

Exercises for Section 4.1

4. Ten percent of the items in a large lot are defective. A sample of six items is drawn from this lot.
- Find the probability that none of the sampled items are defective.
 - Find the probability that one or more of the sampled items is defective.
 - Find the probability that exactly one of the sampled items is defective.
 - Find the probability that fewer than two of the sampled items are defective.

Solution:

Success = item is defective

$p = 0.10$; $n = 6$; $X = \text{number of defective in sample}$

$$\begin{aligned} \text{a) } P(X = 0) &= \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\ &= (0.10)^0 (0.90)^6 = 0.531441 \end{aligned}$$

$$\begin{aligned} \text{b) } P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - 0.531441 = 0.468559 \end{aligned}$$

$$\begin{aligned} \text{c) } P(X = 1) &= \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\ &= 6(0.10)^1 (0.90)^5 = 0.354294 \end{aligned}$$

$$\begin{aligned} \text{d) } P(X < 2) &= P(X = 0) + P(X = 1) \\ &= 0.531441 + .354294 = 0.885735 \end{aligned}$$

10. A distributor receives a large shipment of components. The distributor would like to accept the shipment if 10% or fewer of the components are defective and to return it if more than 10% of the components are defective. She decides to sample 10 components and to return the shipment if more than 1 of the 10 is defective. If the proportion of defectives in the batch is in fact 10%, what is the probability that she will return the shipment?

Solution:

Success = item is defective

$p = 0.10$; $n = 10$; $X = \text{number of defective in sample}$

$$\begin{aligned} P(X > 1) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - .348678 - .387420 = 0.263901 \end{aligned}$$

14. An insurance company offers a discount to homeowners who install smoke detectors in their homes. A company representative claims that 80% or more of policyholders have smoke detectors. You draw a random sample of eight policyholders. Let X be the number of policyholders in the sample who have smoke detectors.
- If exactly 80% of the policyholders have smoke detectors (so the representatives claim is true, but just barely), what is $P(X \leq 1)$?
 - Based on the answer to part (a), if 80% of the policyholders have smoke detectors, would one

policyholder with a smoke detector in a sample of size 8 be an unusually small number?

- If you found that exactly one of the eight sample policyholders had a smoke detector, would this be convincing evidence that the claim is false? Explain.
- If exactly 80% of the policyholders have smoke detectors, what is $P(X \leq 6)$?
- Based on the answer to part (d), if 80% of the policyholders have smoke detectors, would six policyholders with smoke detectors in a sample of size 8 be an unusually small number?
- If you found that exactly six of the eight sample policyholders had smoke detectors, would this be convincing evidence that the claim is false? Explain.

Solution:

Success = having smoke detector

$p = 0.80$; $n = 8$;

$X = \# \text{ of policyholder in sample have smoke detect.}$

- $P(X \leq 1) = P(X = 0) + P(X = 1)$
 $= .00000256 + .00008192 = 0.00008448$
- Yes
- Yes, the probability is 0.00008192.
- $P(X \leq 6) = 1 - P(X = 7) - P(X = 8)$
 $= 1 - .335544 - .167772 = 0.496684$
- No, $P(X = 6) = 0.293601$
- No, $P(X = 6) = 0.293601$

Exercises for Section 4.2

6. One out of every 5000 individuals in a population carries a certain defective gene. A random sample of 1000 individuals is studied.
- What is the probability that exactly one of the sample individuals carries the gene?
 - What is the probability that none of the sample individuals carries the gene?
 - What is the probability that more than two of the sample individuals carry the gene?
 - What is the mean of the number of sample individuals that carry the gene?
 - What is the standard deviation of the number of sample individuals that carry the gene?

Solution:

Success = individual carrying the defective gene

$p = 1/5000$; $n = 1000$; $np = \lambda = 1/5 = 0.2$

$X = \# \text{ of individual in sample have the def. gene}$

- $P(X = 1) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.2} 0.2^1}{1!} = 0.163746$
- $P(X = 0) = e^{-\lambda} = 0.818731$
- $P(X > 2) = 1 - p(0) - p(1) - p(2)$
 $= 1 - .818731 - .163746 - .016375 = 0.001148$

- d) $\mu_X = \lambda = 0.20$
 e) $\sigma_X^2 = \lambda = 0.20$
 $\sigma_X = \sqrt{0.20} = 0.447214$

10. You have received a radioactive mass that is claimed to have a mean decay rate of at least 1 particle per second. If the mean decay rate is less than 1 per second, you may return the product for a refund. Let X be the number of decay events counted in 10 seconds.

