Exercises for Section 5.2

4. Interpolation methods are used to estimate heights above sea level for locations where direct measurements are unavailable. In the article "Transformation of Ellipsoid Heights to Local Leveling Heights" (M. Yanalak and O. Baykal, *Journal Of Surveying Engineering*, 2001:90-103), a second-order polynomial method of interpolation for estimating heights from GPS measurements is evaluated. In a sample of 74 locations, the errors made by the method averaged 3.8 cm, with a standard deviation of 4.8 cm.
   a. Find a 95% confidence interval for the mean error made by this method.
   b. Find a 98% confidence interval for the mean error made by this method.
   c. A surveyor claims that the mean error is between 3.2 and 4.4 cm. With what level of confidence can this statement be made?
   d. Approximately how many locations must be sampled so that a 95% confidence interval will specify the mean to within ± 0.7 cm?
   e. Approximately how many locations must be sampled so that a 98% confidence interval will specify the mean to within ± 0.7 cm?

Solution:

$$n = 74;\ \bar{X} = 3.8mm; s = 4.8mm$$

a) $\alpha = 5\%$

$$\sigma_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{4.8}{\sqrt{74}} = .5580;$$

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} = 3.8 \pm 1.96(.5580) = (2.7064, 4.8936)$$

b) $\alpha = 2\%$

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} = 3.8 \pm 2.33(.5580) = (2.5019, 5.0981)$$

c) $w = \frac{4.4-3.2}{2} = 0.6$

$w = z_{\alpha/2}\sigma_{\bar{X}}; 0.6 = z_{\alpha/2}(.5580);$

$z_{\alpha/2} = 1.0753;\ \alpha/2 = 0.1411;\ \alpha = 0.2822$

Confidence Level $= 1 - \alpha = 71.78\%$

d) $\alpha = 5\%; w = 0.7$

$$n = \frac{{z_{\alpha/2}}^2\sigma^2}{w^2} = \frac{(1.96)^2(4.8)^2}{(0.7)^2} = 180.63 \sim 181$$

e) $\alpha = 2\%; w = 0.7$

$$n = \frac{{z_{\alpha/2}}^2\sigma^2}{w^2} = \frac{(2.33)^2(4.8)^2}{(0.7)^2} = 254.47 \sim 255$$

6. Resistance measurements were made on a sample of 81 wires of a certain types. The sample mean resistance was 17.3 mΩ, and the standard deviation was 1.2 mΩ.
   a. Find a 95% confidence interval for the mean resistance of this type of wire.
   b. Find a 98% confidence interval for the mean resistance of this type of wire.
   c. What is the level of the confidence interval (17.1, 17.5)?
   d. How many wires must be sampled so that a 98% confidence interval will specify the mean to within ± 0.1 mΩ?
   e. How many wires must be sampled so that a 95% confidence interval will specify the mean to within ± 0.1 mΩ?

Solution:

$$n = 81;\ \bar{X} = 17.3m\Omega; s = 1.2m\Omega$$

a) $\alpha = 5\%$

$$\sigma_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{81}} = .1333;$$

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} = 17.3 \pm 1.96(.1333) = (17.0387, 17.5613)$$

b) $\alpha = 2\%$

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} = 17.3 \pm 2.33(.1333) = (16.9898, 17.6102)$$

c) $w = \frac{17.5-17.1}{2} = 0.2$

$w = z_{\alpha/2}\sigma_{\bar{X}}; 0.2 = z_{\alpha/2}(.1333);$

$z_{\alpha/2} = 1.50;\ \alpha/2 = 0.0668;\ \alpha = 0.1336$

Confidence Level $= 1 - \alpha = 86.64\%$

d) $\alpha = 2\%; w = 0.1$

$$n = \frac{{z_{\alpha/2}}^2\sigma^2}{w^2} = \frac{(2.33)^2(1.2)^2}{(0.1)^2} = 779.31 \sim 780$$

e) $\alpha = 5\%; w = 0.1$

$$n = \frac{{z_{\alpha/2}}^2\sigma^2}{w^2} = \frac{(1.96)^2(1.2)^2}{(0.1)^2} = 553.17 \sim 554$$

10. One step in the manufacture of a certain metal clamp involves the drilling of four holes. In a sample of 150 clamps, the average time needed to complete this step was 72 seconds and the standard deviation was 10 seconds.
   a. Find a 95% confidence interval for the mean time needed to complete the step.
   b. Find a 99.5% confidence interval for the mean time needed to complete the step.
   c. What is the confidence level of the interval (71, 73)?
   d. How many clamps must be sampled so that a 95% confidence interval specifies the mean to within ± 1.5 seconds?
   e. How many clamps must be sampled so that a 99.5% confidence interval specifies the mean to within ± 1.5 seconds?

