Chapter 10
Basic Control Charts

Introduction

- In the second half of 1920s, Dr. Walter Shewhart of Bell Lab concluded that there were two components to variations that were displayed in all manuf. processes.
  - The first one was a steady component (random variation) that appeared to be inherent in the process.
  - The second one was an intermittent variation to assignable causes.
  - He concluded that assignable causes could be economically discovered and removed with an effective diagnostic program, but the random causes could not be removed without making basic process changes.
  - Standard control chart based on 3σ limits.

Introduction

- Shewhart control charts came into wide use in the 1940s because of war production efforts.
- Western Electric is credited with the addition of other tests based on sequences or runs.
- 94% of the troubles belong to the system (responsibility of management), 6% special.

10.1 S^4/IEE Application Examples: Control Charts

- Satellite-level metric: The last three-year's ROI for a company was reported monthly in a control chart.
- Transactional 30,000-foot-level metric: One random paid invoice was selected each day from last year's invoices, where the number of days beyond the due date was measured and reported (i.e., days sales outstanding [DSO]). The DSO for each sample was reported in a control chart.
- Transactional 30,000-foot-level metric: The mean and standard deviation of DSOs was tracked using a weekly subgroup. An XmR control chart was used for each chart in lieu of an $\bar{X}$ and s chart.

10.1 S^4/IEE Application Examples: Control Charts

- Manufacturing 30,000-foot-level metric (KPOV): One random sample of a manufactured part was selected each day over the last year. The diameter of the part was measured and plotted in an XmR control chart.
- Transactional and manufacturing 30,000-foot-level cycle time metric (a lean metric): One randomly selected transaction was selected each day over the last year, where the time from order entry to fulfillment was measured and reported in an XmR control chart.
- Transactional and manufacturing 30,000-foot-level inventory metric or satellite-level TOC metric (a lean metric): Inventory was tracked monthly using an XmR control chart.

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10.1 S^4/IEE Application Examples: Control Charts

- Manufacturing 30,000-foot-level quality metric: The number of printed circuit boards produced weekly for a high-volume manufacturing company is similar. The weekly failure rate of printed circuit boards is tracked on an XmR control chart rather than a p chart.
- Transactional 50-foot-level metric (KPIV): An S^4/ IEE project to improve the 30,000-foot-level metrics for DSOs identified a KPIV to the process, the importance of timely calling customers to ensure that they received a company's invoice. A control chart tracked the time from invoicing to when the call was made, where one invoice was selected hourly.

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10.1 S^4/IEE Application Examples: Control Charts

- Product DFSS: An S^4/IEE product DFSS project was to reduce the 30,000-foot-level MTBF (mean time between failures) of a product by its vintage (e. g., laptop computer MTBF rate by vintage of the compute). A control chart tracked the product MTBF by product vintage. Categories of problems for common cause variability were tracked over the long haul in a Pareto chart to identify improvement opportunities for newly developed products.
- S^4/IEE infrastructure 30,000-foot-level metric: A steering committee uses an XmR control chart to track the duration of projects.

10.2 Satellite-level View of the Organization

- Organizations often evaluate their business by comparing their currently quarterly profit (or other measures) to previous quarter or the same period a year ago. → Action plans → Firefighting
- S^4/IEE approach create satellite-level metrics that view the organization as a system. Variation within this system is expected. XmR control chart could be used to assess whether the system is experiencing any special cause, trend, or seasonal issues. Probability plot can be used to determine the expected variability. When change is needed to a common cause response, S^4/IEE projects could be created. (Pulling)

10.3 30,000-ft-level View of Operational and Project Metrics

- 30,000-ft-level control chart gives a macro view of a process KPOV, CTQ, or Y, while a 50-ft-level control chart gives more of a micro view of some aspect of the process (i.e., KPIV or X of the process).
- Control charts at the 50-ft-level are useful in timely identifying special causes. (e.g., temperature)
- Control charts are also useful at a higher level to prevent the attacking of common cause issues. → long sampling frequency → XmR control chart
  - To determine special cause <> common cause
  - To provide long-term view of the capability/performance of the process relative to meeting the needs of customers.

10.3 30,000-ft-level View of Operational and Project Metrics

S[4]/IEE Approach:

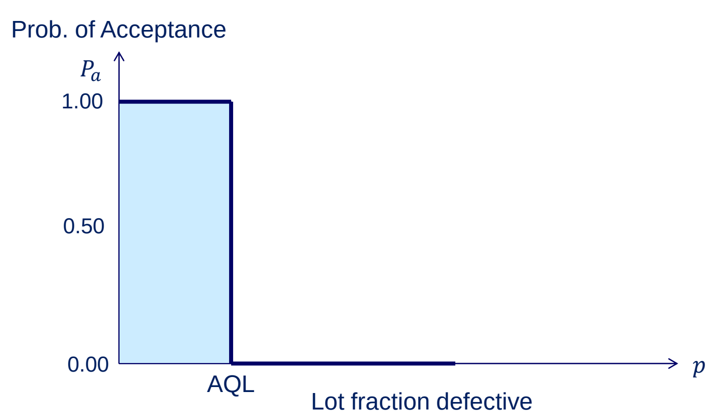
- Identify the problem
- Identify focus items for further investigation (process mapping, cause-and-effect diagram, cause-and-effect matrix, and failure mode and effects analysis (FMEA)
- Monitor the focus items with 30,000-ft-level control charts
- Create a sampling plan to establish a baseline of a process
  - Sampling frequency (less frequent if too many special causes)
- Assess key KPOV relative to the needs of customers. $(C_p, C_{pk}, P_p, P_{pk})$ supplemented with probability plot
- Estimate cost impact

10.4 Acceptable Quality Level (AQL) Sampling Can Be Deceptive

- To show how an AQL pass/fail sample lot test strategy for product is not effective.
- With AQL sampling plans, a lot is inspected to determine whether it should be accepted or rejected.
- Samples must be a random sample from the lot. Two risks:
  - Good lots can be rejected
  - Bad lots can be accepted
  - The operating characteristic (OC) curve for sampling plans quantifies these risks.

10.4 Acceptable Quality Level (AQL) Sampling Can Be Deceptive

- An ideal OC curve



10.4 Acceptable Quality Level (AQL) Sampling Can Be Deceptive

- Acceptable Quality Level (AQL) is the worst quality level that is still considered satisfactory. The probability of accepting an AQL lot should be high.
- Rejectable Quality Level (RQL) or Lot Tolerance Percent Defective (LTPD) is considered to be unsatisfactory quality level. The probability of accepting an RQL lot should be low. This consumer's risk has been standardized as 0.1.
- Indifference Quality Level (IQL) is frequently defined as quality level having probability of acceptance of 0.50 for a sampling plan.