- If the mean decay rate is exactly 1 per second (so that the claim is true, but just barely), what is $P(X \leq 1)$?
- Based on the answer to part (a), if the mean decay rate is 1 particle per second, would one event in 10 seconds be an unusually small number?
- If you counted one decay event in 10 seconds, would this be convincing evidence that the product should be returned? Explain.
- If the mean decay rate is exactly 1 per second, what is $P(X \leq 8)$?
- Based on the answer to part (d), if the mean decay rate is 1 particle per second, would eight events in 10 seconds be an unusually small number?
- If you counted eight decay events in 10 seconds, would this be convincing evidence that the product should be returned? Explain.

Solution:

Success = decay or releasing a particle

Mean = $\lambda = 10$

X = # of particles released in 10 sec.

$$\begin{aligned} \text{a) } P(X \leq 1) &= P(X = 0) + P(X = 1) \\ &= \frac{e^{-10} 10^0}{1!} + \frac{e^{-10} 10^1}{2!} = 0.000499399 \end{aligned}$$

- Yes
- Yes, probability is 0.000453999
- $P(X \leq 8) = p(0) + p(1) + p(2) + p(3) + p(4) + p(5) + p(6) + p(7) + p(8) = 0.33282$
- No
- No, probability is 0.112599

Exercises for Section 4.3

- If $X \sim N(3, 4)$, compute
 - $P(X \geq 3)$
 - $P(1 \leq X < 8)$
 - $P(-1.5 \leq X < 1)$
 - $P(-2 \leq X - 3 < 4)$

Solution:

$$\mu = 3; \sigma = \sqrt{4} = 2$$

- $P(X \geq 3) = P(z \geq 0) = 0.5000$
- $P(1 \leq X < 8) = P(-1 \leq z \leq 2.5)$
 $= P(z \leq 2.5) - P(z \leq -1)$
 $= 0.993790 - 0.158655 = 0.835135$
- $P(-1.5 \leq X < 1) = P(-2.25 \leq z < -1)$
 $= P(z \leq -1) - P(z \leq -2.25)$
 $= 0.158655 - 0.012224 = 0.146431$
- $P(-2 \leq X - 3 < 4) = P(1 \leq X < 7)$
 $= P(-1 \leq z < 2)$
 $= P(z \leq 2) - P(z \leq -1)$
 $= 0.977250 - 0.158655 = 0.818595$

- At a certain university, math SAT scores for the entering freshman class averaged 650 and had a standard deviation of 100. The maximum possible score is 800. Is it possible that the scores of these freshmen are normally distributed? Explain.

Solution:

No, the score was truncated at 800.

- The temperature recorded by a certain thermometer when placed in boiling water (true temperature 100°C) is normally distributed with mean $\mu = 99.8^\circ\text{C}$ and standard deviation 0.1°C .
 - What is the probability that the thermometer reading is greater than 100°C ?
 - What is the probability that the thermometer reading is within $\pm 0.05^\circ\text{C}$ of the true temperature?

Solution:

$$\mu = 99.8; \sigma = 0.1$$

- $P(T \geq 100) = P(z \geq 2) = 1 - P(T < 2)$
 $= 1 - 0.977250 = 0.02275$
- $P(99.95 \leq T \leq 100.05) = P(1.5 \leq z \leq 2.5)$
 $= P(z \leq 2.5) - P(T < 1.5)$
 $= 0.993790 - 0.933193 = 0.060598$

Exercises for Section 4.4

- The article "Stochastic Estimates of Exposure and Cancer Risk from Carbon Tetrachloride Released to the Air from the Rocky Flats Plant" (A. Rood, P. McGavran, et al., *Risk Analysis*, 2001:675-695) models the increase in the risk of cancer due to exposure to carbon tetrachloride as lognormal with $\mu = -15.65$ and $\sigma = 0.79$.
 - Find the mean risk.
 - Find the median risk.
 - Find the standard deviation of the risk.
 - Find the 5th percentile.
 - Find the 95th percentile.

