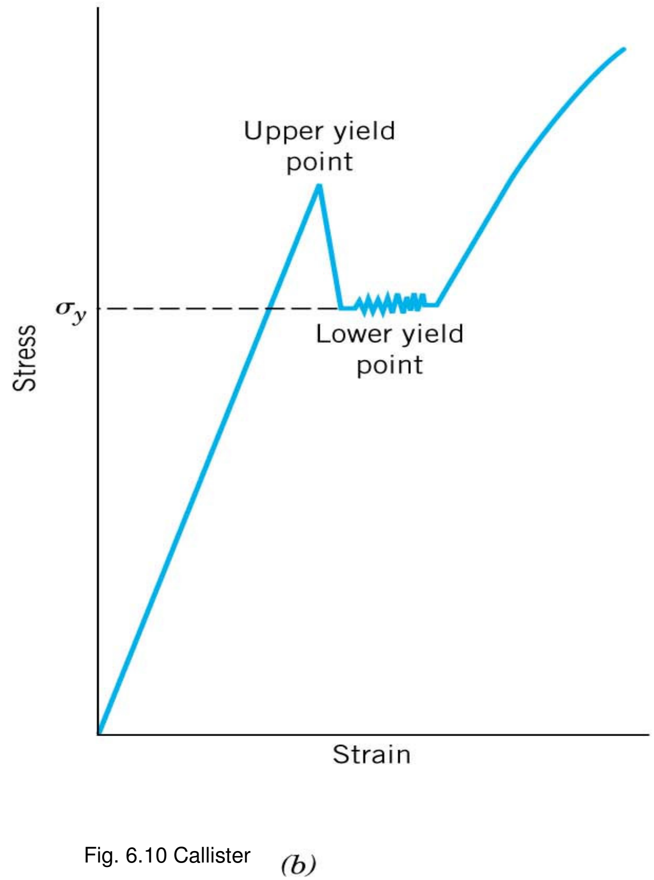
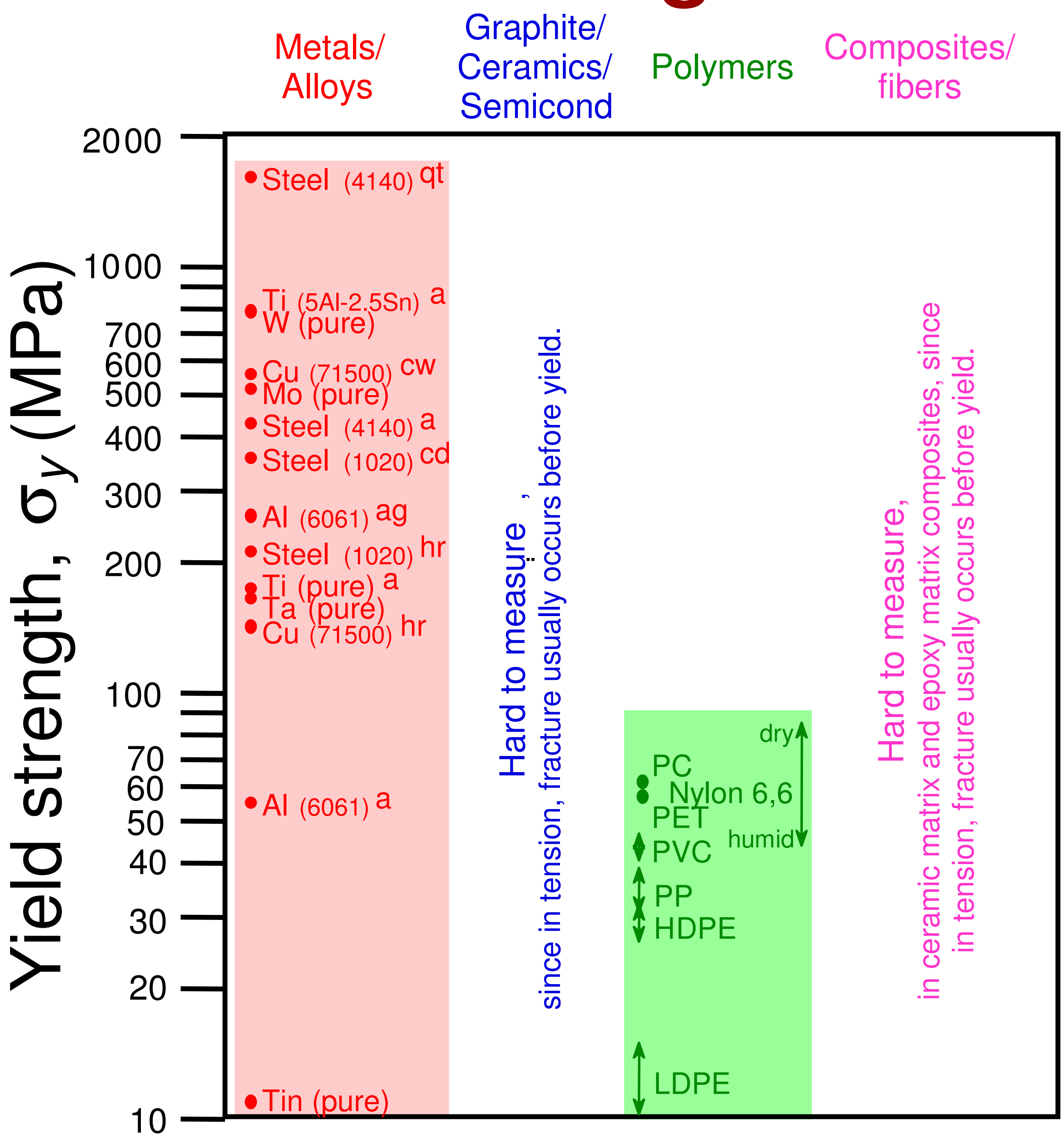


Fig. 6.10 Callister

No offset methods required here.