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10.4 Acceptable Quality Level (AQL) Sampling Can Be Deceptive

- An OC curve describes the probability of acceptance for various values of incoming quality. P_a is the probability that the number of defectives in the sample is equal to or less than the acceptance number for the sampling plan.
- AQL sampling often leads to activities attempting to test quality into product. AQL sampling can reject lots only subject to common-cause variability.
- In lieu of using AQL sampling plan, more useful information can be obtained by using control charts (first to identify special cause issues, then process capability issues).

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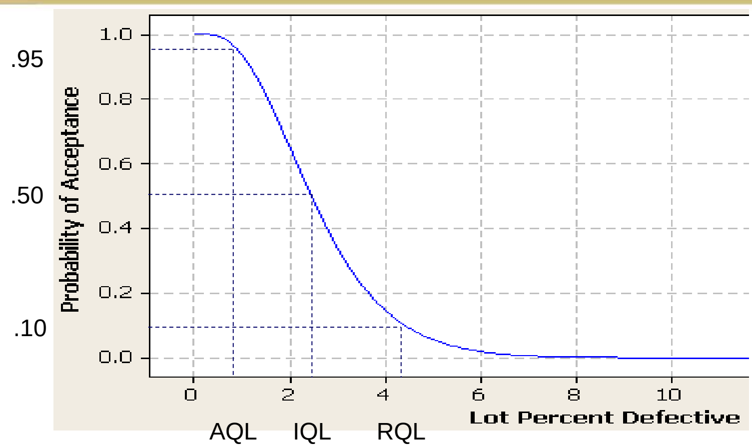
An OC Curve for Acceptance Sampling Plan (n=150, c=3)



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10.5 Example 10.1: Acceptable Quality Level (AQL)

- For N (lot size) = 75 and AQL = 4%, MIL-STD-105E yields for a general inspection level II test plan: n=13, c=1

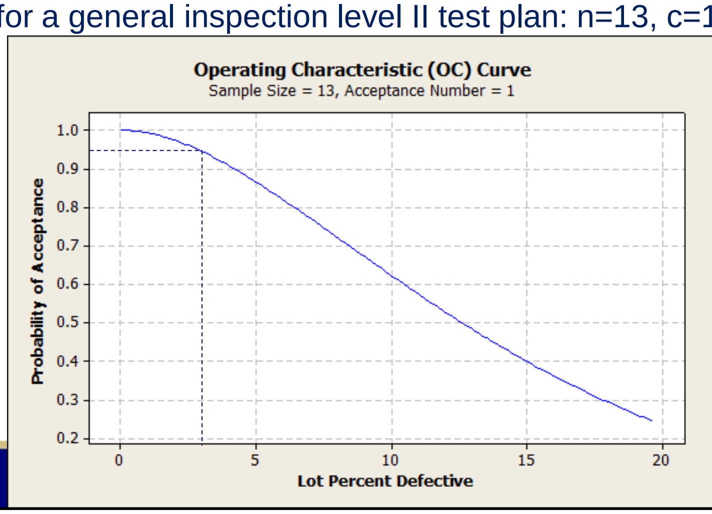

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Fig 10.3 Traditional Selection of Control Charts

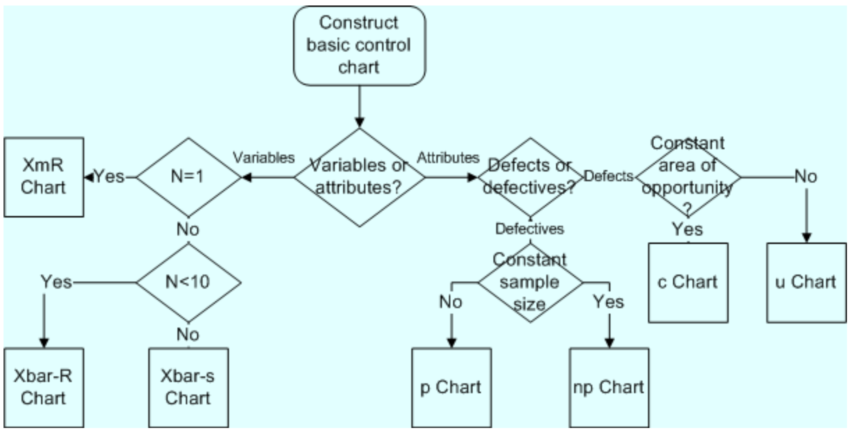


Fig 10.4 Process Control

- A Process is said to be in statistical control when special causes do not exist.

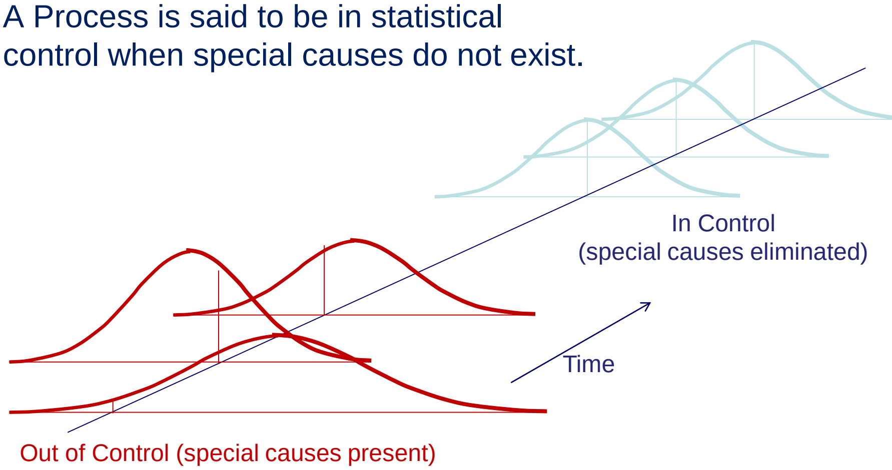
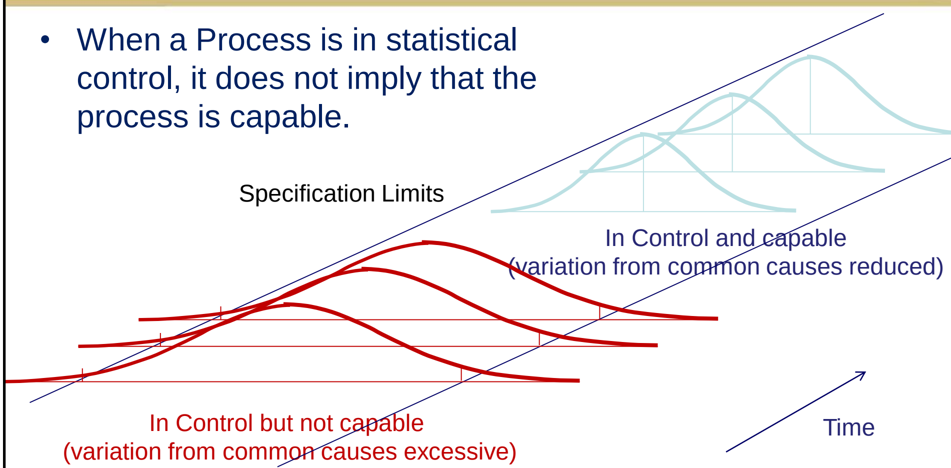