Solution:

$$n = 150;\ \bar{X} = 72sec.;\ s = 10sec.$$

a) $\alpha = 5\%$

$$\sigma_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{150}} = .8165;$$

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} = 72 \pm 1.96(.8165) = (70.3997, 73.6003)$$

b) $\alpha = 0.5\%$

$$\bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} = 72 \pm 2.81(.8165) = (69.7081, 74.2919)$$

c) $w = \frac{73-71}{2} = 1.0$

$w = z_{\alpha/2}\sigma_{\bar{X}}; 1.0 = z_{\alpha/2}(.8165);$

$z_{\alpha/2} = 1.2247;\ \alpha/2 = 0.1103;\ \alpha = 0.2207$

Confidence Level $=1 - \alpha = 77.93\%$

d) $\alpha = 5\%; w = 1.5$

$$n = \frac{{z_{\alpha/2}}^2\sigma^2}{w^2} = \frac{(1.96)^2(10)^2}{(1.5)^2} = 170.73 \sim 171$$

e) $\alpha = 0.5\%; w = 1.5$

$$n = \frac{{z_{\alpha/2}}^2\sigma^2}{w^2} = \frac{(2.81)^2(10)^2}{(1.5)^2} = 350.20 \sim 351$$

Exercises for Section 5.3

2. During a recent drought, a water utility in a certain town sampled 100 residential water bills and found that 73 of the residences had reduced their water consumption over that of the previous year.
   a. Find a 95% confidence interval for the proportion of residences that reduced their water consumption.
   b. Find a 99% confidence interval for the proportion of residences that reduced their water consumption.
   c. Find the sample size needed for a 95% confidence interval to specify the proportion to within ±0.05.
   d. Find the sample size needed for a 99% confidence interval to specify the proportion to within ±0.05.

Solution:

$$n = 100;\ X = 73$$

Traditional approach: $\hat{p} = \frac{X}{n} = \frac{73}{100} = 0.73$

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = .04440$$

a) $\alpha = 5\%$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.73 \pm 1.96(.04440) = (0.6430, 0.8170)$$

b) $\alpha = 1\%$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.73 \pm 2.58(.04440) = (0.6156, 0.8443)$$

c) $\alpha = 5\%; w = 0.05$

$$n = \frac{{z_{\alpha/2}}^2\hat{p}(1-\hat{p})}{w^2} = \frac{(1.96)^2(.73)(1-.73)}{(0.05)^2} = 302.86 \sim 303$$

d) $\alpha = 1\%; w = 0.05$

$$n = \frac{{z_{\alpha/2}}^2\hat{p}(1-\hat{p})}{w^2} = \frac{(2.58)^2(.73)(1-.73)}{(0.05)^2} = 523.10 \sim 524$$

Modified approach: $\tilde{p} = \frac{X+2}{n+4} = \frac{75}{104} = 0.7212$

$$\sigma_{\tilde{p}} = \sqrt{\frac{\tilde{p}\,(1-\tilde{p})}{\tilde{n}}} = .04397$$

a) $\alpha = 5\%$

$$\tilde{p} \pm z_{\alpha/2}\sigma_{\tilde{p}} = 0.7212 \pm 1.96(.04397) = (0.6350, 0.8073)$$

b) $\alpha = 1\%$

$$\tilde{p} \pm z_{\alpha/2}\sigma_{\tilde{p}} = 0.7212 \pm 2.58(.04397) = (0.6079, 0.8344)$$

c) $\alpha = 5\%; w = 0.05$

$$n = \frac{{z_{\alpha/2}}^2\tilde{p}(1-\tilde{p})}{w^2} = \frac{(1.96)^2(.7212)(1-.72)}{(0.05)^2} = 308.99 \sim 309$$

d) $\alpha = 1\%; w = 0.05$

$$n = \frac{{z_{\alpha/2}}^2\tilde{p}(1-\tilde{p})}{w^2} = \frac{(2.58)^2(.7212)(1-.72)}{(0.05)^2} = 533.69 \sim 534$$

4. A In a random sample of 150 households with an Internet connection, 32 said that they had changed their Internet service provider within the past six months.
   a. Find a 95% confidence interval for the proportion of customers who changed their Internet service provider within the past six months.
   b. Find a 99% confidence interval for the proportion of customers who changed their Internet service provider within the past six months.
   c. Find the sample size needed for a 95% confidence interval to specify the proportion to within ±0.05.
   d. Find the sample size needed for a 99% confidence interval to specify the proportion to within ±0.05.