Solution:

$$\mu = -15.65; \sigma = 0.79$$

$$\text{Let } Y = \text{risk}; X = \ln Y \sim N(\mu, \sigma^2)$$

- a) $E(Y) = \exp\left(\mu + \frac{\sigma^2}{2}\right) = \exp\left(-15.65 + \frac{0.79^2}{2}\right)$
 $= \exp(-15.338) = 2.18179 \times 10^{-7}$
- b) At median, $\ln Y = \mu = -15.65$
 $Y = \exp(-15.65) = 1.59695 \times 10^{-7}$
- c) $V(Y) = \exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2)$
 $= \exp[2(-15.65) + 2(0.79^2)]$
 $\quad - \exp[2(-15.65) + (0.79^2)]$
 $= \exp(-30.0518) - \exp(-30.6759)$
 $= 8.88524 \times 10^{-14} - 4.76021 \times 10^{-14}$
 $= 4.12503 \times 10^{-14}$
 $\sigma_Y = \sqrt{V(Y)} = 2.03102 \times 10^{-7}$
- d) $P(z < -1.645) = 0.05$
 $P(X < -16.9494) = 0.05$
 $P(Y < 4.35466 \times 10^{-8}) = 0.05$
- e) $P(z < 1.645) = 0.95$
 $P(X < -14.3505) = 0.95$
 $P(Y < 5.85637 \times 10^{-7}) = 0.95$

6. A manufacturer claims that the tensile strength of a certain composite (in MPa) has the lognormal distribution with $\mu = 5$ and $\sigma = 0.5$. Let X be the strength of a randomly sampled specimen of this composite.
- If the claim is true, what is $P(X < 20)$?
 - Based on the answer to part (a), if the claim is true, would a strength of 20 MPa be unusually small?
 - If you observed a tensile strength of 20 MPa, would this be convincing evidence that the claim is false? Explain.
 - If the claim is true, what is $P(X < 130)$?
 - Based on the answer to part (d), if the claim is true, would a strength of 130 MPa be unusually small?
 - If you observed a tensile strength of 130 MPa, would this be convincing evidence that the claim is false? Explain.

Solution:

$$\mu = 5; \sigma = 0.5$$

Let $X =$ tensile strength; $\ln X \sim N(\mu, \sigma^2)$

- a) $P(X < 20) = P(\ln X < \ln 20)$
 $= P(\ln X < 2.99573) = P(z < -4.00854)$
 $= 3.05482 \times 10^{-5}$
- b) Yes
- c) Yes, the probability is too small.
- d) $P(X < 130) = P(\ln X < \ln 130)$
 $= P(\ln X < 4.86753) = P(z < -0.24931)$
 $= 0.395531$
- e) No
- f) No, the probability is greater than 5%.

Exercises for Section 4.5

4. The distance between flaws on a long cable is exponentially distributed with mean 12 m.
- Find the probability that the distance between two flaws is greater than 15 m.
 - Find the probability that the distance between two flaws is between 8 and 20 m.
 - Find the median distance
 - Find the standard deviation of the distances.
 - Find the 65th percentile of the distances.

Solution:

$$\text{Mean} = \frac{1}{\lambda} = 12; \lambda = \frac{1}{12}$$

Let $X =$ Distance between flaws; $X \sim \text{Exp}(\lambda)$

- a) $P(X > 15) = 1 - P(X < 15)$
 $= 1 - [1 - e^{-\lambda x}] = e^{-\frac{15}{12}} = 0.286505$
- b) $P(8 < X < 20) = P(X < 20) - P(X < 8)$
 $= \left[1 - e^{-\frac{20}{12}}\right] - \left[1 - e^{-\frac{8}{12}}\right] = e^{-\frac{8}{12}} - e^{-\frac{20}{12}}$
 $= 0.324542$
- c) $P(X < \text{Med.}) = 0.50$
 $\left[1 - e^{-\frac{\text{Med.}}{12}}\right] = 0.50$
 $e^{-\frac{\text{Med.}}{12}} = 0.50$
 $-\frac{\text{Med.}}{12} = \ln 0.50$
 $\text{Med.} = 8.31776$
- d) $\lambda = \frac{1}{12}; \sigma_X = \frac{1}{\lambda} = 12$
- e) $P(X < x) = 0.65$
 $\left[1 - e^{-\frac{x}{12}}\right] = 0.65$
 $e^{-\frac{x}{12}} = 0.35$
 $-\frac{x}{12} = \ln 0.35$
 $x = 12.5979$