Fig 10.5 Process Capability

- When a Process is in statistical control, it does not imply that the process is capable.



10.6 Monitoring Processes

- A control charting strategy should be created to separate special-causes events from common-cause events.
- Illustration: A widget with one important dimension, new raw material is supplied daily, measurement is quite expensive.
  - Strategy #1: daily sampling of 5 for $\bar{x}$ and R charts

    → out of control frequently
  - Strategy #2: weekly sampling of 5 for $\bar{x}$ and R charts

    → in control, but nonconformance rate unacceptable

    → Quick fix: sort for satisfactory raw material

    → DOE: identify key process input variable (temp.)

    → Control charts for KPIV (temp.)
- S^4/IEE strategy typically use an XmR chart to identify special cause conditions.

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10.7 Rational Sampling and Rational Subgrouping

- Rational sampling involves the best selection of the what, where, how, and when for measurements. Sampling plans should lead to analyses that give insight.
- Traditionally, rational subgrouping involves the selection of samples that yield relatively homogeneous conditions within the subgroup.
  - For $\bar{x}$ and R charts, the within-subgroup variation defines the control limits (thus the sensitivity of the control charts).
  - Different subgrouping methods can dramatically affect the measured variation within subgroups.
  - $\bar{x}$ charts identify differences between the subgroups, while the R charts identify inconsistency within the subgroups.

  → Consider the source of variation, then organize the subgroups

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10.7 Rational Sampling and Rational Subgrouping

- For high-level metrics, infrequent subgrouping/sampling is preferred to reduce firefighting.
- When process capability/performance improvements are needed, S^4/IEE projects are pulled into the system.

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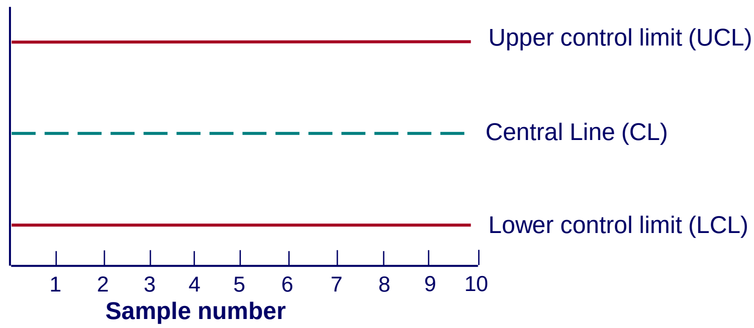
10.8 Statistical Process Control Charts

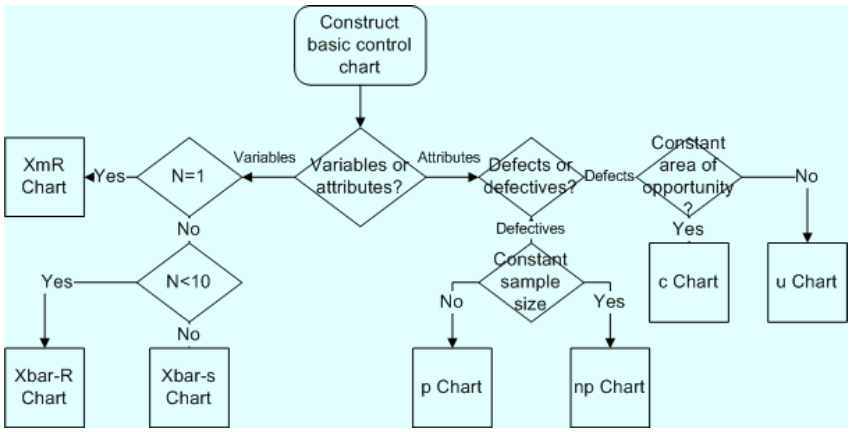
- Shewhart control charts (1931) track processes by plotting data over time in the following form.



10.8 Statistical Process Control Charts

- Control charts can track either variables or attribute process parameters.
- Variable charts: process mean ($\bar{x}$), range (R), standard deviation (s), and individual values (X).
- Attribute charts: proportion nonconforming (p), number of nonconforming items (np), number of nonconformities (c), and nonconformities per unit (u).

Fig 10.3 Traditional Selection of Control Charts



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10.8 Statistical Process Control Charts

- The typical control limits are ±3σ limits, where σ is a function of the sampling plan.
- Typically, 20 data points are needed to initiate a control chart.
- When a point falls outside these limits, the process is said to be out of control (unpredictable). Other control chart patterns are also indications of special causes.
- The process, not the specifications, determines the control limits.
- Variable/continuous data: cycle time, weight, temp., size.
- Rule of thumb: consider data as continuous if at least 10 different values occur and no more than 20% of the data set are repeat values.