Yield Strength : Comparison



σ_y(ceramics)
$>>\sigma_y$(metals)
$>> \sigma_y$(polymers)

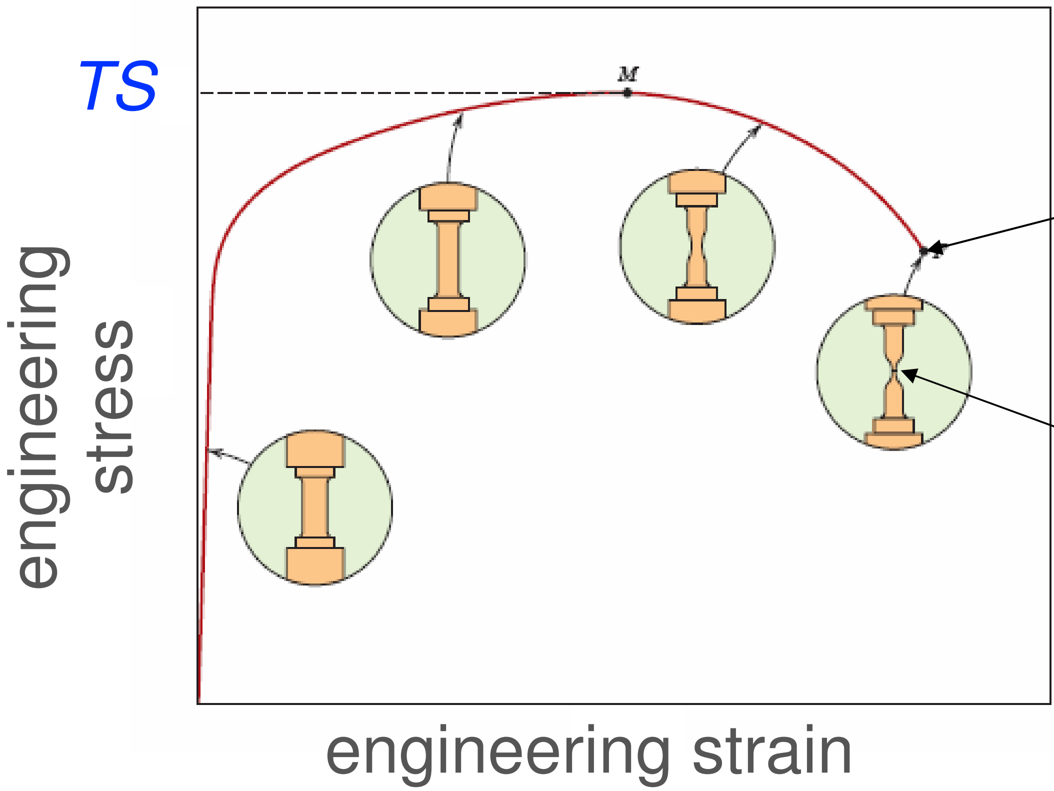
Room *T* values

Based on data in Table B4, *Callister 7e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength, TS

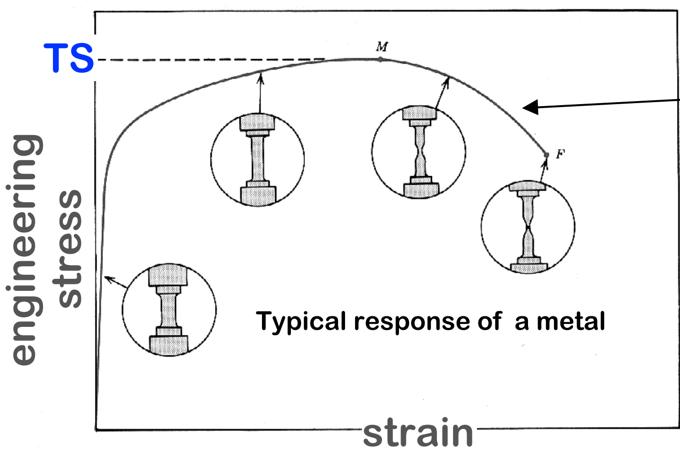
- Maximum stress on engineering stress-strain curve.



Adapted from Fig. 6.11, *Callister 7e.*

- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

True stress and strain



Notice that past maximum stress point, σ decreases.

→ Does this mean that the material is becoming weaker?

Necking leads to smaller cross sectional area!

Recall: Engineering Stress = $\sigma = \dfrac{F}{A_o}$ ← Original cross sectional area!

True Stress = $\sigma_T = \dfrac{F}{A_i}$

True Strain = $\varepsilon_T = \ln \dfrac{l_i}{l_o}$

A_i = instantaneous area

l_i = instantaneous length

If no net volume change (i.e. $A_i \, l_i = A_o \, l_o$)

$$\left.\begin{aligned} \sigma_T &= \sigma(1+\varepsilon) \\ \varepsilon_T &= \ln(1+\varepsilon) \end{aligned}\right\}$$ Only true at the onset of necking