6. A radioactive mass emits particles according to a Poisson process at a mean rate of 2 per second. Let T be the waiting time, in seconds, between emissions.
- What is the mean waiting time?
 - What is the median waiting time?
 - Find $P(T > 2)$.
 - Find $P(T < 0.1)$.
 - Find $P(0.3 < T < 1.5)$.
 - If 3 seconds have elapsed with no emission, what is the probability that there will be an emission within the next second?

Solution:

$$\text{Rate} = \lambda = 2$$

Let $T =$ Time between emissions; $T \sim \text{Exp}(\lambda)$

- a) $\text{Mean} = \frac{1}{\lambda} = 0.5 \text{ sec.}$
- b) $P(T < \text{Med.}) = 0.50$
 $\left[1 - e^{-(\lambda)\text{Med.}}\right] = 0.50$

$$-(2)Med = \ln 0.50$$

$$Med = 0.346574$$

- c) $P(T > 2) = 1 - P(T < 2)$
 $= 1 - [1 - e^{-\lambda t}] = e^{-2(2)} = 0.0183156$
- d) $P(T < 0.1) = [1 - e^{-\lambda t}] = 1 - e^{-2(0.1)}$
 $= 0.181269$
- e) $P(0.3 < T < 1.5) = P(T < 1.5) - P(T < 0.3)$
 $= [1 - e^{-2(1.5)}] - [1 - e^{-2(0.3)}]$
 $= e^{-0.6} - e^{-3.0}$
 $= 0.499025$
- f) From the lack of memory property
 $P(T < 4 | P > 3) = P(T < 1) = 1 - e^{-2(1)}$
 $= 0.864665$

Exercises for Section 4.6

6. If T is a continuous random variable that is always positive (such as a waiting time), with probability density function $f(t)$ and cumulative distribution function $F(t)$, then the hazard function is defined to be the function

$$h(t) = \frac{f(t)}{1 - F(t)}$$

The hazard function is the rate of failure per unit time, expressed as a proportion of the items that have not failed.

- a. If $T \sim Weibull(\alpha, \beta)$, find $h(t)$.
- b. For what values of α is the hazard rate increasing with time? For what values of α is it decreasing?

Solution:

a) $f(t) = \alpha\beta^\alpha t^{\alpha-1} e^{-(\beta t)^\alpha}$
 $F(t) = 1 - e^{-(\beta t)^\alpha}$
 $h(t) = \frac{\alpha\beta^\alpha t^{\alpha-1} e^{-(\beta t)^\alpha}}{1 - [1 - e^{-(\beta t)^\alpha}]} = \frac{\alpha\beta^\alpha t e^{-(\beta t)^\alpha}}{e^{-(\beta t)^\alpha}}$
 $= \alpha\beta^\alpha t^{\alpha-1}$

b) $h'(t) = \frac{\partial h(t)}{\partial t} = (\alpha - 1)\alpha\beta^\alpha t^{\alpha-2}$
 $h(t)$ increasing when $h'(t) > 0$, or $\alpha > 1$;
 $h(t)$ decreasing when $h'(t) < 0$, or $\alpha < 1$.

8. The lifetime of a certain battery is modeled with the Weibull distribution with $\alpha = 2$ and $\beta = 0.1$.
- a. What proportion of batteries will last longer than 10 hours?
- b. What proportion of batteries will last less than 5 hours?
- c. What proportion of batteries will last longer than 20 hours?
- d. The hazard function is defined in Exercise 6. What is the hazard at $t = 10$ hours?