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10.9 Interpretation of Control Chart Patterns

- When a process is in control (predictable), the control chart pattern should exhibit “natural characteristics” as if it were from random data.
- Unnatural patterns classified as “mixture” have an absence of points near the center line. (a combination of 2 different patterns on 1 chart, one at high level and one at low level)
- Unnatural patterns classified as “stratification” have very small up and down variation. (samples are taken consistently from a widely different distribution)
- Unnatural patterns classified as “instability” have points outside the control limits. (something has changed within the process)

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10.9 Interpretation of Control Chart Patterns

Sampling errors

- Type I error: process is stated to be out of control/ unpredictable without special cause (when bad sample was drawn)
- Chance of error increases with the introduction of more criteria when analyzing the charts.
- Type II error: process is stated to be in control/predictable with special cause exists (when good sample was drawn)

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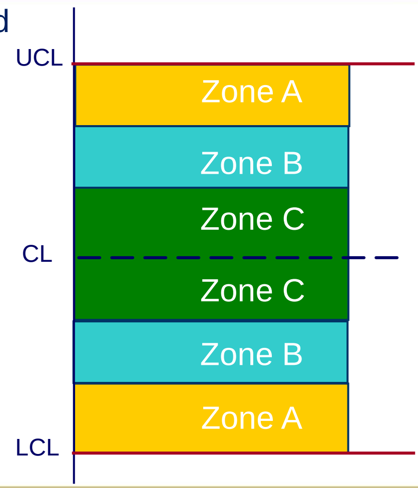
10.9 Interpretation of Control Chart Patterns

- A control chart can be subdivided into three regions. Additional tests for out-of-control conditions could be developed:
  - One point beyond zone A
  - 2 out of 3 points in zone A or beyond
  - 4 out of 5 points in zone B or beyond
  - 8 points in zone C or beyond



10.9 Interpretation of Control Chart Patterns

1. One point beyond zone A
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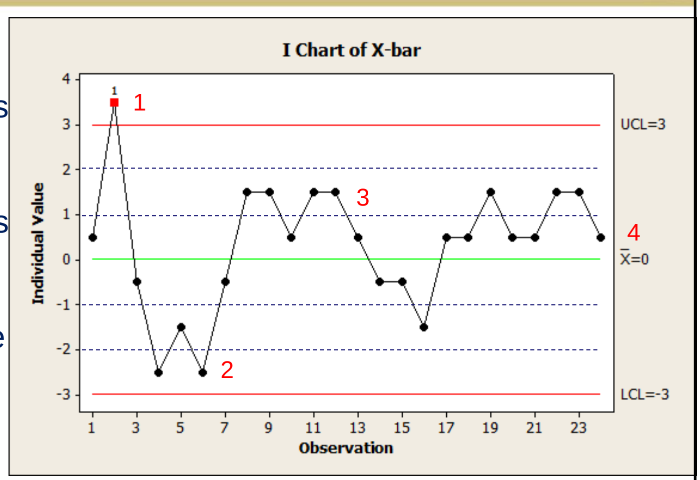
4. 8 points in zone C or beyond



10.9 Interpretation of Control Chart Patterns

- Run tests: A shift has occurred if:
  - At least 10 out of 11 sequential data points are on the same side of the centerline.
  - At least 12 out of 14 sequential data points are on the same side of the centerline.
  - At least 14 out of 17 sequential data points are on the same side of the centerline.
  - At least 16 out of 20 sequential data points are on the same side of the centerline.
- Cost of additional tests: Decreasing average run length(ARL)
- Other patterns within a control chart can tell a story. A cyclic pattern may indicate that samples are being taken from 2 different distributions.

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10.10 $\bar{x} - R$ and $\bar{x} - s$ Charts: Mean and Variability Measurements

- A rational subgrouping of m samples of size n is taken over some period of time. The number of m samples should be at least 20 to 25, where n is often smaller and either 4, 5, or 6.
- For each sample of size n, a mean ($\bar{x}$) and range (R) can be determined.
- For a process variable to be in statistical control, both the mean and range of the process must be in control.
- For a new process, the process mean (μ) is typically not known, it has to be calculated using $\bar{\bar{x}} = \frac{\bar{x}_1+\bar{x}_2+\cdots+\bar{x}_m}{m}$
- The mean range value ($\bar{R}$) is $\bar{R} = \frac{R_1+R_2+\cdots+R_m}{m}$
- For small sample size, a good estimation for the population standard deviation is $\hat{\sigma} = \bar{R}/d_2$

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10.10 $\bar{x} - R$ and $\bar{x} - s$ Charts: Mean and Variability Measurements

- In general, it is better to use the standard deviation from each subgroup when tracking variability. When sample size are of magnitude of 4 to 6, the range is satisfactory and used.
- When the sample size n is moderately large ($n > 10$), the range method for estimating σ loses efficiently. It is best to use $\bar{x} - s$ charts.
- The mean standard deviation value ($\bar{s}$) is $\bar{s} = \frac{s_1 + s_2 + \cdots + s_m}{m}$

  where $s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$
- A good estimation for the population standard deviation is $\hat{\sigma} = \bar{s}/c_4$

10.10 $\bar{x} - R$ and $\bar{x} - s$ Charts: Mean and Variability Measurements

	CL	UCL	LCL
$\bar{x}$ chart	$\bar{\bar{x}}$	$\bar{\bar{x}} + A_2\bar{R}$	$\bar{\bar{x}} - A_2\bar{R}$
		$\bar{\bar{x}} + A_3\bar{s}$	$\bar{\bar{x}} - A_3\bar{s}$
R chart	$\bar{R}$	$D_4\bar{R}$	$D_3\bar{R}$
s chart	$\bar{s}$	$B_4\bar{s}$	$B_3\bar{s}$

- The constants are taken from Table J.
- If successive group values plotted on the s or R charts are in control, control statement can then be made relative to a $\bar{x}$ chart.

10.10 $\bar{x} - R$ and $\bar{x} - s$ Charts: Mean and Variability Measurements

	CL	UCL	LCL
$\bar{x}$ chart	μ	$\mu + A\sigma$	$\mu - A\sigma$
R chart	$d_2\sigma$	$D_2\sigma$	$D_1\sigma$
s chart	$c_4\sigma$	$B_6\sigma$	$B_5\sigma$

- When it is possible to specify the standard values for the process mean (μ) and standard deviation (σ), they can be used to establish the control charts. Care must be exercised when using this approach because the standards may not ne applicable to the process.
- The constants are taken from Table J.

10.11 Example 10.2: $\bar{x} - R$ Chart

- Transactional: Five sequentially paid invoices were randomly selected each hour. The number of days past the invoice due date was tracked using an $\bar{x} - R$ chart.
- Cycle time (manufacturing and transactional): Each hour, five sequential transactions were randomly selected. Cycle time for completing the transactions was tracked using an $\bar{x} - R$ chart.

10.11 Example 10.2: $\bar{x} - R$ Chart

- A grinding machine is to produce treads for a hydraulic system of an aircraft to a diameter of 0.4037±0.0013”.
- Go/no-go thread ring gages are currently used in a 100% test plan to reject non-conforming parts.
- In an attempt to better understand the process variability, variable data were taken for the process.
- Measurements were taken every hour on 5 parts using a visual comparator with accuracy of .0001”.
- The data, sample means, and sample ranges are recorded in Table 10.1. (All data are expressed in units of 0.0001” in excess of 0.4000”.)