Example problem

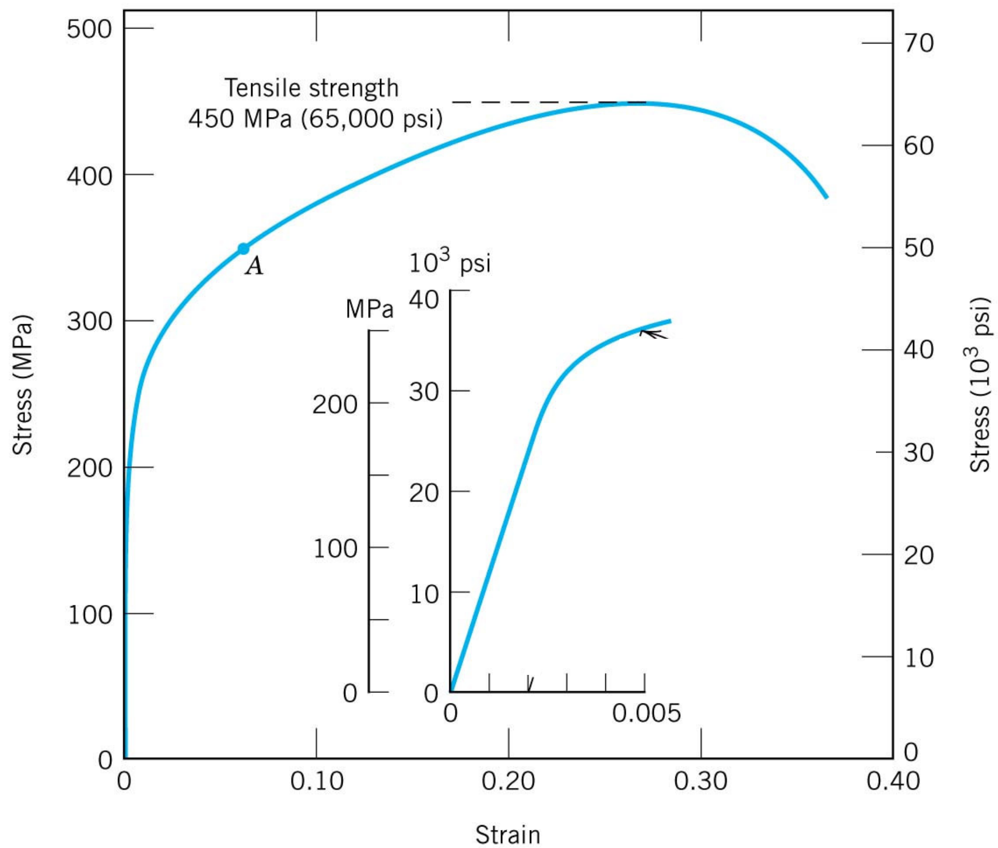
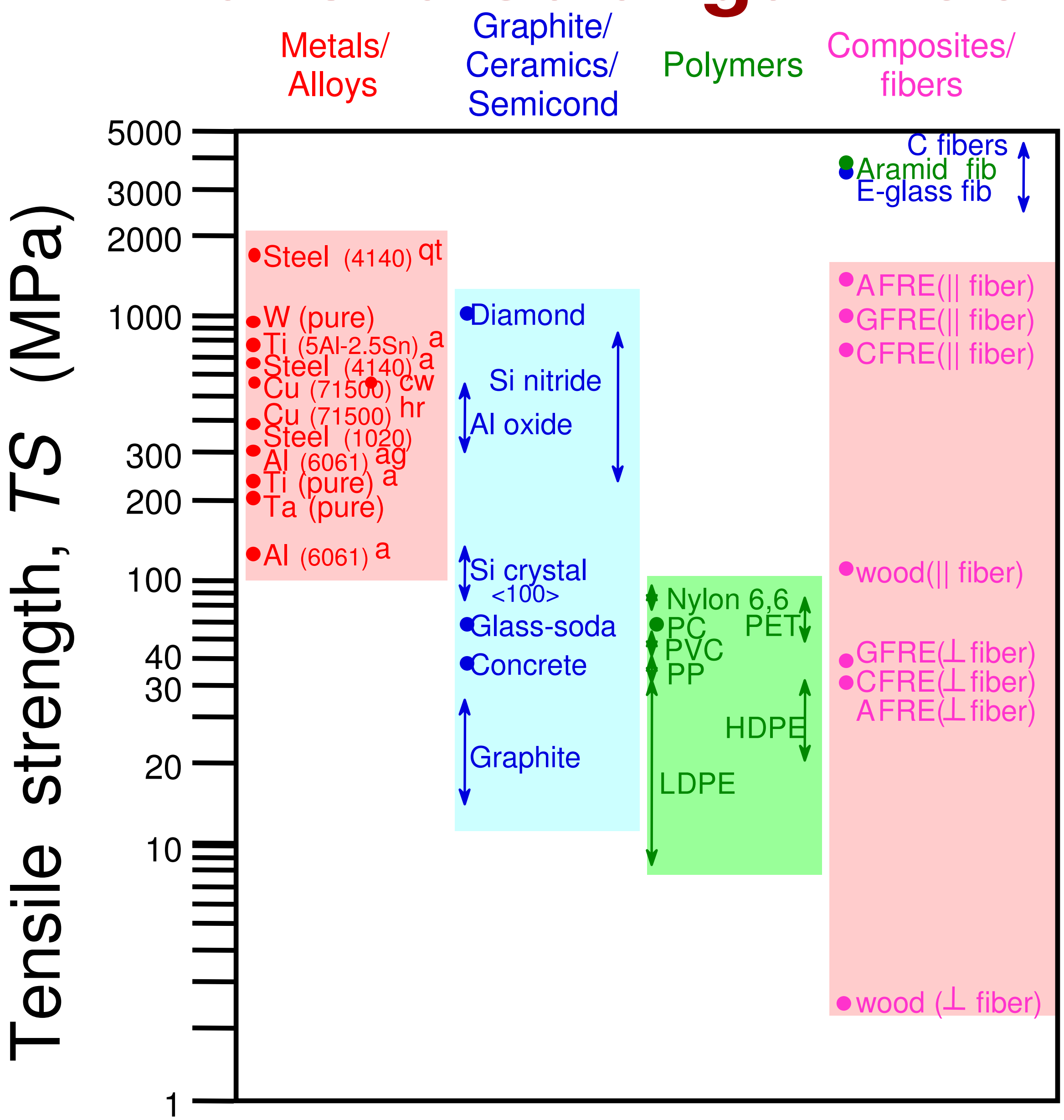


FIGURE 6.12 The stress–strain behavior for the brass specimen discussed in Example Problem 6.3.