Solution:

Let $T =$ Lifetime of battery; $T \sim Weibull(2, 0.1)$

- a) $P(T > 10) = 1 - P(T < 10)$
 $= 1 - [1 - e^{-(\beta t)^\alpha}] = e^{-(\beta t)^\alpha} = e^{-[0.1(10)]^2}$
 $= e^{-1} = 0.367879$
- b) $P(T > 5) = 1 - e^{-(\beta t)^\alpha} = 1 - e^{-[0.1(5)]^2}$
 $= 1 - e^{-0.25} = 0.221199$
- c) $P(T > 20) = 1 - P(T < 20)$
 $= 1 - [1 - e^{-(\beta t)^\alpha}] = e^{-(\beta t)^\alpha} = e^{-[0.1(20)]^2}$
 $= e^{-4} = 0.0183156$
- d) $h(t) = \alpha\beta^\alpha t^{\alpha-1} = 2(0.1)^2 10^{2-1} = 0.2000$

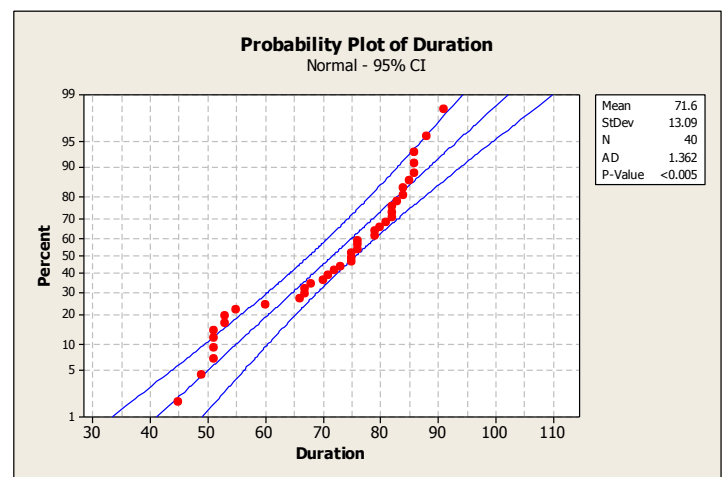
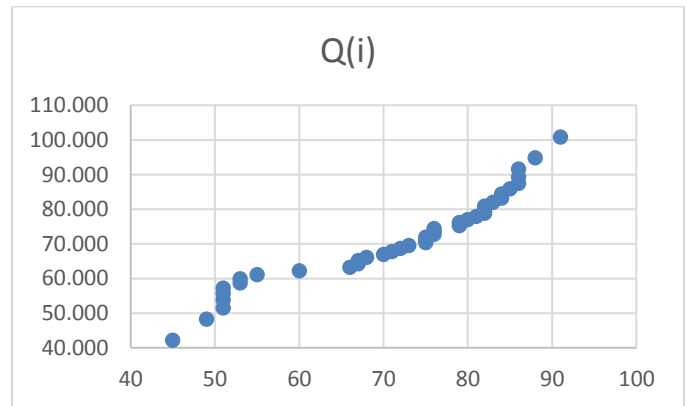
Exercises for Section 4.7

4. Below are the durations (in minutes) of 40 time interval between eruptions of the geyser Old Faithful in Yellowstone National Park.

91 51 79 53 82 51 76 82 84 53
 86 51 85 45 88 51 80 49 82 75
 73 67 68 86 72 75 75 66 84 70
 79 60 86 71 67 81 76 83 76 55