Solution:

$$n = 150;\ X = 32$$

Traditional approach: $\hat{p} = \frac{X}{n} = \frac{32}{150} = 0.2133$

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = .03345$$

a) $\alpha = 5\%$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.2133 \pm 1.96(.03345) = (0.1478, 0.2789)$$

b) $\alpha = 1\%$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.2133 \pm 2.58(.03345) = (0.1272, 0.2995)$$

c) $\alpha = 5\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\hat{p}(1-\hat{p})}{w^2} = \frac{(1.96)^2(.21)(1-.21)}{(0.05)^2} = 257.87 \sim 258$$

d) $\alpha = 1\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\hat{p}(1-\hat{p})}{w^2} = \frac{(2.58)^2(.21)(1-.21)}{(0.05)^2} = 445.39 \sim 446$$

Modified approach: $\tilde{p} = \frac{X+2}{n+4} = \frac{34}{154} = 0.2208$

$$\sigma_{\tilde{p}} = \sqrt{\frac{\tilde{p}\,(1-\tilde{p})}{\tilde{n}}} = .03342$$

a) $\alpha = 5\%$

$$\tilde{p} \pm z_{\alpha/2}\sigma_{\tilde{p}} = 0.2208 \pm 1.96(.03342) = (0.1553, 0.2863)$$

b) $\alpha = 1\%$

$$\tilde{p} \pm z_{\alpha/2}\sigma_{\tilde{p}} = 0.2208 \pm 2.58(.03342) = (0.1347, 0.3069)$$

c) $\alpha = 5\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\tilde{p}(1-\tilde{p})}{w^2} = \frac{(1.96)^2(.22)(1-.22)}{(0.05)^2} = 264.34 \sim 265$$

d) $\alpha = 1\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\tilde{p}(1-\tilde{p})}{w^2} = \frac{(2.58)^2(.22)(1-.22)}{(0.05)^2} = 456.58 \sim 457$$

10. A voltmeter is used to record 100 independent measurements of a known standard voltage. Of the 100 measurements, 85 are within 0.01 V of the true voltage.
    a. Find a 95% confidence interval for the probability that a measurement is within 0.01 V of the true voltage.
    b. Find a 98% confidence interval for the probability that a measurement is within 0.01 V of the true voltage.
    c. Find the sample size needed for a 95% confidence interval to specify the probability to within ±0.05.
    d. Find the sample size needed for a 98% confidence interval to specify the probability to within ±0.05.

Solution:

$$n = 100;\ X = 85$$

Traditional approach: $\hat{p} = \frac{X}{n} = \frac{85}{100} = 0.85$

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = .03571$$

a) $\alpha = 5\%$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.85 \pm 1.96(.03571) = (0.7800, 0.9200)$$

b) $\alpha = 2\%$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.85 \pm 2.33(.03571) = (0.7669, 0.9331)$$

c) $\alpha = 5\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\hat{p}(1-\hat{p})}{w^2} = \frac{(1.96)^2(.85)(1-.85)}{(0.05)^2} = 195.91 \sim 196$$

d) $\alpha = 2\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\hat{p}(1-\hat{p})}{w^2} = \frac{(2.33)^2(.85)(1-.85)}{(0.05)^2} = 276.01 \sim 277$$

Modified approach: $\tilde{p} = \frac{X+2}{n+4} = \frac{87}{104} = 0.8365$

$$\sigma_{\tilde{p}} = \sqrt{\frac{\tilde{p}\,(1-\tilde{p})}{\tilde{n}}} = .03626$$

a) $\alpha = 5\%$

$$\tilde{p} \pm z_{\alpha/2}\sigma_{\tilde{p}} = 0.8365 \pm 1.96(.03626) = (0.7655, 0.9076)$$

b) $\alpha = 2\%$

$$\tilde{p} \pm z_{\alpha/2}\sigma_{\tilde{p}} = 0.8365 \pm 2.33(.03626) = (0.7522, 0.9209)$$

c) $\alpha = 5\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\tilde{p}(1-\tilde{p})}{w^2} = \frac{(1.96)^2(.8365)(1-.84)}{(0.05)^2} = 210.12 \sim 211$$

d) $\alpha = 2\%; w = 0.05$

$$n = \frac{z_{\alpha/2}{}^2\tilde{p}(1-\tilde{p})}{w^2} = \frac{(2.33)^2(.8365)(1-.84)}{(0.05)^2} = 296.01 \sim 297$$

Exercises for Section 5.4

4. Five measurements are taken of the octane rating for a particular type of gasoline. The results (in %) are 87.0, 86.0, 86.5, 88.0, 85.3. Find a 99% confidence interval for the mean octane rating for this type of gasoline.