Calculate/determine the following for a brass specimen that exhibits stress-strain behavior shown on the left.

1) Modulus of elasticity.
2) Yield strength.
3) Maximum load for a cylindrical specimen with d = 12.8mm.
4) Change in length at 345MPa if the initial length is 250mm.

Tensile Strength : Comparison



Room Temp. values

Based on data in Table B4, *Callister 7e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

Tensile properties

C. <u>Ductility</u>: measure of degree of plastic deformation that has been sustained at fracture.

- **Ductile materials** can undergo significant plastic deformation before fracture.
- **Brittle materials** can tolerate only very small plastic deformation.

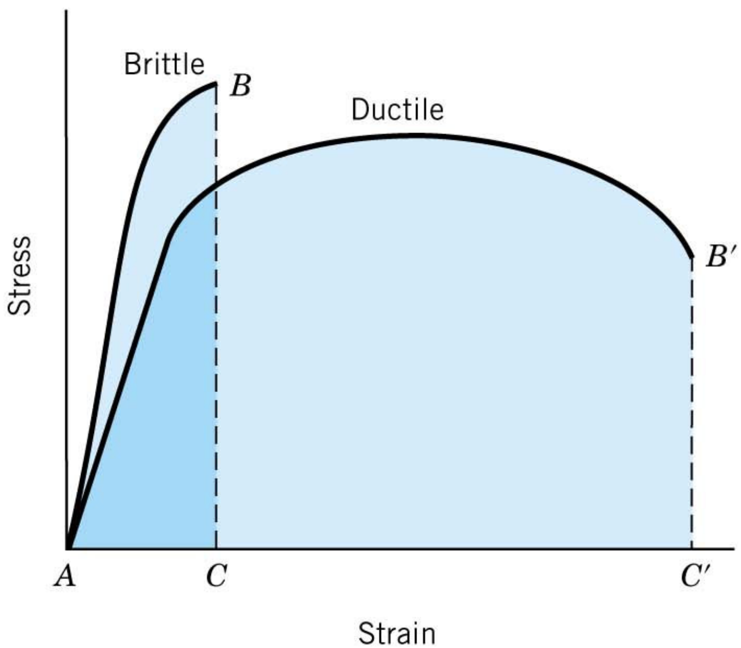
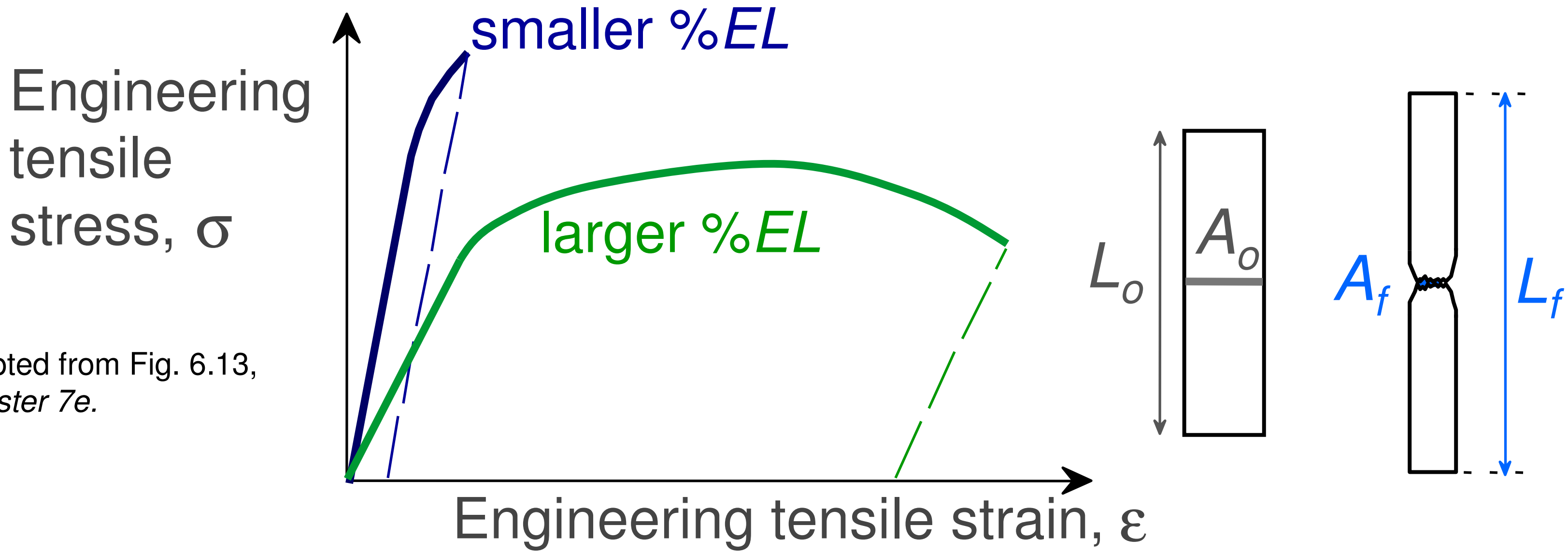


FIGURE 6.13 Schematic representations of tensile stress–strain behavior for brittle and ductile materials loaded to fracture.

Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \text{ x } 100$$



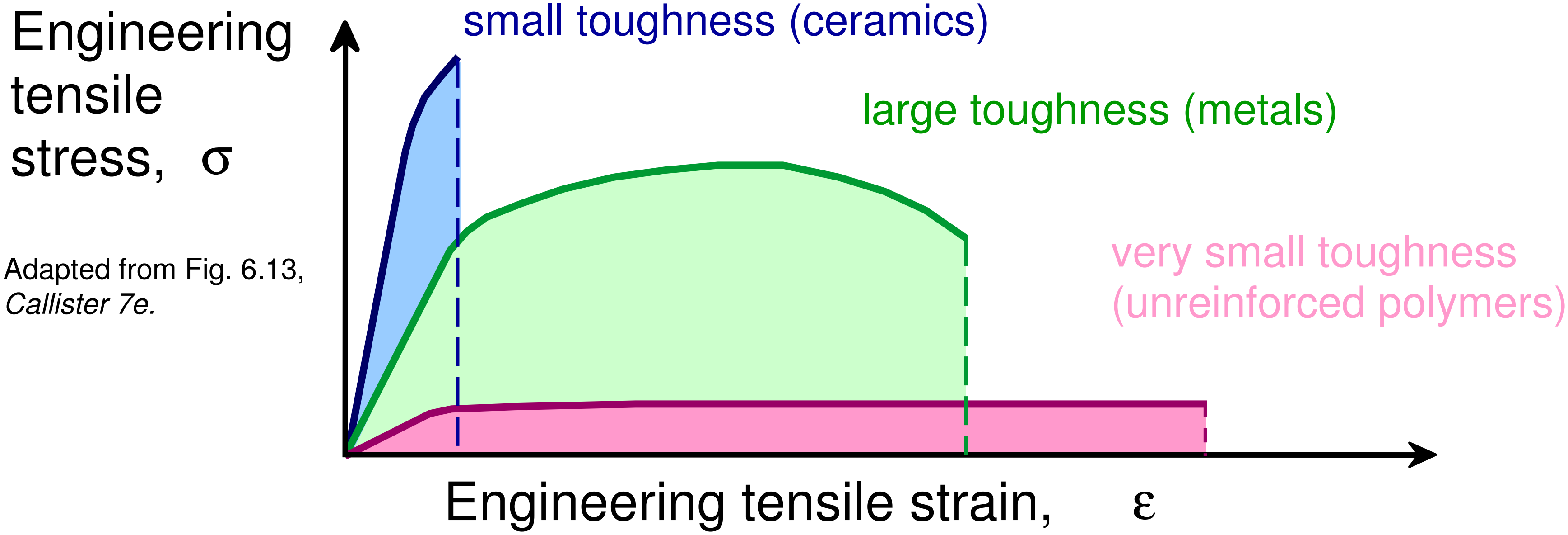
Adapted from Fig. 6.13, *Callister 7e.*

- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \text{ x } 100$$

Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.

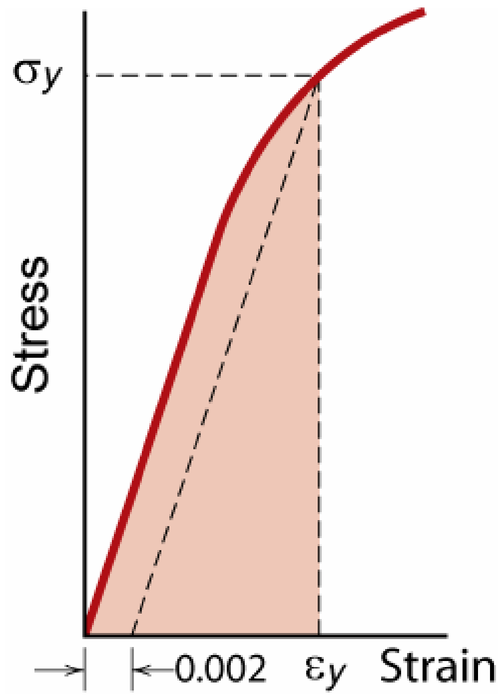


Adapted from Fig. 6.13, *Callister 7e.*

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Resilience, U_r

- Ability of a material to store energy
  - Energy stored best in elastic region



Adapted from Fig. 6.15, *Callister 7e.*

$$U_r = \int_0^{\varepsilon_y} \sigma\, d\varepsilon$$

If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2}\sigma_y \varepsilon_y$$