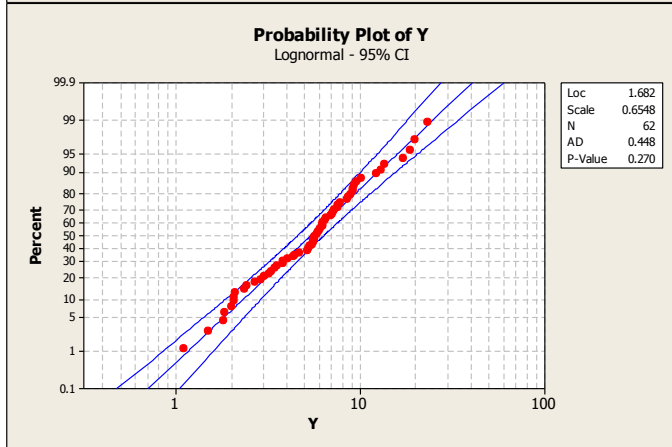
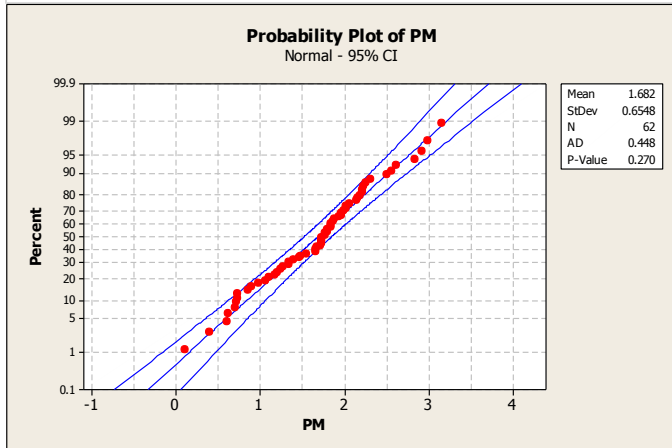
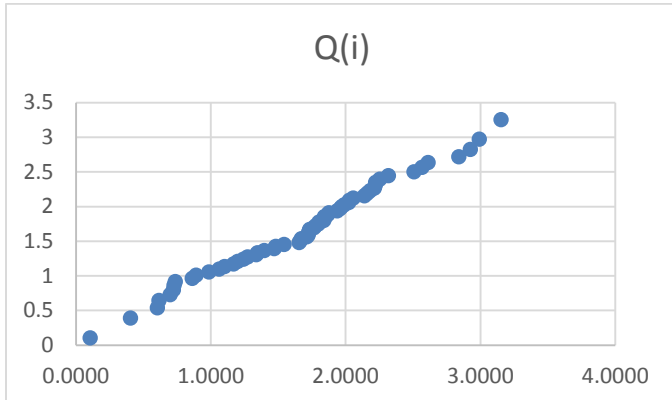
Construct a normal probability plot for these data. Do they appear to come from an approximately normal distribution?

Solution: No



6. Construct a normal probability plot for the logs of the PM data in Table 1.2. Do the logs of the PM data appear to come from a normal population?

Solution: Yes



Exercises for Section 4.8

2. Among all the income tax forms filed in a certain year, the mean tax paid was \$2000, and the standard deviation was \$500. In addition, for 10% of the forms, the tax paid was greater than \$3000. A random sample of 625 tax forms is drawn.
- What is the probability that the average tax paid on the sample forms is greater than \$1980?
 - What is the probability that more than 60 of the sampled forms have a tax of greater than \$3000?

Solution:

- Let X = tax paid

$$\mu = 2000; \sigma = 500$$

$$\text{For sample mean: } \mu_{\bar{X}} = 2000; \sigma_{\bar{X}} = \frac{500}{\sqrt{625}} = 20$$

$$P(\bar{X} > 1980) = P\left(z > \frac{1980 - 2000}{20}\right) = P(z > -1) = 1 - P(z < -1) = 0.841345$$

- b) Let Y = number tax paid more than 3000 in the sample

$$n = 625; p = 10\%$$

$$Y \sim \text{Bin}(625, 10\%)$$

For Normal Approximation:

$$Y' \sim N(62.5, 56.25)$$

$$P(Y > 60) = P(Y' > 60.5)$$

$$= P\left(z > \frac{60.5 - 62.5}{7.5}\right)$$

$$= P(z > -0.266667)$$

$$= 1 - P(z < -0.266667) = 0.605137$$

4. A 500-page book contains 250 sheets of paper. The thickness of the paper used to manufacture the book has mean 0.08 mm and standard deviation 0.01 mm.
- What is the probability that a randomly chosen book is more than 20.2 mm thick (not including the covers)?
 - What is the 10th percentile of book thicknesses?
 - Someone wants to know the probability that a randomly chosen page is more than 0.1 mm thick. Is enough information given to compute this probability? If so, compute the probability. If not, explain why not.