Solution:

$$n = 5;\ \bar{X} = 86.56\%; s = 1.0212\%$$

$\alpha = 1\%$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{1.0212}{\sqrt{5}} = .4567$$

$$\bar{X} \pm t_{4,\alpha/2}s_{\bar{X}} = 86.56 \pm 4.6041(.4567) = (84.4572, 88.6628)$$

6. A chemist made eight independent measurements of the melting point of tungsten. She obtained a sample mean of 3410.14°C and a sample standard deviation of 1.018°C.
   a) Use the Student's t distribution to find a 95% confidence interval for the melting point of tungsten.
   b) Use the Student's t distribution to find a 98% confidence interval for the melting point of tungsten.
   c) If the eight measurements had been 3409.76, 3409.80, 3412.66, 3409.79, 3409.76, 3409.77, 3409.80, 3409.78, would the confidence intervals above be valid? Explain.

Solution:

$$n = 8;\ \bar{X} = 3410.14\,℃; s = 1.018\,℃$$

a) $\alpha = 5\%$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{1.018}{\sqrt{8}} = .3599$$

$$\bar{X} \pm t_{7,\alpha/2} s_{\bar{X}} = 3410.14 \pm 2.3646(.3599) = (3409.29, 3410.99)$$

b) $\alpha = 2\%$

$$\bar{X} \pm t_{7,\alpha/2} s_{\bar{X}} = 3410.14 \pm 2.9980(.3599) = (3409.06, 3411.22)$$

c) No, the data set contains an outlier.

12. Surfactants are chemical agents, such as detergents, that lower the surface tension of a liquid. Surfactants play an important role in the cleaning of contaminated soils. In an experiment to determine the effectiveness of a certain method for removing toluene from sand, the sand was washed with a surfactant and then rinsed with deionized water. Of interest was the amount of toluene that came out in the rinse. In five such experiments, the amounts of toluene removed in the rinse cycle, expressed as a percentage of the total amount originally present, were 5.0, 4.8, 9.0, 10.0, and 7.3. Find a 95% confidence interval for the percentage of toluene removed in the rinse. (This exercise is based on the article "Laboratory Evaluation of the Use of Surfactants for Ground Water Remediation and the Potential for Recycling Them," D. Lee, R. Cody, and B. Hoyle, *Ground Water Monitoring and Remediation*, 2001:49-57.)

Solution:

$$n = 5;\ \bar{X} = 7.22; s = 2.3285$$

$\alpha = 5\%$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{2.3285}{\sqrt{5}} = 1.0413$$

$$\bar{X} \pm t_{4,\alpha/2} s_{\bar{X}} = 7.22 \pm 2.7764(1.0413) = (4.3288, 10.1112)$$

Exercises for Section 5.5

2. In a sample of 20 bolts, the average breaking torque was 89.7 J with a standard deviation of 8.2 J. Assume that the breaking torques are normally distributed.
   a) Find a 99% prediction interval for the breaking torque of a single bolt.
   b) Find a tolerance interval for the breaking torque that includes 95% of the bolts with 99% confidence.

Solution:

$$n = 20;\ \bar{X} = 89.7J; s = 8.2J$$

a) $\alpha = 1\%$

$$\bar{X} \pm t_{n-1,\alpha/2} s \sqrt{1 + \frac{1}{n}}$$
$$= 89.7 \pm (2.86)(8.2)(1.0247) = (65.5510, 113.7390)$$

b) $\bar{X} \pm k_{n,\alpha,\gamma} s = 89.7 \pm 3.1681(8.2) = (63.7216,115.6784)$

4. Six measurements were made of the concentration (in percent) of ash in a certain variety of spinach. The sample mean was 19.35, and the sample standard deviation was 0.577. Assume that the concentrations are normally distributed.
   a. Find a 90% prediction interval for a single measurement.
   b. Find a tolerance interval for the pH that includes 99% of the measurements with 95% confidence.

Solution:

$$n = 6;\ \bar{X} = 19.35\%; s = 0.577\%$$

a) $\alpha = 10\%$

$$\bar{X} \pm t_{n-1,\alpha/2} s \sqrt{1 + \frac{1}{n}}$$
$$= 19.35 \pm (2.0151)(.577)(1.0801) = (18.0942, 20.6058)$$

b) $\bar{X} \pm k_{n,\alpha,\gamma} s = 19.35 \pm 5.7746(.577) = (16.0180, 22.6819)$