Elastic recovery after plastic deformation

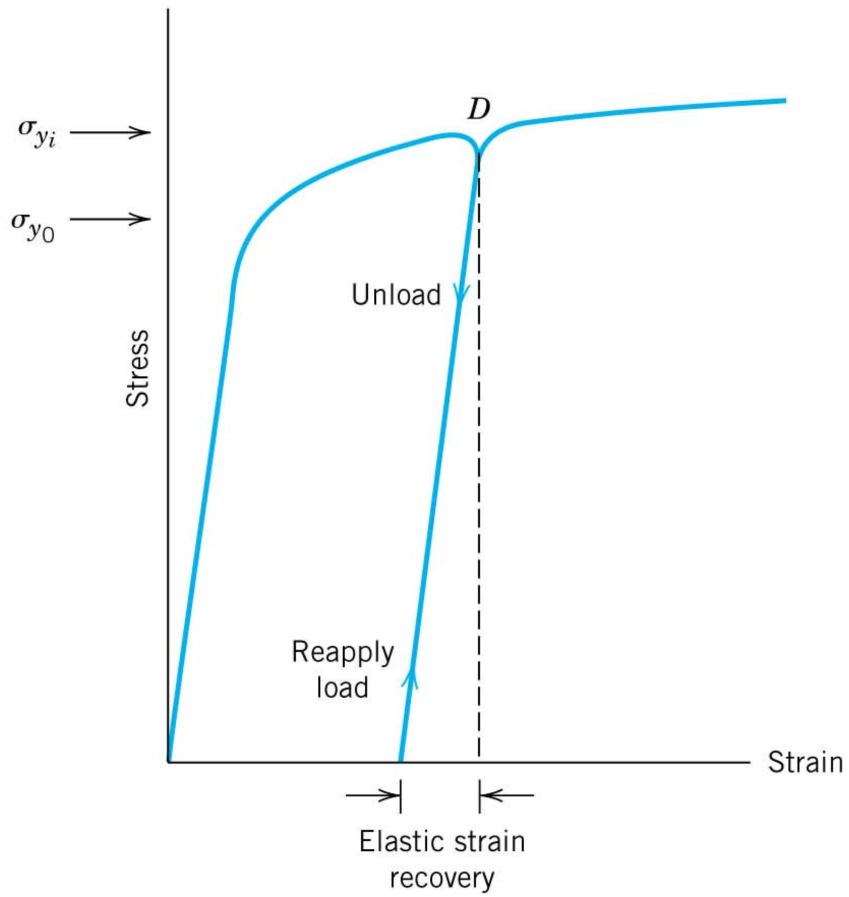
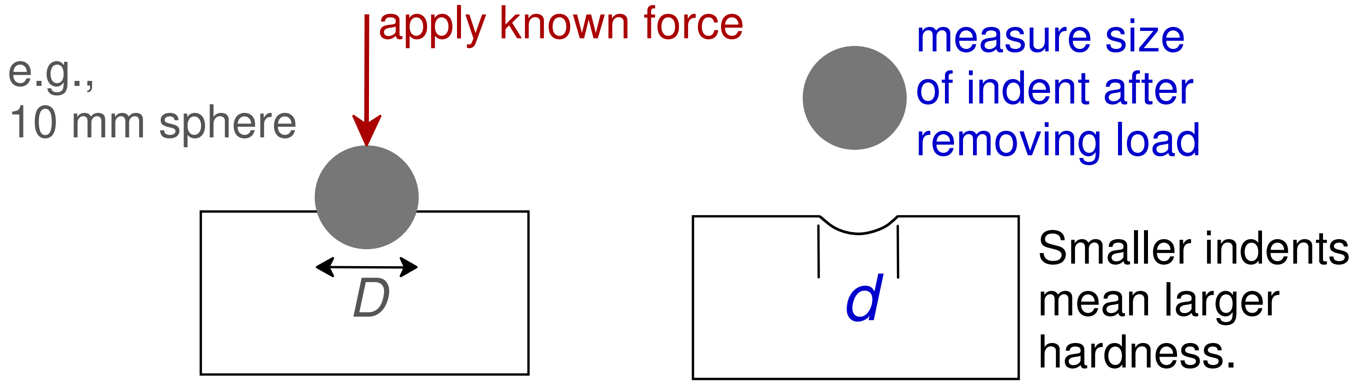


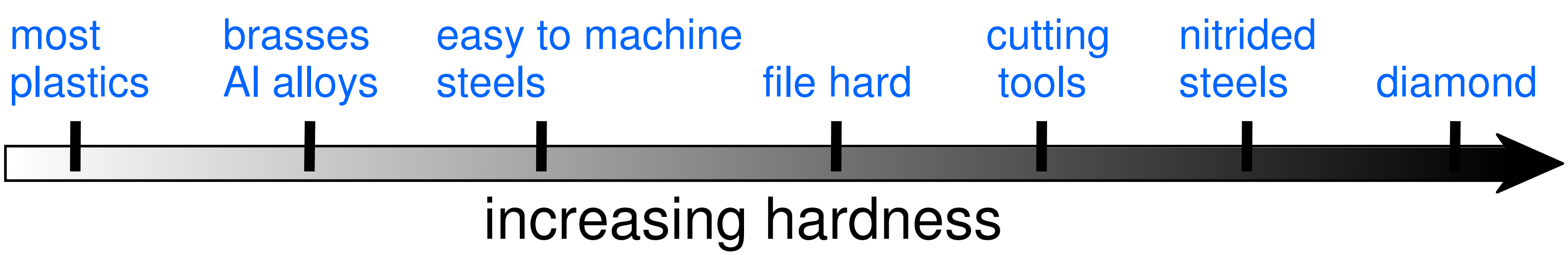
FIGURE 6.17 Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as σ_{y_0}; σ_{y_i} is the yield strength after releasing the load at point D, and then upon reloading.

This behavior is exploited to increase yield strengths of metals: **strain hardening** (also called **cold working**).

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  --resistance to plastic deformation or cracking in compression.
  --better wear properties.