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10.11 Example 10.2: $\bar{x} - R$ Chart
Table 10.1

Sample #	Subgroup Measurements					Sample Mean	Sample Range
1	36	35	34	33	32	34.00	4
2	31	31	34	32	30	31.60	4
3	30	30	32	30	32	30.80	2
4	32	33	33	32	35	33.00	3
5	32	34	37	37	35	35.00	5
6	32	32	31	33	33	32.20	2
7	33	33	36	32	31	33.00	5
8	23	33	36	35	36	32.60	13
9	43	36	35	24	31	33.80	19
10	36	35	36	41	41	37.80	6
11	34	38	35	34	38	35.80	4
12	36	38	39	39	40	38.40	4
13	36	40	35	26	33	34.00	14
14	36	35	37	34	33	35.00	4
15	30	37	33	34	35	33.80	7
16	28	31	33	33	33	31.60	5
17	33	30	34	33	35	33.00	5
18	27	28	29	27	30	28.20	3
19	35	36	29	27	32	31.80	9
20	33	35	35	39	36	35.60	6
						33.55	6.20

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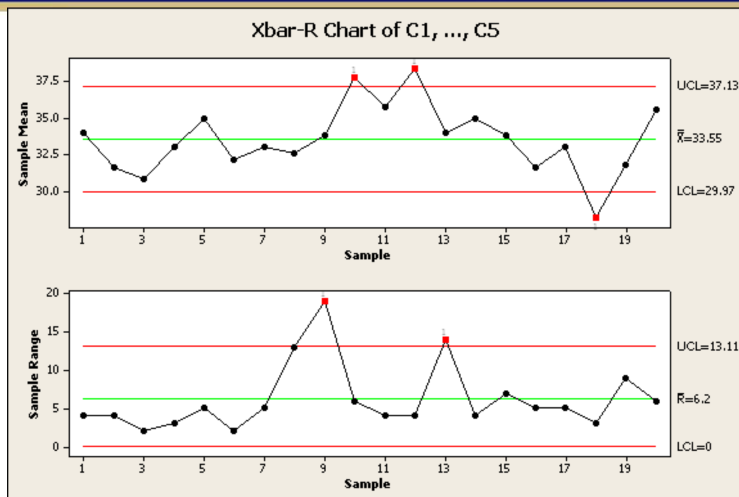
10.11 Example 10.2: $\bar{x} - R$ Chart Fig 10.9



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10.11 Example 10.2: $\bar{x} - R$ Chart

- Both $\bar{x}$ and R charts show lack of control. → Assignable causes (machine setting for $\bar{x}$ chart, operator carelessness for R chart)
- After isolating special causes, these points should be removed to create new control charts with new limits.
- $\bar{x}$ chart indicates that the process mean is shifted from the nominal specification.
- Whenever natural tolerances are found to be consistently within specification limits, consideration should be given to replacing a 100% inspection plan with periodic samples.

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10.12 XmR Charts: Individual Measurements

- For some situations, only a sample size of 1 is achievable.
- A chart of individual values is typically referred to as an I chart or an X chart.
- A moving range chart often accompanies individual charts; hence, the designation $I-MR$ or XmR chart.
- For an individual-measurement control chart, the process average is simply the mean of the m data points,

$$\bar{x} = \frac{\sum_{i=1}^{m} x_i}{m}$$

- Most frequently, adjacent values are used to determine the moving range; larger duration could also be used.

$$MR_1 = |x_2 - x_1|, MR_2 = |x_3 - x_2|, \cdots$$

10.12 XmR Charts: Individual Measurements

- The average moving range ($\overline{MR}$) is

$$\overline{MR} = \frac{\sum_{i=1}^{m-1}(MR_i)}{m-1} = \frac{(MR_1) + (MR_2) + \cdots + (MR_{m-1})}{m-1}$$

- When using 2 adjacent values for MR, charting parameters for the X chart are

$$CL = \bar{x}; UCL = \bar{x} + \frac{3(\overline{MR})}{d_2} = \bar{x} + 2.66(\overline{MR});$$

$$LCL = \bar{x} - \frac{3(\overline{MR})}{d_2} = \bar{x} - 2.66(\overline{MR})$$

- Charting parameters for the MR chart are

$$CL = \overline{MR}; UCL = D_4(\overline{MR}) = 3.267(\overline{MR})$$

10.12 XmR Charts: Individual Measurements

- Some practitioners prefer not to construct MR charts because any information that can be obtained from the MR is contained in the X chart, and the moving ranges are correlated, which can induce patterns of runs or cycles.
- Because of this artificial autocorrelation, the assessment of MR charts should not involve the use of run tests for out-of-control conditions.

10.13 Example 10.3: XmR Charts S^4/IEE Application Examples

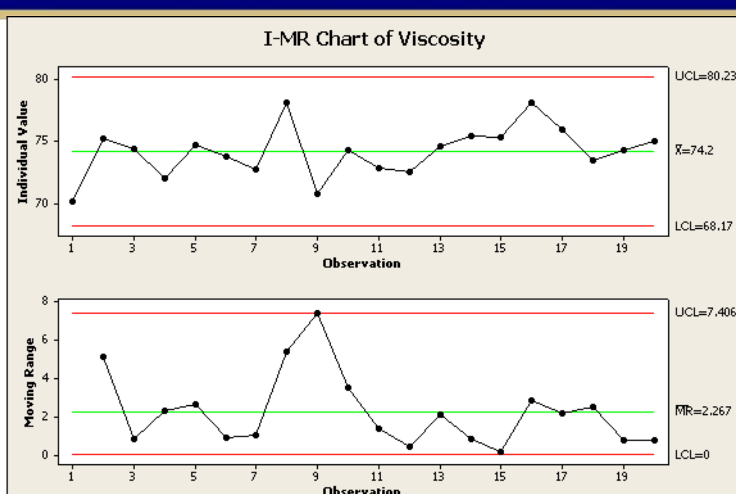
- Transactional: One paid invoice was randomly selected each day. The number of days past the invoice due date was tracked using an XmR chart.
- Cycle time (manufacturing and transactional): One transaction was randomly selected daily. Cycle time for completing the transaction was tracked using an XmR chart.

10.13 Example 10.3: XmR Charts

- The viscosity of a chemical mixing process has the centipoise (cP) measurements for 20 batches.