Solution:

- Let X = thickness of the paper

$$\mu = 0.08; \sigma = 0.01$$

Let S = thickness of the book

$$\text{For } S: \mu_S = 250(0.08) = 20.00;$$

$$\sigma_S^2 = 250(0.01)^2 = 0.0250$$

$$\sigma_S = \sqrt{250(0.01)^2} = \sqrt{0.0250} = 0.158114$$

$$P(S > 20.2) = P\left(z > \frac{20.2 - 20.0}{0.158114}\right)$$

$$= P(z > 1.2649) = 1 - P(z < 1.2649)$$

$$= 0.102952$$

- Let S_{10} = 10th percentile of the book thickness

$$\text{For } S: \mu_S = 250(0.08) = 20.00;$$

$$\sigma_S^2 = 250(0.01)^2 = 0.0250$$

$$\sigma_S = \sqrt{250(0.01)^2} = \sqrt{0.0250} = 0.158114$$

$$P(S < S_{10}) = 10\%$$

$$P(z < -1.28155) = 10\%$$

$$S_{10} = \mu_S + z\sigma_S$$

$$= 20.00 + (-1.28155)(0.158114) = 19.7974$$

- No, because the distribution of the paper thickness is unknown.

8. The amount of warpage in a type of wafer used in the manufacture of integrated circuits has mean 1.3 mm and standard deviation 0.1 mm. A random sample of 200 wafers is drawn.
- What is the probability that the sample mean warpage exceeds 1.305 mm?
 - Find the 25th percentile of the sample mean.
 - How many wafers must be sampled so that the probability is 0.05 that the sample mean exceeds 1.305?

Solution:

- Let X = amount of warpage
 $\mu = 1.3$; $\sigma = 0.1$
 For sample mean:
 $\mu_{\bar{X}} = 1.3$; $\sigma_{\bar{X}} = \frac{0.1}{\sqrt{200}} = 0.007071$
 $P(\bar{X} > 1.305) = P\left(z > \frac{1.305 - 1.3}{0.007071}\right)$
 $= P(z > 0.7071) = 1 - P(z < 0.7071)$
 $= 0.239750$
- Let $\bar{X}_{25} = 25^{\text{th}}$ percentile
 $P(X < \bar{X}_{25}) = 25\%$
 $P(z < -0.6745) = 25\%$
 $\bar{X}_{25} = \mu_{\bar{X}} + z\sigma_{\bar{X}}$
 $= 1.3 + (-0.6745)(0.007071) = 1.295231$
- Let $\bar{X}_{95} = 95^{\text{th}}$ percentile
 $P(X < \bar{X}_{95}) = 95\%$
 $P(z < 1.64485) = 95\%$
 $\bar{X}_{95} = \mu_{\bar{X}} + z\sigma_{\bar{X}}$
 $= 1.3 + (1.64485)\left(\frac{0.1}{\sqrt{n}}\right) = 1.305$
 $n = 1083$

10. A battery manufacturer claims that the lifetime of a certain type of battery has a population mean of 40 hours and a standard deviation of 5 hours. Let \bar{X} represent the mean lifetime of the batteries in a simple random sample of size 100.
- If the claim is true, what is $P(\bar{X} \leq 36.7)$?
 - Based on the answer to part (a), if the claim is true, is a sample mean lifetime of 36.7 hours unusually short?
 - If the sample mean lifetime of the 100 batteries were 36.7 hours, would you find the manufacturer's claim to be plausible? Explain.
 - If the claim is true, what is $P(\bar{X} \leq 39.8)$?
 - Based on the answer to part (d), if the claim is true, is a sample mean lifetime of 39.8 hours unusually short?
 - If the sample mean lifetime of the 100 batteries were 39.8 hours, would you find the manufacturer's claim to be plausible? Explain

Solution:

- Let X = life of the battery

$$\mu = 40; \sigma = 5$$

For sample mean:

$$\mu_{\bar{X}} = 40; \sigma_{\bar{X}} = \frac{5}{\sqrt{100}} = 0.50$$

$$P(\bar{X} \leq 36.7) = P\left(z \leq \frac{36.7 - 40}{0.5}\right)$$

$$= P(z \leq -6.60)$$

$$= 2.06 \times 10^{-11}$$

b) Yes

c) No

$$d) P(\bar{X} \leq 39.8) = P\left(z \leq \frac{39.8 - 40}{0.5}\right)$$

$$= P(z \leq -0.40)$$

$$= 0.344578$$

e) No

f) Yes