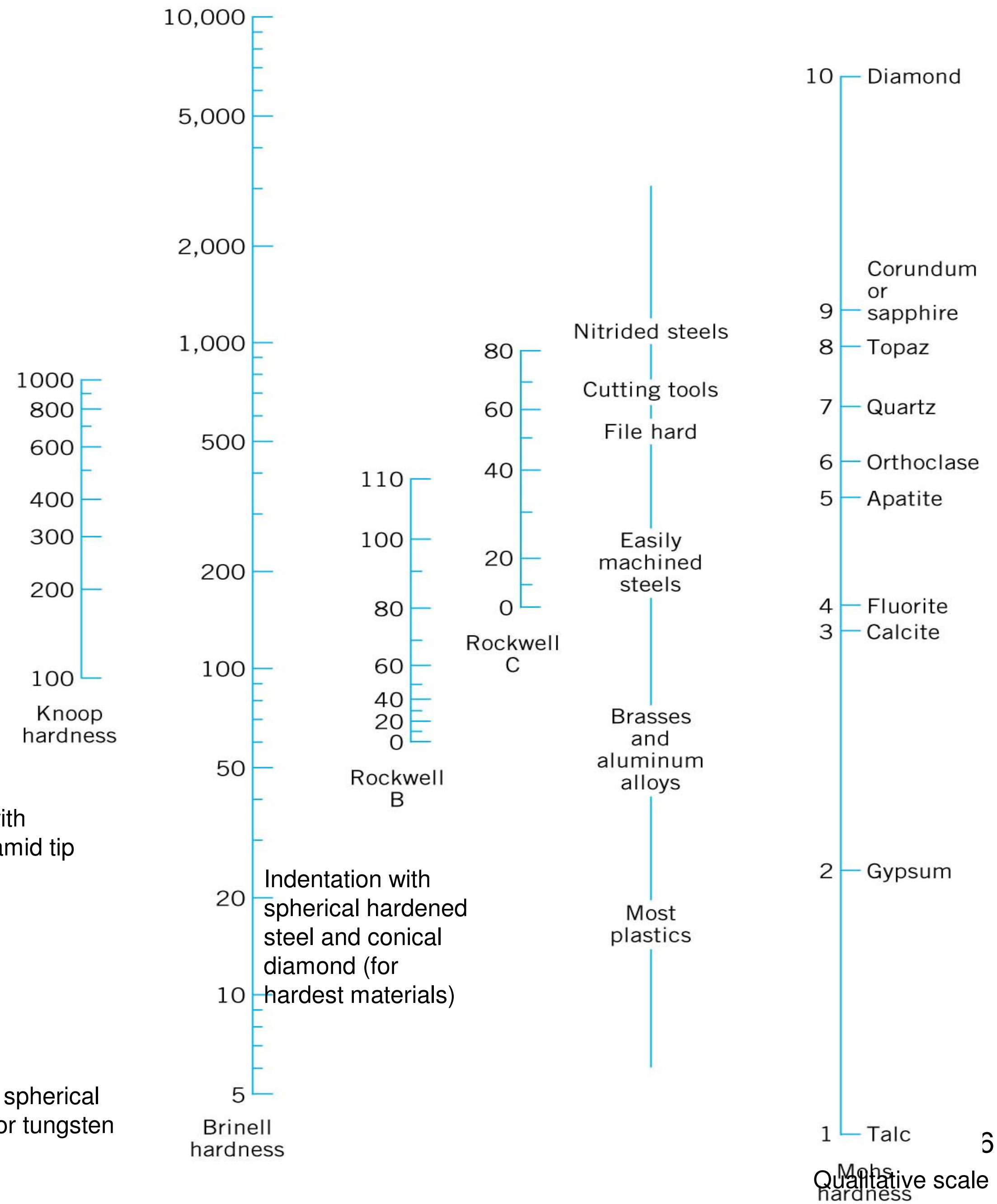
Hardness: Measurement

- Rockwell
  - No major sample damage
  - Each scale runs to 130 but only useful in range 20-100.
  - Minor load   10 kg
  - Major load   60 (A), 100 (B) & 150 (C) kg
    - A = diamond, B = 1/16 in. ball, C = diamond

- HB = Brinell Hardness
  - *TS* (psia) = 500 x HB
  - *TS* (MPa) = 3.45 x HB

Hardness scales

FIGURE 6.18 Comparison of several hardness scales. (Adapted from G. F. Kinney, *Engineering Properties and Applications of Plastics*, p. 202. Copyright © 1957 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)



Indentation with diamond pyramid tip

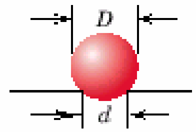
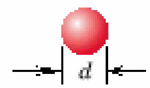
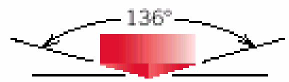
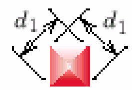
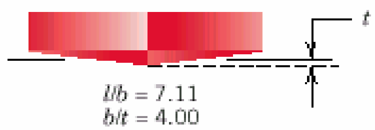
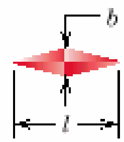
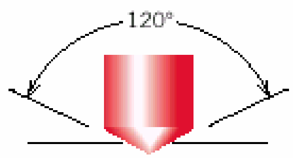
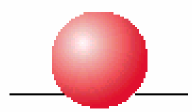
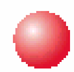
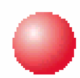
Indentation with spherical hardened steel and conical diamond (for hardest materials)

Indentation with spherical hardened steel or tungsten carbide tip.

Qualitative scale

Hardness: Measurement

Table 6.5 **Hardness Testing Techniques**

Test	*Indenter*	*Shape of Indentation* — *Side View*	*Shape of Indentation* — *Top View*	*Load*	*Formula for Hardness Number*[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$\mathrm{HB} = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$\mathrm{HV} = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$\mathrm{HK} = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone; $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres	 	 	60 kg, 100 kg, 150 kg: Rockwell 15 kg, 30 kg, 45 kg: Superficial Rockwell	

[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

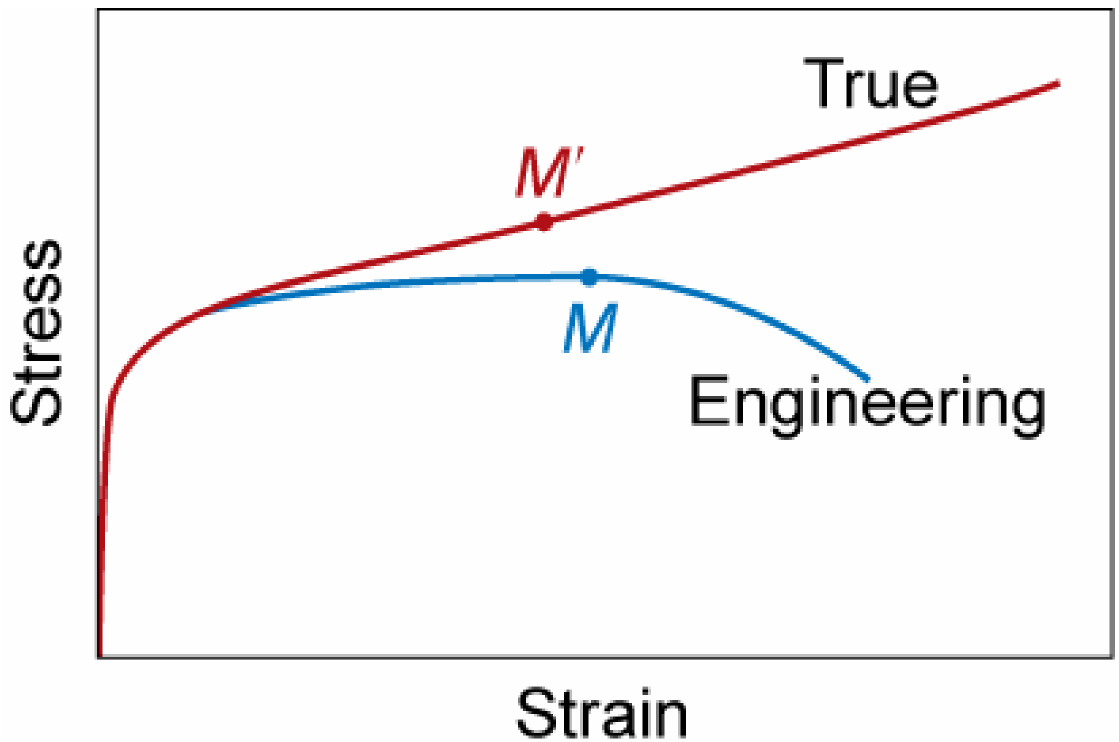
Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials,* Vol. III, *Mechanical Behavior.* Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