Batch #	Viscosity	MR
1	70.10	
2	75.20	5.10
3	74.40	0.80
4	72.07	2.33
5	74.70	2.63
6	73.80	0.90
7	72.77	1.03
8	78.17	5.40
9	70.77	7.40
10	74.30	3.53
11	72.90	1.40
12	72.50	0.40
13	74.60	2.10
14	75.43	0.83
15	75.30	0.13
16	78.17	2.87
17	76.00	2.17
18	73.50	2.50
19	74.27	0.77
20	75.05	0.78
	74.20	2.267

Fig 10.10 XmR control charts



10.13 Example 10.3: XmR Charts

- The XmR charts indicate no out-of-control condition for these data.
- If the 20 batch readings are considered as a random sample, a probability plot could be made to determine the expected range of viscosities experienced by the customer.
- Process capability/performance metric assessments could also be made.

10.14 $\bar{x} - R$ v.s. XmR Charts

- Wheeler (1995) favors the XmR chart for most real-time applications involving periodic data collection.
  - Charting individual values to achieve a timely response to any shift in process location.
  - Charting moving average values when it is more important to know about recent trends.
- For an $\bar{x} - R$ chart, short-term variability is estimated from the variability within a subgroup, while in an XmR chart, variability is estimated from the moving range.

10.15 Attribute Control Charts

- For binomial and Poisson distributions, the standard deviation is dependent on the mean of the data.
- For binomial and Poisson distribution based control charts, it is assumed that the underlying probabilities remain fixed over time when a process is in statistical control.
  - For large sample sizes, batch-to-batch variation can be greater than the prediction.
  - The assumption that "the sum of one or more binomial random variables will follow a binomial distribution" is not true if these random variables have different values.
  - → the classical control chart formulas squeeze limits toward the centerline
  - → process out of control most of the time

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10.15 Attribute Control Charts

- The usual remedy for the problem is to plot the attribute failure rates as individual measurements.
  - The failure rate for the time of interest can be very low. → Use XmR charts to track time between failures
  - The batch sample size could be different.→ Use Z chart.
  - Laney (1997) suggests using $Z\&MR$ charts

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10.16 p Chart: Fraction Nonconforming Measurements

- Consider m rational subgroups where each subgroup has n items with x nonconformities or defective units. The fraction nonconforming (p) for a subgroup is $p = \frac{x}{n}$
- The process average nonconforming $\bar{p} = \frac{\sum_{i=1}^{m} p_i}{m}$ where in general m should be at least 20 to 25.
- The chart parameters for this binomial scenario are

$$CL = \bar{p}, \quad UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \; LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- The LCL cannot be less than zero.

10.16 p Chart: Fraction Nonconforming Measurements

Solutions to unequal sample sizes:

- Use the average sample size
- Adjust the control chart limits for each sample.

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{\sum_{i=1}^{m} n_i}$$

where D_i is the number of nonconformances within the ith sample.

The chart parameters for this binomial scenario are

$$CL = \bar{p}, \quad UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}, \; LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

- Perform a Z transformation on the data.

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10.17 Example 10.4: p Chart S⁴/IEE Application Examples

- Transactional workflow metric (could similarly apply to manufacturing; e.g., inventory or time to complete a manufacturing process): The number of days beyond the due date was measured and reported for all invoices. If an invoice was beyond 30 days late it was considered a failure or defective transaction. The number of nonconformances for total transactions per day was plotted using a p chart.
- Transactional quality metric: The number of defective recorded invoices was measured and reported. The number of defective transactions was compared daily to the total number of transactions using a p chart.

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10.17 Example 10.4: p Chart

- A machine manufactures cardboard cans used to package frozen orange juice. Cans are then inspected whether they will leak when filled with orange juice.
- A p chart is initially established by taking 30 samples of 50 cans at half-hour intervals within the manufacturing process.
- An alternative analysis approach is described in Example 10.5.

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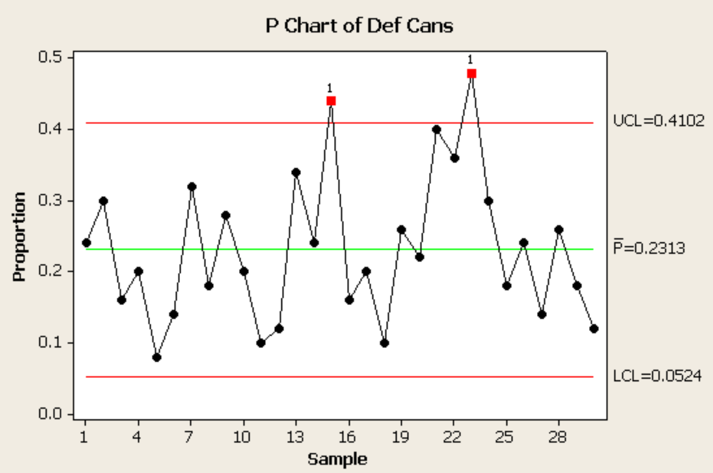
Sample #	No. of Nonconf.	Sample Nonconf. Fraction
1	12	0.24
2	15	0.30
3	8	0.16
4	10	0.20
5	4	0.08
6	7	0.14
7	16	0.32
8	9	0.18
9	14	0.28
10	10	0.20
11	5	0.10
12	6	0.12
13	17	0.34
14	12	0.24
15	22	0.44
16	8	0.16
17	10	0.20
18	5	0.10
19	13	0.26
20	11	0.22
21	20	0.40
22	18	0.36
23	24	0.48
24	15	0.30
25	9	0.18
26	12	0.24
27	7	0.14
28	13	0.26
29	9	0.18
30	6	0.12
		0.231333

Fig 10.11 p Chart Example



10.17 Example 10.4: p Chart

- Two samples are beyond the limits in the p chart. Hence, the process is considered to have out-of-control conditions or is unpredictable.
- If these two points were caused by an adverse condition, the process control limits can be recalculated without the two.
- Whenever out-of-control conditions exist that cannot be explained, these data points should typically not be removed.
- For an in-control process, the magnitude of the average failure rate should be examined for acceptability.
- A reduction in the overall average typically requires a more involved process or design change. Pareto charts and DOE techniques can be a powerful approach.

10.18 np Chart: Number of Nonconforming Items

- An alternative to the p chart when the sample size (n) is constant.
- Instead of the fraction nonconforming (p), the number of nonconforming items is plotted.
- The chart parameters are

$$CL = n\bar{p}, \quad UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \text{ ,}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$\bar{p}$ is determined similar to a p chart.