True Stress & Strain

Note: S.A. changes when sample stretched

- True stress $\sigma_T = F/A_i$
- True Strain $\varepsilon_T = \ln(\ell_i/\ell_o)$

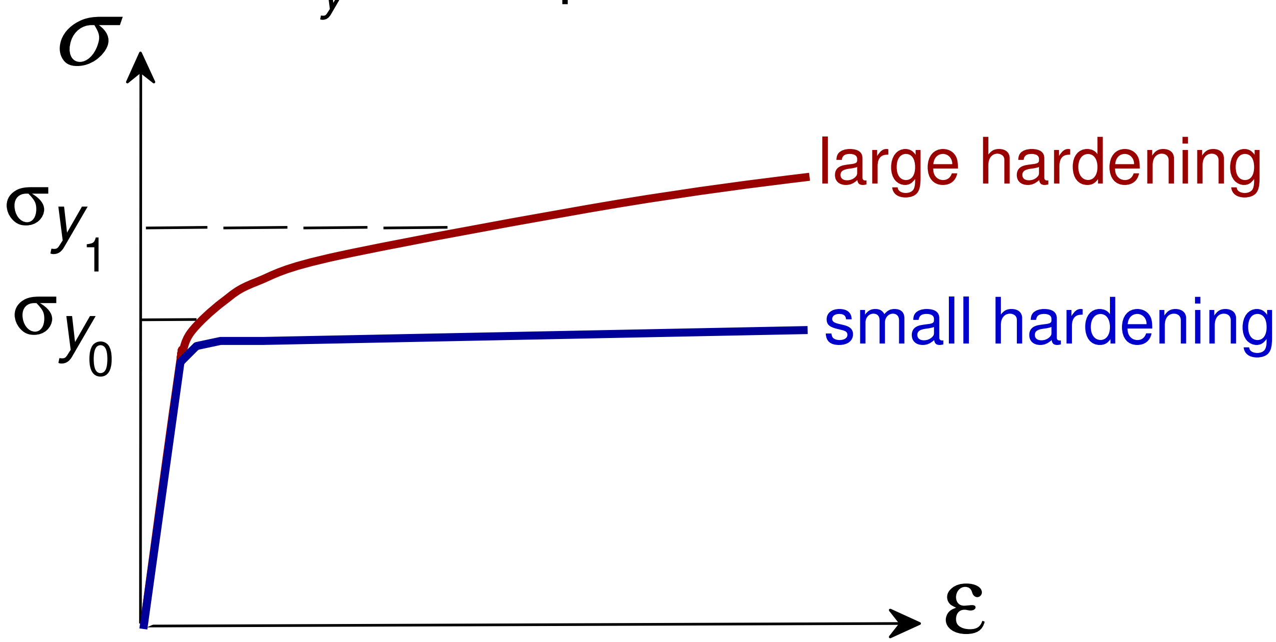
$$\boxed{\begin{aligned}\sigma_T &= \sigma(1+\varepsilon)\\ \varepsilon_T &= \ln(1+\varepsilon)\end{aligned}}$$



Adapted from Fig. 6.16, *Callister 7e.*

Hardening

- An increase in σ_y due to plastic deformation.



- Curve fit to the stress-strain response:

$$\sigma_T = K\left(\varepsilon_T\right)^n$$

"true" stress (F/A)

"true" strain: $\ln(L/L_o)$

hardening exponent:
n = 0.15 (some steels)
to n = 0.5 (some coppers)

Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics
  - Mean

$$\bar{x} = \frac{\overset{n}{\Sigma} x_n}{n}$$

  - Standard Deviation

$$s = \left[\frac{\overset{n}{\Sigma} (x_i - \bar{x})^2}{n-1} \right]^{\frac{1}{2}}$$

where n is the number of data points

Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, N

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is between 1.2 and 4

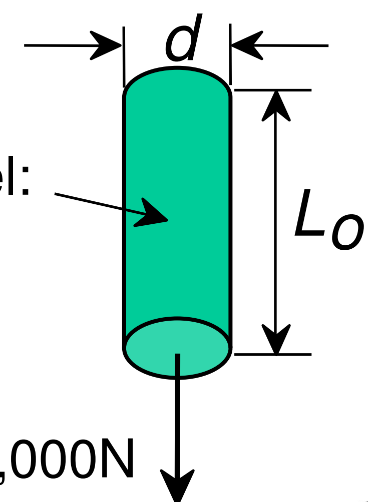
- Example: Calculate a diameter, d, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\sigma_{working} = \frac{220{,}000N}{\pi\left(d^2/4\right)}$$

$N = 5$

1045 plain carbon steel:
$\sigma_y = 310$ MPa
$TS = 565$ MPa



$F = 220{,}000$N

$\boxed{d = 0.067 \text{ m} = 6.7 \text{ cm}}$

Summary

- Stress and strain: These are size-independent measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (*E* or *G*).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.