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10.19 c Chart: Number of Nonconformities S4/IEE Application Example

- Transactional quality metric: The number of daily transactions is constant. The number of defects in filling out invoices was measured and reported, where there can be more than one defect on a transaction. Daily the number of defects on transactions was tracked using a c chart.

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10.19 c Chart: Number of Nonconformities

- In some cases, the number of nonconformities or defects per unit is a more appropriate unit of measure than the fraction nonconforming. The c chart can be used to monitor the processes.
- The Poisson distribution is an appropriate model if the number of opportunities for nonconformities is sufficiently large and the probability of occurrence of a nonconformity at a location is small and constant.
- The chart parameters are

$$CL = \bar{c}, \quad UCL = \bar{c} + 3\sqrt{\bar{c}}\ , \ LCL = \bar{c} - 3\sqrt{\bar{c}}$$

  $\bar{c}$ is the mean occurrences and LCL must be grater or equal to 0.

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10.20 u Chart: Nonconformities per Unit S⁴/IEE Application Example

- Transactional quality metric: The number of daily transactions is not constant. The number of defects in filling out invoices was measured and reported, where there can be more than one defect on a transaction. The number of defects on transactions relative to total transactions was tracked daily using a u chart.

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10.20 u Chart: Nonconformities per Unit

- When the rational subgroup size is not constant, a u chart can be used in lieu of a c chart.
- For a sample size n that has a total number of nonconformities c, $u = c/n$
- The chart parameters are

$$CL = \bar{u},\ \ UCL = \bar{u} + 3\sqrt{\bar{u}/n}\ ,\ LCL = \bar{u} - 3\sqrt{\bar{u}/n}$$

$\bar{u}$ is the mean of the occurrences.

10.21 Median Charts

	CL	UCL	LCL
Median chart		$med + A_2\bar{R}$	$med - A_2\bar{R}$
R chart	$\bar{R}$	$D_4\bar{R}$	$D_3\bar{R}$

- Median charts and $\bar{x} - R$ charts are similar.
- Within a median chart, all points are plotted and the median value is circled. Circled medians are then connected.
- Median charts are considered statistically less sensitive to detecting process instability.

n	2	3	4	5	6	7	8	9
A_2	1.88	1.19	0.80	0.69	0.55	0.51	0.43	0.41

10.22 Example 10.5: Alternatives to p-Chart, np-Chart, c-Chart, and u-Chart Analyses S⁴/IEE Application Examples

- Transactional and manufacturing 30,000-foot-level metric: A company had a large number of transactions completed daily, where the number of daily transactions was similar. The number of defective recorded transactions was measured and reported. It was proposed that daily the number of defective transactions could be compared to the total number of transactions and tracked using a p chart. An XmR chart can be a better alternative for this situation.
- Transactional and manufacturing 30,000-foot-level metric: The number of daily transactions is approximately the same, but not exactly. The number of defects in filling out invoices is large. It was proposed that daily the number of defects on transactions (there can be more than one defect on a transaction) to total transactions could be tracked using a u chart. An XmR chart can be a better alternative for this situation.

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10.22 Example 10.5: Alternatives to p-Chart, np-Chart, c-Chart, and u-Chart Analyses

- Some potential problems with a classical p-chart were presented in 10.15. Various alternatives will be presented in this section.
- The implication of these alternative approaches become more dramatic when n is much larger and varying between samples.
- When creating a p-chart, both the number of opportunities and the number of defects are needed for the calculation of p. An XmR analysis of attribute data needs only response for each sample, such as failure rate, inverse of failure rate, and the total number of failures for each trial.
- Figure 10.12 is an XmR chart with failure rate plotted.

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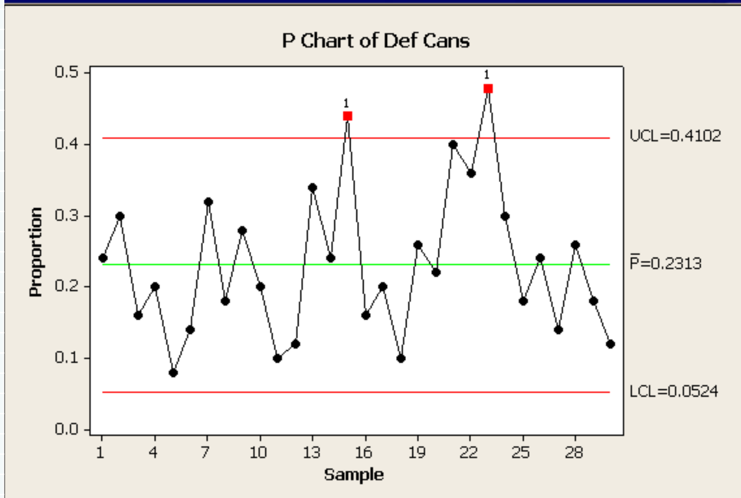
Sample #	No. of Nonconf.	Sample Nonconf. Fraction
1	12	0.24
2	15	0.30
3	8	0.16
4	10	0.20
5	4	0.08
6	7	0.14
7	16	0.32
8	9	0.18
9	14	0.28
10	10	0.20
11	5	0.10
12	6	0.12
13	17	0.34
14	12	0.24
15	22	0.44
16	8	0.16
17	10	0.20
18	5	0.10
19	13	0.26
20	11	0.22
21	20	0.40
22	18	0.36
23	24	0.48
24	15	0.30
25	9	0.18
26	12	0.24
27	7	0.14
28	13	0.26
29	9	0.18
30	6	0.12
		0.231333

Fig 10.11 p-Chart Example



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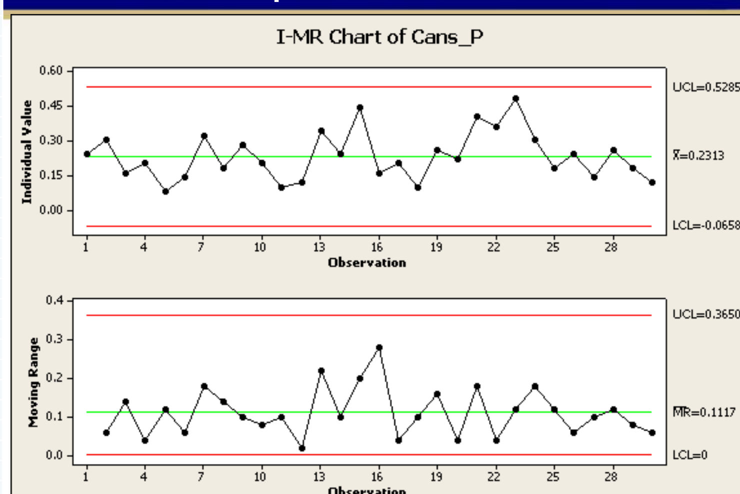
Sample #	Sample Nonconf. Fraction	MR
1	0.24	
2	0.30	0.06
3	0.16	0.14
4	0.20	0.04
5	0.08	0.12
6	0.14	0.06
7	0.32	0.18
8	0.18	0.14
9	0.28	0.10
10	0.20	0.08
11	0.10	0.10
12	0.12	0.02
13	0.34	0.22
14	0.24	0.10
15	0.44	0.20
16	0.16	0.28
17	0.20	0.04
18	0.10	0.10
19	0.26	0.16
20	0.22	0.04
21	0.40	0.18
22	0.36	0.04
23	0.48	0.12
24	0.30	0.18
25	0.18	0.12
26	0.24	0.06
27	0.14	0.10
28	0.26	0.12
29	0.18	0.08
30	0.12	0.06
	0.231333	0.111724

Fig 10.12 p-Chart Alterative Example: XmR Chart



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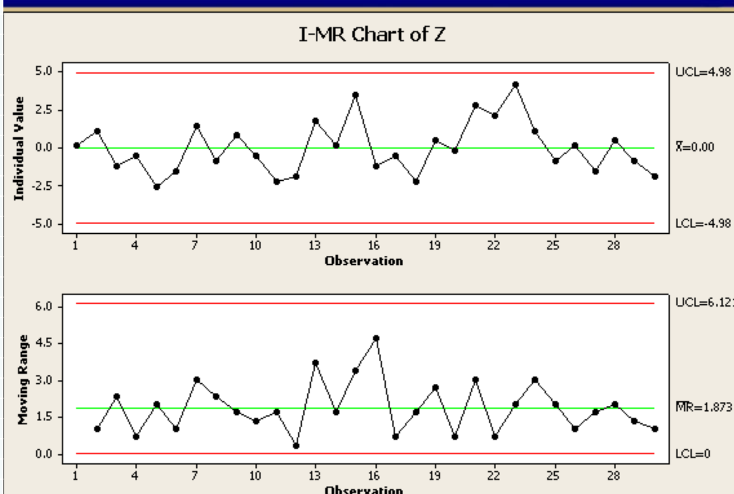
10.22 Example 10.5: Alternatives to p-Chart, np-Chart, c-Chart, and u-Chart Analyses

- The results of the XmR analysis (no out-of-control points) are very different from the p-chart analysis.
- The reason for the differing results is that the XmR analysis considers variability between samples when determining control limits. A p-chart analysis (also for np-chart, c-chart, and u-chart) assumes that dispersion is a function of location and uses theoretical limits.
- A potential issue with the XmR analysis is that there could be differing sample sizes. Laney (1997) suggests using Z&MR chart, where a Z transformation is made of the nonconformance rates.

$$Z_i = \frac{p_i - \bar{p}}{\hat{\sigma}_{p_i}} \quad where \quad \hat{\sigma}_{p_i} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

Fig 10.13 p-Chart Alterative Example: $Z\&MR$ Chart

Sample #	Zi	MR
1	0.1453	
2	1.1514	1.0061
3	-1.1962	2.3476
4	-0.5254	0.6707
5	-2.5376	2.0122
6	-1.5315	1.0061
7	1.4868	3.0183
8	-0.8608	2.3476
9	0.8161	1.6769
10	-0.5254	1.3415
11	-2.2023	1.6769
12	-1.8669	0.3354
13	1.8222	3.6891
14	0.1453	1.6769
15	3.4990	3.3537
16	-1.1962	4.6952
17	-0.5254	0.6707
18	-2.2023	1.6769
19	0.4807	2.6830
20	-0.1900	0.6707
21	2.8283	3.0183
22	2.1576	0.6707
23	4.1698	2.0122
24	1.1514	3.0183
25	-0.8608	2.0122
26	0.1453	1.0061
27	-1.5315	1.6769
28	0.4807	2.0122
29	-0.8608	1.3415
30	-1.8669	1.0061
	0	1.873458



10.23 Charts for Rare Events

- S4/IEE Application Example
  Transactional quality metric: Thirty customers were contacted daily by phone and asked if their shipment was complete. A p- chart frequently bounced off zero and was not very informative. A time between failure recording and tracking using an XmR chart is often more informative.
- Typically, plots for rare events are in the form of c charts.
- A better alternative to the c chart is the XmR chart, which examine the change in failure rate between failure occurrences.

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10.24 Example 10.6: Charts for Rare Events

- A department occasionally experiences a spill. Over the last few years, a spill occurs on the average about once every 7 months.

Date of Occurrence	Time Between Spills	Annual Spill Rate
2/23/90		
1/11/91	322.00 (days)	1.13
9/15/91	247.00	1.48
7/5/92	294.00	1.24
2/17/93	227.00	1.61
9/28/93	223.00	1.64
3/19/94	172.00	2.12
7/12/94	115.00	3.17

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Fig 10.14 c-Chart (Number of Spills each Month)

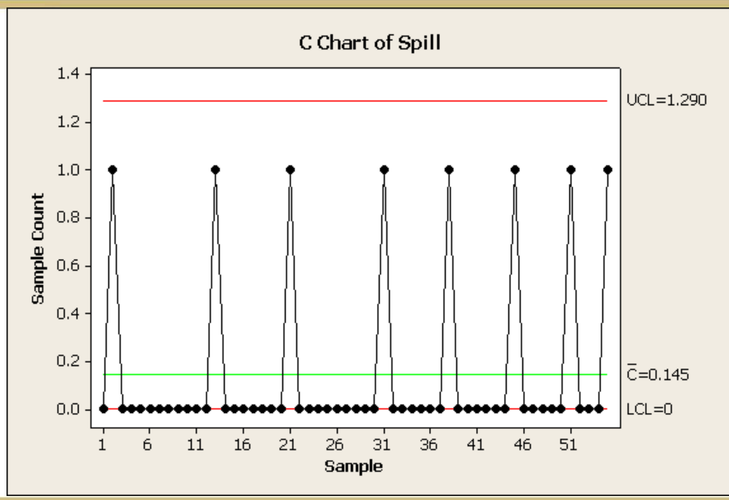

Fig 10.15 XmR-Chart